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CSA0672
Design and analysis of Algorithm.

① Calculate the number of ways to achieve a sum of 15 when rolling four six-sided dice. provide a detailed step-by-step solution.

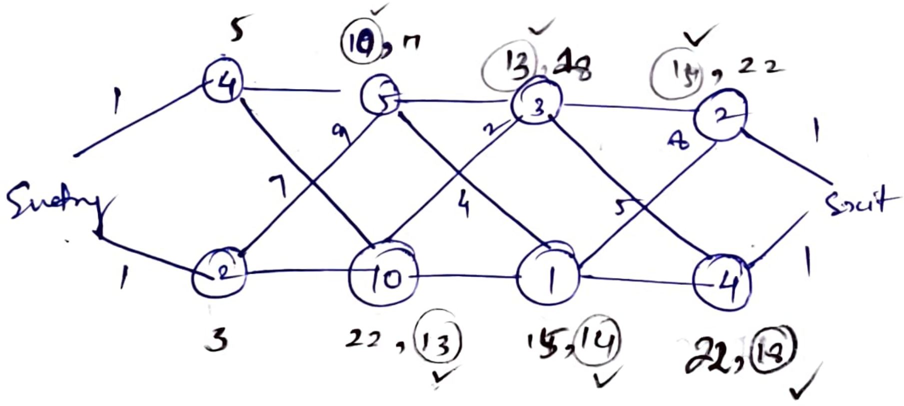
$$\binom{11+4-1}{4-1} = \binom{14}{3} = \frac{14\times13\times12}{3\times2\times1} = 364$$

$$\binom{5-4-1}{4-1} = \binom{8}{3} = \frac{8\times7\times6}{3\times2\times1} = 56$$

since any of four variables $= 4\times56 = 224$

no. of valid solution $\Rightarrow 364-224 = 140$

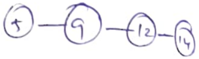
② Two assembly lines have station times as follows: Line1: [4,5,3,2], Line2: [2,10,1,4]. Transfer times between lines are: from line1 to line2: [7,4,5], from line2 to line1: [9,2,8]. Calculate the minimum time to assemble a product.



minimum cost :- 4 5 3 2

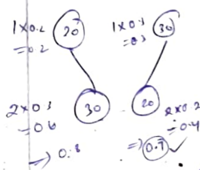
F[i]	5	10	13	15
F[j]	2	13	14	18

path :-



F[i]	1	1	1	1
F[j]	2	2	~~1~~, 2	2

③ Given keys {10, 20, 30, 40} with access probabilities {0.1, 0.2, 0.3, 0.4} respectively. construct the optimal BST. calculate total cost of tree.

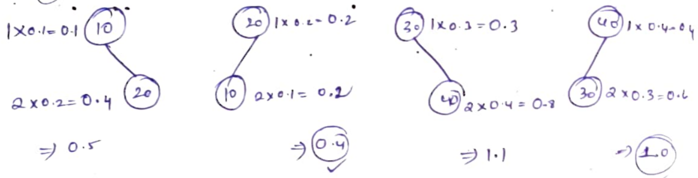


$\{ \overset{1}{10}, \overset{2}{20}, \overset{3}{30}, \overset{4}{40} \}$

$\{ 0.1, 0.2, 0.3, 0.4 \}$

(0,3) = 0.1, 0.2, 0.3 $\boxed{0.6}$

(0,4)



$$\text{cost}(i, j) = \min\{\text{cost}(i, k-1) + \text{cost}(k, j)\} + w_i$$

i) $\text{cost}(0,3) = \min \begin{cases} \text{cost}(0,0) + \text{cost}(1,3) \\ \text{cost}(0,1) + \text{cost}(2,3) \\ \text{cost}(0,2) + \text{cost}(3,3) \end{cases} + 0.6$

k = 1, 2, 3

$= \min \begin{cases} 0 + 0.7 \\ 0.1 + 0.3 \\ 0.4 + 0 \end{cases} + 0.6$

$= \min \begin{cases} 1.3 \\ 1.0 \\ 1.0 ✓ \end{cases}$

j \ i	0	1	2	3	4
0	0	0.1	0.4	0.1	1.1
1		0	0.2	0.7	1.5
2			0	0.3	1.0
3				0	0.4
4					0

ii) $\text{cost}(1,4) = \min \begin{cases} \text{cost}(1,1) + \text{cost}(2,4) \\ \text{cost}(1,2) + \text{cost}(3,4) \\ \text{cost}(1,3) + \text{cost}(4,4) \end{cases} + 0.9$

k = 2, 3, 4

0.2 + 0.3 + 0.4 = 0.9

$= \min \begin{cases} 0 + 1.0 \\ 0.2 + 0.4 \\ 0.7 + 0 \end{cases} + 0.9$

$= \min \begin{cases} 1.9 \\ 1.5 ✓ \\ 1.6 \end{cases}$

0.1 + 0.2 + 0.3 + 0.4

iii) $\text{cost}(i,j) = \min \begin{cases} \text{cost}(0,0) + \text{cost}(1,4) \\ \text{cost}(0,1) + \text{cost}(2,4) \\ \text{cost}(0,2) + \text{cost}(3,4) \\ \text{cost}(0,3) + \text{cost}(4,4) \end{cases} + 1.0$

