(6)

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① If $t_1(n) \in O(g_1(n))$ and $t_2(n) \in O(g_2(n))$, then $t_1(n)+t_2(n) \in O(\max\{g_1(n), g_2(n)\})$. Prove that assertions.

Sol:- Proof: The proof extends to orders of growth the following simple fact about four arbitrary real numbers $a_1, b_1, a_2, b_2$: if $a_1 \le b_1$ and $a_2 \le b_2$, then $a_1+a_2 \le 2\max\{b_1, b_2\}$.

Since $t_1(n) \in O(g_1(n))$, there exist some positive constant $c_1$ and some nonnegative integer $n_1$ such that

$$t_1(n) \le c_1 g_1(n) \text{ for all } n \ge n_1.$$

Similarly, since $t_2(n) \in O(g_2(n))$,

$$t_2(n) \le c_2 g_2(n) \text{ for all } n \ge n_2.$$

Let us denote $c_3 = \max\{c_1, c_2\}$ and consider $n \ge \max\{n_1, n_2\}$ so that we can use both inequalities. Adding them yields the following:

$$\begin{aligned} t_1(n)+t_2(n) &\le c_1 g_1(n) + c_2 g_2(n) \\ &\le c_3 g_1(n) + c_3 g_2(n) \\ &= c_3[g_1(n)+g_2(n)] \\ &\le c_3\, 2\max\{g_1(n), g_2(n)\}. \end{aligned}$$

Hence, $t_1(n)+t_2(n) \in O(\max\{g_1(n), g_2(n)\})$, with the constants $c$ and $n_0$ required by the definition $O$ being $2c_3 = 2\max\{c_1, c_2\}$ and $\max\{n_1, n_2\}$.

The property implies that the algorithm's overall efficiency will be determined by the part with a higher order of growth, i.e., its least efficient part.

$\therefore t_1(n) \in O(g_1(n))$ and $t_2(n) \in O(g_2(n))$, then

$$t_1(n) + t_2(n) \in O(\max\{g_1(n), g_2(n)\}).$$

② Find the Time Complexity of the below recurrence equation:

③ $T(n) = \begin{cases} 2T(n/2)+1 & \text{if } n>1 \\ 1 & \text{otherwise} \end{cases}$

④ $T(n) = \begin{cases} 2T(n-1) & \text{if } n>0 \\ 1 & \text{otherwise} \end{cases}$

Sol:-

$$T(n) = \begin{cases} 2T(n/2)+1 & \text{if } n>1 \\ 1 & \text{otherwise} \end{cases}$$

$T(n) = aT\left(\frac{n}{b}\right) + f(n)$ [∵ from Master's theorem]

Here, $a = 2$ | $f(n) = 1$

$b = 2$ | $K = 0$

$\log_b a = \log_2 2 = 1 > K$

$\log_b a > K$ then

$$T(n) = O(n^{\log_b a})$$
$$= O(n^{\log_2 2}) = O(n^1) = O(n).$$

$$T(n) = 2^n T(n-n)$$
$$= 2^n T(0) \quad [\because T(0) = 1]$$
$$T(n) = 2^n$$

$\therefore$ The time Complexity is $\boxed{T(n) = O(2^n)}$

⑤ Big O Notation: Show that $f(n) = n^2 + 3n + 5$ is $O(n^2)$.

Sol:- Given $f(n) = n^2 + 3n + 5$

A function $f(n)$ is $O(g(n))$ if there exist constants $c > 0$ and $n_0$ such that for all $n \geq n_0$:

$$f(n) \leq c \cdot g(n)$$

$n^2 + 3n + 5 \leq c \cdot n^2$ [$\because$ Divide both sides with "$n^2$"]

$$\Rightarrow 1 + \frac{3}{n} + \frac{5}{n^2} \leq c$$

Let $c = 2$, then

$$\Rightarrow 1 + \frac{3}{n} + \frac{5}{n^2} \leq 2$$

$$\Rightarrow \frac{3}{n} + \frac{5}{n^2} \leq 2 - 1$$

$$\Rightarrow \frac{3}{n} + \frac{5}{n^2} \leq 1$$

$$\Rightarrow \frac{3}{n} + \frac{5}{n^2} \leq \frac{1}{2} + \frac{1}{2}$$

$$\frac{3}{n} \leq \frac{1}{2} \quad \& \quad \frac{5}{n^2} \leq \frac{1}{2}$$

$$T(n) = \begin{cases} 2T(n-1) & \text{if } n>0 \\ 1 & \text{otherwise} \end{cases}$$

