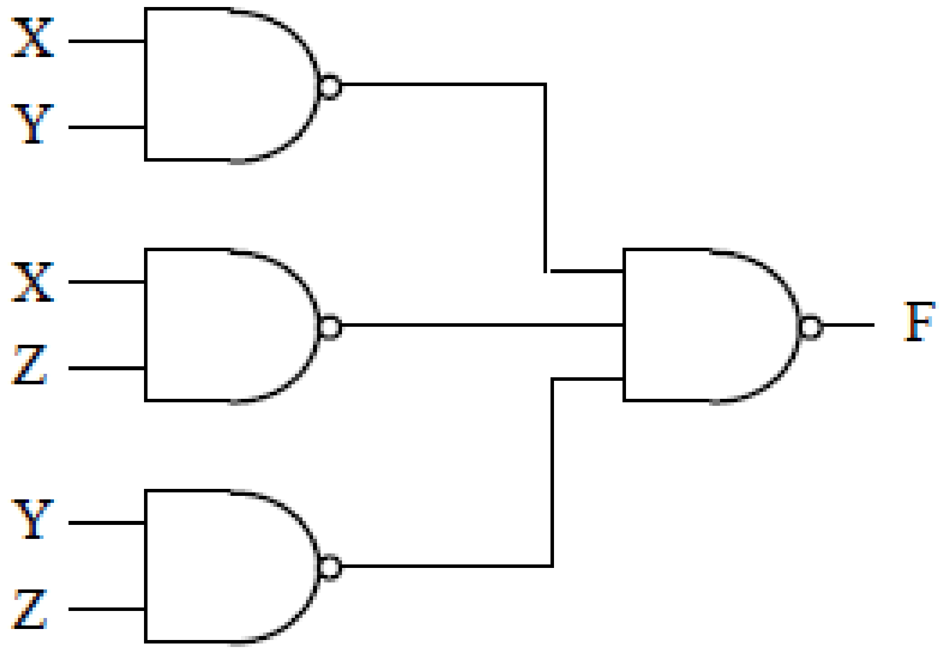
Digital Logic Design

Lecture 07:
Combinational Logic Circuit & Boolean Algebra

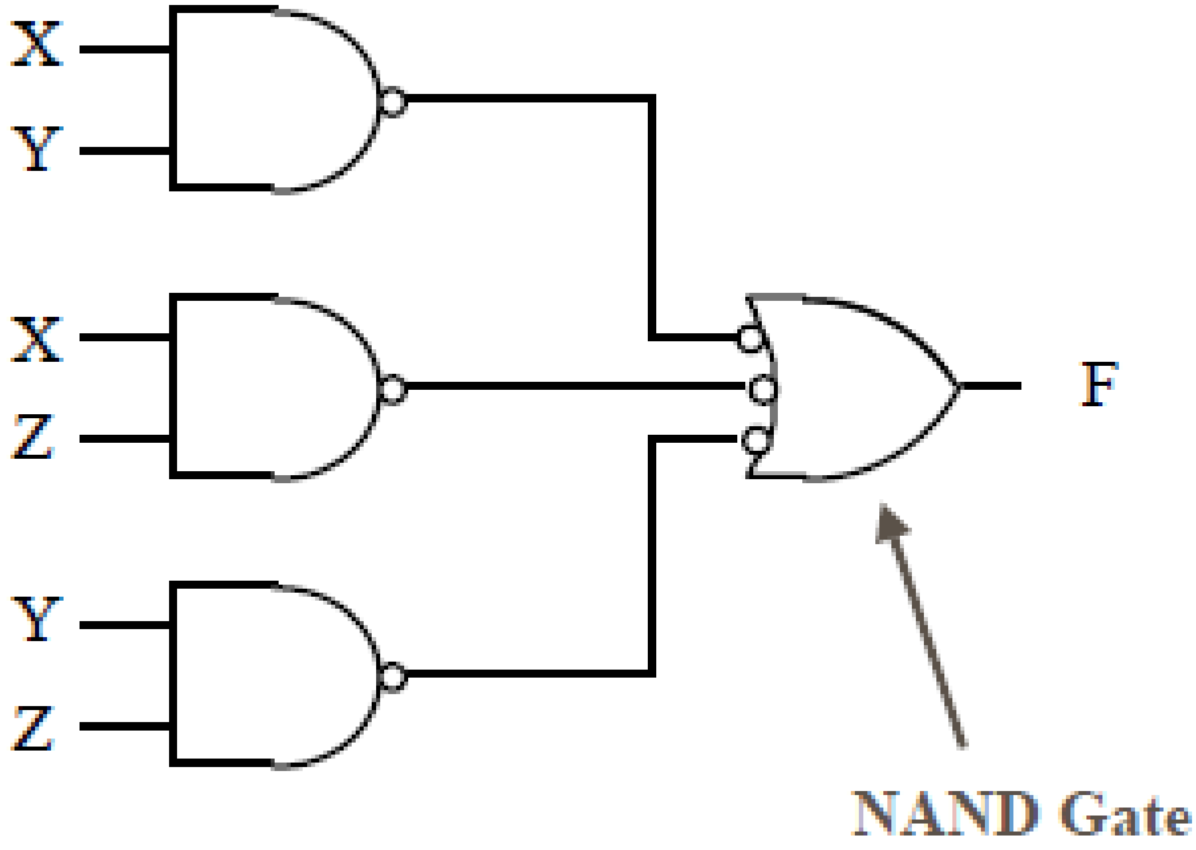
Generalized De Morgan's Theorem

- NOT all variables
- Change . to + and + to .
- NOT the result
- F = X . Y + X . Z + Y . Z
- F = !((!X + !Y) . (!X + !Z) . (!Y + !Z))
- F = !(!(X . Y) . !(X . Z) . !(Y . Z))

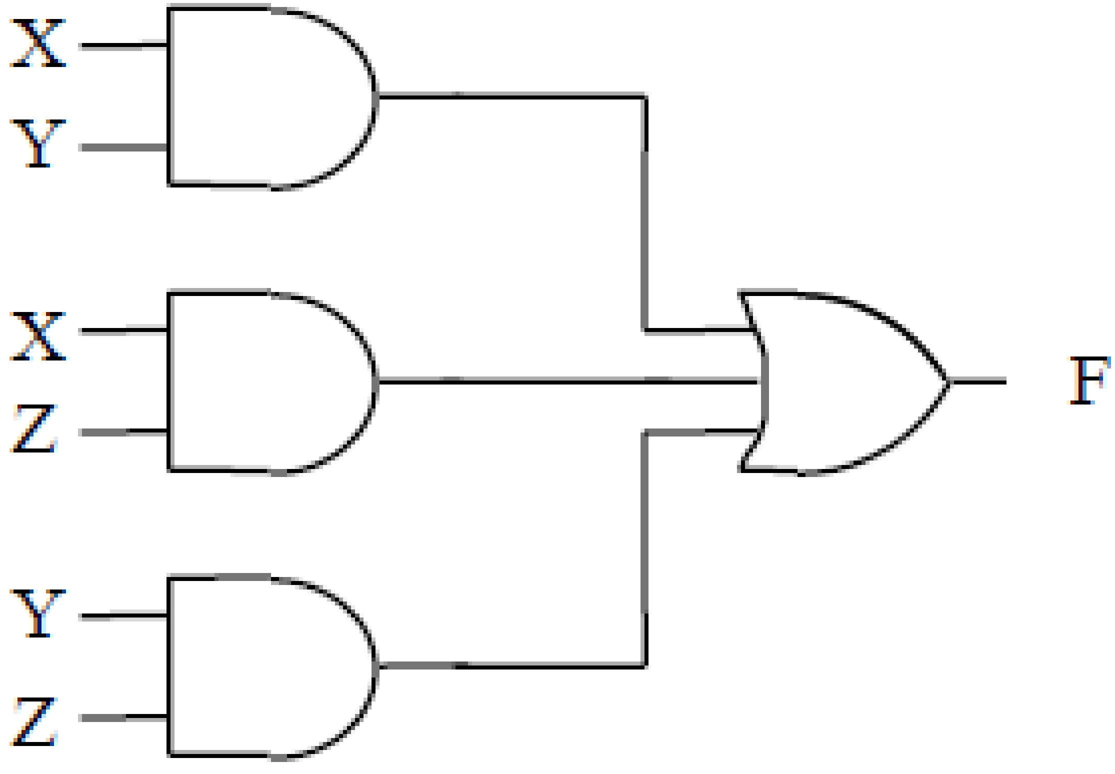
$$F = !(!(X \& Y) \& !(X \& Z) \& !(Y \& Z))$$



$$F = !(!(X \& Y) \& !(X \& Z) \& !(Y \& Z))$$



F = X & Y | X & Z | Y & Z



Boolean Algebra (cont)

- Sum of Products and Product of Sums
- The Duality Principal: *Interchange* AND *with* OR
- Algebraic Manipulations
  - X + XY = X(1+Y) = X
  - XY + XY' = X(Y+Y') = X
  - X+X'Y = (X+X')(X+Y) = X+Y
  - XY+X'Z+YZ = XY+X'Z *-Consensus Theorem*
- Complement of a Function: *Interchange all 0's with 1's in the truth table*

