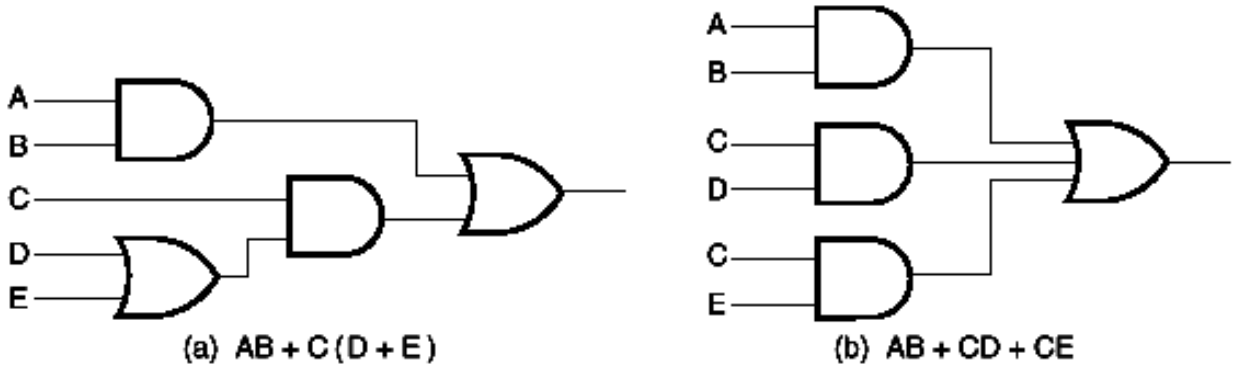
# Digital Logic Design

## Lecture of Week 11-16 March 2011

## Combinational Logic Circuit

# Three- Level and Two- Level Implementation




(a) AB + C (D + E)

(b) AB + CD + CE

## Logic Reduction

- Techniques include
  - Algebraic Manipulation
  - Graphical:
    - Venn Diagram
    - Karnaugh Map (The K-Map)
  - Algorithmic:
    - Quine-McCluskey Method

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## Karnaugh Map (K-Map)

- A graphical map method to simplify Boolean function up to 6 variables
- A diagram made up of squares
- Each square represents one minterm (or maxterm) of a given Boolean function

# Karnaugh Map Examples

**2-variable Karnaugh map**

| A \ B | 0 | 1 |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 0 | 1 |

$F = AB$

**3-variable Karnaugh map**

| A \ BC | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 |
| 1 | 0 | 1 | 1 | 1 |

$F = AB + C$

**4-variable Karnaugh map**

| AB \ CD | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 0 | 1 | 0 | 0 |
| 01 | 0 | 1 | 0 | 0 |
| 11 | 1 | 1 | 1 | 1 |
| 10 | 0 | 1 | 0 | 0 |

$F = AB + \bar{C}D$

Note that the Hamming Distance between adjacent columns or adjacent rows (including cyclic ones) must be 1 for simplification purposes

# Karnaugh Map

**2-variable Karnaugh map**

| A \ B | 0 | 1 | |
|---|---|---|---|
| 0 | 0 | 1 | $\bar{A}$ |
| 1 | 2 | 3 | A |
| | $\bar{B}$ | B | |

**3-variable Karnaugh map**

| A \ BC | $\bar{C}$ 00 | C 01 | C 11 | $\bar{C}$ 10 | |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 3 | 2 | $\bar{A}$ |
| 1 | 4 | 5 | 7 | 6 | A |
| | $\bar{B}$ | | B | | |

**4-variable Karnaugh map**

| | AB \ CD | $\bar{D}$ 00 | D 01 | D 11 | $\bar{D}$ 10 | |
|---|---|---|---|---|---|---|
| $\bar{A}$ | 00 | 0 | 1 | 3 | 2 | $\bar{B}$ |
| | 01 | 4 | 5 | 7 | 6 | B |
| A | 11 | 12 | 13 | 15 | 14 | |
| | 10 | 8 | 9 | 11 | 10 | $\bar{B}$ |
| | | $\bar{C}$ | | C | | |

Adjacent columns or rows allow grouping of minterms (maxterms) for simplification

