Chapter 9

- 9.1 Context and basic steps (most important part)
- 9.2 Example
- 9.3 Multistage decision analysis (example)
- 9.4 Hierarchical decision analysis (example)
- 9.5 Personal vs. institutional decision analysis

Bayesian decision theory

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    - or actions a

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- Expected utility $E[U(x)|d] = \int U(x)p(x|d)dx$
- Choose decision d^*, which maximizes the expected utility

$$d^* = \arg\max_d E[U(x)|d]$$

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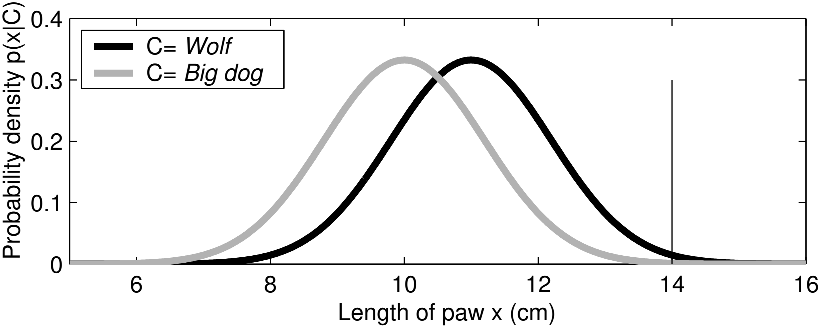
- Helen measures that the length of the paw print is 14 cm and goes home to Google how big paws dogs and wolves have, and tries then to infer which animal has made the paw print



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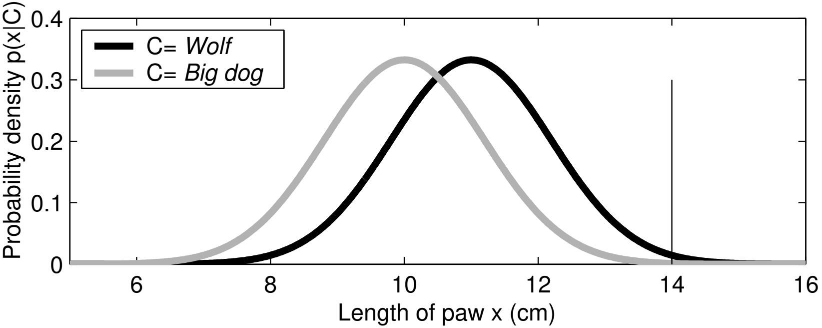

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- Likelihood of wolf is 0.92 (alternative being dog)

Example of decision making

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- Posterior probability of wolf is 10%

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Decision *d*	Wolf	Dog
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Utility matrix $U(x)$

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Utility matrix $U(x)$

	Conditional utility
Action d	$E[U(x)\|d]$
Stay home	0
Go to the forest	-100+0.9

Utilities for different actions

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- Example illustrates that the uncertainties (probabilities) related to all consequences need to be carried on until final decision making

Example of decision making: several choices

- Prof. Gelman has a jar of quarters
  - he first drew a line on the side of the jar and then filled the jar up to the line, and so the number coins was not chosen beforehand
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  - Prof. Gelman does not know the number of coins in the jar
  - Prof. Gelman gives the class a chance to win the coins if they guess the number of coins correctly (someone else has counted the coins without telling Gelman)
  - How should the students make the decision?

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- Multipel parties having different utilities

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- See the book for continuation of the example with additional test for cancer