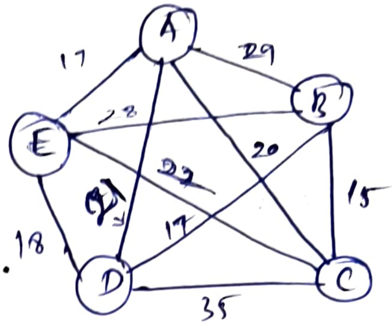
k = 1, 2, 3, 4

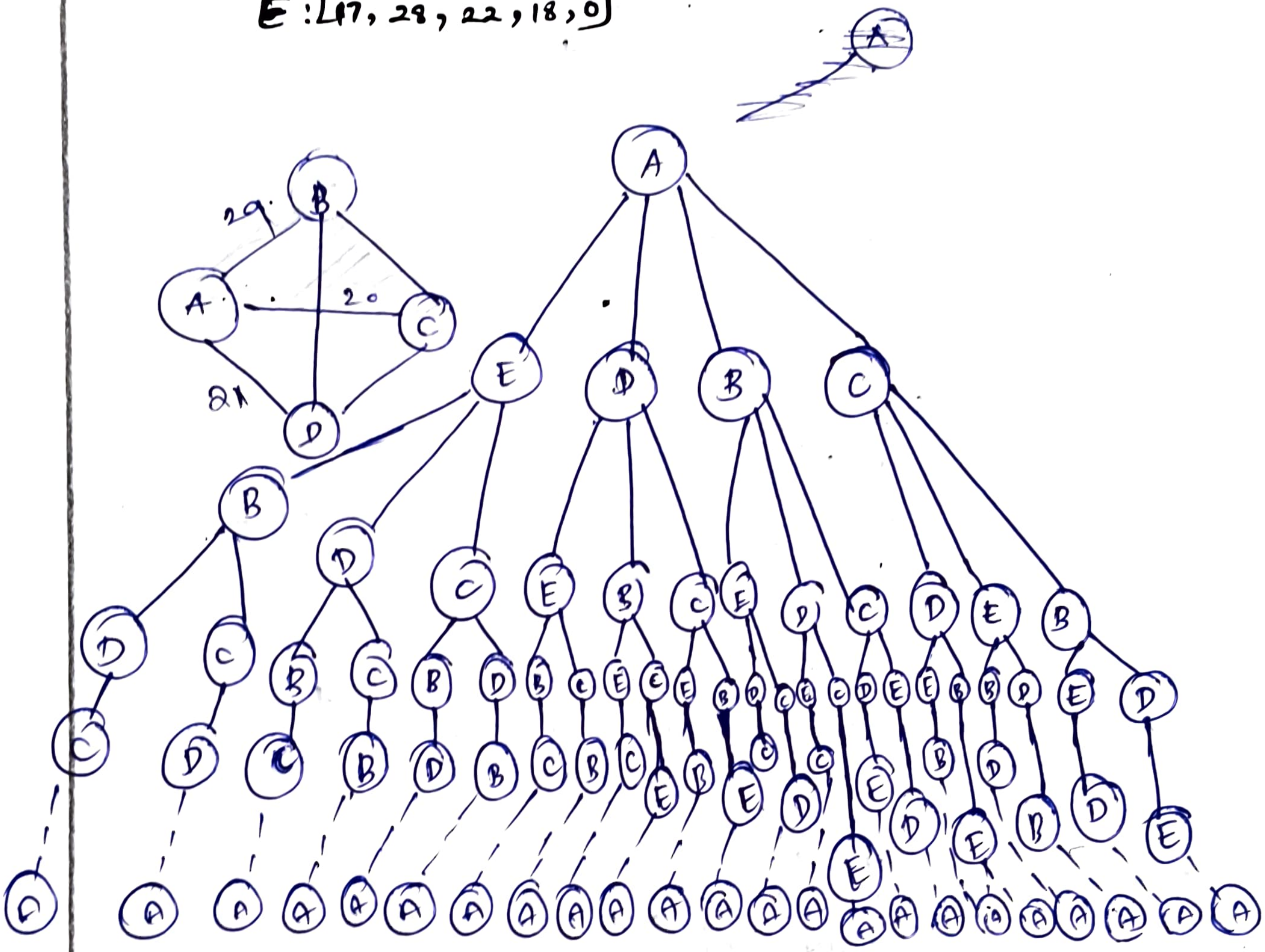
$= \min \begin{cases} 0 + 1.5 \\ 0.1 + 1.0 \\ 0.4 + 0.4 \\ 0.1 + 0 \end{cases} + 1.0$

$= \min \begin{cases} 2.5 \\ 2.1 \\ 1.8 \\ 1.1 ✓ \end{cases}$

④ Solve the TSP for the following 5-city distance matrix using dynamic programming

S→
$$\begin{array}{cccccc} & A & B & C & D & E \\ A: & [0, & 29, & 20, & 21, & 17] \\ B: & [29, & 0, & 15, & 17, & 28] \\ C: & [20, & 15, & 0, & 35, & 22] \\ D: & [21, & 17, & 35, & 0, & 18] \\ E: & [17, & 28, & 22, & 18, & 0] \end{array}$$





⑤ Knapsack with capacity 50 units.

item1 : weight =10 , value = 60

item2: weight =20 , value =100

item3: weight >30 , value -120

Item 4: weight =40 , value =200

item	1	2	3	4
	✓		✗	✓
weight	10	20	30	40
profit	60	100	120	200
$\frac{\text{profit}}{\text{weight}}$	6	5	4	5

