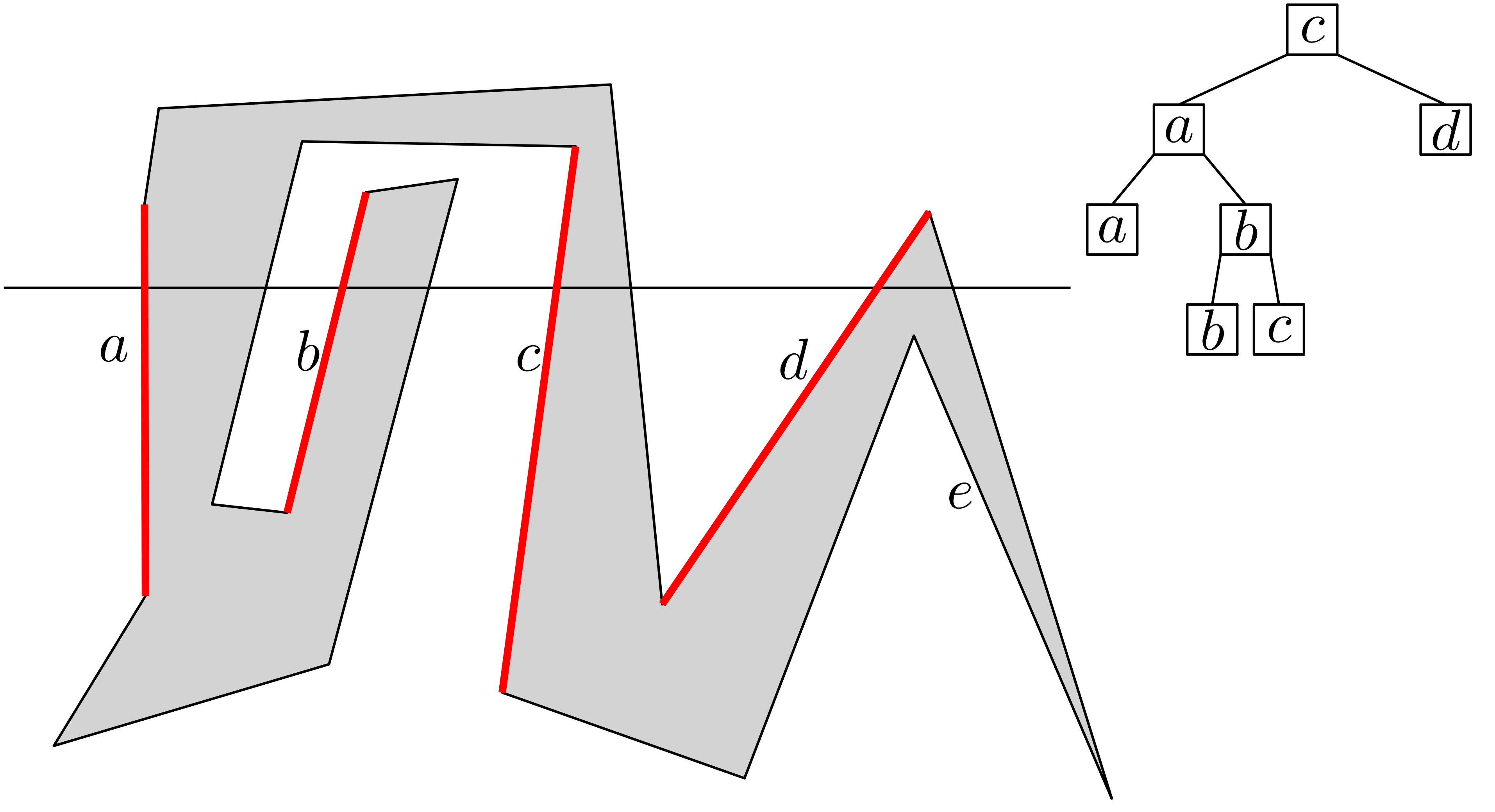
Status structure



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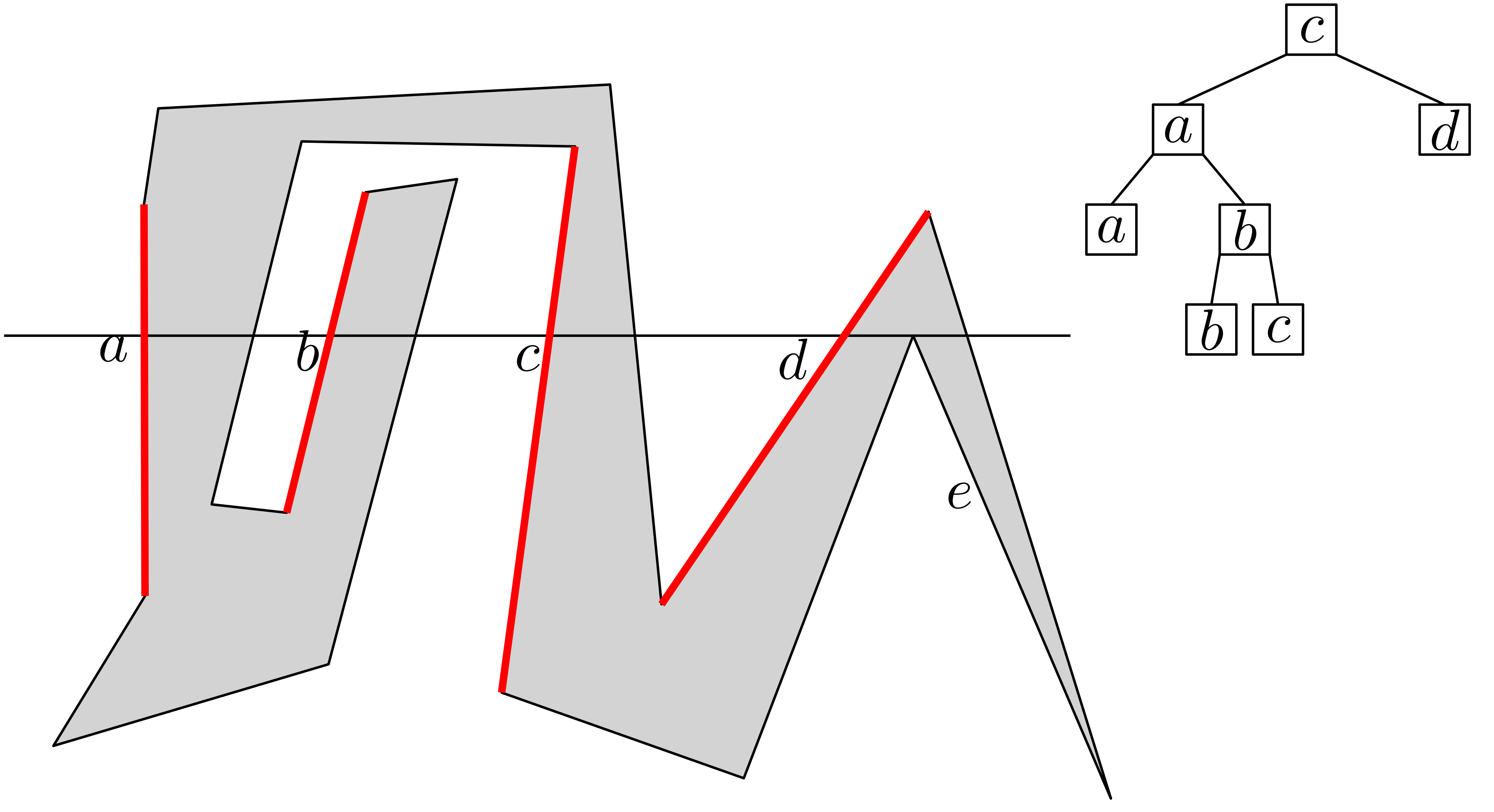

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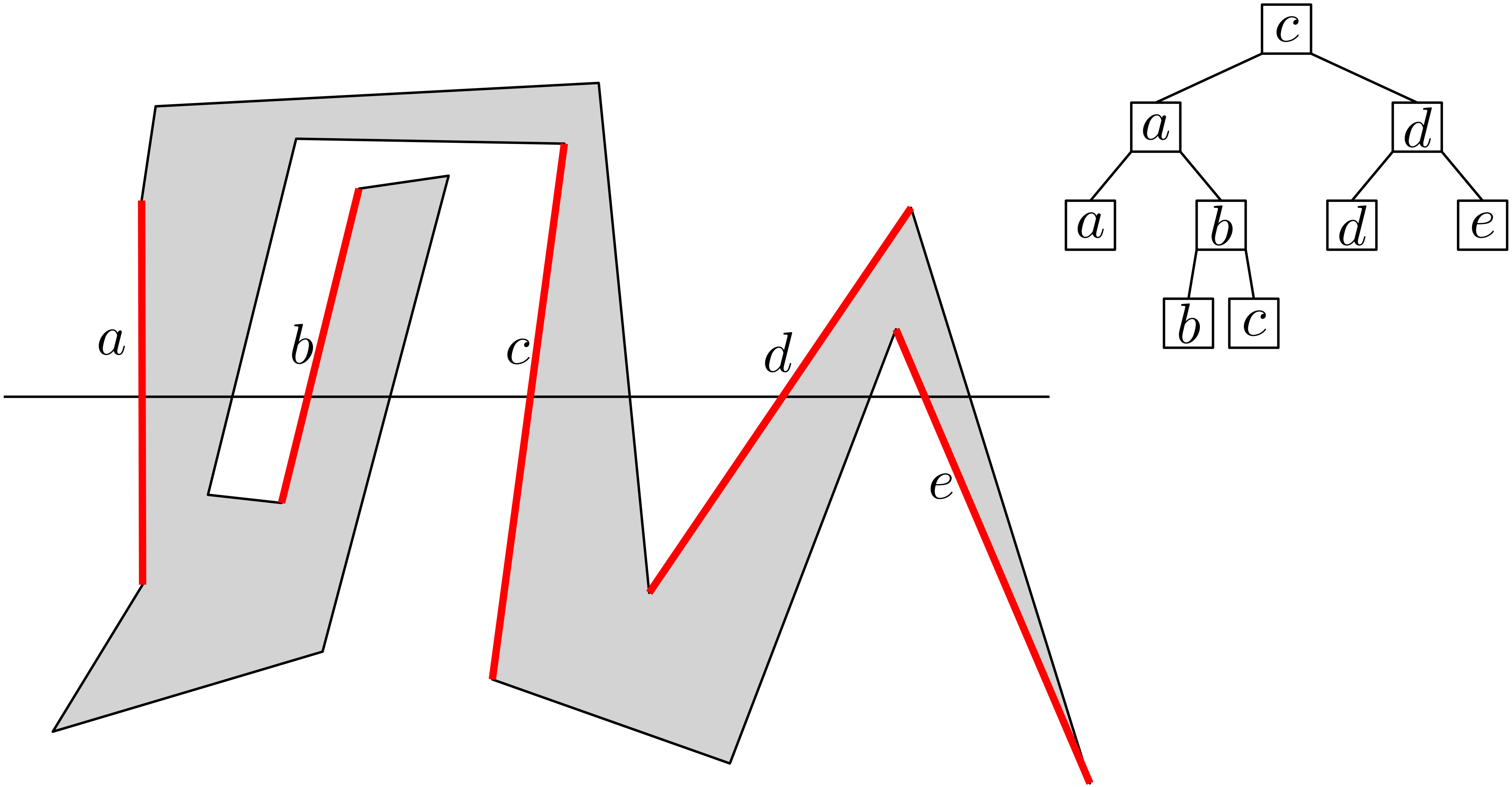

