1.2.6 Binomial Coefficients

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Definition

$$\binom{n}{k} = \frac{n!}{k!(n-k)!} = \frac{n(n-1)...(n-k+1)}{k(k-1)...(1)}.$$

For example,

$$\binom{5}{3} = \frac{5 \cdot 4 \cdot 3}{3 \cdot 2 \cdot 1} = 10,$$

Note how

$$\frac{n!}{(n-k)!} = n(n-1)...(n-k+1)$$

For example, when $n = 6$ and $k = 3$,

$$6 \cdot 5 \cdot 4 = \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1} = \frac{n!}{(n-k)!}$$

The quantity $\binom{n}{k}$, read *"n choose k"*, is a *binomial coefficient*

Operating on Binomial Coefficients

A. Representation by factorials.

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}, \text{ integer } n \leq \text{ integer } k \geq 0$$

B. Symmetry condition.

$$\binom{n}{k} = \binom{n}{n-k}, \text{ integer } n \leq \text{ integer } k.$$

for example:

n	k=0	1	2	3	4	5	6	7	8	9
8	1	8	28	56	70	56	28	8	1	0
9	1	9	36	84	126	126	84	36	9	1

notice the symetry about 4 and 4.5 for n=8 n=9 repsectively

C. Moving in and out of parentheses.

$$\binom{r}{k} = \frac{r}{k}\binom{r-1}{k-1}, \text{ integer } k \neq 0.$$

This makes since because $r! = r(r-1)!$, so the term $n!/k!$ found in subsection A is equal to $\frac{n}{k}\frac{(n-1)!}{(k-1)!}$

D. Addition formula.

E. Summation formulas.

F. The binomial theorem.

G. Negating the upper index.

H. Simplifying products.

I. Sums of products.