Data Science Linear Regression

The textbook for the Data Science course series is freely available online.

Learning Objectives

- How linear regression was originally developed by Galton
- What confounding is and how to detect it
- How to examine the relationships between variables by implementing linear regression in R

Course Overview

There are three major sections in this course: introduction to linear regression, linear models, and confounding.

Introduction to Linear Regression

In this section, you'll learn the basics of linear regression through this course's motivating example, the data-driven approach used to construct baseball teams. You'll also learn about correlation, the correlation coefficient, stratification, and the variance explained.

Linear Models

In this section, you'll learn about linear models. You'll learn about least squares estimates, multivariate regression, and several useful features of R, such as tibbles, lm, do, and broom. You'll learn how to apply regression to baseball to build a better offensive metric.

Confounding

In the final section of the course, you'll learn about confounding and several reasons that correlation is not the same as causation, such as spurious correlation, outliers, reversing cause and effect, and confounders. You'll also learn about Simpson's Paradox.

Section 1 - Introduction to Regression Overview

In the **Introduction to Regression** section, you will learn the basics of linear regression.

After completing this section, you will be able to:

- Understand how Galton developed **linear regression**.
- Calculate and interpret the **sample correlation**.
- **Stratify** a dataset when appropriate.
- Understand what a **bivariate normal distribution** is.
- Explain what the term **variance explained** means.
- Interpret the two **regression lines**.

This section has three parts: **Baseball as a Motivating Example**, **Correlation**, and **Stratification and Variance Explained**.

Motivating Example: Moneyball

The corresponding section of the textbook is the case study on Moneyball

Key points

Bill James was the originator of the **sabermetrics**, the approach of using data to predict what outcomes best predicted if a team would win.

Baseball basics

The corresponding section of the textbook is the section on baseball basics

Key points

- The goal of a baseball game is to score more runs (points) than the other team.
- Each team has 9 batters who have an opportunity to hit a ball with a bat in a predetermined order.
- Each time a batter has an opportunity to bat, we call it a plate appearance (PA).
- The PA ends with a binary outcome: the batter either makes an out (failure) and returns to the bench or the batter doesn't (success) and can run around the bases, and potentially score a run (reach all 4 bases).
- We are simplifying a bit, but there are five ways a batter can succeed (not make an out):

1. Bases on balls (BB): the pitcher fails to throw the ball through a predefined area considered to be hittable (the strike zone), so the batter is permitted to go to first base.
2. Single: the batter hits the ball and gets to first base.
3. Double (2B): the batter hits the ball and gets to second base.
4. Triple (3B): the batter hits the ball and gets to third base.
5. Home Run (HR): the batter hits the ball and goes all the way home and scores a run.

- Historically, the batting average has been considered the most important offensive statistic. To define this average, we define a hit (H) and an at bat (AB). Singles, doubles, triples and home runs are hits. The fifth way to be successful, a walk (BB), is not a hit. An AB is the number of times you either get a hit or make an out; BBs are excluded. The batting average is simply H/AB and is considered the main measure of a success rate.

Bases on Balls or Stolen Bases?

The corresponding section of the textbook is the base on balls or stolen bases textbook section

Key points

The visualization of choice when exploring the relationship between two variables like home runs and runs is a scatterplot.

Code: Scatterplot of the relationship between HRs and runs

```
if(!require(Lahman)) install.packages("Lahman")
```

```
## Loading required package: Lahman
```

```
if(!require(tidyverse)) install.packages("tidyverse")
```

```
## Loading required package: tidyverse
```

```
## -- Attaching packages --------------------------------------------- tidyverse 1.3.0 --
```

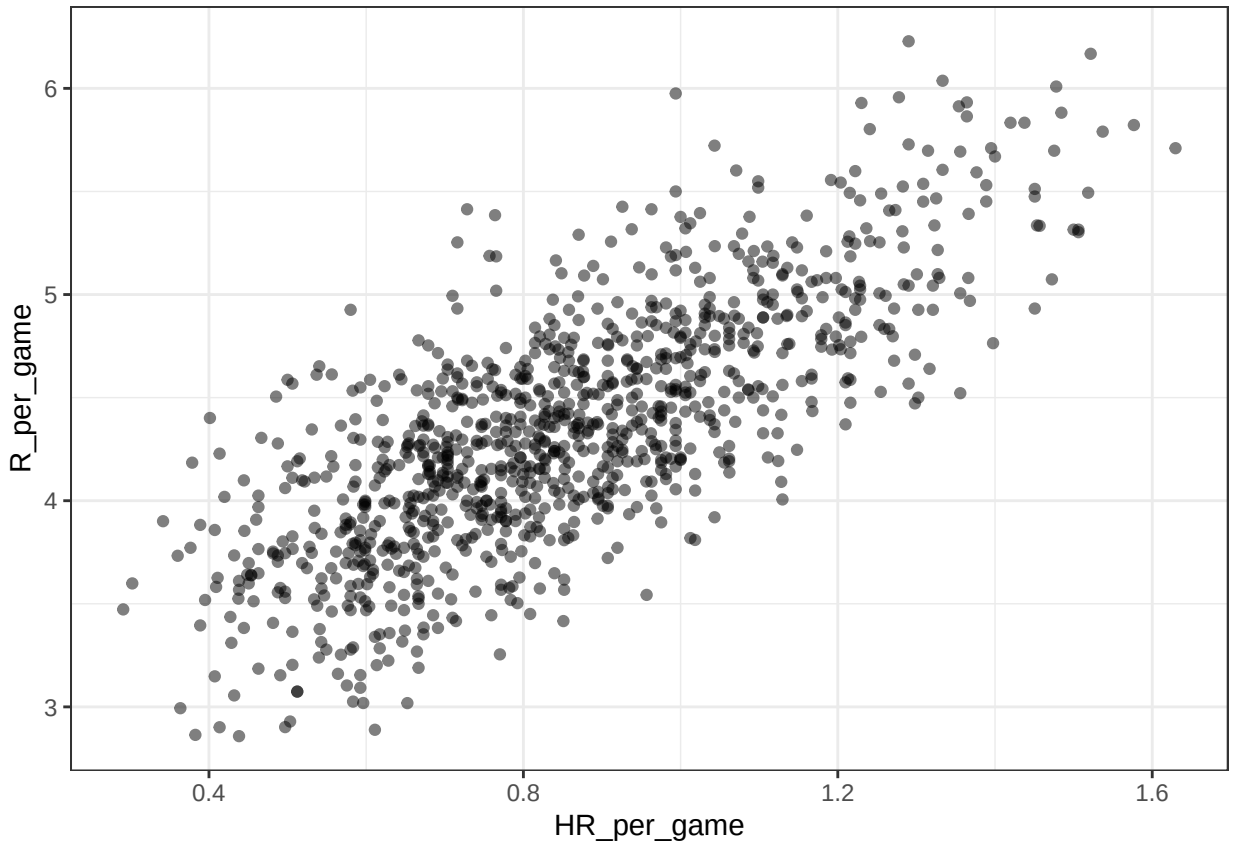
```
## v ggplot2 3.3.2     v purrr   0.3.4
## v tibble  3.0.3     v dplyr   1.0.2
## v tidyr   1.1.2     v stringr 1.4.0
## v readr   1.3.1     v forcats 0.5.0
```

```
## -- Conflicts ------------------------------------------------- tidyverse_conflicts() --
## x dplyr::filter() masks stats::filter()
## x dplyr::lag()    masks stats::lag()
```

```
if(!require(dslabs)) install.packages("dslabs")
```

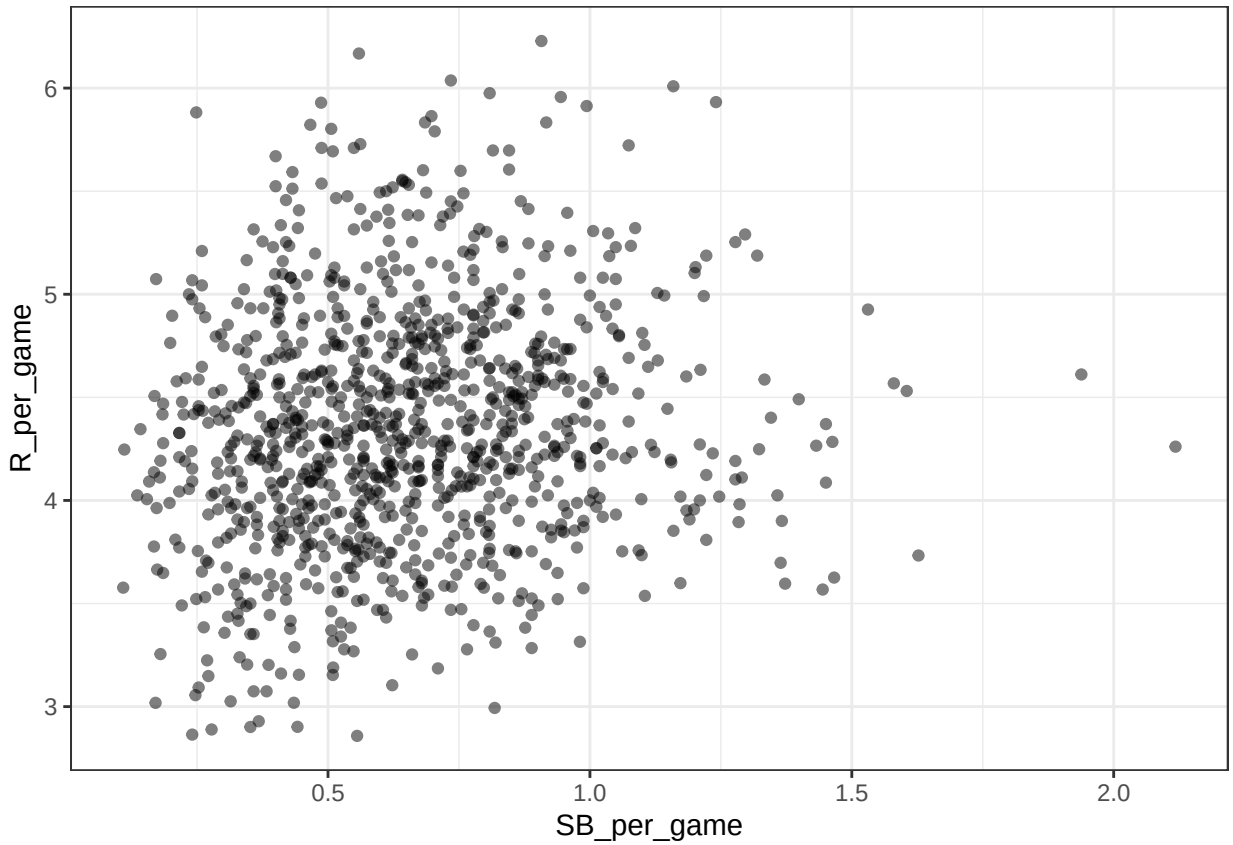
```
## Loading required package: dslabs
```

```
library(Lahman)
library(tidyverse)
library(dslabs)
ds_theme_set()

Teams %>% filter(yearID %in% 1961:2001) %>%
    mutate(HR_per_game = HR / G, R_per_game = R / G) %>%
    ggplot(aes(HR_per_game, R_per_game)) +
    geom_point(alpha = 0.5)
```



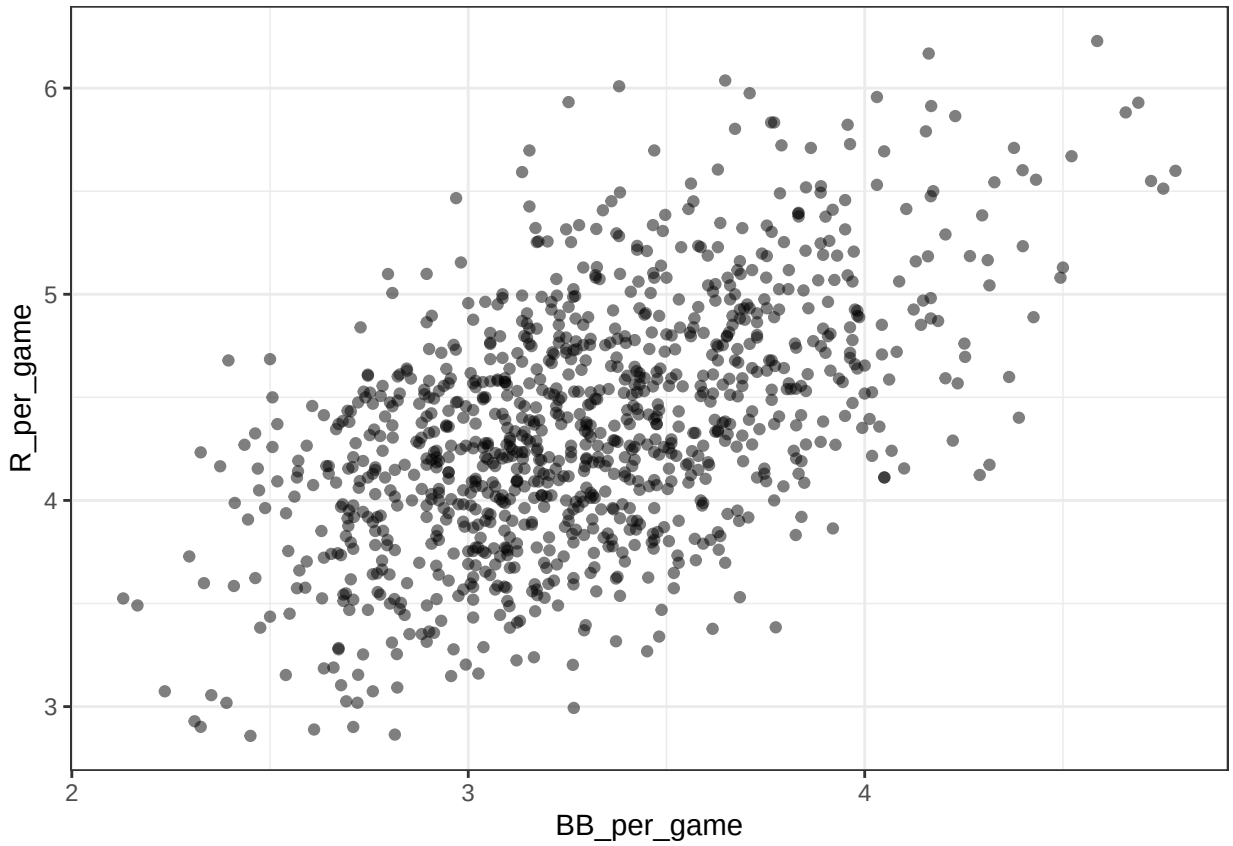
Code: Scatterplot of the relationship between stolen bases and runs

```
Teams %>% filter(yearID %in% 1961:2001) %>%
    mutate(SB_per_game = SB / G, R_per_game = R / G) %>%
    ggplot(aes(SB_per_game, R_per_game)) +
    geom_point(alpha = 0.5)
```



Code: Scatterplot of the relationship between bases on balls and runs

```
Teams %>% filter(yearID %in% 1961:2001) %>%
    mutate(BB_per_game = BB / G, R_per_game = R / G) %>%
    ggplot(aes(BB_per_game, R_per_game)) +
    geom_point(alpha = 0.5)
```



Assessment - Baseball as a Motivating Example

1. What is the application of statistics and data science to baseball called?

- [ ] A. Moneyball
- [X] B. Sabermetrics
- [ ] C. The "Oakland A's Approach"
- [ ] D. There is no specific name for this; it's just data science.

2. Which of the following outcomes is not included in the batting average?

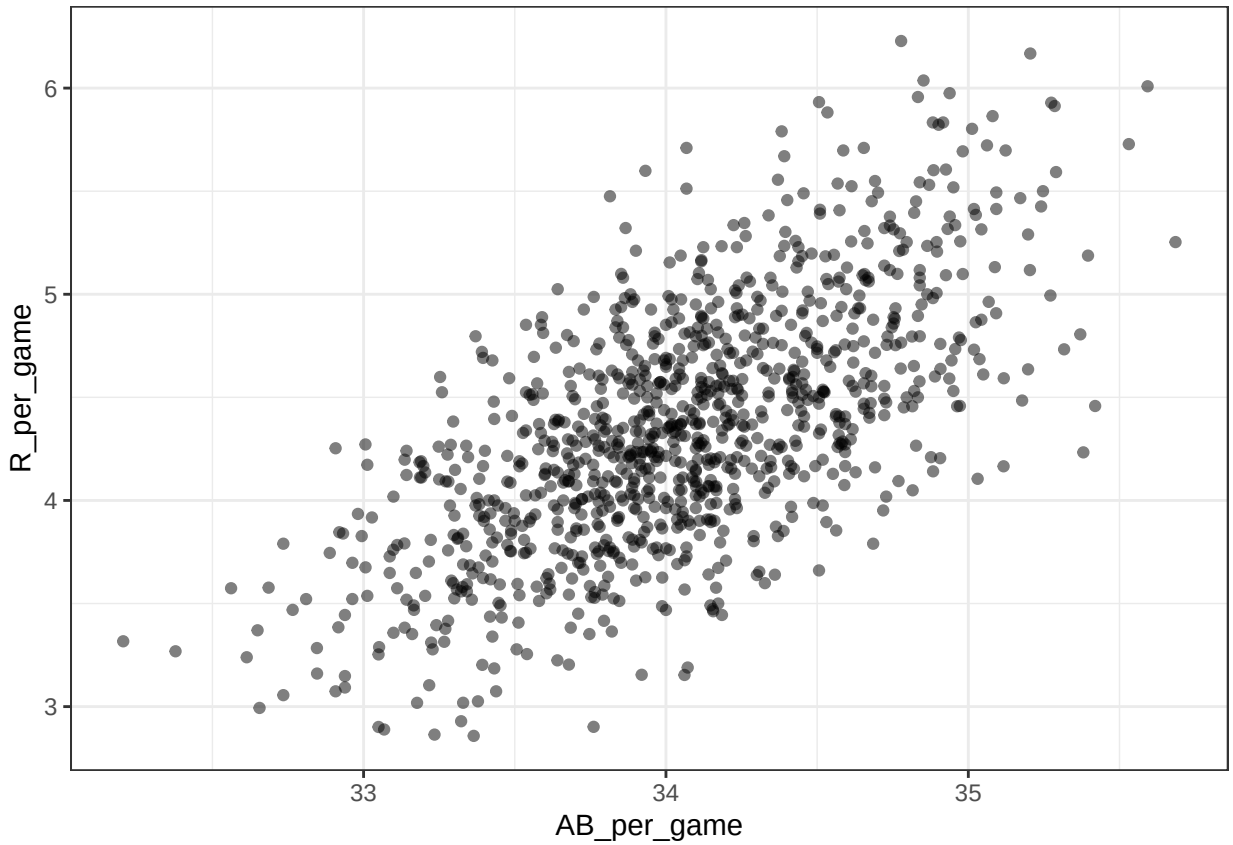
- [ ] A. A home run
- [X] B. A base on balls
- [ ] C. An out
- [ ] D. A single

3. Why do we consider team statistics as well as individual player statistics?

- [X] A. The success of any individual player also depends on the strength of their team.
- [ ] B. Team statistics can be easier to calculate.
- [ ] C. The ultimate goal of sabermetrics is to rank teams, not players.

4. You want to know whether teams with more at-bats per game have more runs per game. What R code below correctly makes a scatter plot for this relationship?

```
Teams %>% filter(yearID %in% 1961:2001 ) %>%
    mutate(AB_per_game = AB/G, R_per_game = R/G) %>%
    ggplot(aes(AB_per_game, R_per_game)) +
    geom_point(alpha = 0.5)
```



- [ ] A.

```
Teams %>% filter(yearID %in% 1961:2001 ) %>%
    ggplot(aes(AB, R)) +
    geom_point(alpha = 0.5)
```

- [X] B.

```
Teams %>% filter(yearID %in% 1961:2001 ) %>%
    mutate(AB_per_game = AB/G, R_per_game = R/G) %>%
    ggplot(aes(AB_per_game, R_per_game)) +
    geom_point(alpha = 0.5)
```

- [ ] C.

```
Teams %>% filter(yearID %in% 1961:2001 ) %>%
    mutate(AB_per_game = AB/G, R_per_game = R/G) %>%
    ggplot(aes(AB_per_game, R_per_game)) +
    geom_line()
```

- [ ] D.

```
Teams %>% filter(yearID %in% 1961:2001 ) %>%
    mutate(AB_per_game = AB/G, R_per_game = R/G) %>%
    ggplot(aes(R_per_game, AB_per_game)) +
    geom_point()
```

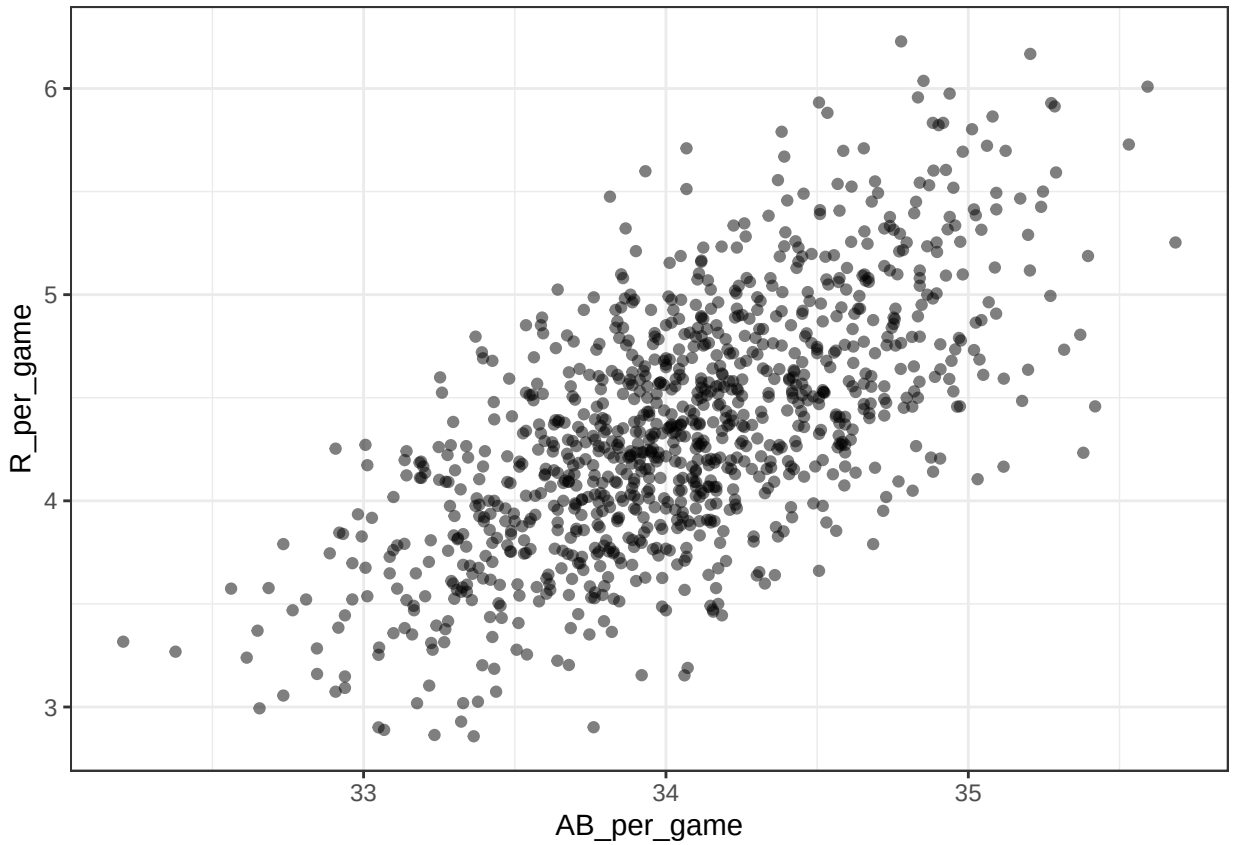
5. What does the variable "SOA" stand for in the Teams table?

Hint: make sure to use the help file (`?Teams`).

- [ ] A. sacrifice out
- [ ] B. slides or attempts
- [X] C. strikeouts by pitchers
- [ ] D. accumulated singles

6. Load the **Lahman** library. Filter the `Teams` data frame to include years from 1961 to 2001. Make a scatterplot of runs per game versus at bats (`AB`) per game.

```
Teams %>% filter(yearID %in% 1961:2001) %>%
  mutate(AB_per_game = AB / G, R_per_game = R / G) %>%
  ggplot(aes(AB_per_game, R_per_game)) +
  geom_point(alpha = 0.5)
```

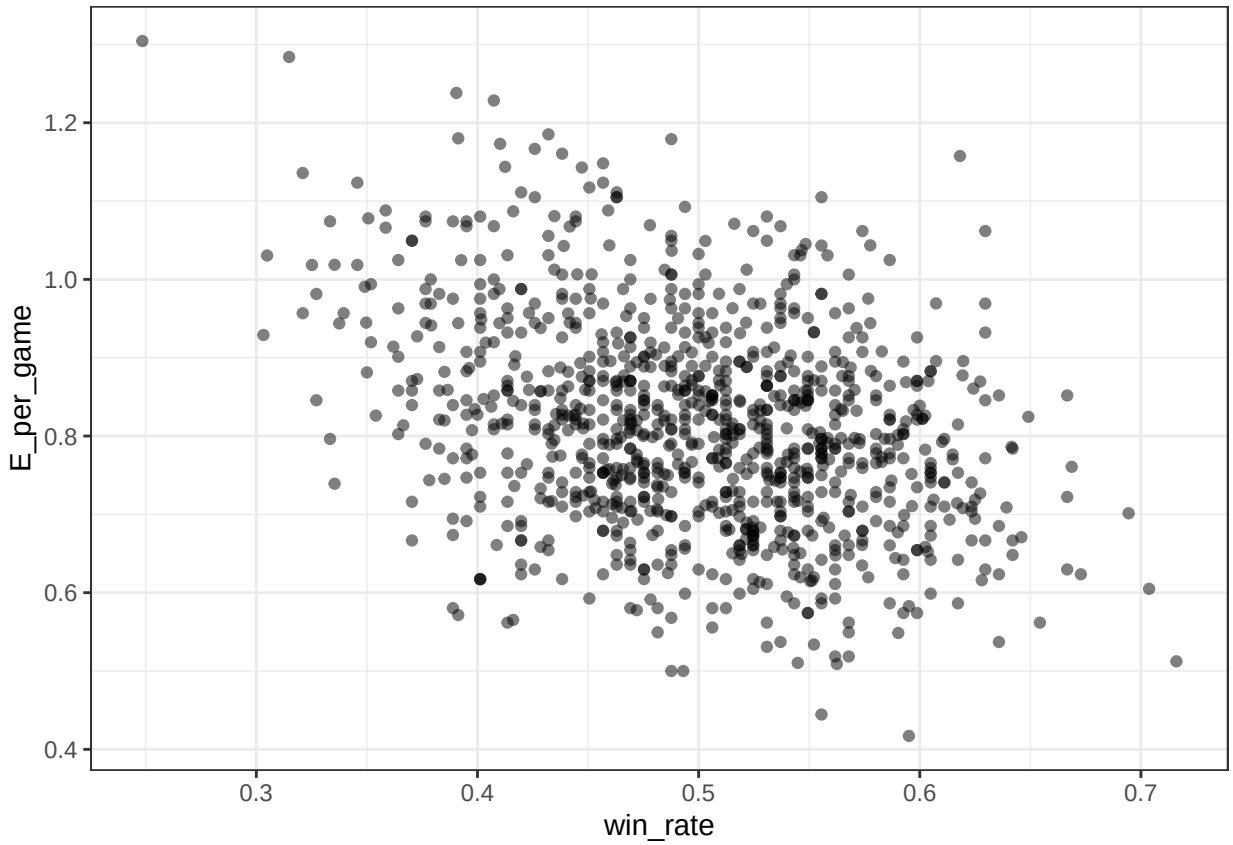


Which of the following is true?

- [ ] A. There is no clear relationship between runs and at bats per game.
- [X] B. As the number of at bats per game increases, the number of runs per game tends to increase.
- [ ] C. As the number of at bats per game increases, the number of runs per game tends to decrease.

