Assignment #3:

Name: Hamza Tanveer

R.g ID: 19-ARID-801

Section: BSCS-5B

Sub. to: Dr. Noman Ijaz

$\longleftrightarrow$

(a) $3x_1 - x_2 + x_3 = 1$

$3x_1 + 6x_2 + 2x_3 = 0$

$3x_1 + 3x_2 + 7x_3 = 4$

Sol:-

→ Condition Check:

$$|3| > |-1| + |1| \Rightarrow 3 > 2 \quad (\text{True})$$
$$|6| > |3| + |2| \Rightarrow 6 > 5 \quad (\text{True})$$
$$|7| > |3| + |3| \Rightarrow 7 > 6 \quad (\text{True})$$

Suppose $x^{(0)} = 0$ so, $x_1^{(0)} = 0$, $x_2^{(0)} = 0$, $x_3^{(0)} = 0$

At n=0:

$$x_1^{(1)} = \frac{1}{3}\left(1 - (-1)x_2^{(0)} - (1)x_3^{(0)}\right) \Rightarrow \boxed{x_1^{(1)} = 0.3333}$$

$$x_2^{(1)} = \frac{1}{6}\left(0 - (3)(0) - (2)(0)\right) \Rightarrow \boxed{x_2^{(1)} = 0}$$

$$x_3^{(1)} = \frac{1}{7}\left(4 - (3)(0) - (3)(0)\right) \Rightarrow \boxed{x_3^{(1)} = 0.5714}$$

$\therefore x_1^{(0)} = 0$, $x_2^{(0)} = 0$

$x_3^{(0)} = 0$

At n= 1:

$$x_1^{(2)} = \frac{1}{3}\left(1 - (-1)(0) - (1)(0.5714)\right) \Rightarrow \boxed{x_1^{(2)} = 0.1429}$$

$$x_2^{(2)} = \frac{1}{6}\left(0 - (3)(0.3333) - (2)(0.5714)\right) \Rightarrow \boxed{x_2^{(2)} = -0.3571}$$

$$x_3^{(2)} = \frac{1}{7}\left(4 - (3)(0.3333) - (3)(0)\right) \Rightarrow \boxed{x_3^{(2)} = 0.4286}$$

$\therefore x_2^{(1)} = 0$, $x_3^{(1)} = 0$

at n=2:

$x_1^{(3)} = \frac{1}{3}(1 - (-1)(-0.3571) - (1)(0.4286)) \Rightarrow \boxed{x_1^{(3)} = 0.0714}$

$x_2^{(3)} = \frac{1}{6}(0 - (3)(0.1429) - (2)(0.4286)) \Rightarrow \boxed{x_2^{(3)} = -0.2143}$

$x_3^{(3)} = \frac{1}{7}(4 - (3)(0.1429) - (3)(-0.3571)) \Rightarrow \boxed{x_3^{(3)} = 0.6632}$

at n=3:

$x_1^{(4)} = \frac{1}{3}(1 - (-1)(-0.2143) - (1)(0.6632)) \Rightarrow \boxed{x_1^{(4)} = 0.0408}$

$x_2^{(4)} = \frac{1}{6}(0 - (3)(0.0714) - (2)(0.6632)) \Rightarrow \boxed{x_2^{(4)} = 0.2568}$

$x_3^{(4)} = \frac{1}{7}(4 - (3)(0.0714) - (3)(-0.2143)) \Rightarrow \boxed{x_3^{(4)} = 0.6327}$

(b)

$10x_1 - x_2 = 9$

$-x_1 + 10x_2 - 2x_3 = 7$

$-2x_2 + 10x_3 = 6$

Solution:-

Check Condition:-

$|10| > |-1| + |0| \Rightarrow 10 > 1$ (true)

$|10| > |-1| + |-2| \Rightarrow 10 > 3$ (true)

$|10| > |0| + |-2| \Rightarrow 10 > 2$ (true)

At n=0:

Suppose $x^{(0)} = 0 \Rightarrow x_1^{(0)} = 0 , x_2^{(0)} = 0 , x_3^{(0)} = 0$

$x_1^{(1)} = \frac{1}{10}(9 - (-1)(0) - (0)(0)) \Rightarrow \boxed{x_1^{(1)} = 0.9}$