M = 50

~~50 − 10 = 40~~

~~40 − 20 = 10~~

~~39 − 4 = 35~~

~~35 − 5 = 30~~

50 − 10 = 40

40 − 40 = 0

$Ei \times Wi = 1 \times 10 + 1 \times 40$

$= 10 + 40$

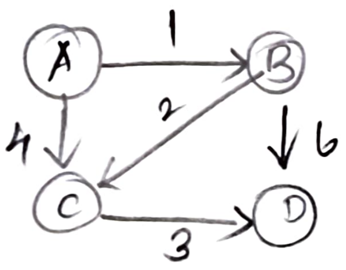
$= 50$ //

$Ei \times pi = 1 \times 60 + 1 \times 200$

$= 60 + 200$

$= 260$ //

⑥ Given the following directed graph with vertices A, B, C, DA, B, C, DA, B, C, D and edges with weights:



using Bellman ford

Initialize

v	A	B	C	D
d	0	∞	∞	∞
p	-	-	-	-

(1)

v	A	B	8	D
A	0	1	4	∞
p	–	A	A	∞

(2)

v	A	B	C	D
d	0	1	3	6
p	-	A	B	B

A → B

A → C

A → D

$A \xrightarrow{1} B$

$A \xrightarrow{1} B \xrightarrow{2} C$

$A \xrightarrow{1} B \xrightarrow{2} C \xrightarrow{3} D$

(4)

Given, sum is exactly 20

$6^5 = 7776$

$x_1 + x_2 + x_3 + x_4 + x_5 = 20$ where $1 \le x_i \le 6$

$y_i = x_i - 1$ for $i = 1, 2, 3, 4, 5$

$(y_1+1) + (y_2+1) + (y_3+1) + (y_4+1) + (y_5+1) = 20$

$y_1 + y_2 + y_3 + y_4 + y_5 = 15$

where $0 \le y_i \le 5$

By "stars and bars"

$$\binom{15+5-1}{5-1} = \binom{19}{4}$$

$$\binom{19}{4} = \frac{19 \times 18 \times 17 \times 16}{4 \times 3 \times 2 \times 1} = 3876$$

→ $y_1 \ge 6$, let $y_1' = y_1 - 6$

$y_1' + y_2 + y_3 + y_4 + y_5 = 9$

$$\binom{9+5-1}{5-1} = \binom{13}{4}$$

$$\binom{13}{4} = \frac{13 \times 12 \times 11 \times 10}{4 \times 3 \times 2 \times 1} = 715$$

There are 5 such variables.

$5 \times 715 = 3575$

→ two variables $y_i, y_j \ge 6$, let $y_i' = y_i - 6$ and $y_j' = y_j - 6$

$y_1' + y_2' + y_3 + y_4 + y_5 = 3$

$$\binom{3+5-1}{5-1} = \binom{7}{4}$$

$$\binom{7}{4} = \frac{7 \times 6 \times 5 \times 4}{4 \times 3 \times 2 \times 1} = 35$$

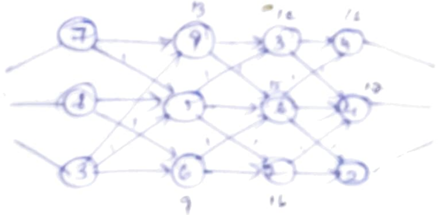
$\binom{5}{2} \times 35 = 10 \times 35 = 350$

using the Inclusion – exclusion principle

$3876 - 3575 + 350 = 651$.

$\frac{651}{7776} = \frac{651}{7776} \approx 0.0837$

(2)



$$F[j] = \min \{ (f_1(j-1) + a_{1j}), (f_2(j-1) + (t_2, j-1) + a_{1j}), (f_3(j-1) + (t_3, j-1) + a_{1j}) \}$$

$$= \min \{ (7+9), (8+1+9), (3+1+9) \} = 13$$

	1	2	3	4
$F_1[j]$	7	13	12	16
$F_2[j]$	8	9	15	17
$F_3[j]$	3	9	16	15

	1	2	3	4
$L_1[j]$	1	3	1.	1
$L_2[j]$	2	3	2	1
$L_3[j]$	3	3	3	1

⑨ Determin OBST :-

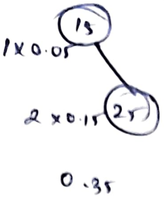
1 2 3 4 5
{15 ; 25, 35, 45, 55}

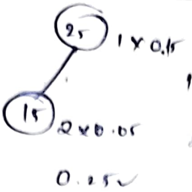
{0.05, 0.15, 0.4, 0.25, 0.15}

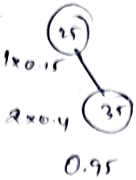
$j - i = 1$
$1 - 0 = 1$ (0,1) (1,1)
$2 - 1 = 1$ (1,2) (2,2)
$3 - 2 = 1$ (2,3) (3,3)
$4 - 3 = 1$ (3,4) (4,4)
$5 - 4 = 1$ (4,5) (5,5)

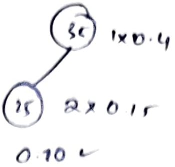
$j - i = 2$
$2 - 0 = 2$ (0,2) (1,2)
$3 - 1 = 1$ (1,3) (2,3)
$4 - 2 = 2$ (2,4) (3,4)
$5 - 3 > 2$ (3,5) (4,5)