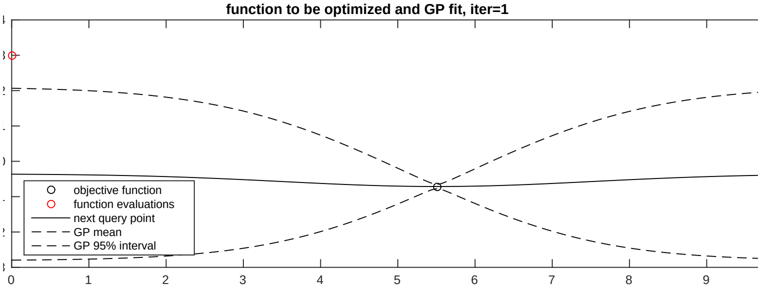
Design of experiment

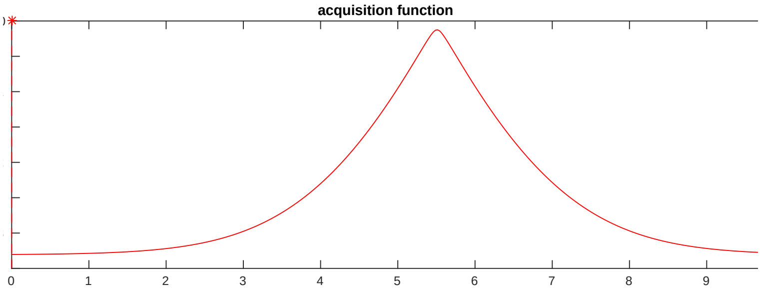
- Which experiment would give most additional information
  - decide values x_{n+1} for the next experiment
  - which values of x_{n+1} would reduce the posterior uncertainty most
- Example
  - Imagine that in bioassay the posterior uncertainty of LD50 is too large
  - which dose should be used in the next experiment to reduce the variance of LD50 as much as possible ?
    - this way less experiments need to be made (and less animals need to be killed)

Bayesian optimization

- Design of experiment
- Used to optimize, for example,
  - machine learning / deep learning model structures, regularization, and learning algorithm parameters
  - material science
  - engines
  - drug testing
  - part of Bayesian inference for stochastic simulators

Bayesian optimization





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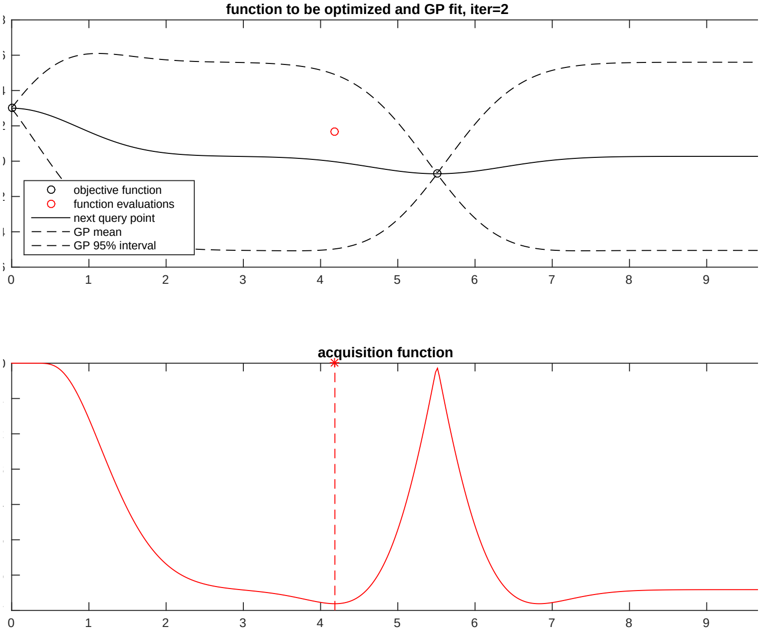

