Inside Class 6

Problem 1

Are the following sets convex, affine, subspaces?

1) $S = \{ x \mid x = \underset{\substack{\downarrow \\ n \times k}}{A} y + c, \text{ for some } y \in \mathbb{R}^k \text{ and } c \text{ constant} \}$

2) The set $\{ x \mid x + S_2 \subseteq S_1 \}$

where $S_1, S_2 \in \mathbb{R}^n$, with S_1 convex

Problem 2

1) Given $P = \{ x \in \mathbb{R}^2 \mid x_1 \geq 0,\ 2x_1 + x_2 \leq 3,\ x_2 \geq 0,\ x_1 + 2x_2 \leq 3 \}$

Is the point $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ a vertex?

2) $\mathbb{P} = \{ x \mid x \geq 0,\ \underset{m \times n}{C}\, x = d \}$ $\quad x \in \mathbb{R}^n$

Prove that the vertices of $\mathbb{P}$ (if they exist) have at least $n-m$ zero elements

3) How many vertices can the polyhedron

$$P = \{ x \mid \underset{m \times n}{A}\, x \leq b \}$$ have?

4) How many vertices does the polyhedron

$$S = \{ x \mid 0 \leq x \leq d \}$$ have?

Problem 3

Prove that, for $x \in R^n$, if the function $f(x)$ is a convex function, then the set $C = \{x | f(x) \leq b\}$ is a convex set, with $b \in R$ a given constant.

Can you find the solution to the following problem (call this P1), by solving an LP?

$$\begin{aligned} \text{minimize}_x \quad & ||x||_1^2 + 2||x||_1 \\ \text{subject to} \quad & Ax = b, \end{aligned} \tag{1}$$

where $x \in R^n$, A is an $m \times n$ matrix and $b \in R^m$. If yes, explain which LP you can solve, if not, explain why.