Date:- 22/06/24

CSA0672 - DAA

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(1) Calculate the number of ways to achieve a sum of 15 when rolling four six-sided dice. Provide a detailed step-by-step solution.

Sol:- No. of solutions

$x_1+x_2+x_3+x_4=15$ where $1 \le x_i \le 6$ into $0 \le y_i \le 5$.

This becomes

$(y_1+1)+(y_2+1)+(y_3+1)+(y_4+1)=15$

$y_1+y_2+y_3+y_4+4=15$

$y_1+y_2+y_3+y_4=11$

Using Inclusion-Exclusion Principle

"stars and bars"

$\binom{11+4-1}{4-1} = \binom{14}{3}$

$\binom{14}{3} = \frac{14 \times 13 \times 12}{3 \times 2 \times 1} = 364$

set $y_1' = y_1 - 6$. then

$y_1' + y_2 + y_3 + y_4 = 5$

$\binom{5+4-1}{4-1} = \binom{8}{3}$

$\binom{8}{3} = \frac{8 \times 7 \times 6}{3 \times 2 \times 1} = 56$

Since any of four variables $= 4 \times 56 = 224$

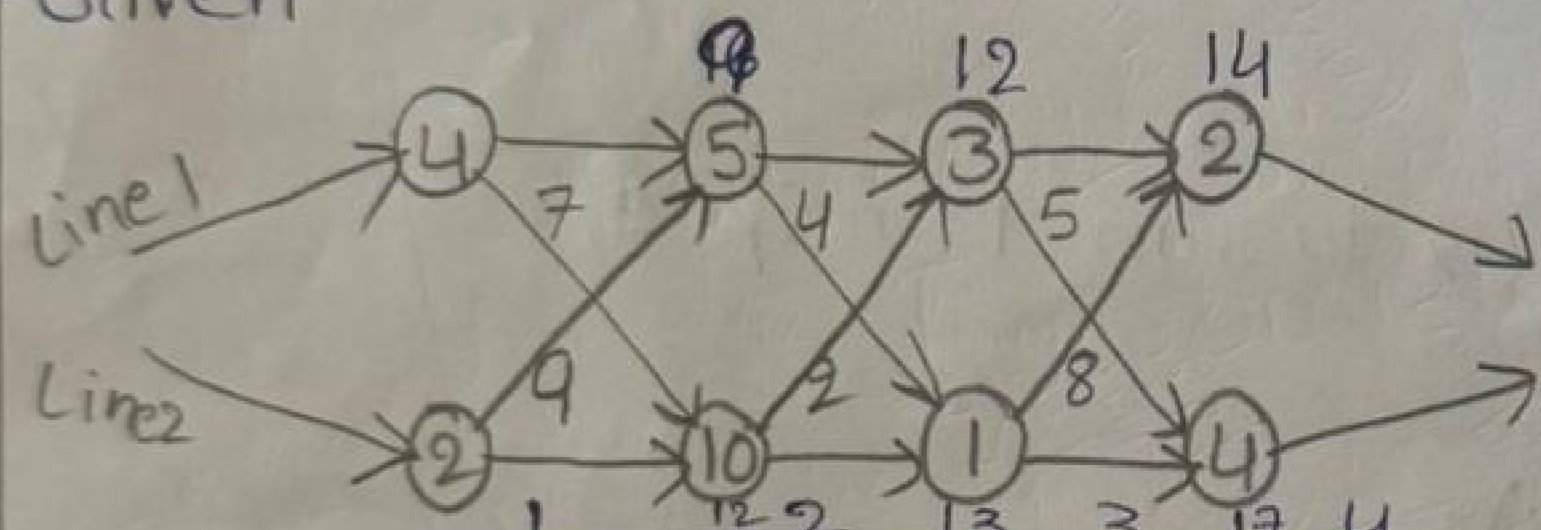
set $y_1' = y_1 - 6$ and $y_2' = y_2 - 6$ then

$y_1' + y_2' + y_3 + y_4 = -1$

$\therefore$ The no. of valid solutions is: $364 - 224 = 140$.

② Two assembly lines have station times as follows: Line 1: [4, 5, 3, 2], Line 2: [2, 10, 1, 4], transfer times between lines are: from Line 1 to Line 2: [7, 4, 5], from Line 2 to Line 1: [9, 2, 8]. Calculate the minimum time to assemble a product.

Sol:- Given



	1	2	3	4
$F_1[j]$	4	9	12	14
$F_2[j]$	2	12	13	17

	1	2	3	4
$L_1[j]$	1	1	1	1
$L_2[j]$	2	2	2	2

$F_1[j] = \min\{(f_1(j-1) + a_{1j}), (f_2(j-1) + (t_{2,j-1}) + a_{1j})\}$

$= \min\{9, [illegible]\} = 9$

$F_2[j] = \min\{(f_1(j-1) + (t_{1,j-1}) + a_{2j}), (f_2(j-1) + (a_{2j})\}$

$= \min\{21, 12\} = 12$

Given keys {10, 20, 30, 40} \ {10, 20, 30, 40} \ {10, 20, 30, 40} with access probabilities {0.1, 0.2, 0.4, 0.3} \ {0.1, 0.2, 0.4, 0.3} \ {0.1, 0.2, 0.4, 0.3} respectively, construct the optimal binary search tree. Calculate the total cost of the tree.

Sol: Given keys with access probabilities

{10, 20, 30, 40} (numbered 1, 2, 3, 4)

{0.1, 0.2, 0.4, 0.3}

i \ j	0	1	2	3	4
0	0	0.1	0.4 [2]	1.1 [3]	1.7 [3]
1		0	0.2	0.8 [3]	1.4 [2]
2			0	0.4	1.0 [3]
3				0	0.3
4					0

j − i = 1

1 − 0 = 1 (0, 1) (1, 1)

2 − 1 = 1 (1, 2) (2, 2)

3 − 2 = 1 (2, 3) (3, 3)

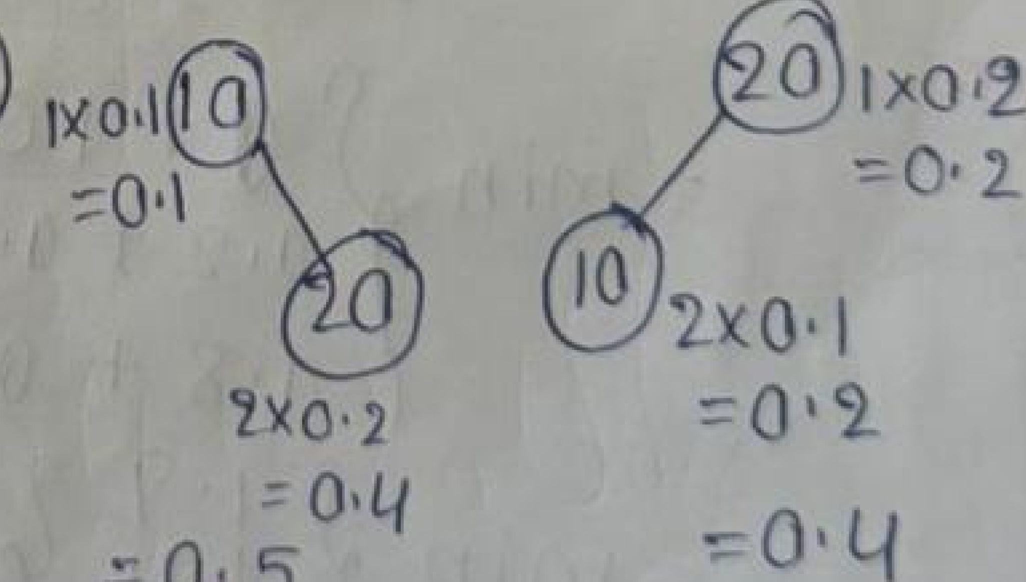
4 − 3 = 1 (3, 4) (4, 4)

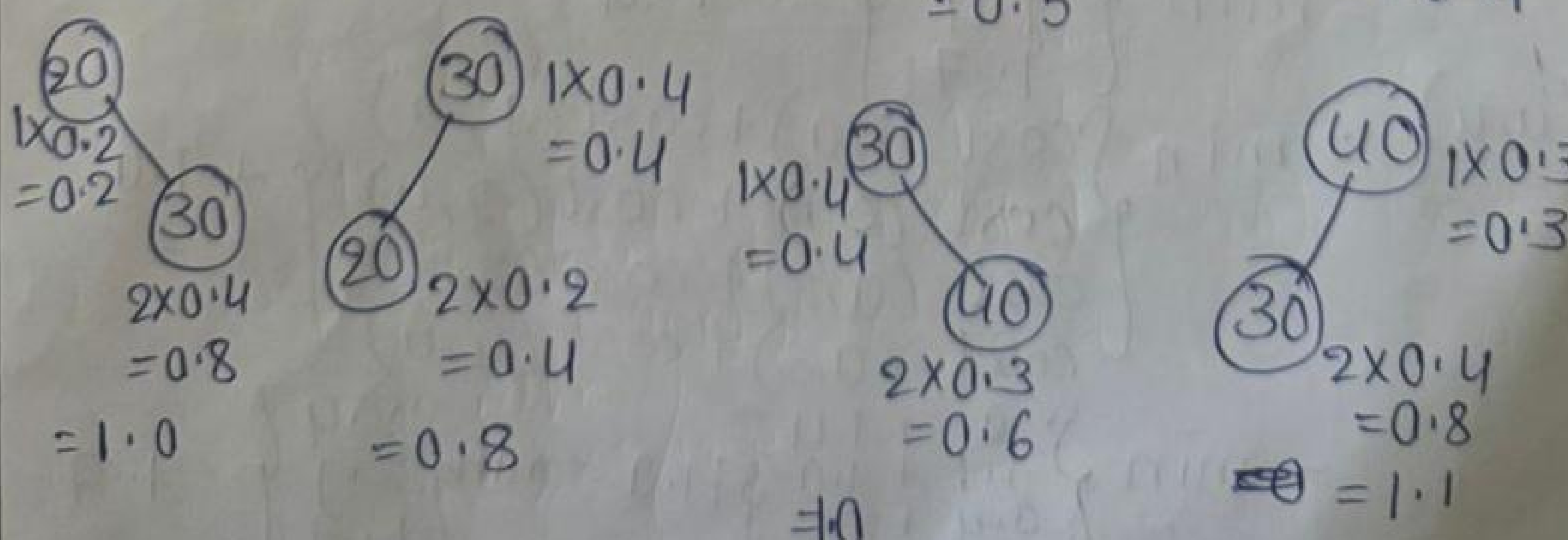
j − i = 2

2 − 0 = 2 (0, 2) ~~(0, 1)~~ (1, 2)

3 − 1 = 2 (1, 3) (2, 3)

4 − 2 = 2 (2, 4) (3, 4)





