

# Chapter 9

- 9.1 Context and basic steps (most important part)
- 9.2 Example
- 9.3 Multistage decision analysis (example)
- 9.4 Hierarchical decision analysis (example)
- 9.5 Personal vs. institutional decision analysis

# Bayesian decision theory

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- Expected utility  $E[U(x)|d] = \int U(x)p(x|d)dx$
- Choose decision  $d^*$ , which maximizes the expected utility

$$d^* = \arg \max_d E[U(x)|d]$$

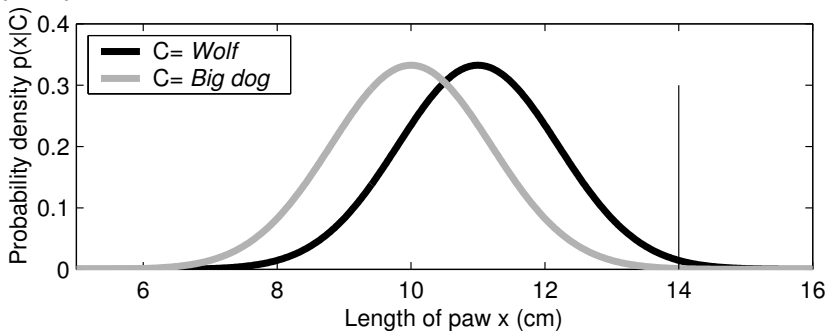
## Example of decision making: 2 choices

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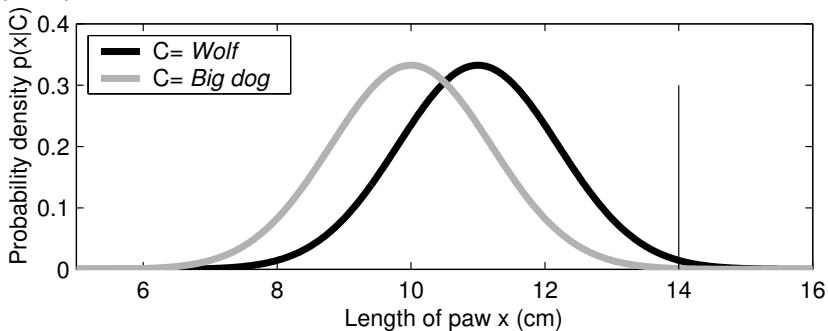
- Helen is going to pick mushrooms in a forest, while she notices a paw print which could be made by a dog or a wolf
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- Likelihood of wolf is 0.92 (alternative being dog)

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- Posterior probability of wolf is 10%

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Stay home	0	0
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Utility matrix  $U(x)$

Action $d$	Conditional utility $E[U(x) d]$
Stay home	0
Go to the forest	-100+0.9

Utilities for different actions

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- Example illustrates that the uncertainties (probabilities) related to all consequences need to be carried on until final decision making

## Example of decision making: several choices

- Prof. Gelman has a jar of quarters
  - he first drew a line on the side of the jar and then filled the jar up to the line, and so the number coins was not chosen beforehand
  - Prof. Gelman does not know the number of coins in the jar



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  - Prof. Gelman does not know the number of coins in the jar
  - Prof. Gelman gives the class a chance to win the coins if they guess the number of coins correctly (someone else has counted the coins without telling Gelman)
  - How should the students make the decision?

