第二章习题三 岳审轩 19020011038

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$f(x) \geqslant 0$ $\int_{-\infty}^{+\infty} f(t)\,dt = 1$

一、1. X 2. √ 3. √ 4. √

二、1. 1 2. $\frac{1}{\pi}$ 0 3. $2^{-\frac{1}{4}}$ 4. $\frac{1}{8}$ 5. $\frac{9}{64}$

三、1. B 2. D 3. D 4. A

四、1. $F(1-0)=1$

$\Rightarrow A=1$

2. $P\{-1<X<\frac{1}{2}\} = F(\frac{1}{2}) - F(-1) = \frac{1}{4}$

$P\{\frac{1}{3}<X\leq 2\} = F(2) - F(\frac{1}{3}) = 1 - \frac{1}{9} = \frac{8}{9}$

3. $f(x) = \begin{cases} 2x, & 0 \leq x < 1 \\ 0, & \text{其它} \end{cases}$

五、1. $\int_{-\infty}^{+\infty} f(x)\,dx = 1$

$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} A\cos x\,dx + \int_{\frac{\pi}{2}}^{+\infty} 0\,dx + \int_{-\infty}^{\frac{\pi}{2}} 0\,dx = 1.$

$A\sin x \Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = 1$

$A = \frac{1}{2}$

2. $F(x) = \begin{cases} \frac{1}{2}\sin x + \frac{1}{2}, & |x| \leq \frac{\pi}{2} \\ 0, & x < \frac{\pi}{2} \\ 1, & x > \frac{\pi}{2} \end{cases}$

3. $P\{0<X<\frac{\pi}{4}\} = F(\frac{\pi}{4}) - F(0) = \frac{1}{2}\cdot\frac{\sqrt{2}}{2} + \frac{1}{2} - \frac{1}{2} = \frac{\sqrt{2}}{4}$

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二、7. $\frac{1}{\sqrt{2\pi}}$ $\quad \frac{1}{2}$ $\quad 0$

8. 4

9. 不变 增大

10. 0.2

六、1. $c = \mu = 3$

2. $P\{2<X\le 5\} = P\{\frac{2-3}{2}<\frac{X-3}{2}\le\frac{5-3}{2}\} = \Phi(1)-\Phi(-\frac{1}{2}) = \Phi(1)+\Phi(\frac{1}{2})-1 = 0.7664$

$P\{|X|>2\} = P\{X>2\}+P\{X<-2\} = P\{\frac{X-3}{2}>\frac{2-3}{2}\}+P\{\frac{X-3}{2}<\frac{-2-3}{2}\}$

$= \Phi(\frac{1}{2}) + \Phi(-\frac{5}{2}) = \Phi(\frac{1}{2})+1-\Phi(\frac{5}{2}) = 0.6977$

3. $P\{\frac{X-3}{2}>\frac{d-3}{2}\}\geq 0.9$

$1-\Phi(\frac{d-3}{2})\geq 0.9$

$\Phi(\frac{d-3}{2})\leq 0.1$

$\Rightarrow \frac{d-3}{2}<0 \Rightarrow d<3$

$\Rightarrow 1-\Phi(\frac{3-d}{2})\leq 0.1$

$\Phi(\frac{3-d}{2})\geq 0.9$

$\frac{3-d}{2}\geq 1.29$

$d\leq 0.42$

七、$X\sim N(168,7^2)$ 设门高度为 h cm.

$P\{X>h\}\leq 0.01$

$P\{\frac{X-168}{7}>\frac{h-168}{7}\}\leq 0.01$

$1-\Phi(\frac{h-168}{7})\leq 0.01$

$\Phi(\frac{h-168}{7})\geq 0.99$

$\frac{h-168}{7}\geq 2.33$

$h\geq 184.31$

$\therefore$ 至少高 184.31 cm

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习题四

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一、1、$f_Y(y)=\begin{cases}1 & ,y\le 0\\ 1-y & ,0<y<1\\ 0 & ,y\ge 1\end{cases}$　　2、$Y\sim N(48,15^2)$

二、1、B　2、A　3、C

三、1、

U	2	1	-1	-2	-3
p_k	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{3}$	$\frac{1}{6}$

Z	9	4	1	0
p_k	$\frac{1}{8}$	$\frac{5}{12}$	$\frac{1}{3}$	$\frac{1}{8}$

2、

X	0	1	2
p_k	$\frac{7}{15}$	$\frac{7}{15}$	$\frac{1}{15}$

X^2	0	1	4
p_k	$\frac{7}{15}$	$\frac{7}{15}$	$\frac{1}{15}$

$P\{X=0\}=\frac{C_8^3}{C_{10}^3}=\frac{7}{15}$

$P\{X=1\}=\frac{C_2^1C_8^2}{C_{10}^3}=\frac{7}{15}$

$P\{X=2\}=\frac{C_2^2C_8^1}{C_{10}^3}=\frac{1}{15}$

四、1、$F_Y(y)=P\{Y\le y\}=P\{-2X+1\le y\}=P\{X\ge\frac{1-y}{2}\}=1-P\{X\le\frac{1-y}{2}\}$