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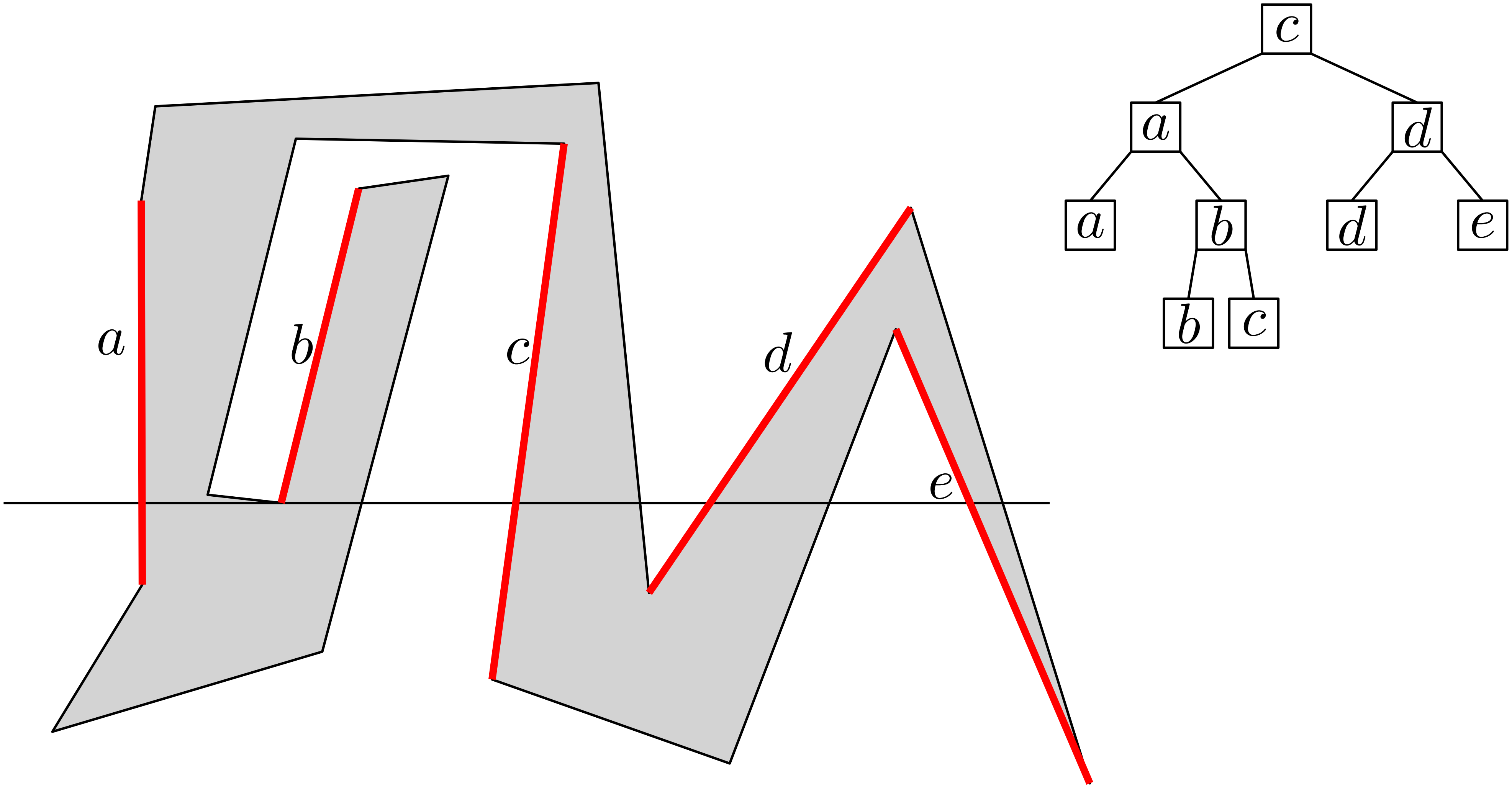
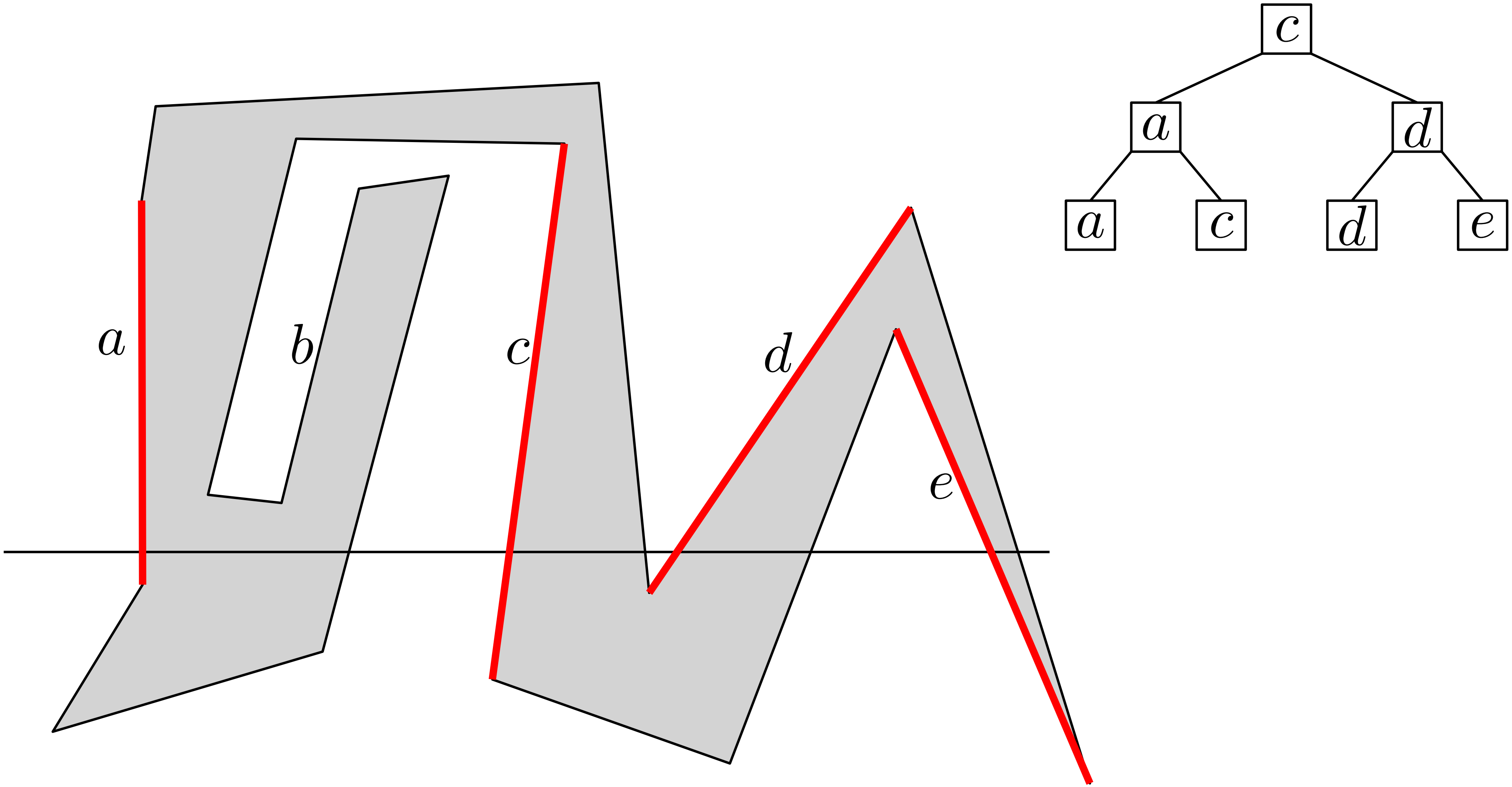
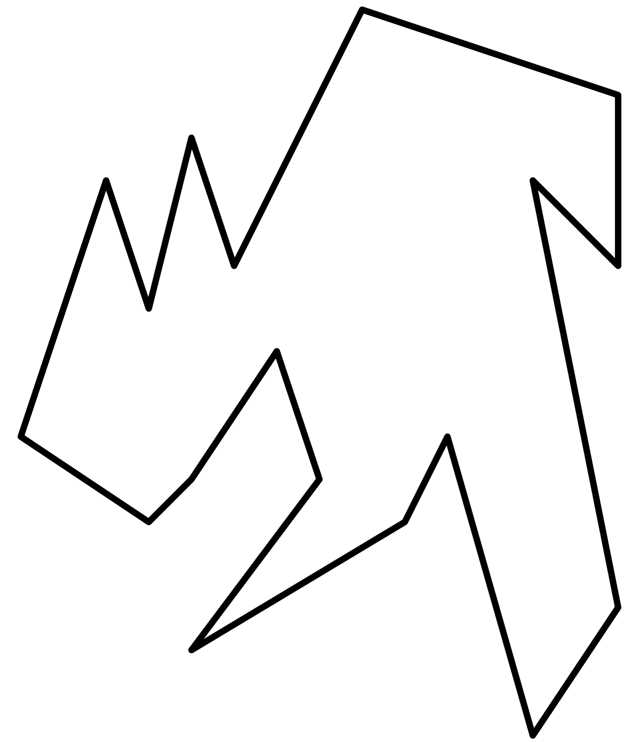