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- See the book for continuation of the example with additional test for cancer

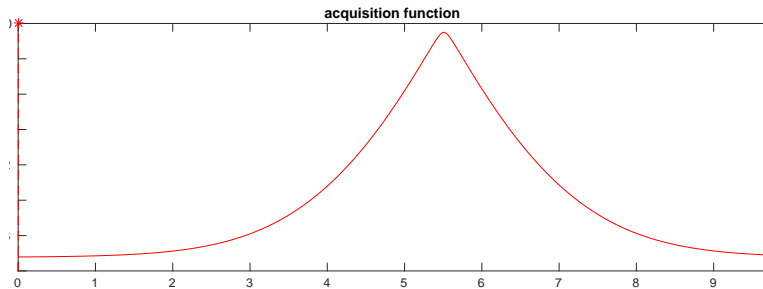
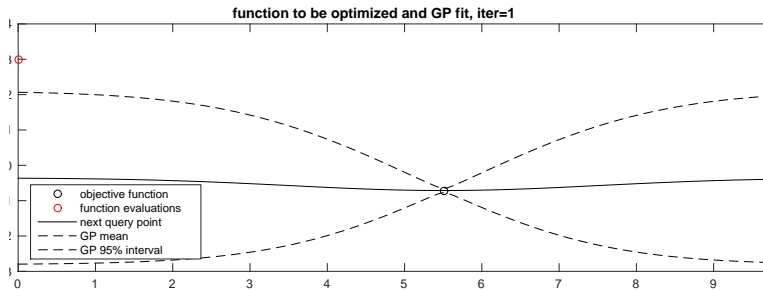
# Design of experiment

- Which experiment would give most additional information
  - decide values  $x_{n+1}$  for the next experiment
  - which values of  $x_{n+1}$  would reduce the posterior uncertainty most
- Example
  - Imagine that in bioassay the posterior uncertainty of LD50 is too large
  - which dose should be used in the next experiment to reduce the variance of LD50 as much as possible ?
    - this way less experiments need to be made (and less animals need to be killed)

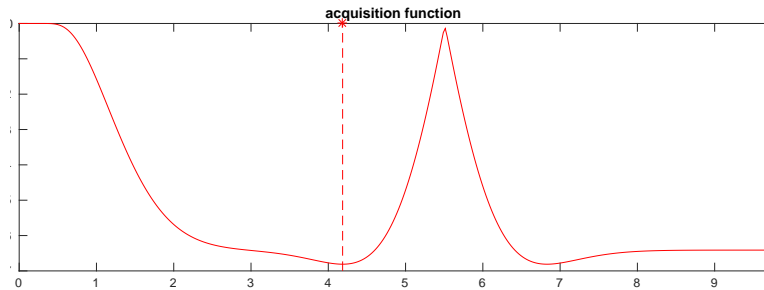
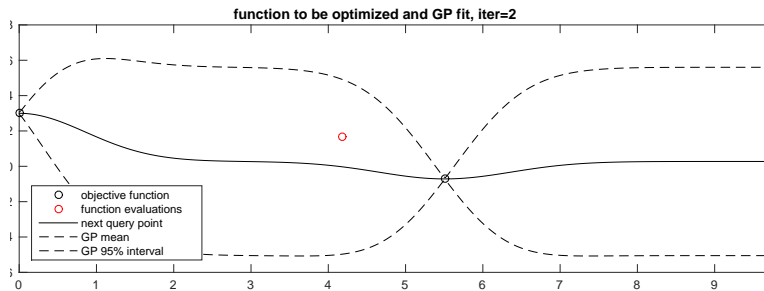
# Bayesian optimization

