TOPIC 6 : BACKTRACKING

1.Discuss the importance of visualizing the solutions of the N-Queens Problem to understand the placement of queens better. Use a graphical representation to show how queens are placed on the board for different values of N. Explain how visual tools can help in debugging the algorithm and gaining insights into the problem's complexity. Provide examples of visual representations for N = 4, N = 5, and N = 8, showing different valid solutions.

Program:

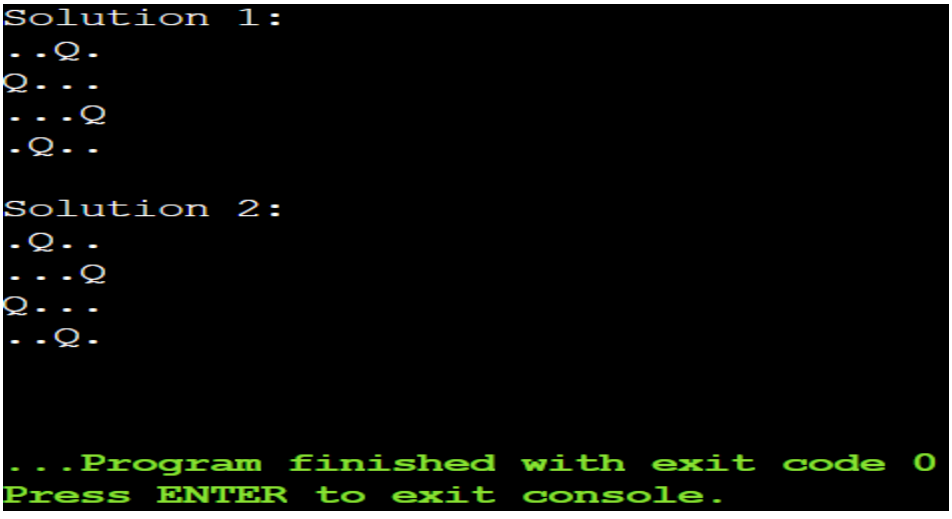
```
N = 4

board = [['.' for _ in range(N)] for _ in range(N)]
solutions = []

row = 0
stack = [(row, 0, [list(r) for r in board])]

while stack:
    row, col, b = stack.pop()

    if row == N:
        solutions.append(["".join(r) for r in b])
continue

    while col < N:
        safe = True

        for i in range(row):
            if b[i][col] == 'Q':
                safe = False
                break

        i, j = row-1, col-1
        while safe and i >= 0 and j >= 0:
            if b[i][j] == 'Q':
                safe = False
                break
            i -= 1
            j -= 1

        i, j = row-1, col+1
```

```
        while safe and i >= 0 and j < N:
            if b[i][j] == 'Q':
                safe = False
                break
            i -= 1
            j += 1

        if safe:
            new_board = [list(r) for r in b]
            new_board[row][col] = 'Q'
            stack.append((row + 1, 0, new_board))
        col += 1

for idx, sol in enumerate(solutions, 1):
    print(f"Solution {idx}:")
    for row in sol:
        print(row)
    print()
```

Sample Input:

N = 4

Output:



2.Discuss the generalization of the N-Queens Problem to other board sizes and shapes, such as rectangular boards or boards with obstacles. Explain how the algorithm can be adapted to handle these variations and the additional constraints they introduce. Provide examples of solving generalized N-Queens Problems for different board configurations, such as an 8×10 board, a 5×5 board with obstacles, and a 6×6 board with restricted positions.