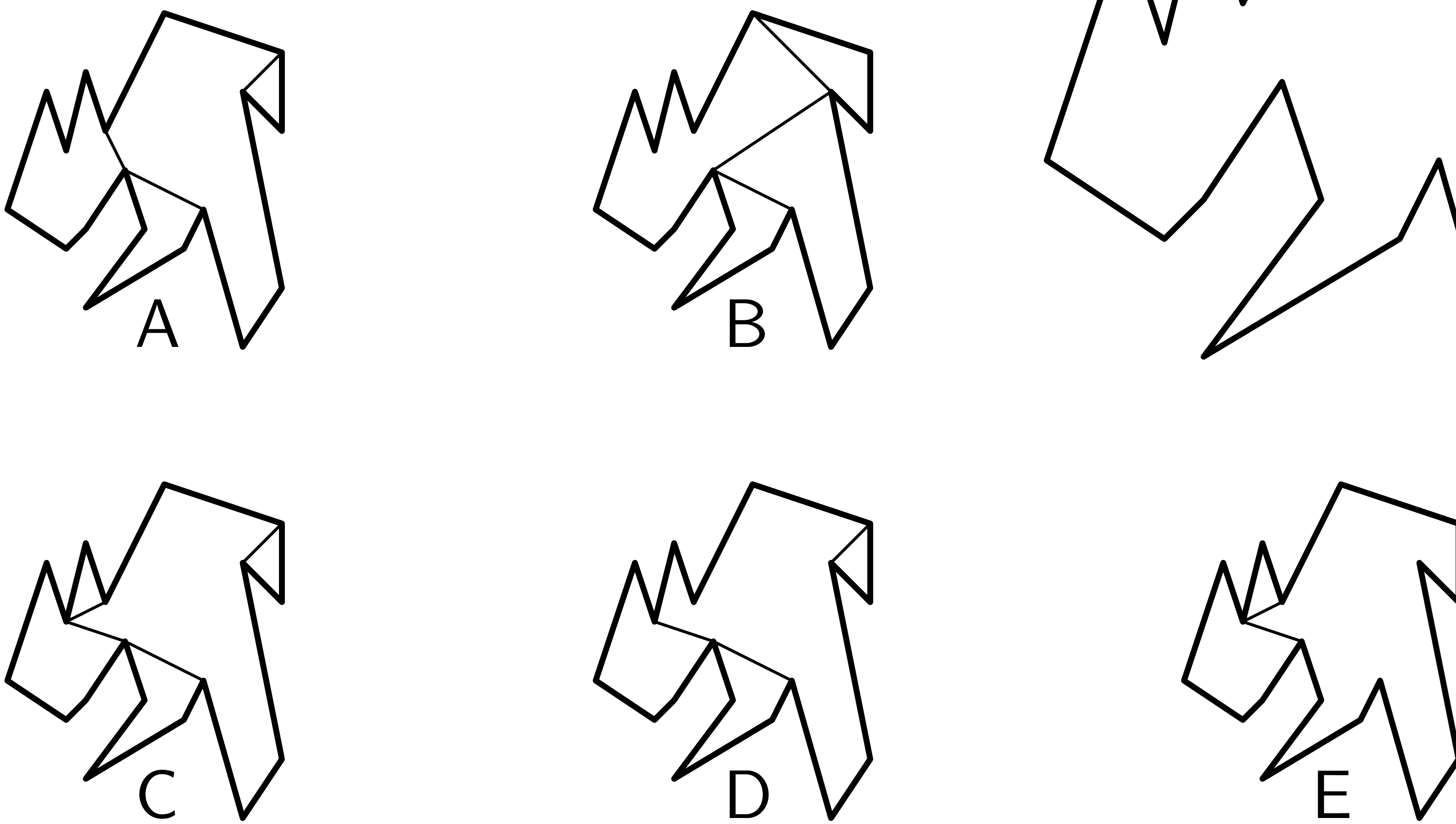
Status structure



What diagonals are introduced after sweeping down?

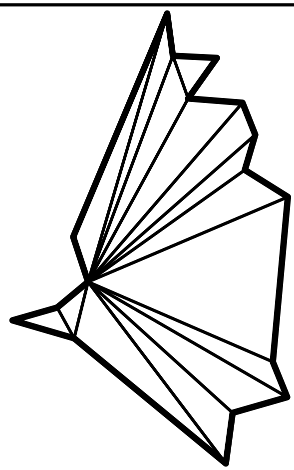
socrative.com → Student login,
Room name: ABRAHAMSEN3464



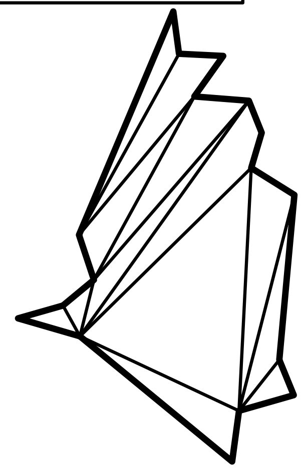


How does the triangulation look?

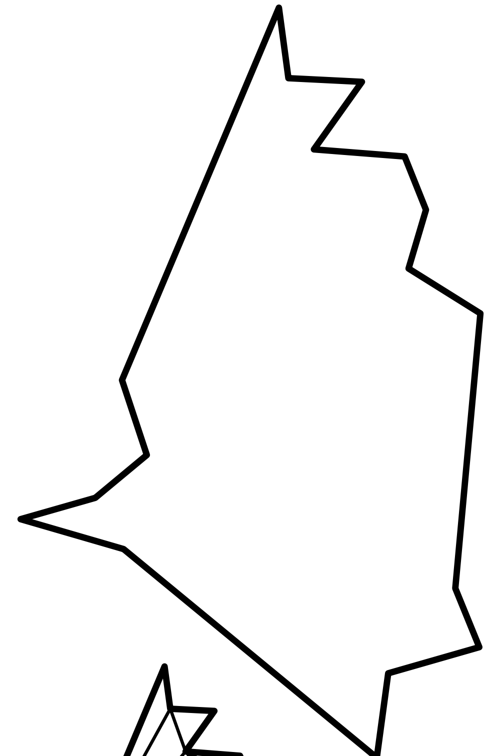
socrative.com → Student login,
Room name: ABRAHAMSEN3464

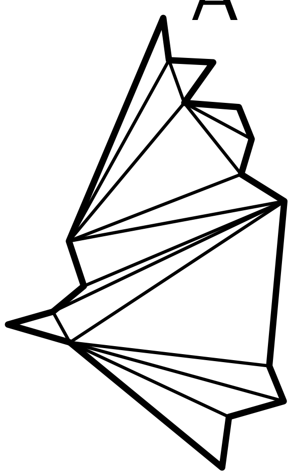


A

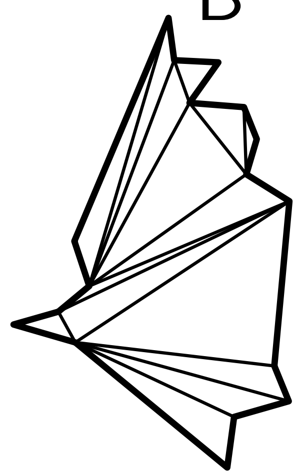


B





C



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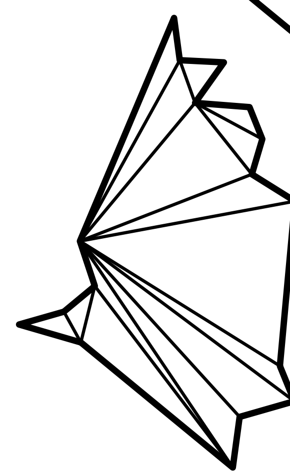


E

At the exam

Focus on the algorithms.

Don't use a lot of time on the Art Gallery Problem or proving that there exists a triangulation.

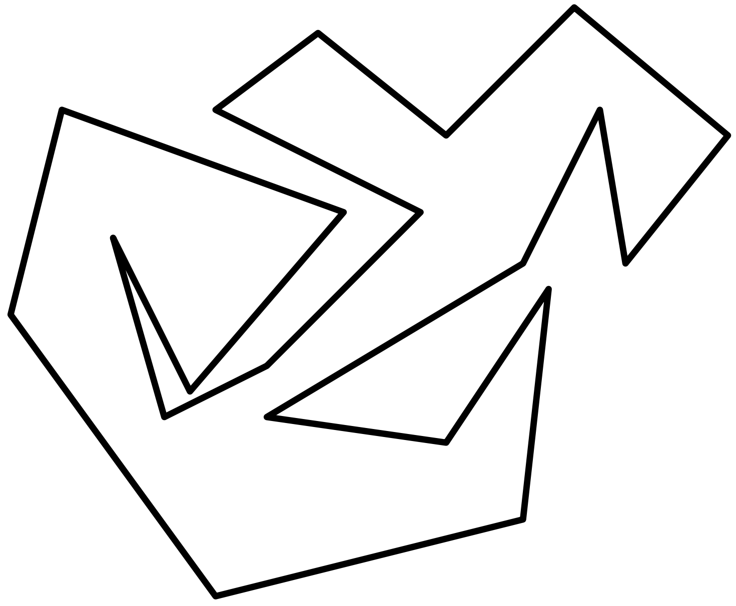
History of Triangulation Algorithms

1979: Garey, Johnson, Preparata, Tarjan. $O(n \log n)$ sweep-line algorithm, similar to this.
1982: Chazelle. $O(n \log n)$ divide-and-conquer algorithm.
1986: Tarjan and Van Wyk. $O(n \log \log n)$ algorithm.
1988: Clarkson, Tarjan, and Van Wyk. Randomized $O(n \log^* n)$ algorithm. Two other algorithms with same running time around the same time.
1990: Chazelle. Optimal $O(n)$ algorithm.