$j-i=3$

$3-0=3 \quad (0,3)\ (1,3)$

$4-1=3 \quad (1,4)\ (2,4)$

$$cost(i,j) = \min\{cost(i,k-1) + cost(k,j)\} + w_i$$

$$cost(0,3) = \min_{k=1,2,3} \begin{Bmatrix} cost(0,0) + cost(1,3) \\ cost(0,1) + cost(2,3) \\ cost(0,2) + cost(3,3) \end{Bmatrix} + 0.7$$

$$= \min \begin{Bmatrix} 0 + 0.8 \\ 0.1 + 0.4 \\ 0.4 + 0 \end{Bmatrix} + 0.7$$

$$= \min \begin{Bmatrix} 1.5 \\ 1.2 \\ 1.1 \end{Bmatrix} = 1.1$$

$$cost(1,4) = \min_{k=2,3,4} \begin{Bmatrix} cost(1,1) + cost(2,4) \\ cost(1,2) + cost(3,4) \\ cost(1,3) + cost(4,4) \end{Bmatrix} + 0.9$$

$$= \min \begin{Bmatrix} 0 + 1.0 \\ 0.2 + 0.3 \\ 0.8 + 0 \end{Bmatrix} + 0.9$$

$$= \min \begin{Bmatrix} 1.9 \\ 1.4 \\ 1.7 \end{Bmatrix} = 1.4$$

$$cost(0,4) = \min \begin{Bmatrix} cost(0,0) + cost(1,4) \\ cost(0,1) + cost(2,4) \\ cost(0,2) + cost(3,4) \\ cost(0,3) + cost(4,4) \end{Bmatrix} + 1.0$$

$$= \min \begin{Bmatrix} 0 + 1.4 \\ 0.1 + 1.0 \\ 0.4 + 0.3 \\ 1.1 + 0 \end{Bmatrix} + 1.0 = \min \begin{Bmatrix} 2.4 \\ 2.1 \\ 1.7 \\ 2.1 \end{Bmatrix} = 1.7$$

Solve the TSP for the following 5-city distance matrix using dp

A : [0, 29, 20, 21, 17]

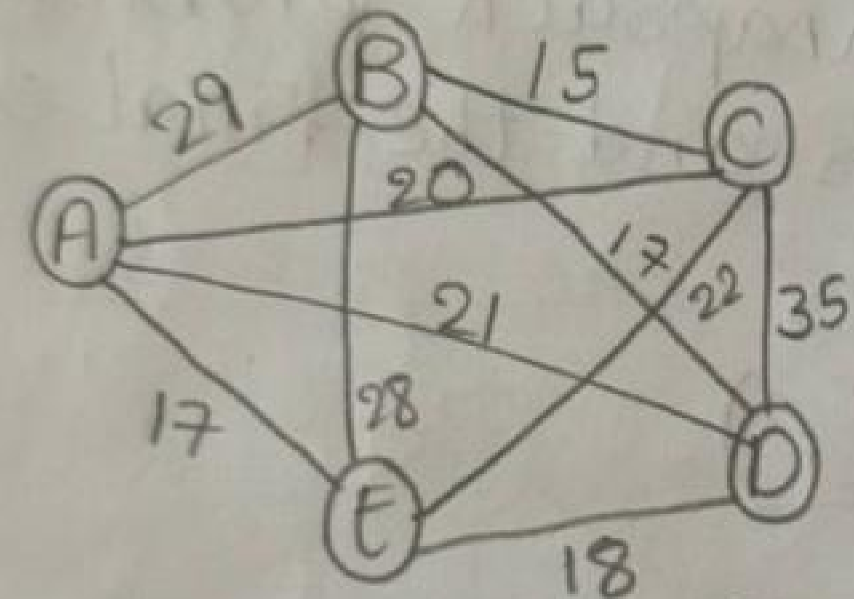
B : [29, 0, 15, 17, 28]

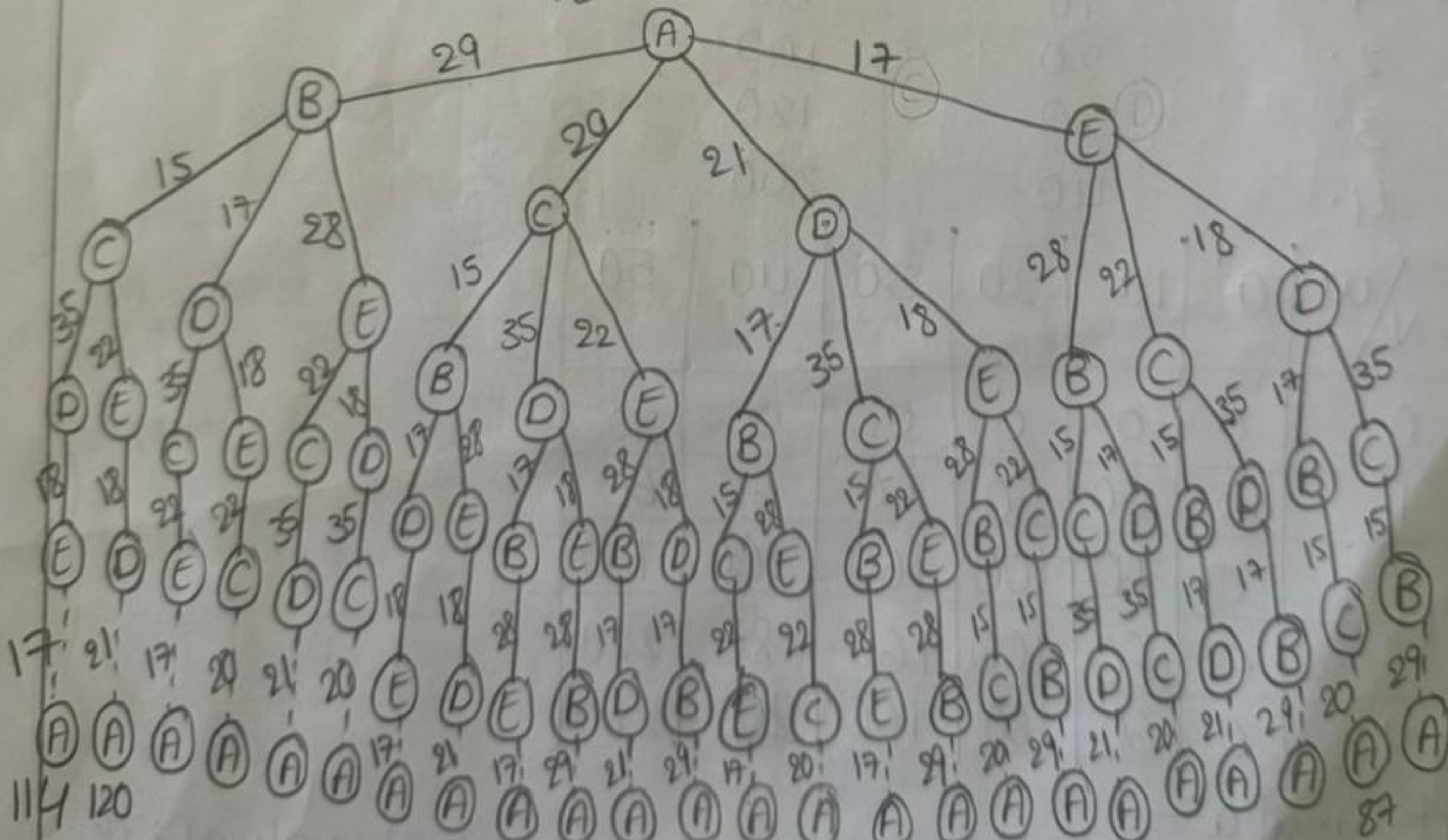
C : [20, 15, 0, 35, 22]

D : [21, 17, 35, 0, 18]

E : [17, 28, 22, 18, 0]

Sol:-





∴ The minimum cost = 87

A — E — D — B — C — A

(5) You have a knapsack with a capacity of 50 units. There are 4 items with the following weights and values:

Item 1: Weight = 10, Value = 60
Item 2: Weight = 20, Value = 100
Item 3: Weight = 30, Value = 120
Item 4: Weight = 40, Value = 200

Determine the maximum value that can be obtained using the 0/1 knapsack problem approach. Show the steps and the final solution.

Sol:- Given capacity 50 units

Item	weight	Value(P)
1.	10	60
2.	20	100
3.	30	120
4.	40	200

V\w	0	10	20	30	40	50
0	0	0	0	0	0	0
1	0	60	60	60	60	60
2	0	60	100	160	160	160
3	0	60	100	160	180	220
4	0	60	100	160	200	260

Formula -

$$V[i, \omega] = \max\{v[i-1, \omega], v[i-1, \omega - \omega[i]] + value[i]\}$$

$$V[4, 50] = \max\{v[3, 50], v[3, \cancel{50} - 200] + value[4]\}$$

$$= \max\{220, 260\} = 260$$

Given the following directed graph with vertices A, B, C, DA, B, C, DA, B, C, D and edges with weights:

A→BA \rightarrow BA→B with weight 1

A→CA \rightarrow CA→C with weight 4

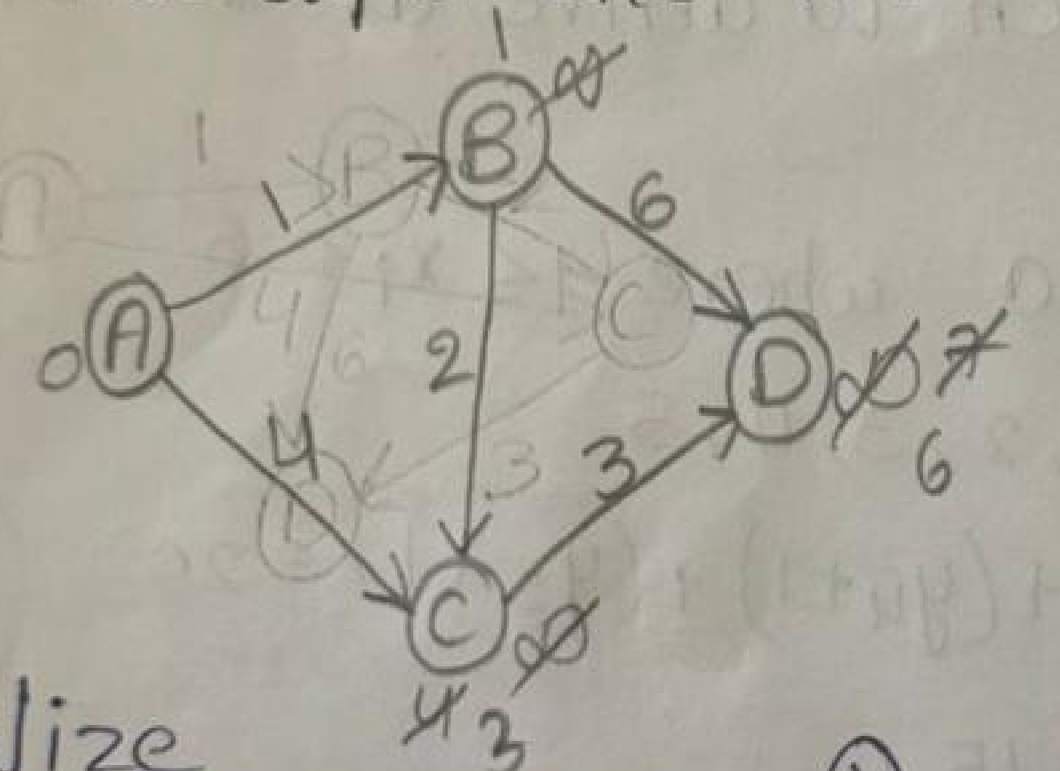
B→CB \rightarrow CB→C with weight 2

B→DB \rightarrow DB→D with weight 6

C→DC \rightarrow DC→D with weight 3

Use the Bellman-Ford algorithm to find the shortest path from vertex AAA to all other vertices. Show the steps and the final distances.

Sol:-



A→B	1
A→C	4
B→C	2
B→D	6
C→D	3

Initialize

V	A	B	C	D
d	0	∞	∞	∞
P	-	-	-	-

(1)

V	A	B	C	D
d	0	1	4	∞
P	-	A	A	B

(2)

V	A	B	C	D
d	0	1	3	7
P	-	A	B	B

(3)

V	A	B	C	D
d	0	1	3	6
P	-	A	B	C

Path	Shortest distance	Shortest path
A - B	1	A - B
A - C	3	A - B - C
A - D	6	A - B - C - D

O/P $\rightarrow A \rightarrow B \rightarrow C \rightarrow D$

(7) Determine the probability of rolling five dice such that the sum is exactly 20. Include a combinatorial approach to arrive at the solution.

