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    - other utility/cost functions possible
  - Calibration
    - α%-posterior interval has the true value in α% cases
    - α%-predictive interval has the true future values in α% cases
    - approximate calibration with shorter intervals for likely true values more important than exact calibration with bad intervals for all possible values.

Frequentist statistics

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  - unbiased estimate for strictly positive parameter can be negative
- Confidence interval is defined to have true value inside the interval in α% cases of repeated data generation from the data generating mechanism
  - doesn't say how likely the true value is inside the interval given the observed data
  - doesn't need be useful to have perfect calibration

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- Lot of machine learning is not pure frequentist or Bayesian

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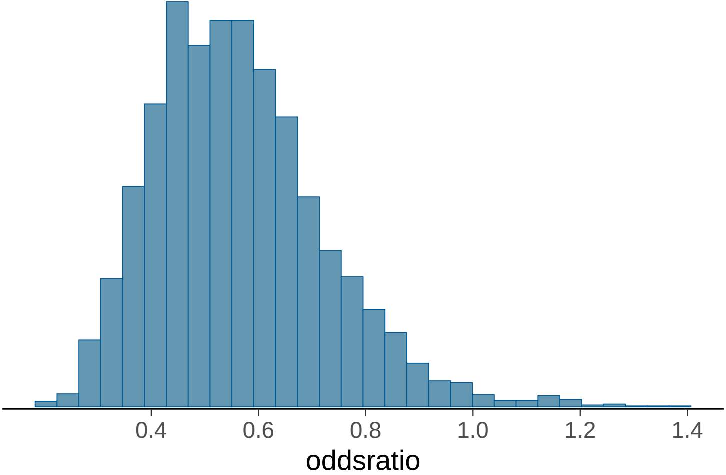
    - reporting just the null hypothesis testing result throws away lot of useful information
    - some Bayesians are also into null hypothesis testing
- Frequentist null hypothesis testing
    - asks what if data is generated from the smaller model
    - doesn't tell whether the more complex model is good enough

Bayesian hypothesis testing

- Instead of hypothesis testing, report full posterior and
  - compare to expert information
  - combine with utility/cost function



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- Instead of hypothesis testing, report full posterior
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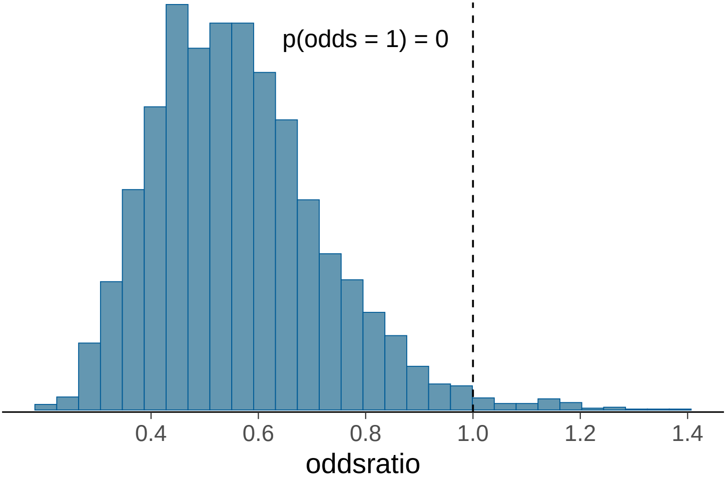
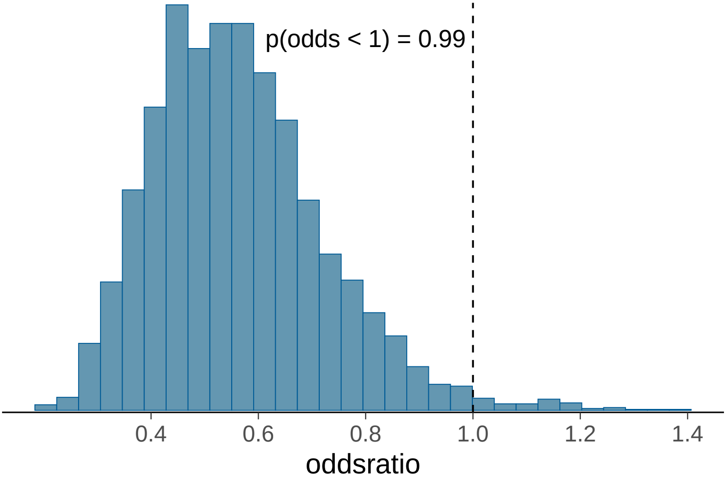


Bayesian hypothesis testing

- Instead of hypothesis testing, report full posterior
  - for continuous posterior we could compute the probability that we know the sign of the effect