x	$\lg^* x$
≤ 1	0
$(1, 2]$	1
$(2, 4]$	2
$(4, 16]$	3
$(16, 65536]$	4
$(65536, 2^{65536}]$	5

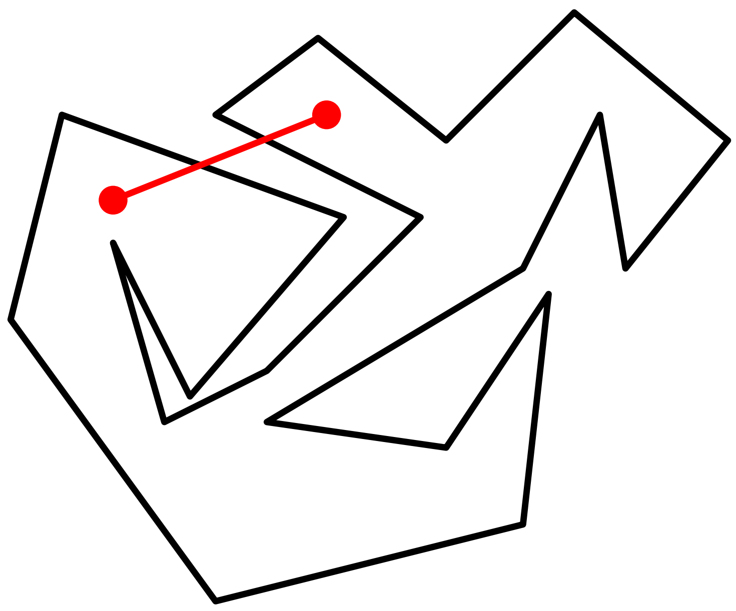
Use of Triangulations

Visibility problems.



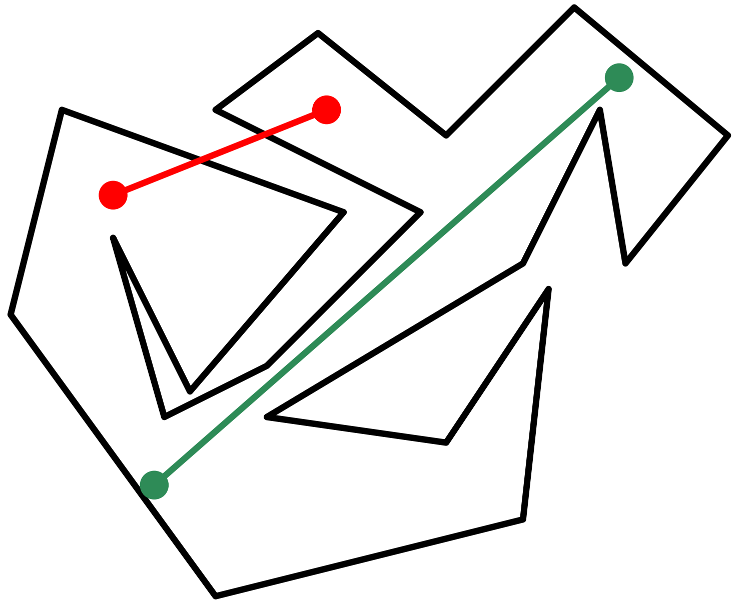
Use of Triangulations

Visibility problems.



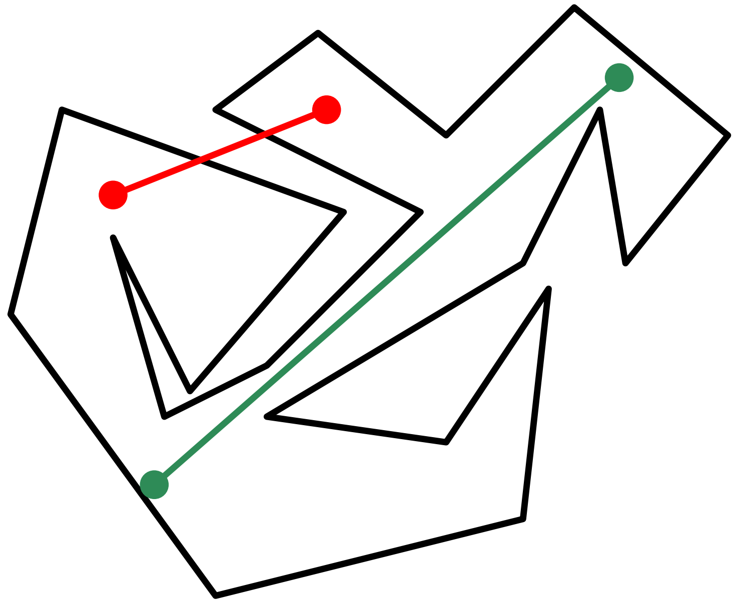
Use of Triangulations

Visibility problems.

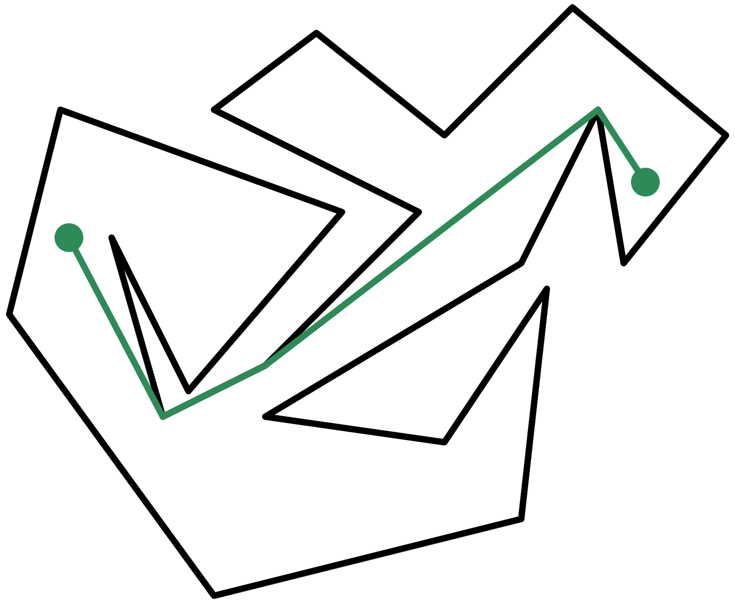


Use of Triangulations

Visibility problems.

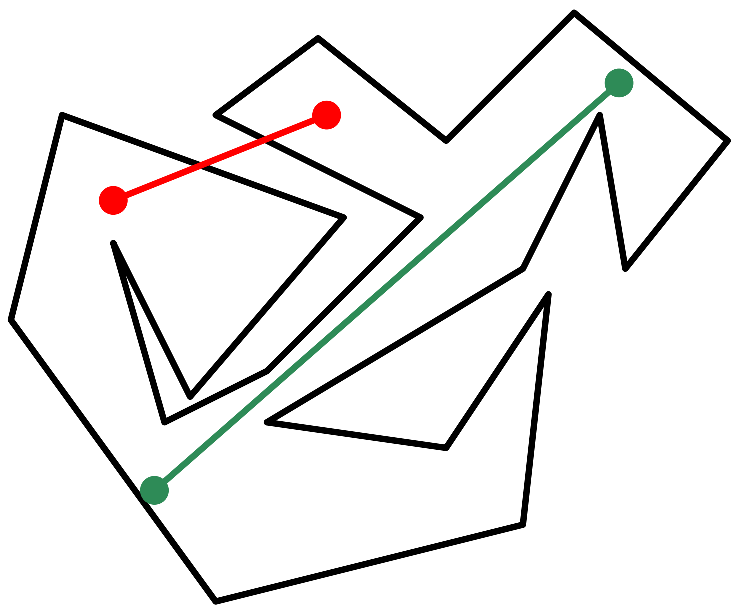


Shortest paths.