Sol:- $6^5 = 7776$

$x_1 + x_2 + x_3 + x_4 + x_5 = 20$ where $1 \le x_i \le 6$

$y_i = x_i - 1$ for $i = 1, 2, 3, 4, 5$

$(y_1+1) + (y_2+1) + (y_3+1) + (y_4+1) + (y_5+1) = 20$

$y_1 + y_2 + y_3 + y_4 + y_5 = 15$

where $0 \le y_i \le 5$

By "stars and bars"

$$\binom{15+5-1}{5-1} = \binom{19}{4}$$

$$\binom{19}{4} = \frac{19 \times 18 \times 17 \times 16}{4 \times 3 \times 2 \times 1} = 3876$$

If $y_1 \ge 6$, let $y_1' = y_1 - 6$

$y_1' + y_2 + y_3 + y_4 + y_5 = 9$

$$\binom{9+5-1}{5-1} = \binom{13}{4}$$

$$\binom{13}{4} = \frac{13\times12\times11\times10}{4\times3\times2\times1} = 715$$

There are 5 such variables,

$5\times715 = 3575$

If two variables $y_i, y_j \geq 6$, let $y_1' = y_1 - 6$ and $y_2' = y_2 - 6$.

$$y_1' + y_2' + y_3 + y_4 + y_5 = 3$$

$$\binom{3+5-1}{5-1} = \binom{7}{4}$$

$$\binom{7}{4} = \frac{7\times6\times5\times4}{4\times3\times2\times1} = 35$$

$$\binom{5}{2}\times35 = 10\times35 = 350$$

Using the inclusion-exclusion principle:

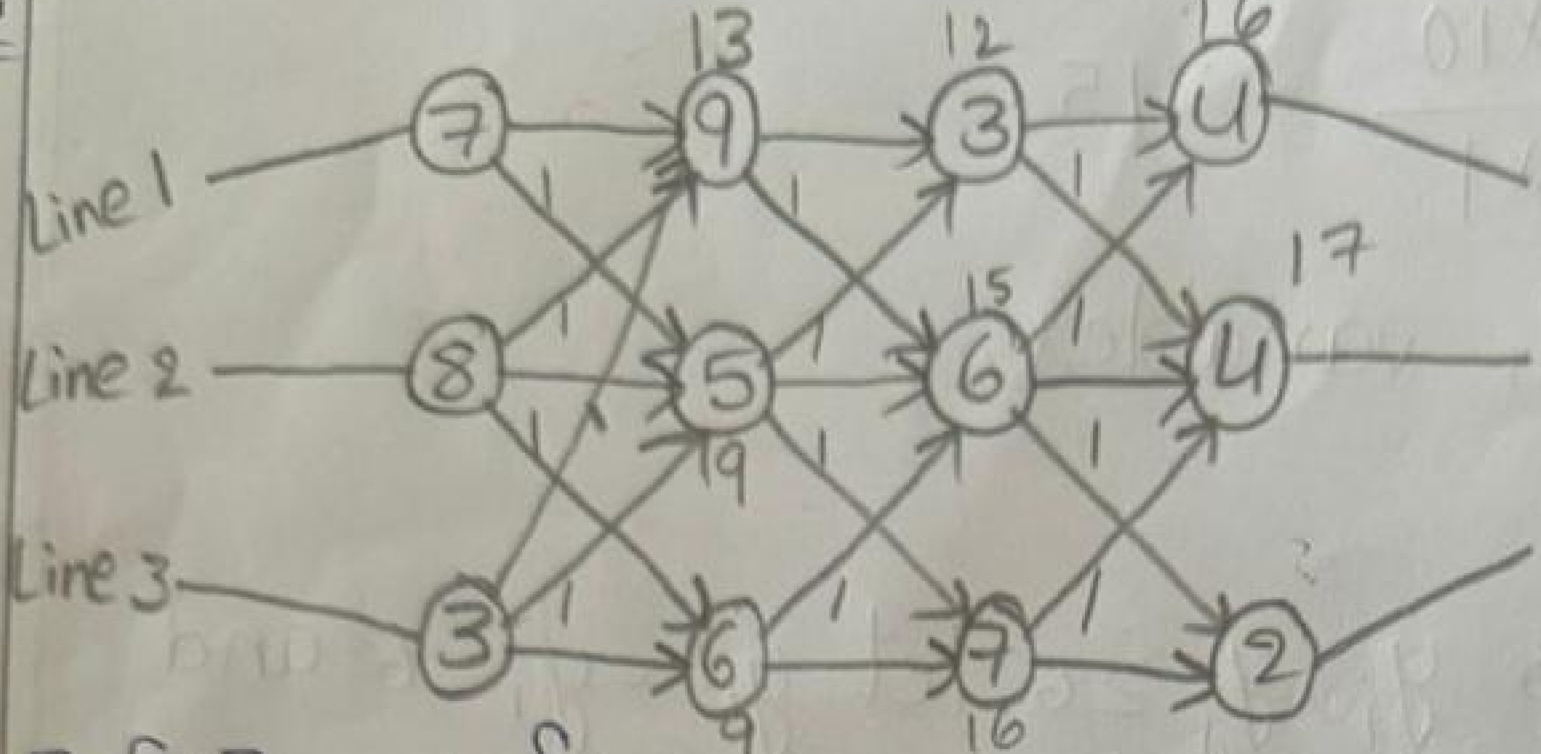
$$3876 - 3575 + 350 = 651$$

$$\frac{651}{7776} = \frac{651}{7776} \approx 0.0837$$

(8) For three assembly lines with station times: Line 1: [7, 9, 3, 4], Line 2: [8, 5, 6, 4], Line 3: [3, 6, 7, 2], and transfer times between lines

given, determine the optimal scheduling and the total minimum assembly time.

Sol:-



$$F_1[j] = \min\{(f_1(j-1)+a_{1j}),\ (f_2(j-1)+(t_{2,j-1})+a_{1j}),\ (f_3(j-1)+(t_{3,j-1})+a_{1j})\}$$

$$= \min\{(7+9),\ (8+1+9),\ (3+1+9)\} = 13$$

	1	2	3	4
$F_1[j]$	~~13~~ 7	13	12	16
$F_2[j]$	8	9	15	17
$F_3[j]$	3	9	16	15

	1	2	3	4
$L_1[j]$	1	3	1	1
$L_1[j]$	2	3	2	1
$L_1[j]$	3	3	3	1

(9) Consider keys {15, 25, 35, 45, 55} \ {15, 25, 35, 45, 55} \ {15, 25, 35, 45, 55} with access probabilities {0.05, 0.15, 0.4, 0.25, 0.15}. Determine the structure of the optimal binary search tree and compute

the expected cost.

Sol:- $\{\overset{1}{15}, \overset{2}{25}, \overset{3}{35}, \overset{4}{45}, \overset{5}{55}\}$

$\{0.05, 0.15, 0.4, 0.25, 0.15\}$

$j-i=1$

$1-0=1$ (0,1) (1,1)

$2-1=1$ (1,2) (2,2)

$3-2=1$ (2,3) (3,3)

$4-3=1$ (3,4) (4,4)

$5-4=1$ (4,5) (5,5)

$j-i=2$

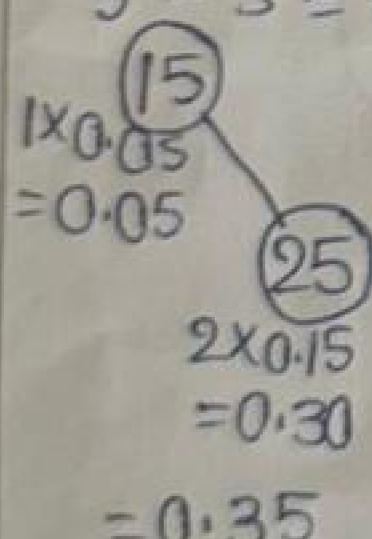
$2-0=2$ (0,2) (1,2)

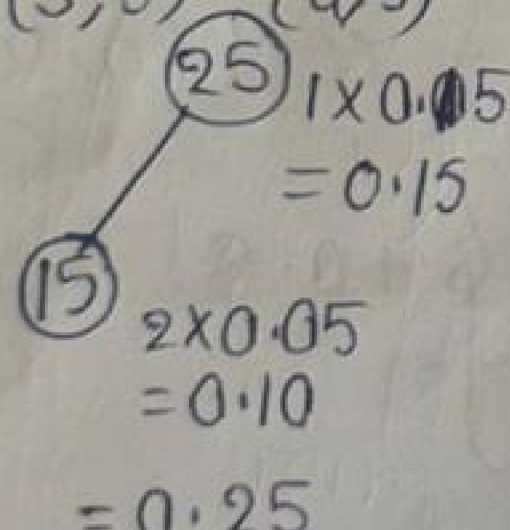
$3-1=2$ (1,3) (2,3)

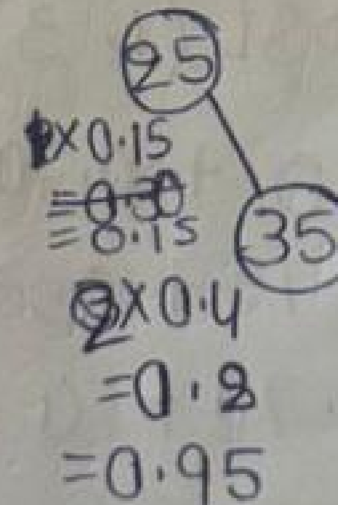
$4-2=2$ (2,4) (3,4)

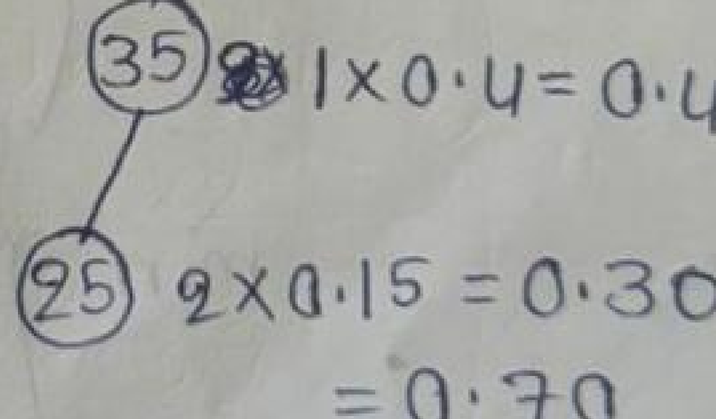
$5-3=2$ (3,5) (4,5)

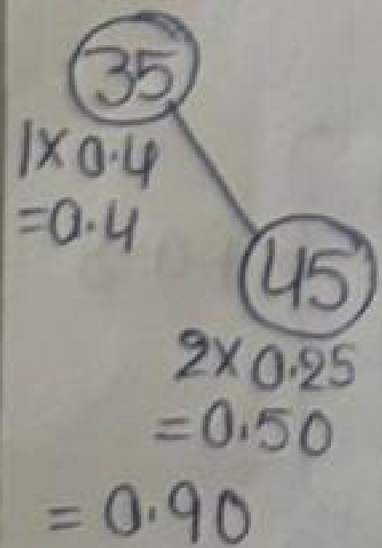
	0	1	2	3	4	5
0	0	0.05	[2] 0.25	[3] 0.85	[3] 1.35	[3] 1.80
1		0	0.15	[3] 0.70	[3] 1.20	[4] 1.80
2			0	0.4	[3] 0.90	[3,4] 1.35
3				0	0.25	[4] 0.55
4					0	0.15
5						0

