Assignment - 12

1) given an array of [4, -2, 5, 3, 10, -5, 2, 8, -3, 6, 7, -4, 1, 9, -1, 0, -6, -8, 11, -9] integers find the maximum and minimum product that can be obtained by multiplying two integers from the array.

Solution:

array is [4, -2, 5, 3, 10, -5, 2, 8, -3, 6, 7, -4, 1, 9, -1, 0, -6, -8, 11, -9]

we need to consider the largest and smallest products that can be formed by selecting two numbers from the array.

1) Sort the array.

sorted array.

[-9, -8, -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11]

2) identify possible candidates for maximum product

3) identify possible candidates for minimum product

calculating maximum product:

- The two largest positive numbers are 10 and 11

$10 \times 11 = 110$

- The two smallest negative numbers are -9 and -8

$-9 \times -8 = 72$

The maximum product is 110

calculating minimum products

The largest positive and negative number is 11 and -9

$11 \times -9 = -99$

The smallest positive negative numbers are

$-9 \times -8 = $

$-9 \times -8 = 72$

-99 is smaller than 72 so

maximum product = 110, and minimum product = -99

2) Demonstrate the Binary search method to search for the key = 23 from the array = {2, 5, 8, 12, 16, 23, 38, 56, 72, 91}.

Solution

given key = 23 and array = {2, 5, 8, 12, 16, 23, 38, 56, 72, 91}

1. initialize Pointers

$low = 0$ and $high = 9$

Calculate $mid = \left[\frac{low + high}{2}\right] = \left[\frac{0+9}{2}\right] = 4$

Compare arr[mid] with key:

$arr[4] = 16$

since $16 < 23$ update $low = mid + 1 = 5$

Calculate $mid = \left(\frac{low + high}{2}\right) = \left[\frac{5+9}{2}\right] = 7$

Compare arr[mid] with key:

$arr[7] = 56$

since $56 > 23$ update $high = mid - 1 = 6$

$mid = \left[\frac{5+6}{2}\right] = 5$

$arr[mid] = arr[5] = 23$

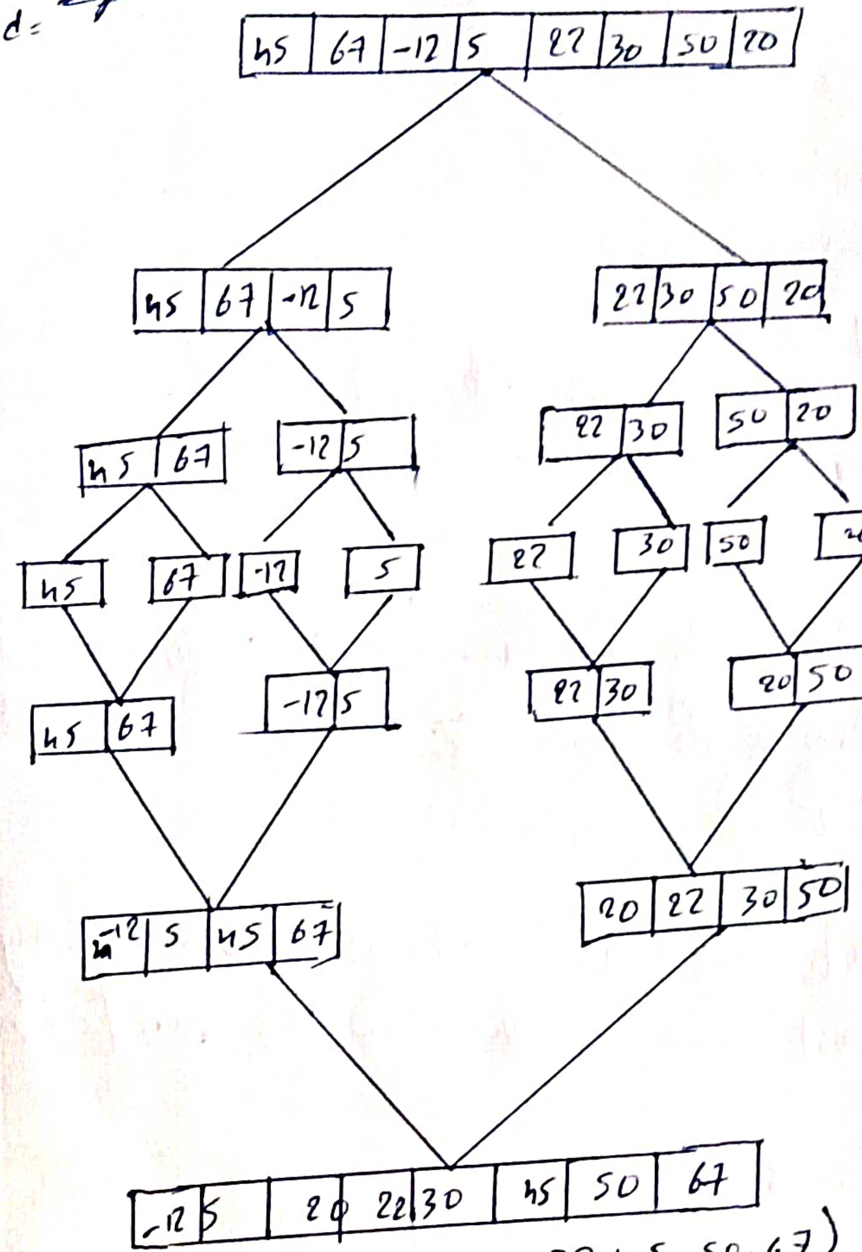
$23 == 23$ The key is found at index 5

∴ The key = 23 is found at index 5.

3) Apply merge sort and other list of 8 elements, Data d = (45, 67, -12, 5, 22, 30, 50, 20). Set up a recurrence relation for the number of key comparisons made by mergesort.

Solution

given d= Merge Sort:



∴ The sorted list = (-12, 5, 20, 23, 30, 45, 50, 67)

4) Find the no of times to perform swapping for selection sort. also estimate the time

Recurrence Relation for Comparisons:

$$T(n) = 2T(n/2) + O(n)$$

if $n=1$, $T(1)=0$. Base case

→ At each level of recursion we make at most n-1 comparisons to merge two halves of size $n/2$ so it becomes

$$T(n) = 2T(n/2) + (n-1)$$

Solving recurrence relation we get

$$T(n) = n\log_2(n) - n + 1$$

$$\therefore T(n) = O(n \log n)$$

∴ The recurrence relation is $T(n) = 2T(n/2) + O(n)$

or more precisely.

$$T(n) = n\log_2(n) - n + 1$$

4) find the no of times to perform solving swapping for selection sort also estimate the time complexity for the order of notation set S (12, 7, 5, -2, 18, 6, 13, 4)

Solution:

The selection sort algorithm always makes exactly n-1 swaps in the worst case, where n is the no of elements in the list.

given $S = \{12, 7, 5, -2, 18, 6, 13, 4\}$:

No. of elements, $n = 8$

no. of swaps $= n - 1 = 8 - 1 = 7$

Time Complexity: The time complexity of selection sort in Big-O notation is $O(n^2)$

So, the number of swaps is 7, and the time complexity is $O(n^2)$.

5) Find the index of the target value 10 using binary search from the following list of elements [2, 4, 6, 8, 10, 12, 14, 16, 18, 20].

given list = [2, 4, 6, 8, 10, 12, 14, 16, 18, 20] and value = 10

$$\begin{array}{cccccccccc} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 2 & 4 & 6 & 8 & 10 & 12 & 14 & 16 & 18 & 20 \end{array}$$

low = 0, and high = 9

$$mid = \frac{low + high}{2} = \frac{0+9}{2} = 4$$

list[4]: mid = 10   mid == value

Since 10 == 10 the target is found at index 4

∴ The target value = 10 is found at index 4.