To solve this problem, we will use iteration method.

$$T(n) = 2T(n-1) \quad \text{--- } ①$$

$n = n-1 \Rightarrow ①$

$$T(n-1) = 2 \cdot T((n-1)-1)$$
$$= 2T(n-2) \quad \text{--- } ②$$

② in ①

$$T(n) = 2(2T(n-2))$$
$$= 2^2 T(n-2) \quad \text{--- } ③$$

$\cancel{n = n-2 \Rightarrow}$

$\cancel{T(n-3) = 2T(n-3-1)}$

$\cancel{=}$

$n = n-2 \Rightarrow ①$

$$T(n-2) = 2T(n-2-1)$$
$$= 2T(n-3) \quad \text{--- } ④$$

④ in ③

$$T(n) = 2^2[2T(n-3)]$$
$$= 2^3 T(n-3) \quad \text{--- } ⑤$$

Continuing this process;

$$T(n) = 2^k T(n-k)$$

$$n - k = 0$$
$$\Rightarrow k = n$$

$n \geq 6$ & $n^2 \geq 10$

$n \geq \sqrt{10} \approx 3.16$

$\therefore \boxed{n = 6}$

$1 + \frac{3}{n} + \frac{5}{n^2} \leq 1 + \frac{1}{2} + \frac{1}{2} = 2$

Thus, for $\boxed{c = 2}$

$n^2 + 3n + 5 \leq 2n^2.$

$\therefore f(n) = n^2 + 3n + 5 \in O(n^2).$ //

⑥ Big Omega Notation: Prove that $g(n) = n^3 + 2n^2 + 4n$ is $\Omega(n^3)$.

Sol:- Given $g(n) = n^3 + 2n^2 + 4n$

A function $g(n)$ is $\Omega(f(n))$ if there exist positive constants $c$ and $n_0$ such that for all $n \geq n_0$:

$g(n) \geq c \cdot f(n)$

$n^3 + 2n^2 + 4n \geq c \cdot n^3$ [Divide with $n^3$]

$1 + \frac{2}{n} + \frac{4}{n^2} \geq c$

Let $\boxed{c = 1}$

$1 + \frac{2}{n} + \frac{4}{n^2} \geq 1$

Thus, for $\boxed{c = 1}$

$n^3 + 2n^2 + 4n \geq 1 \cdot n^3$

$n^3+2n^2+4n \geq n^3$

$\therefore g(n)=n^3+2n^2+4n$ is $\Omega(n^3)$.

⑦ Big Theta Notation: Determine whether $h(n)=4n^2+3n$ is $\Theta(n^2)$ or not.

Sol: Given $h(n)=4n^2+3n$

A function $f(n)$ is $\Theta(g(n))$ if there exist constants $c>0$ and $n_0$ such that for all $n \geq n_0$:

$f(n) \leq c \cdot g(n)$

$4n^2+3n \leq c \cdot n^2$ [Divide with $n^2$]

$4+\frac{3}{n} \leq c$

Let $c=5$, then

$4+\frac{3}{n} \leq 5$

This is true for all $n \geq 1$. Therefore, we can take $c=5$ and $n_0=1$.

Thus, $h(n)=4n^2+3n$ is $\Theta(n^2)$.

⑧ Let $f(n)=n^3-2n^2+n$ and $g(n)=n^2$ show whether $f(n)=\Omega(g(n))$ is true or false and justify your answer.

Sol: Given $f(n)=n^3-2n^2+n$

& $g(n)=n^2$

$f(n) \geq c \cdot g(n)$

$$n^3 - 2n^2 + n \geq c \cdot n^2$$

$$n^3 - 2n^2 + n - c \cdot n^2 \geq 0$$

$$n^3 - (2+c)n^2 + n \geq 0$$

Let $\boxed{c=1}$, then

$$n^3 - (2+1)n^2 + n \geq 0$$

$$n^3 - 3n^2 + n \geq 0$$

$$n^3 - 2n^2 + n \geq c \cdot n^2$$

Thus $\boxed{c=1}$

$$n^3 - 2n^2 + n \geq n^2$$

$\therefore f(n) = n^3 - 2n^2 + n \in \Omega(n^2)$. //

(9) Determine whether $h(n) = n\log n + n$ is in $O(n\log n)$. Prove a rigorous proof for your conclusion.