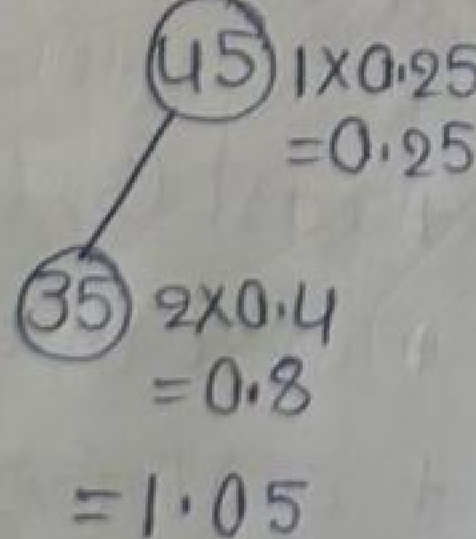


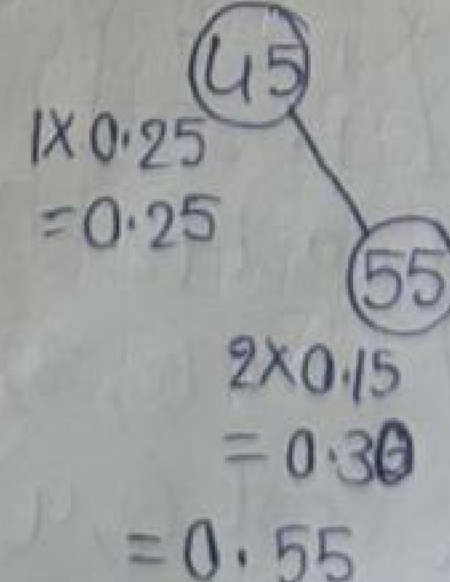


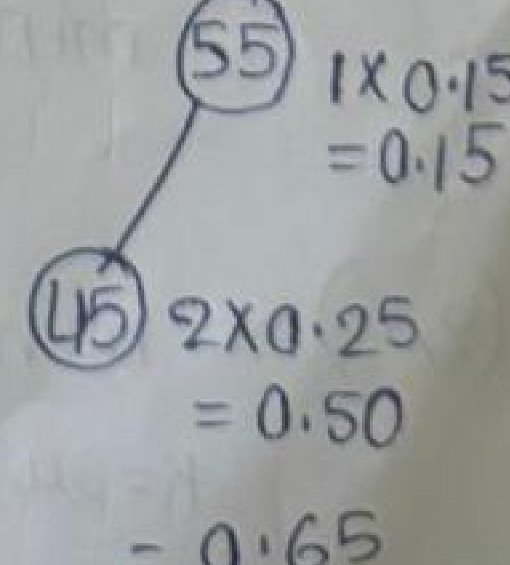












$j-i=3$

$3-0=3$ (0,3) (1,3)

$4-1=3$ (1,4) (2,4)

$5-2=3$ (2,5) (3,5)

$$cost(i,j) = \min\{cost(i,k-1) + cost(k,j)\} + w_i$$

$$cost(0,3) = \min_{k=1,2,3} \begin{cases} cost(0,0) + cost(1,3) \\ cost(0,1) + cost(2,3) \\ cost(0,2) + cost(3,3) \end{cases} + 0.6$$

$$= \min \begin{cases} 0 + 0.70 \\ 0.05 + 0.4 \\ 0.25 + 0 \end{cases} + 0.6$$

$$= \min \begin{cases} 1.30 \\ 1.05 \\ 0.85 \end{cases} = 0.85$$

$$cost(1,4) = \min_{k=2,3,4} \begin{cases} cost(1,1) + cost(2,4) \\ cost(1,2) + cost(3,4) \\ cost(1,3) + cost(4,4) \end{cases} + 0.8$$

$$= \min \begin{cases} 0 + 0.90 \\ 0.15 + 0.25 \\ 0.70 + 0 \end{cases} + 0.8$$

$$= \min \begin{cases} 1.70 \\ 1.20 \\ 1.50 \end{cases} = 1.20$$

$$cost(2,5) = \min_{k=3,4,5} \begin{cases} cost(2,2) + cost(3,5) \\ cost(2,3) + cost(4,5) \\ cost(2,4) + cost(5,5) \end{cases} + 0.8$$

$$= \min \begin{cases} 0 + 0.55 \\ 0.4 + 0.15 \\ 0.90 + 0 \end{cases} + 0.8 = \min \begin{cases} 1.35 \\ 1.35 \\ 1.70 \end{cases} = 1.35$$

~~cost~~ $j - i = 4$

$4 - 0 = 4 \quad (0,4)\ (1,4)$

$5 - 1 = 4 \quad (1,5)\ (2,5)$

$$\text{cost}(0,4) = \min_{K=1,2,3,4} \begin{Bmatrix} \text{cost}(0,0) + \text{cost}(1,4) \\ \text{cost}(0,1) + \text{cost}(2,4) \\ \text{cost}(0,2) + \text{cost}(3,4) \\ \text{cost}(0,3) + \text{cost}(4,4) \end{Bmatrix} + 0.85$$

$$= \min \begin{Bmatrix} 0 + 1.20 \\ 0.05 + 0.90 \\ 0.25 + 0.25 \\ 0.85 + 0 \end{Bmatrix} + 0.85$$

$$= \min \begin{Bmatrix} 2.05 \\ 1.80 \\ 1.35 \\ 1.70 \end{Bmatrix} = \cancel{1.50}\ 1.35$$

$$\text{cost}(1,5) = \min_{K=2,3,4,5} \begin{Bmatrix} \text{cost}(1,1) + \text{cost}(2,5) \\ \text{cost}(1,2) + \text{cost}(3,5) \\ \text{cost}(1,3) + \text{cost}(4,5) \\ \text{cost}(1,4) + \text{cost}(5,5) \end{Bmatrix} + 0.95$$

$$= \min \begin{Bmatrix} 0 + 1.35 \\ 0.15 + 1.35 \\ 0.70 + 0.15 \\ 1.20 + 0 \end{Bmatrix} + 0.95$$

$$= \min \begin{Bmatrix} 2.30 \\ 2.45 \\ 1.80 \\ 2.15 \end{Bmatrix} = 1.80$$

$$\text{cost}(0,5) = \min_{K=1,2,3,4,5} \begin{Bmatrix} \text{cost}(0,0) + \text{cost}(1,5) \\ \text{cost}(0,1) + \text{cost}(2,5) \\ \text{cost}(0,2) + \text{cost}(3,5) \\ \text{cost}(0,3) + \text{cost}(4,5) \\ \text{cost}(0,4) + \text{cost}(5,5) \end{Bmatrix} + 1$$

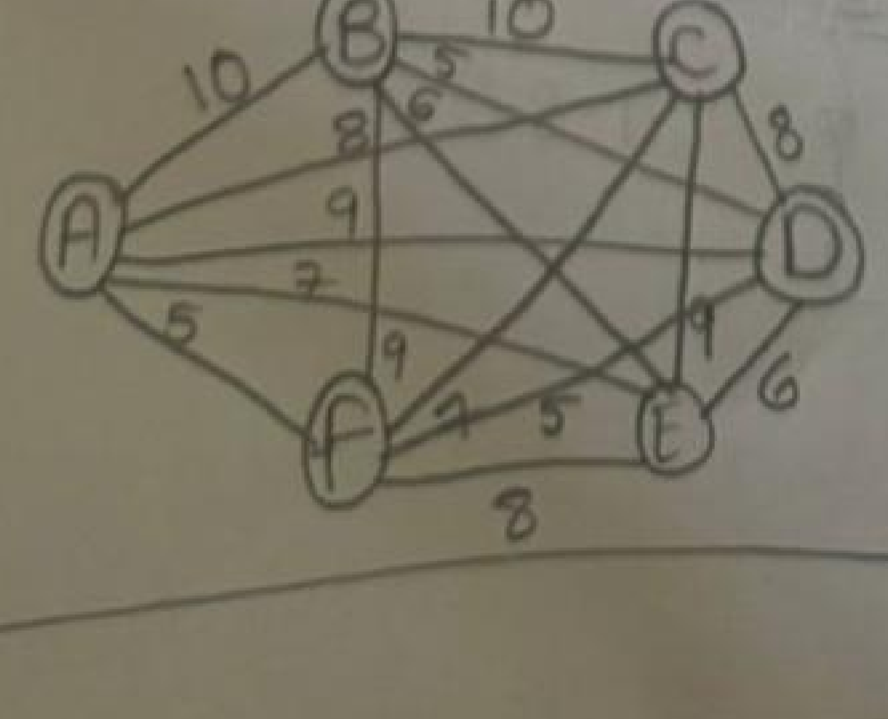
$$= \min \begin{Bmatrix} 0 + 1.80 \\ 0.05 + 1.35 \\ 0.25 + 0.55 \\ 0.85 + 0.15 \\ 1.35 + 0 \end{Bmatrix} + 1$$

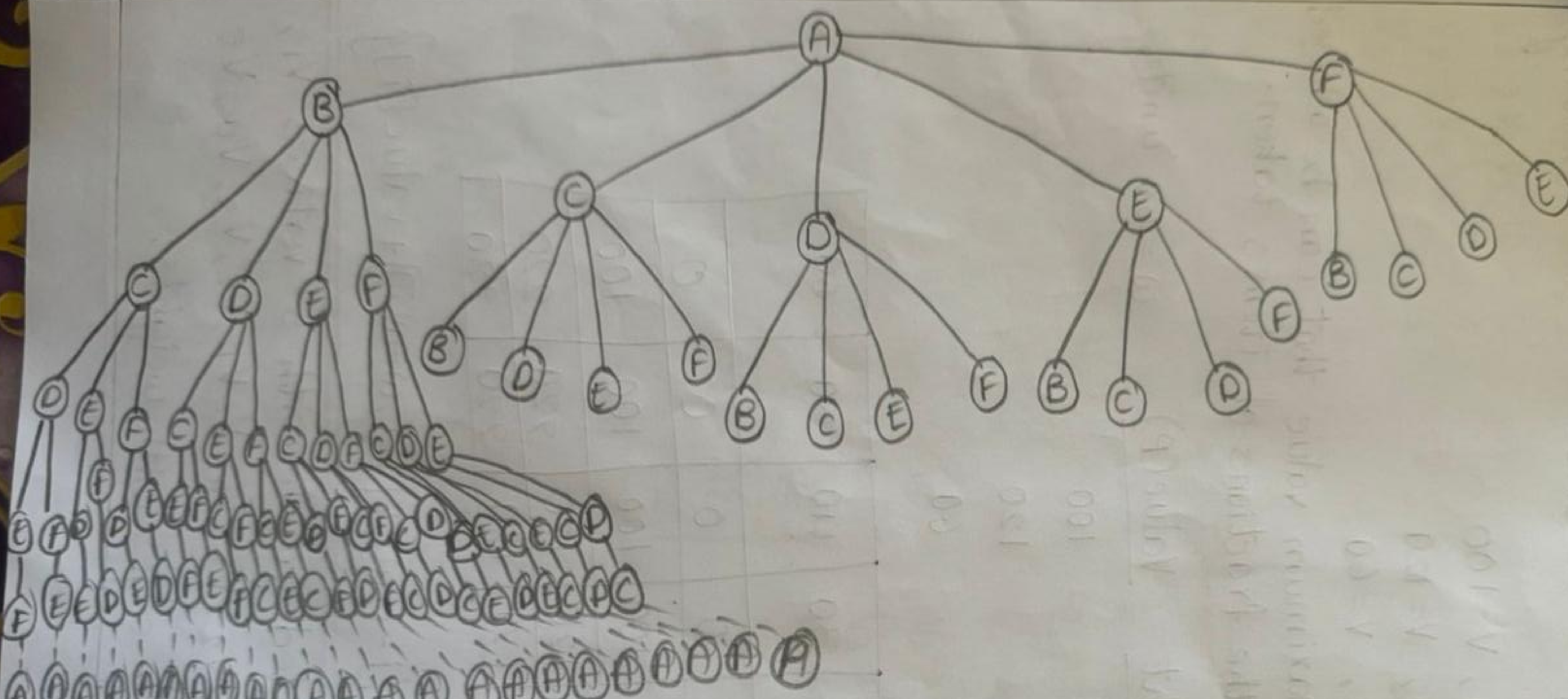
$$= \min \begin{Bmatrix} 2.80 \\ 2.65 \\ 1.80 \\ 2.00 \\ 2.35 \end{Bmatrix} = 1.80$$

(10) Given a distance matrix for 6 cities, find the shortest path using the nearest neighbor heuristic

A: [0, 10, 8, 9, 7, 5]
B: [10, 0, 10, 5, 6, 9]
C: [8, 10, 0, 8, 9, 7]
D: [9, 5, 8, 0, 6, 5]
E: [7, 6, 9, 6, 0, 8]
F: [5, 9, 7, 5, 8, 0]

Sol:-





O/P — A - F - E - B - C - B - A = 47%

(11) Solve the fractional knapsack problem for a knapsack with a capacity of 60 units and the following items:

Item 1: $W = 20$, $V = 100$

Item 2: $W = 30$, $V = 120$

Item 3: $W = 10$, $V = 60$

Calculate the maximum value that can be achieved and describe the fractions of items taken.

