第五章

一、1. $\underline{8}$　2. $\underline{\Phi(x)}$　3. $\underline{\Phi\left(\frac{b-np}{\sqrt{np(1-p)}}\right)-\Phi\left(\frac{a-np}{\sqrt{np(1-p)}}\right)}$

二、1. $\underline{C}$　2. $\underline{C}$

三、1. 设 $X_i=\begin{cases}1, & 第i件是正品\\ 0, & 第i件是次品\end{cases}$

则 $Y_n=X_1+X_2+\cdots+X_n$ 表示 n 件中正品数

$$\frac{Y_n-np}{\sqrt{np(1-p)}}=\frac{Y_n-100\times 90\%}{\sqrt{100\times 90\%\times 10\%}}=\frac{Y_n-90}{3}\ 近似\ N(0,1)$$

$$P\{Y_n\geq 95\}=P\left\{\frac{Y_n-90}{3}\geq\frac{95-90}{3}\right\}=1-\Phi\left(\frac{5}{3}\right)=0.0475$$

2. $\frac{Y_n-np}{\sqrt{np(1-p)}}=\frac{Y_n-0.9n}{0.3\sqrt{n}}$ 近似 $N(0,1)$

$$P\{Y_n\geq 95\}=P\left\{\frac{Y_n-0.9n}{0.3\sqrt{n}}\geq\frac{95-0.9n}{0.3\sqrt{n}}\right\}=1-\Phi\left(\frac{95-0.9n}{0.3\sqrt{n}}\right)=0.99$$

$$\Phi\left(\frac{95-0.9n}{0.3\sqrt{n}}\right)=0.01\Rightarrow\frac{95-0.9n}{0.5\sqrt{n}}=0.5\Rightarrow n=104$$

四、$\frac{\sum_{k=1}^{n}X_k-5000\times 0.5}{\sqrt{5000}\times 0.1}$ 近似 $N(0,1)$

$$P\left\{\sum_{k=1}^{n}X_k>2510\right\}=P\left\{\frac{\sum_{k=1}^{n}X_k-2500}{7.07}>\frac{2510-2500}{7.07}\right\}=1-\Phi(1.41)=0.0793$$

五、$P\{0.4n<\sum X_i<0.6n\}=P\left\{\frac{0.4n-0.5n}{\sqrt{0.5\cdot 0.5\times n}}<\frac{\sum X_i-0.5n}{\sqrt{n\times 0.5\times 0.5}}<\frac{0.6n-0.5n}{\sqrt{n\times 0.5\times 0.5}}\right\}$

$$=P\left\{-\frac{\sqrt{n}}{5}<\frac{\sum X_i-0.5n}{0.5\sqrt{n}}<\frac{\sqrt{n}}{5}\right\}=2\Phi\left(\frac{\sqrt{n}}{5}\right)-1\geq 0.9\Rightarrow\Phi\left(\frac{\sqrt{n}}{5}\right)\geq 0.95$$

$\Rightarrow n\geq 68$

$P\{|X-EX|<\varepsilon\}\geq 1-\frac{DX}{\varepsilon^2}$，令 $EX=0.5n$，$DX=0.25n$，$\varepsilon=0.1n$

则 $P\{0.4n<X<0.6n\}\geq 1-\frac{0.25n}{0.01n^2}\geq 0.9\Rightarrow\frac{0.25n}{0.01n}\leq 0.1\Rightarrow n\geq 250$

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六、以X表示200架分机中同一时刻使用外线的分机数，则$X\sim B(200,0.05)$
设N为所求外线数，
$P\{X\le N\}=0.9 \Rightarrow P\left\{\frac{X-200\times0.05}{\sqrt{200\times0.05\times0.95}}\le\frac{N-200\times0.05}{\sqrt{200\times0.05\times0.95}}\right\}$
$=\Phi\left(\frac{N-10}{3.08}\right)=0.9 \Rightarrow \frac{N-10}{3.08}=1.29 \Rightarrow N=14$
∴至少需要14根外线．

七、设抽样产品中有X件次品，则$X\sim B(N,0.1)$，其中N为样品数
$P\{X>10\}=0.9 \Rightarrow P\left\{\frac{X-N\cdot0.1}{\sqrt{N\times0.1\times0.9}}>\frac{10-N\times0.1}{\sqrt{N\times0.1\times0.9}}\right\}=0.9$
$\Rightarrow 1-\Phi\left(\frac{10-0.1N}{0.3\sqrt{N}}\right)=0.9 \Rightarrow \Phi\left(\frac{10-0.1N}{0.3\sqrt{N}}\right)=0.1$
$\Rightarrow \frac{10-0.1N}{0.3\sqrt{N}}=-1.29 \Rightarrow N=147$
∴至少检查147件产品