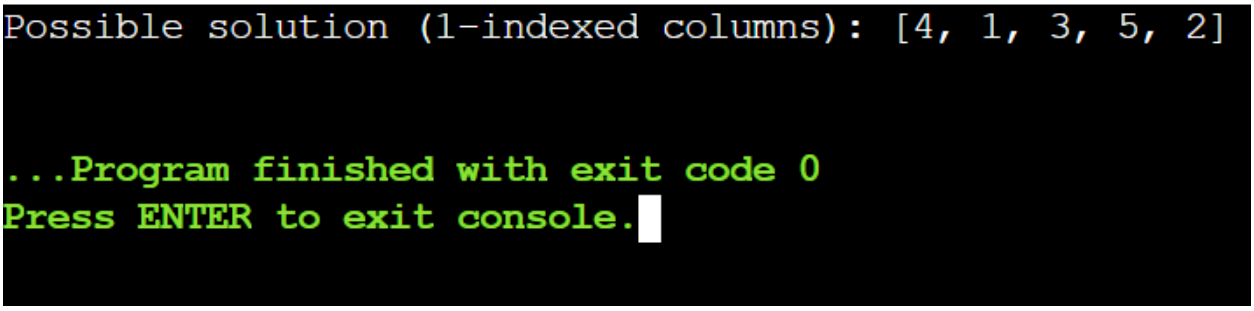
Program:

```
N = 5
obstacles = [(2, 2), (4, 4)]

board = [['.' for _ in range(N)] for _ in range(N)]
for r, c in obstacles:
    board[r-1][c-1] = 'X'

solutions = []
stack = [(0, 0, [list(r) for r in board])]

while stack:
    row, col, b = stack.pop()
    if row == N:

        sol = []
        for r in b:
            for idx, val in enumerate(r):
                if val == 'Q':
                    sol.append(idx+1)
        solutions.append(sol)
        continue

    while col < N:
        if b[row][col] == '.':
            safe = True

            for i in range(row):

                if b[i][col] == 'Q':
                    safe = False
                    break

                j = col - (row - i)
                if 0 <= j < N and b[i][j] == 'Q':
                    safe = False
                    break

                j = col + (row - i)
                if 0 <= j < N and b[i][j] == 'Q':
```

```
                safe = False
                break
        if safe:
            new_board = [list(r) for r in b]
            new_board[row][col] = 'Q'
            stack.append((row+1, 0, new_board))
    col += 1

if solutions:
    print("Possible solution (1-indexed columns):", solutions[0])
else:
    print("No solution found.")
```

Sample Input:
8 rows and 10 columns

Output:



3.Write a program to solve a Sudoku puzzle by filling the empty cells.A sudoku solution must satisfy all of the following rules:Each of the digits 1-9 must occur exactly once in each row.Each of the digits 1-9 must occur exactly once in each column.Each of the digits 1-9 must occur exactly once in each of the 9 3x3 sub-boxes of the grid.The '.' character indicates empty cells.

Program:

```
board = [
    ["5","3",".",".","7",".",".",".",".","."],
    ["6",".",".","1","9","5",".",".",".","."],
    [".","9","8",".",".",".",".",".","6","."],
    ["8",".",".",".",".","6",".",".",".","3"],
    ["4",".",".","8",".","3",".",".",".","1"],
    ["7",".",".",".",".","2",".",".",".","6"],
    [".","6",".",".",".",".",".","2","8","."],
    [".",".",".","4","1","9",".",".",".","5"],
    [".",".",".",".",".","8",".",".","7","9"]
```

```
]

rows = [set() for _ in range(9)]
cols = [set() for _ in range(9)]
boxes = [set() for _ in range(9)]

for i in range(9):
    for j in range(9):
        if board[i][j] != '.':
            rows[i].add(board[i][j])
            cols[j].add(board[i][j])
            boxes[(i//3)*3 + j//3].add(board[i][j])

empty = [(i, j) for i in range(9) for j in range(9) if board[i][j] == '.']
index = 0

while index < len(empty):
    i, j = empty[index]
    found = False
    start = int(board[i][j]) + 1 if board[i][j] != '.' else 1
    for num in range(start, 10):
        num_str = str(num)
        box_index = (i//3)*3 + j//3
        if num_str not in rows[i] and num_str not in cols[j] and num_str not in boxes[box_index]:

            if board[i][j] != '.':
                rows[i].remove(board[i][j])
                cols[j].remove(board[i][j])
                boxes[box_index].remove(board[i][j])
            board[i][j] = num_str
            rows[i].add(num_str)
            cols[j].add(num_str)
            boxes[box_index].add(num_str)
            found = True
            index += 1
            break
    if not found:

        board[i][j] = '.'
        index -= 1
```

```
        pi, pj = empty[index]
        rows[pi].remove(board[pi][pj])
        cols[pj].remove(board[pi][pj])
        boxes[(pi//3)*3 + pj//3].remove(board[pi][pj])

for row in board:
    print(row)
```