Sol:-

Item	Weight	Value(P)
1.	20	100
2.	30	120
3.	10	60

$\omega = 60$ units

V \ ω	0	10	20	30	40	50	60
0	0	0	0	0	0	0	0
1	0	0	100	100	100	100	100
2	0	0	100	120	120	220	220
3	0	10	100	120	180	220	280

Formula -

$$V[i,\omega] = \max\{V[i-1,\omega], V[i-1,\omega-\omega[i]] + value[i]\}$$

(12) Consider a directed graph with 5 vertices V_1, V_2, V_3, V_4, $V_5 V_1$, V_2, V_3, V_4, $V_5 V_1$, V_2, V_3, V_4, V_5 and the following edges with weights:

$V_1 \rightarrow V_2 V_1$ \rightarrow $V_2 V_1 \rightarrow V_2$ with $w - 3$

$V_1 \rightarrow V_3 V_1$ $w - 8$

$V_2 \rightarrow V_3 V_2$ $w - 2$

$V_2 \rightarrow V_4 V_2$ $w - 5$

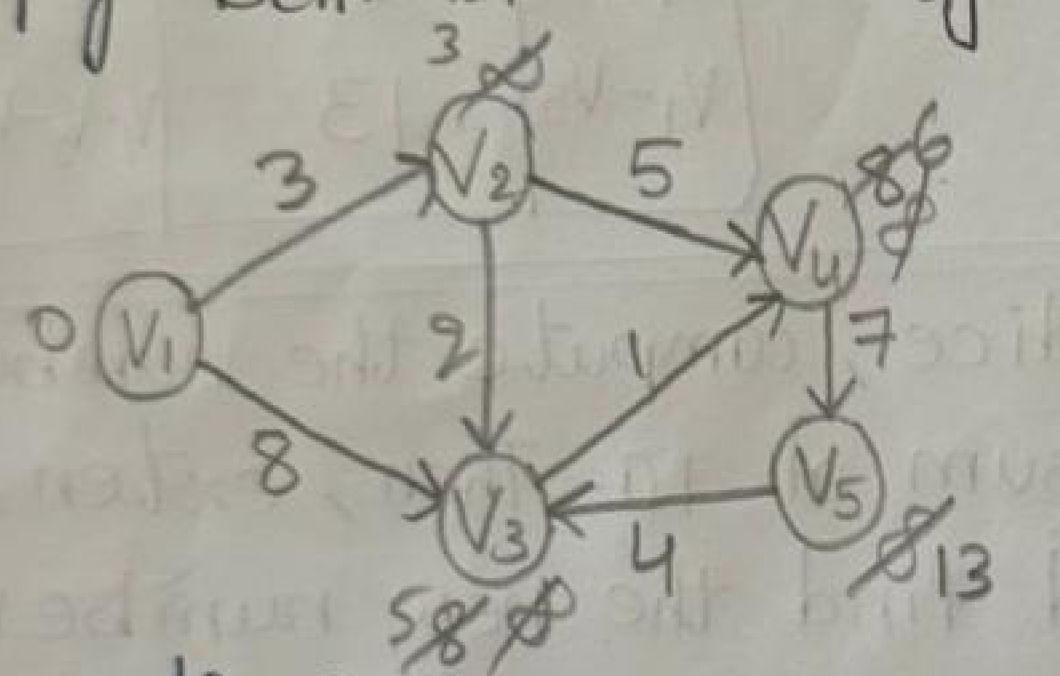
$V_3 \rightarrow V_4 V_3$ $w - 1$

$V_4 \rightarrow V_5 V_4$ $w - 7$

$V_5 \rightarrow V_3 V_5$ $w - 4$

Apply Bellman-Ford algorithm

Sol:-



$V_1 \rightarrow V_2$ 3
$V_1 \rightarrow V_3$ 8
$V_2 \rightarrow V_3$ 2
$V_2 \rightarrow V_4$ 5
$V_3 \rightarrow V_4$ 1
$V_4 \rightarrow V_5$ 7
$V_5 \rightarrow V_3$ 4

Initialize

V	V_1	V_2	V_3	V_4	V_5
d	0	$\propto$	∞	∞	∞
p	-	-	-	-	-

(1)

V	V_1	V_2	V_3	V_4	V_5
d	0	3	8	∞	∞
p	-	V_1	V_1	V_2	V_4

(2)

V	V_1	V_2	V_3	V_4	V_5
d	0	3	5	8	∞
p	-	V_1	V_2	V_2	V_4

(3)

V	V_1	V_2	V_3	V_4	V_5
d	0	3	5	6	13
P	-	V_1	V_2	V_3	V_4

(4)

V	V_1	V_2	V_3	V_4	V_5
d	0	3	5	6	13
P	-	V_1	V_2	V_3	V_4

o/p $\rightarrow V_1 \rightarrow V_2 \rightarrow V_3 \rightarrow V_4 \rightarrow$

Path	shortest distance	Shortest path
$V_1 - V_2$	3	$V_1 - V_2$
$V_1 - V_3$	5	$V_1 - V_2 - V_3$
$V_1 - V_4$	6	$V_1 - V_2 - V_3 - V_4$
$V_1 - V_5$	13	$V_1 - V_2 - V_3 - V_4 - V_5$

(13) Given two eight-sided dice, compute the number of ways to achieve a sum of 10. Then, extend this to three dice and find the new number of ways to get the same sum.

Sol:- We need to count the paris (x, y) such that $x+y=10$ where $1 \leq x, y \leq 8$.

Possible pairs -

$(x, y) = (2, 8)$
$(x, y) = (3, 7)$
$(x, y) = (4, 6)$
$(x, y) = (5, 5)$
$(x, y) = (6, 4)$
$(x, y) = (7, 3)$
$(x, y) = (8, 2)$

No. of ways to achieve a sum of 10 is: 7.

We need to count the no. of triples (x, y, z) such that $x+y+z=10$ where $1 \le x, y, z \le 8$.

1. $x=1$:

   $y+z=9$:

   $(1,1,8), (1,2,7), (1,3,6), (1,4,5)(1,5,4), (1,6,3),$

   $(1,7,2), (1,8,1)$

2. $x=2$:

   $y+z=8$:

   $(2,1,7), (2,2,6), (2,3,5), (2,4,4), (2,5,3), (2,6,2),$

   $(2,7,1)$

3. $x=3$:

   $y+z=7$:

   $(3,1,6), (3,2,5), (3,3,4), (3,4,3), (3,5,2), (3,6,1)$

4. $x=4$:

   $y+z=6$:

   $(4,1,5), (4,2,4), (4,3,3), (4,4,2), (4,5,1)$

5. $x=5$:

   $y+z=5$:

   $(5,1,4), (5,2,3), (5,3,2), (5,4,1)$

6. $x=6$:

   $y+z=4$:

   $(6,1,3), (6,2,2), (6,3,1)$

7. $x=7$:

   $y+z=3$:

   $(7,1,2), (7,2,1)$

8. $x=8$:

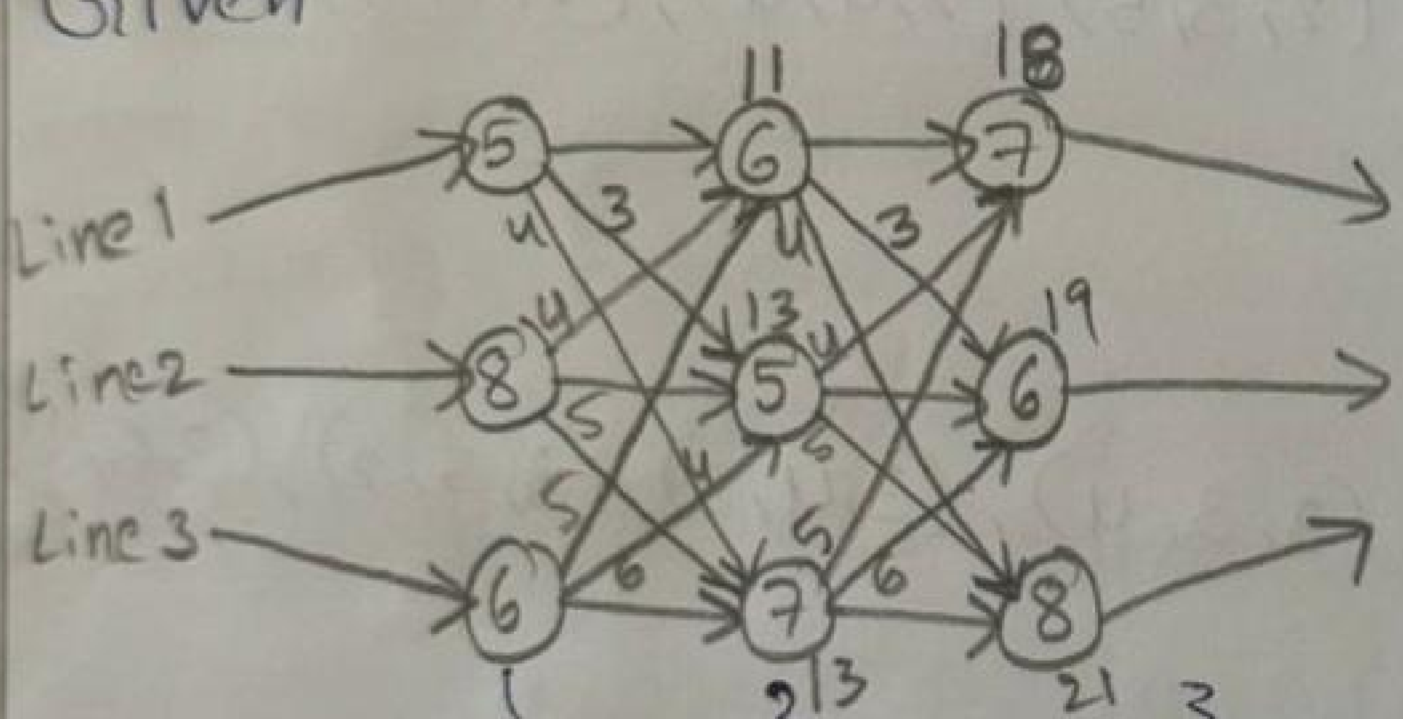
$y+z=2$: $(8,1,1)$

Sum $= 8+7+6+5+4+3+2+1 = 36$

So, the no. of ways to a sum of $10 = 36$ //

(14) Given station times for Line 1: [5, 6, 7], Line 2: [8, 5, 6], and Line 3: [6, 7, 8] and transfer times b/w lines: [3, 4], [4, 5], and [5, 6], calculate the minimum time required to complete the product assembly.