7. Use the filtered `Teams` data frame from Question 6. Make a scatterplot of win rate (number of wins per game) versus number of fielding errors (`E`) per game.

```
Teams %>% filter(yearID %in% 1961:2001) %>%
  mutate(win_rate = W / G, E_per_game = E / G) %>%
  ggplot(aes(win_rate, E_per_game)) +
  geom_point(alpha = 0.5)
```

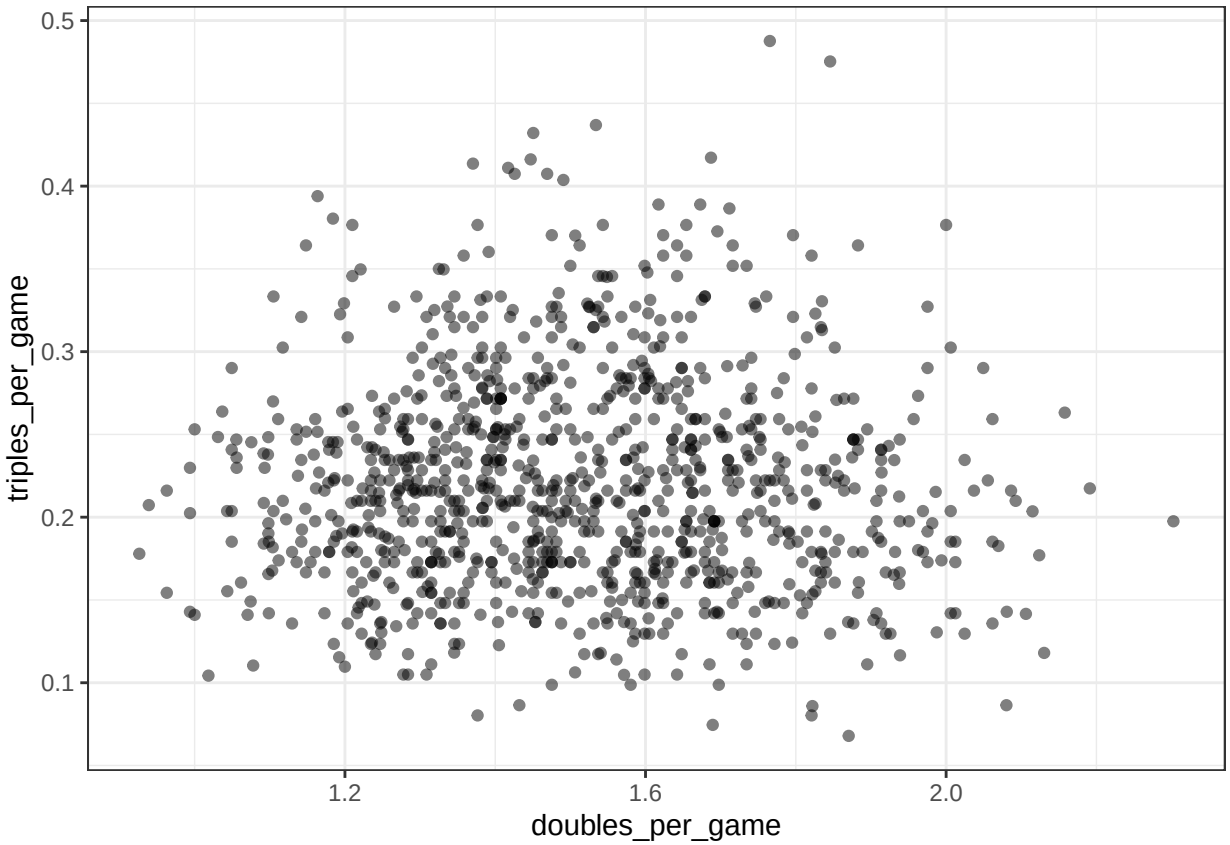


Which of the following is true?

- [ ] A. There is no relationship between win rate and errors per game.
- [ ] B. As the number of errors per game increases, the win rate tends to increase.
- [X] C. As the number of errors per game increases, the win rate tends to decrease.

8. Use the filtered `Teams` data frame from Question 6. Make a scatterplot of triples (`X3B`) per game versus doubles (`X2B`) per game.

```
Teams %>% filter(yearID %in% 1961:2001) %>%
  mutate(doubles_per_game = X2B / G, triples_per_game = X3B / G) %>%
  ggplot(aes(doubles_per_game, triples_per_game)) +
  geom_point(alpha = 0.5)
```



Which of the following is true?

- [X] A. There is no clear relationship between doubles per game and triples per game.
- [ ] B. As the number of doubles per game increases, the number of triples per game tends to increase.
- [ ] C. As the number of doubles per game increases, the number of triples per game tends to decrease.

Correlation

The corresponding textbook section is Case Study: is height hereditary?

Key points

- Galton tried to predict sons' heights based on fathers' heights.
- The mean and standard errors are insufficient for describing an important characteristic of the data: the trend that the taller the father, the taller the son.
- The correlation coefficient is an informative summary of how two variables move together that can be used to predict one variable using the other.

Code

```
# create the dataset
if(!require(HistData)) install.packages("HistData")
```

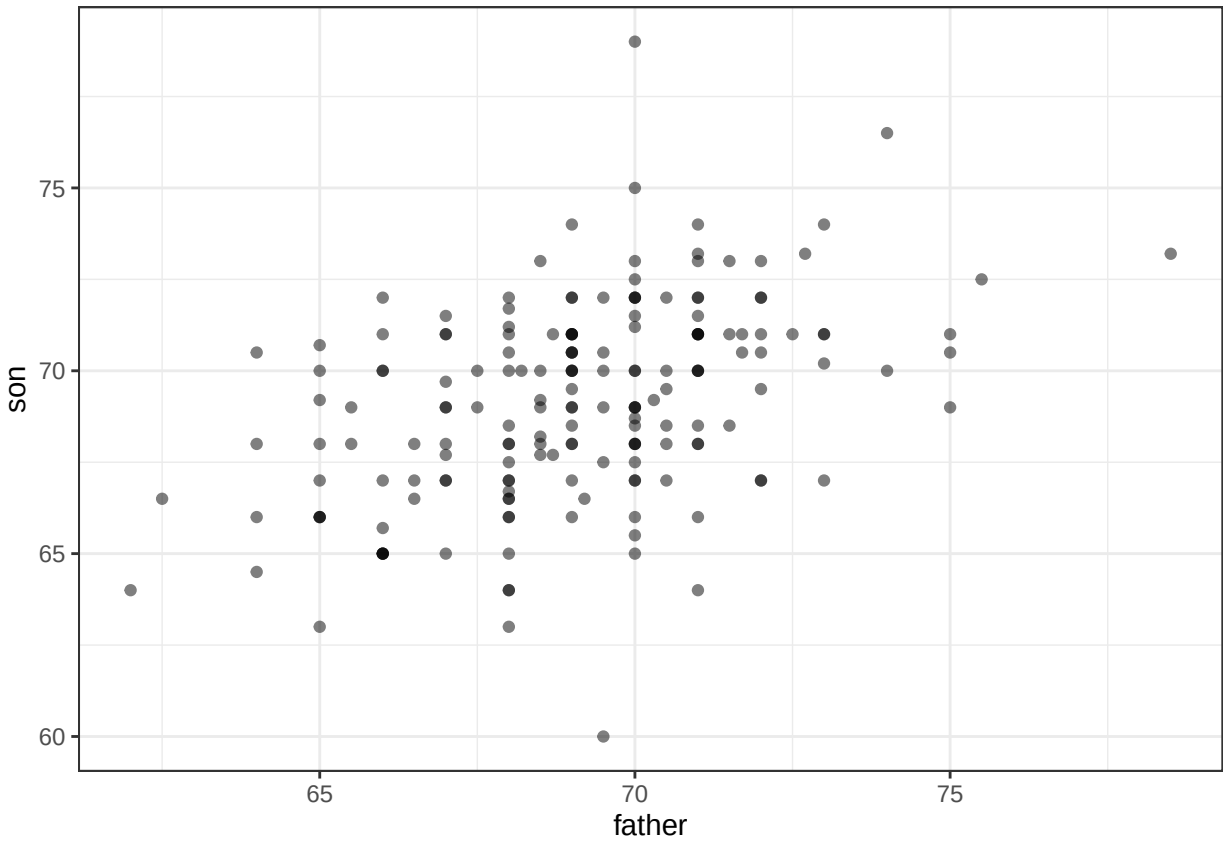
```
## Loading required package: HistData
```

```
library(HistData)
data("GaltonFamilies")
set.seed(1983)
galton_heights <- GaltonFamilies %>%
  filter(gender == "male") %>%
```

```
  group_by(family) %>%
  sample_n(1) %>%
  ungroup() %>%
  select(father, childHeight) %>%
  rename(son = childHeight)

# means and standard deviations
galton_heights %>%
    summarize(mean(father), sd(father), mean(son), sd(son))
```

```
## # A tibble: 1 x 4
##   `mean(father)` `sd(father)` `mean(son)` `sd(son)`
##            <dbl>        <dbl>       <dbl>     <dbl>
## 1           69.1         2.55        69.2      2.71
```

```
# scatterplot of father and son heights
galton_heights %>%
    ggplot(aes(father, son)) +
    geom_point(alpha = 0.5)
```



Correlation Coefficient

The corresponding textbook section is the correlation coefficient

Key points

- The correlation coefficient is defined for a list of pairs $(x_1, y_1), ..., (x_n, y_n)$ as the product of the standardized values: $(\frac{x_i - \mu_x}{\sigma_x})(\frac{y_i - \mu_y}{\sigma_y})$.

- The correlation coefficient essentially conveys how two variables move together.
- The correlation coefficient is always between -1 and 1.

Code

```
rho <- mean(scale(x)*scale(y))
```

```
galton_heights <- GaltonFamilies %>% filter(childNum == 1 & gender == "male") %>% select(father, childH
galton_heights %>% summarize(r = cor(father, son)) %>% pull(r)
```

```
## [1] 0.5007248
```

Sample Correlation is a Random Variable

The corresponding textbook section is Sample correlation is a random variable

Key points

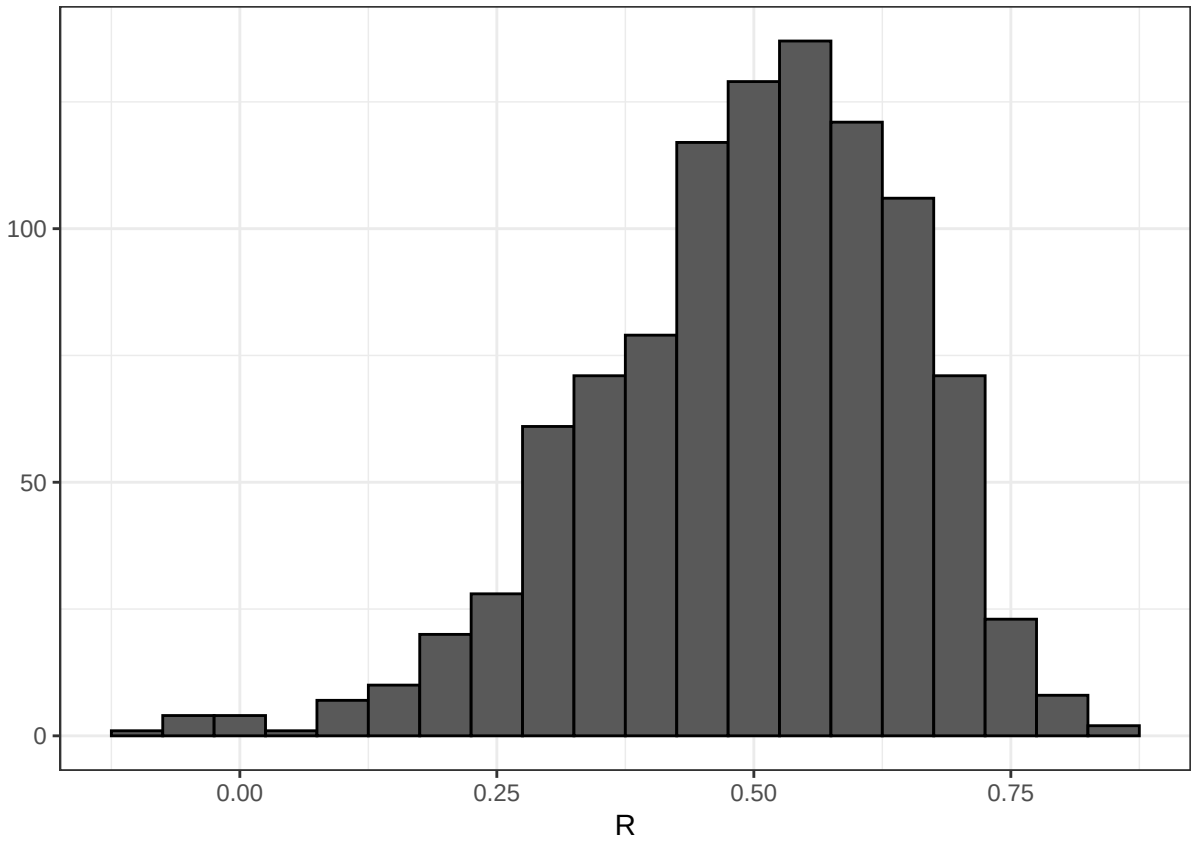
- The correlation that we compute and use as a summary is a random variable.
- When interpreting correlations, it is important to remember that correlations derived from samples are estimates containing uncertainty.
- Because the sample correlation is an average of independent draws, the central limit theorem applies.

Code

```
# compute sample correlation
R <- sample_n(galton_heights, 25, replace = TRUE) %>%
    summarize(r = cor(father, son))
R
```

```
##           r
## 1 0.4787613
```

```
# Monte Carlo simulation to show distribution of sample correlation
B <- 1000
N <- 25
R <- replicate(B, {
    sample_n(galton_heights, N, replace = TRUE) %>%
    summarize(r = cor(father, son)) %>%
    pull(r)
})
qplot(R, geom = "histogram", binwidth = 0.05, color = I("black"))
```



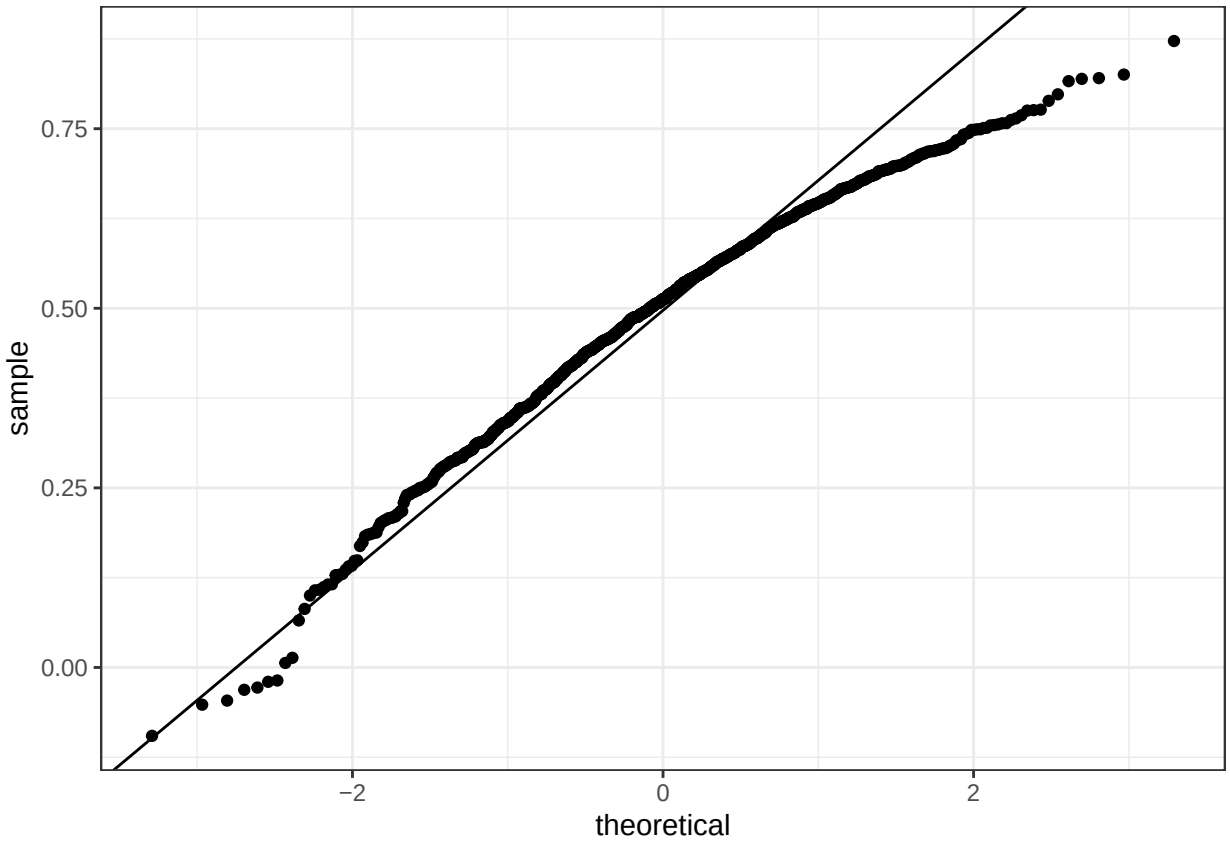

```
# expected value and standard error
mean(R)
```

```
## [1] 0.4970997
```

```
sd(R)
```

```
## [1] 0.1512451
```

```
# QQ-plot to evaluate whether N is large enough
data.frame(R) %>%
    ggplot(aes(sample = R)) +
    stat_qq() +
    geom_abline(intercept = mean(R), slope = sqrt((1-mean(R)^2)/(N-2)))
```



Assessment - Correlation

1. While studying heredity, Francis Galton developed what important statistical concept?

- [ ] A. Standard deviation
- [ ] B. Normal distribution
- [X] C. Correlation
- [ ] D. Probability

2. The correlation coefficient is a summary of what?

- [X] A. The trend between two variables
- [ ] B. The dispersion of a variable
- [ ] C. The central tendency of a variable
- [ ] D. The distribution of a variable

3. Below is a scatter plot showing the relationship between two variables, x and y.

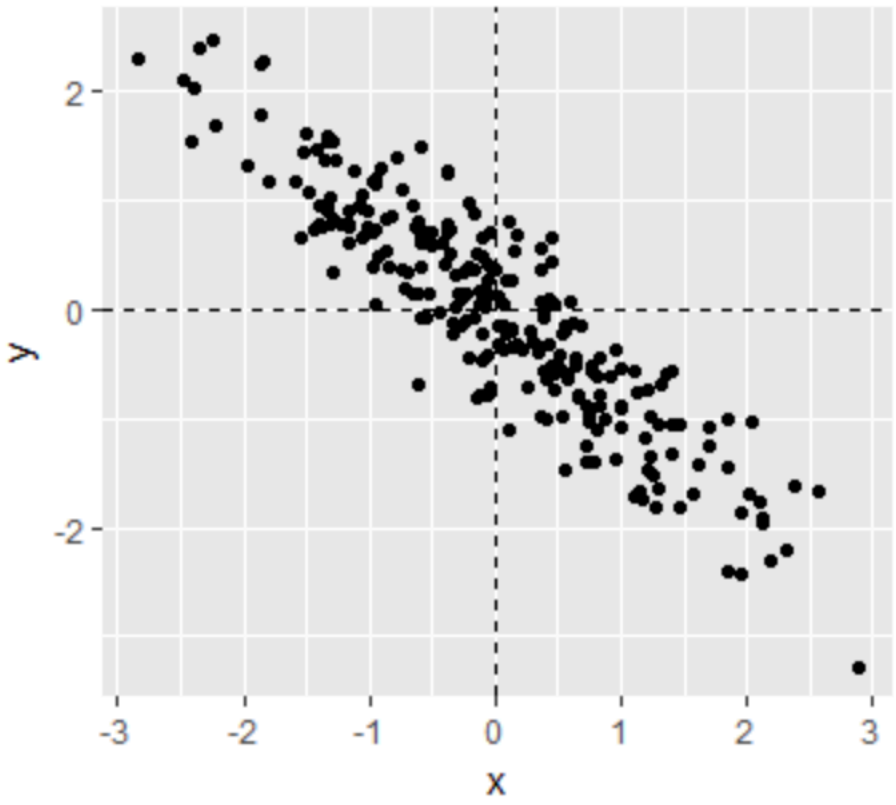


Figure 1: Scatter plot relationship x and y

From this figure, the correlation between x and y appears to be about:

- [X] A. -0.9
- [ ] B. -0.2
- [ ] C. 0.9
- [ ] D. 2

4. Instead of running a Monte Carlo simulation with a sample size of 25 from our 179 father-son pairs, we now run our simulation with a sample size of 50.

Would you expect the **mean** of our sample correlation to increase, decrease, or stay approximately the same?

- [ ] A. Increase
- [ ] B. Decrease
- [X] C. Stay approximately the same

5. Instead of running a Monte Carlo simulation with a sample size of 25 from our 179 father-son pairs, we now run our simulation with a sample size of 50.

Would you expect the **standard deviation** of our sample correlation to increase, decrease, or stay approximately the same?

- [ ] A. Increase
- [X] B. Decrease
- [ ] C. Stay approximately the same

6. If X and Y are completely independent, what do you expect the value of the correlation coefficient to be?

- [ ] A. -1
- [ ] B. -0.5
- [X] C. 0
- [ ] D. 0.5
- [ ] E. 1
- [ ] F. Not enough information to answer the question

7. Load the **Lahman** library. Filter the `Teams` data frame to include years from 1961 to 2001.

What is the correlation coefficient between number of runs per game and number of at bats per game?

```
Teams_small <- Teams %>% filter(yearID %in% 1961:2001)
cor(Teams_small$R/Teams_small$G, Teams_small$AB/Teams_small$G)
```

```
## [1] 0.6580976
```

8. Use the filtered `Teams` data frame from Question 7.

What is the correlation coefficient between win rate (number of wins per game) and number of errors per game?

```
cor(Teams_small$W/Teams_small$G, Teams_small$E/Teams_small$G)
```

```
## [1] -0.3396947
```

9. Use the filtered `Teams` data frame from Question 7.

What is the correlation coefficient between doubles (`X2B`) per game and triples (`X3B`) per game?

```
cor(Teams_small$X2B/Teams_small$G, Teams_small$X3B/Teams_small$G)
```

```
## [1] -0.01157404
```

Anscombe's Quartet/Stratification

There are three links to relevant sections of the textbook:

- Correlation is not always a useful summary
- Conditional expectation
- The regression line

Key points

- Correlation is not always a good summary of the relationship between two variables.
- The general idea of conditional expectation is that we stratify a population into groups and compute summaries in each group.
- A practical way to improve the estimates of the conditional expectations is to define strata of with similar values of x.
- If there is perfect correlation, the regression line predicts an increase that is the same number of SDs for both variables. If there is 0 correlation, then we don't use x at all for the prediction and simply predict the average μ_y. For values between 0 and 1, the prediction is somewhere in between. If the correlation is negative, we predict a reduction instead of an increase.

Code

```
# number of fathers with height 72 or 72.5 inches
sum(galton_heights$father == 72)
```

```
## [1] 8
```

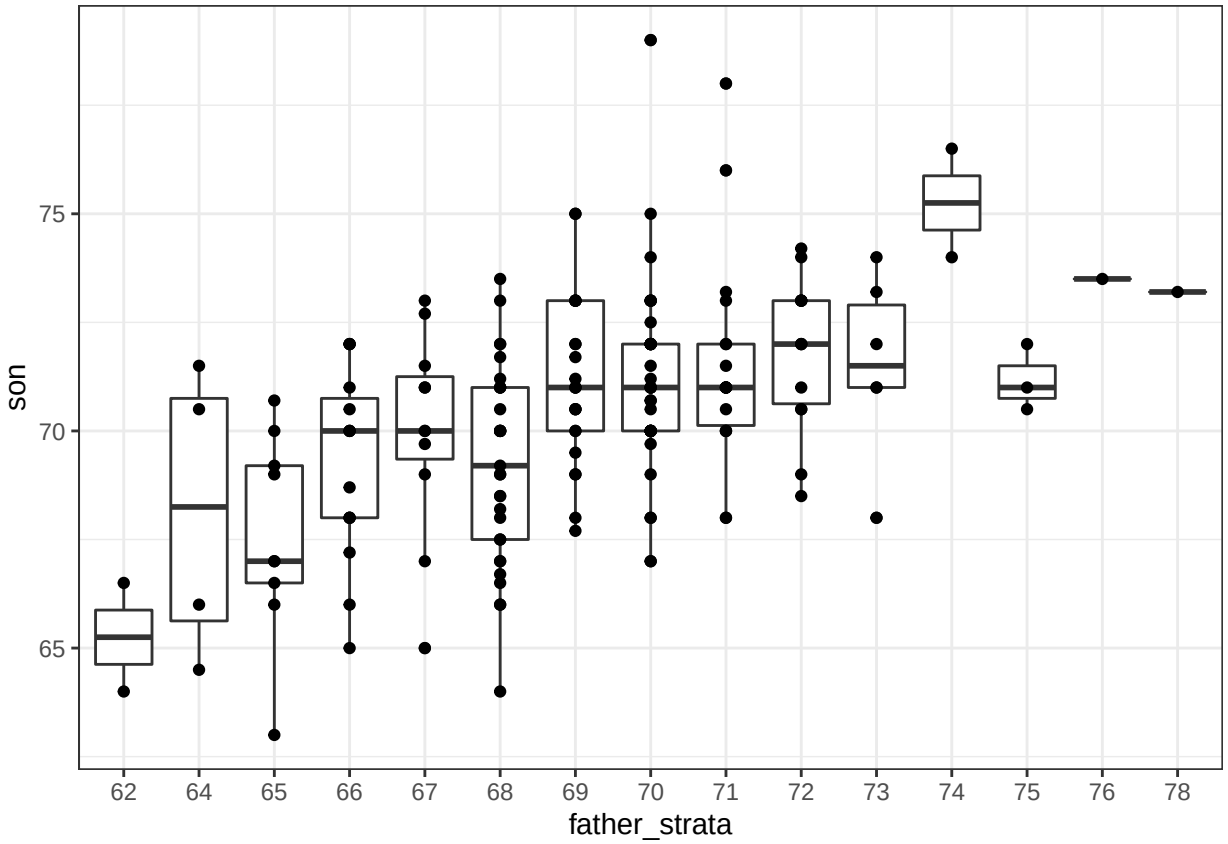
```r
sum(galton_heights$father == 72.5)
```

```
## [1] 1
```

```r
# predicted height of a son with a 72 inch tall father
conditional_avg <- galton_heights %>%
    filter(round(father) == 72) %>%
    summarize(avg = mean(son)) %>%
    pull(avg)
conditional_avg
```

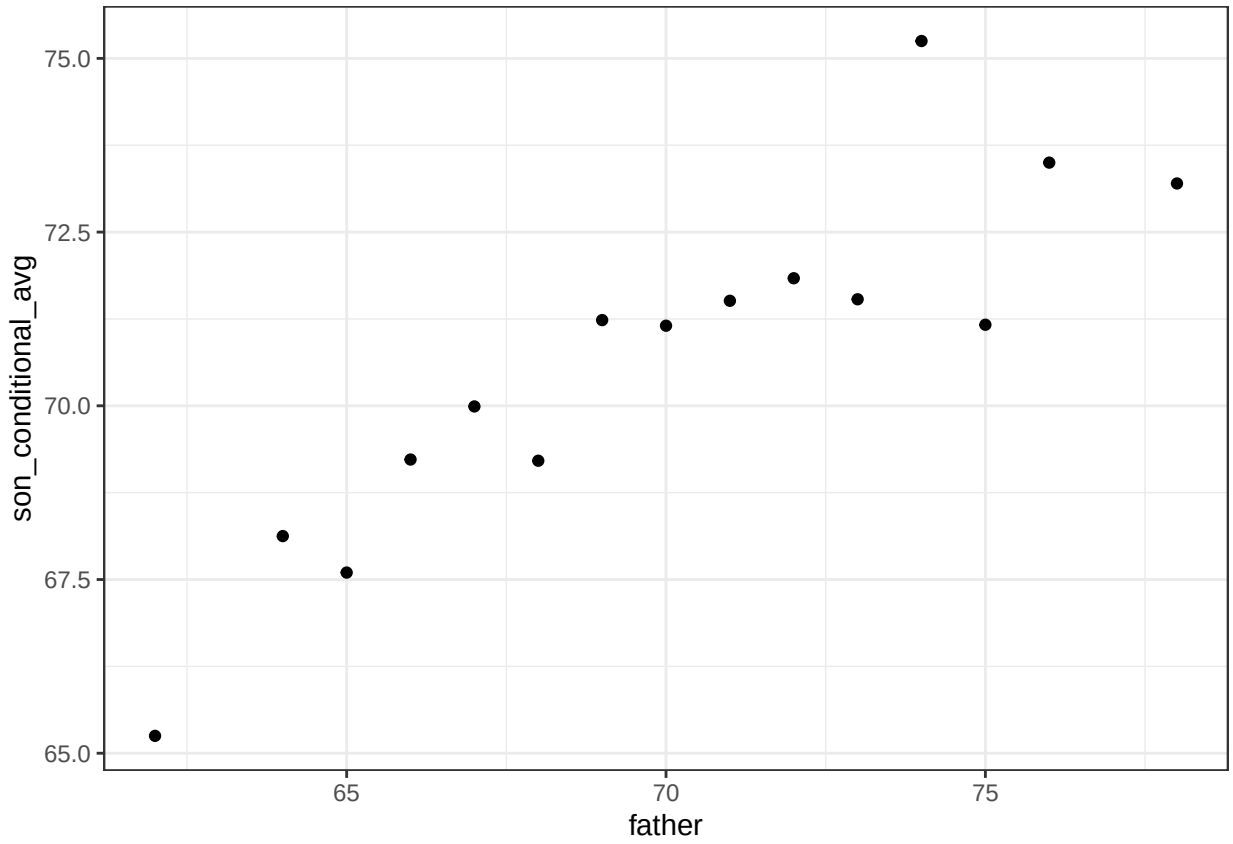
```
## [1] 71.83571
```

```r
# stratify fathers' heights to make a boxplot of son heights
galton_heights %>% mutate(father_strata = factor(round(father))) %>%
    ggplot(aes(father_strata, son)) +
    geom_boxplot() +
    geom_point()
```