- Design of experiment

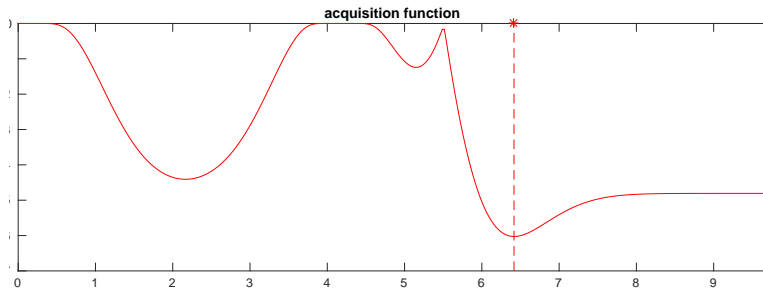
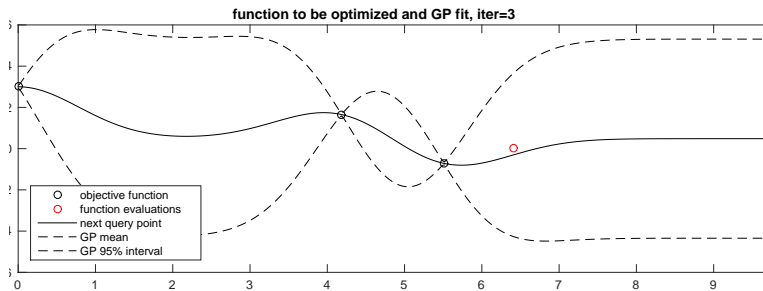
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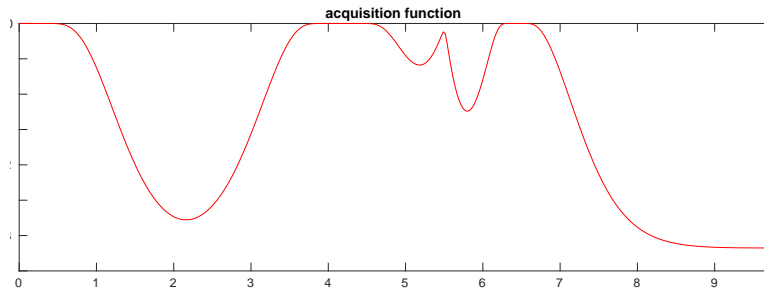
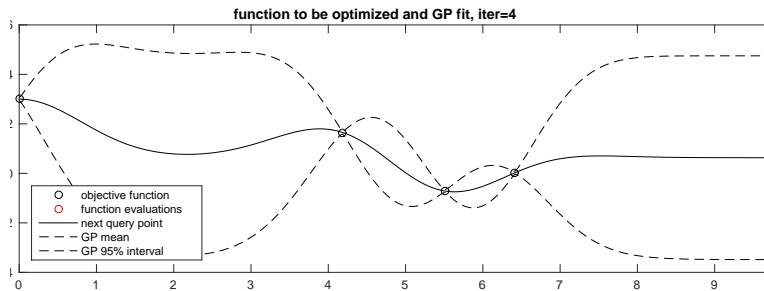


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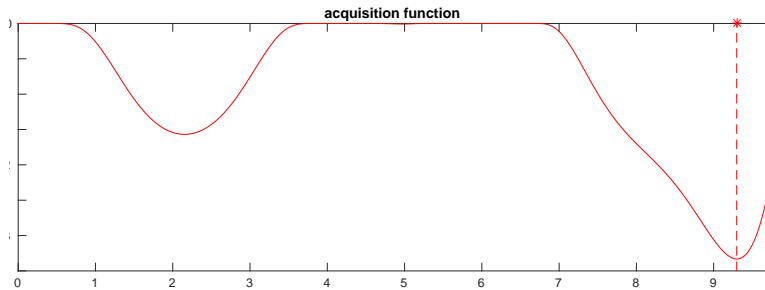
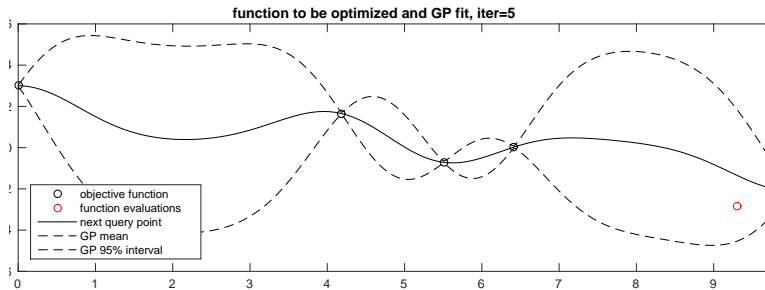