Sample Input:

```
board =
[["5","3",".",".","7",".",".",".",".","."],
["6",".",".","1","9","5",".",".",".","."],
[".","9","8",".",".",".",".",".","6","."],
["8",".",".",".",".","6",".",".",".","3"],
["4",".",".",".","8",".","3",".",".","1"],
["7",".",".",".",".","2",".",".",".","6"],
[".","6",".",".",".",".",".","2","8","."],
[".",".",".","4","1","9",".",".",".","5"],
[".",".",".",".",".","8",".",".","7","9"]]
```

Output:

```
['5', '3', '4', '6', '7', '8', '9', '1', '2']
['6', '7', '2', '1', '9', '5', '3', '4', '8']
['1', '9', '8', '3', '4', '2', '5', '6', '7']
['8', '5', '9', '7', '6', '1', '4', '2', '3']
['4', '2', '6', '8', '5', '3', '7', '9', '1']
['7', '1', '3', '9', '2', '4', '8', '5', '6']
['9', '6', '1', '5', '3', '7', '2', '8', '4']
['2', '8', '7', '4', '1', '9', '6', '3', '5']
['3', '4', '5', '2', '8', '6', '1', '7', '9']


...Program finished with exit code 0
Press ENTER to exit console.
```

4.Write a program to solve a Sudoku puzzle by filling the empty cells.A sudoku solution must satisfy all of the following rules:Each of the digits 1-9 must occur

exactly once in each row.Each of the digits 1-9 must occur exactly once in each column.Each of the digits 1-9 must occur exactly once in each of the 9 3x3 sub-boxes of the grid.The '.' character indicates empty cells.

Program:

```
board = [
   ["5","3",".",".","7",".",".",".",".","."],
   ["6",".",".","1","9","5",".",".",".","."],
   [".","9","8",".",".",".",".",".","6","."],
   ["8",".",".",".","6",".",".",".",".","3"],
   ["4",".",".","8",".","3",".",".",".","1"],
   ["7",".",".",".",".","2",".",".",".","6"],
   [".","6",".",".",".",".",".","2","8","."],
   [".",".",".","4","1","9",".",".",".","5"],
   [".",".",".",".",".","8",".",".","7","9"]
]
def is_valid(board, row, col, num):
   for i in range(9):
      if board[row][i] == num:
         return False
      if board[i][col] == num:
         return False

      if board[row//3*3 + i//3][col//3*3 + i%3] == num:
         return False
   return True

def solve():
   for i in range(9):
      for j in range(9):
         if board[i][j] == '.':
            for num in '123456789':
               if is_valid(board, i, j, num):
                  board[i][j] = num
                  if solve():
                     return True
                  board[i][j] = '.'
            return False
   return True

solve()
```

```
for row in board:
    print(" ".join(row))
```

Sample Input:

```
board =
[["5","3",".",".","7",".",".",".",".","."],
["6",".",".","1","9","5",".",".",".","."],
[".","9","8",".",".",".",".",".","6","."],
["8",".",".",".",".","6",".",".",".","3"],
["4",".",".","8",".","3",".",".",".","1"],
["7",".",".",".",".","2",".",".",".","6"],
[".","6",".",".",".",".",".","2","8","."],
[".",".",".","4","1","9",".",".",".","5"],
[".",".",".",".",".","8",".",".","7","9"]]
```

Output:

```
5 3 4 6 7 8 9 1 2
6 7 2 1 9 5 3 4 8
1 9 8 3 4 2 5 6 7
8 5 9 7 6 1 4 2 3
4 2 6 8 5 3 7 9 1
7 1 3 9 2 4 8 5 6
9 6 1 5 3 7 2 8 4
2 8 7 4 1 9 6 3 5
3 4 5 2 8 6 1 7 9


...Program finished with exit code 0
Press ENTER to exit console.
```

5.You are given an integer array nums and an integer target. You want to build an expression out of nums by adding one of the symbols '+' and '-' before each integer in nums and then concatenate all the integers.For example, if nums = [2, 1], you can add a '+' before 2 and a '-' before 1 and concatenate them to build the expression "+2-1" Return the number of different expressions that you can build, which evaluates to target.