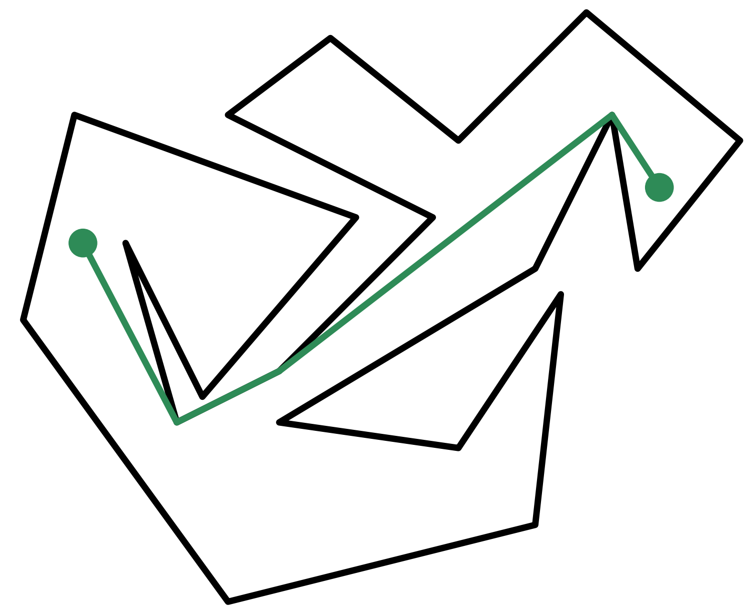


Use of Triangulations

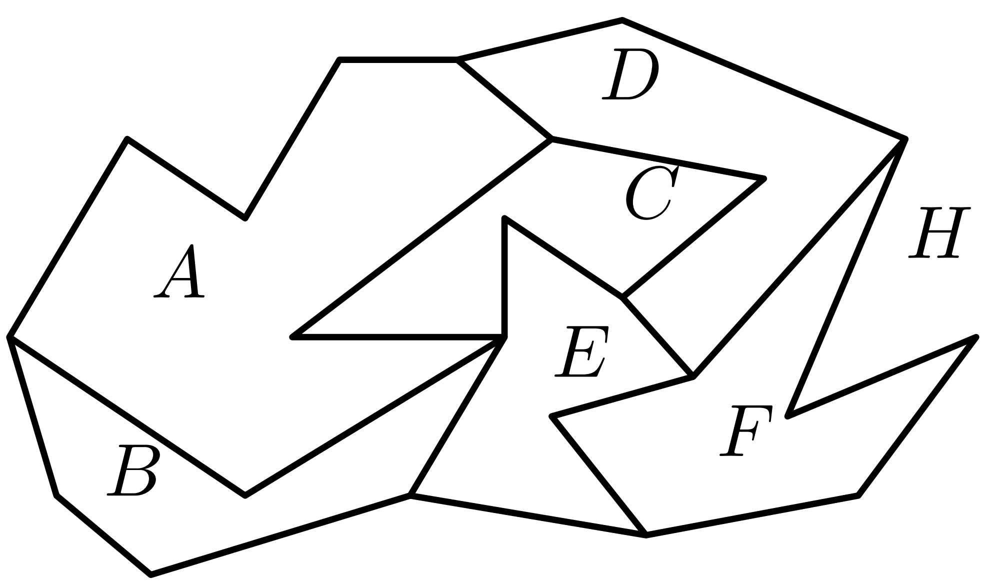
Visibility problems.



Shortest paths.

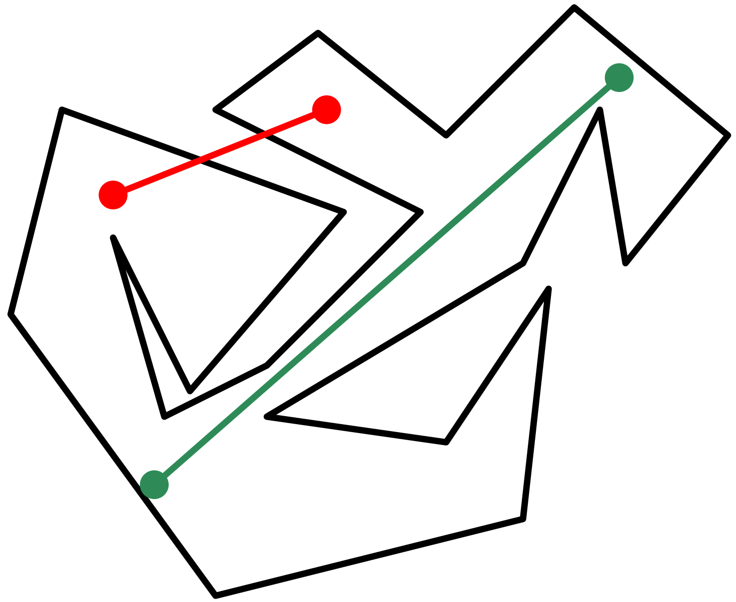


Point location.

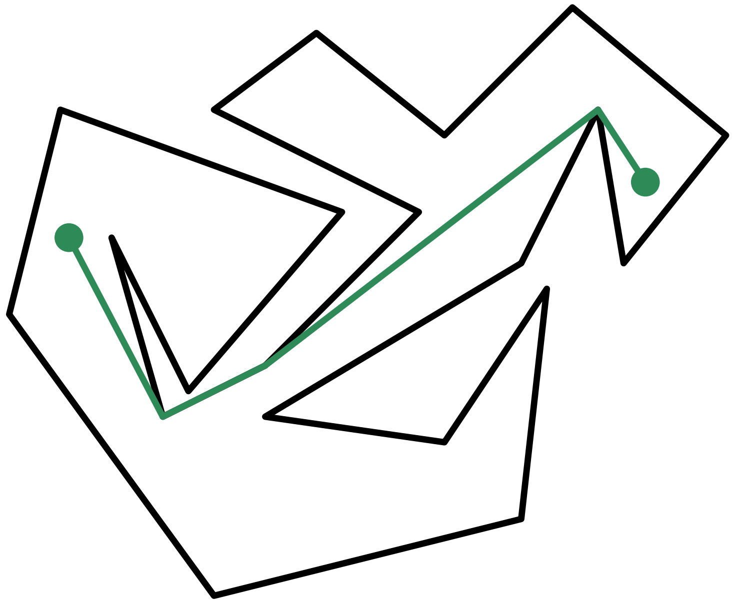


Use of Triangulations

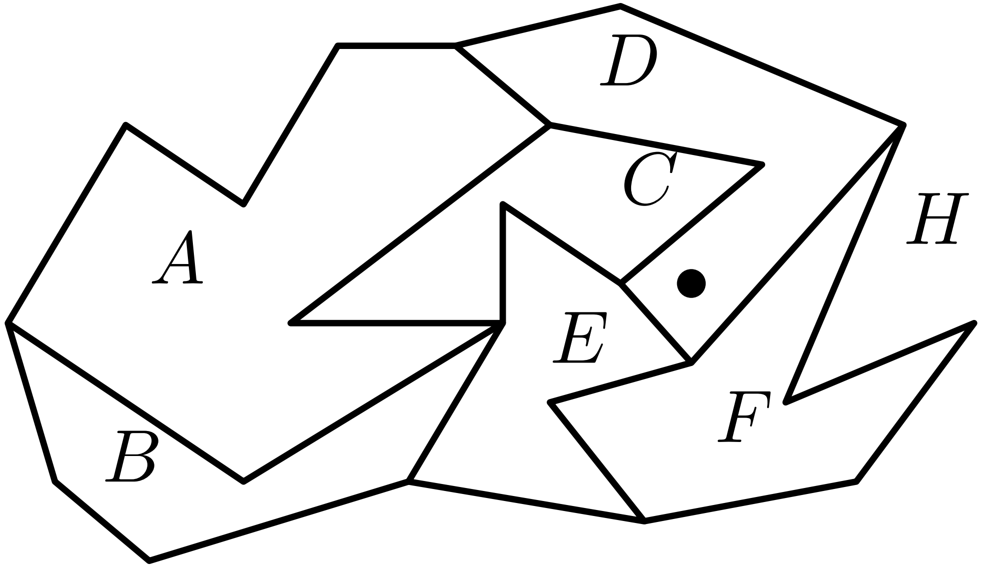
Visibility problems.



Shortest paths.

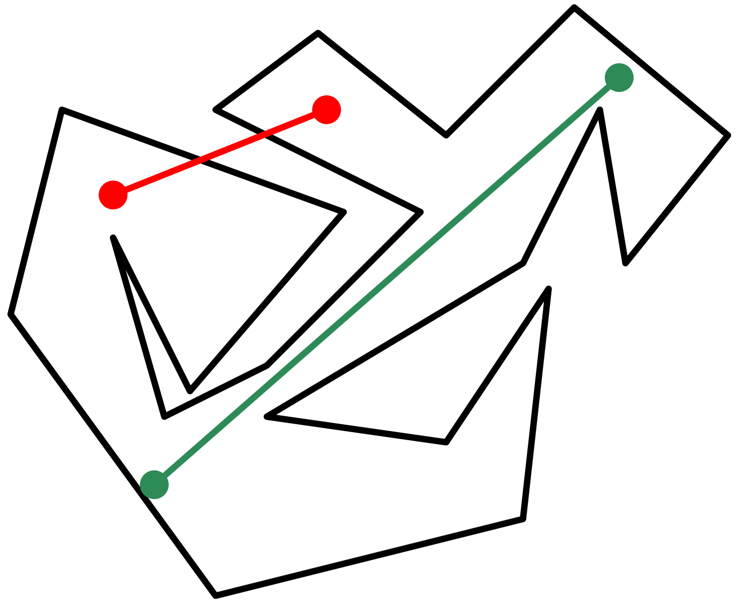


Point location.

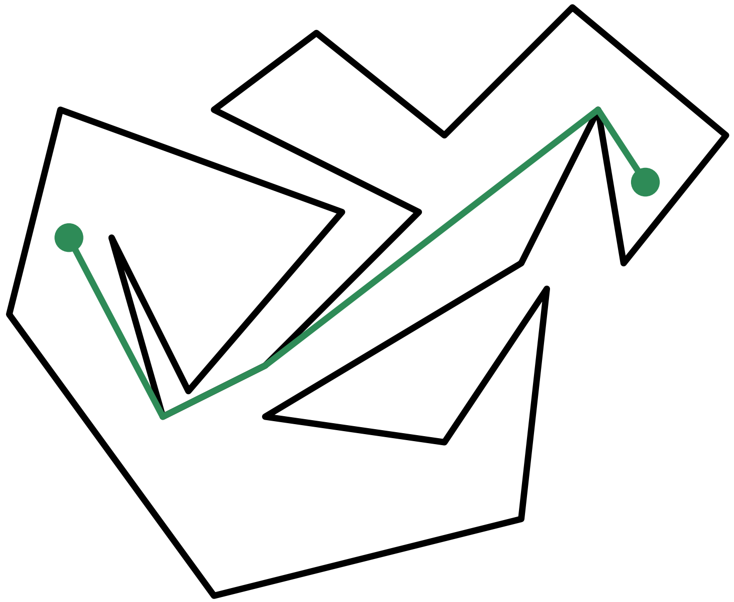


Use of Triangulations

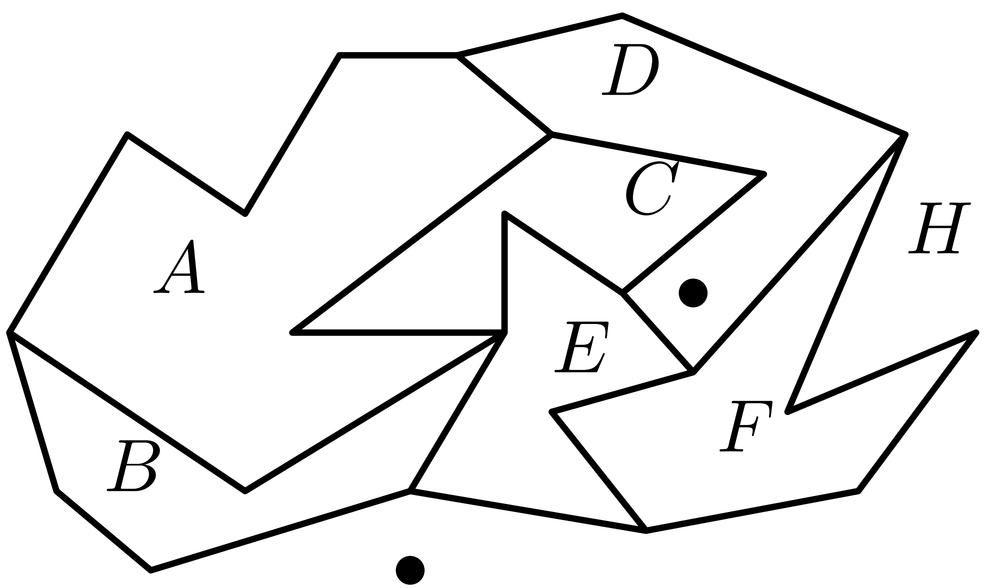
Visibility problems.