$x_2^{(1)} = \frac{1}{10}(7 - (-1)(0) - (-2)(0)) \Rightarrow \boxed{x_2^{(1)} = 0.7}$

$x_3^{(1)} = \frac{1}{10}(6 - (0)(0) - (-2)(0)) \Rightarrow \boxed{x_3^{(1)} = 0.6}$

at n=1:

$x_1^{(2)} = \frac{1}{10}(9 - (-1)(0.7) - (0)(0)) = \boxed{0.97}$

$x_2^{(2)} = \frac{1}{10}(7 - (-1)(0.9) - (-2)(0.6)) = \boxed{0.91}$

$x_3^{(2)} = \frac{1}{10}(6 - (0)(0) - (-2)(0.7)) = \boxed{0.74}$

at n=2:

$x_1^{(3)} = \frac{1}{10}(9 - (-1)(0.91) - (0)(0)) \Rightarrow \boxed{x_1^{(3)} = 0.991}$

$x_2^{(3)} = \frac{1}{10}(7 - (-1)(0.97) - (-2)(0.74)) \Rightarrow \boxed{x_2^{(3)} = 0.945}$

$x_3^{(3)} = \frac{1}{10}(6 - (0)(0) - (-2)(0.91)) \Rightarrow \boxed{x_3^{(3)} = 0.782}$

at n=3:

$x_1^{(4)} = \frac{1}{10}(9 - (-1)(0.945) - (0)(0)) \Rightarrow \boxed{x_1^{(4)} = 0.9945}$

$x_2^{(4)} = \frac{1}{10}(7 - (-1)(0.991) - (-2)(0.74)) \Rightarrow \boxed{x_2^{(4)} = 0.9555}$

$x_3^{(4)} = \frac{1}{10}(6 - (0)(0) - (2)(0.945)) \Rightarrow \boxed{x_3^{(4)} = 0.789}$

(c) $10x_1 + 5x_2 = 6$

$5x_1 + 10x_2 - 4x_3 = 25$

$-4x_2 + 8x_3 - x_4 = -11$

$-x_3 + 5x_4 = -11$

Sol:-

Condition check:

$|10| > |5| + |0| + |0| \Rightarrow 10 > 5 \quad (True)$

$|10| > |5| + |-4| + |0| = \Rightarrow 10 > 9 \quad (True)$

$|8| > |0| + |-4| + |-1| \Rightarrow 8 > 5 \quad (true)$

$|5| > |0| + |0| + |-1| \Rightarrow 5 > 1 \quad (true)$

Suppose $x^{(0)} = 0$ so, $x_1^{(0)} = 0$, $x_2^{(0)} = 0$, $x_3^{(0)} = 0$, $x_4^{(0)} = 0$

At n=0:

$x_1^{(1)} = \frac{1}{10}(6 - 5(0)) \Rightarrow \boxed{x_1^{(1)} = 0.6}$

$x_2^{(1)} = \frac{1}{10}(25 - (5)(0) - (-4)(0)) \Rightarrow \boxed{x_2^{(1)} = 2.5}$

$x_3^{(1)} = \frac{1}{8}(-11 - (-4)(0) - (-1)(0)) \Rightarrow \boxed{x_3^{(1)} = -1.375}$

$x_4^{(1)} = \frac{1}{5}(-11 - (-1)(0)) \Rightarrow \boxed{x_4^{(1)} = -2.2}$

At n=1:

$x_1^{(2)} = \frac{1}{10}(6 - (5)(2.5)) \Rightarrow \boxed{x_1^{(2)} = -0.65}$

$x_2^{(2)} = \frac{1}{10}(25 - (5)(0.6) - (-4)(-1.375)) \Rightarrow \boxed{x_2^{(2)} = 1.65}$

$x_3^{(2)} = \frac{1}{8}(-11 - (-4)(2.5) - (-1)(-2.2)) \Rightarrow \boxed{x_3^{(2)} = -0.4}$

$x_4^{(2)} = \frac{1}{5}(-11 - (-1)(-1.375)) \Rightarrow \boxed{x_4^{(2)} = -2.475}$

At n=2:

$x_1^{(3)} = \frac{1}{10}(6 - (5)(1.65)) \Rightarrow \boxed{x_1^{(3)} = -0.925}$

$x_2^{(3)} = \frac{1}{10}(25 - (5)(-0.65) - (-4)(0.4)) \Rightarrow \boxed{x_2^{(3)} = 2.665}$

