1.2.6 Binomial Coefficients

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Definition

$$\binom{n}{k} = \frac{n!}{k!(n-k)!} = \frac{n(n-1)...(n-k+1)}{k(k-1)...(1)}.$$

For example,

$$\binom{5}{3} = \frac{5 \cdot 4 \cdot 3}{3 \cdot 2 \cdot 1} = 10,$$

Note how

$$\frac{n!}{(n-k)!} = n(n-1)...(n-k+1)$$

For example, when $n = 6$ and $k = 3$,

$$6 \cdot 5 \cdot 4 = \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1} = \frac{n!}{(n-k)!}$$

The quantity $\binom{n}{k}$, read "n choose k", is a *binomial coefficient*

Operating on Binomial Coefficients

A. Representation by factorials.

B. Symmetry condition.

C. Moving in and out of parentheses.

D. Addition formula.

E. Summation formulas.

F. The binomial theorem.

G. Negating the upper index.

H. Simplifying products.

I. Sums of products.