# Karnaugh Map

- 2 Variable
- 3 Variable
- 4 Variable

- Typically limited to a maximum of Six Variables

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## Two-Variable Map

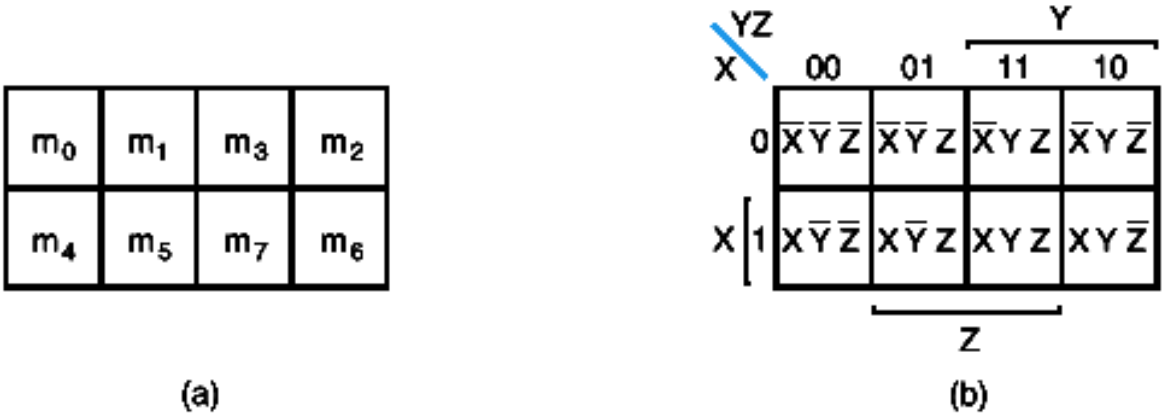
| | |
|---|---|
| $m_0$ | $m_1$ |
| $m_2$ | $m_3$ |

(a)

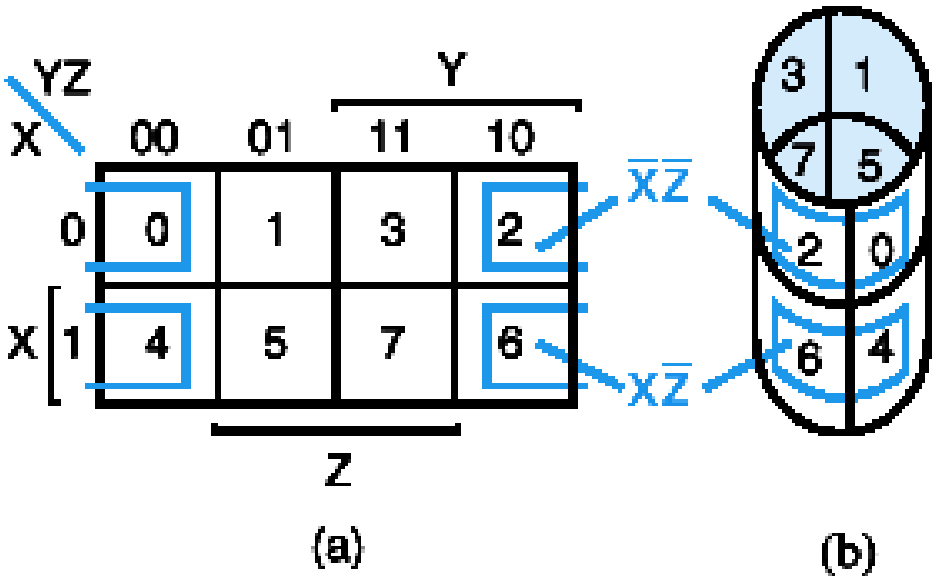
| X \ Y | 0 | 1 |
|---|---|---|
| 0 | $\bar{X}\bar{Y}$ | $\bar{X}Y$ |
| 1 | $X\bar{Y}$ | $XY$ |

(b)

# Three-Variable Map




# Three- Variable Map:
# Flat and on a Cylinder to Show Adjacent Squares



# Three-variable K-Maps

| X \ YZ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 1 | 1 | | |
| 1 | | 1 | 1 | |

```
F = !X & !Y
  | X & Z
```

# Three-variable K-Maps

| X \ YZ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 1 | 1 | | |
| 1 | | 1 | 1 | |

```
F = !X . !Y . !Z
  + !X . !Y . Z
  + X . !Y . Z
  + X . Y . Z
```

```
F = !X . !Y . (!Z + Z)
  + X . Z . (!Y + Y)

  = !X . !Y + X . Z
```

# Three-variable K-Maps

| X \ YZ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | | | | 1 |
| 1 | 1 | 1 | 1 | 1 |

```
F = Y . !Z  + X
```

# Three-variable K-Maps

| X \ YZ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 1 | 1 | 1 | |
| 1 | | 1 | 1 | 1 |

```
F = !X . !Y
    + X . y
    + Z
```

# Three-variable K-Maps

| X \ YZ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 1 | | | 1 |
| 1 | | 1 | 1 | |

```
F = X . Z + !X . !Z
```

# Three-variable K-Maps

| X \ YZ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 1 | | 1 | 1 |
| 1 | 1 | | 1 | 1 |

```
F = Y + !Z
```

# Three-variable K-Maps

| X \ YZ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 0 **1** | 1 | 3 | 2 **1** |
| 1 | 4 | 5 **1** | 7 **1** | 6 |

$$F = m0 + m2 + m5 + m7$$
$$= \Sigma(0,2,5,7)$$

# Four-Variable Map

| | | | |
|---|---|---|---|
| $m_0$ | $m_1$ | $m_3$ | $m_2$ |
| $m_4$ | $m_5$ | $m_7$ | $m_6$ |
| $m_{12}$ | $m_{13}$ | $m_{15}$ | $m_{14}$ |
| $m_8$ | $m_9$ | $m_{11}$ | $m_{10}$ |

(a)

| WX \ YZ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | | | | |
| 01 | | | | |
| 11 | | | | |
| 10 | | | | |

Y, X, W, Z

(b)