Fall 2020
EE 236A
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EE236A Linear Programming
Quiz 3
Tuesday November 10, 2020

NAME: ____________________ **UID**: ____________________

This quiz has 3 questions, for a total of 20 points.

Open book. The exam is for a total of 1:00 hour. **Please, write your name and UID on the top of each sheet.**

Good luck!

Problem	Mark	Total
P1		6
P2		7
P3		7
Total		20

Problem 1 (`6 points`) Prove that the optimization problem in (1) is not feasible.

$$
\begin{aligned}
\min_{x_1,x_2,x_3,x_4,x_5} \quad & \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} \\
\text{subject to} \quad & \begin{bmatrix} 1 & 2 & 0 & 3 & 1 \\ 0 & 2 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 1 \\ 5 \\ 2 \end{bmatrix} \\
& x_i \geq 0 \text{ for i } = 1, \ldots, 5
\end{aligned} \tag{1}
$$

Problem 2 (`7 points`) Show that any $k+2$ points in $\mathbb{R}^k$ can be partitioned into two groups: $(v_i), i \in I$, and $(v_j), j \notin I$, whose convex hulls intersect.

(*Hint:* Argue that the vectors $\begin{pmatrix} v_1 \\ 1 \end{pmatrix}, \ldots, \begin{pmatrix} v_{k+2} \\ 1 \end{pmatrix}$ are linearly dependent, and thus there exists a set of not-all-zero coefficients $a_1, \ldots, a_{k+2}$ such that: $\sum_{i=1}^{k+2} a_i \begin{pmatrix} v_i \\ 1 \end{pmatrix} = \mathbf{0}$.)

Problem 3 (`7 points`): Assume we are given an LP in the standard form, with A an $m \times n$ matrix:

$$\begin{aligned} \min_{x \in \mathbb{R}^n} \quad & c^T x \\ \text{subject to} \quad & Ax = b \\ & x \geq 0 \end{aligned} \tag{2}$$

Recall that, as we derived in class, the associate dual program can be expressed as:

$$\begin{aligned} \max_{y} \quad & -b^T y \\ \text{subject to} \quad & A^T y + c \geq 0 \end{aligned} \tag{3}$$

Assume now that the primal LP has a very large number of variables (i.e. n is very large). A friend of yours claims that we can use the following trick to solve this LP more efficiently: She claims that we can consider a subset of $n_I << n$ variables, say all variables x_i with $i \in I$, where I is a set of indices, and solve instead the following LP:

$$\begin{aligned} \min_{x_I \in \mathbb{R}^{n_I}} \quad & c_I^T x_I \\ \text{subject to} \quad & A_I x_I = b \\ & x_I \geq 0 \end{aligned} \tag{4}$$

where c_I only keeps the values corresponding to the indices in I, and A_I keeps the columns of A corresponding to the indices in I. Assume the optimal solution to the problem in (4) is x_I^* and the corresponding dual optimal variables are y_I^*. Your friend claims that for the dual optimal variables, if $c_i + a_i^T y_I^* \geq 0$ for all $i = 1, \ldots, n$, where a_i is the i^{th} column vector of A, then you can directly find from x_I^* the optimal solution for the problem in (2) x^*. Is your friend right or wrong? Prove your claim.