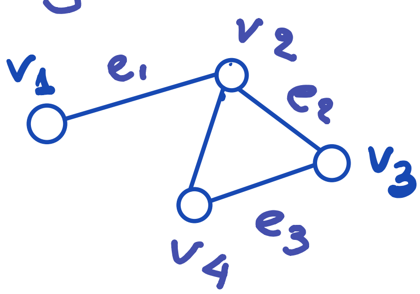
Lecture 11

Today: start module on combinatorial optimization and integerer programming

$$\begin{aligned} \min\ & c^T x \\ \text{st}\ & Ax \leq b \\ & x \in \mathbb{Z}^n \end{aligned}$$

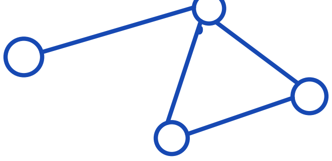
Graph Theory Notation

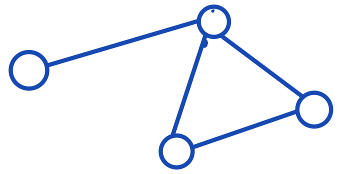
$G = (V, E)$



undirected:

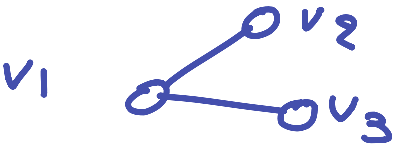
directed:





Undirected graphs

degree of a vertex =



Simple graph:

self loop:

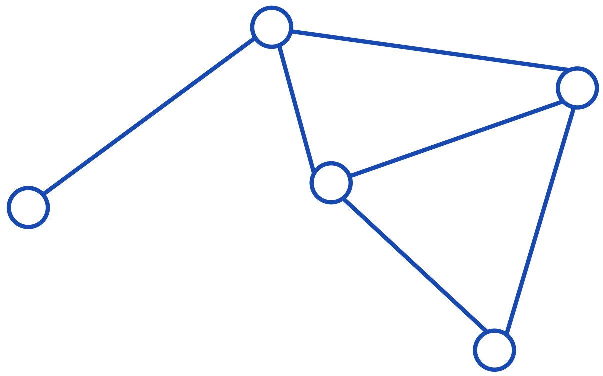
parallel edges:

Consider two graphs
$G=(V,E)$ and $G'=(V',E')$, we say

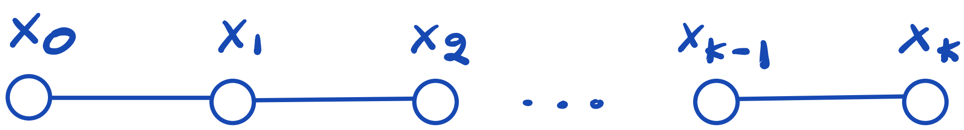
1) if $V' \subseteq V$, and $E' \subseteq E$

2) if $V'=V$, $E' \subseteq E$,

3)

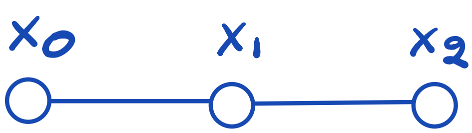


<u>Path</u>:

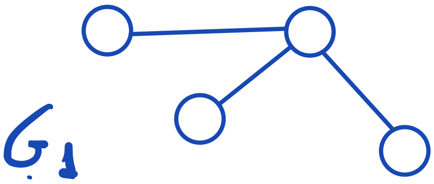


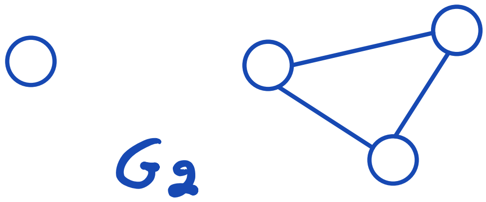
length of a path =

<u>Cycle</u> :



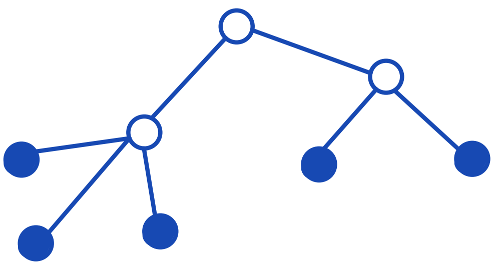
Connected graph:





Acyclic graph:

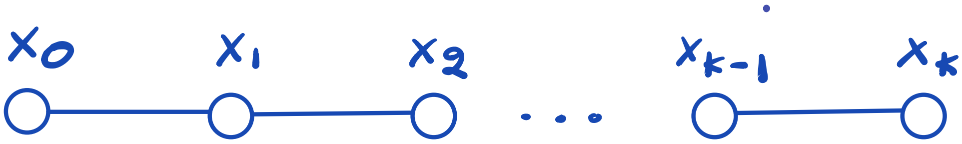
Connected acyclic graph

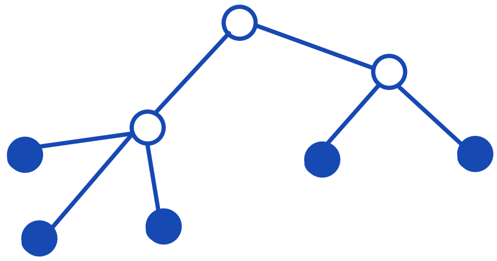


Trees :

1) Each tree has at least two leaves.

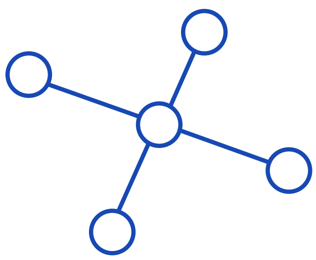
Path:





2) If T is a tree, any two vertices are connected by exactly one path.

3) A tree with n vertices has always $n-1$ edges.



Proof

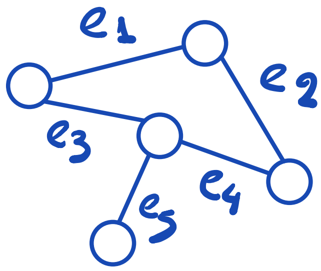
Cut

)

"value" of the cut:

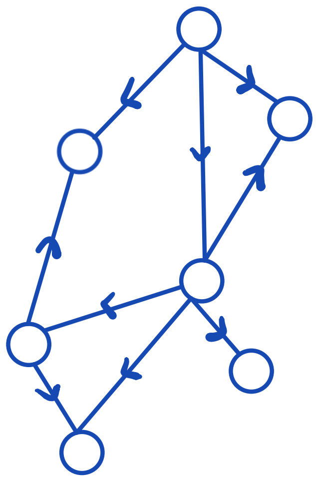
Capacity associated with every edge:

$c: E \to \mathbb{R}^+$



For capacitated graphs
cut value:

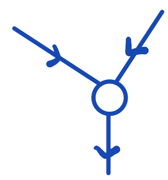
Directed graphs:

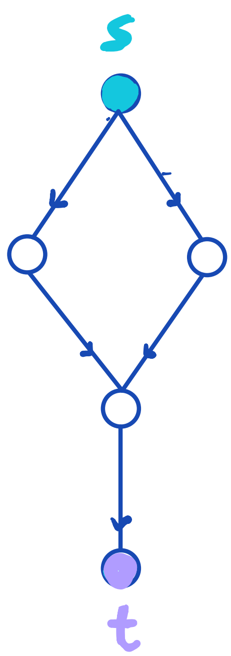


Min-cut max-flow theorem

LP formulation for max-flow

Consider a directed graph $G=(V,E)$, with capacited edges, and two distinct nodes s and t.

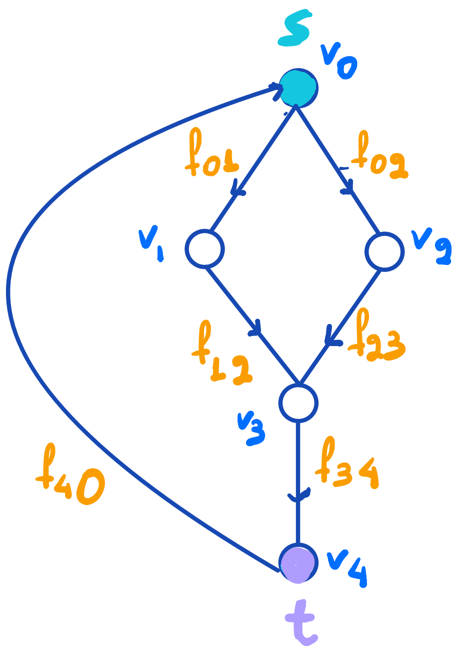




variables:

objective function:

In our example



max flow problem in vector notation