Sol:- Given



	1	2	3
$F_1[j]$	5	11	~~16~~ 18
$F_2[j]$	8	13	19
$F_3[j]$	6	13	21

	1	2	3
$L_1[j]$	1 ←	1 ←	~~8~~ 1 ←
$L_2[j]$	2	2	2
$L_3[j]$	3	3	3

$$F_1[j] = \min\{(f_1(j-1)+a_{1,j}), (f_2(j-1)+(t_{2,j-1})+a_{1,j}), (f_3(j-1)+(t_{3,j-1})+a_{1,j})\}$$

$$= \min\{11, 18, 17\} = 11 \text{ //}$$

Given keys {5, 15, 25, 35, 45, 55} with access probabilities {0.1, 0.05, 0.2, 0.25, 0.3, 0.1}, use dp to find the OBST. Show the steps of your calculation and the resulting cost.

Sol: $\{\overset{1}{5}, \overset{2}{15}, \overset{3}{25}, \overset{4}{35}, \overset{5}{45}, \overset{6}{55}\}$

{0.1, 0.05, 0.2, 0.25, 0.3, 0.1}

$j-i=1$
$1-0=1$
$2-1=1$
$3-2=1$
$4-3=1$
$5-4=1$
$6-5=1$

	0	1	2	3	4	5	6
0	0	0.1	[1] 0.2	[3] 0.55	[3] 1.05	[4] 1.75	[4] 2.05
1		0	0.05	[3] 0.3	[3,4] 0.8	[4] 1.4	[4] 1.7
2			0	0.2	[4] 0.65	[4] 1.25	[4] 1.55
3				0	0.25	[5] 0.8	[5] 1
4					0	0.3	[5] 0.5
5						0	0.1
6							0

$j-i=2$
$2-0=2$ (0,2) (1,2)
$3-1=2$ (1,3) (2,3)
$4-2=2$ (2,4) (3,4)
$5-3=2$ (3,5) (4,5)
$6-4=2$ (4,6) (5,6)

5 $1\times0.1=0.1$
15 $2\times0.05=0.1$
$=0.2$

15 $1\times0.05=0.05$
5 $2\times0.1=0.2$
$=0.25$

15 $1\times0.05=0.05$
25 $2\times0.2=0.4$
$=0.45$

25 $1\times0.2=0.2$
15 $2\times0.05=0.1$
$=0.3$

25 $1\times0.2=0.2$
35 $2\times0.25=0.5$
$=0.7$

35 $1\times0.25=0.25$
25 $2\times0.2=0.4$
$=0.65$

35 $1\times0.25=0.25$
45 $2\times0.3=0.6$
$=0.85$

45 $1\times0.3=0.3$
35 $2\times0.25=0.5$
$=0.8$

45 $1\times0.3=0.3$
55 $2\times0.1=0.2$
$=0.5$

55 $1\times0.1=0.1$
45 $2\times0.3=0.6$
$=0.7$

$j-i=3$

$3-0=3 \quad (0,3)\ (1,3)$

$4-1=3 \quad (1,4)\ (2,4)$

$5-2=3 \quad (2,5)\ (3,5)$

$6-3=3 \quad (3,6)\ (4,6)$

$$cost(i,j) = \min\{cost(i,k-1) + cost(k,j)\} + w_i$$

$$cost(0,3) = \min_{k=1,2,3} \begin{Bmatrix} cost(0,0) + cost(1,3) \\ cost(0,1) + cost(2,3) \\ cost(0,2) + cost(3,3) \end{Bmatrix} + 0.35$$

$$= \min \begin{Bmatrix} 0 + 0.3 \\ 0.1 + 0.65 \\ 0.2 + 0 \end{Bmatrix} + 0.35$$

$$= \min \begin{Bmatrix} 0.65 \\ 1.1 \\ 0.55 \end{Bmatrix} = 0.55$$

$$cost(1,4) = \min_{k=2,3,4} \begin{Bmatrix} cost(1,1) + cost(2,4) \\ cost(1,2) + cost(3,4) \\ cost(1,3) + cost(4,4) \end{Bmatrix} + 0.5$$

$$= \min \begin{Bmatrix} 0 + 0.65 \\ 0.05 + 0.25 \\ 0.3 + 0 \end{Bmatrix} + 0.5$$

$$= \min \begin{Bmatrix} 1.15 \\ 0.8 \\ 0.8 \end{Bmatrix} = 0.8$$

$$cost(2,5) = \min_{k=3,4,5} \begin{Bmatrix} cost(2,2) + cost(3,5) \\ cost(2,3) + cost(4,5) \\ cost(2,4) + cost(5,5) \end{Bmatrix} + 0.75$$

$$= \min \begin{Bmatrix} 0 + 0.8 \\ 0.2 + 0.3 \\ 0.65 + 0 \end{Bmatrix} + 0.75$$

$$= \min \begin{Bmatrix} 1.55 \\ 1.25 \\ 1.4 \end{Bmatrix} = 1.25$$

$$cost(3,6) = \min_{K=4,5,6} \begin{Bmatrix} cost(3,3) + cost(4,6) \\ cost(3,4) + cost(5,6) \\ cost(3,5) + cost(6,6) \end{Bmatrix} + 0.65$$

$$= \min \begin{Bmatrix} 0 + 0.5 \\ 0.25 + 0.1 \\ 0.8 + 0 \end{Bmatrix} + 0.65$$

$$= \min \begin{Bmatrix} 1.15 \\ 1 \\ 1.45 \end{Bmatrix} = 1$$

$j - i = 4$

$4 - 0 = 4$ $(0,4)$ $(1,4)$

$5 - 1 = 4$ $(1,5)$ $(2,5)$

$6 - 2 = 4$ $(2,6)$ $(3,6)$

$$cost(0,4) = \min_{K=1,2,3,4} \begin{Bmatrix} cost(0,0) + cost(1,4) \\ cost(0,1) + cost(2,4) \\ cost(0,2) + cost(3,4) \\ cost(0,3) + cost(4,4) \end{Bmatrix} + 0.6$$

$$= \min \begin{Bmatrix} 0 + 0.8 \\ 0.1 + 0.65 \\ 0.2 + 0.25 \\ 0.55 + 0 \end{Bmatrix} + 0.6 = \min \begin{Bmatrix} 1.4 \\ 1.35 \\ 1.05 \\ 1.15 \end{Bmatrix} = 1.05$$

$$\text{cost}(1,5) = \min_{K=2,3,4,5} \begin{Bmatrix} \text{cost}(1,1) + \text{cost}(2,5) \\ \text{cost}(1,2) + \text{cost}(3,5) \\ \text{cost}(1,3) + \text{cost}(4,5) \\ \text{cost}(1,4) + \text{cost}(5,5) \end{Bmatrix} + 0.8$$

$$= \min \begin{Bmatrix} 0 + 1.25 \\ 0.05 + 0.8 \\ 0.3 + 0.3 \\ 0.8 + 0 \end{Bmatrix} + 0.8$$

$$= \min \begin{Bmatrix} 2.05 \\ 1.65 \\ 1.4 \\ 1.6 \end{Bmatrix} = 1.4$$

$$\text{cost}(2,6) = \min_{K=3,4,5,6} \begin{Bmatrix} \text{cost}(2,2) + \text{cost}(3,6) \\ \text{cost}(2,3) + \text{cost}(4,6) \\ \text{cost}(2,4) + \text{cost}(5,6) \\ \text{cost}(2,5) + \text{cost}(6,6) \end{Bmatrix} + 0.85$$

$$= \min \begin{Bmatrix} 0 + 1 \\ 0.2 + 0.5 \\ 0.65 + 0.1 \\ 1.25 + 0 \end{Bmatrix} + 0.85$$

$$= \min \begin{Bmatrix} 1.85 \\ 1.55 \\ 1.6 \\ 2.1 \end{Bmatrix} = 1.55$$

$j - i = 5$

$5 - 0 = 5 \quad (0,5) \quad (1,5)$

$6 - 1 = 5 \quad (1,6) \quad (2,6)$

$$\text{cost}(0,5) = \min_{K=1,2,3,4,5} \begin{Bmatrix} \text{cost}(0,0) + \text{cost}(1,5) \\ \text{cost}(0,1) + \text{cost}(2,5) \\ \text{cost}(0,2) + \text{cost}(3,5) \\ \text{cost}(0,3) + \text{cost}(4,5) \\ \text{cost}(0,4) + \text{cost}(5,5) \end{Bmatrix} + 0.9$$

$$= \min \begin{cases} 0 + 1.4 \\ 0.1 + 1.25 \\ 0.2 + 0.8 \\ 0.55 + 0.3 \\ 1.05 + 0 \end{cases} + 0.9$$

$$= \min \begin{cases} 2.3 \\ 2.25 \\ 1.9 \\ 1.75 \\ 1.95 \end{cases} = 1.75$$

$$cost(1,6) = \min_{K=2,3,4,5,6} \begin{cases} cost(1,1) + cost(2,6) \\ cost(1,2) + cost(3,6) \\ cost(1,3) + cost(4,6) \\ cost(1,4) + cost(5,6) \\ cost(1,5) + cost(6,6) \end{cases} + 0.9$$

$$= \min \begin{cases} 0 + 1.55 \\ 0.05 + 1 \\ 0.3 + 0.5 \\ 0.8 + 0.1 \\ 1.4 + 0 \end{cases} + 0.9$$

$$= \min \begin{cases} 2.45 \\ 2 \\ 1.7 \\ 1.8 \\ 2.3 \end{cases} = 1.7$$

$$cost(0,6) = \min_{K=1,2,3,4,5,6} \begin{cases} cost(0,0) + cost(1,6) \\ cost(0,1) + cost(2,6) \\ cost(0,2) + cost(3,6) \\ cost(0,3) + cost(4,6) \\ cost(0,4) + cost(5,6) \\ cost(0,5) + cost(6,6) \end{cases} + 1$$

