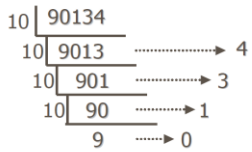
# Digital Logic Design

## Lecture 03:

# Number Systems

## Decimal Number Representation

- Example: 90134 (base-10, used by Homo Sapien)
  - = 90000 + 0 + 100 + 30 + 4
  - = $9*10^4 + 0*10^3 + 1*10^2 + 3*10^1 + 4*10^0$
- How did we get it?




## Generic Number Representation

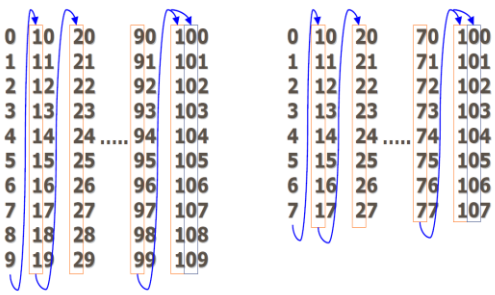
- 90134
  - = $9*10^4 + 0*10^3 + 1*10^2 + 3*10^1 + 4*10^0$
- $A_4\ A_3\ A_2\ A_1\ A_0$ for base-10 (or radix-10)
  - = $A_4*10^4 + A_3*10^3 + A_2*10^2 + A_1*10^1 + A_0*10^0$
  - (A is coefficient; b is base)
- Generalize for a given number N w/ base-b

  $N = A_{n-1}\ A_{n-2} \ldots A_1\ A_0$

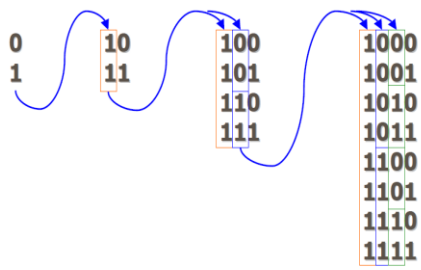
  $N = A_{n-1}*b^{n-1} + A_{n-2}*b^{n-2} + \ldots + A_2*b^2 + A_0*b^0$

  **Note that A < b

## Counting numbers with base-b




## How about base-2



## How about base-2

| | | | |
|---|---|---|---|
| 0 | 10 | 100 | 1000 |
| 1 | 11 | 101 | 1001 |
| | | 110 | 1010 |
| | | 111 | 1011 |
| | | | 1100 |
| | | | 1101 |
| | | | 1110 |
| | | | 1111 |

## How about base-2

| | | | |
|---|---|---|---|
| 0 = 0 | 10 = 2 | 100 = 4 | 1000 = 8 |
| 1 = 1 | 11 = 3 | 101 = 5 | 1001 = 9 |
| | | 110 = 6 | 1010 = 10 |
| | | 111 = 7 | 1011 = 11 |
| | | | 1100 = 12 |
| | | | 1101 = 13 |
| | | | 1110 = 14 |
| | | | 1111 = 15 |

Binary = Decimal

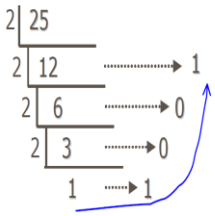
## Information Representation: Binary Numbers

- Radix = 2; Digits $A_i$ can only take one of two values (0 or 1)
  - It is customary to refer to *b*inary dig*it*s as *bit*s

| $(N)_b$ | 2 | 3 | 4 | 5 | ⋯ | 8 | ⋯ | 10 | 11 | 12 | ⋯ | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 0001 | 001 | 01 | 01 | | 01 | | 01 | 01 | 01 | | 1 |
| | 0010 | 002 | 02 | 02 | | 02 | | 02 | 02 | 02 | | 2 |
| | 0011 | 010 | 03 | 03 | | 03 | | 03 | 03 | 03 | | 3 |
| | 0100 | 011 | 10 | 04 | | 04 | | 04 | 04 | 04 | | 4 |
| | 0101 | 012 | 11 | 10 | | 05 | | 05 | 05 | 05 | | 5 |
| | 0110 | 020 | 12 | 11 | | 06 | | 06 | 06 | 06 | | 6 |
| | 0111 | 021 | 13 | 12 | | 07 | | 07 | 07 | 07 | | 7 |
| | 1000 | 022 | 20 | 13 | | 10 | | 08 | 08 | 08 | | 8 |
| | 1001 | 100 | 21 | 14 | | 11 | | 09 | 09 | 09 | | 9 |
| | 1010 | 101 | 22 | 20 | | 12 | | 10 | 0A | 0A | | A |
| | 1011 | 102 | 23 | 21 | | 13 | | 11 | 10 | 0B | | B |
| | 1100 | 110 | 30 | 22 | | 14 | | 12 | 11 | 10 | | C |
| | 1101 | 111 | 31 | 23 | | 15 | | 13 | 12 | 11 | | D |
| | 1110 | 112 | 32 | 24 | | 16 | | 14 | 13 | 12 | | E |
| | 1111 | 120 | 33 | 30 | | 17 | | 15 | 14 | 13 | | F |

## Derive Numbers in Base-2

- Decimal (base-10)
  - $(25)_{10}$
- Binary (base-2)
  - $(11001)_2$
- Exercise




## Base-2

- Decimal (base-10)
  - $(982)_{10}$
- Binary (base-2)
  - $(1111010110)_2$

## Base 8

- Decimal (base-10)
  - $(982)_{10}$
- Octal (base-8)
  - $(1726)_8$

## Base 16

- Decimal (base-10)
  - $(982)_{10}$
- Hexadecimal (base-16)
- So, what do we do when we count to 10??

| | |
|---|---|
| 0 | 0 |
| 1 | 1 |
| 2 | 2 |
| 3 | 3 |
| 4 | 4 |
| 5 | 5 |
| 6 | 6 |
| 7 | 7 |
| 8 | 8 |
| 9 | 9 |
| 10 | a |
| 11 | b |
| 12 | c |
| 13 | d |
| 14 | e |
| 15 | f |

## Base 16

- $(982)_{10} = (3d6)_{16}$
- $(3d6)_{16}$ can be written as $(0011\ 1101\ 0110)_2$
- We use Base-16 (or Hex) a lot in computer world
  - Ex: A 32-bit address can be written as `0xfe8a7d20` (`0x` is an abbreviation of Hex)
  - Or in binary form `1111_1110_1000_1010_0111_1101_0010_0000`