Lecture 9

Duality

Primal	Dual
$\min c^T x$	$\max -b^T \lambda$
st $Ax \le b$	st $A^T \lambda = -c$

1) Weak duality

2) Strong duality

3) Complementary slackness (and KKT conditions)

Primal		Dual
$\min\ c^T x$ s.t. $Ax \leq b$	$\longleftrightarrow$	$\max\ -b^T\lambda$ s.t. $A^T\lambda + c = 0$ $\lambda \geq 0$

Primal	Dual
$\min\ c^T x$ s.t. $Ax \leq b$ $Gx = d$	$\max\ -b^T z - d^T y$ s.t. $A^T z + G^T z + c = 0$ $z \geq 0$

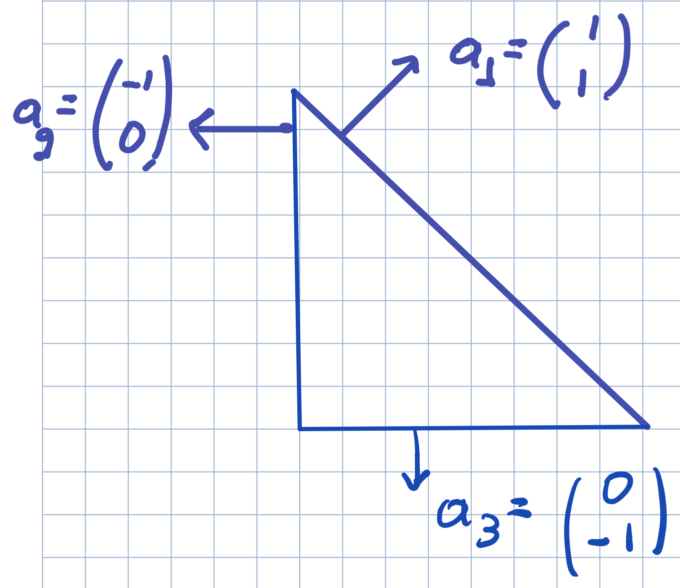
primal

$$\min \; c^T x$$
$$\text{st} \quad Ax = b$$
$$x \geq 0$$

dual

$$\max \; -b^T \lambda_1$$
$$\text{st} \quad A^T \lambda_1 + c = \lambda_2$$
$$\lambda_2 \geq 0$$

Geometric Interpretation.



primal

$$\min \begin{pmatrix} 1 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

s.t.

$$\begin{array}{c} c_1 \\ c_2 \\ c_3 \end{array} \begin{pmatrix} 1 & 1 \\ -1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \leq \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

dual

$$\max -\lambda_1$$

s.t. $\begin{pmatrix} 1 \\ -1 \end{pmatrix} + \begin{pmatrix} \lambda_1 - \lambda_2 \\ \lambda_1 + \lambda_3 \end{pmatrix} = 0$

$\lambda_1, \lambda_2, \lambda_3 \geq 0$

If I find that a vertex x^* of the primal (non-degenerate) is optimal, how can I tell that this is a unique solution or n-?

Example 1

- Express as an LP:

$$\min \|Ax - b\|_\infty, \qquad A \quad m \times n$$

$$x \in \mathbb{R}^n$$

- Find the dual and show it is equivalent to the problem:

$$\max -b^T z$$
$$\text{st} \quad A^T z = 0$$
$$\|z\|_1 \le 1$$

$$\max \quad -b^T z$$
$$\text{st}$$
$$A^T z = 0$$
$$\|z\|_1 \le 1$$

Proof of equivalence

We will show that for each feasible solution of one we ca
find a feasible solution of the other that achieves same obj. valu