ENGG 5202: Assignment #3

Due on Thursday, April 7, 2016

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Problem 1

1.1

$$\begin{aligned} K_3(x, x') &= K_1(x, x') + K_2(x, x') \\ &= \Phi_1(x)\Phi_1(x') + \Phi_2(x)\Phi_2(x') \\ &= [\Phi_1(x), \Phi_2(x)] \cdot [\Phi_1(x'), \Phi_2(x')] \\ \Phi_3(x) &= [\Phi_1(x), \Phi_2(x)] \end{aligned}$$

1.2

$$\begin{aligned} K_3(x, x') &= K_1(x, x')K_2(x, x') \\ &= \Phi_1(x)\Phi_1(x')\Phi_2(x)\Phi_2(x') \\ \Phi_3(x) &= \Phi_1(x)\Phi_2(x) \end{aligned}$$

1.3

$$\begin{aligned} K(x, x') &= 1 + x \cdot x' + 4(x \cdot x')^2 \\ &= 1 + x_1x_1' + x_2x_2' + 4(x_1x_1' + x_2x_2')^2 \\ &= 1 + x_1x_1' + x_2x_2' + 4x_1^2x_1'^2 + 4x_2^2x_2'^2 + 8x_1x_1'x_2x_2' \\ \Phi(x) &= [1 \ x_1 \ x_2 \ 2x_1^2 \ 2x_2^2 \ 2\sqrt{2}x_1x_2] \end{aligned}$$

Problem 2

2.1

The best kernels and corresponding test errors for different datasets are shown in Table 1. Support vectors are plotted in Figure 1 to Figure 3.

Table 1: Choosing kernel for different datasets

Dataset	Best kernel	Test error
Set1	Linear kernel	4.46%
Set2	Radial basis kernel	1.4%
Set3	Radial basis kernel	0%

2.2

Test errors are shown in Table 2.

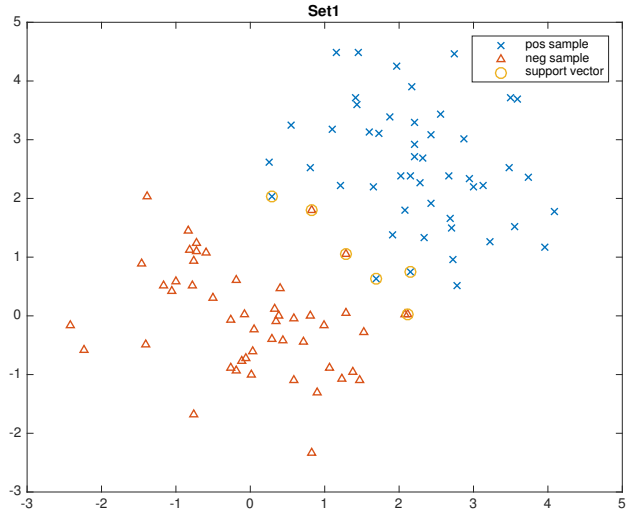

Figure 1: Support vectors of Set1

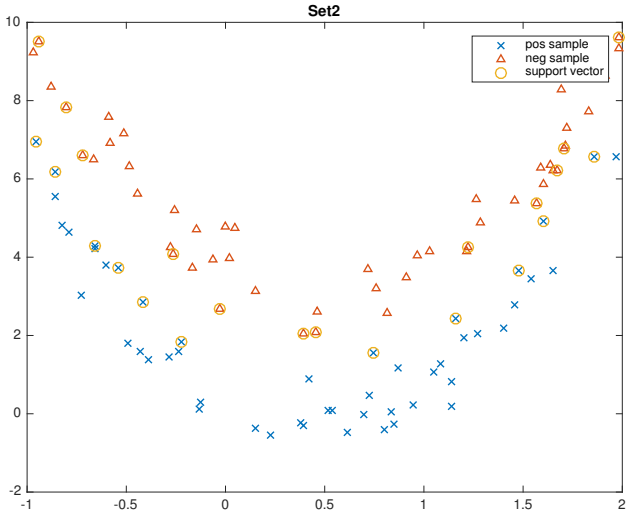

Figure 2: Support vectors of Set2

Table 2: SVM classification error of different kernels

Kernel	Test error
Linear kernel	13.75%
Polynomial kernel	12%
Radial basis kernel	8.5%

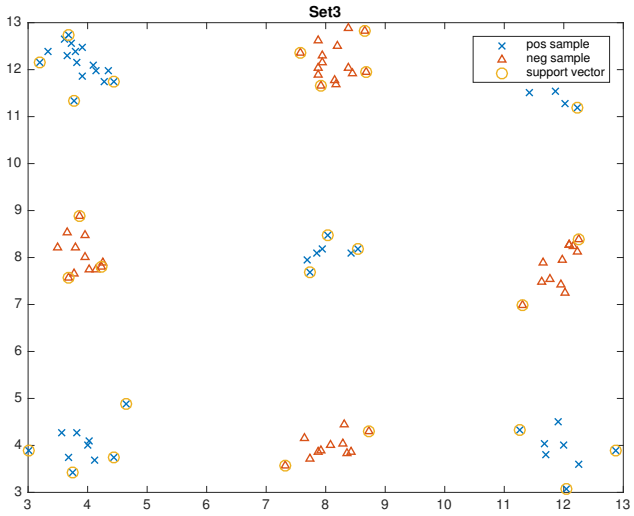

Figure 3: Support vectors of Set3

Problem 3

3.1

The dicision boundary is shown in Figure 4

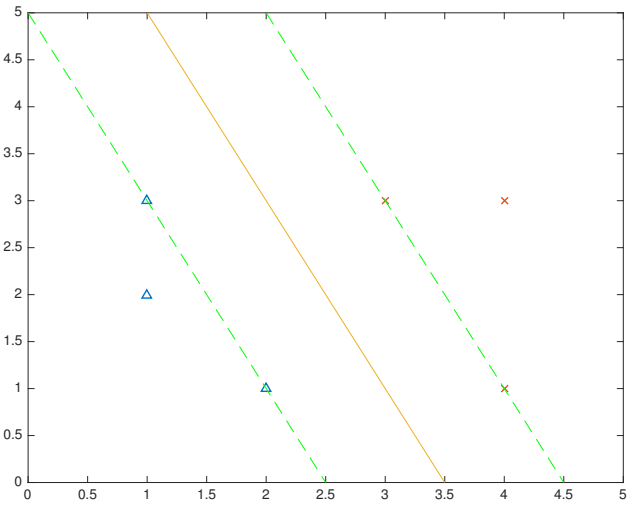

Figure 4: SVM classifier

3.2

Support vectors are these 4 points:

(1, 3)
(2, 1)
(3, 3)
(4, 1)

3.3

Yes, for example, if a negative sample (3, 1) is added, then the number of support vectors will be decreased to 3.

3.4

The leave-one-out cross-validation error is

Problem 4

4.1

4.2

4.3

4.4

4.5