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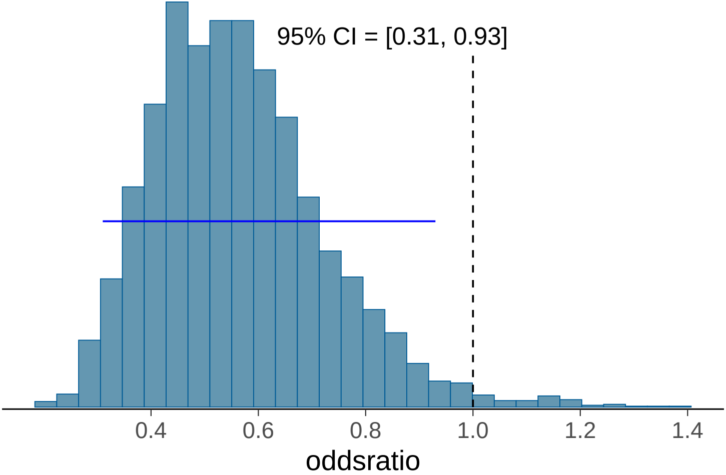
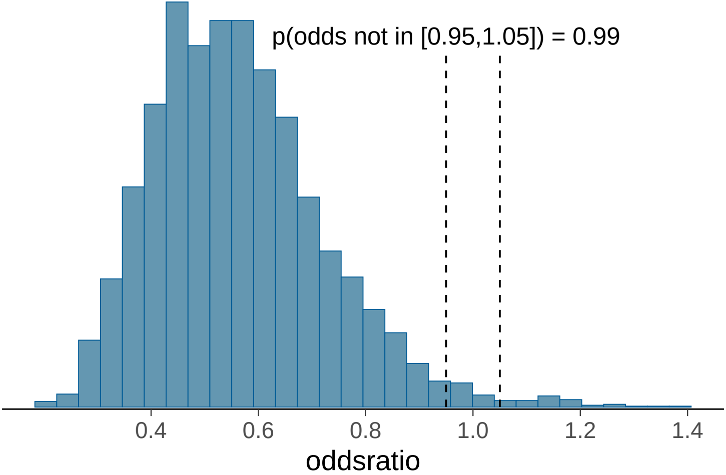
- Instead of hypothesis testing, report full posterior
  - for continuous posterior some people compare whether posterior interval includes null case



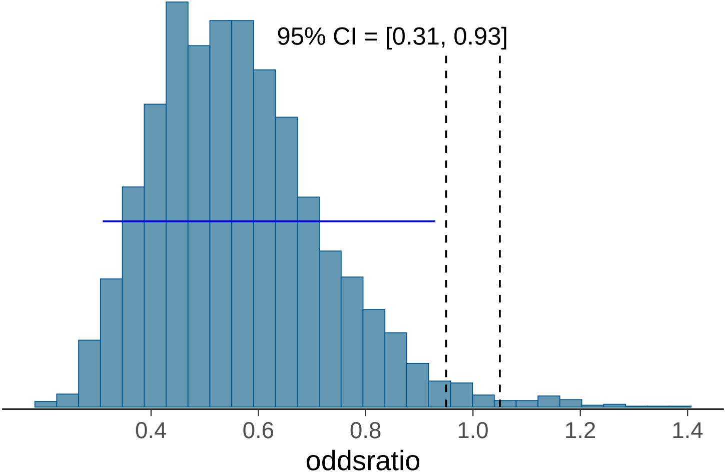
Bayesian hypothesis testing

- Equivalence testing (region of practical equivalence)
  - what is the probability that the effect is closer than ϵ to null, where ϵ is based on what is practically useful effect size



Bayesian hypothesis testing

- Equivalence testing (region of practical equivalence)
  - some people combine posterior interval and region of practical equivalence



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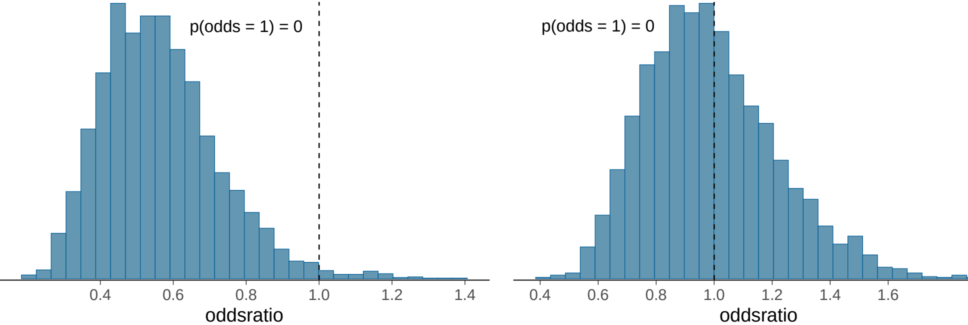