Sol:- Given $h(n) = n\log n + n$

$$f(n) \leq c \cdot g(n)$$

$$n\log n + n \leq c \cdot n\log n$$

$$\log n + 1 \leq c\log n$$

Let $\boxed{c=2}$, then

$$\log n + 1 \leq 2\log n$$

$$\cancel{\log n}\ 1 \leq \log n$$

Therefore, $h(n) = n\log n + n$ is $O(n\log n)$. //

(10) Solve the following recurrence relations and find the order of growth for solutions.

$T(n) = 4T(n/2) + n^2, \ T(1) = 1$

Sol:- Given

$T(n) = 4T(n/2) + n^2, \ T(1) = 1$

By Master's Theorem -

$T(n) = aT\left(\frac{n}{b}\right) + f(n)$

Here, $a = 4$ | $f(n) = n^2$

$b = 2$ | $K = 2$

$\log_b a = \log_2 4 = 2 = K$

$\therefore \log_b a = K$

$p > -1$ then

$= O(n^K \log_n^{p+1})$

$= O(n^2 \log_n^{1+1})$

$= O(n^2 \log n)$.

(11) Given an array of [4, -2, 5, 3, 10, -5, 2, 8, -3, 6, 7, -4, 1, 9, -1, 0, -6, -8, 11, -9] integers, find the maximum and Minimum product that can be obtained by multiplying two integers from the array.

Sol:- Given an array is

array = [4, -2, 5, 3, 10, -5, 2, 8, -3, 6, 7, -4, 1, 9, -1, 0, -6, -8, 11, -9]

Maximum Product -

(1) Product of two largest +ve numbers.

(2) Product of two most negative numbers.

Minimum Product –

① Product of largest +ve and most -ve numbers

② Product of two smallest -ve numbers.

Sorting the Array

Sorted array = [-9, -8, -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11]

Maximum Product Calculation –

① Product of two largest +ve numbers = 10 × 11 = 110

② Product of two most -ve numbers = -9 × -8 = 72

Thus, the maximum product is 110.

Minimum Product Calculation –

① Product of largest +ve & most -ve numbers = 11 × -9 = -99

② Product of two smallest -ve numbers = -9 × -8 = 72

Thus, minimum product is -99.

∴ Maximum Product = 110

Minimum Product = -99.

(12) Demonstrate Binary Search method to search Key = 23, from the array arr[ ] = {2, 5, 8, 12, 16, 23, 38, 56, 72, 91}.

Sol:- Given

array[] = {2, 5, 8, 12, 16, 23, 38, 56, 72, 91}.

| l 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | h 9 |
|---|---|---|---|---|---|---|---|---|---|
| 2 | 5 | 8 | 12 | 16 | 23 | 38 | 56 | 72 | 91 |

$P_1$ (indices 0–4) | $P_2$ (indices 5–9)

$mid = \frac{l+h}{2} = \frac{0+9}{2} = 4 \Rightarrow arr[4] = 16$

P

$Key = 23 > arr[4] = 16$

$P_2 \Rightarrow$

$mid = \frac{l+h}{2} = \frac{5+9}{2} = 7$

$arr[7] = 56$

$Key = 23 < arr[7] = 56$

$P_1 \Rightarrow$

$mid = \frac{5+6}{2} = 5 \Rightarrow arr[5] = 23$

$Key = 23 = arr[5] = 23$

The key = 23 is found at index 5 in array using the Binary Search method.

(13) Apply merge sort and order the list of 8 elements Data d = (45, 67, -12, 5, 22, 30, 50, 20). Set up a recurrence relation for the number of key comparisons made by mergesort.

