

$$18. \text{ 令 } f(x) = x^3 - a, \text{ 则 } f'(x) = 3x^2, x_{k+1} = x_k - \frac{x_k^3 - a}{3x_k^2} = \frac{2}{3}x_k + \frac{a}{3x_k^2}$$

$$\text{令 } \varphi(x) = \frac{2}{3}x + \frac{a}{3x^2}, \text{ 则 } \varphi'(x) = \frac{2}{3} + \frac{a}{3}(-2)\frac{1}{x^3}, \text{ 则 } \varphi'(\sqrt[3]{a}) = 0$$

故该迭代至二阶局部收敛

$$19. \text{ 令 } f(x) = (x^3 - a)^2, \text{ 则 } f'(x) = 2(x^3 - a)3x^2 = 6x^2(x^3 - a)$$

$$x_{k+1} = x_k - \frac{f(x)}{f'(x)} = x_k - \frac{x_k^3 - a}{6x_k^2} = \frac{5}{6}x_k + \frac{a}{6x_k^2}$$

$$\text{令 } \varphi(x) = \frac{5}{6}x + \frac{a}{6x^2}, \varphi'(x) = \frac{5}{6} + \frac{a}{6}(-2)\frac{1}{x^3}, \varphi'(\sqrt[3]{a}) = \frac{1}{2} \neq 0$$

故该迭代仅为线性收敛

$$20. \text{ 令 } f(x) = \frac{1}{x} - a, f'(x) = -\frac{1}{x^2}, \text{ 则 } \varphi(x) = x - \frac{f(x)}{f'(x)} = 2x - a x^2$$

$$\begin{aligned} 21. \text{ 令 } f(x) &= \frac{1}{x^2} - a, f'(x) = -2x^{-3}, \text{ 则 } x_{k+1} = x_k - \frac{f(x)}{f'(x)} \\ &= x_k - \frac{\frac{1}{x_k^2} - a}{-2x_k^{-3}} \\ &= \frac{3}{2}x_k - \frac{a}{2}x_k^3 \end{aligned}$$