Program:

```
nums = [1, 1, 1, 1, 1]
target = 3

dp = {0: 1}
```

```
for num in nums:
    next_dp = {}
    for summ in dp:
        next_dp[summ + num] = next_dp.get(summ + num, 0) + dp[summ]
        next_dp[summ - num] = next_dp.get(summ - num, 0) + dp[summ]
    dp = next_dp

print(dp.get(target, 0))
```

Sample Input:
nums = [1,1,1,1,1], target = 3

Output:

```
5


...Program finished with exit code 0
Press ENTER to exit console.
```

6.Given an array of integers arr, find the sum of min(b), where b ranges over every (contiguous) subarray of arr. Since the answer may be large, return the answer modulo 109 + 7.
Program:

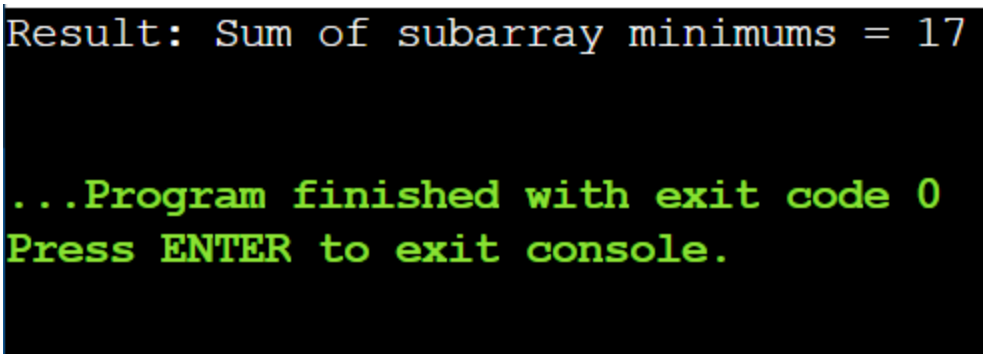
```
arr = [3, 1, 2, 4]
mod = 10**9 + 7

stack = []
prev_less = [0] * len(arr)
next_less = [0] * len(arr)

for i in range(len(arr)):
    count = 1
    while stack and stack[-1][0] > arr[i]:
        count += stack.pop()[1]
    prev_less[i] = count
    stack.append((arr[i], count))
```

```
stack = []

for i in range(len(arr)-1, -1, -1):
    count = 1
    while stack and stack[-1][0] >= arr[i]:
        count += stack.pop()[1]
    next_less[i] = count
    stack.append((arr[i], count))

result = 0
for i in range(len(arr)):
    result = (result + arr[i] * prev_less[i] * next_less[i]) % mod

print("Result: Sum of subarray minimums =", result)
```

Sample Input:

arr = [3,1,2,4]

Output:



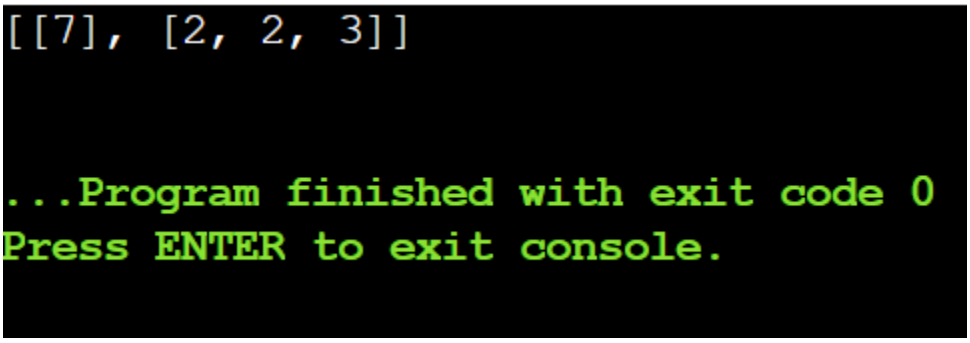
7.Given an array of distinct integers candidates and a target integer target, return a list of all unique combinations of candidates where the chosen numbers sum to target. You may return the combinations in any order.The same number may be chosen from candidates an unlimited number of times. Two combinations are unique if the frequency of at least one of the chosen numbers is different.The test cases are generated such that the number of unique combinations that sum up to target is less than 150 combinations for the given input.

Program:

```
candidates = [2, 3, 6, 7]
target = 7
result = []
stack = [(0, [], target)]
```

```
while stack:
    start, combo, remaining = stack.pop()
    if remaining == 0:
        result.append(combo)
        continue
    for i in range(start, len(candidates)):
        if candidates[i] <= remaining:
            stack.append((i, combo + [candidates[i]], remaining - candidates[i]))

print(result)
```

Sample Input:
candidates = [2,3,6,7], target = 7
Output:



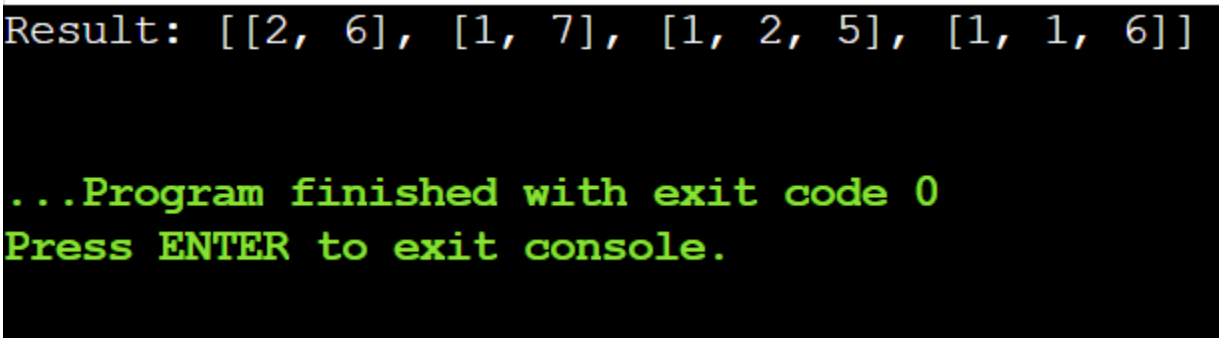
8.Given a collection of candidate numbers (candidates) and a target number (target), find all unique combinations in candidates where the candidate numbers sum to target. Each number in candidates may only be used once in the combination. The solution set must not contain duplicate combinations.
Program:

```
candidates = [10,1,2,7,6,1,5]
target = 8
candidates.sort()
result = []
stack = [(0, [], target)]

while stack:
    start, combo, remaining = stack.pop()
    if remaining == 0:
        result.append(combo)
        continue
    for i in range(start, len(candidates)):
```

```
        if i > start and candidates[i] == candidates[i-1]:
            continue
        if candidates[i] > remaining:
            break
        stack.append((i + 1, combo + [candidates[i]], remaining - candidates[i]))

print(f"Result: {result}")
```

Sample Input:
candidates = [10,1,2,7,6,1,5], target = 8

Output:



9.Given an array nums of distinct integers, return all the possible permutations. You can return the answer in any order.

Program:

```
nums = [1, 2, 3]
result = []
stack = [([], nums)]

while stack:
    perm, remaining = stack.pop()
    if not remaining:
        result.append(perm)
        continue
    for i in range(len(remaining)):
        stack.append((perm + [remaining[i]], remaining[:i] + remaining[i+1:]))

print(f"Result: {result}")
```

Sample Input:
nums = [1,2,3]

Output:

```
Result: [[3, 2, 1], [3, 1, 2], [2, 3, 1], [2, 1, 3], [1, 3, 2], [1, 2, 3]]


...Program finished with exit code 0
Press ENTER to exit console.
```

10.Given a collection of numbers, nums, that might contain duplicates, return all possible unique permutations in any order.