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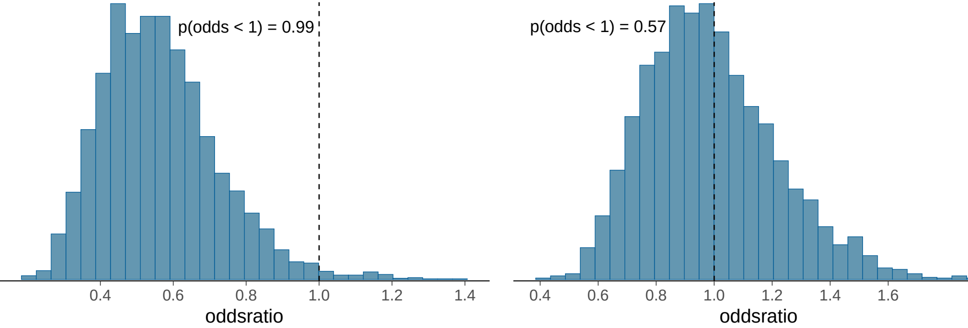

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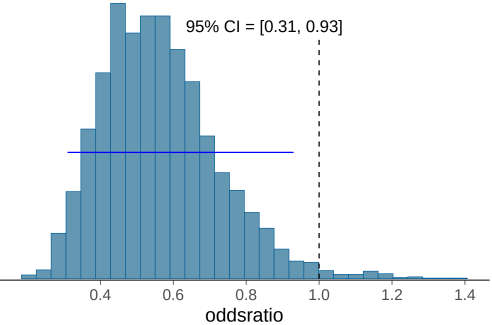

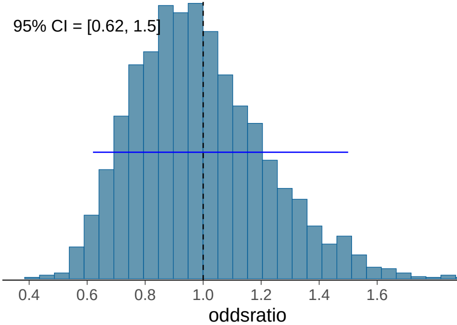

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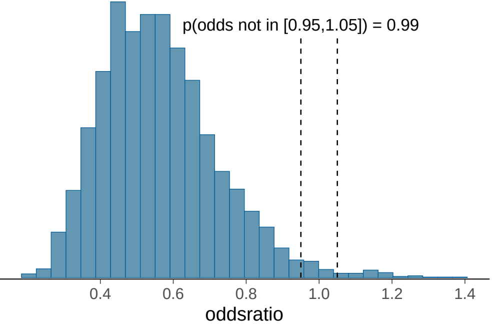

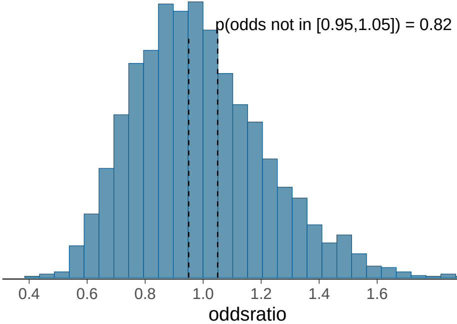

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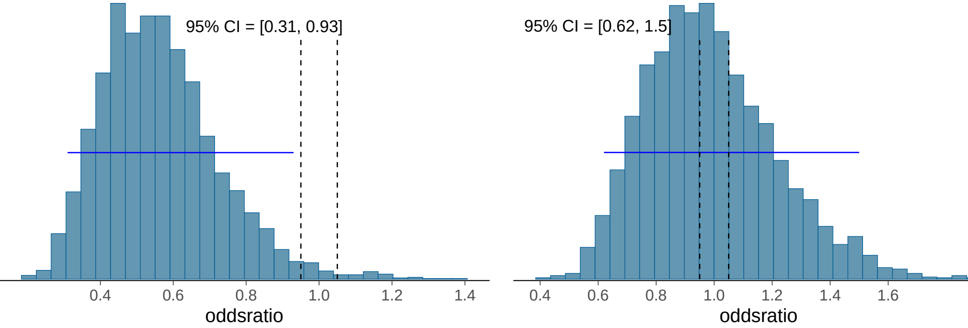

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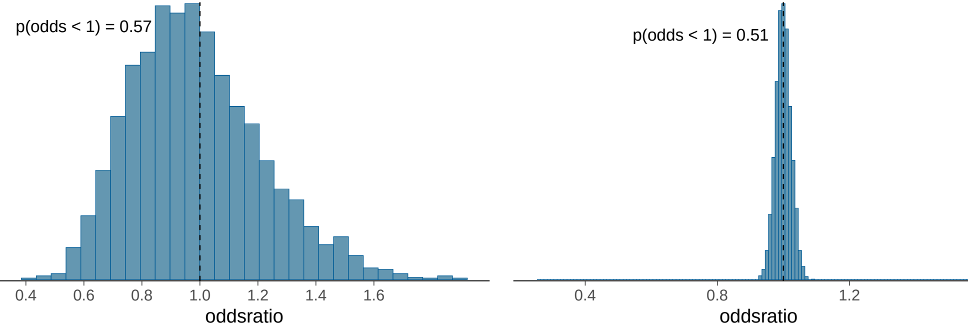