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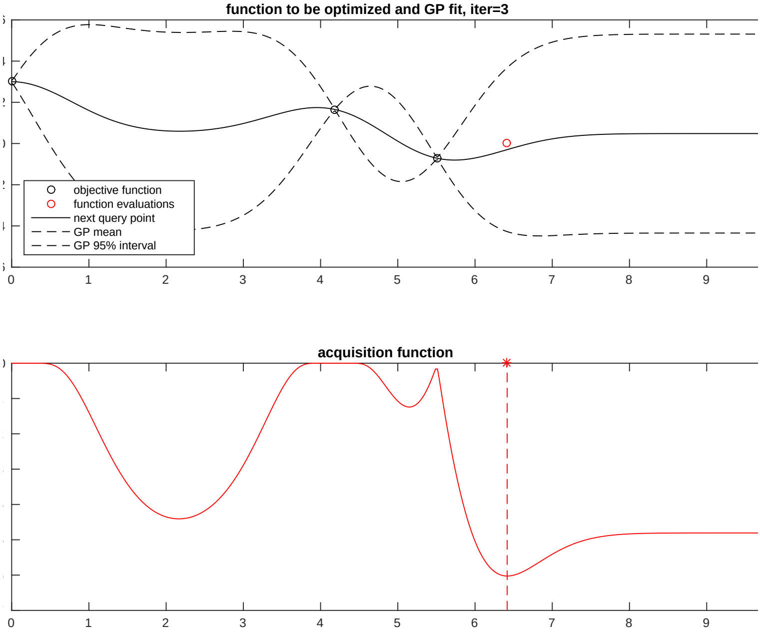

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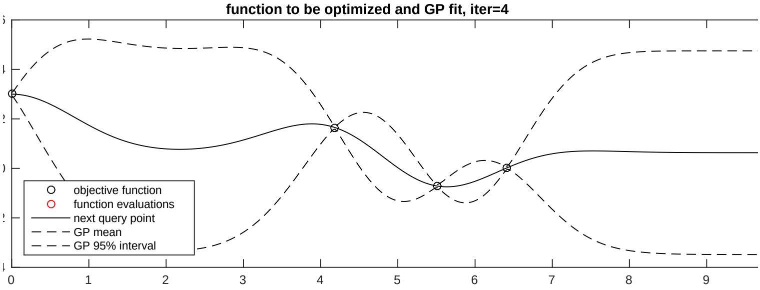

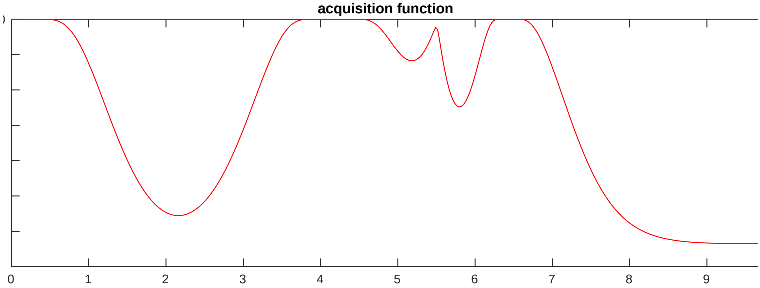

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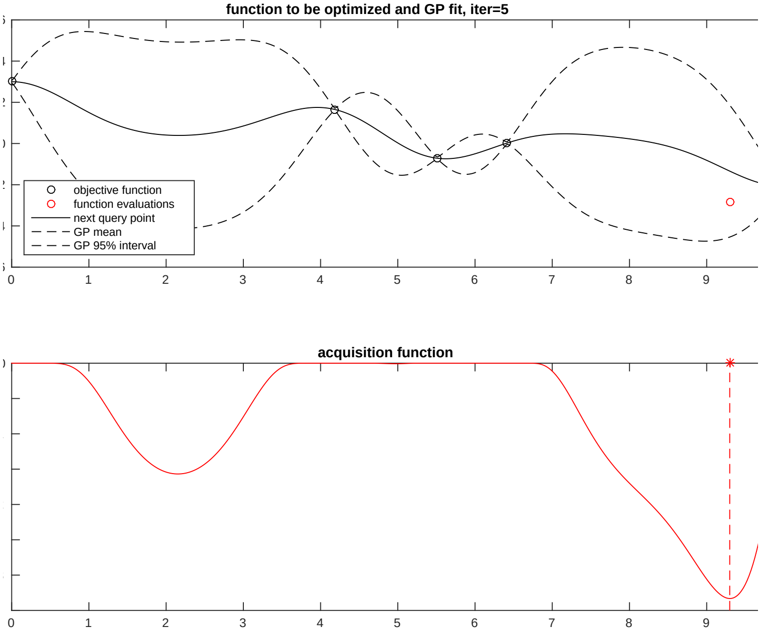