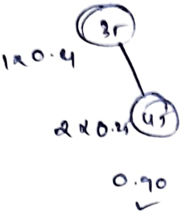
	0	1	2	3	4	5
0	0	0.05	0.25	0.85	1.35	1.80
1		0	0.15	0.70	1.20	1.80
2			0	0.4	0.90	1.35
3				0	0.25	0.55
4					0	0.15
5						0

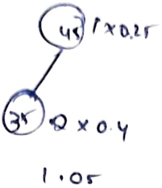


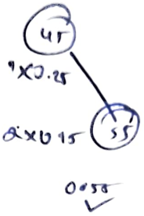


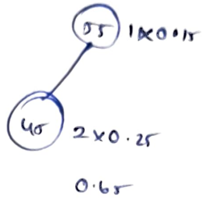












$j-i = 3$

$3-0 = 3$ $(0,3)$ $(1,3)$

$4-1 = 3$ $(1,4)$ $(2,4)$

$5-2 = 3$ $(2,5)$ $(3,5)$

$$\text{cost}(i,j) = \min\{\text{cost}(i,k-1), \text{cost}(k,j)\} + w_i$$

$$\text{cost}(0,3) = \min_{k=1,2,3} \left\{ \begin{array}{l} \text{cost}(0,0) + \text{cost}(1,3) \\ \text{cost}(0,1) + \text{cost}(2,3) \\ \text{cost}(0,2) + \text{cost}(3,3) \end{array} \right\} + 0.6$$

$$= \min \left\{ \begin{array}{l} 0+0.70 \\ 0.05+0.4 \\ 0.25+0 \end{array} \right\} + 0.6$$

$$= \min \left\{ \begin{array}{l} 1.30 \\ 1.05 \\ 0.85 \end{array} \right\} = 0.85$$

$$\text{cost}(1,4) = \min_{k=2,3,4} \left\{ \begin{array}{l} \text{cost}(1,1) + \text{cost}(2,4) \\ \text{cost}(1,2) + \text{cost}(3,4) \\ \text{cost}(1,3) + \text{cost}(4,4) \end{array} \right\} + 0.8$$

$$= \min \left\{ \begin{array}{l} 0+0.90 \\ 0.15+0.25 \\ 0.70+0 \end{array} \right\} + 0.8$$

$$= \min \left\{ \begin{array}{l} 1.70 \\ 1.20 \\ 1.50 \end{array} \right\} = 1.20$$

$$\text{cost}(2,5) = \min_{k=3,4,5} \begin{Bmatrix} \text{cost}(2,2) + \text{cost}(3,5) \\ \text{cost}(2,3) + \text{cost}(4,5) \\ \text{cost}(2,4) + \text{cost}(5,5) \end{Bmatrix} + 0.8$$

$$= \min \begin{Bmatrix} [illegible] + 0.55 \\ 0.4 + 0.15 \\ 0.90 + 0 \end{Bmatrix} + 0.8$$

$$= \min \begin{Bmatrix} 1.35 \\ 1.35 \\ 1.70 \end{Bmatrix} = 1.35$$

~~cost~~ · $j - i = 4$

$4 - 0 = 4$ (0,4) (1,4)

$5 - 1 = 4$ (1,5) (2,5)

$$\text{cost}(0,4) = \min_{k=1,2,3,4} \begin{Bmatrix} \text{cost}(0,0) + \text{cost}(1,4) \\ \text{cost}(0,1) + \text{cost}(2,4) \\ \text{cost}(0,2) + \text{cost}(3,4) \\ \text{cost}(0,3) + \text{cost}(4,4) \end{Bmatrix} + 0.85$$

$$= \min \begin{Bmatrix} 0 + 1.20 \\ 0.05 + 0.90 \\ 0.25 + 0.25 \\ 0.85 + 0 \end{Bmatrix} + 0.85$$

$$= \min \begin{Bmatrix} 2.05 \\ 1.80 \\ 1.35 \\ 1.70 \end{Bmatrix} = 1.35$$

$$\text{cost}(1,5) = \min_{k=2,3,4,5} \begin{Bmatrix} \text{cost}(1,1) + \text{cost}(2,5) \\ \text{cost}(1,2) + \text{cost}(3,5) \\ \text{cost}(1,3) + \text{cost}(4,5) \\ \text{cost}(1,4) + \text{cost}(5,5) \end{Bmatrix} + 0.95$$

$$= \min \begin{Bmatrix} 2.50 \\ 2.45 \\ 1.80 \\ 2.15 \end{Bmatrix} = 1.80$$

$$\text{cost}(0,5) = \min_{k=1,2,3,4,5} \begin{Bmatrix} \text{cost}(0,0) + \text{cost}(1,5) \\ \text{cost}(0,1) + \text{cost}(2,5) \\ \text{cost}(0,2) + \text{cost}(3,5) \\ \text{cost}(0,3) + \text{cost}(4,5) \\ \text{cost}(0,4) + \text{cost}(5,5) \end{Bmatrix} + 1$$

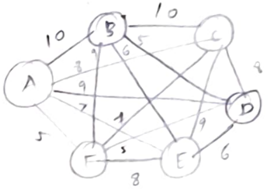
$$= \min \begin{Bmatrix} 2.80 \\ 2.65 \\ 1.80 \\ 2.00 \\ 2.35 \end{Bmatrix} = 1.80$$