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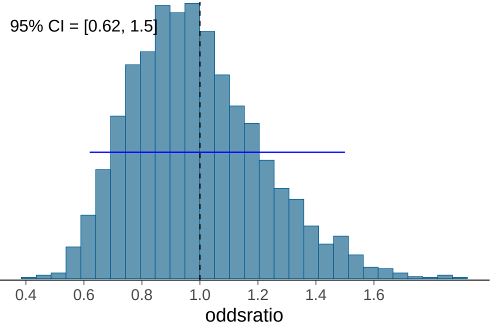

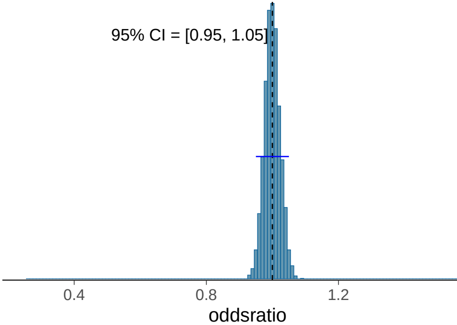

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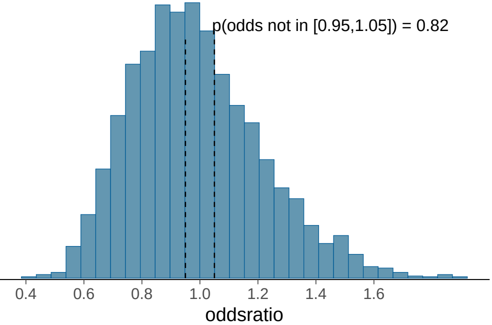

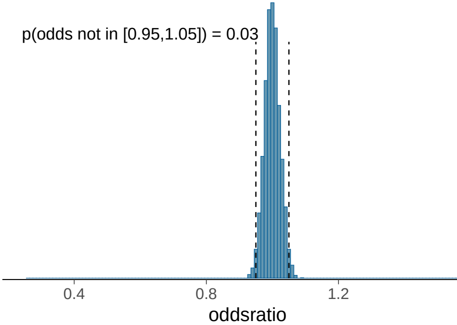

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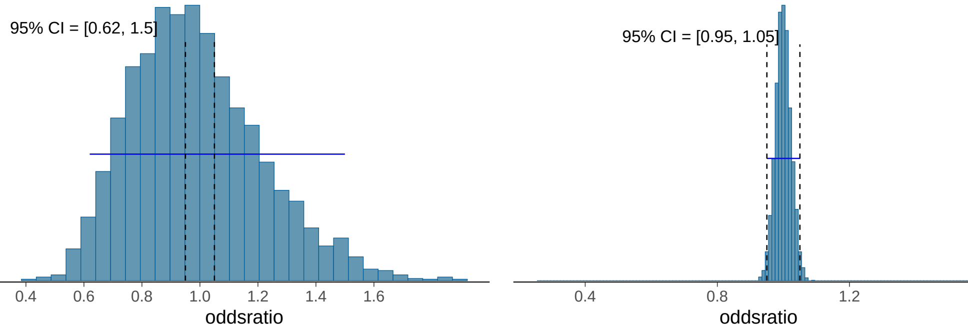
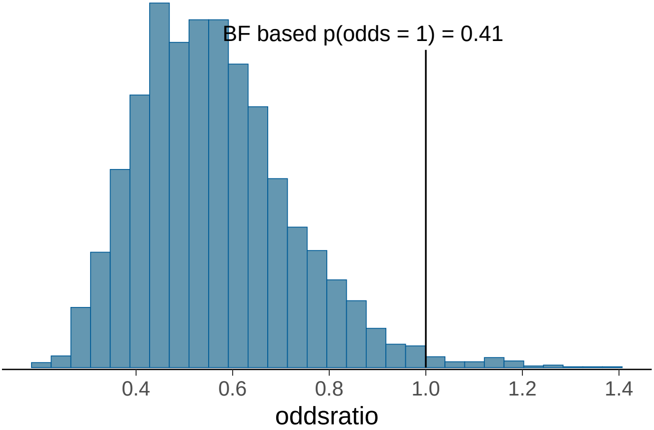


Bayesian hypothesis testing

- Bayes factor
  - null model has, e.g., the treatment effect fixed to 0
  - assumes that there is non-zero probability that the treatment effect can be exactly zero
  - requires posterior inference for the null model, too



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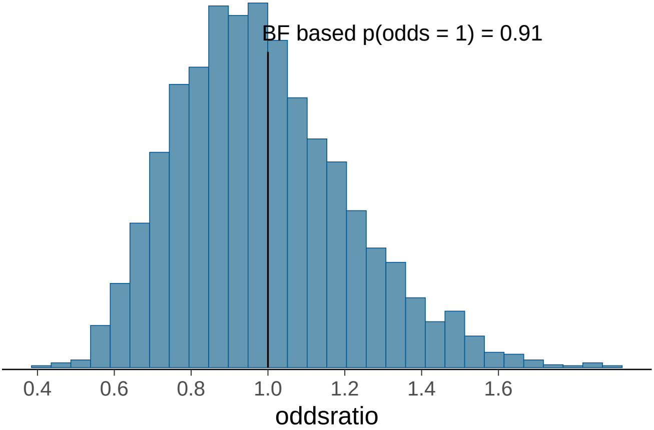

