Fall 2020
EE 236A
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EE236A Linear Programming
Solutions of Quiz 1
Tuesday October 13, 2020

NAME: ___________________ **UID**: ___________________

This quiz has 3 questions, for a total of 20 points.

Open book.
The exam is for a total of 1:00 hour. **Please, write your name and UID on the top of each sheet.**

Good luck!

Problem	Mark	Total
P1		6
P2		7
P3		7
Total		20

Problem 1 (`6 points`) Let x be a real-valued random variable which takes values in $\{a_1, a_2, \ldots, a_n\}$ where $0 < a_1 < a_2 < \cdots < a_n$, and $\Pr(x = a_i) = p_i$.

Consider the problem of determining the probability distribution that maximizes the expected value $\mathbf{E}x$ subject to the constraint that $\Pr(x \geq \alpha) = b$, i.e.,

$$\begin{array}{ll} \text{maximize} & \mathbf{E}x \\ \text{subject to} & \Pr(x \geq \alpha) = b \end{array} \tag{1}$$

where α and b are given ($a_1 < \alpha < a_n$, and $0 \leq b \leq 1$).Write (1) as an LP.

Solution: Given $a_i's$ and α select $k = \text{argmin}_i\{a_i : a_i \geq \alpha\}$. Then problem can be formulated as an LP as follows:

$$\begin{array}{ll} \text{maximize} & p^T a \\ \text{subject to} & \mathbf{1}^T p = 1 \\ & \sum_{i=k}^n p_i = b \\ & p_i \geq 0, \quad \forall i = 1, \ldots, n \end{array} \tag{2}$$

Problem 2 (`7 points`): Can the following problem be expressed using an LP? Explain your approach or why not possible. Consider the n dimensional real vectors $x = [x_1, x_2, \ldots x_n]$ and $z = [z_1, z_2, \ldots z_n]$, we want to

$$\begin{array}{ll} \text{minimize} & ||\alpha x||_2^2 - ||z||_1 \\ \text{subject to} & \max_i x_i^2 \leq \beta, \quad i = 1 \ldots n \\ & -1 \leq z_i \leq 1, \quad i = 1 \ldots n \end{array} \tag{3}$$

where α and β are given real nonnegative constants.

Solution: First, we can change variables x_i^2 to t_i. Also note that $max_i x_i^2 \leq \beta$ implies that $x_i^2 \leq \beta$. As a result we would have $0 \leq t_i \leq \beta$. Then we can note that in the optimal solution z_i's would be positive so we can get rid of the l_1-norm and use lower bound $0 \leq z_i$. Another equivalent way is to define a new variable $0 \leq k_i \leq 1$ and use it instead of $|z_i|$. In the end we can solve the following LP :

$$\begin{array}{ll} \text{minimize} & \alpha^2 \sum_{i=1}^n t_i - \sum_{i=1}^n z_i \\ \text{subject to} & 0 \leq z_i \leq 1, \quad i = 1 \ldots n \\ & 0 \leq t_i \leq \beta, \quad i = 1 \ldots n \end{array} \tag{4}$$

Problem 3 (`7 points`) Formulate the following problem as an LP. Four wireless basestations n_1, n_2, n_3 and n_4 are placed on the circumference of a circle, as depicted in Figure 1. When node i transmits, the two nodes closest to it cannot transmit, because they would cause interference. For example, when basestation 1 transmits, basestations 2 and 4 cannot transmit. Each basestation i transmits at a rate of r_i (the rates r_i are given constants) per time unit; moreover, each basestation needs to

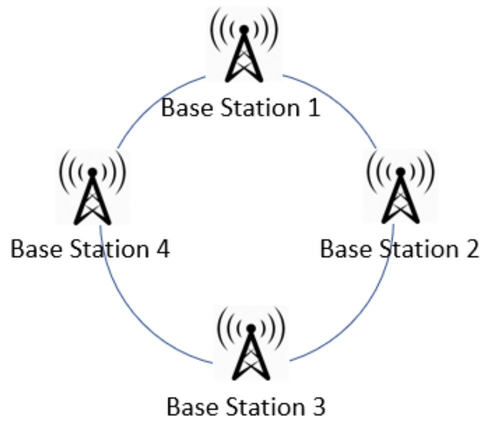

Figure 1: Wireless basestations positioned on the circumference of a circle

transmit for at least 1/8 of each time unit. Write an LP that maximizes the total amount of rate, transmitted from all four basestations during a time unit.

Solution: Let t_i $(i = 1, 2, 3, 4)$ be the fraction of unit time during which basestation i transmits. Then the LP can be formulated as follows.

$$\begin{array}{ll} \text{maximize} & \sum_{i=1}^{4} t_i r_i \\ \text{subject to} & t_i \geq \frac{1}{8}, \quad i = 1, 2, 3, 4 \\ & t_1 + t_2 \leq 1 \\ & t_1 + t_4 \leq 1 \\ & t_2 + t_3 \leq 1 \\ & t_3 + t_4 \leq 1 \end{array} \tag{5}$$

Since $t_1 = t_3$ and $t_2 = t_4$ in the optimal solution, we can also formulate the problem as follows.

$$\begin{array}{ll} \text{maximize} & t_1(r_1 + r_3) + t_2(r_2 + r_4) \\ \text{subject to} & t_i \geq \frac{1}{8}, \quad i = 1, 2 \\ & t_1 + t_2 \leq 1 \end{array} \tag{6}$$

The solution of the LP can be found as follows.

If $r_1 + r_3 \geq r_2 + r_4$, then $t_1^\star = t_3^\star = \frac{7}{8}$ and $t_2^\star = t_4^\star = \frac{1}{8}$.

If $r_1 + r_3 < r_2 + r_4$, then $t_1^\star = t_3^\star = \frac{1}{8}$ and $t_2^\star = t_4^\star = \frac{7}{8}$.