$x_3^{(3)} = \frac{1}{8}(-11 - (-4)(1.65) - (-1)(-2.475)) \Rightarrow \boxed{x_3^{(3)} = -0.8594}$

$x_4^{(3)} = \frac{1}{5}(-11 - (-1)(-0.4)) \Rightarrow \boxed{x_3^{(4)} = -2.28}$

At n=3:

$x_1^{(4)} = \frac{1}{10}(6 - (5)(2.665)) \Rightarrow \boxed{x_1^{(4)} = 0.7325}$

$x_2^{(4)} = \frac{1}{10}(25 - (5)(-0.225) - (-4)(-0.8594)) \Rightarrow \boxed{2.26874}$

$x_3^{(4)} = \frac{1}{8}(-11 - (-4)(2.665) - (-1)(-2.28)) \Rightarrow \boxed{x_3^{(4)} = -0.3275}$

$x_4^{(4)} = \frac{1}{5}(-11 - (-1)(-0.8594)) \Rightarrow \boxed{x_4^{(4)} = -2.37188}$

Q2) Solved by Grauss Siedel Method.

(a)
$$4x_1 + x_2 - x_3 = 5$$
$$-x_1 + 3x_2 + x_3 = -4$$
$$2x_1 + 2x_2 + 5x_3 = -1$$

Sol:
$$|4| > |1| + |-1| \implies 4 > 2 \quad (\text{true})$$
$$|3| > |-1| + |+1| \implies 3 > 2 \quad (\text{true})$$
$$|5| > |2| + |2| \implies 5 > 4 \quad (\text{true})$$

Suppose $x_0^0 = 0$ So, $x_1^{(0)} = 0$, $x_2^{(0)} = 0$, $x_3^{(0)} = 0$

At n=0:
$$x_1^{(1)} = \frac{1}{4}\left(5 - (1)(x_2)^0 - (-1)(x_3)^0\right)$$
$$x^{(1)} = \frac{1}{4}(5 - (1)(0) - (-1)(0)) \implies \boxed{x_1^{(1)} = 1.25}$$
$$x_2^{(1)} = \frac{1}{3}\left(-4 - (-1)x_1^{(1)} - (1)(x_3)^0\right)$$
$$x_2^{(1)} = \frac{1}{3}\left(-4 - (-1)(1.25) - (1)(0)\right) \implies \boxed{x_2^{(1)} = -0.9167}$$
$$x_3^{(1)} = \frac{1}{5}\left(+1 - (2)x_1^{(1)} - (2)(x_2)^{(1)}\right)$$
$$x_3^{(1)} = \frac{1}{5}(+1 - (2)(1.25) - (2)(-0.9167)) \implies \boxed{x_3^{(1)} = 0.0688}$$

At n=1:
$$x_1^{(2)} = \frac{1}{4}\left(5 - (1)(-0.9167) + (1)(0.06688)\right) \implies \boxed{x_1^{(2)} = 1.4958}$$
$$x_2^{(2)} = \frac{1}{3}\left(-4 + (1)(1.4958) - (1)(0.06688)\right) \implies \boxed{x_2^{(2)} = -0.85702}$$
$$x_3^{(2)} = \frac{1}{5}\left(+1 - (2)(1.4958) - (2)(-0.85702)\right) \implies \boxed{x_3^{(2)} = -0.0555}$$

At n=2:
$$x_1^{(3)} = \frac{1}{4}\left(5 - 1(-0.85702) + 1(-0.0555)\right) \implies \boxed{x_1^{(3)} = 1.45038}$$
$$x_2^{(3)} = \frac{1}{3}\left(-4 + 1(1.45038) - 1(-0.0555)\right) \implies \boxed{x_2^{(3)} = -0.8313}$$

$$x_3^{(3)} = \frac{1}{5}(1 - (2)(1.45038) - (2)(-0.8313)) \Rightarrow \boxed{x_3^{(3)} = -0.0476}$$