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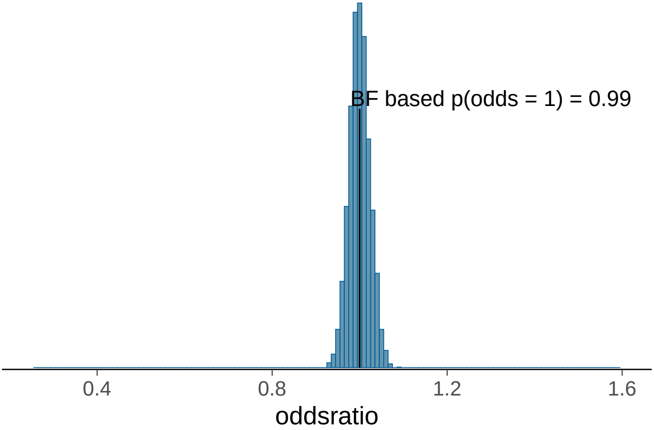

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Bayesian hypothesis testing

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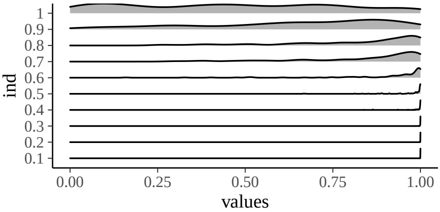
In the beta blockers example

- Leave-one-group-out is not sensible as there are only two groups
- Leave-one-person-out works, but is less efficient than looking at the posterior (see https://avehtari.github.io/modelselection/betablockers.html)

Simulation experiment

p(odds < 1)



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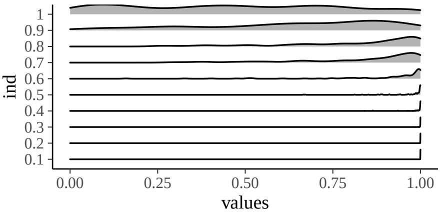
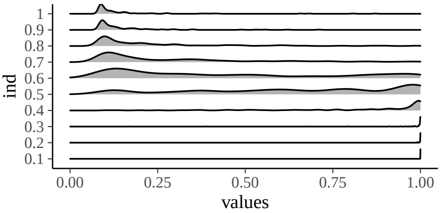


Marginal likelihood comparison



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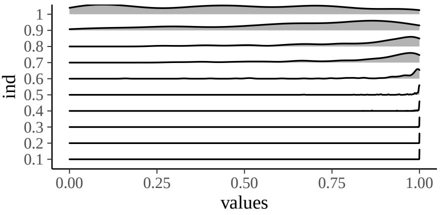

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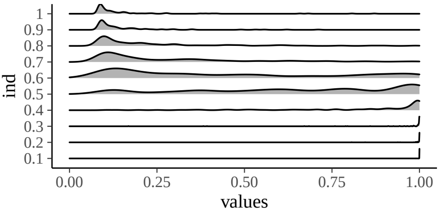
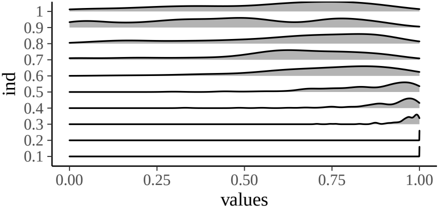


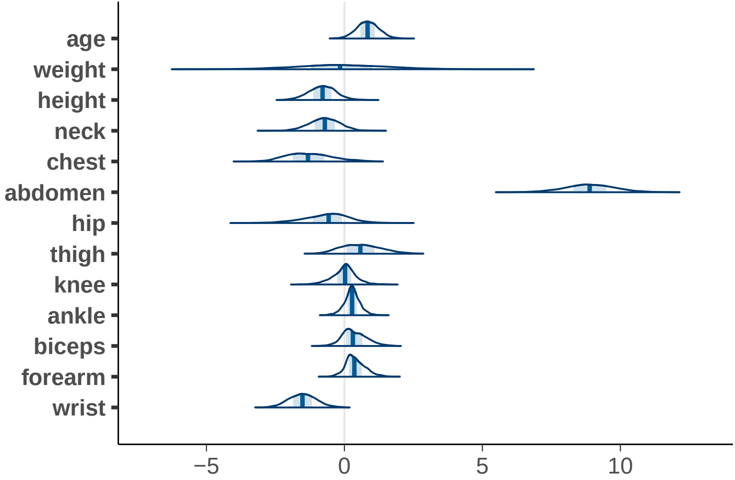
LOO comparison



Hypothesis testing and posterior dependencies

Looking at the marginal posterior(s) can be misleading when there are many parameters

