第四章 习题2

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一、1、1　0　2、0　$\frac{1}{2}$　$\frac{\sqrt{2}}{2}$　3、$3e^{-2}$　81

4、46　5、1　6、3　7、10　8　8、$\frac{8}{9}$　9、$\frac{8}{9}$

二、$EX=\int_{-\infty}^{+\infty} x f(x)\,dx = \int_0^1 x^2\,dx + \int_1^2 x(2-x)\,dx = \frac{1}{3} + \int_1^2 2x\,dx - \int_1^2 x^2\,dx$

$= \frac{1}{3} + 3 - \frac{7}{3} = 1$

$DX = EX^2 - (EX)^2 = EX^2 - 1 = \int_0^1 x^3\,dx + \int_1^2 x^2(2-x)\,dx - 1$

$= \frac{1}{4} + \int_1^2 2x^2\,dx - \int_1^2 x^3\,dx - 1 = \frac{1}{4} + \frac{14}{3} - \frac{15}{4} + 1 = \frac{1}{6}$

三、$f_X(x) = \int_{-\infty}^{+\infty} f(x,y)\,dy = \int_0^x 8xy\,dy = 4x^3$，$f_Y(y) = \int_{-\infty}^{+\infty} f(x,y)\,dx = \int_y^1 8xy\,dx = 4y(1-y^2)$

$x \in [0,1]$ 时

$\therefore f_X(x) = \begin{cases} 4x^3 & 0 \le x \le 1 \\ 0 & \text{其它} \end{cases}$　$f_Y(y) = \begin{cases} 4y(1-y^2) & 0 \le y \le 1 \\ 0 & \text{其它} \end{cases}$

$EX = \int_{-\infty}^{+\infty} x f_X(x)\,dx = \int_0^1 4x^4\,dx$　$EY = \int_{-\infty}^{+\infty} y f_Y(y)\,dy = \int_0^1 4y^2(1-y^2)\,dy$

$= \frac{4}{5}$　$= \frac{8}{15}$

$DX = EX^2 - (EX)^2 = EX^2 - \frac{16}{25}$　$DY = EY^2 - (EY)^2 = \int_0^1 4y^3(1-y^2)\,dy - \frac{64}{225}$

$= \int_0^1 4x^5\,dx - \frac{16}{25} = \frac{2}{75}$　$= 1 - \frac{2}{3} - \frac{64}{225} = \frac{11}{225}$

四、$P\{X=0\} = \frac{8}{10} = \frac{4}{5}$，$P\{X=1\} = \frac{2}{10} \times \frac{8}{9} = \frac{8}{45}$，$P\{X=2\} = \frac{2}{10} \times \frac{1}{9} = \frac{1}{45}$

X	0	1	2
P	$\frac{4}{5}$	$\frac{8}{45}$	$\frac{1}{45}$

X^2	0	1	4
P	$\frac{4}{5}$	$\frac{8}{45}$	$\frac{1}{45}$

$EX = \frac{8}{45} + 2 \times \frac{1}{45} = \frac{10}{45} = \frac{2}{9}$

$EX^2 = \frac{8}{45} + 4 \times \frac{1}{45} = \frac{12}{45} = \frac{4}{15}$

$DX = EX^2 - (EX)^2 = \frac{88}{405}$

五、$F(U) = \begin{cases} 0, & U \le -2 \\ \frac{U^2}{4}, & -2 < U < 2 \\ 1, & U \ge 2 \end{cases}$

X	-1	1
P	$\frac{1}{4}$	$\frac{3}{4}$

Y	-1	1
P	$\frac{3}{4}$	$\frac{1}{4}$

$\because P\{U \le -1, U \ge 1\} = 0$

$\therefore P\{X=1, Y=1\} = 0$

XY	-1	1
P	$\frac{5}{8}$	$\frac{3}{8}$

$E(XY) = -\frac{1}{4}$

X^2	1
P	1

Y^2	1
P	1

$EX = \frac{1}{2}$　$EY = -\frac{1}{2}$

XY	-1	1	
-1	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$
1	0	$\frac{1}{4}$	$\frac{1}{4}$
	$\frac{1}{4}$	$\frac{3}{4}$	

$D(X+Y) = DX + DY + 2\mathrm{Cov}(X,Y)$

$= EX^2 - (EX)^2 + EY^2 - (EY)^2 + 2\mathrm{Cov}(X,Y)$

$= 1 - \frac{1}{4} + 1 - \frac{1}{4} + 2[E(XY) - EXEY]$

$= \frac{3}{2} + 2 \times 0 = \frac{3}{2}$

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第四章 习题三

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一、1、0 0 2、0.5 0 3、25.6 4、$\frac{1}{12}$ 5、6^2 36^2 -6^2 6、-1
7、-0.9 =、D