Sol: Given

d = (45, 67, -12, 5, 22, 30, 50, 20)

| l 0 | 1 | 2 | 3 | 4 | 5 | 6 | h 7 |
|---|---|---|---|---|---|---|---|
| 45 | 67 | -12 | 5 | 22 | 30 | 50 | 20 |

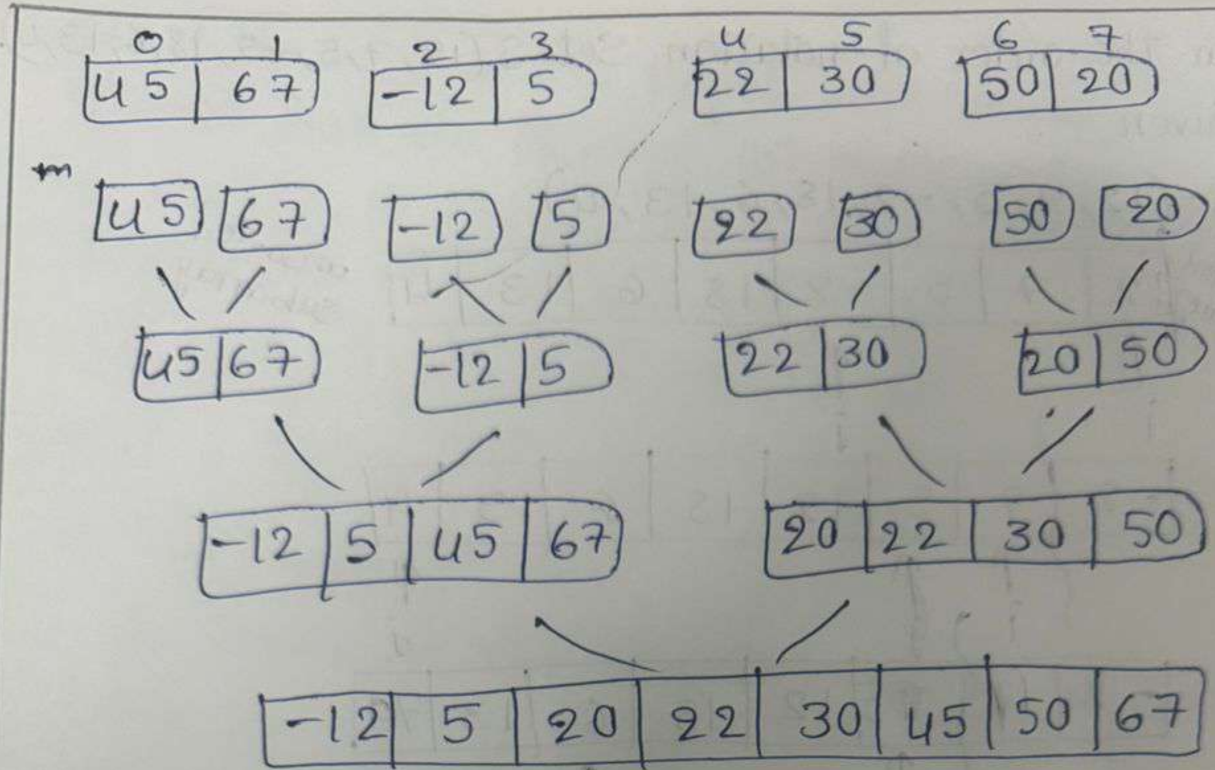
$mid = \frac{l+h}{2} = \frac{0+7}{2} = 3$

| 0 | 1 | 2 | 3 |
|---|---|---|---|
| 45 | 67 | -12 | 5 |

| 4 | 5 | 6 | 7 |
|---|---|---|---|
| 22 | 30 | 50 | 20 |

$mid = \frac{0+3}{2} = 1$

$mid = \frac{4+7}{2} = 5$




**Solving the Recurrence Relation** –

$T(n) = aT\left(\frac{n}{b}\right) + f(n)$

~~Here~~

$T(n) = 2T\left(\frac{n}{2}\right) + O(n)$.

Here, $a = 2$, $b = 2$ | $f(n) = O(n)$

$\log_b a = \log_2 2 = 1$

$n^{\log_b a} = n^1 = n$

$T(n) = O(n \log n)$

$\therefore$ Time Complexity is $\boxed{O(n \log n)}$ //

(14) Find the no. of times to perform swapping for Selection sort. Also estimate the time complexity.

for the order of notation Set S (12, 7, 5, -2, 18, 6, 13, 4)

Sol:- Given

S = (12, 7, 5, -2, 18, 6, 13, 4)

sorted subarray

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 12 | 7 | 5 | -2 | 18 | 6 | 13 | 4 |
| i | | | j | | | | |

unsorted subarray

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| -2 | 7 | 5 | 12 | 18 | 6 | 13 | 4 |
| | i | | | | | | j |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| -2 | 4 | 5 | 12 | 18 | 6 | 13 | 7 |
| | | i | | | j | | |

~~-2 4 5 6, 12 18 7 13 7~~

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| -2 | 4 | 5 | 12 | 18 | 6 | 13 | 7 |
| | | | i | | j | | |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| -2 | 4 | 5 | 6 | 18 | 12 | 13 | 7 |
| | | | | i | | | j |

sorted array

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| -2 | 4 | 5 | 6 | 7 | 12 | 13 | 18 |

Total swaps = 4, bus as per theory, it should be n-1 = 7.