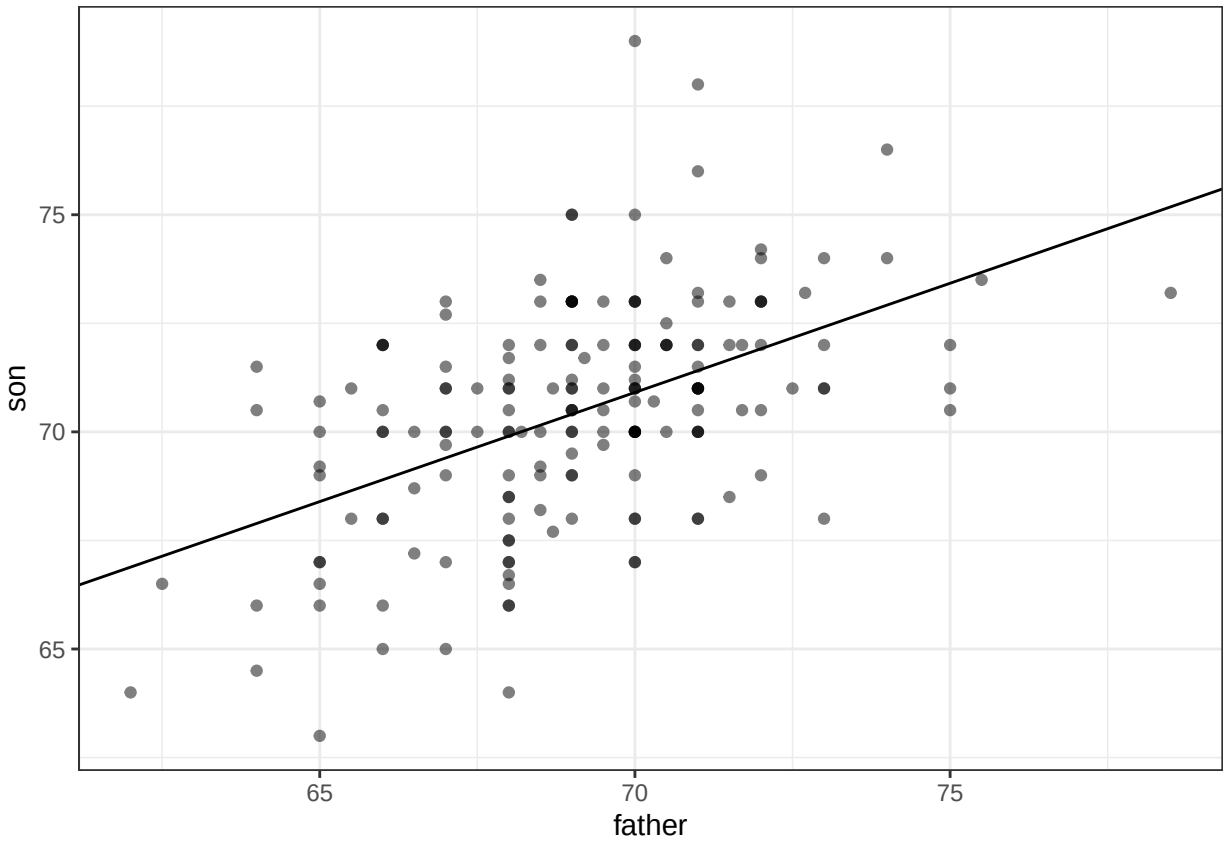



```r
# center of each boxplot
galton_heights %>%
    mutate(father = round(father)) %>%
    group_by(father) %>%
    summarize(son_conditional_avg = mean(son)) %>%
    ggplot(aes(father, son_conditional_avg)) +
    geom_point()
```

```
## `summarise()` ungrouping output (override with `.groups` argument)
```




```
# calculate values to plot regression line on original data
mu_x <- mean(galton_heights$father)
mu_y <- mean(galton_heights$son)
s_x <- sd(galton_heights$father)
s_y <- sd(galton_heights$son)
r <- cor(galton_heights$father, galton_heights$son)
m <- r * s_y/s_x
b <- mu_y - m*mu_x

# add regression line to plot
galton_heights %>%
    ggplot(aes(father, son)) +
    geom_point(alpha = 0.5) +
    geom_abline(intercept = b, slope = m)
```



Bivariate Normal Distribution

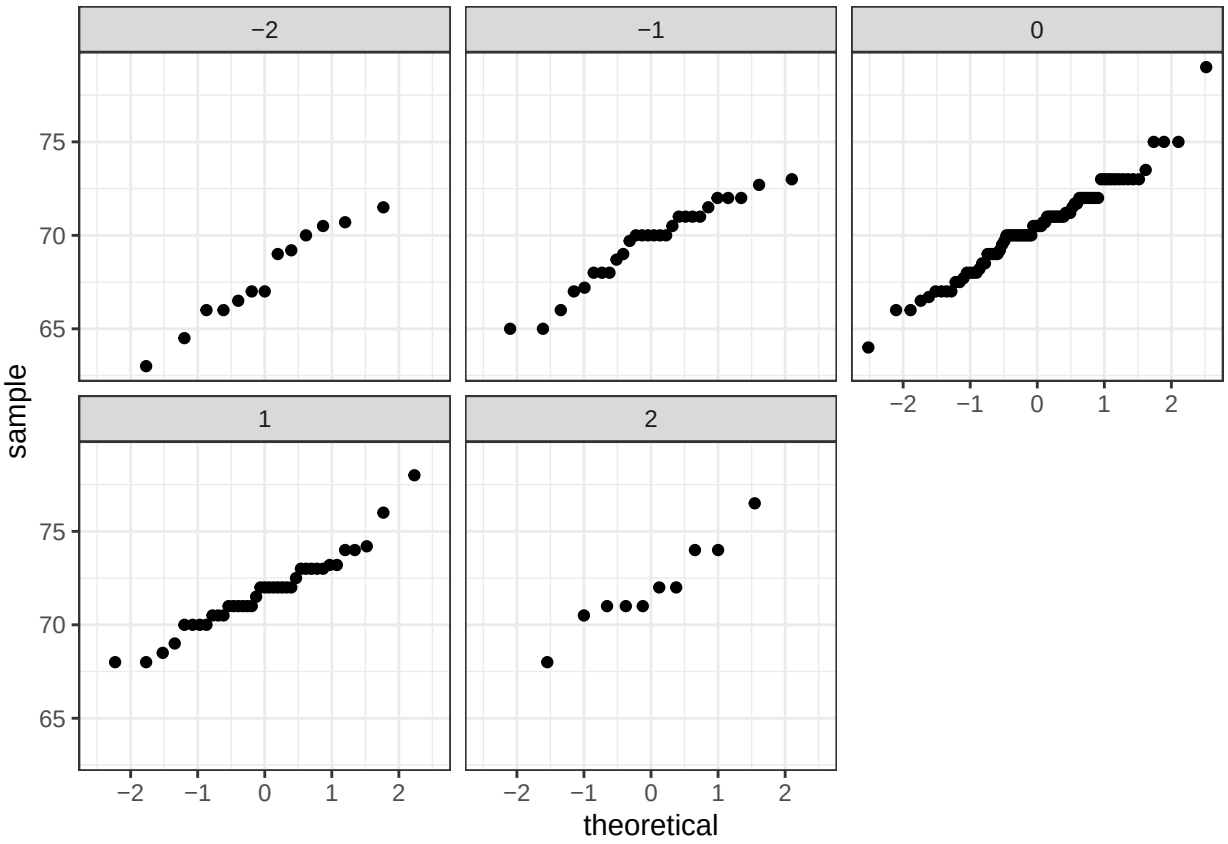
There is a link to the relevant section of the textbook: Bivariate normal distribution (advanced)

Key points

- When a pair of random variables are approximated by the bivariate normal distribution, scatterplots look like ovals. They can be thin (high correlation) or circle-shaped (no correlation).
- When two variables follow a bivariate normal distribution, computing the regression line is equivalent to computing conditional expectations.
- We can obtain a much more stable estimate of the conditional expectation by finding the regression line and using it to make predictions.

Code

```
galton_heights %>%
  mutate(z_father = round((father - mean(father)) / sd(father))) %>%
  filter(z_father %in% -2:2) %>%
  ggplot() +
  stat_qq(aes(sample = son)) +
  facet_wrap( ~ z_father)
```



Variance Explained

There is a link to the relevant section of the textbook: Variance explained

Key points

- Conditioning on a random variable X can help to reduce variance of response variable Y.
- The standard deviation of the conditional distribution is $\text{SD}(Y \mid X = x) = \sigma_y\sqrt{1-\rho^2}$, which is smaller than the standard deviation without conditioning σ_y.
- Because variance is the standard deviation squared, the variance of the conditional distribution is $\sigma_y^2(1-\rho^2)$.
- In the statement "X explains such and such percent of the variability," the percent value refers to the variance. The variance decreases by ρ^2 percent.
- The "variance explained" statement only makes sense when the data is approximated by a bivariate normal distribution.

There are Two Regression Lines

There is a link to the relevant section of the textbook: Warning: there are two regression lines

Key point

There are two different regression lines depending on whether we are taking the expectation of Y given X or taking the expectation of X given Y.

Code

```
# compute a regression line to predict the son's height from the father's height
mu_x <- mean(galton_heights$father)
mu_y <- mean(galton_heights$son)
```

```r
s_x <- sd(galton_heights$father)
s_y <- sd(galton_heights$son)
r <- cor(galton_heights$father, galton_heights$son)
m_1 <-  r * s_y / s_x
b_1 <- mu_y - m_1*mu_x
m_1 # slope 1
```

```
## [1] 0.5027904
```

```r
b_1 # intercept 1
```

```
## [1] 35.71249
```

```r
# compute a regression line to predict the father's height from the son's height
m_2 <-  r * s_x / s_y
b_2 <- mu_x - m_2*mu_y
m_2 # slope 2
```

```
## [1] 0.4986676
```

```r
b_2 # intercept 2
```

```
## [1] 33.96539
```

Assessment - Stratification and Variance Explained, Part 1

1. Look at the figure below. The slope of the regression line in this figure is equal to what, in words?

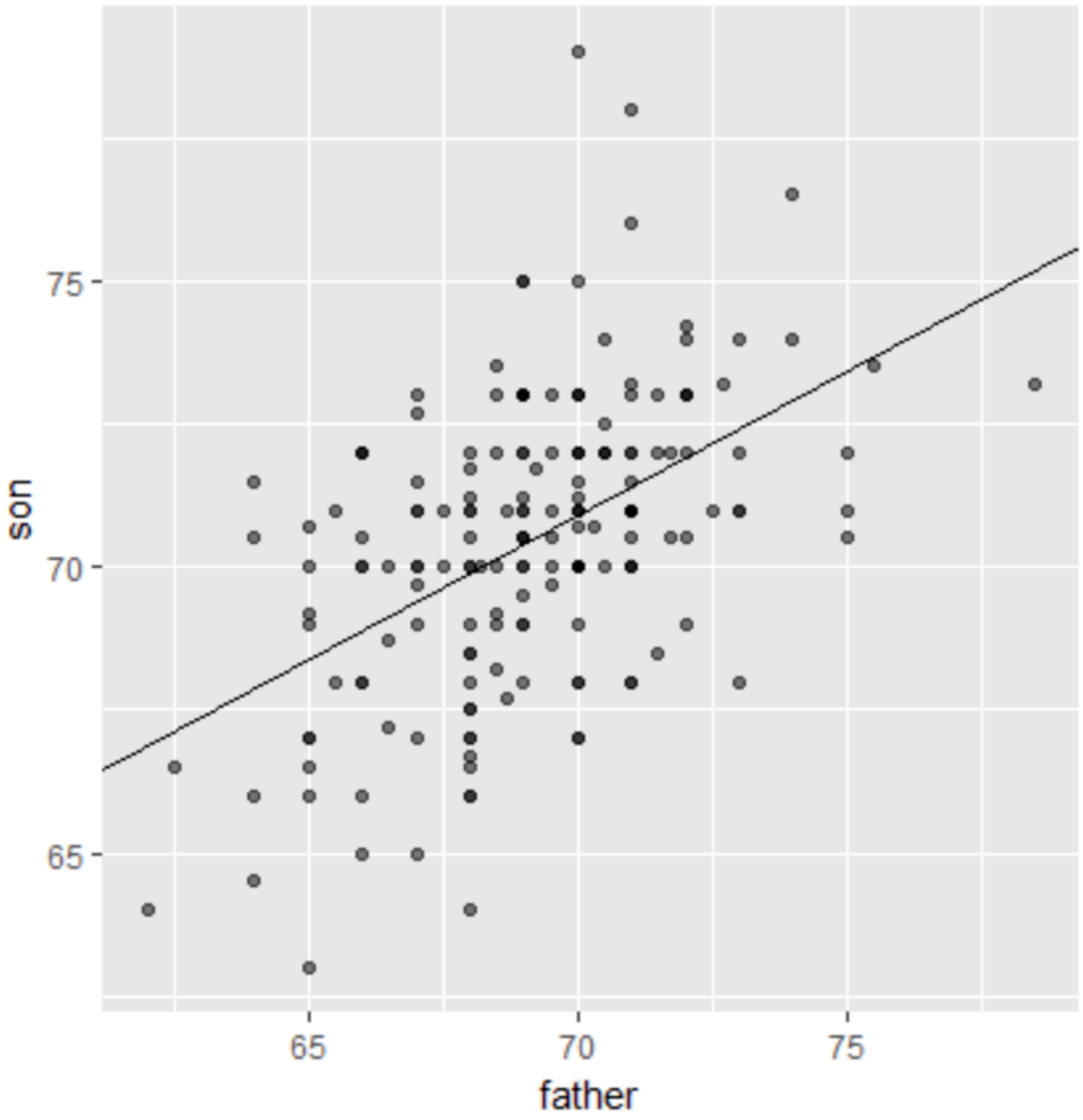


Figure 2: Scatter plot and regression line of son and father heights

- [X] A. Slope = (correlation coefficient of son and father heights) * (standard deviation of sons' heights / standard deviation of fathers' heights)
- [ ] B. Slope = (correlation coefficient of son and father heights) * (standard deviation of fathers' heights / standard deviation of sons' heights)
- [ ] C. Slope = (correlation coefficient of son and father heights) / (standard deviation of sons' heights * standard deviation of fathers' heights)
- [ ] D. Slope = (mean height of fathers) - (correlation coefficient of son and father heights * mean height of sons).

2. Why does the regression line simplify to a line with intercept zero and slope ρ when we standardize our x and y variables? Try the simplification on your own first!

- [ ] A. When we standardize variables, both x and y will have a mean of one and a standard deviation of zero. When you substitute this into the formula for the regression line, the terms cancel out until we have the following equation: $y_i = \rho x_i$.
- [X] B. When we standardize variables, both x and y will have a mean of zero and a standard deviation of one. When you substitute this into the formula for the regression line, the terms cancel out until we have the following equation: $y_i = \rho x_i$.
- [ ] C. When we standardize variables, both x and y will have a mean of zero and a standard deviation of one. When you substitute this into the formula for the regression line, the terms cancel out until we have the following equation: $y_i = \rho + x_i$.

3. What is a limitation of calculating conditional means?

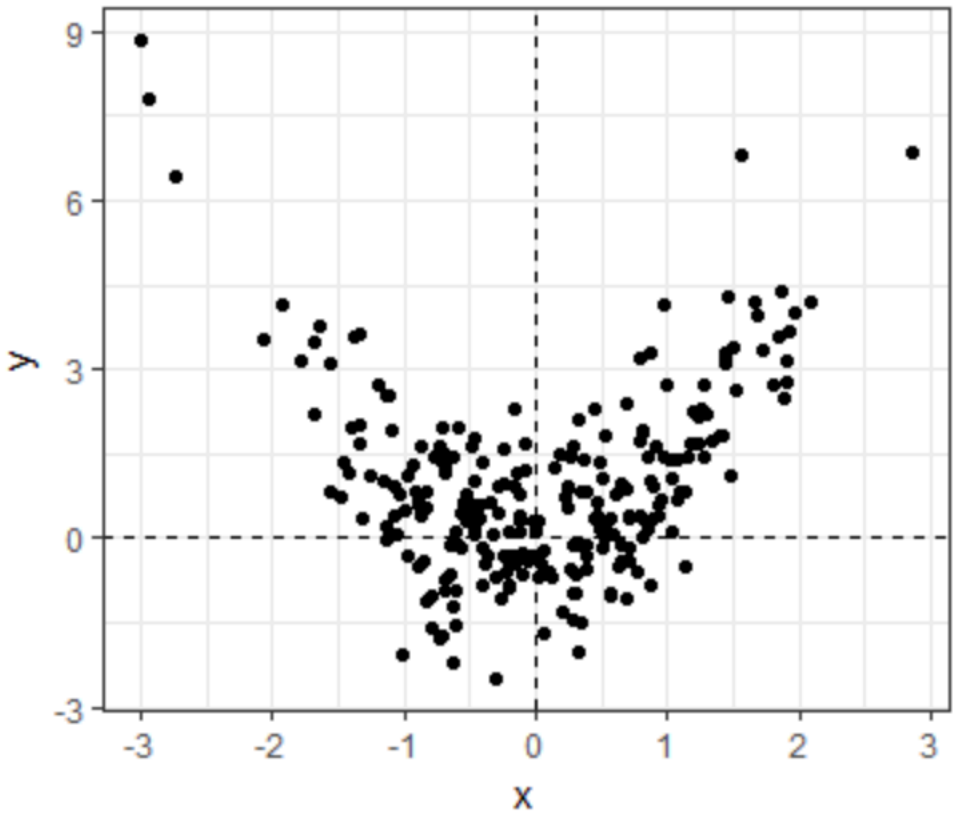
- [X] A. Each stratum we condition on (e.g., a specific father's height) may not have many data points.
- [X] B. Because there are limited data points for each stratum, our average values have large standard errors.
- [X] C. Conditional means are less stable than a regression line.
- [ ] D. Conditional means are a useful theoretical tool but cannot be calculated.

4. A regression line is the best prediction of Y given we know the value of X when:

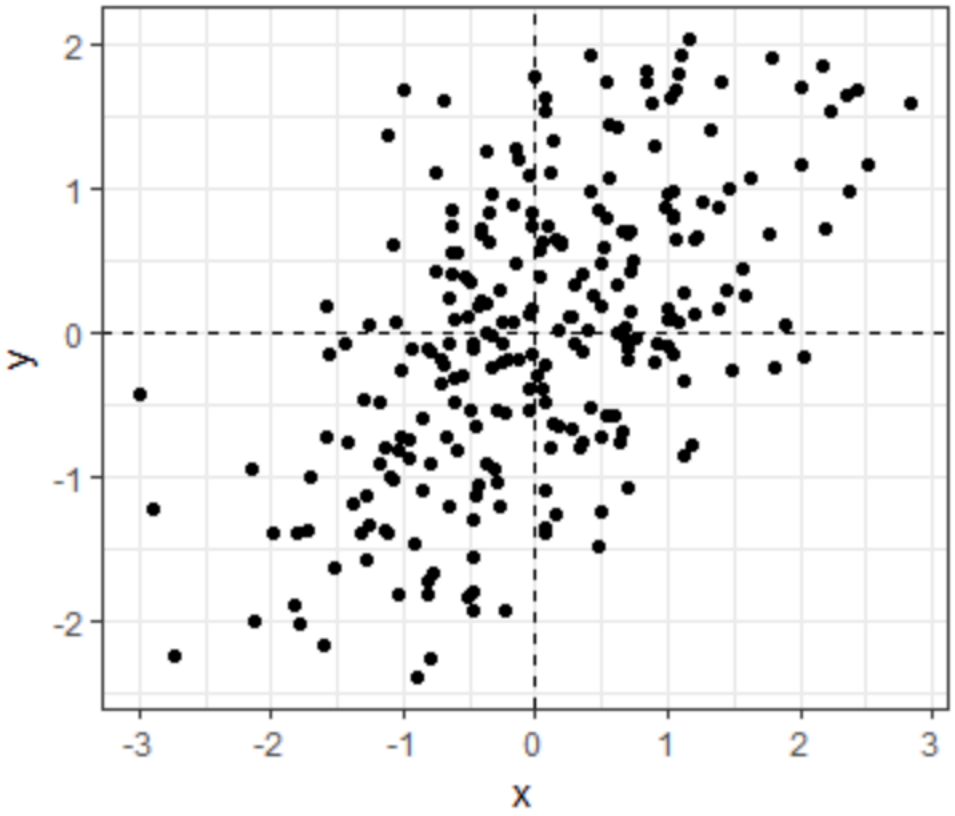
- [X] A. X and Y follow a bivariate normal distribution.
- [ ] B. Both X and Y are normally distributed.
- [ ] C. Both X and Y have been standardized.
- [ ] D. There are at least 25 X-Y pairs.

5. Which one of the following scatterplots depicts an x and y distribution that is NOT well-approximated by the bivariate normal distribution?

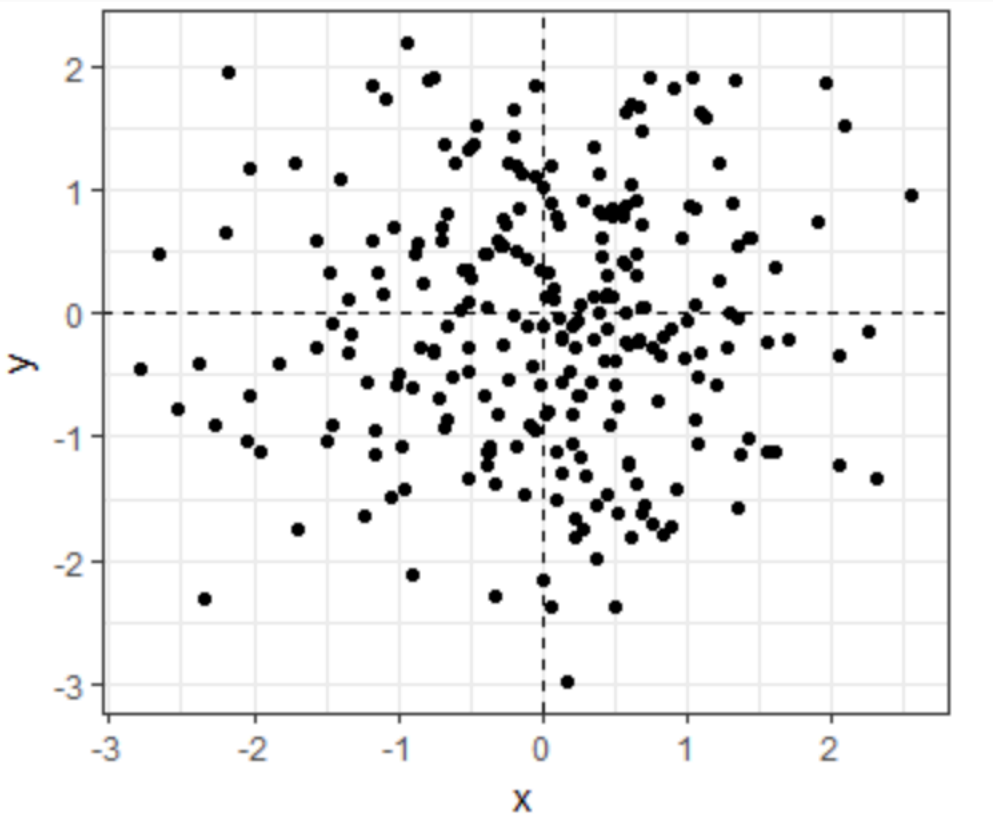
- [X] A.



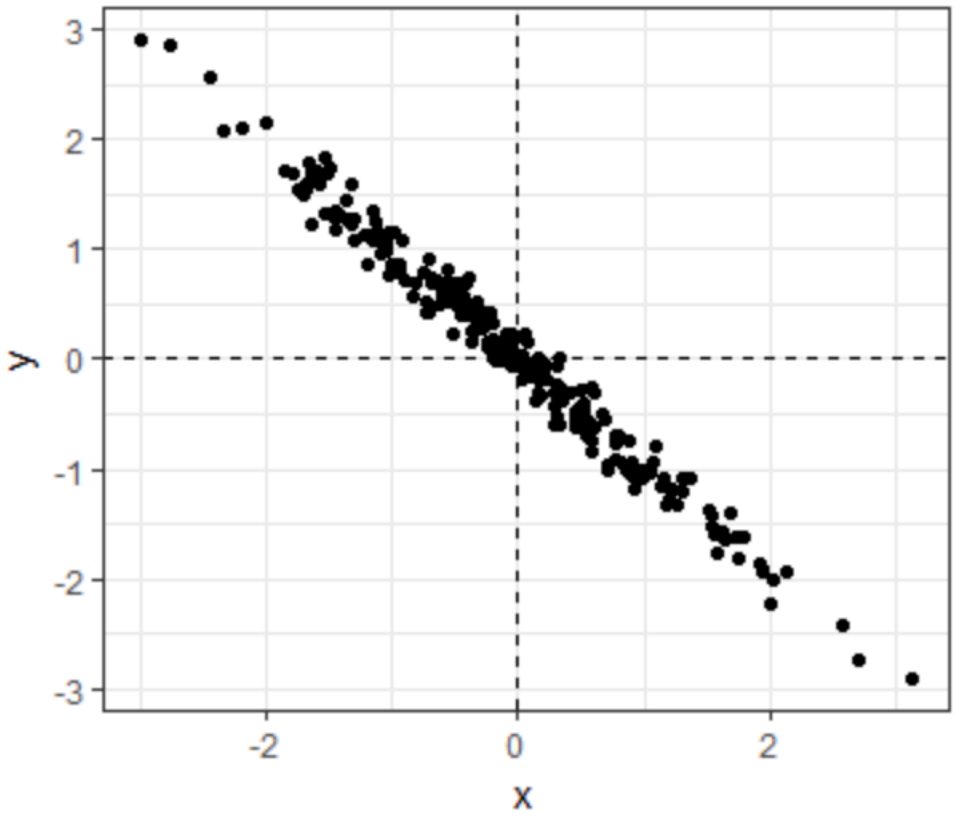
- [ ] B.



- [ ] C.



- [ ] D.



6. We previously calculated that the correlation coefficient between fathers' and sons' heights is 0.5.

Given this, what percent of the variation in sons' heights is explained by fathers' heights?

- [ ] A. 0%
- [X] B. 25%
- [ ] C. 50%
- [ ] D. 75%

When two variables follow a bivariate normal distribution, the variation explained can be calculated as $\rho^2 \times 100$.

7. Suppose the correlation between father and son's height is 0.5, the standard deviation of fathers' heights is 2 inches, and the standard deviation of sons' heights is 3 inches.

Given a one inch increase in a father's height, what is the predicted change in the son's height?

- [ ] A. 0.333
- [ ] B. 0.5
- [ ] C. 0.667
- [X] D. 0.75
- [ ] E. 1
- [ ] F. 1.5

The slope of the regression line is calculated by multiplying the correlation coefficient by the ratio of the standard deviation of son heights and standard deviation of father heights: $\sigma_{son}/\sigma_{father}$.

Assessment - Stratification and Variance Explained, Part 2

In the second part of this assessment, you'll analyze a set of mother and daughter heights, also from `GaltonFamilies`.

Define `female_heights`, a set of mother and daughter heights sampled from `GaltonFamilies`, as follows:

```
set.seed(1989, sample.kind="Rounding") #if you are using R 3.6 or later
```

```
## Warning in set.seed(1989, sample.kind = "Rounding"): non-uniform 'Rounding'
## sampler used
```

```
female_heights <- GaltonFamilies%>%
    filter(gender == "female") %>%
    group_by(family) %>%
    sample_n(1) %>%
    ungroup() %>%
    select(mother, childHeight) %>%
    rename(daughter = childHeight)
```

8. Calculate the mean and standard deviation of mothers' heights, the mean and standard deviation of daughters' heights, and the correlaton coefficient between mother and daughter heights.

Mean of mothers' heights

```
mean(female_heights$mother)
```

```
## [1] 64.125
```

Standard deviation of mothers' heights

```
sd(female_heights$mother)
```

```
## [1] 2.289292
```

Mean of daughters' heights

```
mean(female_heights$daughter)
```

```
## [1] 64.28011
```

Standard deviation of daughters' heights

```
sd(female_heights$daughter)
```

```
## [1] 2.39416
```

Correlation coefficient

```
cor(female_heights$mother, female_heights$daughter)
```

```
## [1] 0.3245199
```

9. Calculate the slope and intercept of the regression line predicting daughters' heights given mothers' heights. Given an increase in mother's height by 1 inch, how many inches is the daughter's height expected to change?

Slope of regression line predicting daughters' height from mothers' heights

```
r <- cor(female_heights$mother, female_heights$daughter)
s_y <- sd(female_heights$daughter)
s_x <- sd(female_heights$mother)
r * s_y/s_x
```

```
## [1] 0.3393856
```

Intercept of regression line predicting daughters' height from mothers' heights

```
mu_y <- mean(female_heights$daughter)
mu_x <- mean(female_heights$mother)
mu_y - (r * s_y/s_x)*mu_x
```

```
## [1] 42.51701
```

Change in daughter's height in inches given a 1 inch increase in the mother's height

```
r * s_y/s_x
```

```
## [1] 0.3393856
```

10. What percent of the variability in daughter heights is explained by the mother's height?

```
r^2*100
```

```
## [1] 10.53132
```

11. A mother has a height of 60 inches.

What is the conditional expected value of her daughter's height given the mother's height?

```
m = r * s_y/s_x
b = mu_y - (r * s_y/s_x)*mu_x
x = 60
m*x+b
```

```
## [1] 62.88015
```

Section 2 - Linear Models Overview

In the **Linear Models** section, you will learn how to do linear regression.