Shortest paths.

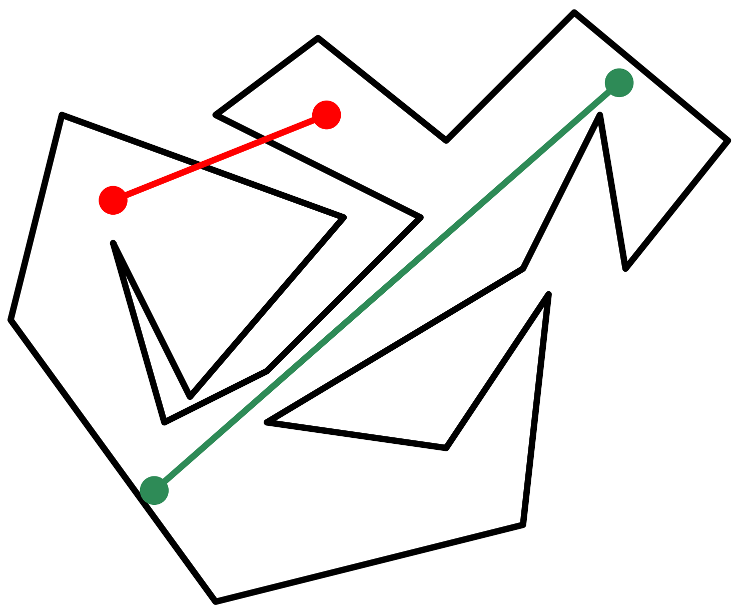


Point location.

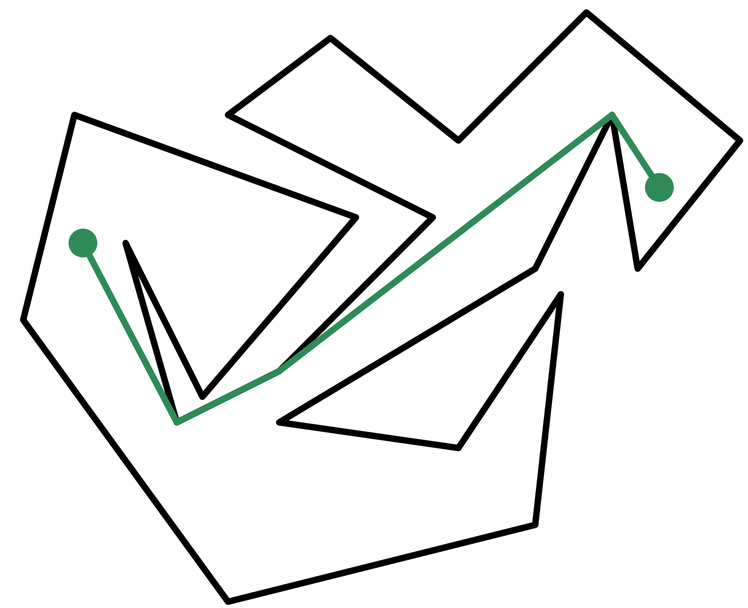


Use of Triangulations

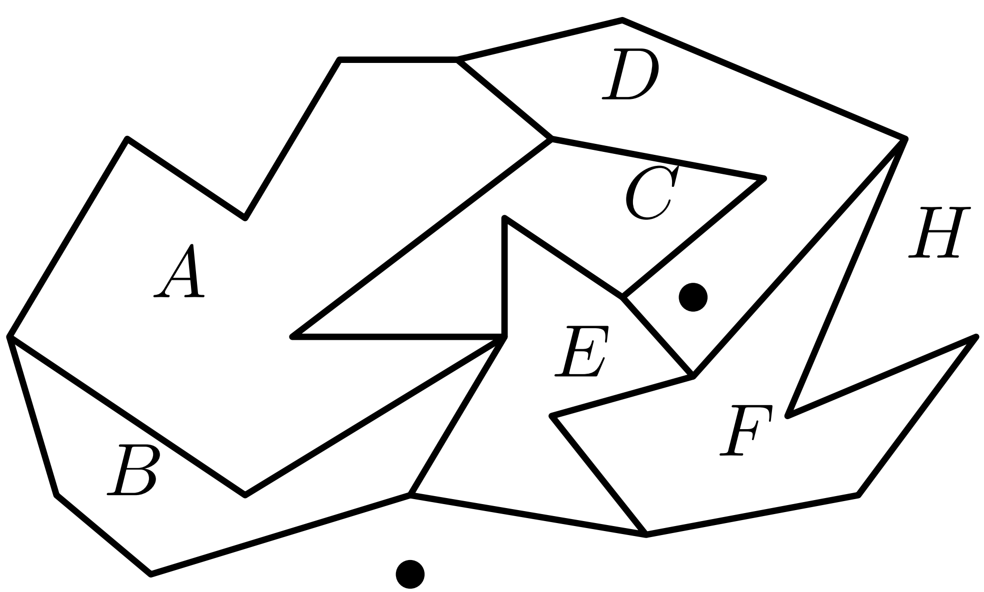
Visibility problems.



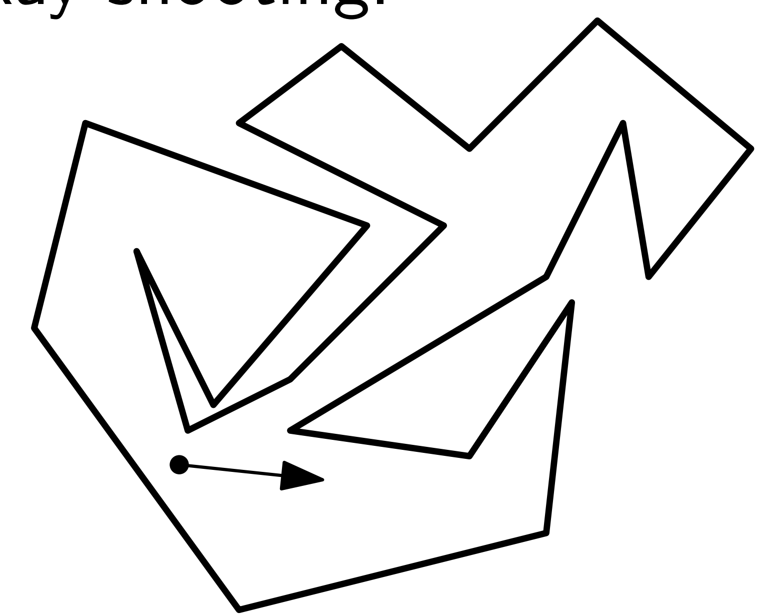
Shortest paths.



Point location.

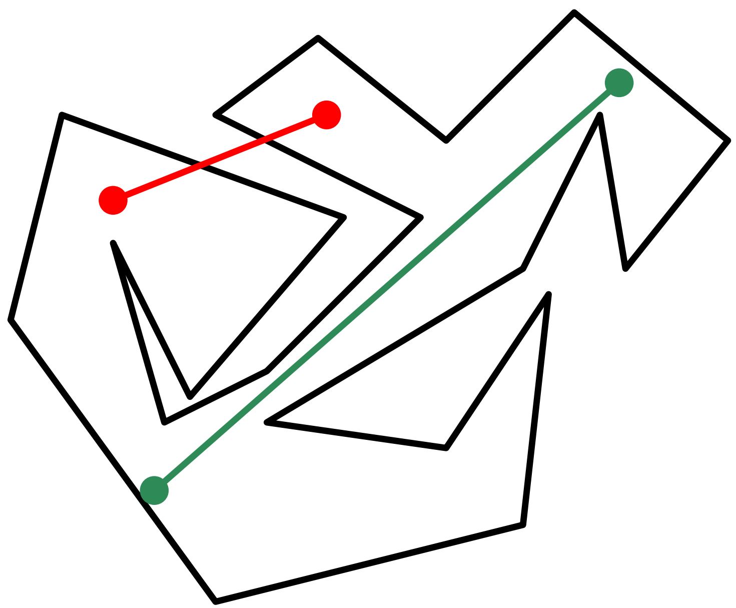


Ray shooting.