At n=3:

$$x_1^{(4)} = \frac{1}{4}(5 - 1(-0.8313) + 1(-0.0476)) \Rightarrow \boxed{x_1^{(4)} = 1.03027}$$

$$x_2^{(4)} = \frac{1}{3}(-4 + 1(1.03027) - 1(-0.0476)) \Rightarrow \boxed{x_2^{(4)} = -0.97404}$$

$$x_3^{(4)} = \frac{1}{5}(1 - (2)(1.03027) - 2(-0.97404)) \Rightarrow \boxed{x_3^{(4)} = 0.17751}$$

b) $-2x_1 + x_2 + \frac{1}{2}x_3 = 4$

$\quad x_1 - 2x_2 - \frac{1}{2}x_3 = -4$

$\qquad x_2 + 2x_3 = 0$

Sol:-

Check:-

$\quad |-2| > |1| + |\frac{1}{2}| \Rightarrow 2 > 1.5 \ (True)$

$\quad |-2 > (1| + |-\frac{1}{2}| \Rightarrow 2 > 1.5 \ (True)$

$\quad |2| > |1| \qquad \Rightarrow \quad 2 > 1 \quad (true)$

$\qquad$ Suppose $x^{(0)} = 0 \Rightarrow x_1^{(0)} = 0, \ x_2^{(0)} = 0, \ x_3^{(0)} = 0$

At n=0:

$$x_1^{(1)} = -\frac{1}{2}(4 - (1)(0) - (\frac{1}{2})(0)) \Rightarrow \boxed{x_1^{(1)} = -2}$$

$$x_2^{(1)} = -\frac{1}{2}(-4 - (1)(-2) - (-\frac{1}{2})(0)) \Rightarrow \boxed{x_2^{(1)} = 1}$$

$$x_3^{(1)} = +\frac{1}{2}(0 - (1)(1)) \Rightarrow x_3^{(1)} = \boxed{x_3^{(1)} = -0.5}$$

At n=1

$$x_1^{(2)} = -\tfrac{1}{2}\left(4 - (1)(1) - (\tfrac{1}{2})(-0.5)\right) \Rightarrow x_1^{(2)} = -1.625$$

$$x_2^{(2)} = -\tfrac{1}{2}\left(-4 - (1)(-1.625) - (-\tfrac{1}{2})(-0.5)\right) \Rightarrow x_2^{(2)} = 1.3125$$

$$x_3^{(2)} = \tfrac{1}{2}\left(0 - (1)(1.3125)\right) \Rightarrow x_3^{(2)} = -0.6563$$

At n=2:

$$x_1^{(3)} = -\tfrac{1}{2}\left(4 - (1)(1.3125) - (\tfrac{1}{2})(-0.6563)\right) \Rightarrow x_1^{(3)} = -1.5078$$

$$x_2^{(3)} = -\tfrac{1}{2}\left(-4 - (1)(-1.5078) - (-\tfrac{1}{2})(-0.6563)\right) \Rightarrow x_2^{(3)} = 1.4101$$

$$x_3^{(3)} = \tfrac{1}{2}\left(0 - (1)(1.4101)\right) \Rightarrow x_3^{(3)} = -0.7051$$

At n=3:

$$x_1^{(4)} = -\tfrac{1}{2}\left(4 - (1)(1.4101) - (\tfrac{1}{2})(-0.7051)\right) \Rightarrow x_1^{(4)} = -1.4712$$

$$x_2^{(4)} = -\tfrac{1}{2}\left(-4 - (1)(-1.4712) - (-\tfrac{1}{2})(-0.7051)\right) \Rightarrow x_2^{(4)} = 1.4407$$

$$x_3^{(4)} = \tfrac{1}{2}\left(0 - (1)(1.4407)\right) \Rightarrow x_3^{(4)} = -0.7204$$

(C)

$$4x_1 + x_2 - x_2 + x_4 = -2$$
$$x_1 + 4x_2 - x_3 - x_4 = -1$$
$$-x_1 - x_2 + 5x_3 + x_4 = 0$$
$$x_1 - x_2 + x_3 + 3x_4 = 1$$

Sol:-

$$|4| > |1| + |-1| + |1| \Rightarrow 4 > 3 \quad (\text{true})$$
$$|4| > |1| + |-1| + |-1| \Rightarrow 4 > 3 \quad (\text{true})$$
$$|5| > |-1| + |-1| + |1| \Rightarrow 5 > 3 \quad (\text{true})$$
$$|3| > |-1| + |-1| + |1| \Rightarrow 3 > 3 \quad (\text{false})$$

As condition is false, so it isn't solvable further.