Marginal posteriors of coefficients



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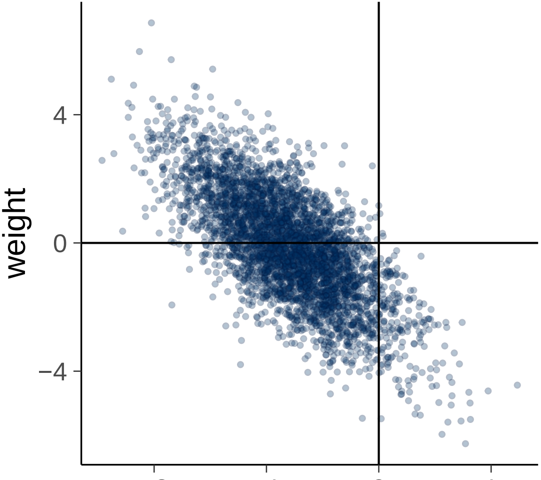
Bivariate marginal of weight and height



Hypothesis testing and posterior dependencies

In bodyfat example, starting from full model

- BF in favor of removing weight (p=0.92)
- LOO in favor of removing weight (p=0.99)

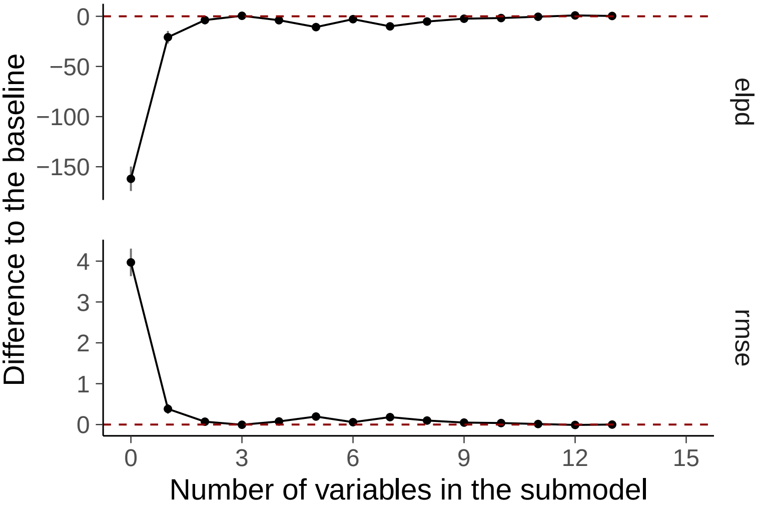
In bodyfat example, starting from model y $\sim$ abdomen

- BF in favor of adding weight (p=1.0)
- LOO in favor of adding weight (p=1.0)

Variable selection

More elaborate approaches are needed for variable selection

See Lecture 9.3 on projection predictive variable selection



Common statistical tests as Bayesian models

Most common statistical tests are linear models

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paired *t*-test	mean of diffs	`stan_glm((y1 - y2) ~ 1)`
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See longer list and illustrations (with `lm`) at
https://lindeloev.github.io/tests-as-linear/
and
in the forthcoming *Regression and other stories* book

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  - Censoring and truncation

Chapter 14: Introduction to regression models

- Justification of conditional modeling
  - if joint model factorizes $p(y, x|\theta, \phi) = p(y|x, \theta)p(x|\phi)$
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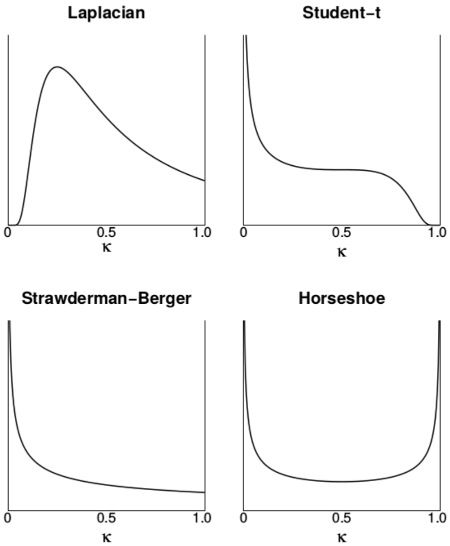
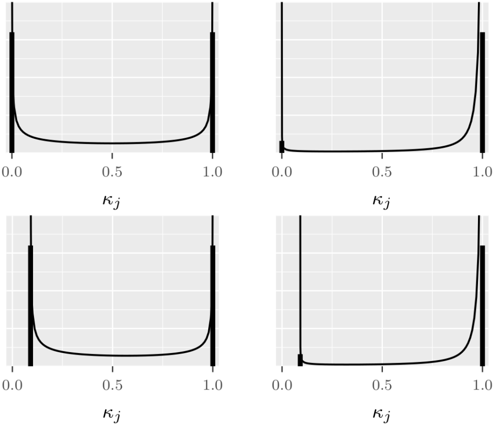
  - empirically better results obtained with more sparse priors
  - it's best to separate selection of sensible prior, good posterior inference, and the decision analysis of which variables are important

Sparse priors



from Carvalho, Polson, Scott (2009).