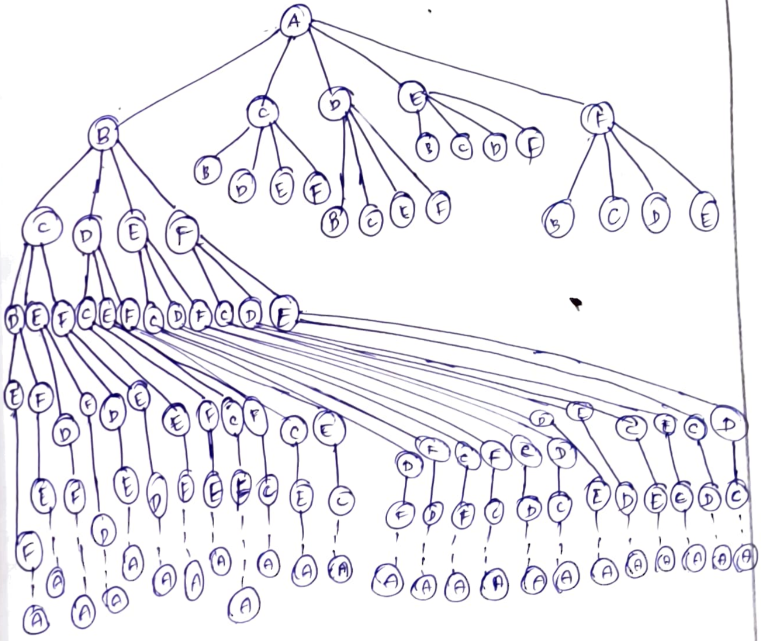
~~= Min { 2.~~

⑩ TSP

Given that,

6 cities.





O/P Ⓐ — Ⓕ — Ⓔ — Ⓓ — Ⓒ — Ⓑ — Ⓐ = 4.7

⑪

i	wi	val (p)
1	20	100
2	30	120
3	10	60

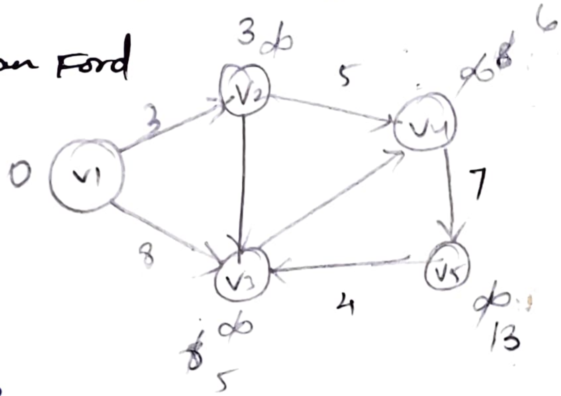
M = 60 units

v\w	0	10	20	30	40	50	60
0	0	0	0	0	0	0	0
1	0	0	100	100	100	100	100
2	0	0	100	120	120	220	220
3	0	10	100	120	180	220	280

Formula

$$v[i, w] = \max\{v[i-1, w], v[i-1, w-w[i]] + value[i]\}$$

⑫ **Bellman Ford**



v1 → v2 3
v1 → v3 8
v2 → v3 2
v2 → v4 5
v3 → v4 1
v4 → v5 7
v5 → v3 4

Initialize

v	v1	v2	v3	v4	v5
d	0	∞	∞	∞	∞
p	—	—	—	—	—

①

v	v1	v2	v3	v4	v5
d	0	3	8	∞	∞
p	–	v1	v1	v2 \ v4	

②

v	v1	v2	v3	v4	v5
d	0	3	5	8	∞
p	–	v1	v2	v2	v4

two Eight sided dice, sum of 10. Then Extend this to three dice and find a new no. of. ways to get the same sum

we have to count the pairs (x,y) such that

$x+y=10$ where $1 \leq x, y \leq 8$

possible pairs :-

$(x,y) = (2,8)$
$(x,y) = (3,7)$
$(x,y) = (4,6)$
$(x,y) = (5,5)$
$(x,y) = (6,4)$
$(x,y) = (7,3)$
$(x,y) = (8,2)$

No. of. ways to achieve a sum of 10 is 7.

we need to count the no. of. triples (x,y,z) such that $x+y+z=10$ where $1 \leq x, y, z \leq 8$.

1) $x=1$

$y+z=9$

$(1,1,8)$, $(1,2,7)$, $(1,3,6)$, $(1,4,5)$, $(1,5,4)$, $(1,6,3)$, $(1,7,2)$, $(1,8,1)$

2) $x=2$

$y+z=8$:

$(2,1,7)$, $(2,2,6)$, $(2,3,5)$, $(2,4,4)$, $(2,5,3)$, $(2,6,2)$, $(2,7,1)$

3) $x=3$

$y+z=7$

$(3,1,4)$, $(3,2,5)$, $(3,3,4)$, $(3,4,3)$, $(3,5,2)$, $(3,6,1)$

4) $x=4$

$y+z=6$:

$(4,1,5)$ $(4,2,4)$ $(4,3,3)$ $(4,4,2)$, $(4,5,1)$

5) $x=5$

$y+z=5$

$(5,1,4)$, $(5,2,3)$, $(5,3,2)$, $(5,4,1)$

6) $x=6$

$y+z=4$

(6,1,3), (6,2,2), (6,3,1)

7) $x=7$

$y+z=3$

(7,1,2) (7,2,1)