Program:

```
nums = [1, 1, 2]
nums.sort()
result = []
stack = [([], [True]*len(nums))]

while stack:
    perm, used = stack.pop()
    if len(perm) == len(nums):
        result.append(perm)
        continue
    for i in range(len(nums)):
        if not used[i]:
            continue
        if i > 0 and nums[i] == nums[i-1] and used[i-1]:
            continue
        new_used = used[:]
        new_used[i] = False
        stack.append((perm + [nums[i]], new_used))

print(f"Result: {result}")
```

Sample Input:

nums = [1,1,2]

Output:

```
Result: [[2, 1, 1], [1, 2, 1], [1, 1, 2]]


...Program finished with exit code 0
Press ENTER to exit console.
```

11.You and your friends are assigned the task of coloring a map with a limited number of colors. The map is represented as a list of regions and their adjacency relationships. The rules are as follows: At each step, you can choose any uncolored region and color it with any available color. Your friend Alice follows the same strategy immediately after you, and then your friend Bob follows suit. You want to maximize the number of regions you personally color. Write a function that takes the map's adjacency list representation and returns the maximum number of regions you can color before all regions are colored. Write a program to implement the Graph coloring technique for an undirected graph. Implement an algorithm with minimum number of colors. edges = [(0, 1), (1, 2), (2, 3), (3, 0), (0, 2)] No. of vertices, n = 4

Program:

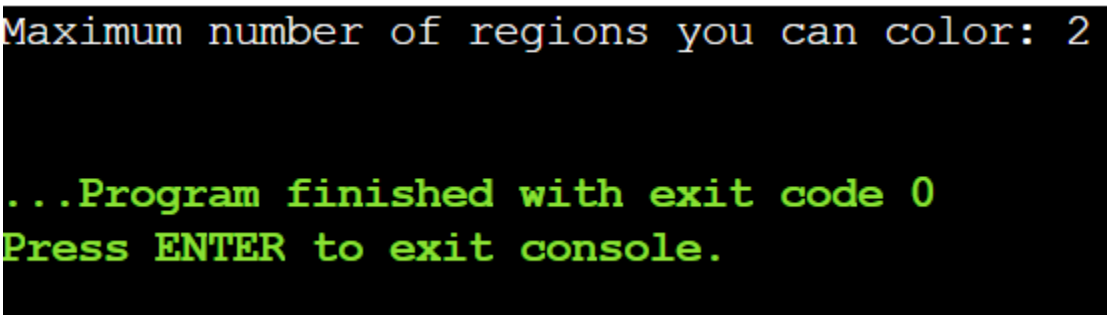
```
n = 4
edges = [(0, 1), (1, 2), (2, 3), (3, 0), (0, 2)]
k = 3

graph = [[] for _ in range(n)]
for u, v in edges:
    graph[u].append(v)
    graph[v].append(u)

colors = [-1] * n
my_turn_count = 0

players = ["You", "Alice", "Bob"]
turn = 0  # Start with your turn

def can_color(vertex, c):
    for neighbor in graph[vertex]:
        if colors[neighbor] == c:
            return False
    return True
```

```
uncolored = list(range(n))

while uncolored:
    colored_this_turn = False
    for vertex in uncolored:
        for c in range(k):
            if can_color(vertex, c):
                colors[vertex] = c
                if players[turn] == "You":
                    my_turn_count += 1
                uncolored.remove(vertex)
                colored_this_turn = True
                break
        if colored_this_turn:
            break
    turn = (turn + 1) % 3

print(f"Maximum number of regions you can color: {my_turn_count}")
```

Sample Input:

- Number of vertices: n = 4
- Edges: [(0, 1), (1, 2), (2, 3), (3, 0), (0, 2)]
- Number of colors: k = 3

