Digital Logic Design

Lecture 08:
Combinational Logic Circuit &
Boolean Algebra

Canonical (Standard) POS Function

$$F(A,B,C) = (\overline{A}+\overline{B}+\overline{C})(\overline{A}+\overline{B}+C)(A+\overline{B}+\overline{C})(A+\overline{B}+C)$$

$$= M7 \cdot M6 \cdot M3 \cdot M2$$

$$F(A,B,C) = \prod M(2,3,6,7) = \text{zero}-\text{set}(2,3,6,7)$$

$$F(A,B,C,D) = (\overline{A}+B+\overline{C}+\overline{D})(A+\overline{B}+\overline{C}+D)(A+B+C+\overline{D})$$

$$= M11 \cdot M6 \cdot M1$$

$$F(A,B,C,D) = \prod M(1,6,11) = \text{zero}-\text{set}(1,6,11)$$

Convert a Boolean to Canonical SOP

- Expand the Boolean eqn into a SOP
- Take each product term w/ a missing literal A, "AND" (•) it with $(A+\bar{A})$

Convert a Boolean to Canonical SOP

$$F = \overline{A}\,\overline{B} + BC \ \text{ in } \mathcal{B}^3$$

$$\Rightarrow F(A,B,C) = \overline{ABC} + \overline{A}\,\overline{B}C + \overline{A}BC + ABC$$

$$= \sum m(0,1,3,7)$$

	A	B	C	F	
$\overline{A}\,\overline{B}\,\overline{C}$	0	0	0	1	0
$\overline{A}\,\overline{B}C$	0	0	1	1	1
$\overline{A}B\overline{C}$	0	1	0	0	
$\overline{A}BC$	0	1	1	1	3
$A\overline{B}\,\overline{C}$	1	0	0	0	
$A\overline{B}C$	1	0	1	0	
$AB\overline{C}$	1	1	0	0	
ABC	1	1	1	1	7

Minterms listed as 1's

Convert a Boolean to Canonical SOP

$$F = \overline{A}\,\overline{B} + BC \text{ in } \mathcal{B}^4$$

$$\begin{aligned}\Rightarrow F(A,B,C,D) &= \overline{A}\,\overline{B}\,\overline{C}\,\overline{D} + \overline{A}\,\overline{B}\,\overline{C}D + \overline{A}\,\overline{B}C\overline{D} + \overline{A}\,\overline{B}CD \\ &\quad + \overline{A}BC\overline{D} + \overline{A}BCD + ABC\overline{D} + ABCD \\ &= \sum m(0,1,2,3,6,7,14,15)\end{aligned}$$

$$F = AB + \overline{B}(\overline{A} + \overline{C}) \text{ in } \mathcal{B}^3$$

$$\begin{aligned}\Rightarrow F(A,B,C) &= \overline{A}\,\overline{B}\,\overline{C} + \overline{A}\,\overline{B}C + A\overline{B}\,\overline{C} + AB\overline{C} + ABC \\ &= \sum m(0,1,4,6,7)\end{aligned}$$

Convert a Boolean to Canonical POS

- Expand Boolean eqn into a POS
  - Use distributive property
- Take each sum term w/ a missing variable A and OR it with $A \cdot \bar{A}$

Convert a Boolean to Canonical POS

$$F = \overline{AB} + BC \text{ in } \mathcal{B}^3$$

$$F = \overline{AB} + BC$$

$$F = (A+\bar{B}+C)(\bar{A}+B+C)(\bar{A}+B+\bar{C})(\bar{A}+\bar{B}+C)$$

$$= \prod M(2,4,5,6)$$

	A	B	C	F	
$\bar{A}\bar{B}\bar{C}$	0	0	0	1	
$\bar{A}\bar{B}C$	0	0	1	1	
$\bar{A}B\bar{C}$	0	1	0	0	2
$\bar{A}BC$	0	1	1	1	
$A\bar{B}\bar{C}$	1	0	0	0	4
$A\bar{B}C$	1	0	1	0	5
$AB\bar{C}$	1	1	0	0	6
ABC	1	1	1	1	