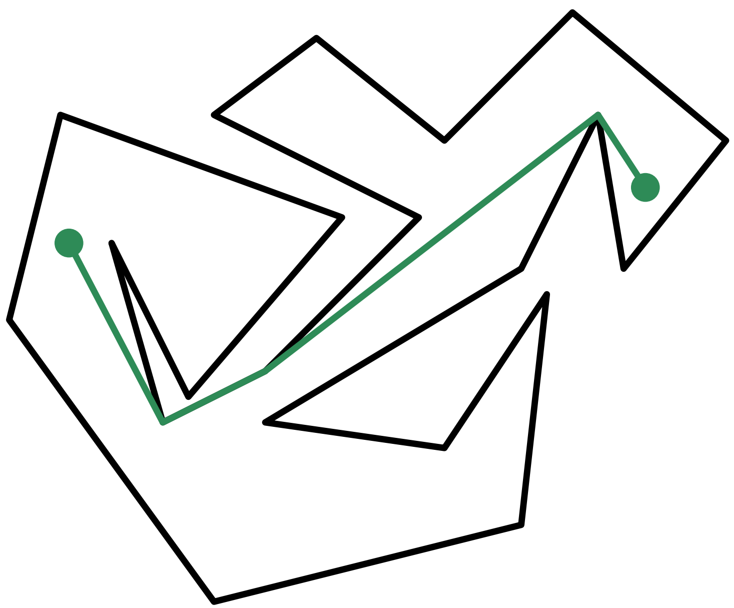


Use of Triangulations

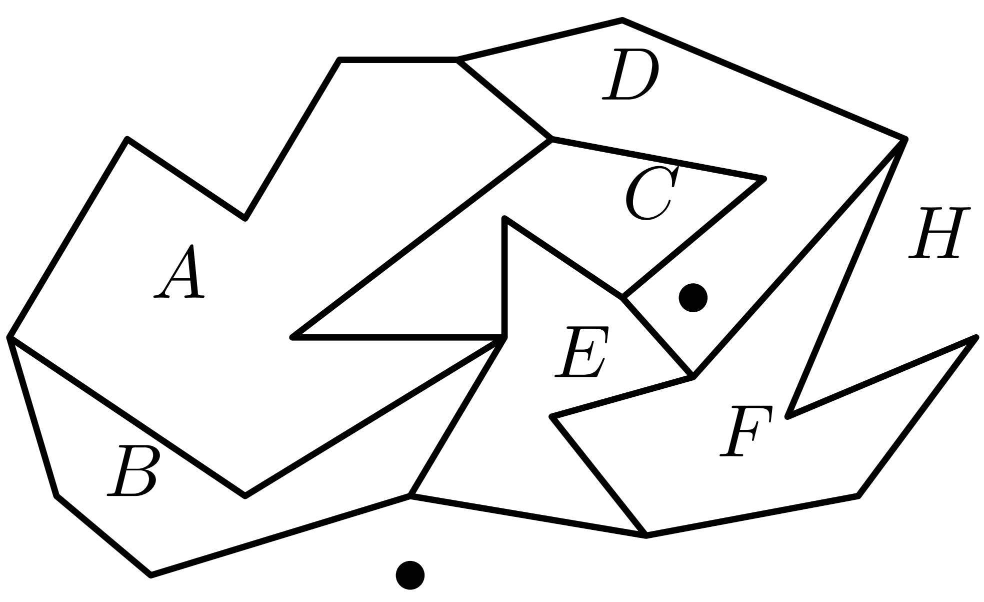
Visibility problems.



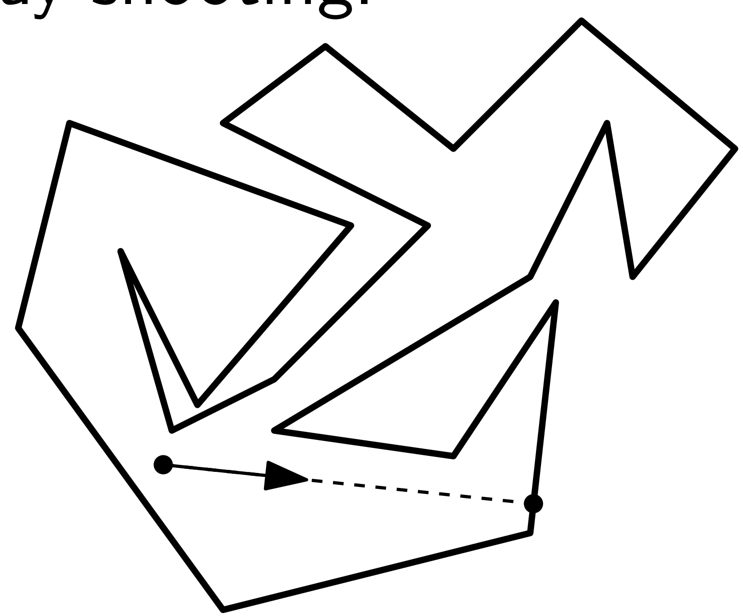
Shortest paths.



Point location.

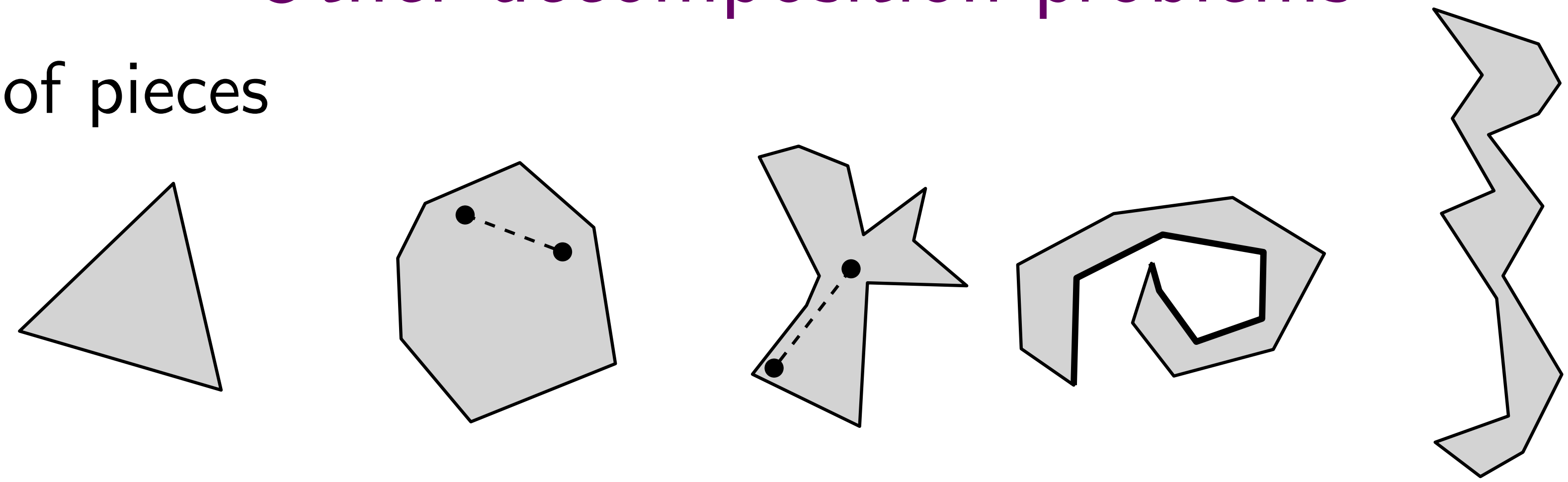


Ray shooting.



