Fall 2020
EE 236A
Christina Fragouli
TAs: Mine Dogan and Kaan Ozkara

Linear Programming

Homework 5
Due: 9 AM, Dec. 4, 2020

Problem 1 (`3 points`): Consider a graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$. A *matching* on $\mathcal{G}$ is a collection of $\mathcal{M} \subseteq \mathcal{E}$ such that, no two edges in $\mathcal{M}$ share a vertex. In other words, each vertex in $\mathcal{V}$ has at most one connected edge in $\mathcal{M}$. A *maximal matching* is a matching $\mathcal{M}$ such that, if any other edge in $\mathcal{E} \setminus \mathcal{M}$ is added to $\mathcal{M}$ it no longer becomes a valid matching.

Assume that you have an algorithm that takes as input an arbitrary graph $\mathcal{G}$, and outputs a maximal matching $\mathcal{M}$. Propose a heuristic that takes as input $\mathcal{G}$ and $\mathcal{M}$, and outputs a valid vertex cover $\mathcal{C}$ (a cover is a subset of vertices $\mathcal{C} \subseteq \mathcal{V}$ such that each edge in $\mathcal{E}$ is incident to at least one vertex in $\mathcal{C}$). The size of the output vertex cover should be within an approximation factor of 2.

Problem 2 (`6 points`):

(a) Use the simplex procedure to solve the following problem

$$\begin{array}{ll} \text{minimize} & z = x - y \\ \text{subject to} & -x + y \geq -4 \\ & -x - y \geq -6 \\ & x, y \geq 0. \end{array}$$

(b) Draw a graphical representation of the problem in X-Y space and indicate the path of the simplex steps.

(c) Repeat the problem above but using the new objective function $z = -x + y$. This problem has multiple solutions, so find all the vertex solutions and write down an expression for the full set of solutions.

(d) Solve the following problem, and graph the path followed by the simplex method:

$$\begin{array}{ll} \text{minimize} & z = -x - y \\ \text{subject to} & 2x - y \geq -2 \\ & -x + y \geq -1 \\ & x, y \geq 0. \end{array}$$

Problem 3 (`4 points`):

Consider the following LP:

$$\begin{array}{ll} \text{minimize} & z = x_1 - x_2 \\ \text{subject to} & 0 \leq x_i \leq \frac{1}{2}, \quad i = 1,2,3 \\ & \sum_{i=1}^{3} x_i = 1 \end{array}$$

Given an initial feasible point $(1/2, 1/2, 0)$, use the simplex method to find an optimal solution to this LP.

Problem 4 (`4 points`):

(a) Demonstrate that

$$\begin{array}{ll} \text{minimize} & z = -3x_1 + 4x_2 \\ \text{subject to} & -x_1 - x_2 \geq -1 \\ & -2x_1 + x_2 \geq 2 \\ & x_1, x_2 \geq 0. \end{array}$$

is infeasible using the Phase I procedure.

(b) Demonstrate that

$$\begin{array}{ll} \text{minimize} & z = -2x_1 + x_2 \\ \text{subject to} & 2x_1 - x_2 \geq 1 \\ & x_1 + 2x_2 \geq 2 \\ & x_1, x_2 \geq 0. \end{array}$$

is unbounded using the Phase I procedure.

Problem 5 (`3 points`): Solve the following linear program using the simplex algorithm with Bland's pivoting rule. Start the algorithm at the extreme point $x = (2, 2, 0)$, with active set $I = \{3, 4, 5\}$.

$$\begin{array}{ll} \text{minimize} & x_1 + x_2 - x_3 \\ \text{subject to} & \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \leq \begin{bmatrix} 0 \\ 0 \\ 0 \\ 2 \\ 2 \\ 2 \\ 4 \end{bmatrix}. \end{array}$$