Fluid Flow Equations:

- $F = \tau_{ij} \cdot [Area\ that\ stress\ is\ acting\ over]$
- Newton's Law of Viscosity:
  - $\tau_{yx} = \eta \frac{dv_x}{dy} = \eta\dot{\gamma}$. Viscosity is constant for Newtonian fluids
- Power law model (for shear thinning fluids at high shear rates)
  - $\eta(\dot{\gamma}) = K\dot{\gamma}^{n-1},\ \log\eta = \log(K) + (n-1)\log(\dot{\gamma})$

Fluid Statics equations

- Hydrostatic pressure equations. Valid for different positions in the same fluid
  - $P_2 - P_1 = -\rho g(z_2 - z_1)$
  - $\phi_i = P_i + \rho g z_i$
- Archimedes Principle. Valid for floating objects
  - $W = F_B = \rho_f g(Vol_{dis})$
  - For the same object floating in different Fluids:
    - $F_{B1} = \rho_{f1} g(Vol_{dis1}) = W = F_{B2} = \rho_{f2} g(Vol_{dis2})$
- Surface Tension
  - $\Delta P = \frac{2\gamma_T}{R}$

Conservation of Mass

- $\int \frac{d\rho}{dt} dVol = \dot{m}_{in} - \dot{m}_{out}$
- For incompressible fluid, simplifies to:
  - $\frac{dVol}{dt} = Q_{in} - Q_{out} = A_{in}V_{in} - A_{out}V_{out}$
- For Steady state, simplifies to:
  - $\dot{m}_{in} = \dot{m}_{out}$
- For Steady state and incompressible flow, simplifies to:
  - $Q_{in} = Q_{out} = A_{in}V_{in} = A_{out}V_{out}$
- Average velocity term for a non-flat velocity profile:
  - $V_{avg} = \frac{1}{A}\int(\vec{v} \cdot \vec{n})dA$

Conservation of Energy

- $\frac{P}{\rho} + \frac{V^2}{2} + gz = constant$
- Torricelli's Theorem: $V = \sqrt{2gH}$

Conservation of Momentum

- $\frac{d}{dt}(m\vec{v}) = \sum_{in}(m\vec{v})_{in} - \sum_{out}(m\vec{v})_{out} + \sum \vec{F}$

Review Questions:
Part 1. Fundamental fluid properties, units and dimensions

- What is a fluid? What distinguishes a fluid from a solid?
    - Fluids deform under a shear force
- What are the main features that characterize how molecules are arranged in solids, liquids, and gases?
    - Solids have a lattice structure, liquids have weaker forces, which are dependent on the separation, and gases have very weak interactions
- What are the dimensions associated with common physical quantities? What is meant by an FLt System and a MLt system?
    - Mass, Length, time, Temperature
    - FLt systems have units in dimensions of Force length and time, MLt systems have them defined in
- What are some common systems of units? What units are associated with common measures?
    - In the English system of units there's English engineering, English absolute, and British gravitational. (absolute and gravitational are scientific). For metric there's SI and CGS (both scientific). However, both have an engineering version
- What is gc? Why is it important?
    - Gc is a conversion factor to go from units of MLt^-2 to F. Scientific unit systems have a Gc value of one, while engineering systems have some other value
- Define density. What are its units? What are some ways that it can be measured?
    - Mass per unit volume; M/L^3. Through a hydrometer!
- Define specific gravity. What are its units?
    - The density divided by the density of water at 4 degrees Celsius. It is unitless
- Define pressure. What are its units? What is meant by gauge pressure?
    - Pressure is a type of stress. F/A. absolute pressure – atmospheric pressure
- How is temperature defined? What are its units?
    - It is a measure of internal energy. T (K, R, *C, *F)
- What does it mean when we assume a fluid behaves as a continuum?
    - We assume that the entire volume of liquid has a constant value, instead of reality where there are points of mass (molecules) with lots of space in-between
- Define shear stress. How do you determine the relevant local shear stress? What are its units
    - A stress acting on a fluid in the normal direction due to flow in the same direction that the segment of fluid you are looking at is moving it
- What is Newton's law of viscosity? Write an equation expressing this law. Define each term and its units.
    - $\tau_{yx} = \eta \frac{dv_x}{dy}$
    - Tau y-x is the shear stress from the y direction due to flow in the x direction n is the viscosity, dvx/dy is the velocity gradient of the flow

- Define viscosity. How is it typically measured and what are its units?
    - Viscosity is a quantity measuring the magnitude of internal friction in a fluid resisting flow
    - It is measured through Couette or Poiseuille flows. Units of M/Lt or Ft/L^2
- What are the dominant physical processes that give rise to viscosity in liquids? In gases? How do these processes explain the temperature dependence of viscosity?
    - Liquids: Interactions between molecules that break and reform. An increase in temperature weakens these forces and decreases viscosity
    - Gases: Collisions between molecules. An increase in temperature causes more collisions and viscosity goes up
- What is meant by a Newtonian fluid? What is a non-Newtonian fluid?
    - Newtonian fluid: constant viscosity. Non-Newtonian fluids have viscosities that differ by shear rate
- What is meant by shear thinning, shear thickening, and Bingham-plastic behavior?
    - Shear thinning: decreasing viscosity with increases in shear rate
    - Shear thickening: increasing viscosity with increases in shear rate
    - Bingham-plastic: acts like a solid until a certain shear rate is achieved when it ats like a fluid
- What is a power law model? When is it typically applicable? How do you determine the power law parameters K & n and what are their units?
    - A power law model is a model used to describe shear thinning fluids at high shear rates. K and n are determined by plotting data on a log-log scale and creating a power trendline. Log(K) is the y intercept and (n – 1) is the slope.
    - n is unitless, and K has units of Pressure * time ^ n
- What are representative values for atmospheric pressure? The viscosity of water?
    - Atmospheric Pressure 1.01325*10^5 Pa, 1.01325 bar, 14.696 psi
    - Viscosity of water: 10^-6 Pa s at 293.15 K