Output:



12.You are given an undirected graph represented by a list of edges and the number of vertices n. Your task is to determine if there exists a Hamiltonian cycle in the graph. A Hamiltonian cycle is a cycle that visits each vertex exactly once and returns to the starting vertex. Write a function that takes the list of edges and the number of vertices as input and returns true if there exists a Hamiltonian cycle in the graph, otherwise return false. Example: Given edges = [(0, 1), (1, 2), (2, 3), (3, 0), (0, 2), (2, 4), (4, 0)] and n = 5

Program:

```
n = 5
```

```
edges = [(0, 1), (1, 2), (2, 3), (3, 0), (0, 2), (2, 4), (4, 0)]

graph = [[] for _ in range(n)]
for u, v in edges:
    graph[u].append(v)
    graph[v].append(u)

path = [0]  # start from vertex 0
visited = [False] * n
visited[0] = True

def is_safe(v, pos):

    if v not in graph[path[pos - 1]]:
        return False

    if visited[v]:
        return False
    return True

def hamiltonian(pos):
    if pos == n:

        return path[-1] in graph[path[0]]

    for v in range(1, n):
        if is_safe(v, pos):
            path.append(v)
            visited[v] = True
            if hamiltonian(pos + 1):
                return True

            path.pop()
            visited[v] = False
    return False

exists = hamiltonian(1)

if exists:
    print(f"Hamiltonian Cycle Exists: True (Example cycle: {path + [path[0]]})")
```

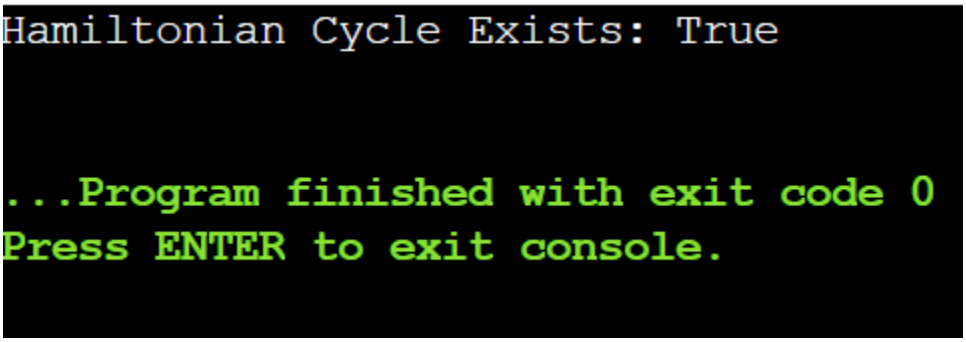
```
else:
    print("Hamiltonian Cycle Exists: True")
```

Sample Input:

• Number of vertices: n = 5

• Edges: [(0, 1), (1, 2), (2, 3), (3, 0), (0, 2), (2, 4), (4, 0)]

Output:



13.You are given an undirected graph represented by a list of edges and the number of vertices n. Your task is to determine if there exists a Hamiltonian cycle in the graph. A Hamiltonian cycle is a cycle that visits each vertex exactly once and returns to the starting vertex. Write a function that takes the list of edges and the number of vertices as input and returns true if there exists a Hamiltonian cycle in the graph, otherwise return false. Example:edges = [(0, 1), (1, 2), (2, 3), (3, 0), (0, 2)] and n = 4

Program:

```
n = 4
edges = [(0, 1), (1, 2), (2, 3), (3, 0), (0, 2)]

graph = [[] for _ in range(n)]
for u, v in edges:
    graph[u].append(v)
    graph[v].append(u)

path = [0]
visited = [False] * n
visited[0] = True

def is_safe(v, pos):
    return v in graph[path[pos-1]] and not visited[v]

def hamiltonian(pos):
```

```
    if pos == n:
        return path[-1] in graph[path[0]]
    for v in range(1, n):
        if is_safe(v, pos):
            path.append(v)
            visited[v] = True
            if hamiltonian(pos + 1):
                return True
            path.pop()
            visited[v] = False
    return False

exists = hamiltonian(1)

print(f"Hamiltonian Cycle Exists: {exists}" + (f" (Example cycle: {' -> '.join(map(str, path + [path[0]]))})" if exists else ""))
```