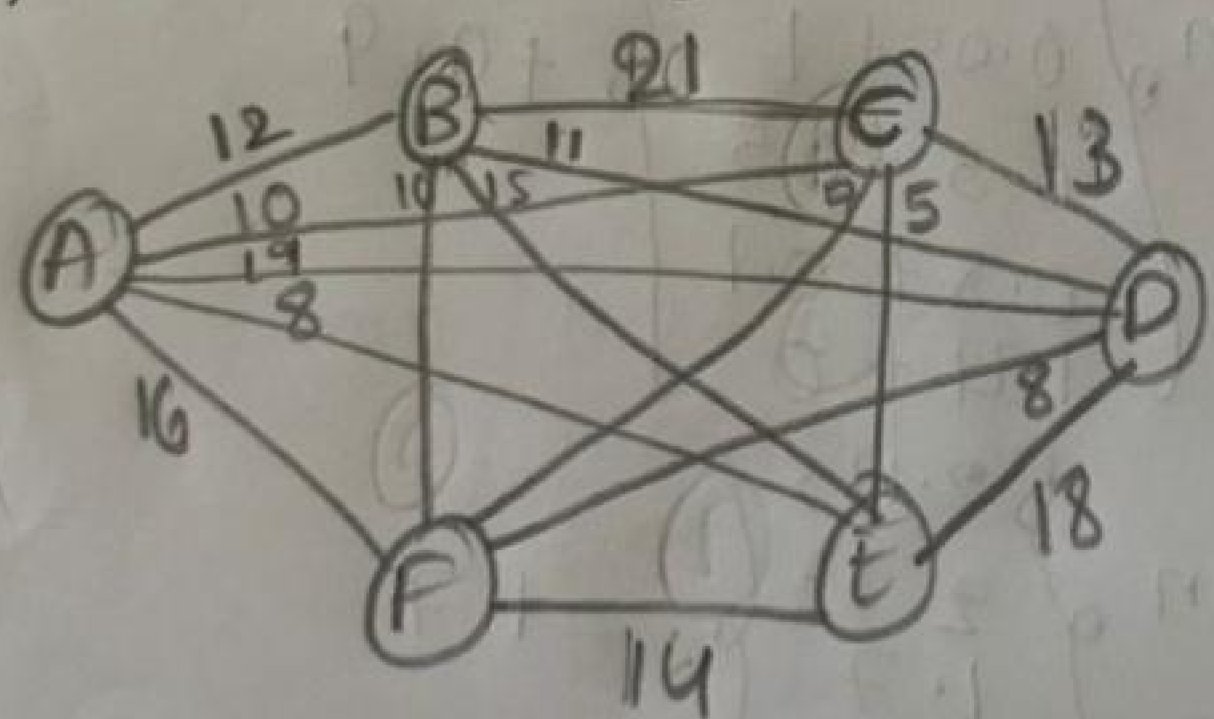
$$= \min \begin{cases} 0 + 1.7 \\ 0.1 + 1.55 \\ 0.2 + 1 \\ 0.55 + 0.5 \\ 1.05 + 0.1 \\ 1.75 + 0 \end{cases} + 1$$

$$= \min \begin{Bmatrix} 2.7 \\ 2.65 \\ 2.2 \\ 2.05 \\ 2.15 \\ 2.75 \end{Bmatrix} = 2.05.$$

(16) Extend the following distance matrix to 7 cities and solve the TSP

A: [0, 12, 10, 19, 8, 16]
B: [12, 0, 21, 11, 15, 10]
C: [10, 21, 0, 13, 5, 7]
D: [19, 11, 13, 0, 18, 8]
E: [8, 15, 5, 18, 0, 14]
F: [16, 10, 7, 8, 14, 0]

Sol:-



O/P → A – B – C – D – E – F = 83.

...n a knapsack capacity of 70 units and the ...lowing items:

...tem 1 : W = 25, V = 80

Item 2 : W = 35, V = 90

Item 3 : W = 45, V = 120

Item 4 : W = 30, V = 70

Use dp to solve 0/1 Knapsack problem.

Sol:-

V \ W	0	25	35	45	30	70
0	0	0	0	0	0	0
1	0	80	80	80	80	80
2	0	80	90	90	90	90
3	0	80	90	120	120	90
4	0	80	90	80	120	150

(18) For a graph

A → BA, ω − 1

A → CA, ω − 4

B → CB, ω − 3

B → DB, ω − 2

B → EB, ω = 2

D → BD, ω − 1

D → CD, ω − 5

E → DE, ω − 3

Use Bellman-Ford and solve it.

Sol:-

A → B = −1   B → D = 2   D → C = 5

A → C = 4   B → E = 2   E → D = −3

B → C = 3   D → B = 1

V	A	B	C	D	E
d	0	∞	∞	∞	∞
P	-	-	-	-	-

(1)

V	A	B	C	D	E
d	0	-1	4	∞	∞
P	-	A	A	-	-

(2)

V	A	B	C	D	E
d	0	-1	4	∞	1
P	-	A	A.	-	B

(3)

V	A	B	C	D	E
d	0	-1	4	3	1
P	-	A	A	E	B

(4)

V	A	B	C	D	E
d	0	-1	4	3	1
P	-	A	A	E	B

Path	distance	Shortest path
A	0	A
B	-1	A→B
C	4	A→C
D	3	A→E→D
E	1	A→B→C

(19) Find the expected value of the sum of outcomes when rolling 3 four-sided dice. Show your calculations and reasoning.

Sol:- Sum - 3(1+1+1)

sum

$4 = \frac{3}{64}$ (1+1+2, 1+2+1, 2+1+1)

$5 = \frac{6}{64}$ (1+1+3, 1+2+2, 1+3+1, 2+1+2, 2+2+1, 3+1+1)

$6 = \frac{10}{64}$ (1+2+3, 1+3+2, 2+1+3, 2+2+2, 2+3+1, 3+1+2, 3+2+1, 1+4+1, 2+2+2, 2+3+1)

$=7 \begin{pmatrix} 1+3+3,\ 2+2+3,\ 2+3+2,\ 3+1+3,\ 3+2+2,\ 3+3+1, \\ 1+4+2,\ 2+3+2,\ 2+4+1,\ 3+2+2,\ 3+3+1,\ 4+1+2 \end{pmatrix}$

$8 = 12/64.$

$9 = 10/64$

$10 = 6/64$

$11 = 3/64$

$12 = 1/64$

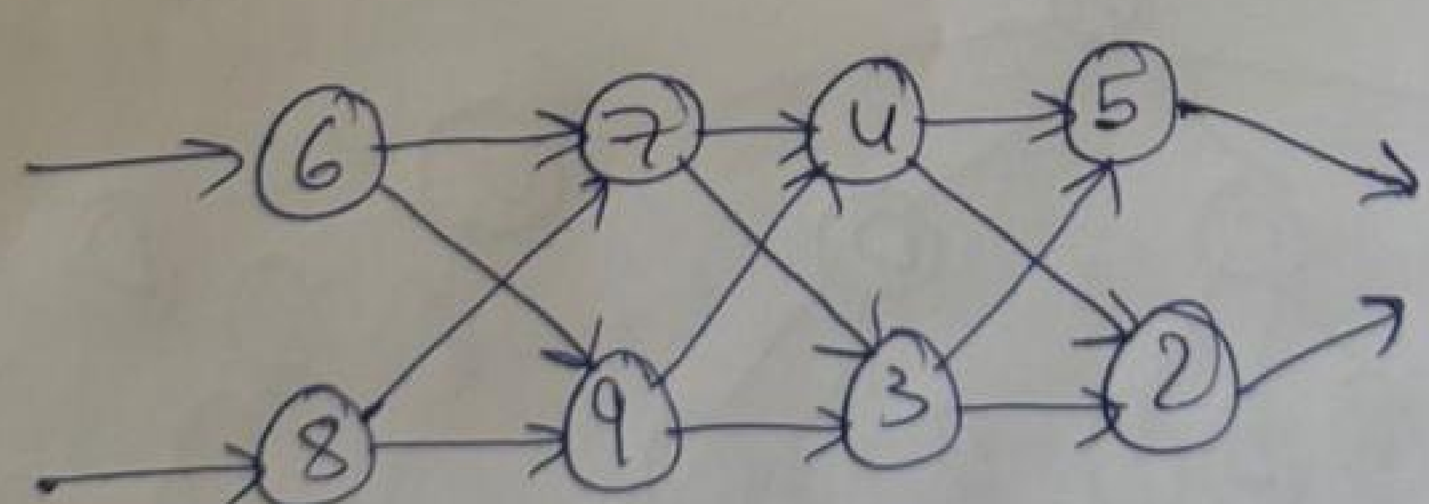
Σ(sum * probability)

$$= \left(3\times\frac{1}{64}\right)+\left(4\times\frac{3}{64}\right)+\left(5\times\frac{6}{64}\right)+\left(6\times\frac{10}{64}\right)+\left(7\times\frac{12}{64}\right)+\left(8\times\frac{12}{64}\right)$$
$$+\left(9\times\frac{10}{64}\right)+\left(10\times\frac{6}{64}\right)+\left(11\times\frac{3}{64}\right)+\left(12\times\frac{1}{64}\right)$$

$$= \frac{480}{64} = 7.5$$

(20) Calculate min. time for Line 1: [6, 7, 4, 5], Line 2: [8, 9, 3, 2] with transfer lines [4, 5, 6] 1 to 2 and [6, 5, 4] 2 to 1.

Sol:-



	1	2	3	4
$f_1[j]$	6	13	17	12
$f_2[j]$	8	17	20	22

	1	2	3	4
$L_1[j]$	1	1	1	1
$L_2[j]$	2	2	2	2

(21) Keys {10, 20, 30} have probabilities {0.2, 0.5, 0.3} do OBST.

Sol:- K = {10, 20, 30}

V = {0.2, 0.5, 0.3}

j - i = 3

3 - 0 = (0.3)

$$cost(0,3) = min \begin{Bmatrix} 2.1 \\ 1.5 \\ 1.1 \end{Bmatrix}$$

	0	1	2	3
0	0	0.2	[0.7]	[1.1]
1		0	0.5	[1.1]
2			0	0.3
3				0

(22) Using 5 cities

A : [0, 14, 4, 10, 20]

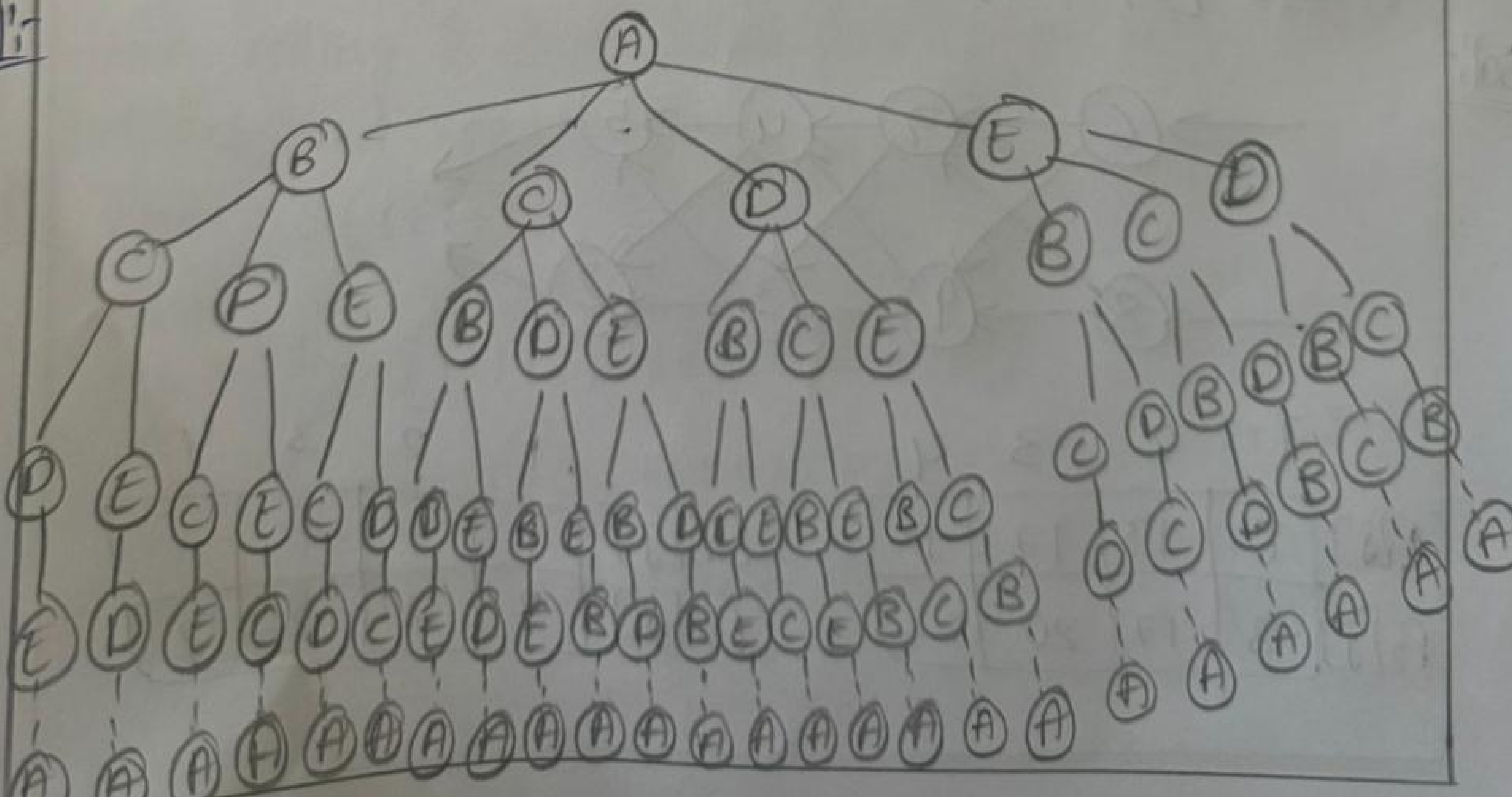
B : [14, 0, 7, 8, 7]

C : [4, 7, 0, 12, 6]

D : [10, 8, 12, 0, 15]

E : [20, 7, 6, 15, 0]

Sol:-



Knapsack 0/1 50 units

I-1 = W=10, V=50

I-2 = W=20, V=70

I-3 = W=30, V=90

I-4 = W=25, V=60

I-5 = W=15, V=40.