After completing this section, you will be able to:

- Use **multivariate regression** to adjust for confounders.
- Write **linear models** to describe the relationship between two or more variables.
- Calculate the **least squares estimates** for a regression model using the **lm** function.
- Understand the differences between **tibbles** and **data frames**.
- Use the `do()` function to bridge R functions and the tidyverse.
- Use the `tidy()`, `glance()`, and `augment()` functions from the **broom** package.
- Apply linear regression to **measurement error models**.

This section has four parts: **Introduction to Linear Models, Least Squares Estimates, Tibbles, do, and broom**, and **Regression and Baseball**.

Confounding: Are BBs More Predictive?

The textbook for this section is available here

Key points

- Association is not causation!
- Although it may appear that BB cause runs, it is actually the HR that cause most of these runs. We say that BB are **confounded** with HR.
- Regression can help us account for confounding.

Code

```r
# find regression line for predicting runs from BBs
bb_slope <- Teams %>%
  filter(yearID %in% 1961:2001 ) %>%
  mutate(BB_per_game = BB/G, R_per_game = R/G) %>%
  lm(R_per_game ~ BB_per_game, data = .) %>%
  .$coef %>%
  .[2]
bb_slope
```

```
## BB_per_game
##   0.7353288
```

```r
# compute regression line for predicting runs from singles
singles_slope <- Teams %>%
  filter(yearID %in% 1961:2001 ) %>%
  mutate(Singles_per_game = (H-HR-X2B-X3B)/G, R_per_game = R/G) %>%
  lm(R_per_game ~ Singles_per_game, data = .) %>%
  .$coef  %>%
  .[2]
singles_slope
```

```
## Singles_per_game
##        0.4494253
```

```r
# calculate correlation between HR, BB and singles
Teams %>%
  filter(yearID %in% 1961:2001 ) %>%
  mutate(Singles = (H-HR-X2B-X3B)/G, BB = BB/G, HR = HR/G) %>%
  summarize(cor(BB, HR), cor(Singles, HR), cor(BB,Singles))
```

```
##   cor(BB, HR) cor(Singles, HR) cor(BB, Singles)
## 1   0.4039313       -0.1737435      -0.05603822
```

Stratification and Multivariate Regression

The textbook for this section is available here

Key points

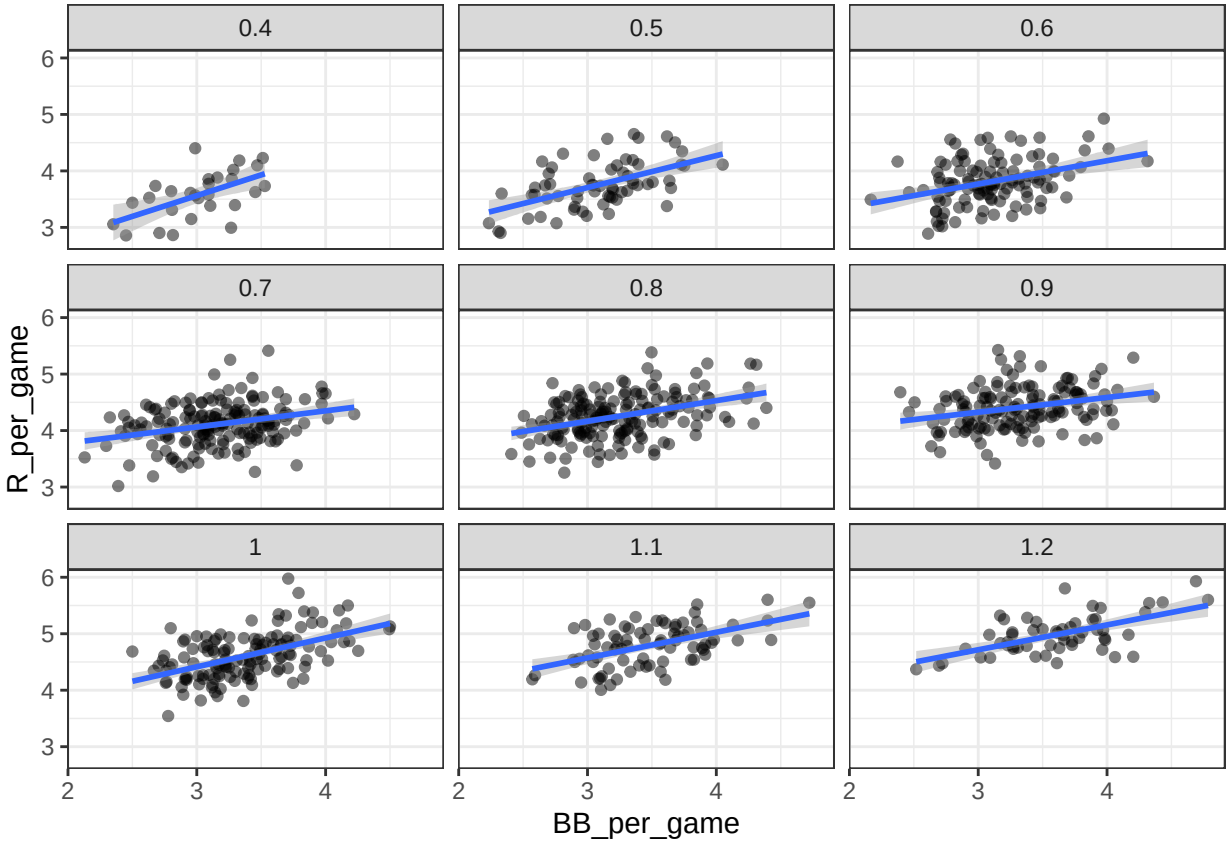
- A first approach to check confounding is to keep HRs fixed at a certain value and then examine the relationship between BB and runs.
- The slopes of BB after stratifying on HR are reduced, but they are not 0, which indicates that BB are helpful for producing runs, just not as much as previously thought.

Code

```r
# stratify HR per game to nearest 10, filter out strata with few points
dat <- Teams %>% filter(yearID %in% 1961:2001) %>%
  mutate(HR_strata = round(HR/G, 1),
         BB_per_game = BB / G,
         R_per_game = R / G) %>%
  filter(HR_strata >= 0.4 & HR_strata <=1.2)

# scatterplot for each HR stratum
dat %>%
  ggplot(aes(BB_per_game, R_per_game)) +
  geom_point(alpha = 0.5) +
```

```
  geom_smooth(method = "lm") +
  facet_wrap( ~ HR_strata)
```

```
## `geom_smooth()` using formula 'y ~ x'
```




```
# calculate slope of regression line after stratifying by HR
dat %>%
  group_by(HR_strata) %>%
  summarize(slope = cor(BB_per_game, R_per_game)*sd(R_per_game)/sd(BB_per_game))
```

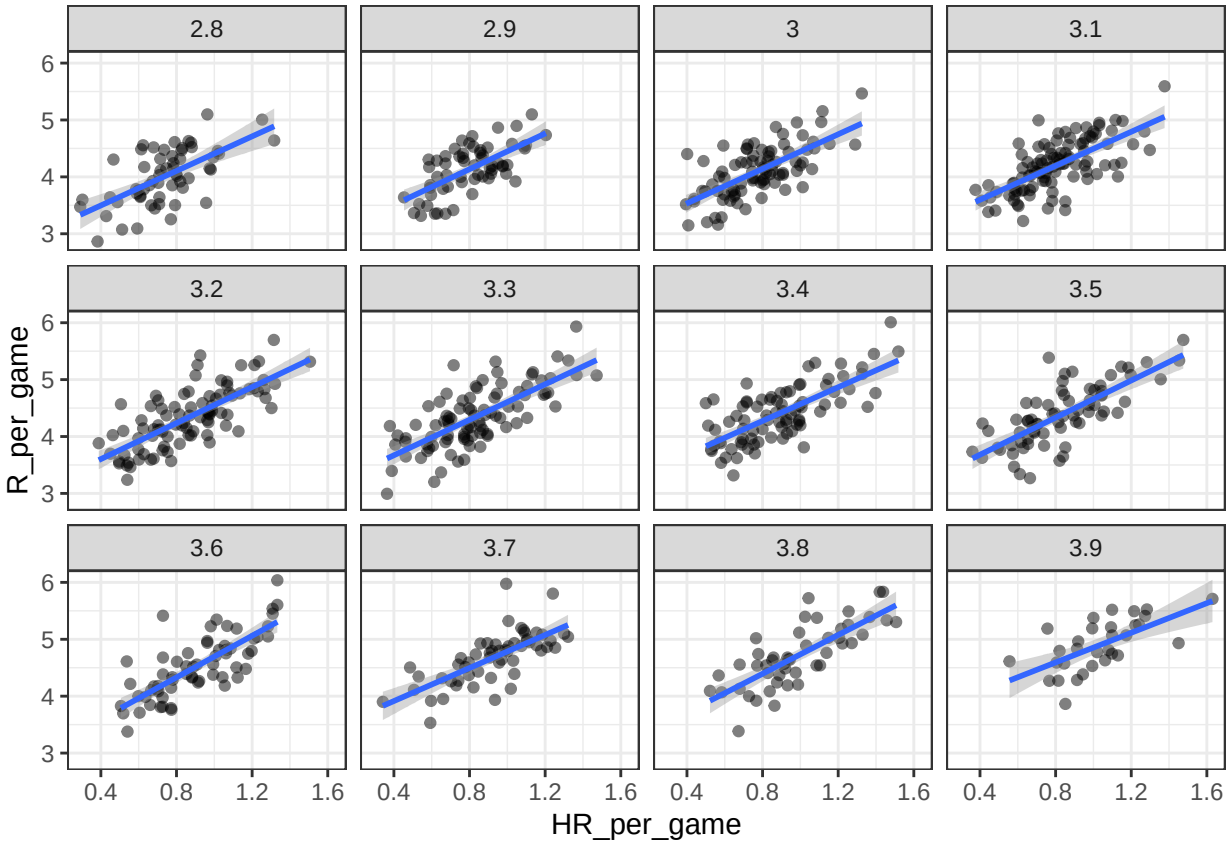
```
## `summarise()` ungrouping output (override with `.groups` argument)
```

```
## # A tibble: 9 x 2
##   HR_strata slope
##       <dbl> <dbl>
## 1       0.4 0.734
## 2       0.5 0.566
## 3       0.6 0.412
## 4       0.7 0.285
## 5       0.8 0.365
## 6       0.9 0.261
## 7       1   0.511
## 8       1.1 0.454
## 9       1.2 0.440
```

```
# stratify by BB
dat <- Teams %>% filter(yearID %in% 1961:2001) %>%
  mutate(BB_strata = round(BB/G, 1),
```

```
          HR_per_game = HR / G,
          R_per_game = R / G) %>%
  filter(BB_strata >= 2.8 & BB_strata <=3.9)

# scatterplot for each BB stratum
dat %>% ggplot(aes(HR_per_game, R_per_game)) +
  geom_point(alpha = 0.5) +
  geom_smooth(method = "lm") +
  facet_wrap( ~ BB_strata)
```

```
## `geom_smooth()` using formula 'y ~ x'
```




```
# slope of regression line after stratifying by BB
dat %>%
  group_by(BB_strata) %>%
  summarize(slope = cor(HR_per_game, R_per_game)*sd(R_per_game)/sd(HR_per_game))
```

```
## `summarise()` ungrouping output (override with `.groups` argument)
```

```
## # A tibble: 12 x 2
##    BB_strata slope
##        <dbl> <dbl>
##  1       2.8  1.52
##  2       2.9  1.57
##  3       3    1.52
##  4       3.1  1.49
##  5       3.2  1.58
##  6       3.3  1.56
```

```
##  7       3.4  1.48
##  8       3.5  1.63
##  9       3.6  1.83
## 10       3.7  1.45
## 11       3.8  1.70
## 12       3.9  1.30
```

Linear Models

The textbook for this section is available here

Key points

- "Linear" here does not refer to lines, but rather to the fact that the conditional expectation is a linear combination of known quantities.
- In Galton's model, we assume Y (son's height) is a linear combination of a constant and X (father's height) plus random noise. We further assume that ϵ_i are independent from each other, have expected value 0 and the standard deviation σ which does not depend on i.
- Note that if we further assume that ϵ is normally distributed, then the model is exactly the same one we derived earlier by assuming bivariate normal data.
- We can subtract the mean from X to make β_0 more interpretable.

Assessment: Introduction to Linear Models

1. When we stratified our regression lines for runs per game vs. bases on balls by the number of home runs, what happened?

- [X] A. The slope of runs per game vs. bases on balls within each stratum was reduced because we removed confounding by home runs.
- [ ] B. The slope of runs per game vs. bases on balls within each stratum was reduced because there were fewer data points.
- [ ] C. The slope of runs per game vs. bases on balls within each stratum increased after we removed confounding by home runs.
- [ ] D. The slope of runs per game vs. bases on balls within each stratum stayed about the same as the original slope.

2. We run a linear model for sons' heights vs. fathers' heights using the Galton height data, and get the following results:

```
> lm(son ~ father, data = galton_heights)

Call:
lm(formula = son ~ father, data = galton_heights)

Coefficients:
(Intercept)    father
    35.71       0.50
```

Interpret the numeric coefficient for "father."

- [ ] A. For every inch we increase the son's height, the predicted father's height increases by 0.5 inches.
- [X] B. For every inch we increase the father's height, the predicted son's height grows by 0.5 inches.
- [ ] C. For every inch we increase the father's height, the predicted son's height is 0.5 times greater.

3. We want the intercept term for our model to be more interpretable, so we run the same model as before but now we subtract the mean of fathers' heights from each individual father's height to create a new variable centered at zero.

```
galton_heights <- galton_heights %>%
    mutate(father_centered=father - mean(father))
```

We run a linear model using this centered fathers' height variable.

```
> lm(son ~ father_centered, data = galton_heights)

Call:
lm(formula = son ~ father_centered, data = galton_heights)

Coefficients:
(Intercept)    father_centered
    70.45             0.50
```

Interpret the numeric coefficient for the intercept.

- [X] A. The height of a son of a father of average height is 70.45 inches.
- [ ] B. The height of a son when a father's height is zero is 70.45 inches.
- [ ] C. The height of an average father is 70.45 inches.

4. Suppose we fit a multivariate regression model for expected runs based on BB and HR:

$E[R|BB = x_1, HR = x_2] = \beta_0 + \beta_1 x_1 + \beta_2 x_2$

Suppose we fix $BB = x_1$. Then we observe a linear relationship between runs and HR with intercept of:

- [ ] A. β_0
- [ ] B. $\beta_0 + \beta_2 x_2$
- [X] C. $\beta_0 + \beta_1 x_1$
- [ ] D. $\beta_0 + \beta_2 x_1$

5. Which of the following are assumptions for the errors ϵ_i in a linear regression model? Check ALL correct answers.

- [X] A. The ϵ_i are independent of each other
- [X] B. The ϵ_i have expected value 0
- [X] C. The variance of ϵ_i is a constant

Least Squares Estimates (LSE)

The textbook for this section is available here

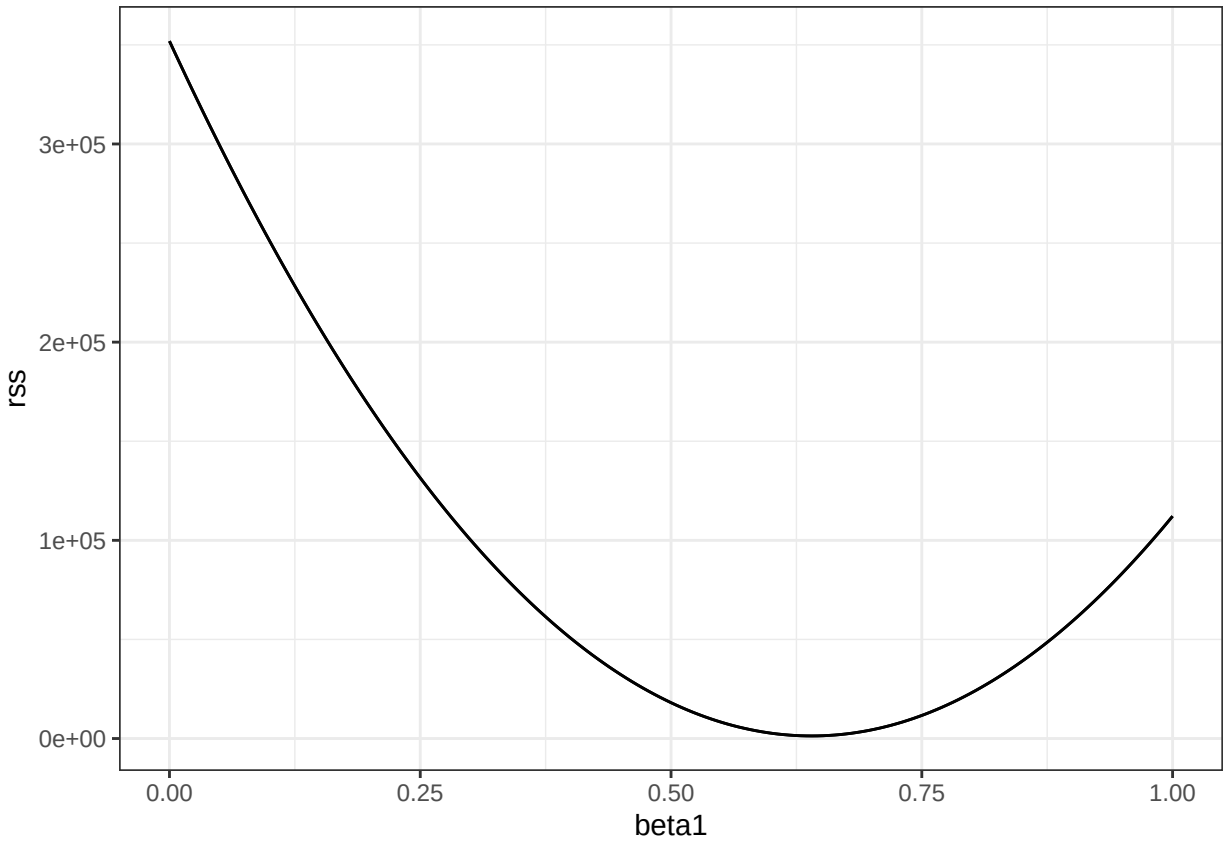
Key points

- For regression, we aim to find the coefficient values that minimize the distance of the fitted model to the data.
- Residual sum of squares (RSS) measures the distance between the true value and the predicted value given by the regression line. The values that minimize the RSS are called the least squares estimates (LSE).
- We can use partial derivatives to get the values for β_0 and β_1 in Galton's data.

Code

```
# compute RSS for any pair of beta0 and beta1 in Galton's data
set.seed(1983)
galton_heights <- GaltonFamilies %>%
  filter(gender == "male") %>%
  group_by(family) %>%
  sample_n(1) %>%
  ungroup() %>%
```

```
  select(father, childHeight) %>%
  rename(son = childHeight)
rss <- function(beta0, beta1, data){
    resid <- galton_heights$son - (beta0+beta1*galton_heights$father)
    return(sum(resid^2))
}

# plot RSS as a function of beta1 when beta0=25
beta1 = seq(0, 1, len=nrow(galton_heights))
results <- data.frame(beta1 = beta1,
                      rss = sapply(beta1, rss, beta0 = 25))
results %>% ggplot(aes(beta1, rss)) + geom_line() +
  geom_line(aes(beta1, rss))
```



The lm Function

The textbook for this section is available here

Key points

- When calling the `lm()` function, the variable that we want to predict is put to the left of the ~ symbol, and the variables that we use to predict is put to the right of the ~ symbol. The intercept is added automatically.
- LSEs are random variables.

Code

```
# fit regression line to predict son's height from father's height
fit <- lm(son ~ father, data = galton_heights)
```

```
fit
```

```
##
## Call:
## lm(formula = son ~ father, data = galton_heights)
##
## Coefficients:
## (Intercept)       father
##     38.7646       0.4411
```

```
# summary statistics
summary(fit)
```

```
##
## Call:
## lm(formula = son ~ father, data = galton_heights)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -9.4228 -1.7022  0.0333  1.5670  9.3567
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 38.76457    5.41093   7.164 2.03e-11 ***
## father       0.44112    0.07825   5.637 6.72e-08 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.659 on 177 degrees of freedom
## Multiple R-squared:  0.1522, Adjusted R-squared:  0.1474
## F-statistic: 31.78 on 1 and 177 DF,  p-value: 6.719e-08
```

LSE are Random Variables

The textbook for this section is available here

Key points

- Because they are derived from the samples, LSE are random variables.
- β_0 and β_1 appear to be normally distributed because the central limit theorem plays a role.
- The t-statistic depends on the assumption that ϵ follows a normal distribution.

Code

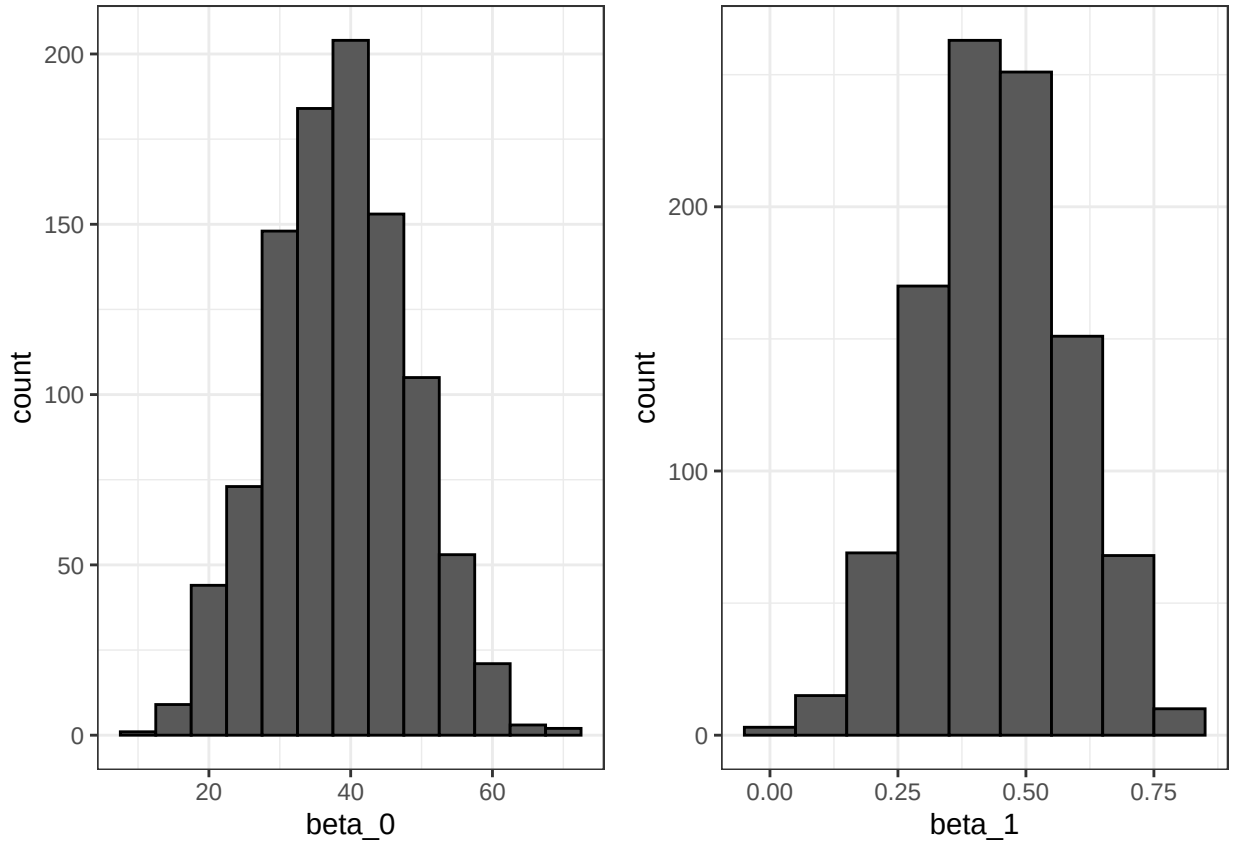
```
# Monte Carlo simulation
B <- 1000
N <- 50
lse <- replicate(B, {
  sample_n(galton_heights, N, replace = TRUE) %>%
    lm(son ~ father, data = .) %>%
    .$coef
})
lse <- data.frame(beta_0 = lse[1,], beta_1 = lse[2,])

# Plot the distribution of beta_0 and beta_1
if(!require(gridExtra)) install.packages("gridExtra")
```

```
## Loading required package: gridExtra
```

```
##
## Attaching package: 'gridExtra'
```

```
## The following object is masked from 'package:dplyr':
##
##     combine
```

```
library(gridExtra)
p1 <- lse %>% ggplot(aes(beta_0)) + geom_histogram(binwidth = 5, color = "black")
p2 <- lse %>% ggplot(aes(beta_1)) + geom_histogram(binwidth = 0.1, color = "black")
grid.arrange(p1, p2, ncol = 2)
```




```
# summary statistics
sample_n(galton_heights, N, replace = TRUE) %>%
  lm(son ~ father, data = .) %>%
  summary %>%
  .$coef
```

```
##               Estimate Std. Error  t value     Pr(>|t|)
## (Intercept) 34.4729422  8.6021831 4.007464 0.0002129225
## father       0.4990193  0.1240572 4.022493 0.0002030210
```

```
lse %>% summarize(se_0 = sd(beta_0), se_1 = sd(beta_1))
```

```
##       se_0      se_1
## 1 9.683973 0.1411404
```

Advanced Note on LSE

Although interpretation is not straight-forward, it is also useful to know that the LSE can be strongly correlated, which can be seen using this code:

```
lse %>% summarize(cor(beta_0, beta_1))
```

```
##   cor(beta_0, beta_1)
## 1          -0.9993386
```

However, the correlation depends on how the predictors are defined or transformed.

Here we standardize the father heights, which changes x_i to $x_i - \bar{x}$.

```
B <- 1000
N <- 50
lse <- replicate(B, {
      sample_n(galton_heights, N, replace = TRUE) %>%
      mutate(father = father - mean(father)) %>%
      lm(son ~ father, data = .) %>% .$coef
})
```

Observe what happens to the correlation in this case:

```
cor(lse[1,], lse[2,])
```

```
## [1] 0.1100929
```

Predicted Variables are Random Variables

The textbook for this section is available here

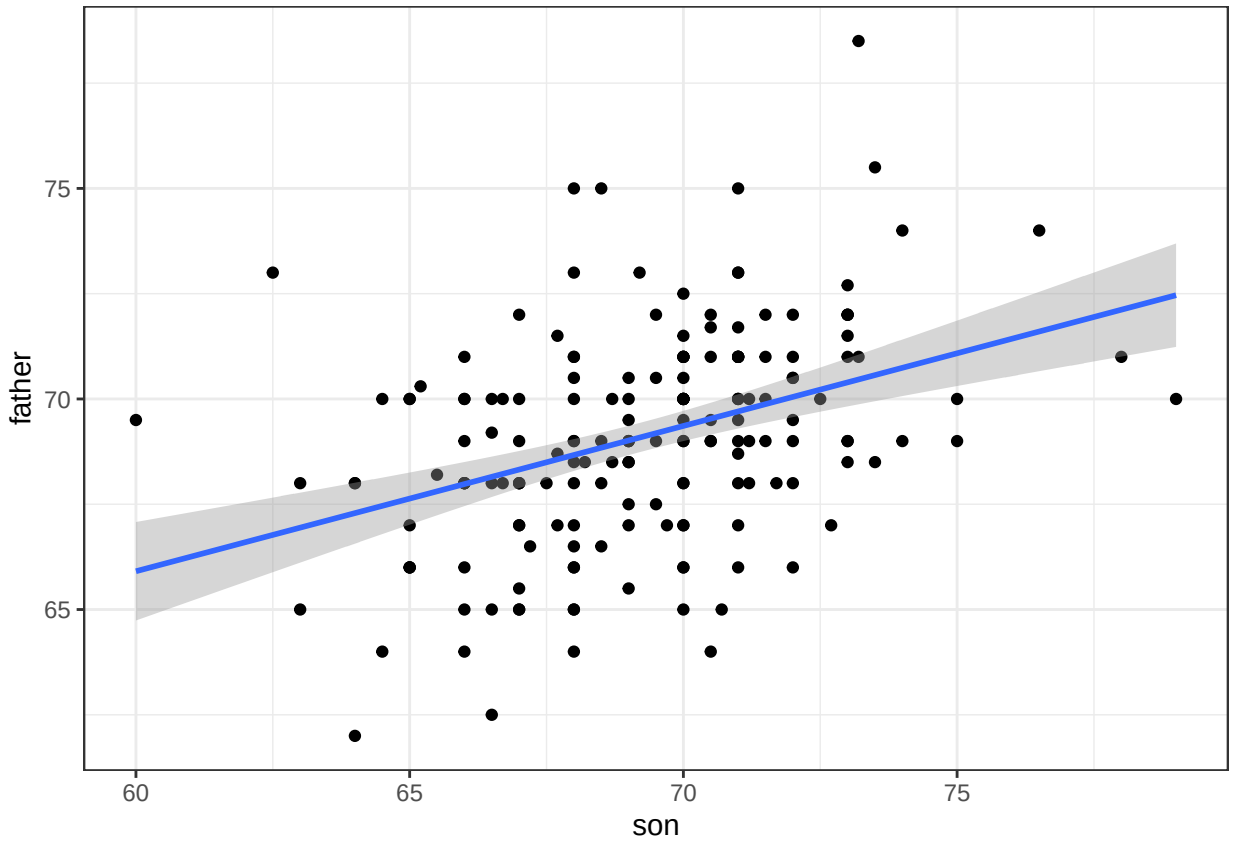
Key points

- The predicted value is often denoted as $\hat{Y}$, which is a random variable. Mathematical theory tells us what the standard error of the predicted value is.
- The `predict()` function in R can give us predictions directly.

