22. (1) $P(x)=a_0+a_1x+a_2x^2+a_3x^3$

$P'(x)=a_1+2a_2x+3a_3x^2$

$$\begin{cases} 0=P(0)=a_0 \\ 0=P'(0)=a_1 \\ 1=P(1)=a_0+a_1+a_2+a_3 \\ 1=P'(1)=2a_2+3a_3 \end{cases} \Rightarrow \begin{cases} a_0=0 \\ a_1=0 \\ a_2=2 \\ a_3=-1 \end{cases}$$

$P(x)=2x^2-x^3$

(2) 由 $P(0)=0$, $P'(0)=0$ 构造 $P_1(x)=ax^2$.

由 $P(1)=1$ 得 $P_1(x)=x^2$

设 $P_3(x)=x^2+bx^2(x-1)$, $P_3'(x)=2x+3bx^2-2bx$

由 $P_3'(1)=1$ 得 $b=-1$

$\therefore P_3(x)=x^2+x^2(-1)(x-1)=2x^2-x^3$

24. 由 $P(0)=0$, $P(1)=1$, $P(2)=2$, $P(3)=3$ 构建 Lagrange 插值多项式 $P_3(x)=x$,

设 $P_4(x)=x+b(x-0)(x-1)(x-2)(x-3)$

$P_4'(x)=4bx^3-18bx^2+22bx-6b+1$

由 $P_4'(2)=0$ 得 $b=\frac{1}{2}$

$\therefore P_4(x)=\frac{1}{2}x^4-3x^3+\frac{11}{2}x^2-2x$

33. $S(1) = 2 = 2+b+c$ ... ①

$$S'(x) = \begin{cases} 3x^2+2x, & 0<x<1 \\ 6x^2+2bx+c, & 1<x<2 \end{cases}$$

$S'(1) = 3+2 = 6+2b+c$ ... ②

由①② 得 $b=-2$, $c=3$

因此 $S''(x) = \begin{cases} 6x+2, & 0<x<1 \\ 12x-4, & 1<x<2 \end{cases}$

$S_-''(1) = 8$, $S_+''(1) = 8$ $\Rightarrow S_-''(1) = S_+''(1)$

故 $b=-2$, $c=3$

35. 令 $f(x,y) = (2x+4y-11)^2+(3x-5y-3)^2+(x+2y-6)^2+(2x+y-7)^2$

$\frac{\partial f}{\partial x} = 2(2x+4y-11)\cdot 2 + 2(3x-5y-3)\cdot 3 + 2(x+2y-6) + 2(2x+y-7)\cdot 2$

$= 8x+16y-44+18x-30y-18+2x+4y-12+8x+4y-28$

$= 36x-6y-102$

$\frac{\partial f}{\partial y} = 2(2x+4y-11)\cdot 4 + 2(3x-5y-3)(-5) + 2(x+2y-6)\cdot 2 + 2(2x+y-7)$

$= 16x+32y-88-30x+50y+30+4x+8y-24+4x+2y-14$

$= -6x+92y-96=0$

令 $\begin{cases} \frac{\partial f}{\partial x} = 36x-6y-102=0 \\ \frac{\partial f}{\partial y} = -6x+92y-96=0 \end{cases} \Rightarrow \begin{cases} x = \frac{830}{273} \\ y = \frac{113}{91} \end{cases}$

3].

$$\begin{bmatrix} 5 & 157 & 5327 \\ 157 & 5327 & 192331 \\ 5327 & 192331 & 7277699 \end{bmatrix} \begin{bmatrix} a \\ 0 \\ b \end{bmatrix} = \begin{bmatrix} 271.4 \\ 9776.1 \\ 369321.5 \end{bmatrix}$$

$$\begin{cases} 5a + 5327b = 271.4 \\ 5327a + 7277699b = 369321.5 \end{cases} \Rightarrow \begin{cases} a = 0.97 \\ b = 0.05 \end{cases}$$

$$P_2(x) = 0.97 + 0.05x^2$$

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