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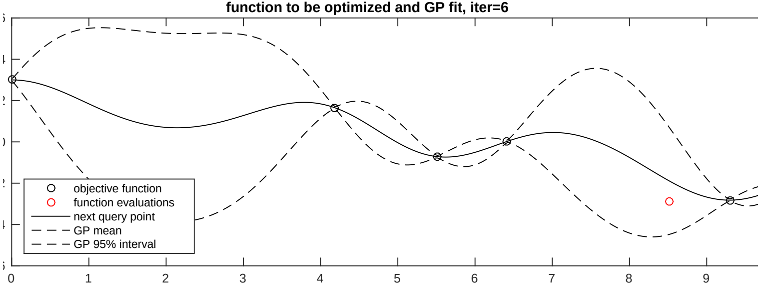

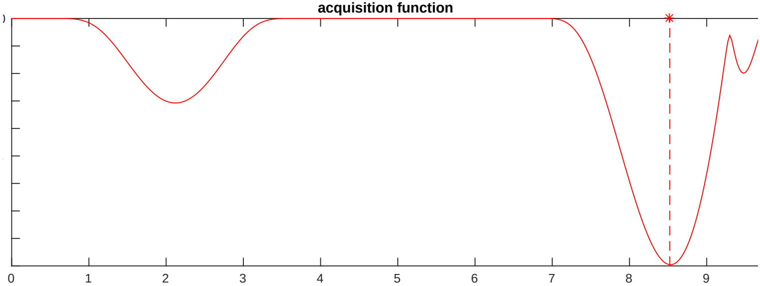

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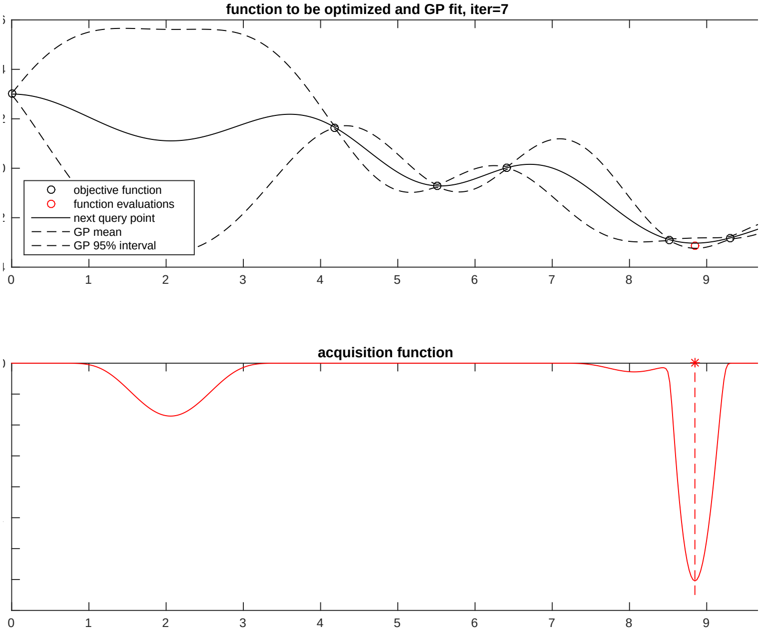

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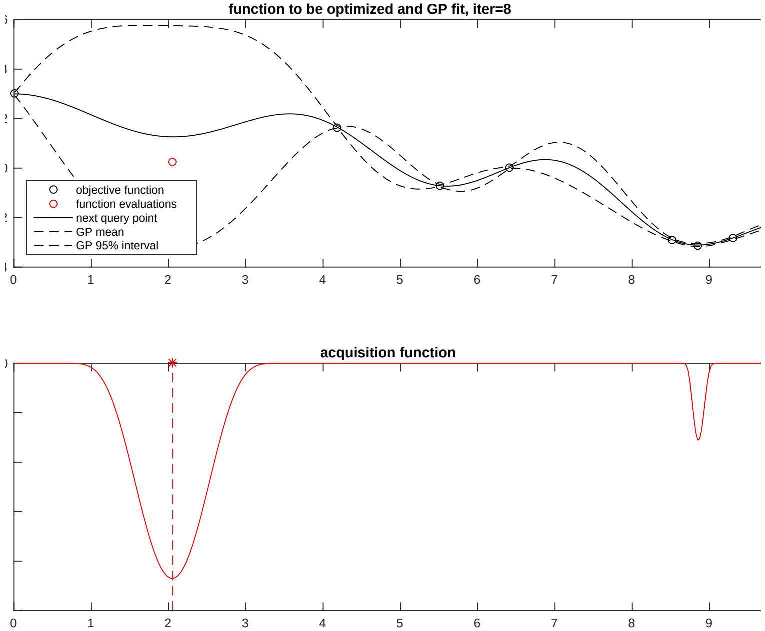

