

1.2.6 Binomial Coefficients

February 10, 2021

Definition

$$\binom{n}{k} = \frac{n!}{k!(n-k)!} = \frac{n(n-1)\dots(n-k+1)}{k(k-1)\dots(1)}.$$

For example,

$$\binom{5}{3} = \frac{5 \cdot 4 \cdot 3}{3 \cdot 2 \cdot 1} = 10,$$

Note how

$$\frac{n!}{(n-k)!} = n(n-1)\dots(n-k+1)$$

For example, when $n = 6$ and $k = 3$,

$$6 \cdot 5 \cdot 4 = \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1} = \frac{n!}{(n-k)!}$$

The quantity $\binom{n}{k}$, read " n choose k ", is a *binomial coefficient*

Operating on Binomial Coefficients

A. Representation by factorials.

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}, \text{ integer } n \geq \text{integer } k \geq 0$$

B. Symmetry condition.

$$\binom{n}{k} = \binom{n}{n-k}, \text{ integer } n \geq \text{integer } k.$$

| n | k=0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|-----|---|----|----|-----|-----|----|----|---|---|
| 8 | 1 | 8 | 28 | 56 | 70 | 56 | 28 | 8 | 1 | 0 |
| 9 | 1 | 9 | 36 | 84 | 126 | 126 | 84 | 36 | 9 | 1 |

notice the symmetry about 4 and 4.5 for n=8 n=9 respectively

- C. Moving in and out of parentheses.
- D. Addition formula.
- E. Summation formulas.
- F. The binomial theorem.
- G. Negating the upper index.
- H. Simplifying products.
- I. Sums of products.