Fall 2020
EE 236A
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EE236A Linear Programming
Quiz 2
Tuesday October 27, 2020

NAME: ____________________ **UID**: ____________________

This quiz has 3 questions, for a total of 20 points.

Open book.
The exam is for a total of 1:00 hour. **Please, write your name and UID on the top of each sheet.**

Good luck!

Problem	Mark	Total
P1		6
P2		7
P3		7
Total		20

Problem 1 (`6 points`) The following two questions are not related to each other.

1) (3 points) Consider the set

$$C = \{x \in \mathbb{R}^n \mid \|x\|_\infty \leq n\} \tag{1}$$

Argue that this set is a pointed polyhedron in $\mathbb{R}^n$, by expressing it through a set of linear inequalities in $\mathbb{R}^n$. Find what form the vertices of this polyhedron have. How many vertices are there?

2) (3 points) Is the point $x_0 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ a vertex in the polyhedron P described next?

$$P = \left\{ x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \in \mathbb{R}^3 \mid x_i \geq 0, \quad x_1 + x_2 \geq 2, \quad x_1 + x_3 \geq 2, \quad 3x_1 + 2x_2 + x_3 \geq 6 \right\} \tag{2}$$

Problem 2 (`7 points`): Which of the following statements are true and which are false? Give a brief justification (answers without justification do not get points).

- (i) (2 points) Consider m points in $\mathbb{R}^n$ with $m > n$. The convex hull of these points is a polyhedron with exactly m vertices.
- (ii) (2 points) The minimal face of a polyhedron is always a single point.
- (iii) (3 points) Let $P \subseteq \mathbb{R}^{\mathrm{d}}$ and $Q \subset \mathbb{R}^e$ be polytopes. Then the following set $\mathbb{S} \subseteq \mathbb{R}^{d+e+1}$ is also a polytope, where:

$$\mathbb{S} = \left\{ z \in \mathbb{R}^{d+e+1} | z = \begin{pmatrix} x \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ y \\ 1 \end{pmatrix}, \text{with } x \in P \text{ and } y \in Q \right\} \tag{3}$$

Problem 3 (`7 points`) Assume $x_1, x_2, y_1, y_2, z \in \mathbb{R}^n$, A is a given $m \times n$ matrix, λ is a constant and c is a given column vector of dimension $n \times 1$. Prove that the following two programs, (4) and (5), are equivalent.

$$\begin{array}{rl} \min_{x_1,x_2,y_1,y_2} & -c^T(x_1 - x_2) \\ \text{subject to} & A(x_1 - x_2) + y_1 - y_2 = 0 \\ & y_1 + y_2 = \lambda \mathbf{1} \\ & x_1 \geq 0, x_2 \geq 0, y_1 \geq 0, y_2 \geq 0 \end{array} \tag{4}$$

$$\begin{array}{rl} \max_z & c^T z \\ \text{subject to} & \|Az\|_\infty \leq \lambda \end{array} \tag{5}$$