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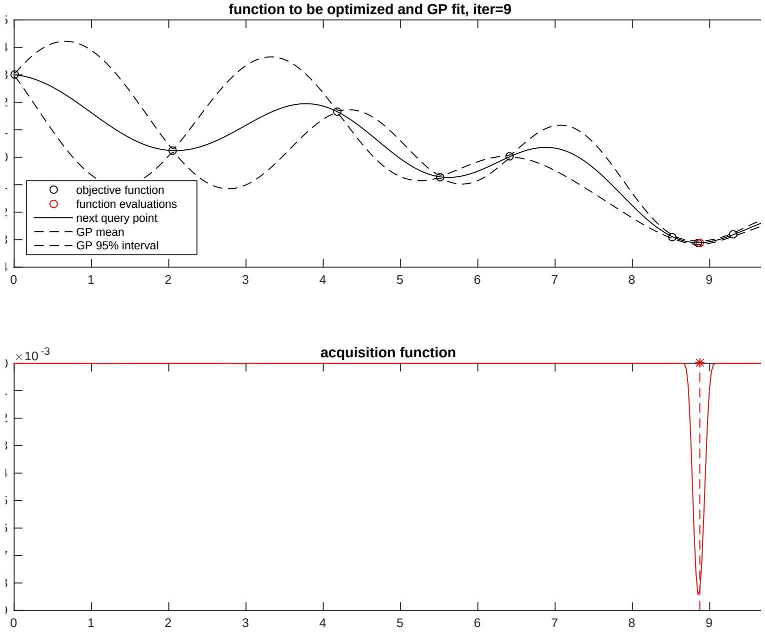

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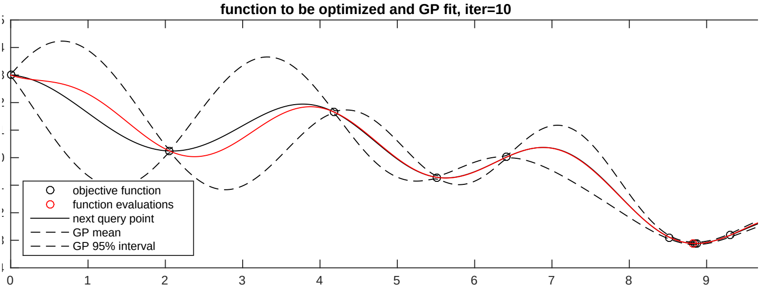

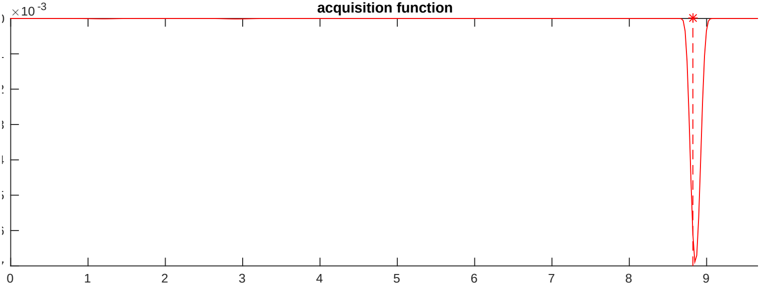

Model selection as decision problem

- Expected utility of using the model in the future