Part 2. Fluid statics

- How does pressure vary with depth in a fluid subjected to gravity forces? Write an equation expressing this relationship.
    - Pressure increases with depth
    - $P_2 - P_1 = -\rho g(z_2 - z_1)$
- How does pressure vary in the horizontal plane in a fluid subjected to gravity forces?
    - It remains the same
- How does a manometer work?
    - A pipe is connected to two different pressure sources for comparison. Relations between those pressures can be made using hydrostatic potential equations for the different fluids

- Why would someone use an inclined tube manometer instead of a straight U-tube?
    - For very low changes in pressure, it can be more easily measured since the change in height can be more easily recorded. This is because pressure does not change horizontally, so at lower angles, the change in height will be more pronounced by the horizontal length achieved.
- What is the buoyancy force (Archimedes' Principle)?
    - Buoyancy force is defined as the density of the fluid * gravitational constant * volume of displaced fluid
- What is a hydrometer? How does it work?
    - A hydrometer is a way of measuring specific gravity by using buoyancy
    - It compares the volume displaced in that fluid to the volume displaced in
- What is surface tension?
    - Work needed to change interfacial area
- What is the Young-Laplace equation and how is it used?
    - $\boxed{\Delta P = \gamma_T\left(\frac{1}{R_1} + \frac{1}{R_2}\right)}$ used to find surface tension due to a pressure difference

Part 3. Conservation of Mass

- What is the general form of a conservation law?
    - Accumulation = input – output + generation - consumption
- Write a generalized equation for the conservation of mass.
    - $\int_{VOL} \frac{\partial \rho}{\partial t}\, dVOL + \int_{A} \rho(\underline{v} \cdot \underline{n})\, dA = 0$
- Write a generalized equation for a velocity vector in Cartesian coordinates.
    - u * i hat + v * j hat + w * k hat
- Write the continuity equation for incompressible flow in Cartesian coordinates.
    - $\boxed{\underline{\nabla} \cdot (\rho \underline{v}) = 0}$
- What is the conservation of mass for steady-state incompressible flow?
    - Conservation of volume
- What is the mass flow rate? Mass flux? The volume flow rate? The average velocity? What are their units?
    - Mass flow rate: density * velocity * area; Mass/time
    - Mass flux: density * velocity; Mass/time Length^2
    - Volume flow rate: velocity * area; L^3/time
    - Average velocity: $\boxed{V_{AVG} = \bar{V} = \frac{Q}{A} = \frac{1}{A}\int(\underline{v} \cdot \underline{n})\, dA}$ m/s

Part 4. Conservation of Energy, Idealized Bernoulli's Equation

- What forms of energy are accounted for in the conservation of energy equation?
    - **Kinetic, gravitational,** internal, mechanical work, heat, **energy crossing boundary with mass flow**

- Write the Idealized Bernoulli's Equation. Under what conditions is it valid?
    - $\boxed{\frac{P_{IN}-P_{OUT}}{\rho} + \frac{1}{2}\left(V_{IN}^2 - V_{OUT}^2\right) + g\left(z_{IN}-z_{OUT}\right) = 0}$ BERNOULLI'S EQUATION (IDEALIZED FORM)
    - Incompressible, steady state, inviscid/no losses, away from walls
- How does a Pitot probe work?
    - By facing a tube towards the direction of moving water and letting the velocity difference from the pipe to the stream move the water column up
- How does a Venturi flow meter work?
    - By comparing the flow rates at two different cross-sectional areas
- What is the "head" form of Idealized Bernoulli's equation?
    - The Bernoulli's equation but with the entire equation divided by gravity. It is useful for civil engineering
- What is the problem with analyzing the draining of a tank using the Idealized Bernoulli's equation?
    - The assumptions to not consider fluid specific qualities such as viscosity, limiting the use case. Also, Bernoulli's equation is used for steady-state conditions, and that wasn't at steady state

Part 5. Macroscopic Conservation of Momentum

- What is linear momentum? Write an expression for the macroscopic conservation of linear momentum.
    - P=m*v. $\underbrace{\frac{d}{dt}(m\tilde{v})}_{\text{ACCUMULATION OF MOMENTUM}} = \underbrace{\sum_{IN}(\dot{m}\hat{v})_{IN}}_{\text{INFLOW OF MOMENTUM}} - \underbrace{\sum_{OUT}(\dot{m}\hat{v})_{OUT}}_{\text{OUTFLOW OF MOMENTUM}} + \underbrace{\sum \tilde{F}}_{\text{SUM OF FORCES}}$
- How is the conservation of linear momentum applied to calculate forces on pipe bends and for flow through nozzles?
    - Atmospheric pressure acting on the control volume has to be considered, which leads to gauge pressures being more useful
- What is angular momentum and how does it compare with linear momentum?
    - Associates with torques instead of linear forces