

$$11. y_{n+1} = y_n + \frac{h}{6} (k_1 + 2k_2 + 2k_3 + k_4) \dots \textcircled{1}$$

$$k_1 = f(x_n, y_n) = 8 - 3y_n \dots \textcircled{2}$$

$$k_2 = f(x_n + \frac{h}{2}, y_n + \frac{h}{2} k_1) = 0.7(8 - 3y_n) \dots \textcircled{3}$$

$$k_3 = f(x_n + \frac{h}{2}, y_n + \frac{h}{2} k_2) = 0.79(8 - 3y_n) \dots \textcircled{4}$$

$$k_4 = f(x_n + h, y_n + h k_3) = 0.526(8 - 3y_n) \dots \textcircled{5}$$

将②③④⑤代入①式得  $y_{n+1} = 0.5494 y_n + 1.2016$

取  $y_0, y_1, y_2$  分别对应  $x_0 = 0, x_1 = 0.2, x_2 = 0.4$

$$\text{则 } y_1 = 0.5494 \times 2 + 1.2016 = 2.3004$$

$$y_2 = 0.5494 \times 2.3004 + 1.2016 = 2.4654$$

$$18. y_1'(0) = 3y_1(0) + 2y_2(0) = 2, y_2'(0) = 4y_1(0) + y_2(0) = 1$$

$$y_1(0.1) = y_1(0) + h f(0,0) = 0.2, y_2(0.1) = y_2(0) + h g(0,0) = 1.1$$

$$y_1(0.1) = y_1(0) + \frac{h}{2} [f(0,0) + f(0.1,0.2)]$$

$$= 0 + \frac{0.1}{2} [2 + 3 \times 0.2 + 2 \times 1.1]$$

$$= 0.24$$

$$y_2(0.1) = y_2(0) + \frac{h}{2} [g(0,0) + g(0.1,1.1)]$$

$$= 1 + \frac{0.1}{2} [1 + 4 \times 0.2 + 1.1]$$

$$= 1.145$$

19. 由  $\begin{cases} y'' = 2y^3 \\ y'(1) = y(1) = -1 \end{cases}$  易得  $y' = \frac{1}{2} y^4 - \frac{3}{2}$

$$y_{1.1} = y_1 + \frac{0.1}{6} (k_1 + 2k_2 + 2k_3 + k_4)$$

$$k_1 = f(1, -1) = -1$$

$$k_2 = f(1.05, -1.05) = -0.89225$$

$$k_3 = f(1.05, -0.95539) = -1.00916$$

$$k_4 = f(1.1, -1.100916) = -0.76551$$

$$y_{1.1} = -1 + \frac{0.1}{6} [-1 + 2(-0.89225) + 2(-1.00916) - 0.76551]$$

$$= -1 + \frac{0.1}{6} [-1 - 1.7845 - 2.01832 - 0.76551]$$

$$= -0.907294$$