$=1-F_X(\frac{1-y}{2})$

$f_Y(y)=-F_X'(\frac{1-y}{2})(\frac{1-y}{2})'=\frac{1}{2}f_X(\frac{1-y}{2})$

$=\begin{cases}\frac{1}{2}3(\frac{1-y}{2})^2 & -1<y<1\\ 0 & 其它\end{cases}=\begin{cases}\frac{3}{8}(y-1)^2 & ,-1<y<1\\ 0 & ,其它\end{cases}$

$F_Y(y)=\begin{cases}0, & y\le -1\\ \frac{1}{8}(y-1)^3+1 & -1<y<1\\ 1 & y\ge 1\end{cases}$

2、 $f_X(x)=\begin{cases}\frac{1}{\pi}, & -\frac{\pi}{2}<x<\frac{\pi}{2}\\ 0, & 其它\end{cases}$

$F_Y(y)=P\{Y\le y\}=P\{\tan X\le y\}=P\{X\le \arctan y\}=F_X(\arctan y)$

$f_Y(y)=f_X(\arctan y)\frac{1}{1+y^2}=\frac{1}{\pi}\frac{1}{(1+y)^2}=\frac{1}{\pi(1+y)^2}$

$F_Y(y)=\frac{1}{\pi}\arctan y+\frac{1}{2}$

$P\{Y>0\}=1-P\{Y\le 0\}=\frac{1}{2}$

3、(1) $F_Y(y)=P\{e^X\le y\}=P\{X\le \ln y\}=F_X(\ln y)$

$f_Y(y)=f_X(\ln y)\frac{1}{y}=\frac{1}{\sqrt{2\pi}y}e^{-\frac{(\ln y)^2}{2}}$

(2) $F_Y(y)=P\{2X^2+1\le y\}=P\{X^2\le\frac{y-1}{2}\}=F_{X^2}(\frac{y-1}{2})$

$f_Y(y)=f_{X^2}(\frac{y-1}{2})\cdot\frac{1}{2}=\begin{cases}\frac{1}{2}\frac{1}{\sqrt{2\pi}}\frac{1}{\sqrt{\frac{y-1}{2}}}e^{-\frac{y-1}{4}} & y>1\\ 0 & y\le 1\end{cases}$

$=\begin{cases}\frac{1}{2\sqrt{\pi(y-1)}}e^{-\frac{y-1}{4}}, & y>1\\ 0, & y\le 1\end{cases}$

(3) $F_Y(y)=P\{|X|\le y\}=P\{-y\le X\le y\}=F_X(y)-F_X(-y)$

$=2F_X(y)-1$

$f_Y(y)=2f_X(y)=\sqrt{\frac{2}{\pi}}e^{-\frac{y^2}{2}}$

4. $f_X=\begin{cases}2e^{-2x}, & x>0\\ 0, & x\le 0\end{cases}$

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$F_Y = P\{1-e^{-2X} \le y\} = F_X(-\frac{1}{2}\ln(1-y))$

$f_Y(y) = f_X(-\frac{1}{2}\ln(1-y))(-\frac{1}{2}\ln(1-y))'$

$= \begin{cases} 2(1-y) \cdot \frac{1}{2} \frac{1}{1-y} = 1 & (0<y<1) \\ 0 & \text{其它} \end{cases}$

$F_Y(y) = \begin{cases} 0, & y \le 0 \\ x, & 0<y<1 \\ 1, & y \ge 1 \end{cases} \quad \therefore Y \sim U(0,1)$

5、$S = \pi R^2$

$R^2 \ge 0 \Rightarrow F_S(s) = 0 \quad (s \le 0)$

当$s>0$时，$F_S(s) = P\{\pi R^2 \le s\} = P\{-\sqrt{\frac{s}{\pi}} \le R \le \sqrt{\frac{s}{\pi}}\} = F_R(\sqrt{\frac{s}{\pi}}) - F_R(-\sqrt{\frac{s}{\pi}})$

$f_S(s) = \frac{1}{2\sqrt{\pi s}}[f_R(\sqrt{\frac{s}{\pi}}) + f_R(-\sqrt{\frac{s}{\pi}})] = \begin{cases} \frac{1}{2\sqrt{\pi s}}, & 0<s<\pi \\ 0, & \text{其它} \end{cases}$

五、

Y	1	-1
p_k	$\frac{2}{3}$	$\frac{1}{3}$