Best case = $O(n^2)$

Average case = $O(n^2)$

Worst Case = $O(n^2)$.

(15) Find the index of the target value 10 using binary search from the following list of elements [2, 4, 6, 8, 10, 12, 14, 16, 18, 20].

Sol: Given list of elements arr[] =

$$\begin{array}{cccccccccc} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ l & & & & & & & & & h \\ \end{array}$$

$$[2, 4, 6, 8, 10, 12, 14, 16, 18, 20]$$

$$mid = \frac{l+h}{2} = \frac{0+9}{2} = 4$$

$$arr[4] = 10$$

$\therefore$ The index of the target value 10 in the list is 4.

(16) Sort the following elements using Merge sort divide-and-conquer strategy [38, 27, 43, 3, 9, 82, 10, 15, 88, 52, 60, 5] and analyze complexity of the algorithm.

Sol: Given elements

$$\begin{array}{cccccccccccc} l & & & & & & & & & & & h \\ 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ \end{array}$$

$$[38, 27, 43, 3, 9, 82, 10, 15, 88, 52, 60, 5]$$

$$mid = \frac{l+h}{2} = \frac{0+11}{2} = 5$$

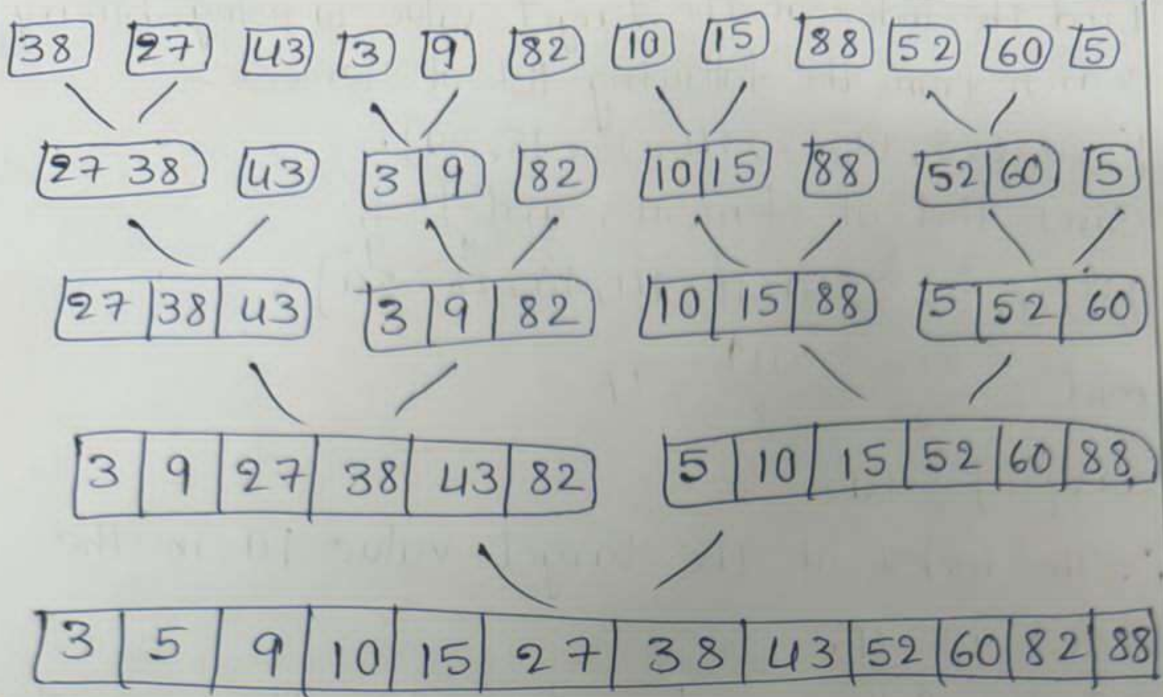
$$\begin{array}{cccccc} 0 & 1 & 2 & 3 & 4 & 5 \\ \end{array} \quad \begin{array}{cccccc} 6 & 7 & 8 & 9 & 10 & 11 \\ \end{array}$$