Code

```
# plot predictions and confidence intervals
galton_heights %>% ggplot(aes(son, father)) +
  geom_point() +
  geom_smooth(method = "lm")
```

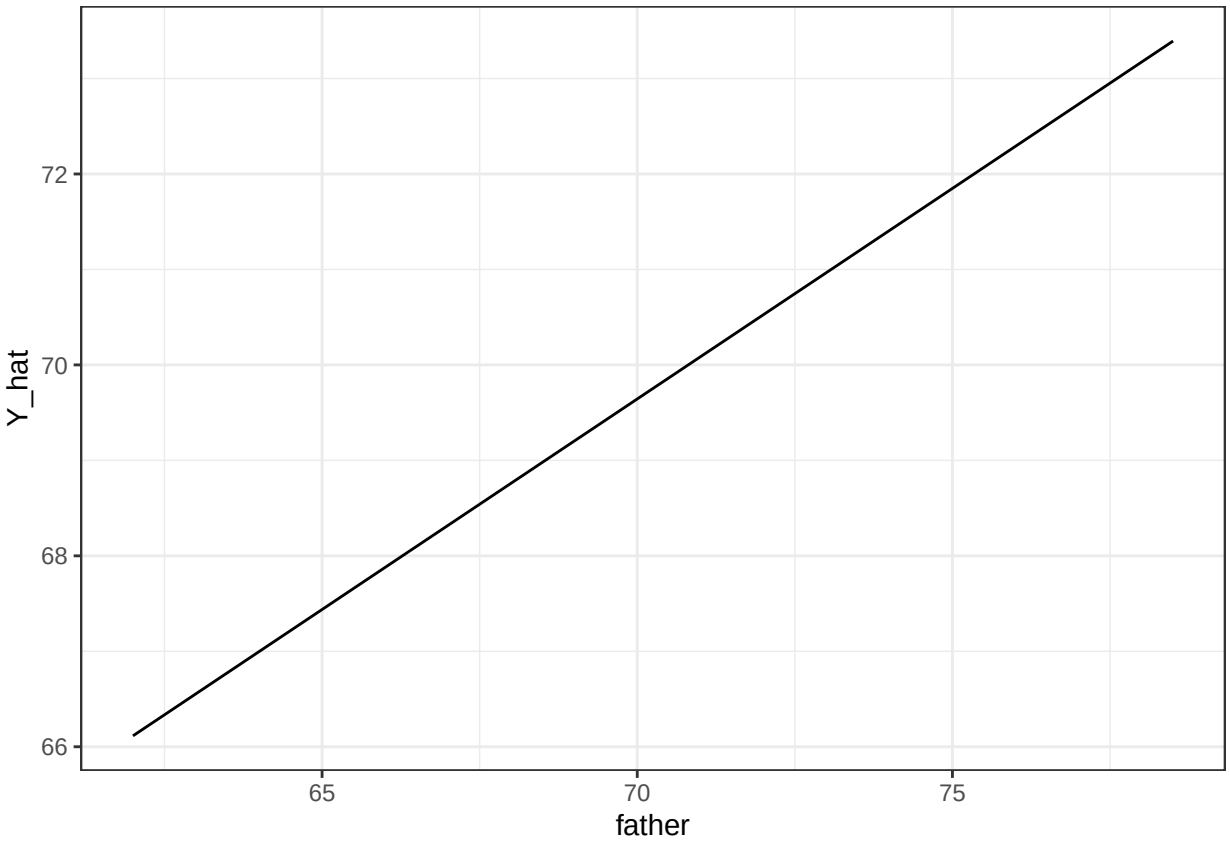
```
## `geom_smooth()` using formula 'y ~ x'
```




```
# predict Y directly
fit <- galton_heights %>% lm(son ~ father, data = .)
Y_hat <- predict(fit, se.fit = TRUE)
names(Y_hat)
```

```
## [1] "fit"            "se.fit"         "df"             "residual.scale"
```

```
# plot best fit line
galton_heights %>%
  mutate(Y_hat = predict(lm(son ~ father, data=.))) %>%
  ggplot(aes(father, Y_hat))+
  geom_line()
```



Assessment - Least Squares Estimates (LSE), part 1

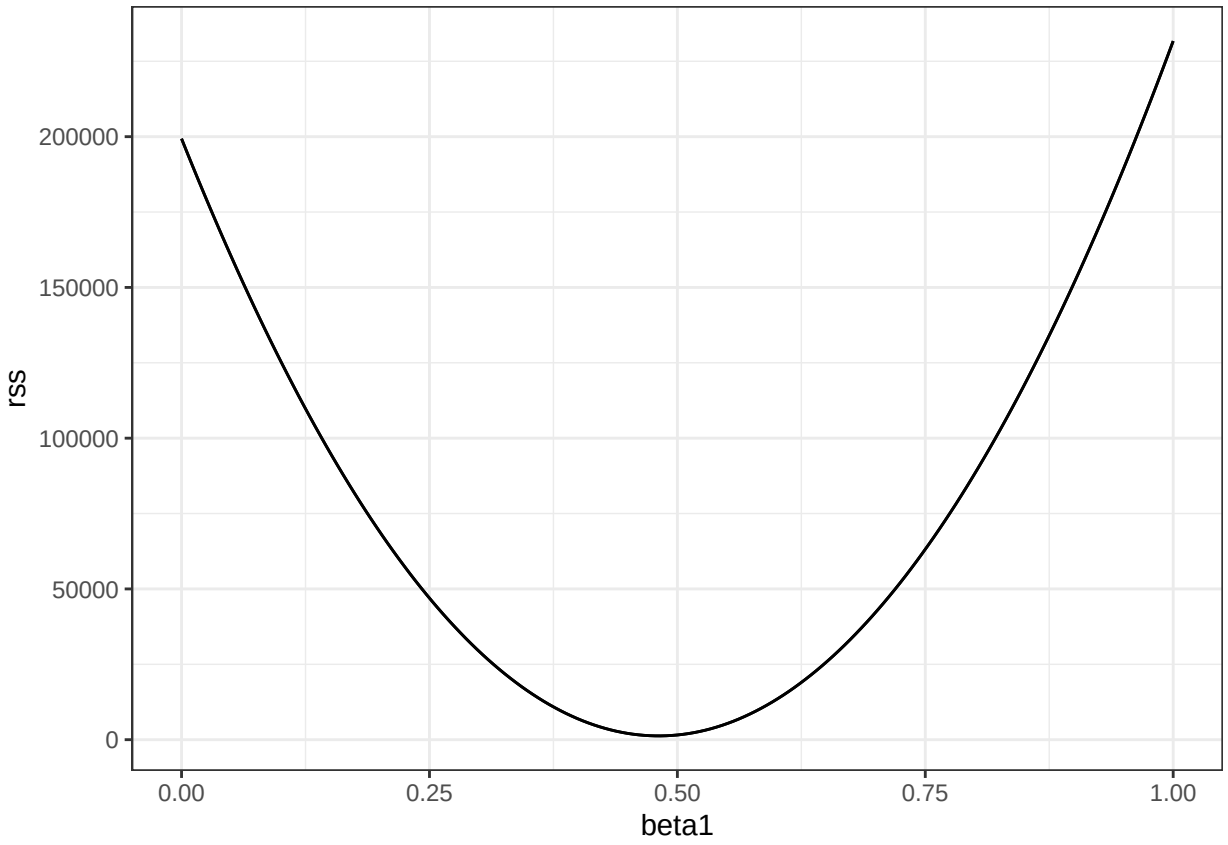
1. The following code was used in the video to plot RSS with $\beta_0 = 25$.

```
beta1 = seq(0, 1, len=nrow(galton_heights))
results <- data.frame(beta1 = beta1,
                      rss = sapply(beta1, rss, beta0 = 25))
results %>% ggplot(aes(beta1, rss)) + geom_line() +
  geom_line(aes(beta1, rss), col=2)
```

In a model for sons' heights vs fathers' heights, what is the least squares estimate (LSE) for β_1 if we assume $\hat{\beta}_0$ is 36? Hint: modify the code above to do your analysis.

```
# compute RSS for any pair of beta0 and beta1 in Galton's data
set.seed(1983)
galton_heights <- GaltonFamilies %>%
  filter(gender == "male") %>%
  group_by(family) %>%
  sample_n(1) %>%
  ungroup() %>%
  select(father, childHeight) %>%
  rename(son = childHeight)
rss <- function(beta0, beta1, data){
    resid <- galton_heights$son - (beta0+beta1*galton_heights$father)
    return(sum(resid^2))
}

# plot RSS as a function of beta1 when beta0=36
```

```
beta1 = seq(0, 1, len=nrow(galton_heights))
results <- data.frame(beta1 = beta1,
                      rss = sapply(beta1, rss, beta0 = 36))
results %>% ggplot(aes(beta1, rss)) + geom_line() +
  geom_line(aes(beta1, rss))
```



- [ ] A. 0.65
- [X] B. 0.5
- [ ] C. 0.2
- [ ] D. 12

2. The least squares estimates for the parameters $\beta_0, \beta_1, \ldots, \beta_n$ **minimize** the residual sum of squares.

3. Load the `Lahman` library and filter the `Teams` data frame to the years 1961-2001.

Run a linear model in R predicting the number of runs per game based on the number of bases on balls and the number of home runs. Remember to first limit your data to 1961-2001.

What is the coefficient for bases on balls?

```
fit <- Teams %>% filter(yearID %in% 1961:2001) %>%
mutate(R_per_game = R / G, BB_per_game = BB / G, HR_per_game = HR / G) %>%
lm(R_per_game ~ BB_per_game + HR_per_game, data = .)

summary(fit)
```

```
##
## Call:
## lm(formula = R_per_game ~ BB_per_game + HR_per_game, data = .)
##
```

```
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.87325 -0.24507 -0.01449  0.23866  1.24218
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  1.74430    0.08236   21.18   <2e-16 ***
## BB_per_game  0.38742    0.02701   14.34   <2e-16 ***
## HR_per_game  1.56117    0.04896   31.89   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.3484 on 1023 degrees of freedom
## Multiple R-squared:  0.6503, Adjusted R-squared:  0.6496
## F-statistic: 951.2 on 2 and 1023 DF,  p-value: < 2.2e-16
```

- [X] A. 0.39
- [ ] B. 1.56
- [ ] C. 1.74
- [ ] D. 0.027

The coefficient for bases on balls is 0.39; the coefficient for home runs is 1.56; the intercept is 1.74; the standard error for the BB coefficient is 0.027.

4. We run a Monte Carlo simulation where we repeatedly take samples of N = 100 from the Galton heights data and compute the regression slope coefficients for each sample:

```
B <- 1000
N <- 100
lse <- replicate(B, {
    sample_n(galton_heights, N, replace = TRUE) %>%
    lm(son ~ father, data = .) %>% .$coef
})

lse <- data.frame(beta_0 = lse[1,], beta_1 = lse[2,])
```

What does the central limit theorem tell us about the variables beta_0 and beta_1?

- [X] A. They are approximately normally distributed.
- [X] B. The expected value of each is the true value of β_0 and β_1 (assuming the Galton heights data is a complete population).
- [ ] C. The central limit theorem does not apply in this situation.
- [ ] D. It allows us to test the hypothesis that $\beta_0 = 0$ and $\beta_0 = 1$

5. In an earlier video, we ran the following linear model and looked at a summary of the results.

```
$\beta_0$
> mod <- lm(son ~ father, data = galton_heights)
> summary(mod)

Call:
lm(formula = son ~ father, data = galton_heights)

Residuals:
   Min     1Q  Median     3Q    Max
-5.902  -1.405   0.092  1.342  8.092

Coefficients:
```

```
                Estimate  Std. Error  t value     Pr(>|t|)
(Intercept)     35.7125     4.5174       7.91    2.8e-13 ***
father           0.5028     0.0653       7.70    9.5e-13 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
$\beta_0$
```

What null hypothesis is the second p-value (the one in the father row) testing?

- [ ] A. $\beta_1 = 1$, where β_1 is the coefficient for the variable "father."
- [ ] B. $\beta_1 = 0.503$, where β_1 is the coefficient for the variable "father."
- [X] C. $\beta_1 = 0$, where β_1 is the coefficient for the variable "father."

6. Which R code(s) below would properly plot the predictions and confidence intervals for our linear model of sons' heights?

Select ALL that apply.

- [ ] A.

```
        galton_heights %>% ggplot(aes(father, son)) +
    geom_point() +
    geom_smooth()
```

- [X] B.

```
        galton_heights %>% ggplot(aes(father, son)) +
    geom_point() +
    geom_smooth(method = "lm")
```

- [X] C.

```
        model <- lm(son ~ father, data = galton_heights)
predictions <- predict(model, interval = c("confidence"), level = 0.95)
data <- as.tibble(predictions) %>% bind_cols(father = galton_heights$father)

ggplot(data, aes(x = father, y = fit)) +
    geom_line(color = "blue", size = 1) +
    geom_ribbon(aes(ymin=lwr, ymax=upr), alpha=0.2) +
    geom_point(data = galton_heights, aes(x = father, y = son))
```

- [ ] D.

```
        model <- lm(son ~ father, data = galton_heights)
predictions <- predict(model)
data <- as.tibble(predictions) %>% bind_cols(father = galton_heights$father)

ggplot(data, aes(x = father, y = fit)) +
    geom_line(color = "blue", size = 1) +
    geom_point(data = galton_heights, aes(x = father, y = son))
```

Assessment - Least Squares Estimates, part 2

In Questions 7 and 8, you'll look again at female heights from GaltonFamilies.

Define `female_heights`, a set of mother and daughter heights sampled from `GaltonFamilies`, as follows:

```
set.seed(1989, sample.kind="Rounding") #if you are using R 3.6 or later
```

```
## Warning in set.seed(1989, sample.kind = "Rounding"): non-uniform 'Rounding'
## sampler used
```

```
options(digits = 3)    # report 3 significant digits

female_heights <- GaltonFamilies %>%
    filter(gender == "female") %>%
    group_by(family) %>%
    sample_n(1) %>%
    ungroup() %>%
    select(mother, childHeight) %>%
    rename(daughter = childHeight)
```

7. Fit a linear regression model predicting the mothers' heights using daughters' heights.

What is the slope of the model?

```
fit <- lm(mother ~ daughter, data = female_heights)
fit$coef[2]
```

```
## daughter
##     0.31
```

What the intercept of the model?

```
fit$coef[1]
```

```
## (Intercept)
##        44.2
```

8. Predict mothers' heights using the model.

What is the predicted height of the first mother in the dataset?

```
predict(fit)[1]
```

```
##    1
## 65.6
```

What is the actual height of the first mother in the dataset?

```
female_heights$mother[1]
```

```
## [1] 67
```

We have shown how BB and singles have similar predictive power for scoring runs. Another way to compare the usefulness of these baseball metrics is by assessing how stable they are across the years. Because we have to pick players based on their previous performances, we will prefer metrics that are more stable. In these exercises, we will compare the stability of singles and BBs.

Before we get started, we want to generate two tables: one for 2002 and another for the average of 1999-2001 seasons. We want to define per plate appearance statistics, keeping only players with more than 100 plate appearances. Here is how we create the 2002 table:

```
bat_02 <- Batting %>% filter(yearID == 2002) %>%
    mutate(pa = AB + BB, singles = (H - X2B - X3B - HR)/pa, bb = BB/pa) %>%
    filter(pa >= 100) %>%
    select(playerID, singles, bb)
```

9. Now compute a similar table but with rates computed over 1999-2001. Keep only rows from 1999-2001 where players have 100 or more plate appearances, calculate each player's single rate and BB rate

per season, then calculate the average single rate (`mean_singles`) and average BB rate (`mean_bb`) per player over those three seasons.

How many players had a single rate `mean_singles` of greater than 0.2 per plate appearance over 1999-2001?

```
bat_99_01 <- Batting %>% filter(yearID %in% 1999:2001) %>%
    mutate(pa = AB + BB, singles = (H - X2B - X3B - HR)/pa, bb = BB/pa) %>%
    filter(pa >= 100) %>%
    group_by(playerID) %>%
    summarize(mean_singles = mean(singles), mean_bb = mean(bb))
```

```
## `summarise()` ungrouping output (override with `.groups` argument)
```

```
sum(bat_99_01$mean_singles > 0.2)
```

```
## [1] 46
```

How many players had a BB rate `mean_bb` of greater than 0.2 per plate appearance over 1999-2001?

```
sum(bat_99_01$mean_bb > 0.2)
```

```
## [1] 3
```

10. Use `inner_join()` to combine the 'bat_02 table with the table of 1999-2001 rate averages you created in the previous question.

What is the correlation between 2002 singles rates and 1999-2001 average singles rates?

```
dat <- inner_join(bat_02, bat_99_01)
```

```
## Joining, by = "playerID"
```

```
cor(dat$singles, dat$mean_singles)
```

```
## [1] 0.551
```

What is the correlation between 2002 BB rates and 1999-2001 average BB rates?

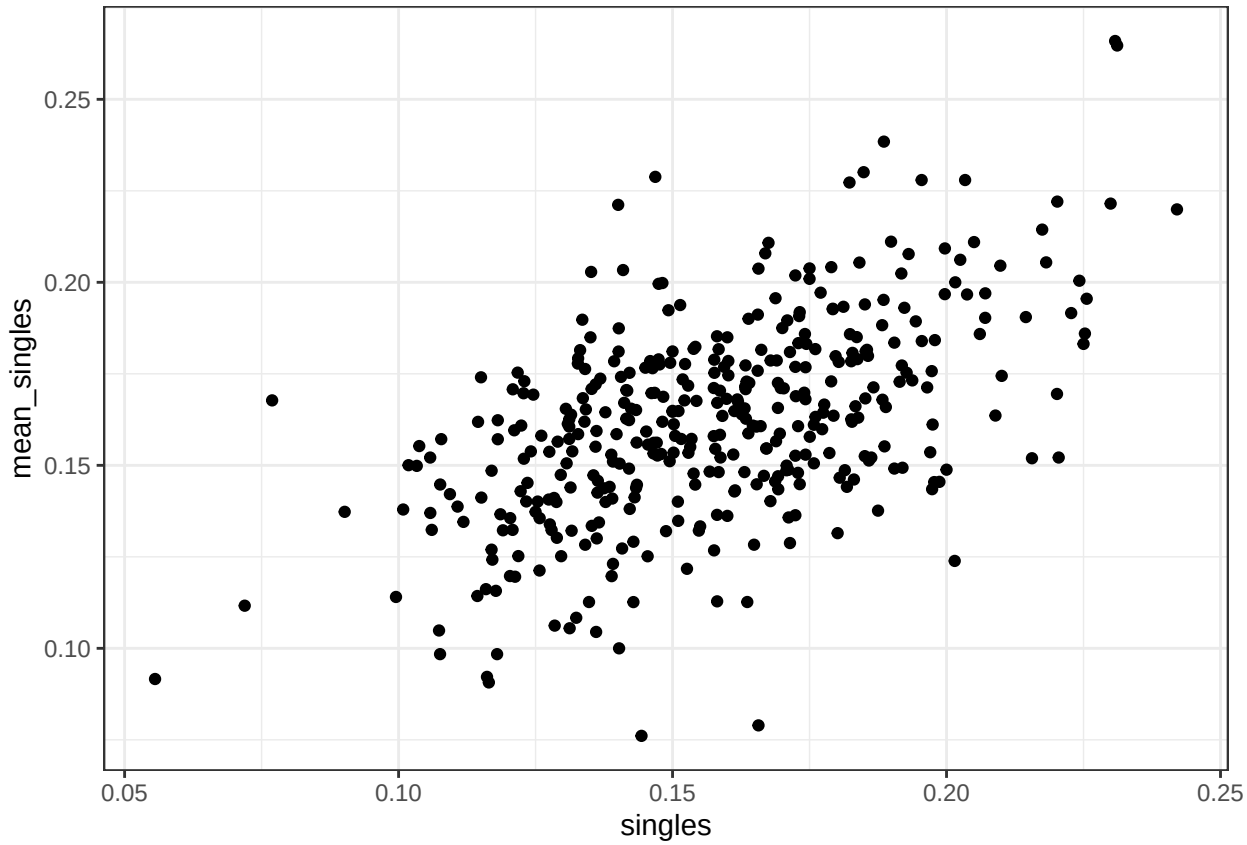
```
cor(dat$bb, dat$mean_bb)
```

```
## [1] 0.717
```

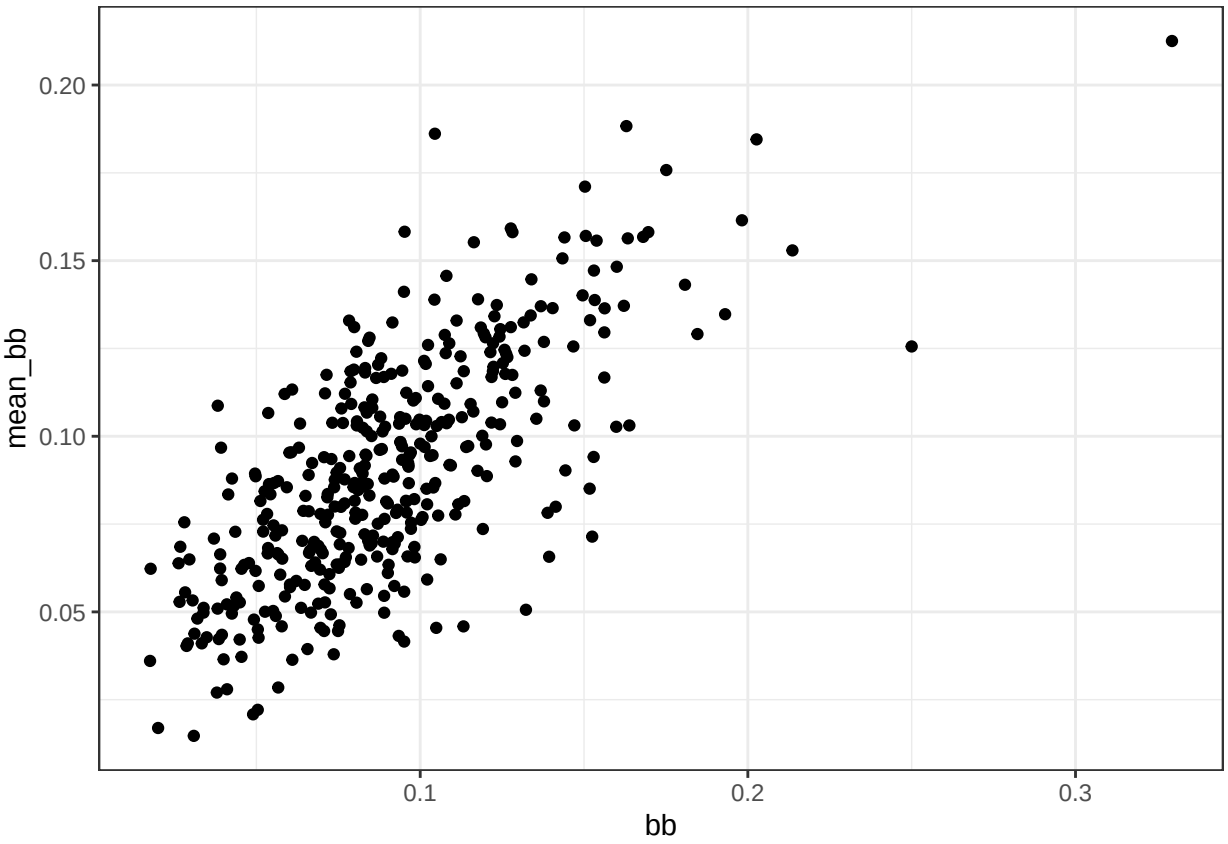
11. Make scatterplots of `mean_singles` versus `singles` and `mean_bb` versus `bb`.

Are either of these distributions bivariate normal?

```
dat %>%
    ggplot(aes(singles, mean_singles)) +
    geom_point()
```




```
dat %>%
    ggplot(aes(bb, mean_bb)) +
    geom_point()
```



- [ ] A. Neither distribution is bivariate normal.
- [ ] B. `singles` and `mean_singles` are bivariate normal, but `bb` and `mean_bb` are not.
- [ ] C. `bb` and `mean_bb` are bivariate normal, but `singles` and `mean_singles` are not.
- [X] D. Both distributions are bivariate normal.

12. Fit a linear model to predict 2002 `singles` given 1999-2001 `mean_singles`.

What is the coefficient of `mean_singles`, the slope of the fit?

```
fit_singles <- lm(singles ~ mean_singles, data = dat)
fit_singles$coef[2]
```

```
## mean_singles
##        0.588
```

Fit a linear model to predict 2002 `bb` given 1999-2001 `mean_bb`.

What is the coefficient of `mean_bb`, the slope of the fit?

```
fit_bb <- lm(bb ~ mean_bb, data = dat)
fit_bb$coef[2]
```

```
## mean_bb
##   0.829
```

Advanced dplyr: Tibbles

The textbook for this section is available here

Key points

- Tibbles can be regarded as a modern version of data frames and are the default data structure in the tidyverse.
- Some functions that do not work properly with data frames do work with tibbles.

Code

```
# stratify by HR
dat <- Teams %>% filter(yearID %in% 1961:2001) %>%
  mutate(HR = round(HR/G, 1),
         BB = BB/G,
         R = R/G) %>%
  select(HR, BB, R) %>%
  filter(HR >= 0.4 & HR<=1.2)

# calculate slope of regression lines to predict runs by BB in different HR strata
dat %>%
  group_by(HR) %>%
  summarize(slope = cor(BB,R)*sd(R)/sd(BB))
```

```
## `summarise()` ungrouping output (override with `.groups` argument)
```

```
## # A tibble: 9 x 2
##      HR slope
##   <dbl> <dbl>
## 1   0.4 0.734
## 2   0.5 0.566
## 3   0.6 0.412
## 4   0.7 0.285
## 5   0.8 0.365
## 6   0.9 0.261
## 7   1   0.511
## 8   1.1 0.454
## 9   1.2 0.440
```

```
# use lm to get estimated slopes - lm does not work with grouped tibbles
dat %>%
  group_by(HR) %>%
  lm(R ~ BB, data = .) %>%
  .$coef
```

```
## (Intercept)          BB
##       2.198       0.638
```

```
# inspect a grouped tibble
dat %>% group_by(HR) %>% head()
```

```
## # A tibble: 6 x 3
## # Groups:   HR [5]
##      HR    BB     R
##   <dbl> <dbl> <dbl>
## 1   0.9  3.56  4.24
## 2   0.7  3.97  4.47
## 3   0.8  3.37  4.69
## 4   1.1  3.46  4.42
## 5   1    2.75  4.61
## 6   0.9  3.06  4.58
```

```
dat %>% group_by(HR) %>% class()
```

```
## [1] "grouped_df" "tbl_df"     "tbl"        "data.frame"
```

Tibbles: Differences from Data Frames

Key points

- Tibbles are more readable than data frames.
- If you subset a data frame, you may not get a data frame. If you subset a tibble, you always get a tibble.
- Tibbles can hold more complex objects such as lists or functions.
- Tibbles can be grouped.

