# Data Science Inference and Modeling

The textbook for the Data Science course series is freely available online.

This course corresponds to the textbook chapters Statistical Inference and Statistical Models.

## Learning Objectives

- The concepts necessary to define estimates and margins of errors of populations, parameters, estimates, and standard errors in order to make predictions about data
- How to use models to aggregate data from different sources
- The very basics of Bayesian statistics and predictive modeling

## Course Overview

### Section 1: Parameters and Estimates

You will learn how to estimate population parameters.

### Section 2: The Central Limit Theorem in Practice

You will apply the central limit theorem to assess how close a sample estimate is to the population parameter of interest.

### Section 3: Confidence Intervals and p-Values

You will learn how to calculate confidence intervals and learn about the relationship between confidence intervals and p-values.

### Section 4: Statistical Models

You will learn about statistical models in the context of election forecasting.

### Section 5: Bayesian Statistics

You will learn about Bayesian statistics through looking at examples from rare disease diagnosis and baseball.

### Section 6: Election Forecasting

You will learn about election forecasting, building on what you've learned in the previous sections about statistical modeling and Bayesian statistics.

**Section 7: Association Tests**

You will learn how to use association and chi-squared tests to perform inference for binary, categorical, and ordinal data through an example looking at research funding rates.

## Introduction to Inference

The textbook for this section is available here

In this course, we will learn:

- *statistical inference*, the process of deducing characteristics of a population using data from a random sample
- the statistical concepts necessary to define *estimates* and *margins of errors*
- how to *forecast future results* and estimate the precision of our forecast
- how to calculate and interpret *confidence intervals and p-values*

### Key points

- Information gathered from a small random sample can be used to infer characteristics of the entire population.
- Opinion polls are useful when asking everyone in the population is impossible.
- A common use for opinion polls is determining voter preferences in political elections for the purposes of forecasting election results.
- The *spread* of a poll is the estimated difference between support two candidates or options.

## Section 1 Overview

Section 1 introduces you to parameters and estimates.

After completing Section 1, you will be able to:

- Understand how to use a sampling model to perform a poll.
- Explain the terms **population**, **parameter**, and **sample** as they relate to statistical inference.
- Use a sample to estimate the population proportion from the sample average.
- Calculate the expected value and standard error of the sample average.

## Sampling Model Parameters and Estimates

The textbook for this section is available here and here; first part

### Key points

- The task of statistical inference is to estimate an unknown population parameter using observed data from a sample.
- In a sampling model, the collection of elements in the urn is called the *population*.
- A *parameter* is a number that summarizes data for an entire population.
- A *sample* is observed data from a subset of the population.
- An *estimate* is a summary of the observed data about a parameter that we believe is informative. It is a data-driven guess of the population parameter.
- We want to predict the proportion of the blue beads in the urn, the parameter $p$ . The proportion of red beads in the urn is $1 - p$ and the *spread* is $2p - 1$.

- The sample proportion is a random variable. Sampling gives random results drawn from the population distribution.

*Code: Function for taking a random draw from a specific urn*

The **dslabs** package includes a function for taking a random draw of size $n$ from the urn:

```
if(!require(tidyverse)) install.packages("tidyverse")
```

```
## Loading required package: tidyverse
```

```
## -- Attaching packages ---------------------------------------------------------------------------------
```

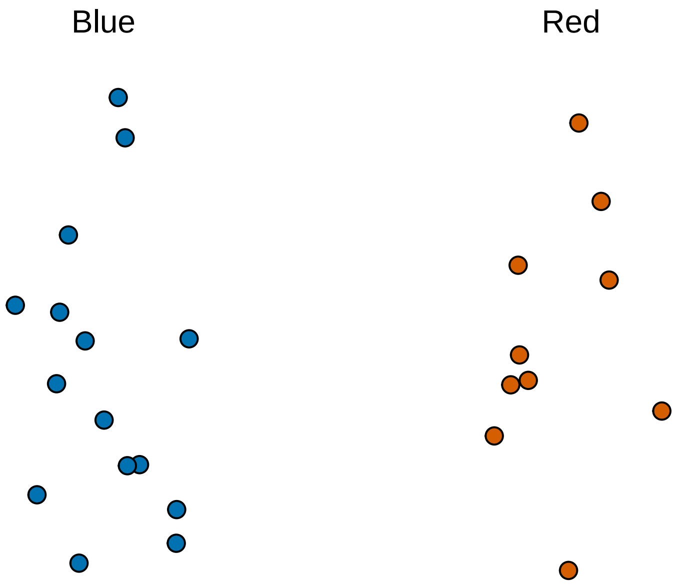
```
## v ggplot2 3.3.2     v purrr   0.3.4
## v tibble  3.0.3     v dplyr   1.0.1
## v tidyr   1.1.1     v stringr 1.4.0
## v readr   1.3.1     v forcats 0.5.0
```

```
## -- Conflicts ------------------------------------------------------------------------------------------
## x dplyr::filter() masks stats::filter()
## x dplyr::lag()    masks stats::lag()
```

```
if(!require(dslabs)) install.packages("dslabs")
```

```
## Loading required package: dslabs
```

```
library(tidyverse)
library(dslabs)
take_poll(25)    # draw 25 beads
```