So, the no of ways to a sum of 10 = 36

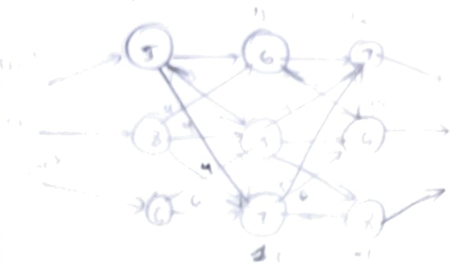
8) $x=8$

$y+z=2$

(8,1,1)

sum = 8+7+6+5+4+3+2+1 = 36

⑭ Assembly line scheduling



	1	2	3
$f_1[j]$	5	11	17
$f_2[j]$	8	13	19
$f_3[j]$	6	13	21

	1	2	3
$l_1[j]$	1 ←	1 ←	1
$l_2[j]$	2	2	2
$l_3[j]$	3	3	3

$$F[j] = \min\{f_1(j-1)+a_{1j}),(f_2(j-1)+(t_{2,j-1})+a_{1j}),$$

$$(f_3(j-1)+(t_{3,j-1})+a_{1j})\}$$

$$= \min\{11, 11, 17\} = 11$$

⑮ OBST

key = {5, 15, 25, 35, 45, 55}

prob = {0.1, 0.05, 0.2, 0.15, 0.1, 0.1}

j − i = 1	j − i = 2
1 − 0 = 1	2 − 0 = 2
2 − 1 = 1	3 − 1 = 2
3 − 2 = 1	4 − 2 = 2
4 − 3 = 1	5 − 3 = 2
5 − 4 = 1	6 − 4 = 2
6 − 5 = 1	

j − i = 3

3 − 0 = 3

4 − 1 = 3

5 − 2 = 3

6 − 3 = 3

$$\text{cost}(i,j) = \min\{(\text{cost}(i,k-1) + \text{cost}(k,j)\} \text{ for } \ldots$$

$$\text{cost}(0,3) = \min\left\{\begin{array}{l}\text{cost}(0,0) + \text{cost}(1,3)\\ \text{cost}(0,1) + \text{cost}(2,3)\\ \text{cost}(0,2) + \text{cost}(3,3)\end{array}\right\} + 0.35$$

$k = 1, 2, 3$

$$= \min\left\{\begin{array}{l}0.65\\ 1.1\\ 0.55\end{array}\right\} = 0.55$$

$$\text{cost}(1,4) = \min\left\{\begin{array}{l}\text{cost}(1,1) + \text{cost}(2,4)\\ \text{cost}(1,2) + \text{cost}(3,4)\\ \text{cost}(1,3) + \text{cost}(4,4)\end{array}\right\} + 0.5$$

$k = 2, 3, 4$

$$= \min\left\{\begin{array}{l}1.15\\ 0.9\\ 0.8\end{array}\right\} = 0.8$$

$$\text{cost}(2,5) = \min\left\{\begin{array}{l}\text{cost}(2,2) + \text{cost}(3,5)\\ \text{cost}(2,3) + \text{cost}(4,5)\\ \text{cost}(2,4) + \text{cost}(5,5)\end{array}\right\} + 0.55$$

$k = 3, 4, 5$

$$= \min\left\{\begin{array}{l}1.45\\ 1\\ 1.45\end{array}\right\} = 1$$

$j - i = 4$

$4 - 0 = 4$

$5 - 1 = 4$

$6 - 2, 4$

$$\text{cost}(0,4) = \left\{ \begin{array}{l} \text{cost}(0,0) + \text{cost}(1,4) \\ \text{cost}(0,1) + \text{cost}(2,4) \\ \text{cost}(0,2) + \text{cost}(3,4) \\ \text{cost}(0,3) + \text{cost}(4,4) \end{array} \right\} + 0.6$$

$k = 1, 2, 3, 4$

$$= \left\{ \begin{array}{l} 1.4 \\ 1.35 \\ 1.05 \\ 1.15 \end{array} \right\} = 1.05$$

$$\text{cost}(1,5) = \min \left\{ \begin{array}{l} \text{cost}(1,1) + \text{cost}(2,5) \\ \text{cost}(1,2) + \text{cost}(3,5) \\ \text{cost}(1,3) + \text{cost}(4,5) \\ \text{cost}(1,4) + \text{cost}(5,5) \end{array} \right\} + 0.8$$

$k = 2, 3, 4, 5$

$$= \left\{ \begin{array}{l} 2.05 \\ 1.65 \\ 1.4 \\ 1.6 \end{array} \right\} = 1.4$$

$$\text{cost}(2,6) = \min \left\{ \begin{array}{l} \text{cost}(2,2) + \text{cost}(3,6) \\ \text{cost}(2,3) + \text{cost}(4,6) \\ \text{cost}(2,4) + \text{cost}(5,6) \\ \text{cost}(2,5) + \text{cost}(6,6) \end{array} \right\} + 0.85$$

$k = 3, 4, 5, 6$

$$= \min \left\{ \begin{array}{l} 1.85 \\ 1.55 \\ 1.4 \\ 2.1 \end{array} \right\} = 1.85$$

$j - i - 5$

$5 - 0 = 5$

$6 - 1 = 5$

$$\text{cost}(0,5) = \min \left\{ \begin{array}{l} \text{cost}(0,0) + \text{cost}(1,5) \\ \text{cost}(0,1) + \text{cost}(2,5) \\ \text{cost}(0,2) + \text{cost}(3,5) \\ \text{cost}(0,4) + \text{cost}(5,5) \end{array} \right\} + 0.9$$

