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(1) $\alpha_{k+1}(i) = P(O_1 O_2 \cdots O_{k+1}, q_{k+1}=S_i)$

$= \sum_j P(O_1 O_2 \cdots O_{k+1}, q_k=S_j, q_{k+1}=S_i)$

$= \sum_j P(O_1 O_2 \cdots O_k, q_k=S_j)\, a_{ji}\, b_i(O_{k+1})$ 式①

N:状态空间,K:样本总数

∵ $\alpha_1(i) = P(O_1, q_1=S_i) = \pi_i b_i(O_1)$, $i \in [1,N]$

$q_1=S_i$ 即状态值为 S_i

$\alpha_j(i)$ 为观测O,状态S之概率,因为齐次Markov,

∴ $\alpha_{k+1}(i) = \sum_{j=1}^{N} \alpha_k(j) \cdot a_{ji} \cdot b_i(O_{k+1})$ 式②

即第k+1次观测为O_i,状态为S_i的概率 α_{k+1} 等于第k次所有 $\alpha_k(j)$ 的概率,乘从j转移到i的概率,乘观测与状态对应概率。因为对于第k次,有

∵ $\alpha_k(j) = P(O_1 O_2 \cdots O_k, q_k=S_j)$

代入式①

$\alpha_{k+1}(i) = \sum_{j=1}^{N} P(O_1 O_2 \cdots O_k, q_k=S_j)\, a_{ji}\, b_i(O_{k+1})$

$= \sum_{j=1}^{N} (\alpha_k(j) \cdot a_{ji}) \cdot b_i(O_{k+1})$

= 式②

代入结果与式②使用HMM的齐次Markov假设得到结果一致,得证

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Date

2. 推导 $\delta_k(j)$，

$$\delta_k(j) = \max P(q_1 \cdots q_{k-1}, q_k = S_j, O_1 O_2 \cdots O_k)$$

$$= \max\left[\max P(q_1 q_2 \cdots q_{k-1}, q_{k-1} = S_i, O_1 O_2 \cdots O_{k-1}) \cdot P(S_j | S_i) \cdot P(S_j | O_k)\right]$$

2. 推导 $\delta_k(j)$，由于齐次 Markov 假设

$$\delta_k(j) = \max P(q_1 \cdots q_{k-1}, q_k = S_j, O_1 O_2 \cdots O_k)$$

$$= \max(\delta_{k-1}(i) \cdot P(S_j | S_i) \cdot P(S_j | O_k))$$

$$= \max_i (\max P(q_1 \cdots q_{k-1} = S_i, O_1 \cdots O_{k-1}) \cdot a_{ij} \cdot b_j(O_k))$$

3. 推导 BW

$$\xi_k(i,j) = P(q_k = S_i, q_{k+1} = S_j | O_1 O_2 \cdots O_k)$$

$$\gamma_k(i) = P(q_k = S_i | O_1 \cdots O_k)$$

① from state S_i to state S_j equals to

$$P(q_1 = S_i, q_2 = S_j | O_1 O_2 \cdots O_k) + \cdots + P(q_{k-1} = S_i, q_k = S_j | O_1 O_2 \cdots O_k)$$

$$= \xi_1(i,j) + \xi_2(i,j) + \cdots + \xi_{k-1}(i,j)$$

$$= \sum_{k=1}^{k-1} \xi_k(i,j)$$

② ~~out~~ out of state S_i，从S_i转移到别的状态的概率.

$$P(q_{k=1} = S_i | O_1 O_2 \cdots O_k) + \cdots + P(q_{k=k-1} = S_i | O_1 O_2 O_3 \cdots O_K)$$

$$= \gamma_1(i) + \cdots + \gamma_{k-1}(i) = \sum_{k=1}^{k-1} \gamma_k(i)$$

Date

(3) when V_m occurs in state s_i, $O_k = O_m$.

V_m means the observation is V_m.

∴ $\sum_{k=1}^{K} P(q_k = s_i \mid O_1 \cdots O_m)$, k is when $O_k = V_m$

$= \sum_{k=1}^{K} \gamma_k(i)$

(4) Expect frequency of in state S_i at time $K = \gamma_k(i)$)

$\Gamma_k(i)$ is a probability, and k is a integer, k can not equals to $\Gamma_k(i)$,

I find the paper in

"cse.buffalo.edu/~jcorso/t/CSE555/files/lecture_hmm.pdf"

and it is

"Expected frequency in state S_i at time $k=1$: $\gamma_1(i)$"

So, $\pi_i = P(q_1 = S_i \mid O_1 O_2 \cdots O_k)$

$= \Gamma_1(i)$

论文链接：https://cse.buffalo.edu/~jcorso/t/CSE555/files/lecture_hmm.pdf