Quiz 1. Shengze Ye 20541895 9

problem 1

$\{a_1, a_2 \cdots a_n\}$ $0 < a_1 < a_2 < \cdots < a_n$,

$Pr(X = a_i) = p_i$ $\sum_{i=1}^{n} p_i = 1$

we want maximize $\mathbb{E}X$

subject to $Pr(X \geq \alpha) = b$

$a_1 < \alpha < a_n$ and $0 \leq b \leq 1$

$\mathbb{E}(X) = \sum_{i=1}^{n} a_i p_i$

$Pr(X \geq \alpha) = p_j + p_{j+1} + \cdots + p_n = b$

where $a_j \geq \alpha$ and $a_{j-1} < \alpha$

$\therefore$ the problem could be formulated as

$$\Rightarrow \text{maximize} \quad \sum_{i=1}^{n} a_i p_i$$

$$\text{s.t.} \quad \sum_{i=1}^{n} p_i = 1$$

$$p_j + p_{j+1} + \cdots + p_n = b$$

$$\alpha \leq a_j$$

$$\alpha > a_{j-1}$$

problem 2:

$x = [x_1, x_2, \cdots x_n]$ $\quad z = [z_1, z_2 \cdots z_n]$

$$\text{minimize} \quad \|\alpha x\|_2^2 - \|z\|_1$$
$$\text{s.t} \quad \max_i x_i^2 \le \beta$$
$$-1 \le z_i \le 1$$

$$\|\alpha x\|_2^2 = \alpha^2 x^T x = \alpha^2 \sum_{i=1}^{n} x_i^2$$
$$\|z\|_1 = \sum_{i=1}^{n} |z_i|$$

let $t_i = x_i^2$ $\quad t = (t_1, t_2 \cdots t_n)$

$$\therefore \Rightarrow \text{minimize} \quad \alpha^2 \sum_{i=1}^{n} t_i - \sum_{i=1}^{n} |z_i|$$
$$\text{s.t} \quad t_i \le \beta \qquad \text{for all } i = 1, \cdots, n$$
$$-1 \le z_i \le 1$$
$$t_i \ge 0$$

let $p_i = |z_i|$ $\quad p = (p_1, \cdots p_n)$

The problem could be formulated as

$$\therefore \Rightarrow \text{minimize} \quad \alpha^2 \sum_{i=1}^{n} t_i - \sum_{i=1}^{n} p_i$$
$$t_i \le \beta$$
$$t_i \ge 0$$
$$p_i \ge 0 \qquad \text{for } i = 1, 2, \cdots n$$
$$p_i \le 1$$

we can Solve the problem

we want to minimize it, so we need $\alpha^2 \sum_{i=1}^{n} t_i$ as small as possible

and make $\sum_{i=1}^{n} p_i$ as large as possible

$\because t_i = x_i^2 \geq 0$ $\therefore$ the minimized value for

$$\alpha^2 \sum_{i=1}^{n} t_i = 0$$

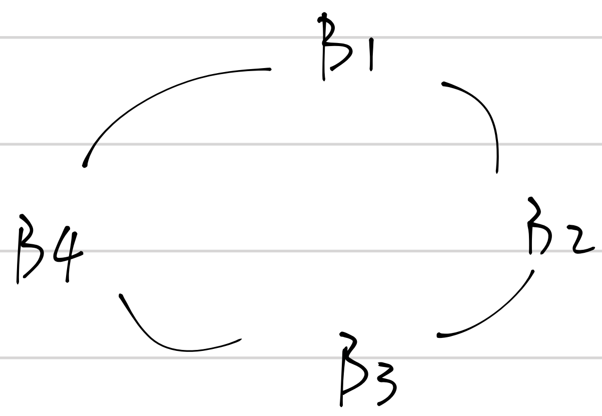
$\because p_i = |z_i| \geq 0$ and $p_i = |z_i| \leq 1$