$k = 1, 2, 3, 4, 5$

$$= \min \left\{ \begin{array}{l} 2.3 \\ 2.25 \\ 1.9 \\ 1.75 \\ 1.95 \end{array} \right\} = 1.75$$

$$\text{cost}(1,6) = \min \left\{ \begin{array}{l} \text{cost}(1,1) + \text{cost}(2,6) \\ \text{cost}(1,2) + \text{cost}(3,6) \\ \text{cost}(1,3) + \text{cost}(4,6) \\ \text{cost}(1,4) + \text{cost}(5,6) \\ \text{cost}(1,5) + \text{cost}(6,6) \end{array} \right\} + 0.9$$

$k = 2, 3, 4, 5, 6$

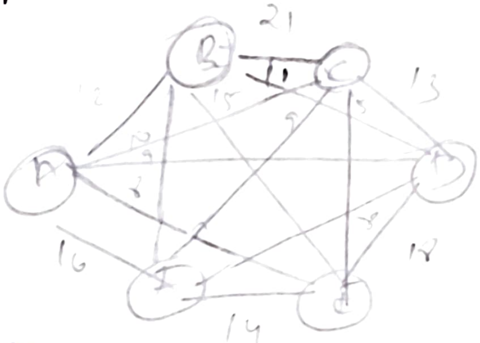
15.) Assembly line

$$= \min \left\{ \begin{matrix} 2 \\ 1.7 \\ 1.8 \\ 2.3 \end{matrix} \right\} = 1.7$$

$$\text{cost }(0,6) = \min_{k=1,2,3,4,5,6} \left\{ \begin{matrix} \text{cost }(0,0) + \text{cost }(1,6) \\ \text{cost }(0,1) + \text{cost }(2,6) \\ \text{cost }(0,2) + \text{cost }(3,6) \\ \text{cost }(0,3) + \text{cost }(4,6) \\ \text{cost }(0,4) + \text{cost }(5,6) \\ \text{cost }(0,5) + \text{cost }(6,6) \end{matrix} \right\} + 1$$

$$= \min \left\{ \begin{matrix} 2.7 \\ 2.65 \\ 2.2 \\ 2.05 \\ 2.15 \\ 2.75 \end{matrix} \right\} = 2.05$$

⑯ TSP



O/P $A \rightarrow B \rightarrow C \rightarrow D \rightarrow E \rightarrow F = 83$ //

⑰ Knapsack problem

v|w	0	25	35	45	30	70
0	0	0	0	0	0	0
1	0	80	80	80	80	80
2	0	80	90	90	90	90
3	0	80	90	120	120	90
4	0	80	90	80	120	150

⑱ Bellman ford

V	A	B	C	D	E
d	0	∞	∞	∞	∞
p	-	-	-	-	-

①

V	A	B	C	D	E
d	0	-1	4	∞	∞
p	-	A	A	-	-

②

V	A	B	C	D	E
d	0	-1	4	∞	1
p	-	A	A	-	B

③

V	A	B	C	D	E
d	0	-1	4	3	1
p	-	A	A	E	B

④

V	A	B	C	D	E
d	0	-1	4	8	1
p	-	A	A	E	B

A → B
A → C
A → E → D
A → B → C

⑲ 3 four sided dice.

sum = 3(1+1+1)

sum E.

4 = 3/64 (1+1+2, 1+2+1, 2+1+1)

5 = 6/64 (1+1+3, 1+2+2, 1+3+1, 2+1+2, 2+2+1, 3+1+1)

6 = 10/64 (1+2+3, 1+3+2, 2+1+3, 2+2+2, 2+3+1, 3+1+2, 3+2+1, 1+4+1, 2+2+2, 2+3+1)

7 = 12/64.

9 = 10/64
10 = 6/64
11 = 3/64
12 = 1/64

Σ(sum * probability)

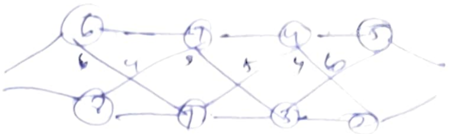
$= 3 \times 1/64 + 4 \times 3/64 + 5 \times 6/64 + 6 \times 10/64 +$

$7 \times 12/64 + 8 \times 12/64 + 9 \times 10/64 + 10 \times 6/64$

$+ 11 \times 3/64 + 12 \times 1/64$

$= \frac{480}{64} = 7.5$ /

⑳ Assembly line scheduling



	1	2	3	4
F1[j]	6	13	17	12
F2[j]	8	17	20	22

	1	2	3	4
l1[j]	1	1	1	1
l2[j]	2	2	2	2

㉑ OBST

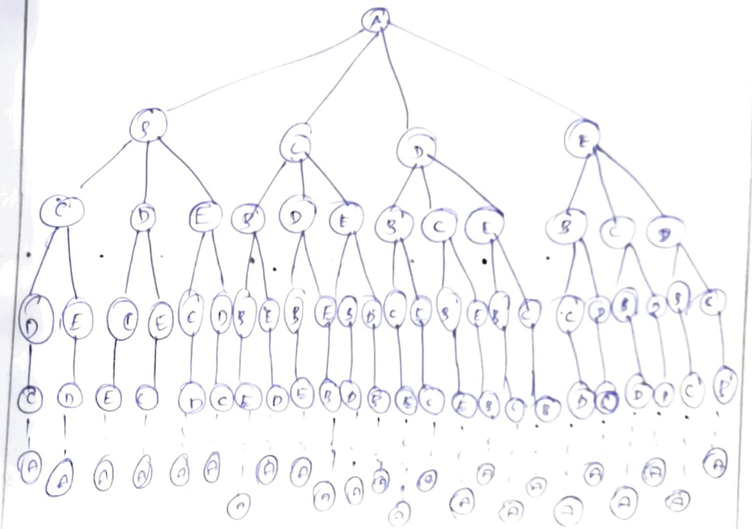
K = {10, 20, 30}
v = {0.2, 0.5, 0.3}

j - i = 3
3 - 0 = (0,3)