第六章

$\chi^2(n)$ 定义：$X_1 X_2 \cdots X_n$ 是来自总体 $N(0,1)$ 的样本，则

$$\chi^2 = X_1^2 + X_2^2 + \cdots + X_n^2 \sim \chi^2(n)$$

期望：n　方差：$2n$

$t(n)$ 定义：设 $X \sim N(0,1)$　$Y \sim \chi^2(n)$，X 和 Y 独立，则

$$t = \frac{X}{\sqrt{Y/n}} \sim t(n)$$

$F(m,n)$ 定义：$U \sim \chi^2(m)$　$V \sim \chi^2(n)$，U 和 V 独立，则

$$F = \frac{U/m}{V/n} \sim F(m,n)$$

Thm1：$\dfrac{\bar{X}-\mu}{\sigma/\sqrt{n}} \sim N(0,1)$

Thm2：$\dfrac{(n-1)S^2}{\sigma^2} \sim \chi^2(n-1)$，$S^2$ 与 $\bar{X}$ 相互独立

Thm3：$\dfrac{\bar{X}-\mu}{S/\sqrt{n}} \sim t(n-1)$

一、1、$\underline{0.6}$ $\underline{0.3}$ $\underline{0.24}$　2、$\underline{N(0,1)}$ $\underline{t(n-1)}$ $\underline{\chi^2(n-1)}$ $\underline{\chi^2(n)}$

3. $\underline{N(\mu, \sigma^2/n)}$ $\underline{\mu}$ $\underline{\sigma/\sqrt{n}}$ $\underline{\sigma^2}$ $\underline{\frac{2\sigma^4}{n-1}}$

4、$\underline{0.8293}$　5、$\underline{\lambda}$ $\underline{\frac{n}{n-1}\lambda}$　6. $\underline{1.645}$ $\underline{1.3722}$ $\underline{40.646}$ $\underline{2.91}$

7. $\underline{F(n,1)}$　8、$\underline{6}$

二、D

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三、(1) $X_i \sim N(0, 0.5^2) \Rightarrow 2X_i \sim N(0,1) \Rightarrow \sum_{i=1}^{10}(2X_i)^2 \sim \chi^2(10)$

$P\{\sum_{i=1}^{10} X_i^2 \geq 4\} = P\{\sum_{i=1}^{10} 4X_i^2 \geq 16\} = 0.1$

(2) $S^2 = \frac{1}{10-1}\sum_{i=1}^{10}(X_i - \bar{X})^2$

$$\frac{(10-1)S^2}{\sigma^2} = 4\sum_{i=1}^{10}(X_i-\bar{X})^2 \sim \chi^2(9)$$

$$P\{\sum_{i=1}^{10}(X_i-\bar{X})^2 \geq 2.85\} = P\{\sum_{i=1}^{10} 4(X_i-\bar{X})^2 \geq 11.4\} = 0.25$$

四、设 $X \sim (\mu, \sigma^2)$，则 $Y_1 \sim N(\mu, \sigma^2/6)$

$Y_2 \sim (\mu, \sigma^2/3)$ 且 Y_1 与 Y_2 相互独立.

$$\frac{2S^2}{\sigma^2} = \frac{(3-1)\frac{1}{3-1}\sum_{i=7}^{9}(X_i - Y_2)^2}{\sigma^2} \sim \chi^2(3-1)$$

$$Y_1 - Y_2 \sim N(0, \frac{\sigma^2}{2}) \Rightarrow \sqrt{2}\frac{Y_1 - Y_2}{\sigma} \sim N(0,1)$$

$$\cancel{\frac{\frac{\sqrt{2}}{\sigma}(Y_1 - Y_2)}{\sqrt{\sum_{i=7}^{9}(X_i - Y_2)^2}}} \quad \therefore \frac{\frac{\sqrt{2}}{\sigma}(Y_1 - Y_2)}{\sqrt{\frac{2S^2}{\sigma^2} \times \frac{1}{2}}} \sim t(2)$$

$$\therefore Z = \frac{\sqrt{2}(Y_1 - Y_2)}{S} \sim t(2)$$

五、$2\bar{X} \sim (2\mu, 2\sigma^2/n)$

$X_i + X_{n+i} \sim N(2\mu, 2\sigma^2)$

$\therefore \dfrac{X_i + X_{n+i} - 2\bar{X}}{\sqrt{2\sigma^2(1-\frac{1}{n})}} \sim N(0,1)$

$\dfrac{1}{2\sigma^2(1-\frac{1}{n})} \sum_{i=1}^{n} (X_i + X_{n+i} - 2\bar{X})^2 \sim \chi^2(n)$

$\dfrac{n}{2\sigma^2(n-1)} EY = n$

$EY = 2(n-1)\sigma^2$

六、$X_1 + X_2 + X_3 \sim N(0,3)$

$\frac{1}{\sqrt{3}}(X_1+X_2+X_3) \sim N(0,1)$

$\frac{1}{3}(X_1+X_2+X_3)^2 \sim \chi^2(1)$

同理 $\frac{1}{3}(X_4+X_5+X_6)^2 \sim \chi^2(1)$

$\therefore \frac{1}{3}[(X_1+X_2+X_3)^2 + (X_4+X_5+X_6)^2] \sim \chi^2(2)$

$\Rightarrow \frac{1}{3} Y \sim \chi^2(2)$

$\Rightarrow C = \frac{1}{3}$