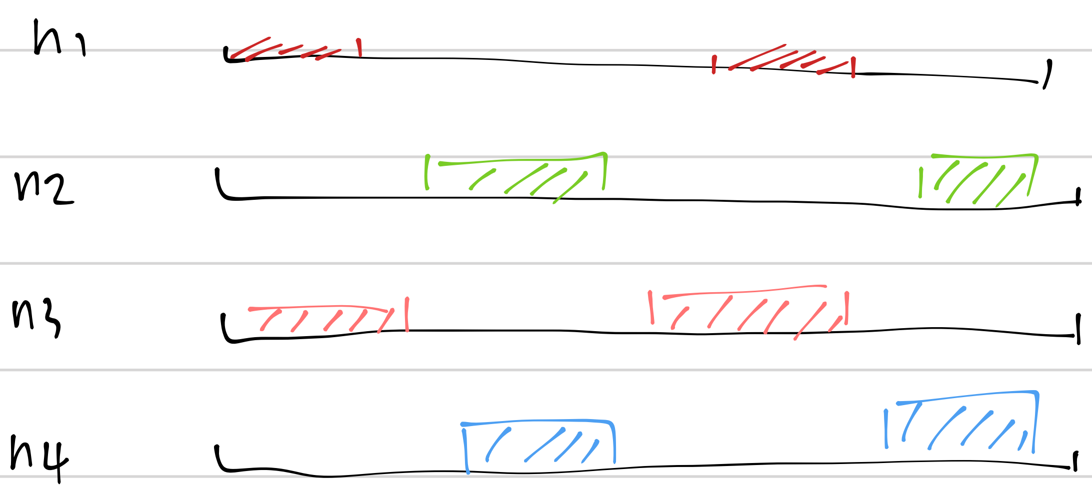
$\therefore$ we can set $z_i = \pm 1$ for $i=1, \cdots n$

$\therefore$ $\|z\|_1 = n$

The minimized value is $0-n = -n$

problem 3 Formulat as LP





unit time

let k_1, k_2, k_3, k_4 denote the # time slots that n_1, n_2, n_3, n_4 has

le $t_{n_i j s}$ denote the start time of the jth time slot for n_i station

$t_{n_i j e}$ denote the end time of the jth time slot for n_i station

for n_i, $j = 1, 2, \dots k_i$

$\therefore$ For each station, the total time assign to it is $\sum_{j=1}^{k_i} (t_{n_i j e} - t_{n_i j s})$

$\therefore$ we want to maximize

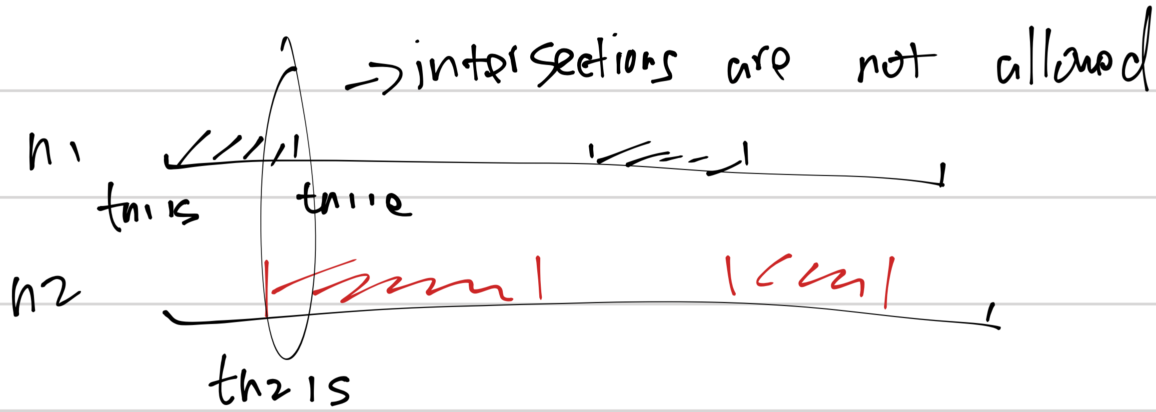
$$\sum_{i=1}^{4} r_i \cdot \sum_{j=1}^{k_i} (t_{n_i j e} - t_{n_i j s})$$

The constraints are

$t_{n_i j e} \geq t_{n_i j s}$ for any i, j

$\sum_{j=1}^{k_i} (t_{n_i j e} - t_{n_i j s}) \geq \frac{1}{8}$

(transmit at least $\frac{1}{8}$ each)



for any j, k $(t_{n_i j s} - t_{n_{i'} k s})(t_{n_{i'} k s} - t_{n_i j s}) \geq 0$

$i = 1$ $\quad i' = 2, 4$

$i = 2$ $\quad i' = 1, 3$

$i = 3$ $\quad i' = 2, 4$

$i = 4$ $\quad i' = 1, 3$

solution: if $r_1+r_3 \geq r_2+r_4$

we assign $\frac{7}{8}$ unit time to r_1, r_3

assign $\frac{1}{8}$ unit time to r_2, r_4 (1)

if $r_1+r_3 < r_2+r_4$

we assign $\frac{7}{8}$ unit time to r_2, r_4

assign $\frac{1}{8}$ unit time to r_1, r_3 (2)

for (1) optimal value:

$$(r_1+r_3)\cdot\frac{7}{8}+(r_2+r_4)\cdot\frac{1}{8}$$

for (2) optimal value:

$$(r_1+r_3)\cdot\frac{1}{8}+(r_2+r_4)\cdot\frac{7}{8}$$