Sol:-

	0	10	20	30	25	15	50
0	0	0	0	0	0	0	0
1	0	50	50	50	50	50	50
2	0	50	70	70	70	70	70
3	0	50	70	90	90	90	160
4	0	50	70	90	90	90	160
5	0	50	70	90	90	90	160

(24) Bellman-Ford

1→21, W=4

1→31, W=5

2→32, W=-2

3→43, W=3

4→24, W=-10

Sol:-

1→2 -4

1→3 - 5

2→3 -3

4→2=-10

V	1	2	3	4
d	0	∞	∞	∞
P	-	-	-	-

V	1	2	3	4
d	0	4	5	∞
P	-	1	1	-

V	1	2	3	4
d	0	4	2	∞
P	-	1	2	-

V	1	2	3	4
d	0	4	2	5
P	-	1	2	3

Vertex	Dist	path
1	0	1
2	4	$1 \to 2$
3	2	$1 \to 2 \to 3$
4	5	$1 \to 2 \to 3 \to 4$

(25) Roll six six-sided dice. Determine the no. of ways to get a sum of 18, ensuring that at least one die shows a 6.

Sol:- $x + x^2 + x^3 + x^4 + x^5 + x^6$

$$\Rightarrow x(1 + x + x^2 + x^3 + x^4 + x^5)$$

$$= \frac{x(1+x^6)}{1-x}$$

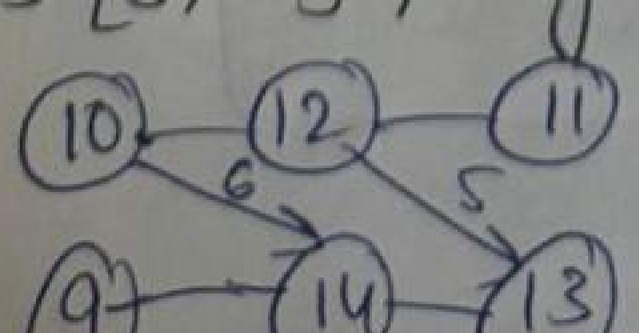
For six dice

$$\left(\frac{x(1+x^6)}{1-x}\right)^6 = x^6(1-x^6)^6(1-x)^{-6} = (x^{18})$$

$$= 340.4$$

(26) Given Line 1: [10, 12, 11], Line 2: [9, 14, 13], Transfer lines [6, 5], by 2 units.

Sol:-



...e Reduction

L1	6	5
L2	30	30

After Red(2)

L1	4	5
L2	28	27

(27) For keys (8, 12, 16, 20, 24) with access possibilities {0.2, 0.05, 0.4, 0.25, 0.1}. Determine OBST using dp.

Sol'n (8, 12, 16, 20, 24)

(0.2, 0.05, 0.4, 0.25, 0.1)

	0	1	2	3	4	5
0	0	0.2	0.3	0.7	1.45	1.8
1		0	0.05	0.5	1	1.3
2			0	0.4	0.9	1.2
3				0	0.25	0.05
4					0	0.1
5						0

$j-i=0$

$j-i=1$

$j-i=2$

$2-0=[0,2]$

$3-1=[1,3]$

$4-2=[2,4]$

$5-3=[3,5]$

$$cost(1,5) = \min\begin{Bmatrix} cost(1,1)+cost(2,5) \\ cost(1,2)+cost(3,5) \\ cost(1,3)+cost(4,5) \\ cost(1,4)+cost(5,5) \end{Bmatrix} + 0.8$$

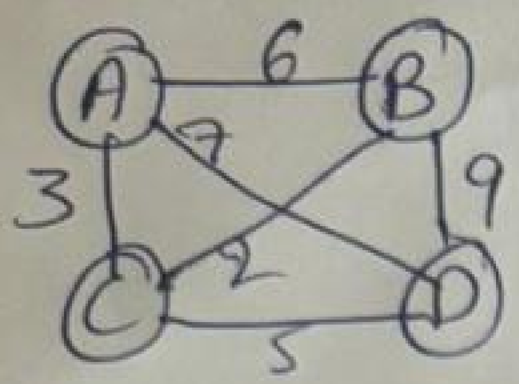
$$= \min\begin{Bmatrix} 0+1.2 \\ 0.05+0.45 \\ 0.5+0.1 \\ 1+0 \end{Bmatrix} + 0.8 = \min\begin{Bmatrix} 2 \\ 1.3 \\ 1.4 \\ 1.8 \end{Bmatrix} = 1.3$$

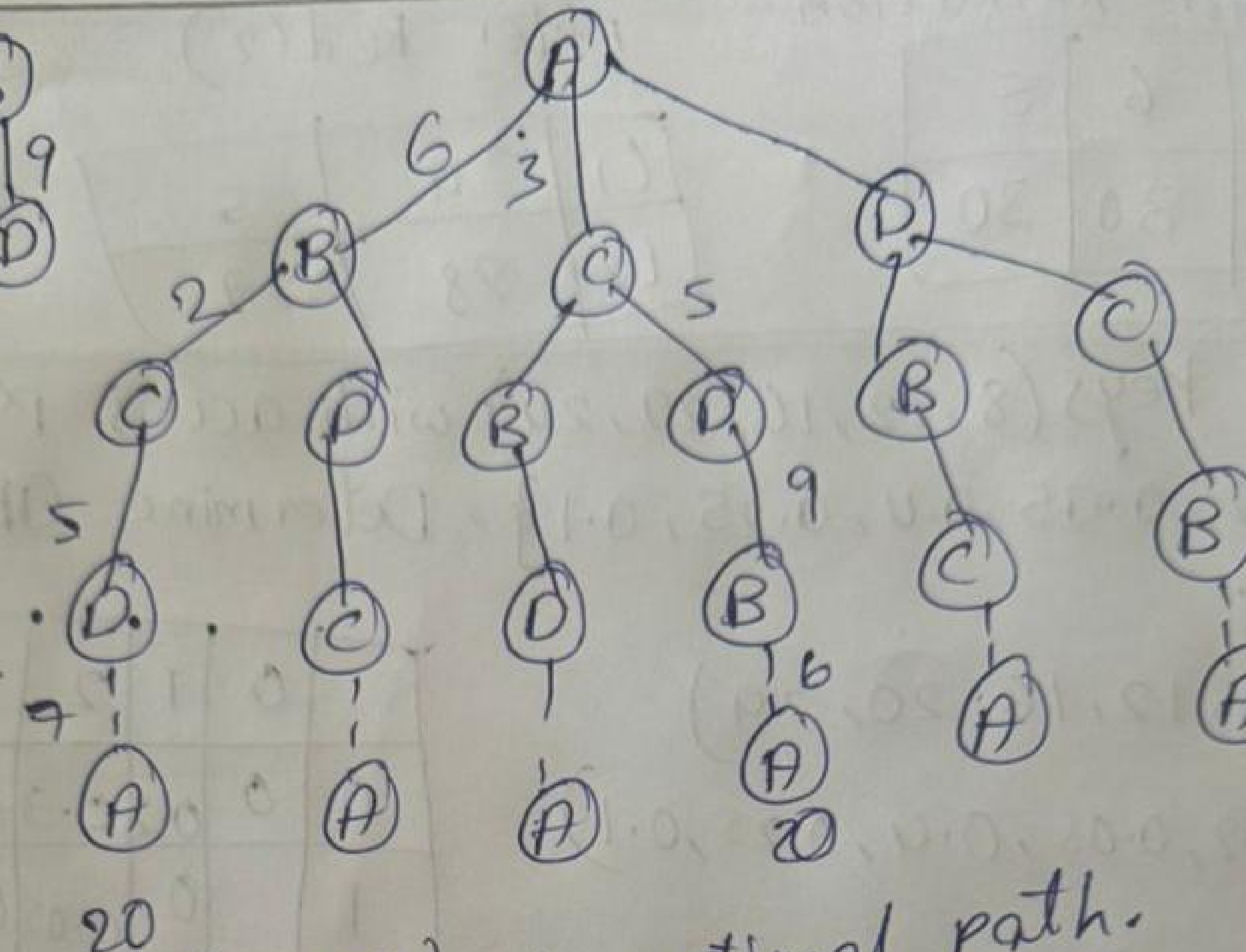
(28) Solve TSP 4 cities

A: [0, 6, 3, 7] C: [3, 2, 0, 5]

B: [6, 0, 2, 9] D: [7, 9, 5, 0]

Sol:-





A – B – C – D – A – 20 } Min optimal path.
A – D – C – B – A – 20

(29) Knapsack 0/1 50 units

I_1, w = 10, v = 60

I_2, w = 20, v = 100

I_3, w = 30, v = 120

I_4, w = 40, v = 200

Sol:-

v\W	0	10	20	30	40	50
0	0	0	0	0	0	0
1	0	60	60	60	60	60
2	0	60	100	160	160	160
3	0	60	100	120	180	180
4	0	60	100	120	200	260

llman -Ford

A→BA, ω=6

A→DA, ω=7

B→CB, ω=5

B→EB, ω=4

B→DB, ω=8

C→BC, ω=-2

D→CD, ω=-3

D→ED, ω=-9

E→FE, ω=7

F→CF, ω=2

Sol:-

V	A	B	C	D	E	F
d	0	∞	∞	∞	∞	∞
P	-	-	-	-	-	-

(1)

V	A	B	C	D	E	F
d	0	6	4	7	2	9
P	-	A	D	A	B	E

(2)

V	A	B	C	D	E	F
d	0	2	4	7	2	9
P	-	C	D	A	B	E

(3)

V	A	B	C	D	E	F
d	0	2	4	7	2	9
P	-	C	D	A	B	E

(4)

V	A	B	C	D	E	F
d	0	2	4	7	2	9
P	-	C	D	A	B	E

(5)

V	A	B	C	D	E	F
d	0	2	4	7	2	9
P	-	C	D	A	B	E

Verten	Dist	Path
A	0	A
B	2	A-D-C-B
C	4	A-D-C
D	7	A-D
E	2	A-D-C-B-E
F	9	A-D-C-B-E-F