三、1、$f_X(x)=\int_{-\infty}^{+\infty}f(x,y)dy=\int_0^1(2-x-y)dy=-x+\frac{3}{2}$ ($x\in[0,1]$时)

$f_Y(y)=\int_{-\infty}^{+\infty}f(x,y)dx=\int_0^1(2-x-y)dx=-y+\frac{3}{2}$ ($x\in[0,1]$时)

$EX=\int_{-\infty}^{+\infty}xf_X(x)dx=\int_0^1(-x^2+\frac{3}{2}x)dx=\frac{5}{12}$

$EY=\int_{-\infty}^{+\infty}yf_Y(y)dy=\frac{5}{12}$

$EXY=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}xyf(x,y)dxdy=\int_0^1\int_0^1xy(2-x-y)dxdy=\frac{1}{6}$

$Cov(X,Y)=EXY-EXEY=-\frac{1}{144}$

$EX^2=\int_{-\infty}^{+\infty}x^2f_X(x)dx=\int_0^1(-x^3+\frac{3}{2}x^2)dx=\frac{1}{4}$

$DX=EX^2-(EX)^2=\frac{11}{144}$

$EY^2=\int_{-\infty}^{+\infty}y^2f_Y(y)dy=\frac{1}{4}$

$DY=EY^2-(EY)^2=\frac{11}{144}$

$\rho_{XY}=\frac{Cov(X,Y)}{\sqrt{DX}\sqrt{DY}}=-\frac{1}{11}$

$D(2X-3Y)=4DX+9DY-12Cov(X,Y)=\frac{155}{144}$

2、$\rho_{XY}=-\frac{1}{11}$，相关

$f(x,y)\neq f_X(x)\cdot f_Y(y)$，不独立

四、$X^2+Y^2=1$，设 $X=\cos\theta$，$Y=\sin\theta$，$\theta\sim U(-\pi,\pi)$

$EX=\frac{1}{2\pi}\int_{-\pi}^{\pi}\cos\theta d\theta=0$，$EY=\frac{1}{2\pi}\int_{-\pi}^{\pi}\sin\theta d\theta=0$

$EX^2=\frac{1}{2\pi}\int_{-\pi}^{\pi}\cos^2\theta d\theta=\frac{1}{2}$，$EY^2=\frac{1}{2\pi}\int_{-\pi}^{\pi}\sin^2\theta d\theta=\frac{1}{2}$

$DX=EX^2-(EX)^2=\frac{1}{2}$，$DY=EY^2-(EY)^2=\frac{1}{2}$

$EXY=\frac{1}{2\pi}\int_{-\pi}^{\pi}\cos\theta\sin\theta d\theta=0$，$Cov(X,Y)=EXY-EXEY=0\Rightarrow\rho_{XY}=0$ 不相关

$P\{0<X<\frac{1}{2}\}\neq0$，$P\{0<Y<\frac{1}{2}\}\neq0\Rightarrow P\{0<X<\frac{1}{2}\}P\{0<Y<\frac{1}{2}\}\neq0$

而 $\because X^2+Y^2=1$ $\therefore P\{0<X<\frac{1}{2},0<Y<\frac{1}{2}\}=0$ 不独立

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五、 $Cov(X, X-aY) = Cov(Y, X-aY)$

$Cov(X,X) - a\,Cov(X,Y) = Cov(Y,X) - a\,Cov(Y,Y)$

$DX + a = -1 - aDY$

$4 + a = -1 - 4a$

$a = -1$

$Cov(X,Z) = Cov(X, X+Y) = DX + Cov(X,Y) = 4 - 1 = 3$

六、

XX	1	0	
1	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{6}$
0	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{5}{6}$
	$\frac{1}{4}$	$\frac{3}{4}$	

$P(AB) = P(B|A)P(A) = \frac{1}{12}$

$P(B) = \frac{P(AB)}{P(A|B)} = \frac{1}{6}$

X	1	0
	$\frac{1}{4}$	$\frac{3}{4}$

Y	1	0
	$\frac{1}{6}$	$\frac{5}{6}$

$EX = \frac{1}{4}$

$EY = \frac{1}{6}$

XY	1	0
	$\frac{1}{24}$	$\frac{23}{24}$

X^2	1	0
	$\frac{1}{4}$	$\frac{3}{4}$

Y^2	1	0
	$\frac{1}{6}$	$\frac{5}{6}$

$EX^2 = \frac{1}{4}$

$EY^2 = \frac{1}{6}$

$EXY = \frac{1}{24}$

$DX = EX^2 - (EX)^2 = \frac{3}{16}$, $DY = EY^2 - (EY)^2 = \frac{5}{36}$

$Cov(X,Y) = EXY - EX\,EY = \frac{1}{24} - \frac{1}{24} = 0$.

$\rho_{XY} = 0$