[38 | 27 | 43 | 3 | 9 | 82]   [10 | 15 | 88 | 52 | 60 | 5]

$$mid = \frac{0+5}{2} = 2$$

[38 | 27 | 43]   [3 | 9 | 82]   [10 | 15 | 88]   [52 | 60 | 5]

[38 | 27]   [43]   [3 | 9]   [82]   [10 | 15]   [88]   [52 | 60]   [5]




Algorithm -

MergeSort(l, h)

{

If ($l < h$)

{

Mid = $(l+h)/2$; — 1

Merge sort (l, mid) — $n/2$

Merge sort (mid+1, h)

$n/2$

Merge (l, mid, h) — n

}

}

$T(n) = 2T(n/2) + n$

$a = 2$ | $K = 1$

$b = 2$

$\log_b a = \log_2 2 = 1 = k$

$p > -1 \quad O(n^k \log_n^{p+1})$

$= (n^1 \log_n^{1+1})$

$= n \log_n^2$

$\therefore$ Time Complexity $= O(n \log n)$.

Space Complexity $= O(n)$.

(17) Sort the array 64, 34, 25, 12, 22, 11, 90 using Bubble Sort. What is the time Complexity of Selection Sort in the best, worst and average cases?

Sol: Given array

I -

| | | | | | | |
|---|---|---|---|---|---|---|
| 64 | 34 | 25 | 12 | 22 | 11 | 90 |

| | | | | | | |
|---|---|---|---|---|---|---|
| 34 | 64 | 25 | 12 | 22 | 11 | 90 |

| | | | | | | |
|---|---|---|---|---|---|---|
| 34 | 25 | 64 | 12 | 22 | 11 | 90 |

| | | | | | | |
|---|---|---|---|---|---|---|
| 34 | 25 | 12 | 64 | 22 | 11 | 90 |

| | | | | | | |
|---|---|---|---|---|---|---|
| 34 | 25 | 12 | 22 | 64 | 11 | 90 |

| | | | | | | |
|---|---|---|---|---|---|---|
| 34 | 25 | 12 | 22 | 11 | 64 | 90 |

I-2

| | | | | | | |
|---|---|---|---|---|---|---|
| 34 | 25 | 12 | 22 | 11 | 64 | 90 |

| | | | | | | |
|---|---|---|---|---|---|---|
| 25 | 34 | 12 | 22 | 11 | 64 | 90 |

| | | | | | | |
|---|---|---|---|---|---|---|
| 25 | 12 | 34 | 22 | 11 | 64 | 90 |

| | | | | | | |
|---|---|---|---|---|---|---|
| 25 | 12 | 22 | 34 | 11 | 64 | 90 |

| | | | | | | |
|---|---|---|---|---|---|---|
| 25 | 12 | 22 | 11 | 34 | 64 | 90 |

I-3

| | | | | | | |
|---|---|---|---|---|---|---|
| 25 | 12 | 22 | 11 | 34 | 64 | 90 |

| | | | | | | |
|---|---|---|---|---|---|---|
| 12 | 25 | 22 | 11 | 34 | 64 | 90 |

| | | | | | | |
|---|---|---|---|---|---|---|
| 12 | 22 | 25 | 11 | 34 | 64 | 90 |

| | | | | | | |
|---|---|---|---|---|---|---|
| 12 | 22 | 11 | 25 | 34 | 64 | 90 |

I-4

| | | | | | | |
|---|---|---|---|---|---|---|
| 12 | 22 | 11 | 25 | 34 | 64 | 90 |

| | | | | | | |
|---|---|---|---|---|---|---|
| 12 | 22 | 11 | 25 | 34 | 64 | 90 |

| | | | | | | |
|---|---|---|---|---|---|---|
| 12 | 11 | 22 | 25 | 34 | 64 | 90 |

I-5

| | | | | | | |
|---|---|---|---|---|---|---|
| 12 | 11 | 22 | 25 | 34 | 64 | 90 |

| | | | | | | |
|---|---|---|---|---|---|---|
| 11 | 12 | 22 | 25 | 34 | 64 | 90 |

Time complexity of selection sort is -

Best - $O(n)$

Averge - $O(n^2)$

Worst - $O(n^2)$.//

(18) Sort the array 64, 25, 12, 22, 11 using selection Sort. What is the time complexity of Selection sort in the best, average and worst cases?