Code

```
# inspect data frame and tibble
head(Teams)
```

```
##   yearID lgID teamID franchID divID Rank  G Ghome  W  L DivWin WCWin LgWin
## 1   1871   NA    BS1       BNA  <NA>    3 31    NA 20 10   <NA>  <NA>     N
## 2   1871   NA    CH1       CNA  <NA>    2 28    NA 19  9   <NA>  <NA>     N
## 3   1871   NA    CL1       CFC  <NA>    8 29    NA 10 19   <NA>  <NA>     N
## 4   1871   NA    FW1       KEK  <NA>    7 19    NA  7 12   <NA>  <NA>     N
## 5   1871   NA    NY2       NNA  <NA>    5 33    NA 16 17   <NA>  <NA>     N
## 6   1871   NA    PH1       PNA  <NA>    1 28    NA 21  7   <NA>  <NA>     Y
##   WSWin   R   AB   H X2B X3B HR BB SO SB CS HBP SF  RA  ER  ERA CG SHO SV
## 1  <NA> 401 1372 426  70  37  3 60 19 73 16  NA NA 303 109 3.55 22   1  3
## 2  <NA> 302 1196 323  52  21 10 60 22 69 21  NA NA 241  77 2.76 25   0  1
## 3  <NA> 249 1186 328  35  40  7 26 25 18  8  NA NA 341 116 4.11 23   0  0
## 4  <NA> 137  746 178  19   8  2 33  9 16  4  NA NA 243  97 5.17 19   1  0
## 5  <NA> 302 1404 403  43  21  1 33 15 46 15  NA NA 313 121 3.72 32   1  0
## 6  <NA> 376 1281 410  66  27  9 46 23 56 12  NA NA 266 137 4.95 27   0  0
##   IPouts  HA HRA BBA SOA   E DP    FP                   name
## 1    828 367   2  42  23 243 24 0.834    Boston Red Stockings
## 2    753 308   6  28  22 229 16 0.829 Chicago White Stockings
## 3    762 346  13  53  34 234 15 0.818  Cleveland Forest Citys
## 4    507 261   5  21  17 163  8 0.803    Fort Wayne Kekiongas
## 5    879 373   7  42  22 235 14 0.840        New York Mutuals
## 6    747 329   3  53  16 194 13 0.845  Philadelphia Athletics
##                          park attendance BPF PPF teamIDBR teamIDlahman45
## 1          South End Grounds I         NA 103  98      BOS            BS1
## 2      Union Base-Ball Grounds         NA 104 102      CHI            CH1
## 3 National Association Grounds         NA  96 100      CLE            CL1
## 4               Hamilton Field         NA 101 107      KEK            FW1
## 5     Union Grounds (Brooklyn)         NA  90  88      NYU            NY2
## 6     Jefferson Street Grounds         NA 102  98      ATH            PH1
##   teamIDretro
## 1         BS1
## 2         CH1
## 3         CL1
## 4         FW1
## 5         NY2
## 6         PH1
```

```
as_tibble(Teams)
```

```
## # A tibble: 2,925 x 48
##    yearID lgID  teamID franchID divID  Rank     G Ghome     W     L DivWin WCWin
##     <int> <fct> <fct>  <fct>    <chr> <int> <int> <int> <int> <int> <chr>  <chr>
##  1   1871 NA    BS1    BNA      <NA>      3    31    NA    20    10 <NA>   <NA>
##  2   1871 NA    CH1    CNA      <NA>      2    28    NA    19     9 <NA>   <NA>
##  3   1871 NA    CL1    CFC      <NA>      8    29    NA    10    19 <NA>   <NA>
##  4   1871 NA    FW1    KEK      <NA>      7    19    NA     7    12 <NA>   <NA>
##  5   1871 NA    NY2    NNA      <NA>      5    33    NA    16    17 <NA>   <NA>
##  6   1871 NA    PH1    PNA      <NA>      1    28    NA    21     7 <NA>   <NA>
##  7   1871 NA    RC1    ROK      <NA>      9    25    NA     4    21 <NA>   <NA>
##  8   1871 NA    TRO    TRO      <NA>      6    29    NA    13    15 <NA>   <NA>
##  9   1871 NA    WS3    OLY      <NA>      4    32    NA    15    15 <NA>   <NA>
## 10   1872 NA    BL1    BLC      <NA>      2    58    NA    35    19 <NA>   <NA>
## # ... with 2,915 more rows, and 36 more variables: LgWin <chr>, WSWin <chr>,
## #   R <int>, AB <int>, H <int>, X2B <int>, X3B <int>, HR <int>, BB <int>,
## #   SO <int>, SB <int>, CS <int>, HBP <int>, SF <int>, RA <int>, ER <int>,
## #   ERA <dbl>, CG <int>, SHO <int>, SV <int>, IPouts <int>, HA <int>,
## #   HRA <int>, BBA <int>, SOA <int>, E <int>, DP <int>, FP <dbl>, name <chr>,
## #   park <chr>, attendance <int>, BPF <int>, PPF <int>, teamIDBR <chr>,
## #   teamIDlahman45 <chr>, teamIDretro <chr>
```

```
# Note that the function was formerly called as.tibble()

# subsetting a data frame sometimes generates vectors
class(Teams[,20])
```

```
## [1] "integer"
```

```
# subsetting a tibble always generates tibbles
class(as_tibble(Teams[,20]))
```

```
## [1] "tbl_df"     "tbl"        "data.frame"
```

```
# pulling a vector out of a tibble
class(as_tibble(Teams)$HR)
```

```
## [1] "integer"
```

```
# access a non-existing column in a data frame or a tibble
Teams$hr
```

```
## NULL
```

```
as_tibble(Teams)$HR
```

```
##    [1]   3  10   7   2   1   9   3   6   6  14   0   1   7   0   2   4   4   5
##   [19]   0   0   9   0   6  13   0   5   4   8   2   1   1  17   4   2   7   6
##   [37]   2   2  15   0   2   0   2   7   7   5   0   0   0   0   9   8   4   2
##   [55]   6   2   7   2   4   0   6   4   9   1   2   3   5   3   2   8   2  20
##   [73]   3   4   8  12   5   4   3  20   4   7   7   8   5   8  12   5  12   7
##   [91]  17  11   5   7  18   4  15  15  20   5  19   9   5  11  18  11  12  16
##  [109]   8   5  34  13   8  15  34  13  14  24   6  20   3  21  13   7   2  39
##  [127]  32  17  16  36  19 142  10  16  40  36  26  31  20   6  17   0  22  22
##  [145]  26  14   7  21   2   7  11  32   0   8   2   6   4  23  17  14  22  54
##  [163]  26  25  19  16  21  30  20   6   5  17   8   8  16  24  53  45  53  19
##  [181]  20  21  18  21  26  16  20  30  23  31  25  53  80  14  37  55  33  27
```

```
##  [199]  48  21  29  47  20  39  47  19  25  56  77  12  32  51  34  19  14  55
##  [217]  31  16  14  36  30  20  47  42  79  25  36  52  62  18  22  52  43  44
##  [235]  42  58  25  20   2  13  43  34  31  58  67  31  27  21  16  27  15  25
##  [253]  66  24  23  49  20  35  31  48  14  24  30  23  52  53  60  40  22  20
##  [271]  28  17  13  46  55  21  29  57  19  30  30  34  26  44  26  18  39  50
##  [289]  38  45  37  27  45  65  32  29  32  19  61  80  37  10  23  33  42 103
##  [307]  65  61  37  42  45  40  48  54  59  25  41  54  55  36  29  34  32  61
##  [325]  27  39  55  23  28  36  34  20  28  37  40  49  27  37  45  19  24  45
##  [343]  38  22  16  40  31  40  25  32  36  12  17  53  18  19  18  32  34  33
##  [361]  14  13  36  17  27  39  27  13  12  40  23  31  29  47  47  26  48  33
##  [379]  33  23  29  26  36  24  37  32  28  32  18  38  12  29  26  19  35  24
##  [397]  29  39  33  33  42  19  14  14   6  18  33  22   6  38   5  18  29  10
##  [415]  47  48  15  25  14   9  28  31  12  20  18  32  12  34  12   8  17  26
##  [433]  15  24  14  22  21  27  11  31  27  31  23  15  10  24  10  29  29  17
##  [451]  11  12  27  18  13  39  23  24  16  22  16  20  22  13  25  16   7  20
##  [469]  16  12  10  14  17  32  12  12  20   9  26  18  18  22   5  13  15  11
##  [487]  11  23  15  22  12  19  10  18  12  14  28  17   3  19  14  18  19  20
##  [505]  13  21  11  25  20  17   8  20  16  14   4  20  22  10  19  26  16  21
##  [523]  12  25  10  15   9  43  25  31   7  34  23   9  28  31  20  19  22  33
##  [541]  12  15   9  35  28  37  20  54  21  20  30  41  25  35  60  49  17  26
##  [559]  16  29  32  35  17  43  21  12  19  47  18  22  43  39  19  27  20  17
##  [577]  39  32  24  59  27  16  24  30   8  33  73  35  18  15  19  32  18  42
##  [595]  31  35  38  19  52  42  16  10  25  33  39  30  12  29  62  18  34  17
##  [613]  26  33  18  36  14  36  14  17  40  25  50  53  15  20  23  28  17  24
##  [631]  31  16  58  24  20  19  23  20  12  14  28  22  17  46  14  16  17  42
##  [649]  35  19  42  20  14  25  12  14  25  22  18  17  26  13  25  39  27  17
##  [667]  38   9  15  26   4  15  10  13   8  21  15   9  13  13  20  22  25  15
##  [685]   5  27   4  33  25  24  25  21  20  24  23  40  45  35  42  17  31  18
##  [703]  24  22  28  23  37  34  18  35  30  46 115  44  64  16  50  32  36  17
##  [721]  59  61  35  37  20  42  58  75 134  82  88  37  67  83  42  45  56  32
##  [739]  45  42  45  32  54  80  95 111 116  52  98 107  45  34  62  32  42  90
##  [757]  45  59  41  85 105  53 112  49  82  63  26  30  72  25  41  66  36  41
##  [775]  35  95  98  63  94  44  67  67  22  41  64  41  38  86  44  52  50 114
##  [793] 110  76 100  78 110 109  56  32  40  16  32  66  35  27  36  73 121  61
##  [811]  75  44  72  90  43  28  39  37  36  74  29  26  51 109 158  56  57  54
##  [829]  55  84  29  38  66  52  24  92  32  34  62 118 133  89  85  52  63 113
##  [847]  40  28  99  33  37 139  34  62 110 136 142 122 153  60  46 100  48  47
##  [865] 122  66  63 171  74  72  82 143 152 125 126  86  75 104  57  37  71  34
##  [883]  27  84  21  71  43 101 155 118  81  41  76  60  49  53 110  63  36  69
##  [901]  47  78  80 116 160 172 122  48  67  76  61  50  62  54  43  72  34  50
##  [919]  57  82 144 139  60  39  64  57  60  51  79  83  71 101  55 100  74 126
##  [937] 135 144  56  52  62 104  51  69  59  75  74  88  73  93 106 123 104 112
##  [955]  92  66  73  86  32  86  33  67  60  76  82 123  94  97 182  72 103  60
##  [973]  79  88  62 100  37  63  67  96  73 103 150 111 174  94 103  47  71  94
##  [991]  47  98  61  54  67  65 110 113 137 125 174  98  40  65  92  91  85 124
## [1009]  78  56  64  91  98  85 124 116 166  98  49  63  91  98  44 145  93  59
## [1027]  73  86  89 101 134  91 155 105  75  76 118 119  52 124 101  48  47  99
## [1045]  64 103  81  95 151  85  64  56  91  70  52 103  62  68  25  75  66  50
## [1063]  76 109 108  33  44  54  98  60  40  57  39  39  33  52  43  55  77  81
## [1081] 100  26  66  42  78  70  47  69  56  79  23  71  51  70  60  93  96  36
## [1099]  55  70  72 100  33  50  57 101  22  57  56  65  77 114  93  33  56  72
## [1117]  63  64  27 109  55  44  37  56  65  79 108 121 136  40  80  60  84  81
## [1135]  60 103  83  85  53  71  95 112 103 221 115  61  60 156  90 115  42 121
## [1153]  91  95  55  87 104 155  78 164 139  68  91 108  63 105  31 131 152 103
```

```
## [1171]  43  97  86 112  88 147 115  82 122 126 117 102  81 161 194 148  93 161
## [1189]  99 164 114 133 159 100 125 138 106 102  76 127 184 130  86 103  88 140
## [1207] 104 179 140 102 108 137  86  95  54 113 153 110  80 107 104 148 103 151
## [1225] 129  89  93  92  82  97  50 101 208  74 137 166 160 108 156 176 139 116
## [1243] 115  99 112 140  69  52 123 186  94 159 147 156  90 139 186 133  94 102
## [1261]  76 119  81  54 137 201 116 164 181 148 130 121 182 169 175 132  91 143
## [1279]  80  91 139 179 128 142 221 153 150 112 177 145 190 121 110 124 112  87
## [1297] 153 147 106 147 187 140 116 166 199 157 145 117  92 132 111 108 155 101
## [1315] 182 123 161 109 138 172 167 164 124 134 170 111 121 109 125  97 163 161
## [1333] 167 160 117 148 177 153 113 112 167 118 163 123 124 112 119 140 127 150
## [1351] 110 126 170 193  99 120 130 138 147 149 112 138 176 158 150 180  90 189
## [1369] 157 167 188 240 103 128 183 103 119 156 146  92 126 167 180 209 105 116
## [1387] 137 140 185 181 199 139 142 108 204 137 132 146 171 114 127 122 169 148
## [1405]  62  95  95 110 225 139 188  96 126 108 197 128 138 162 186 106 145 130
## [1423] 164 157  70 166 102  79 221 159 162 103 130 121 165 109 125 125 165  92
## [1441] 125 134 183 156 162  97 110  78 150 196 149 107 144 111 159 109 136 207
## [1459] 175 145 122  87 140 149 155 179 112  70 108 144 162  98 117 158 181 108
## [1477] 126 158 138 158 114  89 128 109 131 152  93  69  82 131 100  83 103  91
## [1495] 140 115 115  80 133 125  83  71 130 106  75 185  66  67 105 109  81  94
## [1513] 100  80 108  73 124 141 175 197  88 112 142 171 119 182 104  98  97 163
## [1531] 125  94 109 148 137 119  99 125 136  90 148 160 179 203 114 123 179 191
## [1549] 183 148 129  97  87 153 126 136 111 120 171 101 130 172 165 113 138 153
## [1567] 158 161  96 138 128 138 109 179  71  80  95 116 104  88  97  98 160 123
## [1585] 154  96 140  95  86 144 100 124  78 108 133 124  91 122 134  78  98  93
## [1603]  88  91 103 105 134  98 110 102 150  70  56 206 119 147  93 111 117 137
## [1621] 158 157 134 114 110 120 145 125 131  85 147 134 154 112 161  75 110 120
## [1639] 116 109  95 135 110 135 131 131 110  89 139 111 120  86 101  96 132  95
## [1657] 114  99  93  83  99 107 124 134  55  94  95 124 153 125  84 118 118 121
## [1675] 146  98 110 101 151 125 138  78  84  81 134  82 119 134  63  73 105 141
## [1693]  85 101  66  65  91  81  88  94 120 102 113 110 110  64  85  63  80 139
## [1711] 148 213 131 192 111 181 100 166 114 146 191 123 125 138 184  88 117 186
## [1729] 133 120 133 134  96 135 100 123 154 172 108 106  72 136 106 129  70  98
## [1747] 149  82 173 121 125  86 100 133 115  75  97 117  79 132  98 126 181 194
## [1765] 164 127 135 132 138 164  49 116 183 112 185 143 150  74 108 119 148  93
## [1783] 132 125 100 140  95 144 156 162 106  91 107 113  89 143  75 115 148  99
## [1801] 203 114 189  61 137 117 116  67 104  80 101 124 126  64  88  90  97  76
## [1819]  57  64  39  65  45  61  82  47  96  81 100  57 104  69  55  32  89  63
## [1837]  50  49  61 146 179 136 186 136 102  82 109 177  74 132 138 148 216 133
## [1855] 161  97 149 112 134  81 130 133  67 115 106 130 168 142 154 157 140 107
## [1873]  86 156  97 109 146 141 132 102 153 112 121 125 121  93 111 142  83 106
## [1891] 167 111 160 181 150 172 136 106 123 187  79 117 102 114  96  96 130 107
## [1909] 158 147  98 109 129 112  75 120 143 126 214 162 153 146 150 114 116 202
## [1927] 121 154 129 141 101 118 176 134 155 141  80 109 171 115  87 129 158 138
## [1945] 169 144 167 121 155 144 157 198 125 137 130 196 127 110 188 148 163 154
## [1963] 111 136 158 114  58 184 181 152 211 174 172 173 209 192 187 225 122 168
## [1981] 125 196 163 120 196 192 199 169 131 113 161 205  94 194 215  96 137 124
## [1999] 124 132 113 122 134 143  96 121  99 151 113 107 148 152 156 106 110  94
## [2017] 148 113  71 112 158 128 129 108 145  94 124 128 127 116  97 101  89 117
## [2035] 126 100 130 147 127 123  95 120 134 141  73 122 142 162 132 106 147 106
## [2053] 136 125 110 172  94 100 129 100 128 114 147 172 164 103 138 123 107 152
## [2071]  73 110 167 141 170 126 115 139 159 164  79 209  79 117 108 140 116  95
## [2089] 147 117 159 111 126 121 126 141  68 177 133 138 148  84  88 110 104  99
## [2107] 127 182  96  75  72 104  82 102 163  93 142 118 106 135 149 105  94 159
## [2125] 163 169 157 114 114 162 161 137 141 142 178  94 138 125 130 121 125 122
```

```
## [2143] 178 158 158 156 110 153 161 168 118 181 159 137 139 120 120 121 109 124
## [2161] 167 125 161  94 120 100 115 103  99 108 139 117 113  80  80  92 153 123
## [2179] 108 124 115 168 173 175 186 146 158 161 207 200 159 144 109 119 140 120
## [2197] 128 118 122 125 169  94 125 116 182 152 107 138 140 197 257 209 192 195
## [2215] 175 191 218 221 204 150 129 123 150 118 178 148 162 147 243 132 138 147
## [2233] 245 153 142 221 177 161 174 196 185 158 127 142 220 239 176 136 133 158
## [2251] 174 132 135 172 161 153 197 116 129 152 264 172 144 187 147 147 159 215
## [2269] 214 205 198 212 138 198 183 165 114 166 134 159 152 115 147 207 136 149
## [2287] 126 107 167 234 161 223 111 201 221 158 216 197 203 176 162 189 209 209
## [2305] 223 212 128 168 151 187 165 105 163 193 181 235 161 171 153 244 188 194
## [2323] 145 230 212 236 179 179 184 167 216 183 200 221 161 177 160 249 150 211
## [2341] 177 116 178 205 198 239 144 168 157 198 226 235 162 173 244 158 208 174
## [2359] 136 198 214 194 176 212 213 139 166 208 152 206 209 164 131 203 147 199
## [2377] 164 161 161 169 235 199 121 246 195 152 165 164 165 177 217 200 169 192
## [2395] 152 124 146 167 140 155 139 167 162 223 160 205 165 142 136 152 198 175
## [2413] 133 230 187 150 152 235 152 238 220 172 182 158 198 153 157 191 162 124
## [2431] 196 155 144 230 124 176 166 163 128 139 180 196 137 239 190 162 135 178
## [2449] 169 222 242 235 194 184 202 201 148 187 150 203 135 191 151 242 185 189
## [2467] 215 142 139 136 183 214 145 227 145 191 184 189 199 200 194 222 207 150
## [2485] 168 128 161 126 147 149 175 134 229 175 155 167 139 130 130 128 170 157
## [2503] 260 136 117 160 222 164 192 236 166 217 196 157 203 182 174 124 159 153
## [2521] 180 143 210 200 175 216 141 161 172 163 184 190 183 199 164 171 176 142
## [2539] 166 190 151 204 178 171 177 201 167 102 123 129 231 118 201 177 171 213
## [2557] 148 171 153 131 141 187 179 165 123 159 130 172 173 235 184 187 171 160
## [2575] 200 208 167 120 159 137 198 111 180 172 125 214 153 154 124  94 174 180
## [2593] 194 126 117 173 149 160 212 184 161 158 161 190 183 159 142 144 173 145
## [2611] 182 172 244  95 135 224 125 141 160 122 160 199 224 209 156 180 139 133
## [2629] 211 177 149 188 128 173 152 152 108 121 155 120 182 142 201 128 109 166
## [2647] 126 132 101 162 150 160 162 257 149 172 173 191 203 154 148 183 154 163
## [2665] 169 149  95 129 155 117 185 103 222 108 114 153 107  91 109 121 162 172
## [2683] 210 186 154 165 149 214 165 211 137 172 136 166 163 146 131 187 116 137
## [2701] 202 131 245 139 195 158 170 121 149 103 159 175 200 198 194 130 181 212
## [2719] 178 148 172 155 171 159 176 148 112 164 138  95 157 151 144 130 186 140
## [2737] 161 146 188 107 125 165 176 185 161 118 123 211 123 155 157 131 142 186
## [2755] 155 163  95 155 134 122 150 128 147 125 146 125 156 109 136 132 105 117
## [2773] 111 177 152 154 100 217 161 136 171 167 141 186 151 230 139 176 187 120
## [2791] 145 156 212 177 146 130 140 148 198 136 137 167 172 232 177 190 122 253
## [2809] 208 168 199 164 185 204 211 198 147 156 189 128 194 200 183 218 169 161
## [2827] 153 177 223 130 225 216 215 221 203 220 165 232 168 186 223 219 212 192
## [2845] 187 238 193 186 221 194 224 206 241 224 234 174 151 189 200 128 196 228
## [2863] 237 222 215 176 175 188 208 182 167 172 216 210 135 205 155 214 235 128
## [2881] 218 166 267 170 227 186 157 162 176 133 205 150 194 217 191 220 249 213
## [2899] 245 182 256 227 223 224 149 288 162 220 279 146 250 307 306 242 257 215
## [2917] 163 219 239 167 210 217 223 247 231
```

```
# create a tibble with complex objects
tibble(id = c(1, 2, 3), func = c(mean, median, sd))
```

```
## # A tibble: 3 x 2
##      id func
##   <dbl> <list>
## 1     1 <fn>
## 2     2 <fn>
## 3     3 <fn>
```

do

The textbook for this section is available here

Key points

- The `do()` function serves as a bridge between R functions, such as `lm()`, and the tidyverse.
- We have to specify a column when using the `do()` function, otherwise we will get an error.
- If the data frame being returned has more than one row, the rows will be concatenated appropriately.

Code

```
# use do to fit a regression line to each HR stratum
dat %>%
    group_by(HR) %>%
    do(fit = lm(R ~ BB, data = .))
```

```
## # A tibble: 9 x 2
## # Rowwise:
##      HR fit
##   <dbl> <list>
## 1   0.4 <lm>
## 2   0.5 <lm>
## 3   0.6 <lm>
## 4   0.7 <lm>
## 5   0.8 <lm>
## 6   0.9 <lm>
## 7   1   <lm>
## 8   1.1 <lm>
## 9   1.2 <lm>
```

```
# using do without a column name gives an error
dat %>%
    group_by(HR) %>%
    do(lm(R ~ BB, data = .))
```

```
# define a function to extract slope from lm
get_slope <- function(data){
  fit <- lm(R ~ BB, data = data)
  data.frame(slope = fit$coefficients[2],
             se = summary(fit)$coefficient[2,2])
}

# return the desired data frame
dat %>%
  group_by(HR) %>%
  do(get_slope(.))
```

```
## # A tibble: 9 x 3
## # Groups:   HR [9]
##      HR slope     se
##   <dbl> <dbl>  <dbl>
## 1   0.4 0.734 0.208
## 2   0.5 0.566 0.110
## 3   0.6 0.412 0.0974
## 4   0.7 0.285 0.0705
## 5   0.8 0.365 0.0653
## 6   0.9 0.261 0.0754
```

```
## 7   1   0.511 0.0749
## 8   1.1 0.454 0.0855
## 9   1.2 0.440 0.0801
```

```
# not the desired output: a column containing data frames
dat %>%
  group_by(HR) %>%
  do(slope = get_slope(.))
```

```
## # A tibble: 9 x 2
## # Rowwise:
##      HR slope
##   <dbl> <list>
## 1   0.4 <df[,2] [1 x 2]>
## 2   0.5 <df[,2] [1 x 2]>
## 3   0.6 <df[,2] [1 x 2]>
## 4   0.7 <df[,2] [1 x 2]>
## 5   0.8 <df[,2] [1 x 2]>
## 6   0.9 <df[,2] [1 x 2]>
## 7   1   <df[,2] [1 x 2]>
## 8   1.1 <df[,2] [1 x 2]>
## 9   1.2 <df[,2] [1 x 2]>
```

```
# data frames with multiple rows will be concatenated appropriately
get_lse <- function(data){
  fit <- lm(R ~ BB, data = data)
  data.frame(term = names(fit$coefficients),
    estimate = fit$coefficients,
    se = summary(fit)$coefficient[,2])
}

dat %>%
  group_by(HR) %>%
  do(get_lse(.))
```

```
## # A tibble: 18 x 4
## # Groups:   HR [9]
##       HR term        estimate     se
##    <dbl> <fct>          <dbl>  <dbl>
##  1   0.4 (Intercept)    1.36  0.631
##  2   0.4 BB             0.734 0.208
##  3   0.5 (Intercept)    2.01  0.344
##  4   0.5 BB             0.566 0.110
##  5   0.6 (Intercept)    2.53  0.305
##  6   0.6 BB             0.412 0.0974
##  7   0.7 (Intercept)    3.21  0.225
##  8   0.7 BB             0.285 0.0705
##  9   0.8 (Intercept)    3.07  0.213
## 10   0.8 BB             0.365 0.0653
## 11   0.9 (Intercept)    3.54  0.252
## 12   0.9 BB             0.261 0.0754
## 13   1   (Intercept)    2.88  0.255
## 14   1   BB             0.511 0.0749
## 15   1.1 (Intercept)    3.21  0.300
## 16   1.1 BB             0.454 0.0855
```

```
## 17   1.2 (Intercept)    3.40  0.291
## 18   1.2 BB             0.440 0.0801
```

broom

The textbook for this section is available here

Key points

- The **broom** package has three main functions, all of which extract information from the object returned by **lm** and return it in a **tidyverse** friendly data frame.
- The `tidy()` function returns estimates and related information as a data frame.
- The functions `glance()` and `augment()` relate to model specific and observation specific outcomes respectively.

Code

```
# use tidy to return lm estimates and related information as a data frame
if(!require(broom)) install.packages("broom")
```

```
## Loading required package: broom
```

```
library(broom)
fit <- lm(R ~ BB, data = dat)
tidy(fit)
```

```
## # A tibble: 2 x 5
##   term        estimate std.error statistic  p.value
##   <chr>          <dbl>     <dbl>     <dbl>    <dbl>
## 1 (Intercept)    2.20     0.113       19.4 1.12e-70
## 2 BB             0.638    0.0344      18.5 1.35e-65
```

```
# add confidence intervals with tidy
tidy(fit, conf.int = TRUE)
```

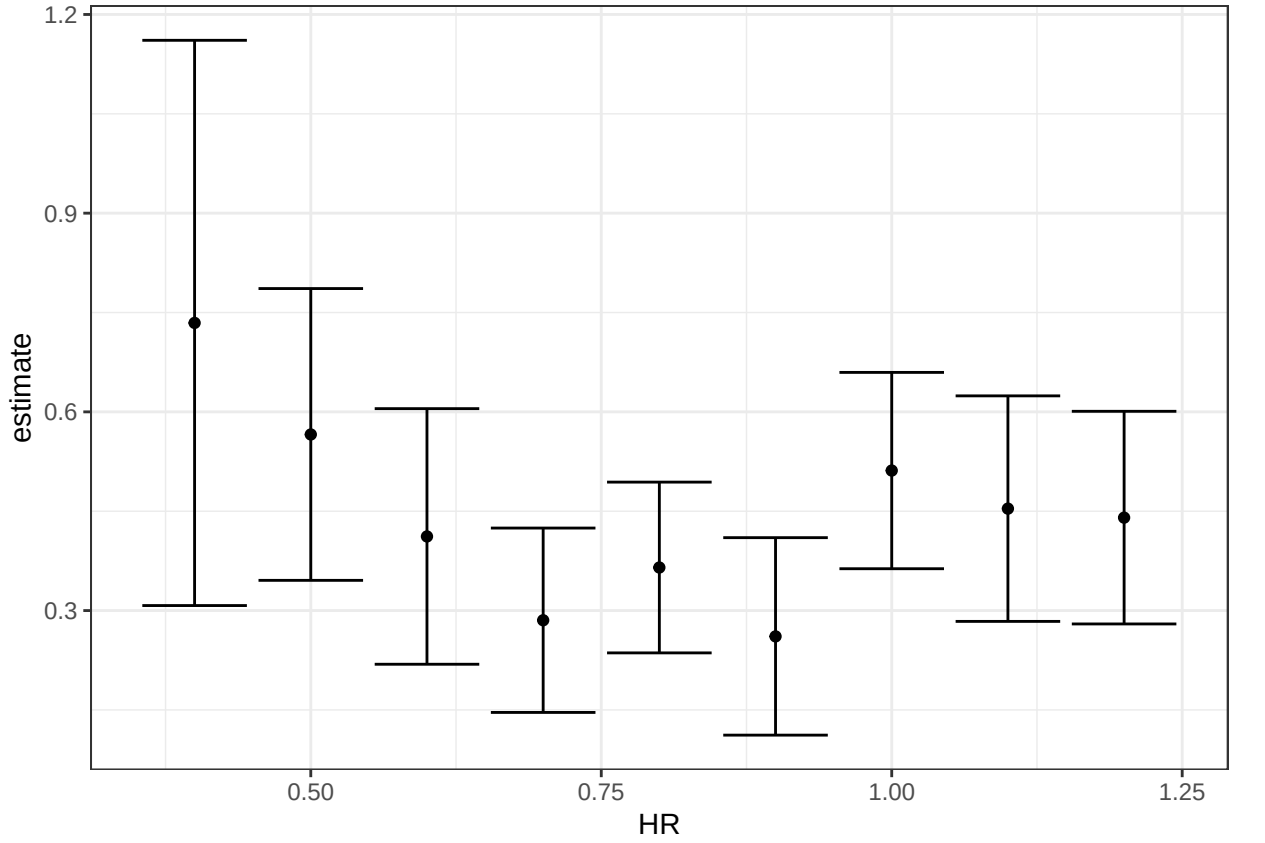
```
## # A tibble: 2 x 7
##   term        estimate std.error statistic  p.value conf.low conf.high
##   <chr>          <dbl>     <dbl>     <dbl>    <dbl>    <dbl>     <dbl>
## 1 (Intercept)    2.20     0.113       19.4 1.12e-70    1.98      2.42
## 2 BB             0.638    0.0344      18.5 1.35e-65    0.570     0.705
```

```
# pipeline with lm, do, tidy
dat %>%
  group_by(HR) %>%
  do(tidy(lm(R ~ BB, data = .), conf.int = TRUE)) %>%
  filter(term == "BB") %>%
  select(HR, estimate, conf.low, conf.high)
```

```
## # A tibble: 9 x 4
## # Groups:   HR [9]
##      HR estimate conf.low conf.high
##   <dbl>    <dbl>    <dbl>     <dbl>
## 1   0.4    0.734    0.308     1.16
## 2   0.5    0.566    0.346     0.786
## 3   0.6    0.412    0.219     0.605
## 4   0.7    0.285    0.146     0.425
## 5   0.8    0.365    0.236     0.494
## 6   0.9    0.261    0.112     0.410
## 7   1      0.511    0.363     0.660
```

```
## 8   1.1    0.454    0.284     0.624
## 9   1.2    0.440    0.280     0.601
```

```
# make ggplots
dat %>%
  group_by(HR) %>%
  do(tidy(lm(R ~ BB, data = .), conf.int = TRUE)) %>%
  filter(term == "BB") %>%
  select(HR, estimate, conf.low, conf.high) %>%
  ggplot(aes(HR, y = estimate, ymin = conf.low, ymax = conf.high)) +
  geom_errorbar() +
  geom_point()
```




```
# inspect with glance
glance(fit)
```

```
## # A tibble: 1 x 12
##   r.squared adj.r.squared sigma statistic  p.value    df logLik   AIC   BIC
##       <dbl>         <dbl> <dbl>     <dbl>    <dbl> <dbl>  <dbl> <dbl> <dbl>
## 1     0.266         0.265 0.454      343. 1.35e-65     1  -596. 1199. 1214.
## # ... with 3 more variables: deviance <dbl>, df.residual <int>, nobs <int>
```

Assessment - Tibbles, do, and broom, part 1

1. As seen in the videos, what problem do we encounter when we try to run a linear model on our baseball data, grouping by home runs?

