

Bayesian Data Analysis - Assignment 1

General information

- The recommended tool in this course is R (with the IDE RStudio). You can download R [here](#) and RStudio [here](#). There are tons of tutorials, videos and introductions to R and RStudio online. You can find some initial hints [here](#).
- The format of the report should follow the instruction in the R markdown template that can be found [here](#).
- Report all results in a single, **anonymous** *.pdf -file and return it to [peer-grade.io](#).
- Many of the exercises can be checked using the R package `markmyassignment`. Information on how to install and use the package can be found [here](#).
- The course has its own R package with data and functionality to simplify coding. To install the package just run the following:

```
1. install.packages("devtools")
2. devtools::install_github("avehtari/BDA_course_Aalto",
  subdir = "rpackage")
```

- Many of the exercises can be checked automatically using the R package `markmyassignment`. Information on how to install and use the package can be found [here](#).
- Additional self study exercises and solutions for each chapter in BDA3 can be found [here](#).
- If you have any suggestions or improvements to the course material, please feel free to submit a Pull Request to the public repository!

Information on this assignment

The exercises of this assignment are not necessarily related to chapter 1, but rather work to test whether or not you have sufficient knowledge to participate in the course. There are three pen and paper exercises and one computer task. The maximum amount of points from this assignment is 2.

Reading instructions: Chapter 1 in BDA3, see reading instructions [here](#)

Grading instructions: The grading will be done in peergrade. All grading questions and evaluations for exercise 1 can be found [here](#)

To use `markmyassignment` for this assignment, run the following code in R:

```
> library(markmyassignment)
> exercise_path <-
+   "https://github.com/avehtari/BDA_course_Aalto/blob/master/exercises/tests/ex1.yml"
> set_assignment(exercise_path)
```

1. **(Basic probability theory notation and terms)**. This can be trivial or you may need to refresh your memory on these concepts. Note that some terms may be different names for the same concept. Explain the following terms with one sentence:

- probability
- probability mass
- probability density
- probability mass function (pmf)
- probability density function (pdf)
- probability distribution
- discrete probability distribution
- continuous probability distribution
- cumulative distribution function (cdf)
- likelihood

2. **(Basic computer skills)** This task deals with elementary plotting and computing skills needed during the rest of the course. You can use either R or Python, although R is the recommended language and we will only guarantee support in R. For documentation in R, just type `?{function name here}`.

- a) Plot the density function of Beta-distribution, with mean $\mu = 0.2$ and variance $\sigma^2 = 0.01$. The parameters α and β of the Beta-distribution are related to the mean and variance according to the following equations

$$\alpha = \mu \left(\frac{\mu(1-\mu)}{\sigma^2} - 1 \right), \quad \beta = \frac{\alpha(1-\mu)}{\mu}.$$

Hint! Useful R functions: `seq()`, `plot()` and `dbeta()`. Later on we will also use the more flexible `ggplot2` for plotting.

- b) Take a sample of 1000 random numbers from the above distribution and plot a histogram of the results. Compare visually to the density function.

Hint! Useful R functions: `rbeta()` and `hist()`

- c) Compute the sample mean and variance from the drawn sample. Verify that they match (roughly) to the true mean and variance of the distribution.

Hint! Useful R functions: `mean()` and `var()`

- d) Estimate the central 95%-interval of the distribution from the drawn samples.

Hint! Useful R functions: `quantile()`

3. **(Bayes' theorem)** A group of researchers has designed a new inexpensive and painless test for detecting lung cancer. The test is intended to be an initial screening test for the population in general. A positive result (presence of lung cancer) from the test would be followed up immediately with medication, surgery or more extensive and expensive test. The researchers know from their studies the following facts:

- Test gives a positive result in 98% of the time when the test subject has lung cancer.

- Test gives a negative result in 96 % of the time when the test subject does not have lung cancer.
- In general population approximately one person in 1000 has lung cancer.

The researchers are happy with these preliminary results (about 97% success rate), and wish to get the test to market as soon as possible. How would you advise them? Base your answer on elementary probability calculus.

4. **(Bayes' theorem)** We have three boxes, A, B, and C. There are

- 2 red balls and 5 white balls in the box A,
- 4 red balls and 1 white ball in the box B, and
- 1 red ball and 3 white balls in the box C.

Consider a random experiment in which one of the boxes is randomly selected and from that box, one ball is randomly picked up. After observing the color of the ball it is replaced in the box it came from. Suppose also that on average box A is selected 40% of the time and box B 10% of the time (i.e. $P(A) = 0.4$).

- What is the probability of picking a red ball?
- If a red ball was picked, from which box it most probably came from?

Implement two functions in R that computes the probabilities. Below is an example of how the functions should be named and work if you want to check them with `markmyassignment`. Note that this is a test case, you need to change the number in the matrix.

```
> boxes <- matrix(c(2,2,1,5,5,1), ncol = 2,
+   dimnames = list(c("A", "B", "C"), c("red", "white")))
> boxes

   red white
A    2     5
B    2     5
C    1     1

> p_red(boxes)

[1] 0.3928571

> p_box(boxes)

[1] 0.29090909 0.07272727 0.63636364
```

5. **(Bayes' theorem)** Assume that on average fraternal twins (two fertilized eggs) occur once in 150 births and identical twins (single egg divides into two separate embryos) once in 400 births (**Note!** This is not the true values, see Exercise 1.5 in BDA3). American male singer-actor Elvis Presley (1935 – 1977) had a twin brother who died in birth. What is the probability that Elvis was an identical twin? Assume that an equal number of boys and girls are born on average.

Implement this as a function in R that computes the probability. Below is an example of how the functions should be named and work if you want to check your result with `markmyassignment`.

```
> p_identical_twin(fraternal_prob = 1/125, identical_prob = 1/300)
```

```
[1] 0.4545455
```

```
> p_identical_twin(1/100, 1/500)
```

```
[1] 0.2857143
```