Digital Logic Design

Lecture 06: Combinational Logic Circuit & Boolean Algebra

What is Boolean Algebra

- An algebra dealing with
  - Binary variables by alphabetical letters
  - Logic operations: OR, AND, XOR, etc
- Consider the following Boolean equation

$$F(X,Y,Z)=\overline{X}\cdot Y+\overline{Y\cdot\overline{Z}+Z}$$

- A Boolean function can be represented by a truth table which list all combinations of 1's and 0's for each binary value

Fundamental Operators

- NOT
  - Unary operator
  - Complements a Boolean variable represented as A', ~A, or Ā
- OR
  - Binary operator
  - A ORed with B is represented as A + B
- AND
  - Binary operator
  - A ANDed with B is represented as AB or A·B
  - Can perform logical multiplication

Precedence of Operators

- Precedence of Operator Evaluation (Similar to decimal arithmetic)
  - () : Parentheses
  - NOT
  - AND
  - OR

$$F=A\cdot\overline{(B+\overline{C}\cdot D)}+\overline{A}\cdot\overline{\overline{B}+E}$$

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Function Evaluation

$$F=A\cdot\overline{(B+\overline{C}\cdot D)}+\overline{A}\cdot\overline{\overline{B}+E}$$

ABCDE=00000

$$F=0\cdot\overline{(0+\overline{0}\cdot 0)}+\overline{0}\cdot\overline{\overline{0}+0}=0\cdot\overline{(0+1\cdot 0)}+\overline{0}\cdot\overline{1+0}$$

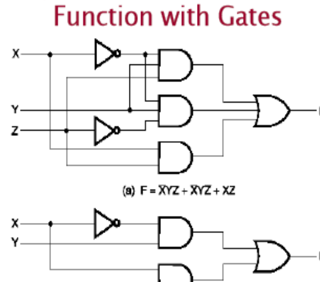
$$=0\cdot\overline{(0+0)}+1\cdot\overline{1}=0\cdot 1+1\cdot 0=0$$

ABCDE=10000

$$F=1\cdot\overline{(0+\overline{0}\cdot 0)}+\overline{1}\cdot\overline{\overline{0}+0}=1\cdot\overline{(0+1\cdot 0)}+0\cdot\overline{1+0}$$

$$=1\cdot\overline{(0+0)}+0\cdot\overline{1}=1\cdot 1+0\cdot 0=1$$

Implementation of Boolean Function with Gates



(a) $F=\overline{X}YZ+\overline{X}Y\overline{Z}+XZ$

(b) $F=\overline{X}Y+XZ$

Boolean Variables

- A multi-dimensional space spanned by a set of n Boolean variables is denoted by $\mathcal{B}^n$
- A literal is an instance (e.g. A) of a variable or its complement ($\bar{A}$)

Basic Identities of Boolean Algebra

$X+0=X$	(Identity)
$X+1=1$	
$X+X=X$	(Idempotent Law)
$X+\overline{X}=1$	(Complement)
$\overline{\overline{X}}=X$	(Involution Law)
$X+Y=Y+X$	(Commutative)
$X+(Y+Z)=(X+Y)+Z$	(Associative)
$X(Y+Z)=XY+XZ$	(Distributive)
$\overline{X+Y}=\overline{X}\,\overline{Y}$	(DeMorgan's Law)
$X+XY=X$	(Absorption Law)
$X+\overline{X}Y=X+Y$	(Simplification)
$XY+\overline{X}Z+YZ=XY+\overline{X}Z$	(Consensus Theorem)

Derivation of Simplification

$$X+\overline{X}Y$$
$$=X\cdot(1+Y)+\overline{X}Y$$
$$=X+XY+\overline{X}Y$$
$$=X+(X+\overline{X})Y$$
$$=X+Y$$
$$\therefore X+\overline{X}Y=X+Y$$

Derivation of Consensus Theorem

$$XY+\overline{X}Z+YZ$$
$$=XY+\overline{X}Z+YZ\cdot(X+\overline{X})$$
$$=XY+\overline{X}Z+XYZ+\overline{X}YZ$$
$$=XY(1+Z)+\overline{X}Z(1+Y)$$
$$=XY+\overline{X}Z$$
$$\therefore XY+\overline{X}Z+YZ=XY+\overline{X}Z$$

Duality Principle

- A Boolean equation remains valid if we take the dual of the expressions on both sides of the equals sign
- Dual of expressions
  - Interchange 1's and 0's
  - Interchange AND (•) and OR (+)

Duality Principle

$X+0=X$	$X\cdot 1=X$
$X+1=1$	$X\cdot 0=0$
$X+X=X$	$X\cdot X=X$
$X+\overline{X}=1$	$X\cdot\overline{X}=0$
$X+Y=Y+X$	$X\cdot Y=Y\cdot X$
$X(Y+Z)=XY+XZ$	$X+Y\cdot Z=(X+Y)\cdot(X+Z)$
$\overline{X+Y}=\overline{X}\cdot\overline{Y}$	$\overline{X\cdot Y}=\overline{X}+\overline{Y}$
$X+X\cdot Y=X$	$X\cdot(X+Y)=X$
$X+\overline{X}\cdot Y=X+Y$	$X\cdot(\overline{X}+Y)=X\cdot Y$
$XY+\overline{X}Z+YZ=XY+\overline{X}Z$	$(X+Y)(\overline{X}+Z)(Y+Z)=(X+Y)(\overline{X}+Z)$

DeMorgan's Law

$$\overline{A \cdot B} = \overline{A} + \overline{B}$$

$$\overline{A + B} = \overline{A} \cdot \overline{B}$$