Regularized horseshoe



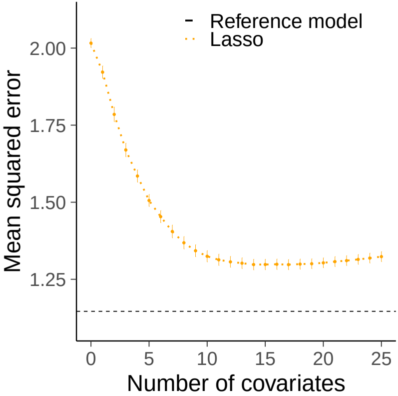
for more see

- Piironen and Vehtari (2017). Sparsity information and regularization in the horseshoe and other shrinkage priors. In Electronic Journal of Statistics, 11(2):5018-5051. Online
- https://betanalpha.github.io/assets/case_studies/bayes_sparse_regression.html

Projpred selection vs. Lasso

See projpred in lecture 9.3

Same simulated regression data as in lecture 9,3,
$n = 50$, $p = 500$, $p_{\text{rel}} = 150$, $\rho = 0.5$



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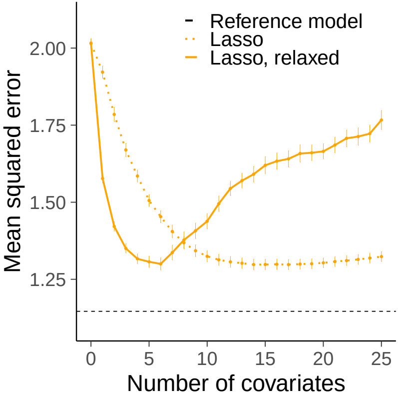

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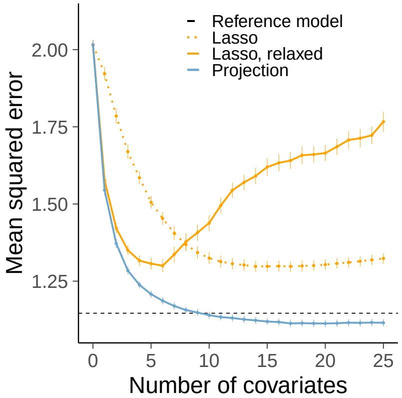

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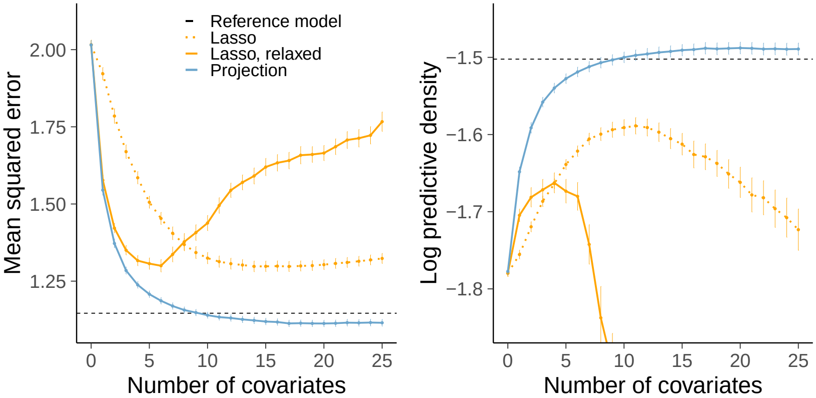

Chapter 15: Hierarchical linear models

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- With probabilistic programming computation is also easy
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`y ~ 1 + x`	fixed / population effect; pooled model
`y ~ 1 + (0 + x | g)`	random / group effects
`y ~ 1 + x + (1 + x | g)`	mixed effects; hierarchical model

- ANOVA in section 15.6 (see also `stan_aov`)

Chapter 16: Generalized linear models

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  - Hierarchical GLM natural extension
  - 16.3 Weakly informative priors section is excellent although the recommendation on using Cauchy has changed (see https://github.com/stan-dev/stan/wiki/Prior-Choice-Recommendations)

Chapter 17: Models for robust inference

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normal	$\rightarrow$	t-distribution
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  - rstanarm doesn't have t-distribution for outcome, but brms has

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- Extends the data collection modelling from Chapter 8
- Useful terms

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  - Missing not at random (MNAR)
    missingness depends on missing values
- Multiple imputation
  1. make a model predicting missing data
  2. sample repeatedly from the missing data model to generate multiple imputed data sets
  3. make usual inference for each imputed data set
  4. combine results

Chapter 21: Gaussian process models

- Gaussian process is
  - infinite dimensional extension of normal distribution
  - useful prior for non-linear functions
  - for any finite number of variables, the marginal is multivariate normal $f_1, \ldots, f_n \sim \mathrm{N}\left(\mu(x_1, \ldots, x_n), K(x_1, \ldots, x_n)\right)$