- [ ] A. There is not enough data in some levels to run the model.
- [X] B. The `lm()` function does not know how to handle grouped tibbles.

- [ ] C. The results of the `lm()` function cannot be put into a tidy format.

2. Tibbles are similar to what other class in R?

- [ ] A. Vectors
- [ ] B. Matrices
- [X] C. Data frames
- [ ] D. Lists

3. What are some advantages of tibbles compared to data frames?

- [X] A. Tibbles display better.
- [X] B. If you subset a tibble, you always get back a tibble.
- [X] C. Tibbles can have complex entries.
- [X] D. Tibbles can be grouped.

4. What are two advantages of the do command, when applied to the tidyverse?

- [ ] A. It is faster than normal functions.
- [ ] B. It returns useful error messages.
- [X] C. It understands grouped tibbles.
- [X] D. It always returns a data.frame.

5. You want to take the tibble `dat`, which we used in the video on the `do()` function, and run the linear model R ~ BB for each strata of HR. Then you want to add three new columns to your grouped tibble: the coefficient, standard error, and p-value for the BB term in the model.

You've already written the function `get_slope()`, shown below.

```
get_slope <- function(data) {
  fit <- lm(R ~ BB, data = data)
  sum.fit <- summary(fit)

  data.frame(slope = sum.fit$coefficients[2, "Estimate"],
             se = sum.fit$coefficients[2, "Std. Error"],
             pvalue = sum.fit$coefficients[2, "Pr(>|t|)"])
}
```

What additional code could you write to accomplish your goal?

```
dat %>%
  group_by(HR) %>%
  do(get_slope(.))
```

```
## # A tibble: 9 x 4
## # Groups:   HR [9]
##      HR slope     se   pvalue
##   <dbl> <dbl>  <dbl>    <dbl>
## 1   0.4 0.734 0.208  1.54e- 3
## 2   0.5 0.566 0.110  3.02e- 6
## 3   0.6 0.412 0.0974 4.80e- 5
## 4   0.7 0.285 0.0705 7.93e- 5
## 5   0.8 0.365 0.0653 9.13e- 8
## 6   0.9 0.261 0.0754 7.12e- 4
## 7   1   0.511 0.0749 3.00e-10
## 8   1.1 0.454 0.0855 1.03e- 6
## 9   1.2 0.440 0.0801 1.07e- 6
```

- [ ] A.

```
dat %>%
  group_by(HR) %>%
  do(get_slope)
```

- [X] B.

```
dat %>%
  group_by(HR) %>%
  do(get_slope(.))
```

- [ ] C.

```
dat %>%
  group_by(HR) %>%
  do(slope = get_slope(.))
```

- [ ] D.

```
dat %>%
  do(get_slope(.))
```

6. The output of a broom function is always what?

- [X] A. A data.frame
- [ ] B. A list
- [ ] C. A vector

7. You want to know whether the relationship between home runs and runs per game varies by baseball league.

You create the following dataset:

```
dat <- Teams %>% filter(yearID %in% 1961:2001) %>%
  mutate(HR = HR/G,
         R = R/G) %>%
  select(lgID, HR, BB, R)
```

What code would help you quickly answer this question?

```
dat %>%
  group_by(lgID) %>%
  do(tidy(lm(R ~ HR, data = .), conf.int = T)) %>%
  filter(term == "HR")
```

```
## # A tibble: 2 x 8
## # Groups:   lgID [2]
##   lgID  term  estimate std.error statistic  p.value conf.low conf.high
##   <fct> <chr>    <dbl>     <dbl>     <dbl>    <dbl>    <dbl>     <dbl>
## 1 AL    HR        1.90    0.0734      25.9 1.29e-95     1.75      2.04
## 2 NL    HR        1.76    0.0671      26.2 1.16e-95     1.62      1.89
```

- [X] A.

```
dat %>%
  group_by(lgID) %>%
  do(tidy(lm(R ~ HR, data = .), conf.int = T)) %>%
  filter(term == "HR")
```

- [ ] B.

```
dat %>%
  group_by(lgID) %>%
  do(glance(lm(R ~ HR, data = .)))
```

- [ ] C.

```
dat %>%
  do(tidy(lm(R ~ HR, data = .), conf.int = T)) %>%
  filter(term == "HR")
```

- [ ] D.

```
dat %>%
  group_by(lgID) %>%
  do(mod = lm(R ~ HR, data = .))
```

Assessment - Tibbles, do, and broom, part 2

We have investigated the relationship between fathers' heights and sons' heights. But what about other parent-child relationships? Does one parent's height have a stronger association with child height? How does the child's gender affect this relationship in heights? Are any differences that we observe statistically significant?

The `galton` dataset is a sample of one male and one female child from each family in the `GaltonFamilies` dataset. The `pair` column denotes whether the pair is father and daughter, father and son, mother and daughter, or mother and son.

Create the `galton` dataset using the code below:

```
data("GaltonFamilies")
set.seed(1) # if you are using R 3.5 or earlier
set.seed(1, sample.kind = "Rounding") # if you are using R 3.6 or later
```

```
## Warning in set.seed(1, sample.kind = "Rounding"): non-uniform 'Rounding' sampler
## used
```

```
galton <- GaltonFamilies %>%
    group_by(family, gender) %>%
    sample_n(1) %>%
    ungroup() %>%
    gather(parent, parentHeight, father:mother) %>%
    mutate(child = ifelse(gender == "female", "daughter", "son")) %>%
    unite(pair, c("parent", "child"))

galton
```

```
## # A tibble: 710 x 8
##    family midparentHeight children childNum gender childHeight pair
##    <fct>            <dbl>    <int>    <int> <fct>        <dbl> <chr>
##  1 001               75.4        4        2 female        69.2 fath~
##  2 001               75.4        4        1 male          73.2 fath~
##  3 002               73.7        4        4 female        65.5 fath~
##  4 002               73.7        4        2 male          72.5 fath~
##  5 003               72.1        2        2 female        68   fath~
##  6 003               72.1        2        1 male          71   fath~
##  7 004               72.1        5        5 female        63   fath~
##  8 004               72.1        5        2 male          68.5 fath~
```

```
##  9 005               69.1        6        5 female        62.5 fath~
## 10 005               69.1        6        1 male          72   fath~
## # ... with 700 more rows, and 1 more variable: parentHeight <dbl>
```

8. Group by `pair` and summarize the number of observations in each group.

How many father-daughter pairs are in the dataset?

How many mother-son pairs are in the dataset?

```
galton %>%
    group_by(pair) %>%
    summarize(n = n())
```

```
## `summarise()` ungrouping output (override with `.groups` argument)
```

```
## # A tibble: 4 x 2
##   pair                n
##   <chr>           <int>
## 1 father_daughter   176
## 2 father_son        179
## 3 mother_daughter   176
## 4 mother_son        179
```

9. Calculate the correlation coefficients for fathers and daughters, fathers and sons, mothers and daughters and mothers and sons.

Which pair has the **strongest** correlation in heights?

```
galton %>%
    group_by(pair) %>%
    summarize(cor = cor(parentHeight, childHeight)) %>%
    filter(cor == max(cor))
```

```
## `summarise()` ungrouping output (override with `.groups` argument)
```

```
## # A tibble: 1 x 2
##   pair         cor
##   <chr>      <dbl>
## 1 father_son 0.430
```

Which pair has the **weakest** correlation in heights?

```
galton %>%
    group_by(pair) %>%
    summarize(cor = cor(parentHeight, childHeight)) %>%
    filter(cor == min(cor))
```

```
## `summarise()` ungrouping output (override with `.groups` argument)
```

```
## # A tibble: 1 x 2
##   pair         cor
##   <chr>      <dbl>
## 1 mother_son 0.343
```

Question 10 has two parts. The information here applies to both parts.

Use `lm()` and the **broom** package to fit regression lines for each parent-child pair type. Compute the least squares estimates, standard errors, confidence intervals and p-values for the `parentHeight` coefficient for each pair.

10a. What is the estimate of the father-daughter coefficient?

```
galton %>%
    group_by(pair) %>%
    do(tidy(lm(childHeight ~ parentHeight, data = .), conf.int = TRUE)) %>%
    filter(term == "parentHeight", pair == "father_daughter") %>%
    pull(estimate)
```

```
## [1] 0.345
```

For every 1-inch increase in mother's height, how many inches does the typical son's height increase?

```
galton %>%
    group_by(pair) %>%
    do(tidy(lm(childHeight ~ parentHeight, data = .), conf.int = TRUE)) %>%
    filter(term == "parentHeight", pair == "mother_son") %>%
    pull(estimate)
```

```
## [1] 0.381
```

10b. Which sets of parent-child heights are significantly correlated at a p-value cut off of .05?

```
galton %>%
    group_by(pair) %>%
    do(tidy(lm(childHeight ~ parentHeight, data = .), conf.int = TRUE)) %>%
    filter(term == "parentHeight" & p.value < .05)
```

```
## # A tibble: 4 x 8
## # Groups:   pair [4]
##   pair       term      estimate std.error statistic   p.value conf.low conf.high
##   <chr>      <chr>        <dbl>     <dbl>     <dbl>     <dbl>    <dbl>     <dbl>
## 1 father_da~ parentHe~    0.345    0.0599      5.77   3.56e-8    0.227     0.464
## 2 father_son parentHe~    0.443    0.0700      6.33   1.94e-9    0.305     0.581
## 3 mother_da~ parentHe~    0.394    0.0720      5.47   1.56e-7    0.252     0.536
## 4 mother_son parentHe~    0.381    0.0784      4.86   2.59e-6    0.226     0.535
```

- [X] A. father-daughter
- [X] B. father-son
- [X] C. mother-daughter
- [X] D. mother-son

Which of the following statements are true?

- [X] A. All of the confidence intervals overlap each other.
- [ ] B. At least one confidence interval covers zero.
- [X] C. The confidence intervals involving mothers' heights are larger than the confidence intervals involving fathers' heights.
- [ ] D. The confidence intervals involving daughters' heights are larger than the confidence intervals involving sons' heights.
- [X] E. The data are consistent with inheritance of height being independent of the child's gender.
- [X] F. The data are consistent with inheritance of height being independent of the parent's gender.

Building a Better Offensive Metric for Baseball

The textbook for this section is available here

Code

```
# linear regression with two variables
fit <- Teams %>%
  filter(yearID %in% 1961:2001) %>%
```

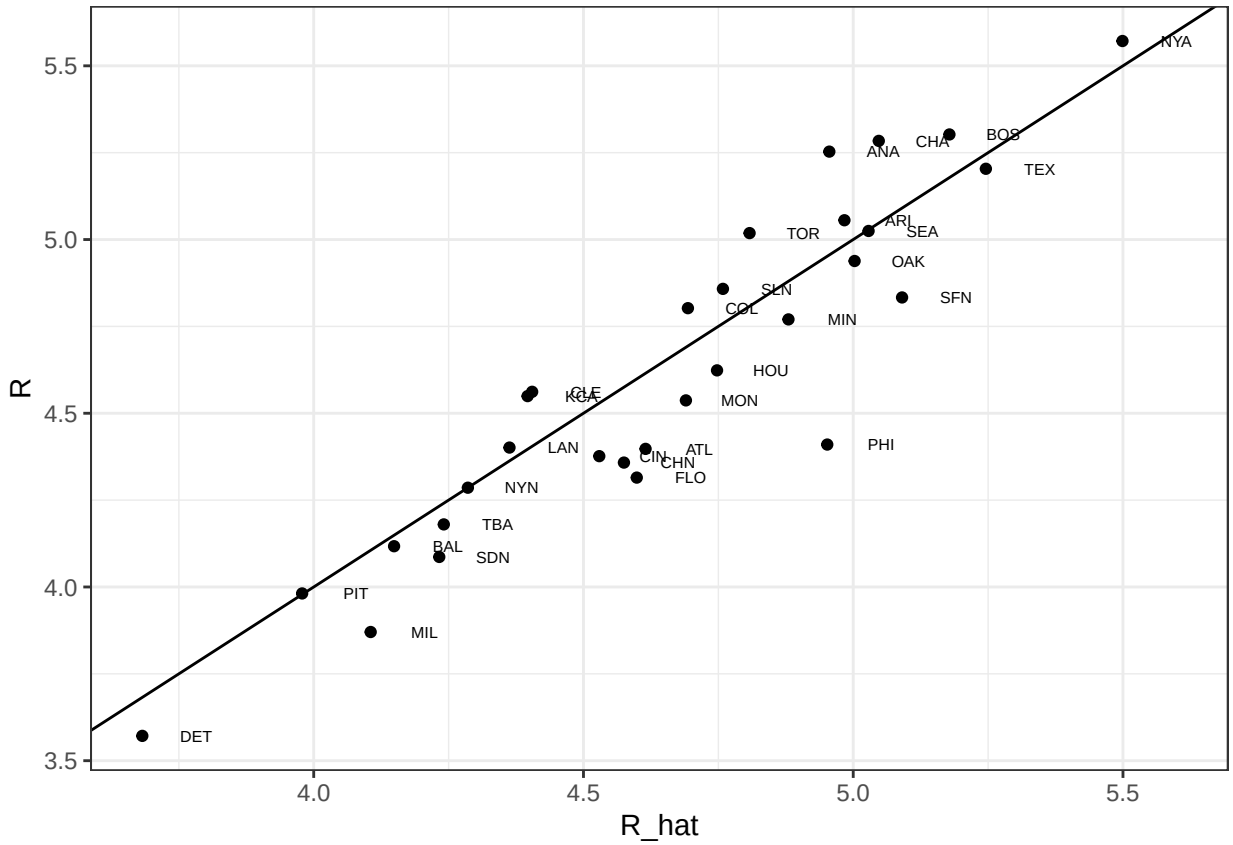
```
  mutate(BB = BB/G, HR = HR/G,  R = R/G) %>%
  lm(R ~ BB + HR, data = .)
  tidy(fit, conf.int = TRUE)
```

```
## # A tibble: 3 x 7
##   term        estimate std.error statistic   p.value conf.low conf.high
##   <chr>          <dbl>     <dbl>     <dbl>     <dbl>    <dbl>     <dbl>
## 1 (Intercept)    1.74     0.0824      21.2 7.62e- 83    1.58      1.91
## 2 BB             0.387    0.0270      14.3 1.20e- 42    0.334     0.440
## 3 HR             1.56     0.0490      31.9 1.78e-155    1.47      1.66
```

```
# regression with BB, singles, doubles, triples, HR
fit <- Teams %>%
  filter(yearID %in% 1961:2001) %>%
  mutate(BB = BB / G,
         singles = (H - X2B - X3B - HR) / G,
         doubles = X2B / G,
         triples = X3B / G,
         HR = HR / G,
         R = R / G) %>%
  lm(R ~ BB + singles + doubles + triples + HR, data = .)
coefs <- tidy(fit, conf.int = TRUE)
coefs
```

```
## # A tibble: 6 x 7
##   term        estimate std.error statistic   p.value conf.low conf.high
##   <chr>          <dbl>     <dbl>     <dbl>     <dbl>    <dbl>     <dbl>
## 1 (Intercept)   -2.77     0.0862     -32.1 4.76e-157   -2.94     -2.60
## 2 BB             0.371    0.0117      31.6 1.87e-153    0.348     0.394
## 3 singles        0.519    0.0127      40.8 8.67e-217    0.494     0.544
## 4 doubles        0.771    0.0226      34.1 8.44e-171    0.727     0.816
## 5 triples        1.24     0.0768      16.1 2.12e- 52    1.09      1.39
## 6 HR             1.44     0.0243      59.3 0.           1.40      1.49
```

```
# predict number of runs for each team in 2002 and plot
Teams %>%
  filter(yearID %in% 2002) %>%
  mutate(BB = BB/G,
         singles = (H-X2B-X3B-HR)/G,
         doubles = X2B/G,
         triples =X3B/G,
         HR=HR/G,
         R=R/G)  %>%
  mutate(R_hat = predict(fit, newdata = .)) %>%
  ggplot(aes(R_hat, R, label = teamID)) +
  geom_point() +
  geom_text(nudge_x=0.1, cex = 2) +
  geom_abline()
```



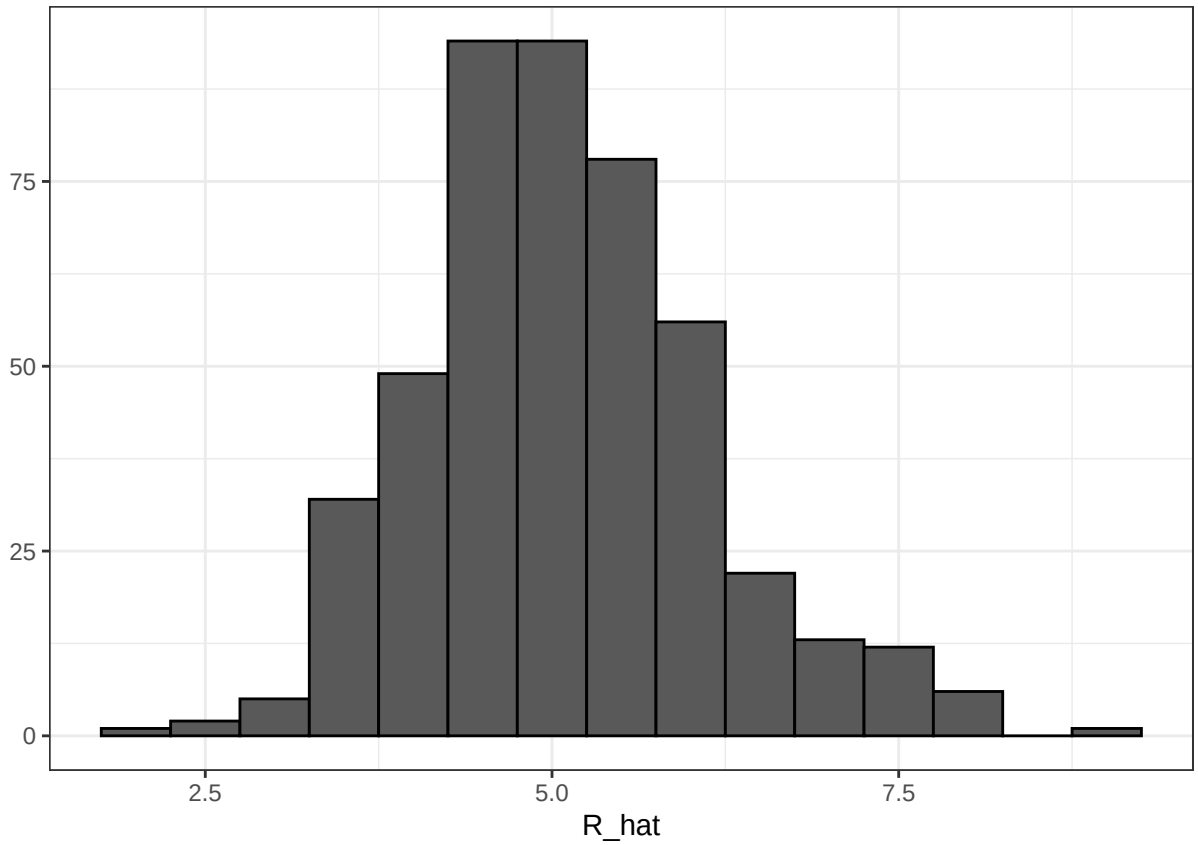

```
# average number of team plate appearances per game
pa_per_game <- Batting %>% filter(yearID == 2002) %>%
  group_by(teamID) %>%
  summarize(pa_per_game = sum(AB+BB)/max(G)) %>%
  pull(pa_per_game) %>%
  mean
```

```
## `summarise()` ungrouping output (override with `.groups` argument)
```

```
# compute per-plate-appearance rates for players available in 2002 using previous data
players <- Batting %>% filter(yearID %in% 1999:2001) %>%
  group_by(playerID) %>%
  mutate(PA = BB + AB) %>%
  summarize(G = sum(PA)/pa_per_game,
    BB = sum(BB)/G,
    singles = sum(H-X2B-X3B-HR)/G,
    doubles = sum(X2B)/G,
    triples = sum(X3B)/G,
    HR = sum(HR)/G,
    AVG = sum(H)/sum(AB),
    PA = sum(PA)) %>%
  filter(PA >= 300) %>%
  select(-G) %>%
  mutate(R_hat = predict(fit, newdata = .))
```

```
## `summarise()` ungrouping output (override with `.groups` argument)
```

```
# plot player-specific predicted runs
qplot(R_hat, data = players, geom = "histogram", binwidth = 0.5, color = I("black"))
```




```
# add 2002 salary of each player
players <- Salaries %>%
  filter(yearID == 2002) %>%
  select(playerID, salary) %>%
  right_join(players, by="playerID")

# add defensive position
position_names <- c("G_p","G_c","G_1b","G_2b","G_3b","G_ss","G_lf","G_cf","G_rf")
tmp_tab <- Appearances %>%
  filter(yearID == 2002) %>%
  group_by(playerID) %>%
  summarize_at(position_names, sum) %>%
  ungroup()
pos <- tmp_tab %>%
  select(position_names) %>%
  apply(., 1, which.max)
```

```
## Note: Using an external vector in selections is ambiguous.
## i Use `all_of(position_names)` instead of `position_names` to silence this message.
## i See <https://tidyselect.r-lib.org/reference/faq-external-vector.html>.
## This message is displayed once per session.
```

```
players <- data_frame(playerID = tmp_tab$playerID, POS = position_names[pos]) %>%
  mutate(POS = str_to_upper(str_remove(POS, "G_"))) %>%
  filter(POS != "P") %>%
  right_join(players, by="playerID") %>%
  filter(!is.na(POS)  & !is.na(salary))
```

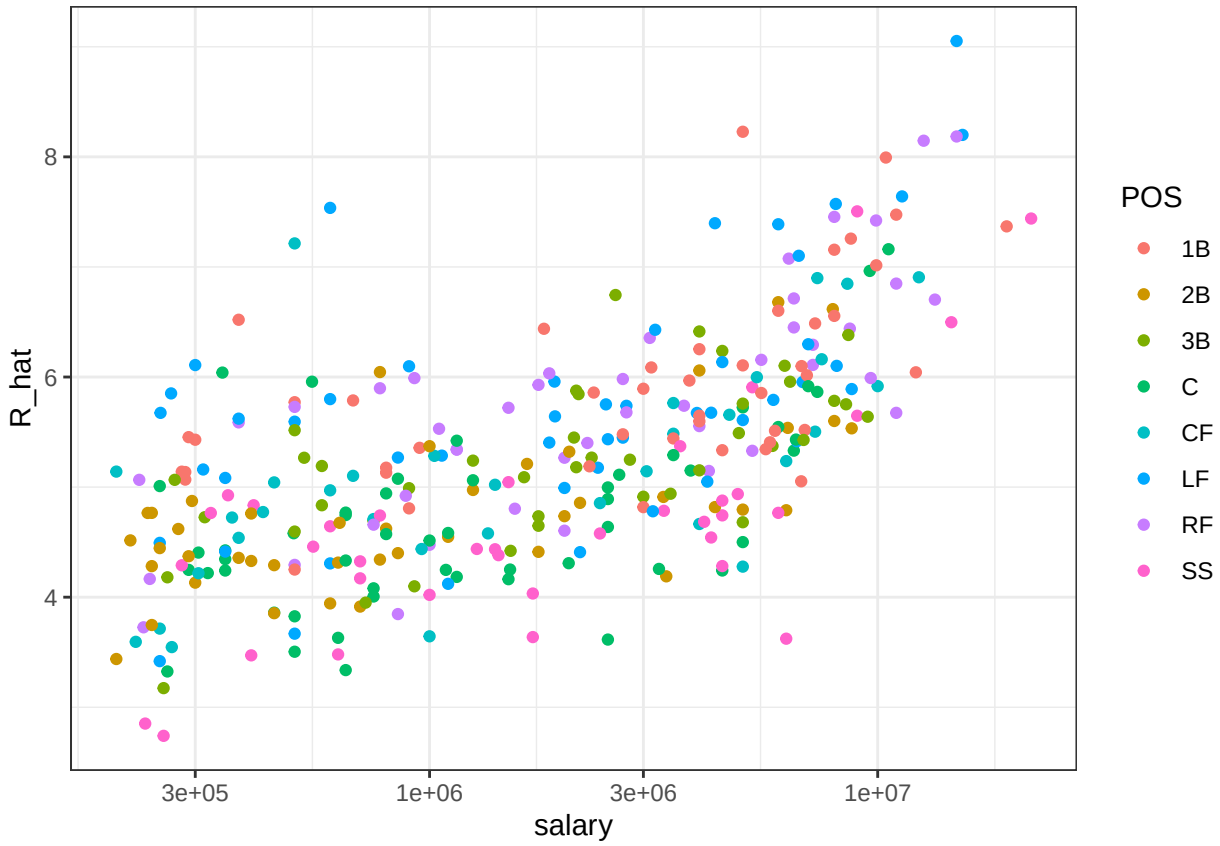
```
## Warning: `data_frame()` is deprecated as of tibble 1.1.0.
## Please use `tibble()` instead.
## This warning is displayed once every 8 hours.
## Call `lifecycle::last_warnings()` to see where this warning was generated.
```

```
# add players' first and last names
players <- Master %>%
  select(playerID, nameFirst, nameLast, debut) %>%
  mutate(debut = as.Date(debut)) %>%
  right_join(players, by="playerID")

# top 10 players
players %>% select(nameFirst, nameLast, POS, salary, R_hat) %>%
  arrange(desc(R_hat)) %>%
  top_n(10)
```

```
## Selecting by R_hat
```

```
##    nameFirst    nameLast POS   salary R_hat
## 1      Barry       Bonds  LF 15000000  9.05
## 2       Todd      Helton  1B  5000000  8.23
## 3      Manny     Ramirez  LF 15462727  8.20
## 4      Sammy        Sosa  RF 15000000  8.19
## 5      Larry      Walker  RF 12666667  8.15
## 6      Jason      Giambi  1B 10428571  7.99
## 7    Chipper       Jones  LF 11333333  7.64
## 8      Brian       Giles  LF  8063003  7.57
## 9     Albert      Pujols  LF   600000  7.54
## 10     Nomar Garciaparra  SS  9000000  7.51
```

```
# players with a higher metric have higher salaries
players %>% ggplot(aes(salary, R_hat, color = POS)) +
  geom_point() +
  scale_x_log10()
```