Maxterms listed as 0's

Convert a Boolean to Canonical SOP

$$F = \overline{AB} + BC \ \text{ in } \mathcal{B}^3$$

$$\Rightarrow F(A,B,C) = \overline{ABC} + \overline{A}\,\overline{B}C + \overline{A}BC + ABC$$

$$= \sum m(0,1,3,7)$$

	A	B	C	F	
$\bar{A}\bar{B}\bar{C}$	0	0	0	1	← 0
$\bar{A}\bar{B}C$	0	0	1	1	← 1
$\bar{A}B\bar{C}$	0	1	0	0	
$\bar{A}BC$	0	1	1	1	← 3
$A\bar{B}\bar{C}$	1	0	0	0	
$A\bar{B}C$	1	0	1	0	
$AB\bar{C}$	1	1	0	0	
ABC	1	1	1	1	← 7

Minterms listed as 1's

Convert a Boolean to Canonical SOP

$$F = AB + \overline{B}(\overline{A} + \overline{C}) \text{ in } \mathcal{B}^3$$

$$\Rightarrow F(A, B, C) = \overline{A}\,\overline{B}\,\overline{C} + \overline{A}\,\overline{B}C + A\overline{B}\,\overline{C} + AB\overline{C} + ABC$$

$$= \sum m(0, 1, 4, 6, 7)$$

Interchange Canonical SOP and POS

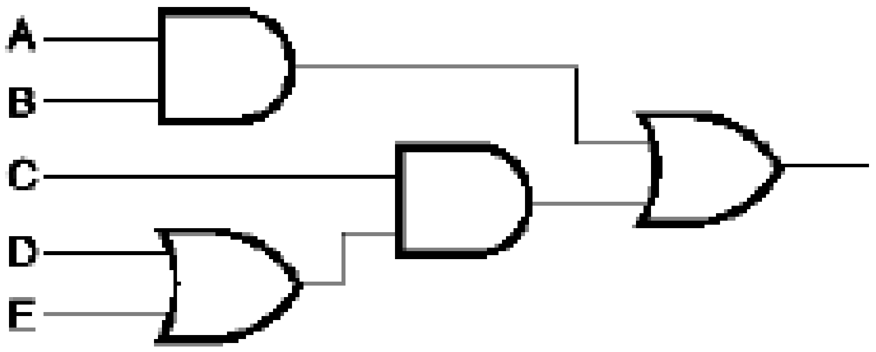
- For the same Boolean eqn
  - Canonical SOP form is complementary to its canonical POS form
  - Use missing terms to interchange Σ and Π
- Examples
  - $F(A,B,C) = \Sigma\, m(0,1,4,6,7)$

  Can be re-expressed by

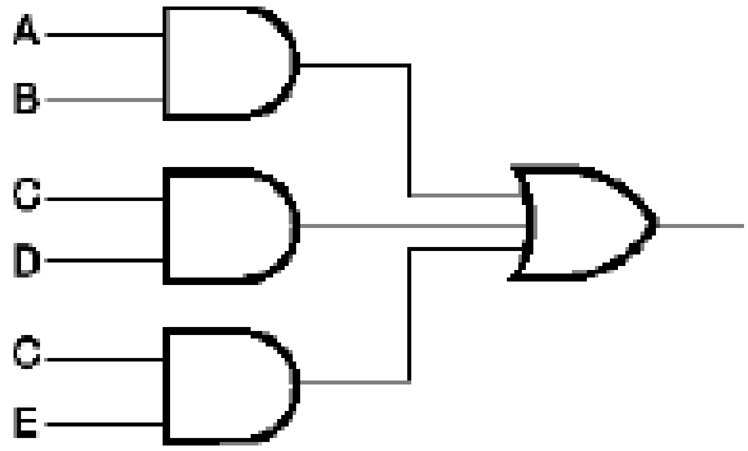
  - $F(A,B,C) = \Pi\, M(2,3,5)$

    Where 2, 3, 5 are the missing minterms in the canonical SOP form

Three- Level and Two- Level Implementation



(a) AB + C (D + E)



(b) AB + CD + CE