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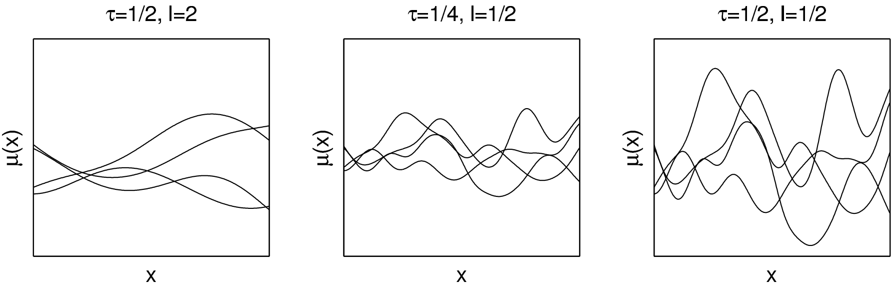

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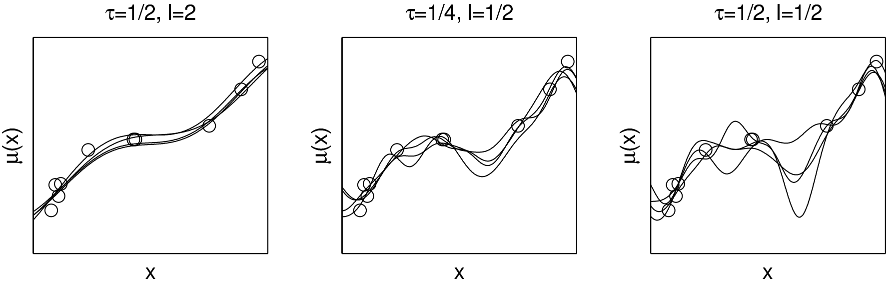
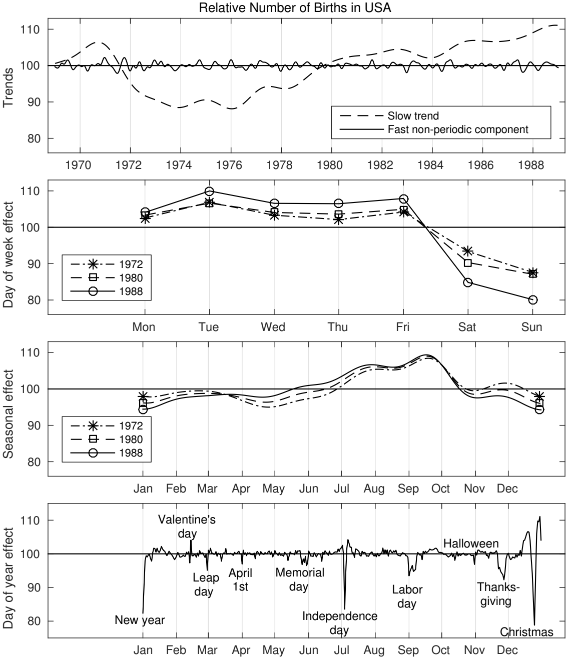


Chapter 21: Gaussian process models

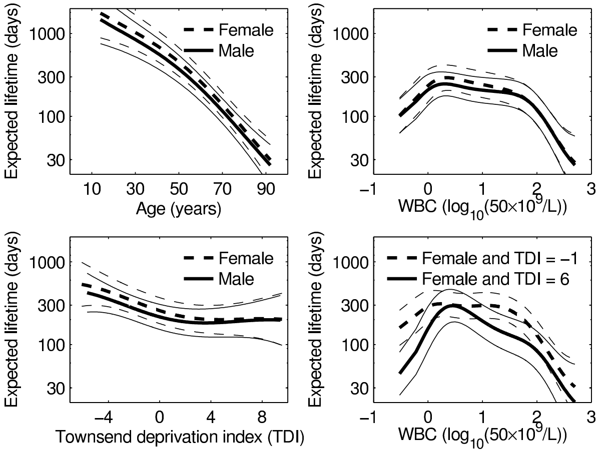
- Conditional on covariance function parameter the posterior is just multivariate normal
  - need to make inference for covariance function parameters given the marginal likelihood
  - the exact computation of the marginal likelihood scales $O(N^3)$

- Easy to make additive models
  $y_t(t) = f_1(t) + f_2(t) + f_3(t) + f_4(t) + f_5(t) + \epsilon_t$



Chapter 21: Gaussian process models

- For non-Gaussian outcome models similar extension as GLMs
- Survival model example:



GPs in Stan

- GP specific software (e.g. GPy, GPflow, GPyTorch) scale computationally better for GPs than Stan
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- In case of non-Gaussian outcome models, sampling of latent variables can be slow (Laplace integration over the latents coming)
- Instead of covariance matrix based approach, for low dimensional cases faster to use basis function representation
  - e.g. `stan_glm(y ~ s(x, bs="gp"))`