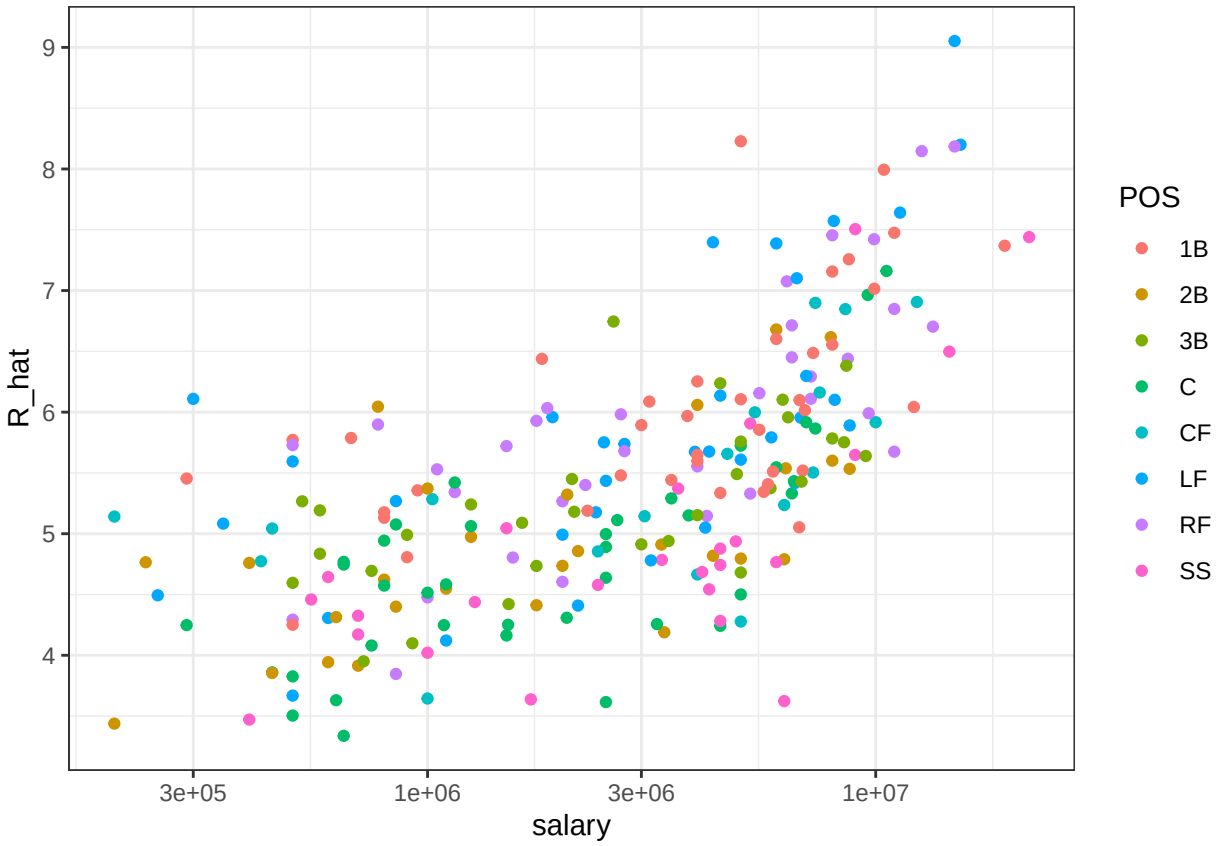

```
# remake plot without players that debuted after 1998
if(!require(lubridate)) install.packages("lubridate")
```

```
## Loading required package: lubridate
```

```
##
## Attaching package: 'lubridate'
```

```
## The following objects are masked from 'package:base':
##
##     date, intersect, setdiff, union
```

```
library(lubridate)
players %>% filter(year(debut) < 1998) %>%
 ggplot(aes(salary, R_hat, color = POS)) +
  geom_point() +
  scale_x_log10()
```



Building a Better Offensive Metric for Baseball: Linear Programming

A way to actually pick the players for the team can be done using what computer scientists call linear programming. Although we don't go into this topic in detail in this course, we include the code anyway:

```
if(!require(reshape2)) install.packages("reshape2")
```

```
## Loading required package: reshape2
```

```
##
## Attaching package: 'reshape2'
```

```
## The following object is masked from 'package:tidyr':
##
##     smiths
```

```
if(!require(lpSolve)) install.packages("lpSolve")
```

```
## Loading required package: lpSolve
```

```
library(reshape2)
library(lpSolve)

players <- players %>% filter(debut <= "1997-01-01" & debut > "1988-01-01")
constraint_matrix <- acast(players, POS ~ playerID, fun.aggregate = length)
```

```
## Using R_hat as value column: use value.var to override.
```

```
npos <- nrow(constraint_matrix)
constraint_matrix <- rbind(constraint_matrix, salary = players$salary)
```

```
constraint_dir <- c(rep("==", npos), "<=")
constraint_limit <- c(rep(1, npos), 50*10^6)
lp_solution <- lp("max", players$R_hat,
                  constraint_matrix, constraint_dir, constraint_limit,
                  all.int = TRUE)
```

This algorithm chooses these 9 players:

```
our_team <- players %>%
  filter(lp_solution$solution == 1) %>%
  arrange(desc(R_hat))
our_team %>% select(nameFirst, nameLast, POS, salary, R_hat)
```

```
##   nameFirst    nameLast POS   salary R_hat
## 1     Larry      Walker  RF 12666667  8.15
## 2     Nomar Garciaparra  SS  9000000  7.51
## 3      Luis    Gonzalez  LF  4333333  7.40
## 4      Mike      Piazza   C 10571429  7.16
## 5       Jim     Edmonds  CF  7333333  6.90
## 6      Phil       Nevin  3B  2600000  6.75
## 7      Greg    Colbrunn  1B  1800000  6.44
## 8     Terry    Shumpert  2B   775000  6.04
```

We note that these players all have above average BB and HR rates while the same is not true for singles.

```
my_scale <- function(x) (x - median(x))/mad(x)
players %>% mutate(BB = my_scale(BB),
                   singles = my_scale(singles),
                   doubles = my_scale(doubles),
                   triples = my_scale(triples),
                   HR = my_scale(HR),
                   AVG = my_scale(AVG),
                   R_hat = my_scale(R_hat)) %>%
    filter(playerID %in% our_team$playerID) %>%
    select(nameFirst, nameLast, BB, singles, doubles, triples, HR, AVG, R_hat) %>%
    arrange(desc(R_hat))
```

```
##   nameFirst    nameLast      BB singles doubles triples     HR   AVG R_hat
## 1     Larry      Walker  1.0605  0.6554   0.922   1.562  1.566 2.835 2.904
## 2     Nomar Garciaparra  0.0274  1.6371   3.118   0.336  0.625 3.197 2.244
## 3      Luis    Gonzalez  0.7046  0.0000   1.413   0.537  1.355 1.829 2.133
## 4      Mike      Piazza  0.3129 -0.0547  -0.242  -1.274  2.035 1.252 1.891
## 5       Jim     Edmonds  1.8074 -1.1409   0.674  -0.674  1.264 0.579 1.621
## 6      Phil       Nevin  0.4909 -0.6479   0.764  -1.098  1.548 0.728 1.463
## 7      Greg    Colbrunn  0.2703  0.6546   0.784   0.585  0.475 1.375 1.148
## 8     Terry    Shumpert -0.1576  0.1221   1.326   3.908 -0.123 0.859 0.744
```

On Base Plus Slugging (OPS)

Key point

The on-base-percentage plus slugging percentage (OPS) metric is:

$$\frac{\text{BB}}{\text{PA}} + \frac{(\text{Singles}+2\text{Doubles}+3\text{Triples}+4\text{HR})}{\text{AB}}$$

Regression Fallacy

The textbook for this section is available here

Key points

- Regression can bring about errors in reasoning, especially when interpreting individual observations.
- The example showed in the video demonstrates that the **"sophomore slump"** observed in the data is caused by regressing to the mean.

Code

The code to create a table with player ID, their names, and their most played position:

```
playerInfo <- Fielding %>%
    group_by(playerID) %>%
    arrange(desc(G)) %>%
    slice(1) %>%
    ungroup %>%
    left_join(Master, by="playerID") %>%
    select(playerID, nameFirst, nameLast, POS)
```

The code to create a table with only the ROY award winners and add their batting statistics:

```
ROY <- AwardsPlayers %>%
    filter(awardID == "Rookie of the Year") %>%
    left_join(playerInfo, by="playerID") %>%
    rename(rookie_year = yearID) %>%
    right_join(Batting, by="playerID") %>%
    mutate(AVG = H/AB) %>%
    filter(POS != "P")
```

The code to keep only the rookie and sophomore seasons and remove players who did not play sophomore seasons:

```
ROY <- ROY %>%
    filter(yearID == rookie_year | yearID == rookie_year+1) %>%
    group_by(playerID) %>%
    mutate(rookie = ifelse(yearID == min(yearID), "rookie", "sophomore")) %>%
    filter(n() == 2) %>%
    ungroup %>%
    select(playerID, rookie_year, rookie, nameFirst, nameLast, AVG)
```

The code to use the spread function to have one column for the rookie and sophomore years batting averages:

```
ROY <- ROY %>% spread(rookie, AVG) %>% arrange(desc(rookie))
ROY
```

```
## # A tibble: 102 x 6
##    playerID  rookie_year nameFirst nameLast rookie sophomore
##    <chr>           <int> <chr>     <chr>     <dbl>     <dbl>
##  1 mccovwi01        1959 Willie    McCovey   0.354     0.238
##  2 suzukic01        2001 Ichiro    Suzuki    0.350     0.321
##  3 bumbral01        1973 Al        Bumbry    0.337     0.233
##  4 lynnfr01         1975 Fred      Lynn      0.331     0.314
##  5 pujolal01        2001 Albert    Pujols    0.329     0.314
##  6 troutmi01        2012 Mike      Trout     0.326     0.323
##  7 braunry02        2007 Ryan      Braun     0.324     0.285
##  8 olivato01        1964 Tony      Oliva     0.323     0.321
##  9 hargrmi01        1974 Mike      Hargrove  0.323     0.303
```

```
## 10 darkal01         1948 Al        Dark      0.322     0.276
## # ... with 92 more rows
```

The code to calculate the proportion of players who have a lower batting average their sophomore year:

```
mean(ROY$sophomore - ROY$rookie <= 0)
```

```
## [1] 0.686
```

The code to do the similar analysis on all players that played the 2013 and 2014 seasons and batted more than 130 times (minimum to win Rookie of the Year):

```
two_years <- Batting %>%
    filter(yearID %in% 2013:2014) %>%
    group_by(playerID, yearID) %>%
    filter(sum(AB) >= 130) %>%
    summarize(AVG = sum(H)/sum(AB)) %>%
    ungroup %>%
    spread(yearID, AVG) %>%
    filter(!is.na(`2013`) & !is.na(`2014`)) %>%
    left_join(playerInfo, by="playerID") %>%
    filter(POS!="P") %>%
    select(-POS) %>%
    arrange(desc(`2013`)) %>%
    select(nameFirst, nameLast, `2013`, `2014`)
```

```
## `summarise()` regrouping output by 'playerID' (override with `.groups` argument)
```

```
two_years
```

```
## # A tibble: 312 x 4
##    nameFirst nameLast `2013` `2014`
##    <chr>     <chr>     <dbl>  <dbl>
##  1 Miguel    Cabrera   0.348  0.313
##  2 Hanley    Ramirez   0.345  0.283
##  3 Michael   Cuddyer   0.331  0.332
##  4 Scooter   Gennett   0.324  0.289
##  5 Joe       Mauer     0.324  0.277
##  6 Mike      Trout     0.323  0.287
##  7 Chris     Johnson   0.321  0.263
##  8 Freddie   Freeman   0.319  0.288
##  9 Yasiel    Puig      0.319  0.296
## 10 Yadier    Molina    0.319  0.282
## # ... with 302 more rows
```

The code to see what happens to the worst performers of 2013:

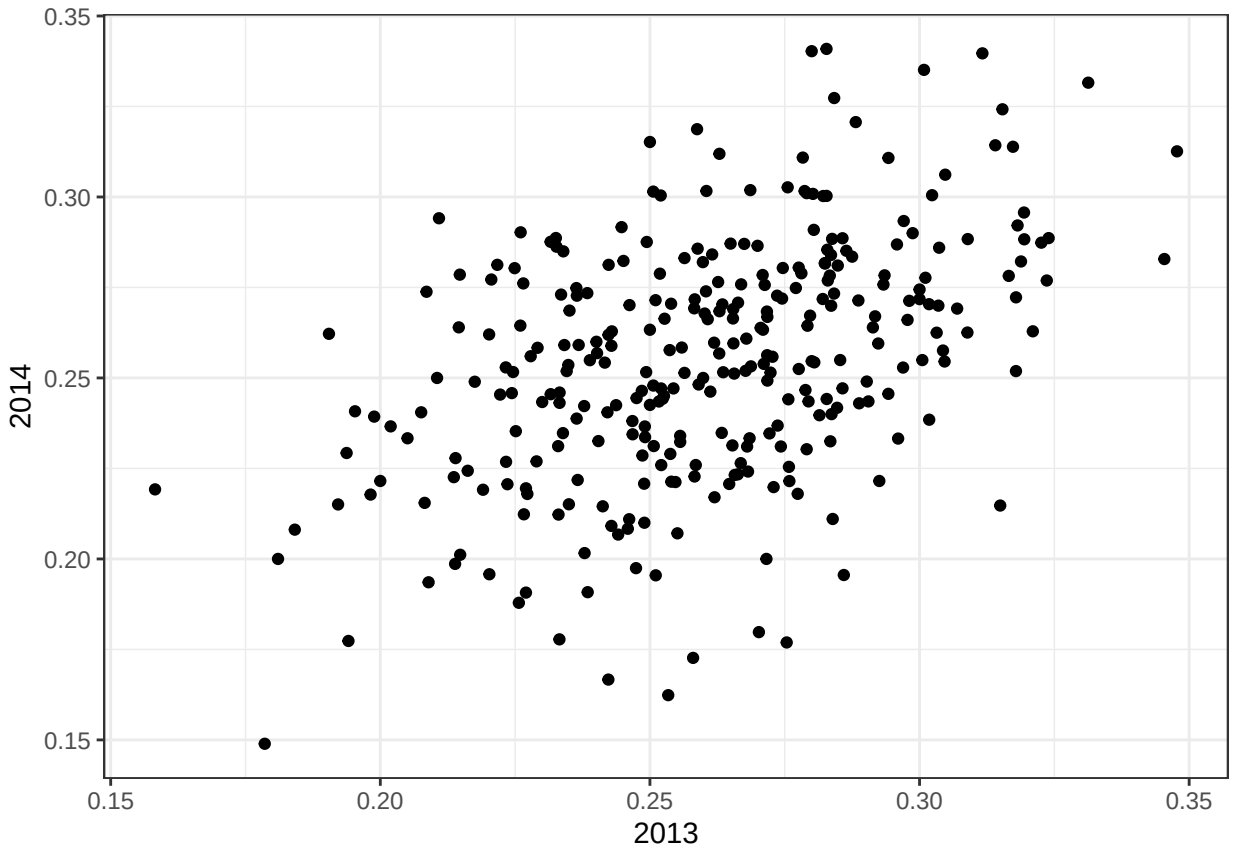
```
arrange(two_years, `2013`)
```

```
## # A tibble: 312 x 4
##    nameFirst nameLast  `2013` `2014`
##    <chr>     <chr>      <dbl>  <dbl>
##  1 Danny     Espinosa   0.158  0.219
##  2 Dan       Uggla      0.179  0.149
##  3 Jeff      Mathis     0.181  0.2
##  4 B. J.     Upton      0.184  0.208
##  5 Adam      Rosales    0.190  0.262
##  6 Aaron     Hicks      0.192  0.215
```

```
##  7 Chris     Colabello  0.194  0.229
##  8 J. P.     Arencibia  0.194  0.177
##  9 Tyler     Flowers    0.195  0.241
## 10 Ryan      Hanigan    0.198  0.218
## # ... with 302 more rows
```

The code to see the correlation for performance in two separate years:

```
qplot(`2013`, `2014`, data = two_years)
```




```
summarize(two_years, cor(`2013`,`2014`))
```

```
## # A tibble: 1 x 1
##   `cor(\`2013\`, \`2014\`)`
##                       <dbl>
## 1                     0.460
```

Measurement Error Models

The textbook for this section is available here

Key points

- Up to now, all our linear regression examples have been applied to two or more random variables. We assume the pairs are bivariate normal and use this to motivate a linear model.
- Another use for linear regression is with **measurement error models**, where it is common to have a non-random covariate (such as time). Randomness is introduced from measurement error rather than sampling or natural variability.

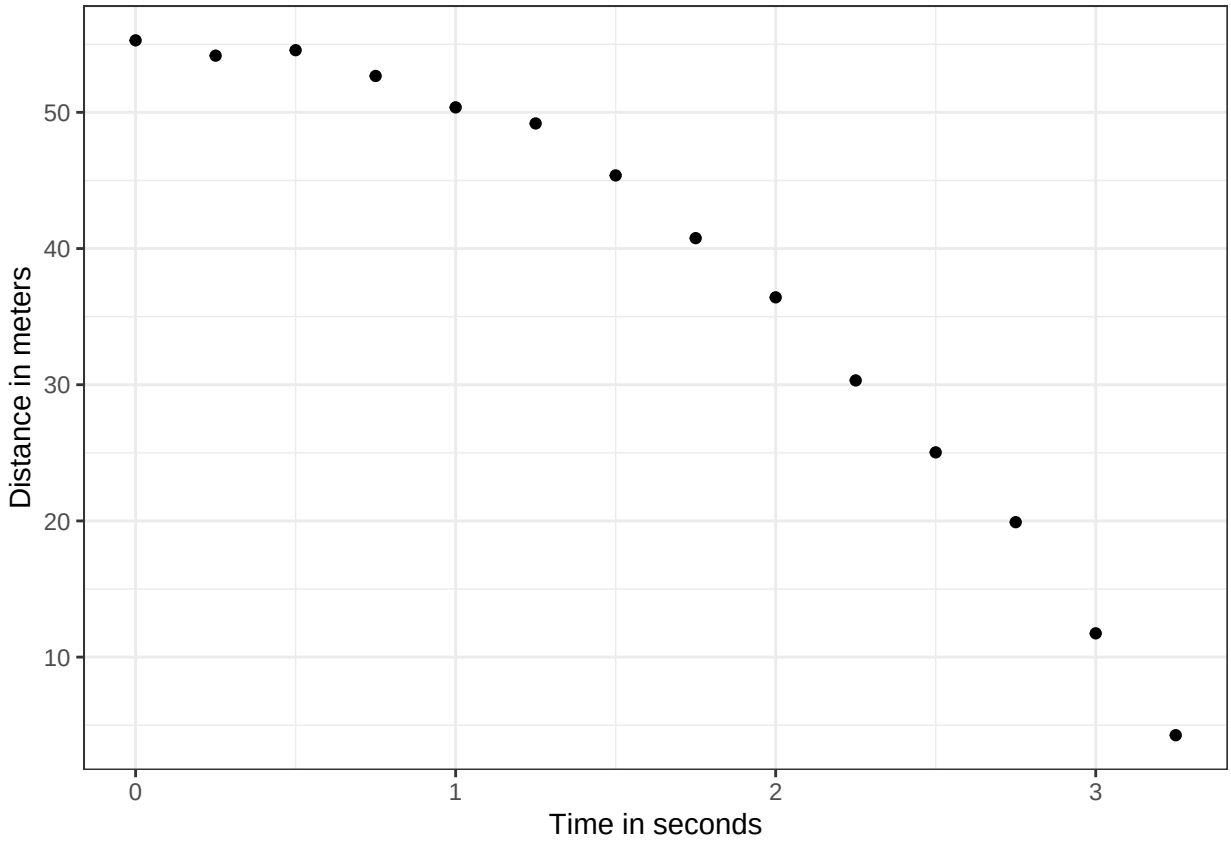
Code

The code to use **dslabs** function **rfalling_object** to generate simulations of dropping balls:

```
falling_object <- rfalling_object()
```

The code to draw the trajectory of the ball:

```
falling_object %>%
    ggplot(aes(time, observed_distance)) +
    geom_point() +
    ylab("Distance in meters") +
    xlab("Time in seconds")
```



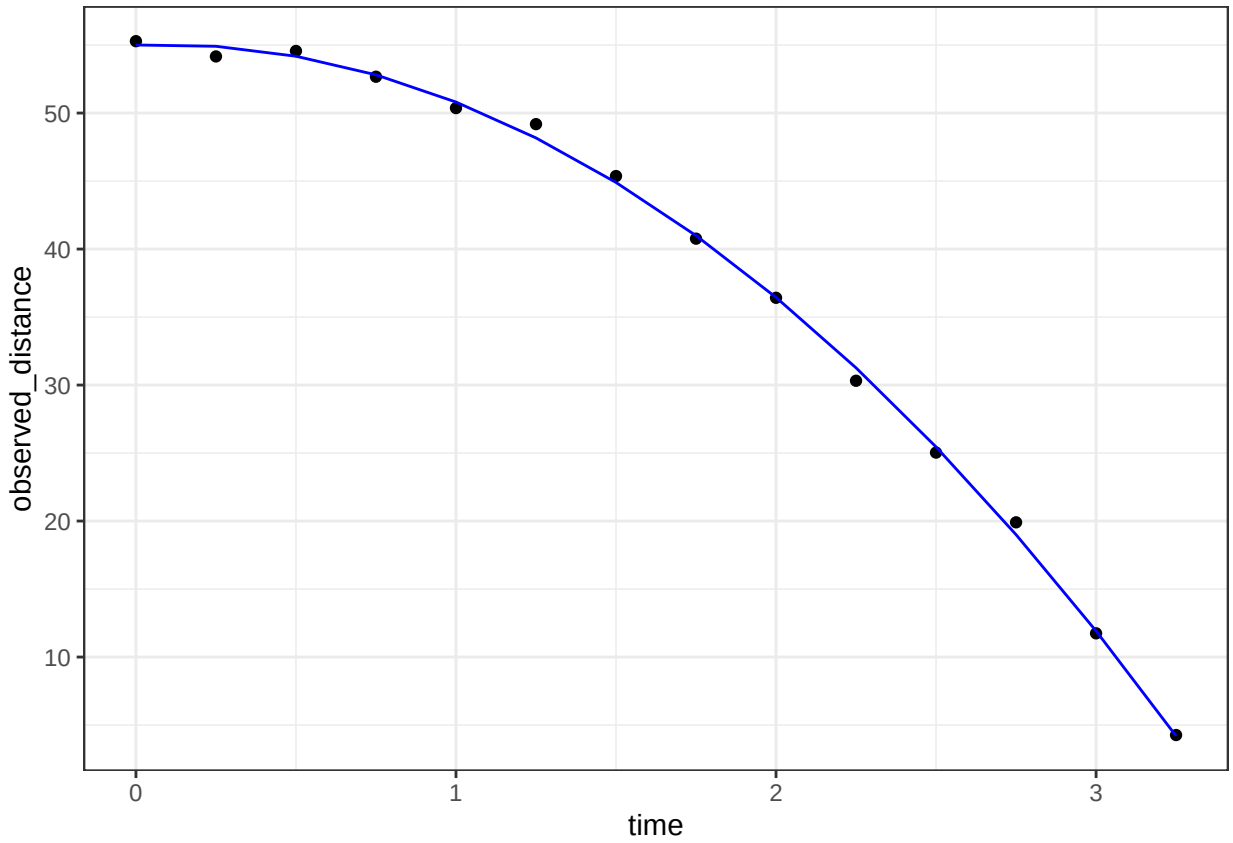
The code to use the `lm()` function to estimate the coefficients:

```
fit <- falling_object %>%
    mutate(time_sq = time^2) %>%
    lm(observed_distance~time+time_sq, data=.)

tidy(fit)
```

```
## # A tibble: 3 x 5
##   term        estimate std.error statistic  p.value
##   <chr>          <dbl>     <dbl>     <dbl>    <dbl>
## 1 (Intercept)   55.0       0.434    127.   9.15e-19
## 2 time           0.888     0.620      1.43 1.80e- 1
## 3 time_sq       -5.08      0.184    -27.7  1.61e-11
```

The code to check if the estimated parabola fits the data:

```
augment(fit) %>%
    ggplot() +
    geom_point(aes(time, observed_distance)) +
    geom_line(aes(time, .fitted), col = "blue")
```



The code to see the summary statistic of the regression:

```
tidy(fit, conf.int = TRUE)
```

```
## # A tibble: 3 x 7
##   term        estimate std.error statistic  p.value conf.low conf.high
##   <chr>          <dbl>     <dbl>     <dbl>    <dbl>    <dbl>     <dbl>
## 1 (Intercept)   55.0       0.434    127.   9.15e-19   54.1       56.0
## 2 time           0.888     0.620      1.43 1.80e- 1   -0.476      2.25
## 3 time_sq       -5.08      0.184    -27.7  1.61e-11   -5.49      -4.68
```

Assessment 12 - Building a Better Offensive Metric for Baseball

1. What is the final linear model we use to predict runs scored per game?

- [ ] A. `lm(R ~ BB + HR)`
- [ ] B. `lm(HR ~ BB + singles + doubles + triples)`
- [X] C. `lm(R ~ BB + singles + doubles + triples + HR)`
- [ ] D. `lm(R ~ singles + doubles + triples + HR)`

2. We want to estimate runs per game scored by individual players, not just by teams. What summary metric do we calculate to help estimate this?

Look at the code from the video for a hint:

```
pa_per_game <- Batting %>%
  filter(yearID == 2002) %>%
  group_by(teamID) %>%
  summarize(pa_per_game = sum(AB+BB)/max(G)) %>%
  .$pa_per_game %>%
  mean
```

- [ ] A. `pa_per_game`: the mean number of plate appearances per team per game for each team
- [ ] B. `pa_per_game`: the mean number of plate appearances per game for each player
- [X] C. `pa_per_game`: the number of plate appearances per team per game, averaged across all teams

3. Imagine you have two teams. Team A is comprised of batters who, on average, get two bases on balls, four singles, one double, and one home run. Team B is comprised of batters who, on average, get one base on balls, six singles, two doubles, and one triple.

Which team scores more runs, as predicted by our model?

- [ ] A. Team A
- [X] B. Team B
- [ ] C. Tie
- [ ] D. Impossible to know

Assessment 13 - On Base Plus Slugging (OPS)

1. The on-base-percentage plus slugging percentage (OPS) metric gives the most weight to:

- [ ] A. Singles
- [ ] B. Doubles
- [ ] C. Triples
- [X] D. Home Runs

Assessment 14 - Regression Fallacy

1. What statistical concept properly explains the "sophomore slump"?

- [X] A. Regression to the mean
- [ ] B. Law of averages
- [ ] C. Normal distribution

Assessment 15 - Measurement Error Models

1. In our model of time vs. observed_distance, the randomness of our data was due to:

- [ ] A. sampling
- [ ] B. natural variability
- [X] C. measurement error

2. Which of the following are important assumptions about the measurement errors in this experiment?

- [X] A. The measurement error is random
- [X] B. The measurement error is independent
- [X] C. The measurement error has the same distribution for each time i

3. Which of the following scenarios would violate an assumption of our measurement error model?

- [ ] A. The experiment was conducted on the moon.
- [X] B. There was one position where it was particularly difficult to see the dropped ball.
- [ ] C. The experiment was only repeated 10 times, not 100 times.

Section 3 - Confounding Overview

In the Confounding section, you will learn what is perhaps the most important lesson of statistics: that correlation is not causation.

After completing this section, you will be able to:

- Identify examples of spurious correlation and explain how data dredging can lead to spurious correlation.
- Explain how outliers can drive correlation and learn to adjust for outliers using Spearman correlation.
- Explain how reversing cause and effect can lead to associations being confused with causation.
- Understand how confounders can lead to the misinterpretation of associations.
- Explain and give examples of Simpson's Paradox.

This section has one part: Correlation is Not Causation.

The textbook for this section is available here

Assessment 1 - Correlation is Not Causation: Spurious Correlation

1. In the video, we ran one million tests of correlation for two random variables, X and Y.

How many of these correlations would you expect to have a significant p-value ($p>0.05$), just by chance?

- [ ] A. 5,000
- [X] B. 50,000
- [ ] C. 100,000
- [ ] D. It's impossible to know

2. Which of the following are examples of p-hacking?

- [X] A. Looking for associations between an outcome and several exposures and only reporting the one that is significant.
- [X] B. Trying several different models and selecting the one that yields the smallest p-value.
- [X] C. Repeating an experiment multiple times and only reporting the one with the smallest p-value.
- [ ] D. Using a Monte Carlo simulations in an analysis.

Assessment 2 - Correlation is Not Causation: Outliers

1. The Spearman correlation coefficient is robust to outliers because:

- [ ] A. It drops outliers before calculating correlation.
- [ ] B. It is the correlation of standardized values.
- [X] C. It calculates correlation between ranks, not values.

Assessment 3 - Correlation is Not Causation: Reversing Cause and Effect

1. Which of the following may be examples of reversed cause and effect?

- [X] A. Past smokers who have quit smoking may be more likely to die from lung cancer.
- [ ] B. Tall fathers are more likely to have tall sons.
- [X] C. People with high blood pressure tend to have a healthier diet.
- [X] D. Individuals in a low social status have a higher risk of schizophrenia.

Assessment 4 - Correlation is Not Causation: Confounders

1. What can you do to determine if you are misinterpreting results because of a confounder?

- [ ] A. Nothing, if the p-value says the result is significant, then it is.
- [X] B. More closely examine the results by stratifying and plotting the data.

- [ ] C. Always assume that you are misinterpreting the results.
- [ ] D. Use linear models to tease out a confounder.

2. Look again at the admissions data using ?admissions. What important characteristic of the table variables do you need to know to understand the calculations used in this video? Select the best answer.

- [ ] A. The data is from 1973.
- [ ] B. The columns "major" and "gender" are of class character, while "admitted" and "applicants" are numeric.
- [ ] C. The data is from the "dslabs" package.
- [X] D. The column "admitted" is the percent of student admitted, while the column "applicants" is the total number of applicants.

3. In the example in the video, major selectivity confounds the relationship between UC Berkley admission rates and gender because:

- [ ] A. It was harder for women to be admitted to UC Berkeley.
- [X] B. Major selectivity is associated with both admission rates and with gender, as women tended to apply to more selective majors.
- [ ] C. Some majors are more selective than others
- [ ] D. Major selectivity is not a confounder.

Assessment 5 - Simpson's Paradox

1. Admission rates at UC Berkeley are an example of Simpson's Paradox because:

- [X] A. It appears that men have higher a higher admission rate than women, however, after we stratify by major, we see that on average women have a higher admission rate than men.
- [ ] B. It was a paradox that women were being admitted at a lower rate than men.
- [ ] C. The relationship between admissions and gender is confounded by major selectivity.