cost (0,3) = min {2.1, 1.5, 1.1}

	0	1	2	3
0	0	0.2	0.7	1.1
1		0	0.5	1.1
2			0	0.3
3				0

㉒ TSP



23 Knapsack

	0	10	20	30	21	11	50
0	0	0	0	0	0	0	0
1	0	50	50	50	50	50	50
2	0	50	70	70	70	70	70
3	0	50	70	90	90	90	160
4	0	50	70	90	90	90	160
5	0	50	70	90	90	90	160

24 Bellman Ford

v	1	2	3	4
d	0	∞	∞	∞
p	–	–	–	–

①

v	1	2	3	4
d	0	4	5	∞
p	–	1	1	–

②

v	1	2	3	4
d	0	4	2	∞
p	–	1	2	–

③

v	1	2	3	4
d	0	4	2	5
p	–	1	2	3

1

1 → 2

1 → 2 → 3

1 → 2 → 3 → 4

㉕ Roll six six-sided dice, to get a sum of 18, ensuring that atleast one die shows a 6.

$$x + x^2 + x^3 + x^4 + x^5 + x^6$$

$$x(1 + x + x^2 + x^3 + x^4 + x^5)$$

$$= \frac{x(1+x^6)}{1-x}$$

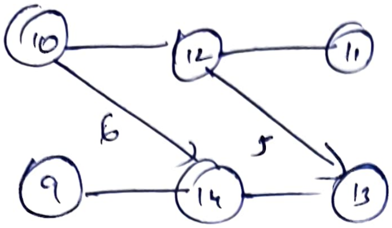
For six dice

$$\left(\frac{x-(1-x^6)}{1-x}\right)^6 = x^6(1-x^6)^6(1-x)^{-6}$$

$$= x^{x^4}$$

$$= 340$$

㉖ Assembly line



Before Reduction

4	6	5
C8	30	30

After Reduction (e)

4	4	5
C2	28	27

㉗ OBST

<8, 12, 16, 20, 24>

<0.2, 0.05, 0.4, 0.25, 0.1>

$j-i=0$

$j-i=1$

$j-i=2$

$2-0=(0,2)$

$3-1=(1,3)$

$4-2=(2,4)$

$5-3=(3,5)$

I -

I	0	1	2	3	4	5
0	0	0.2	0.3	0.7	1.45	1.8
1		0	0.05	0.5	1	1.3
2			0	0.4	0.9	1.2
3				0	0.25	0.05
4					0	0.1
5						0

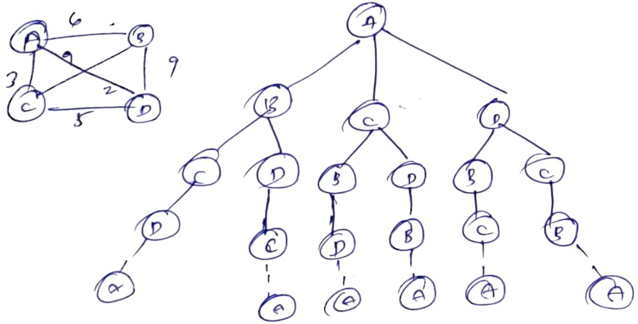
$$\text{cost}(1,5)=\min\left\{\begin{array}{l}\text{cost}(1,1)+\text{cost}(2,5)\\ \text{cost}(1,2)+\text{cost}(3,5)\\ \text{cost}(1,3)+\text{cost}(4,5)\\ \text{cost}(1,4)+\text{cost}(5,5)\end{array}\right\}+0.8$$

$$=\min\left\{\begin{array}{c}2\\ 1.3\\ 1.4\\ 1.8\end{array}\right\}=1.3$$

㉙ **Knapsack**

v|w	0	10	20	30	40	50
0	0	0	0	0	0	0
1	0	60	60	60	60	60
2	0	60	100	160	160	160
3	0	60	100	120	180	180
4	0	60	100	120	200	260

㉘ **TSP**



A → B → C → D → A – 20
A → D → C → B → A – 20 } min optimal path

㉚ **Bellman Ford**

v	A	B	C	D	E	F
d	0	∞	∞	∞	∞	∞
p	–	–	–	–	–	–

①

v	A	B	C	D	E	F
d	0	6	4	7	2	9
p	–	A	D	A	B	E

②

v	A	B	C	D	E	F
d	0	2	4	7	2	9
p	–	C	D	A	B	E

③

v	A	B	C	D	E	F
d	0	2	4	7	2	9
p	–	C	D	A	B	E

④

v	A	B	C	D	E	F
d	0	2	4	7	2	9
p	–	C	D	A	B	E

⑤

v	A	B	C	D	E	F
d	0	2	4	7	2	9
p	–	C	D	A	B	E

vertex	Dist	path
A	0	A
B	2	A → D → C → B
C	4	A → D → C
D	7	A → D
E	2	A → D → C → B → E
F	9	A → D → C → B → E → F