## The Sample Average

The textbook for this section is available here and here

**Key points**

- Many common data science tasks can be framed as estimating a parameter from a sample.
- We illustrate statistical inference by walking through the process to estimate $p$. From the estimate of $p$, we can easily calculate an estimate of the spread, $2p - 1$.
- Consider the random variable $X$ that is 1 if a blue bead is chosen and 0 if a red bead is chosen. The proportion of blue beads in $N$ draws is the average of the draws $X_1, ..., X_N$.
- $\overline{X}$ is the *sample average*. In statistics, a bar on top of a symbol denotes the average. $\overline{X}$ is a random variable because it is the average of random draws - each time we take a sample, $\overline{X}$ is different.

$\overline{X} = \frac{X_1 + X_2 + ... + X_N}{N}$

- The number of blue beads drawn in N draws, $N\overline{X}$, is $N$ times the proportion of values in the urn. However, we do not know the true proportion: we are trying to estimate this parameter $p$.

## Polling versus Forecasting

The textbook for this section is available here

**Key points**

- A poll taken in advance of an election estimates $p$ for that moment, not for election day.
- In order to predict election results, forecasters try to use early estimates of $p$ to predict $p$ on election day. We discuss some approaches in later sections.

## Properties of Our Estimate

The textbook for this section is available here

**Key points**

- When interpreting values of $\overline{X}$, it is important to remember that $\overline{X}$ is a random variable with an expected value and standard error that represents the sample proportion of positive events.
- The expected value of $\overline{X}$ is the parameter of interest $p$. This follows from the fact that $\overline{X}$ is the sum of independent draws of a random variable times a constant $1/N$.

$E(\overline{X}) = p$

- As the number of draws $N$ increases, the standard error of our estimate $\overline{X}$ decreases. The standard error of the average of $\overline{X}$ over $N$ draws is:

$SE(\overline{X}) = \sqrt{p(1-p)/N}$

- In theory, we can get more accurate estimates of $p$ by increasing $N$. In practice, there are limits on the size of $N$ due to costs, as well as other factors we discuss later.
- We can also use other random variable equations to determine the expected value of the sum of draws $E(S)$ and standard error of the sum of draws $SE(S)$.

$E(S) = Np$

$SE(S) = \sqrt{Np(1-p)}$

## Assessment 1.1: Parameters and Estimates

1. Polling - expected value of S

Suppose you poll a population in which a proportion **p** of voters are Democrats and **1−p** are Republicans. Your sample size is **N=25**. Consider the random variable **S**, which is the total number of Democrats in your sample.

What is the expected value of this random variable **S**?

Possible Answers - [ ] A. $E(S)=25(1-p)$ - [X] B. $E(S)=25p$ - [ ] C. $E(S)=\sqrt{(25p(1-p))}$ - [ ] D. $E(S)=p$

2. Polling - standard error of S

Again, consider the random variable S, which is the total number of Democrats in your sample of 25 voters. The variable p describes the proportion of Democrats in the sample, whereas 1−p describes the proportion of Republicans.

What is the standard error of S?

Possible Answers - [ ] A. $SE(S)=25p(1-p)$ - [ ] B. $SE(S)=\sqrt{25p}$ - [ ] C. $SE(S)=25(1-p)$ - [X] D. $SE(S)=\sqrt{(25p(1-p))}$

3. Polling - expected value of X-bar

Consider the random variable **S/N**, which is equivalent to the sample average that we have been denoting as $\bar{\mathbf{X}}$. The variable **N** represents the sample size and **p** is the proportion of Democrats in the population.

What is the expected value of $\bar{\mathbf{X}}$?

Possible Answers - [X] A. $E(\bar{X})=p$ - [ ] B. $E(\bar{X})=Np$ - [ ] C. $E(\bar{X})=N(1-p)$ - [ ] D. $E(\bar{X})=1-p$

4. Polling - standard error of X-bar

What is the standard error of the sample average, $\bar{\mathbf{X}}$?

The variable **N** represents the sample size and **p** is the proportion of Democrats in the population.

Possible Answers - [ ] A. $SE(\bar{X})=\sqrt{(Np(1-p))}$ - [X] B. $SE(\bar{X})=\sqrt{(p(1-p)/N)}$ - [ ] C. $SE(\bar{X})=\sqrt{(p(1-p))}$ - [ ] D. $SE(\bar{X})=\sqrt{N}$

5. se versus p

Write a line of code that calculates the standard error se of a sample average when you poll 25 people in the population. Generate a sequence of 100 proportions of Democrats p that vary from 0 (no Democrats) to 1 (all Democrats).

Plot se versus p for the 100 different proportions.

Instructions - Use the seq function to generate a vector of 100 values of p that range from 0 to 1. - Use the sqrt function to generate a vector of standard errors for all values of p. - Use the plot function to generate a plot with p on the x-axis and se on the y-axis.