

Bayesian Data Analysis - Assignment 4

General information

- The recommended tool in this course is R (with the IDE R-Studio). You can download R [here](#) and R-Studio [here](#). There are tons of tutorials, videos and introductions to R and R-Studio online. You can find some initial hints [here](#).
- You can write the report with your preferred software, but the outline of the report should follow the instruction in the R markdown template that can be found [here](#).
- Report all results in a single, **anonymous** *.pdf -file and return it to [peer-grade.io](#).
- Many of the exercises can be checked using the R package `markmyassignment`. Information on how to install and use the package can be found [here](#).
- The course has its own R package with data and functionality to simplify coding. To install the package just run the following:

```
1. install.packages("remotes")
2. remotes::install_github("avehtari/BDA_course_Aalto",
  subdir = "rpackage")
```

- Many of the exercises can be checked automatically using the R package `markmyassignment`. Information on how to install and use the package can be found [here](#).
- Additional self study exercises and solutions for each chapter in BDA3 can be found [here](#).
- We collect common questions regarding installation and technical problems in a course Frequently Asked Questions (FAQ). This can be found [here](#).
- If you have any suggestions or improvements to the course material, please feel free to create an issue or submit a pull request to the public repository!!

Information on this assignment

This exercise is related to Chapters 3 and 10. The maximum amount of points from this assignment is 6.

Reading instructions: Chapters 3 and 10 in BDA3, see reading instructions [here](#) and [here](#)

Grading instructions: The grading will be done in peergrade. All grading questions and evaluations for exercise 4 can be found [here](#)

Reporting accuracy: As many significant digits as justified by the Monte Carlo error and posterior accuracy.

To use markmyassignment for this assignment, run the following code in R:

```
> library(markmyassignment)
> exercise_path <-
+   "https://github.com/avehtari/BDA_course_Aalto/blob/master/exercises/tests/ex4.yml"
> set_assignment(exercise_path)
> # To check your code/functions, just run
> mark_my_assignment()
```

1. (**Bioassay model and importance sampling**). In this exercise, you will use a dose-response relation model that is used in Section 3.7 of the course book and in the chapter reading instructions [here](#). The used likelihood is the same, but we will use a different prior distribution on the parameters α and β .

- a) Construct a bivariate normal distribution as prior distribution for (α, β) . The marginal distributions are $\alpha \sim N(0, 2^2)$, $\beta \sim N(10, 10^2)$ with correlation $\text{corr}(\alpha, \beta) = 0.5$. Report the mean and covariance of the bivariate normal distribution.

Hint! The mean and covariance of the bivariate normal distribution are a length-2 vector and a 2×2 matrix. The elements of the covariance matrix can be computed using the relation of correlation and covariance.

- b) Implement a function in R for computing the **logarithm** of the density of the prior distribution in a) for arbitrary values of α and β . Below is an example of how the function should be named and work if you want to check them with `markmyassignment`.

```
> alpha <- 3
> beta <- 9
> p_log_prior(alpha, beta)
[1] -6.296435
```

Hint! Use function `dmvnorm` from the `aaltobda` package. We use logarithms for better numerical accuracy in later questions.

- c) Implement a function in R for computing the **logarithm** of the density of the posterior for arbitrary values of α and β and data x , y and n . Below is an example of how the function should be named and work if you want to check them with `markmyassignment`.

```
> library(aaltobda)
> data("bioassay")
> alpha <- 3
> beta <- 9
> p_log_posterior(alpha, beta, x = bioassay$x, y = bioassay$y, n = bioassay$n)
[1] -15.78798
```

Hint! Equation (3.16) in the course book. The **logarithm** of the prior density was already implemented in b). For computing the **logarithm** of the likelihood, use the `bioassaylp` function from the `aaltobda` package. The data can be loaded with the R command `data("bioassay")`.

Hint! Logarithm of the product of two densities is the sum of the log-densities, i.e. $\log ab = \log a + \log b$.

- d) Plot the posterior density in a grid of points ($\alpha \in [-4, 4]$, $\beta \in [-10, 30]$) using the `bioassay_posterior_density_plot` function from the `aaltobda` package. Pass the `p_log_posterior` function you implemented in c) as argument to `bioassay_posterior_density_plot`.
- e) Implement two functions in: 1) A function for computing the log importance ratios (log importance weights) for draws from the prior distribution in 1a) when the target distribution is the posterior distribution. 2) A function for

exponentiating the log importance ratios and normalizing them to sum to one. Below is a test example, the functions can also be tested with `mark-myassignment`. **Note!** The values below are *only* a test case, you need to use correct draws for `alpha` and `beta` in the final report.

```
> alpha <- c(1.896, -3.6, 0.374, 0.964, -3.123, -1.581)
> beta <- c(24.76, 20.04, 6.15, 18.65, 8.16, 17.4)
> log_importance_weights(alpha, beta)
[1] -8.95 -23.47 -6.02 -8.13 -16.61 -14.57
> normalized_importance_weights(alpha, beta)
[1] 0.045 0.000 0.852 0.103 0.000 0.000
```

Hint! Equation (10.3) in the course book.

- f) Sample draws of α and β from the prior distribution from 1a). Implement a function for computing the posterior mean using importance sampling, and report the obtained mean.

Hint! Use the function `rmvnorm` from the `aaltobda` package. Below is an example how the function would work with the example values for `alpha` and `beta` above. **Note!** The values below are *only* a test case, you need to use correct draws for `alpha` and `beta` in the final report.

```
> alpha
[1] 1.896 -3.600 0.374 0.964 -3.123 -1.581
> beta
[1] 24.76 20.04 6.15 18.65 8.16 17.40
> posterior_mean(alpha, beta)
[1] 0.503 8.275
```

- g) Using the importance ratios, compute the effective sample size S_{eff} and report it. If S_{eff} is less than 1000, repeat f) with more draws. **Note!** The values below are *only* a test case, you need to use correct draws for `alpha` and `beta` in the final report.

```
> S_eff(alpha, beta)
[1] 1.354
```

Hint! Equation (10.4) in the course book.

- **Note!** *BDA3 1st (2013) and 2nd (2014) printing have an error for $\tilde{w}(\theta^s)$ used in the effective sample size equation (10.4). The normalized weights equation should not have the multiplier S (the normalized weights should sum to one). Errata for the book http://www.stat.columbia.edu/~gelman/book/errata_bda3.txt. The later printings and slides have the correct equation.*

- h) Use importance resampling without replacement to obtain a posterior sample of size 1000 of α and β and plot a scatterplot of the obtained posterior sample.
- i) Using the posterior sample obtained via importance resampling, report an estimate for $p(\beta > 0 | x, n, y)$, that is, the probability that the drug is harmful.

- j) Using the posterior sample obtained via importance resampling, draw a histogram of the draws from the posterior distribution of the LD50 conditional on $\beta > 0$.

Hint! See Figure 3.4 and corresponding section in the course book. You can plot a basic histogram with R using the library `ggplot2` and the command `ggplot() + geom_histogram(aes(ld50))`