# Bayesian data analysis – reading instructions ch 10

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## Chapter 10

Outline of the chapter 10

- 10.1 Numerical integration (overview)
- 10.2 Distributional approximations (overview, more in Chapter 4 and 13)
- 10.3 Direct simulation and rejection sampling (overview)
- 10.4 Importance sampling (used in PSIS-LOO discussed later)
- 10.5 How many simulation draws are needed? (Important! Ex 10.1 and 10.2)
- 10.6 Software (can be skipped)
- 10.7 Debugging (can be skipped)

Sections 10.1-10.4 give overview of different computational methods. Some of then have been already used in the book.

Section 10.5 is very important and related to the exercises.

Demos

- demo10\_1: Rejection sampling
- demo10\_2: Importance sampling
- demo10\_3: Sampling-importance resampling

Find all the terms and symbols listed below. When reading the chapter, write down questions related to things unclear for you or things you think might be unclear for others.

- unnormalized density
- target distribution
- log density
- overflow and underflow
- numerical integration
- quadrature
- simulation methods
- Monte Carlo
- stochastic methods
- deterministic methods
- distributional approximations
- crude estimation
- direct simulation
- grid sampling
- rejection sampling
- importance sampling
- importance ratios/weights

# **Draws and sample**

A group of draws is a sample. A sample can consist of one draw, and thus some people use the word sample for both single item and for the group. For clarity, we prefer separate words for single item (draw) and for the group (sample).

#### Quadrature

Sometimes 'quadrature' is used to refer generically to any numerical integration method (including Monte Carlo), sometimes it is used to refer just to deterministic numerical integration methods.

# Rejection sampling

Rejection sampling is mostly used as a part of fast methods for univariate sampling. For example, sampling from the normal distribution is often made using Ziggurat method, which uses a proposal distribution resembling stairs.

Rejection sampling is also commonly used for truncated distributions, in which case all draws from the truncated part are rejected.

### **Importance sampling**

Popularity of importance sampling is increasing. It is used, for example, as part of other methods as particle filters and pseudo marginal likelihood approaches, and to improve distributional approximations (including variational inference in machine learning).

Importance sampling is useful in importance sampling leave-one-out cross-validation. Cross-validation is discussed in Chapter 7 and importance sampling leave-one-out cross-validation is discussed in the article

• Aki Vehtari, Andrew Gelman and Jonah Gabry (2016). Practical Bayesian model evaluation using leave-one-out cross-validation and WAIC. In Statistics and Computing, 27(5):1413–1432. arXiv preprint arXiv:1507.04544 <a href="http://arxiv.org/abs/1507.04544">http://arxiv.org/abs/1507.04544</a>

After the book was published, we have developed Pareto smoothed importance sampling which is more stable than plain importance sampling and has very useful Pareto-k diagnostic to check the reliability

 Aki Vehtari, Daniel Simpson, Andrew Gelman, Yuling Yao, and Jonah Gabry (2019). Pareto smoothed importance sampling. arXiv preprint arXiv:1507.02646. <a href="http://arxiv.org/abs/1507.02646">http://arxiv.org/abs/1507.02646</a>.

#### Importance sampling effective sample size

BDA3 1st (2013) and 2nd (2014) printing have an error for  $\tilde{w}(\theta^s)$  used in the effective sample size equation 10.4. The normalized weights equation should not have the multiplier S (the normalized weights should sum to one). Errata for the book http://www.stat.columbia.edu/~gelman/book/errata\_bda3.txt.

# **Buffon's needles**

Computer simulation of Buffon's needle dropping method for estimating the value of  $\pi$  https://mste. illinois.edu/activity/buffon/.