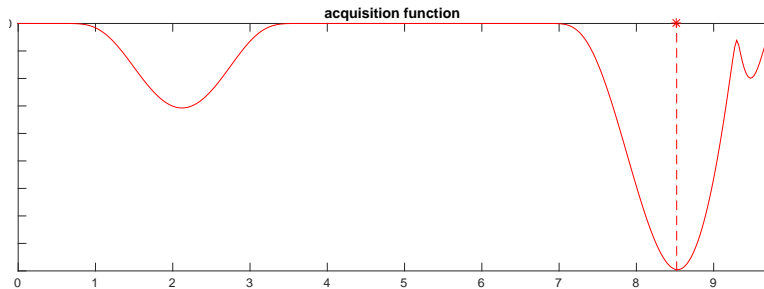
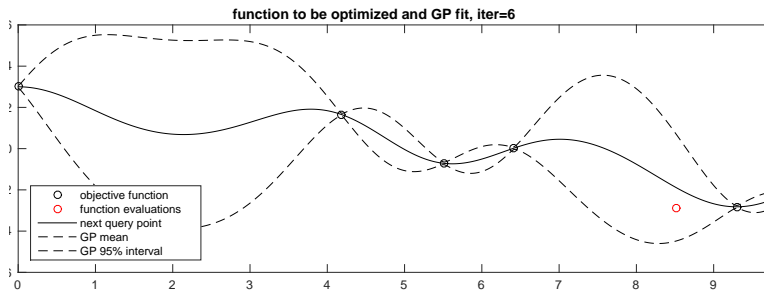
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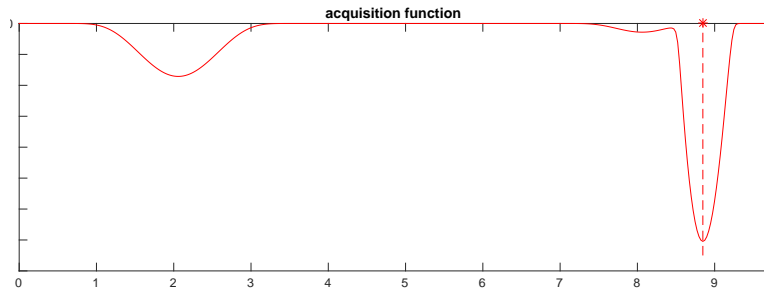
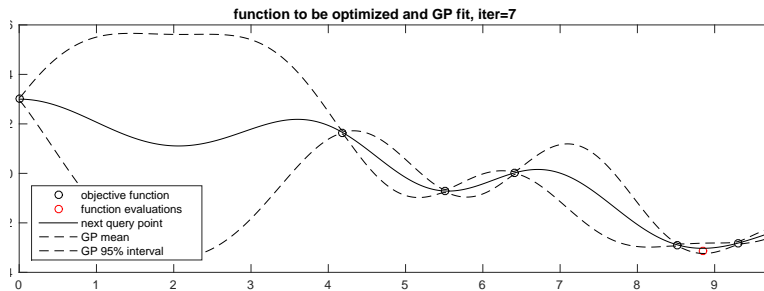
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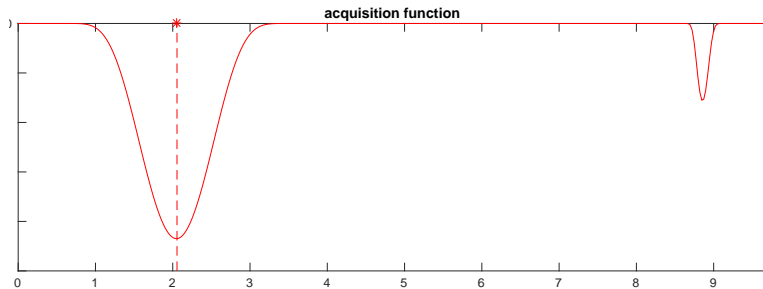
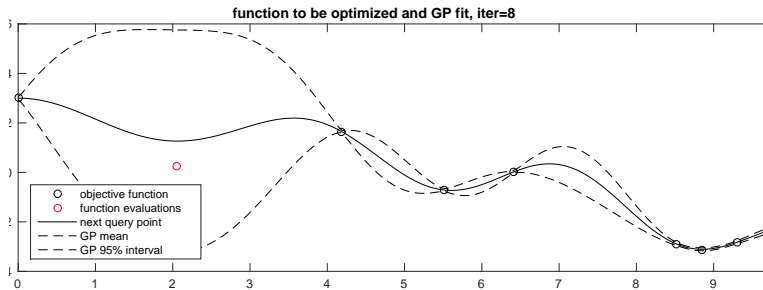