Sample Input:

• Number of vertices: n = 4

• Edges: [(0, 1), (1, 2), (2, 3), (3, 0), (0, 2)]

Output:

```
Hamiltonian Cycle Exists: True (Example cycle: 0 -> 1 -> 2 -> 3 -> 0)


...Program finished with exit code 0
Press ENTER to exit console.
```

14.You are tasked with designing an efficient coading to generate all subsets of a given set S containing n elements. Each subset should be outputted in lexicographical order. Return a list of lists where each inner list is a subset of the given set. Additionally, find out how your coading handles duplicate elements in S. A = [1, 2, 3] The subsets of [1, 2, 3] are: [], [1], [2], [3], [1, 2], [1, 3], [2, 3], [1, 2, 3]

Program:

```
A = [1, 2, 3]

A.sort()

result = [[]]

for num in A:
```

```
    new_subsets = [curr + [num] for curr in result]
    result.extend(new_subsets)

result = list(map(list, sorted(set(map(tuple, result)))))

print(f"Subsets: {result}")
print("Handling of duplicates: If A contained duplicates (e.g., [1, 2, 2]), subsets would include duplicates unless duplicates are removed.")
```

Sample Input:
Set: A = [1, 2, 3]

Output:

```
Subsets: [[], [1], [1, 2], [1, 2, 3], [1, 3], [2], [2, 3], [3]]
Handling of duplicates: If A contained duplicates (e.g., [1, 2, 2]), subsets would include duplicates unless duplica
tes are removed.


...Program finished with exit code 0
Press ENTER to exit console.
```

15.Write a program to implement the concept of subset generation. Given a set of unique integers and a specific integer 3, generate all subsets that contain the element 3. Return a list of lists where each inner list is a subset containing the element 3 E = [2, 3, 4, 5], x = 3, The subsets containing 3 : [3], [2, 3], [3, 4], [3,5], [2, 3, 4], [2, 3, 5], [3, 4, 5], [2, 3, 4, 5] Given an integer array nums of unique elements, return all possible subsets(the power set). The solution set must not contain duplicate subsets. Return the solution in any order.

Program:

```
E = [2, 3, 4, 5]
x = 3

E.sort()

all_subsets = [[]]
for num in E:
    all_subsets += [curr + [num] for curr in all_subsets]

subsets_with_x = [s for s in all_subsets if x in s]

print(f"Subsets containing {x}: {subsets_with_x}")
```

Sample Input:
nums = [1,2,3]

Output:

```
Subsets containing 3: [[3], [2, 3], [3, 4], [2, 3, 4], [3, 5], [2, 3, 5], [3, 4, 5], [2, 3, 4, 5]]


...Program finished with exit code 0
Press ENTER to exit console.
```

16.You are given two string arrays words1 and words2. A string b is a subset of string a if every letter in b occurs in a including multiplicity. For example, "wrr" is a subset of "warrior" but is not a subset of "world". A string a from words1 is universal if for every string b in words2, b is a subset of a. Return an array of all the universal strings in words1. You may return the answer in any order.

Program:

```
words1 = ["amazon","apple","facebook","google","leetcode"]
words2 = ["e","o"]

from collections import Counter

max_freq = Counter()
for b in words2:
    freq = Counter(b)
    for char in freq:
        max_freq[char] = max(max_freq.get(char, 0), freq[char])

universal_words = []
for a in words1:
    freq_a = Counter(a)
    if all(freq_a.get(char,0) >= count for char, count in max_freq.items()):
        universal_words.append(a)

print(f"Universal words: {universal_words}")
```

Sample Input:

words1 = ["amazon","apple","facebook","google","leetcode"], words2 = ["e","o"]

Output:

```
Universal words: ['facebook', 'google', 'leetcode']


...Program finished with exit code 0
Press ENTER to exit console.
```