SOP Form

- A product of literals is called a product term or a cube (e.g. $\bar{A}\cdot B\cdot C$ in $\mathcal{B}^3$, or $B\cdot C$ in $\mathcal{B}^3$)
- Sum–Of–Product (SOP) Form: OR of product terms, e.g. $\bar{A}B+AC$
- A minterm is a product term in which every literal (or variable) appears in $\mathcal{B}^n$
  - $\bar{A}BC$ is a minterm in $\mathcal{B}^3$ but not in $\mathcal{B}^4$. ABCD is a minterm in $\mathcal{B}^4$.
- A canonical (or standard) SOP function:
  - a sum of minterms corresponding to the input combination of the truth table for which the function produces a "1" output.

Minterms in $\mathcal{B}^3$

			m_0	m_1	m_2	m_3	m_4	m_5	m_6	m_7
A	B	C	$\bar{A}\bar{B}\bar{C}$	$\bar{A}\bar{B}C$	$\bar{A}B\bar{C}$	$\bar{A}BC$	$A\bar{B}\bar{C}$	$A\bar{B}C$	$AB\bar{C}$	ABC
0	0	0	1	0	0	0	0	0	0	0
0	0	1	0	1	0	0	0	0	0	0
0	1	0	0	0	1	0	0	0	0	0
0	1	1	0	0	0	1	0	0	0	0
1	0	0	0	0	0	0	1	0	0	0
1	0	1	0	0	0	0	0	1	0	0
1	1	0	0	0	0	0	0	0	1	0
1	1	1	0	0	0	0	0	0	0	1

Canonical (Standard) SOP Function

$$F(A,B,C) = \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + A\bar{B}\bar{C} + A\bar{B}C$$

$$= m0 + m1 + m4 + m5$$

$$F(A,B,C) = \sum m(0,1,4,5) = \text{one-set}(0,1,4,5)$$

$$F(A,B,C,D) = \bar{A}B\bar{C}\bar{D} + A\bar{B}\bar{C}D + ABC\bar{D}$$

$$= m4 + m9 + m14$$

$$F(A,B,C,D) = \sum m(4,9,14) = \text{one-set}(4,9,14)$$

Product of Sums Design

Maxterms:

A maxterm is NOT a minterm

maxterm M0 = NOT minterm m0

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M0  =  !m0
    =  !(!X . !Y)
    =  !!X + !!Y
    =  X + Y
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POS form (dual of SOP form)

- A sum of literals is called a sum term (e.g. $\bar{A}+B+C$ in $\mathcal{B}^3$, or $(B+C)$ in $\mathcal{B}^3$)
- Product-Of-Sum (POS) Form: AND of sum terms, e.g. $(\bar{A}+B)(A+C)$
- A maxterm is a sum term in which every literal (or variable) appears in $\mathcal{B}^n$
  - $(\bar{A}+B+C)$ is a maxterm in $\mathcal{B}^3$ but not in $\mathcal{B}^4$. $A+B+C+D$ is a maxterm in $\mathcal{B}^4$.
- A canonical (or standard) POS function:
  - a product of maxterms corresponding to the input combination of the truth table for which the function produces a "0" output.

Product of Sums Design

X	Y	minterms	maxterms
0	0	m0 = !X . !Y	M0 = !m0 = X + Y
0	1	m1 = !X . Y	M1 = !m1 = X + !Y
1	0	m2 = X . !Y	M2 = !m2 = !X + Y
1	1	m3 = X . Y	M3 = !m3 = !X + !Y

Maxterms in $\mathcal{B}^3$

A	B	C	M_0 $A+B+C$	M_1 $A+B+\overline{C}$	M_2 $A+\overline{B}+C$	M_3 $A+\overline{B}+\overline{C}$	M_4 $\overline{A}+B+C$	M_5 $\overline{A}+B+\overline{C}$	M_6 $\overline{A}+\overline{B}+C$	M_7 $\overline{A}+\overline{B}+\overline{C}$
0	0	0	0	1	1	1	1	1	1	1
0	0	1	1	0	1	1	1	1	1	1
0	1	0	1	1	0	1	1	1	1	1
0	1	1	1	1	1	0	1	1	1	1
1	0	0	1	1	1	1	0	1	1	1
1	0	1	1	1	1	1	1	0	1	1
1	1	0	1	1	1	1	1	1	0	1
1	1	1	1	1	1	1	1	1	1	0