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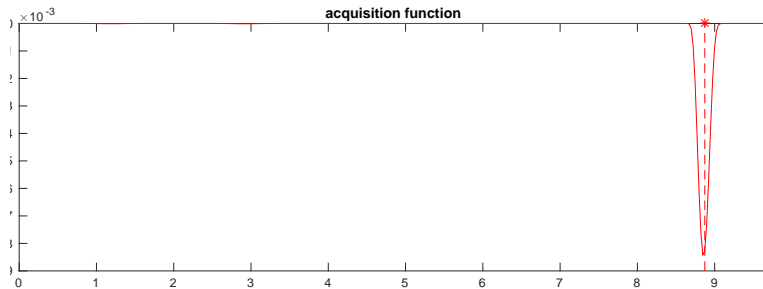
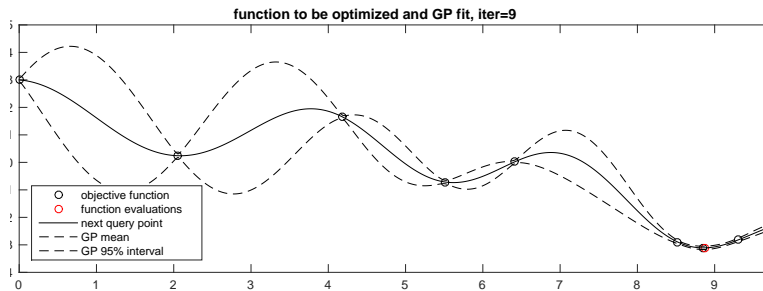
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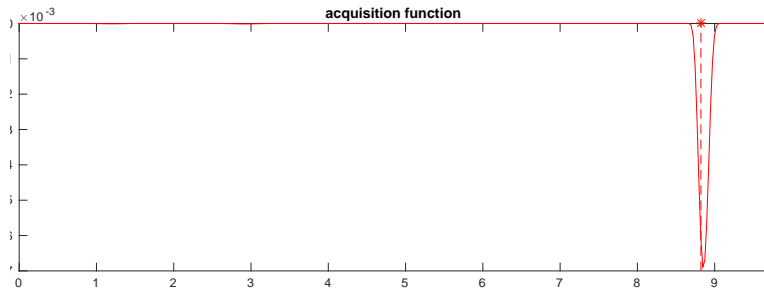
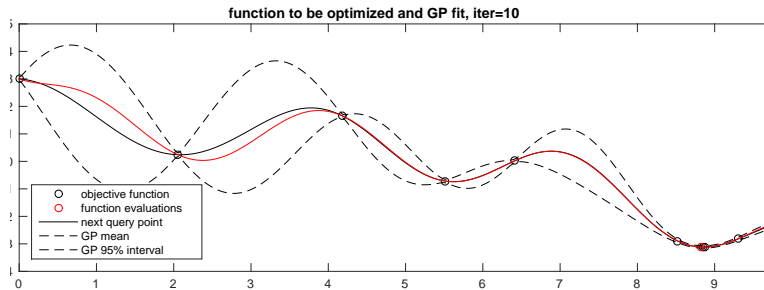
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# Model selection as decision problem

- Expected utility of using the model in the future