Sol:- Given array

sorted subarray

| | | | | |
|---|---|---|---|---|
| 64 | 25 | 12 | 22 | 11 |

unsorted subarray

i j

| | | | | |
|---|---|---|---|---|
| 11 | 25 | 12 | 22 | 64 |

i j

| | | | | |
|---|---|---|---|---|
| 11 | 12 | 25 | 22 | 64 |

i j

| | | | | |
|---|---|---|---|---|
| 11 | 12 | 22 | 25 | 64 |

sorted array

Time Complexity

Best - $O(n)$

Average - $O(n^2)$

Worst - $O(n^2)$.//

(19) Sort the following elements using insertion sort using Brute force Approach strategy [38, 27, 43, 3, 9, 82, 10, 15, 88, 52, 60, 5] and analyze complexity of the algorithm.

Sol:- Given

38 27 43 3 9 82 10 15 88 52 60 5

27 38 43 3 9 82 10 15 88 52 60 5

3 27 38 43 9 82 10 15 88 52 60 5

3 9 27 38 43 82 10 15 88 52 60 5

3 9 ~~27~~ 10 27 38 43 82 15 88 52 60 5

3 9 10 15 27 38 43 82 88 52 60 5

3 9 10 15 27 38 43 52 82 88 60 5

3 9 10 15 27 38 43 52 82 60 88 5

3 9 10 15 27 38 43 52 60 82 88 5

3 5 9 10 15 27 38 43 52 60 82 88

Time Complexity -

Best case - $O(n)$ - This occurs when the array is already sorted. The inner loop will run only once.

Avg Case - $O(n^2)$ - The list is randomly ordered,

worst case - $O(n^2)$ - If the list is in reverse

Space Complexity -

$O(1)$ - Insertion Sort

(20) Given an array of [4, -2, 5, 3, 10, -5, 2, 8, -3, 6, 7, -4, 1, 9, -1, 0, -6, -8, 11, -9] integers, sort the fallowing elements using insertion sort using Brute Force approach strategy analyse complexity of algorithm.

**Sol:** Given array

= [4, -2, 5, 3, 10, -5, 2, 8, -3, 6, 7, -4, 1, 9, -1, 0, -6, -8, 11, -9]

4 -2 5 3 10 -5 2 8 -3 6 7 -4 1 9 -1 0 -6 -8 11 -9
i j

-2 4 5 3 10 -5 2 8 -3 6 7 -4 1 9 -1 0 -6 -8 11 -9
i j

-2 4 5 3 10 -5 2 8 -3 6 7 -4 1 9 -1 0 -6 -8 11 -9
i j

-2 4 3 5 10 -5 2 8 -3 6 7 -4 1 9 -1 0 -6 -8 11 -9
i j

-2 3 4 5 10 -5 2 8 -3 6 7 -4 1 9 -1 0 -6 -8 11 -9
i j

-2 3 4 5 -5 10 2 8 -3 6 7 -4 1 9 -1 0 -6 -8 11 -9
i j

-2 3 4 -5 5 10 2 8 -3 6 7 -4 1 9 -1 0 -6 -8 11 -9
i j

-2 3 -5 4 5 10 2 8 -3 6 7 -4 1 9 -1 0 -6 -8 11 -9
i j

-2 -5 3 4 5 10 2 8 -3 6 7 -4 1 9 -1 0 -6 -8 11 -9
i j

-5 -2 3 4 5 10 2 8 -3 6 7 -4 1 9 -1 0 -6 -8 11 -9
i j

-5 -2 3 4 5 2 10 8 -3 6 7 -4 1 9 -1 0 -6 -8 11 -9

i j

-5 -4 -3 -2 1 2 3 4 5 6 7 8 9 -1 10 0 -6 -8 11 -9

-5 -4 -3 -2 -1 1 2 3 4 5 6 7 8 9 10 0 -6 -8 11 -9

-5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10 -6 -8 11 -9

-6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10 -8 11 -9

-8 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10 11 -9

-9 -8 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10 11

Time campexity -

Best case $(O(n))$ - This occurs when the array is already sorted. The inner loop will run only once for each element.

Average case$(O(n^2))$ - The list is randomly ordered

Worst case $(O(n^2))$ - If the list is in reverse arder.