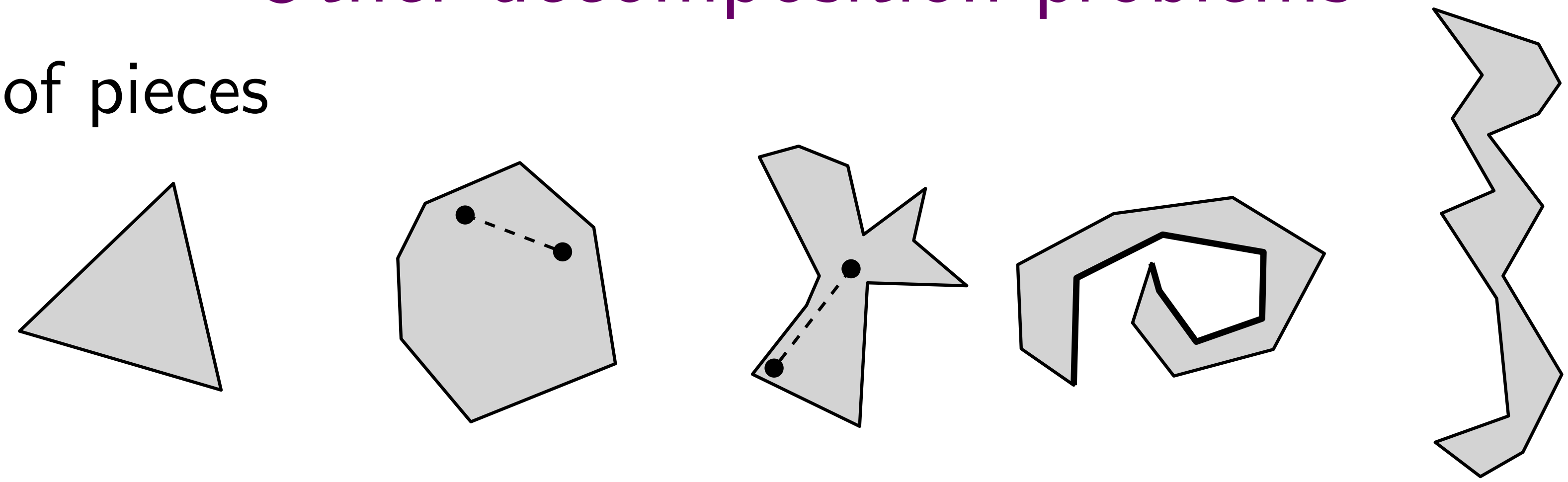
Other decomposition problems

Type of pieces

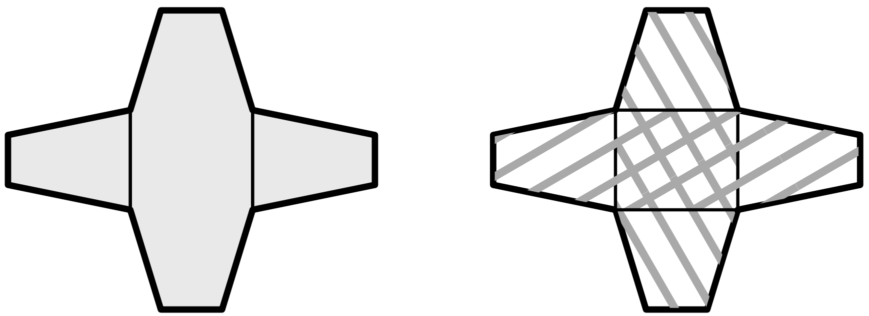


Other decomposition problems

Type of pieces

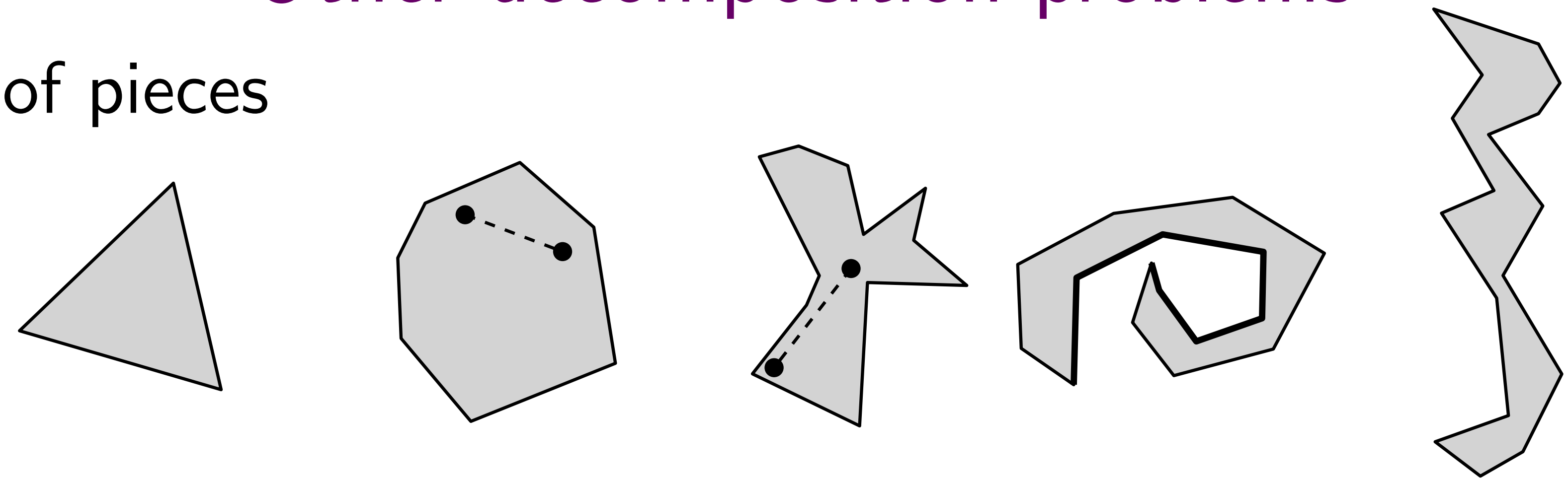


Partition vs. covering

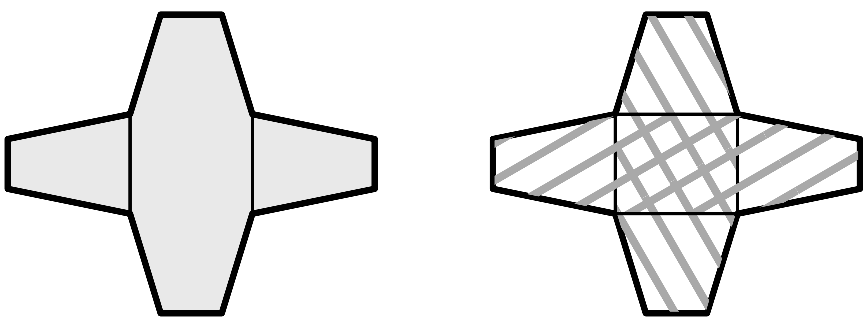


Other decomposition problems

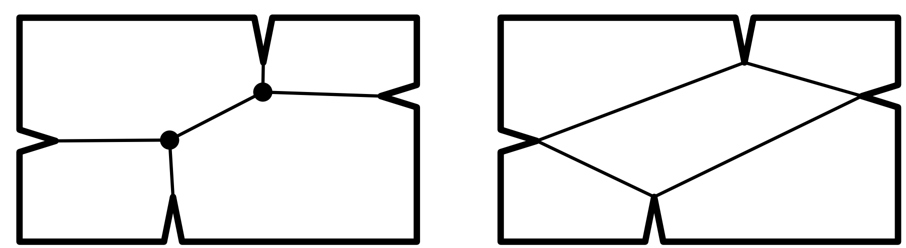
Type of pieces



Partition vs. covering

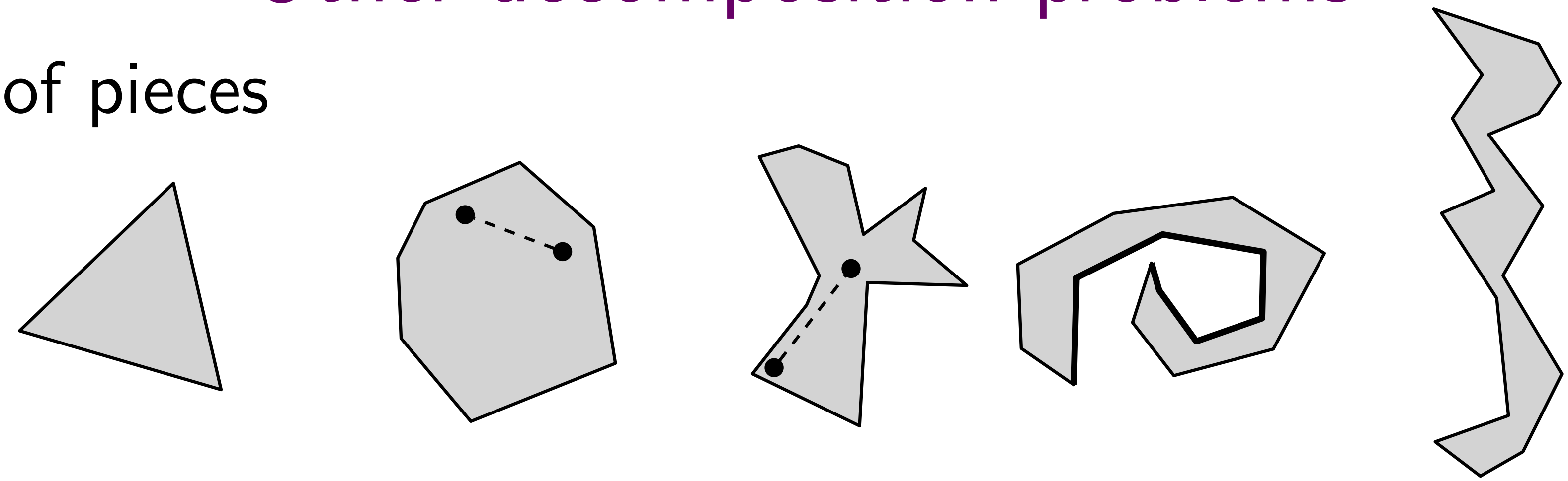


With or without Steiner points

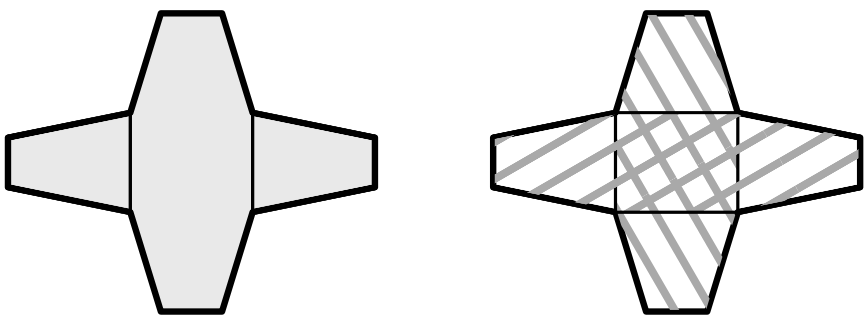


Other decomposition problems

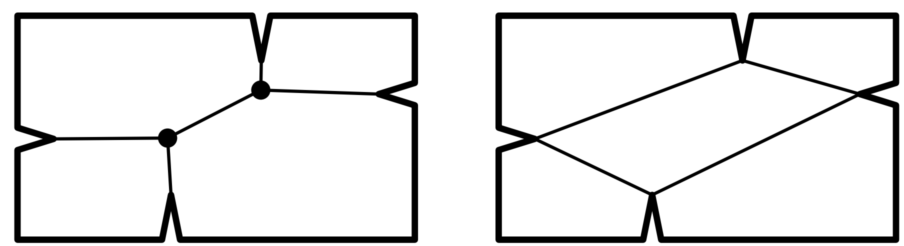
Type of pieces



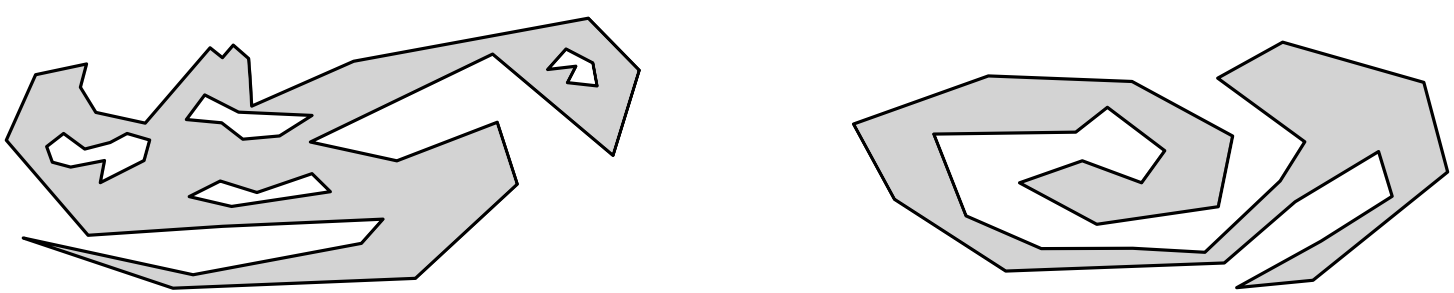
Partition vs. covering



With or without Steiner points

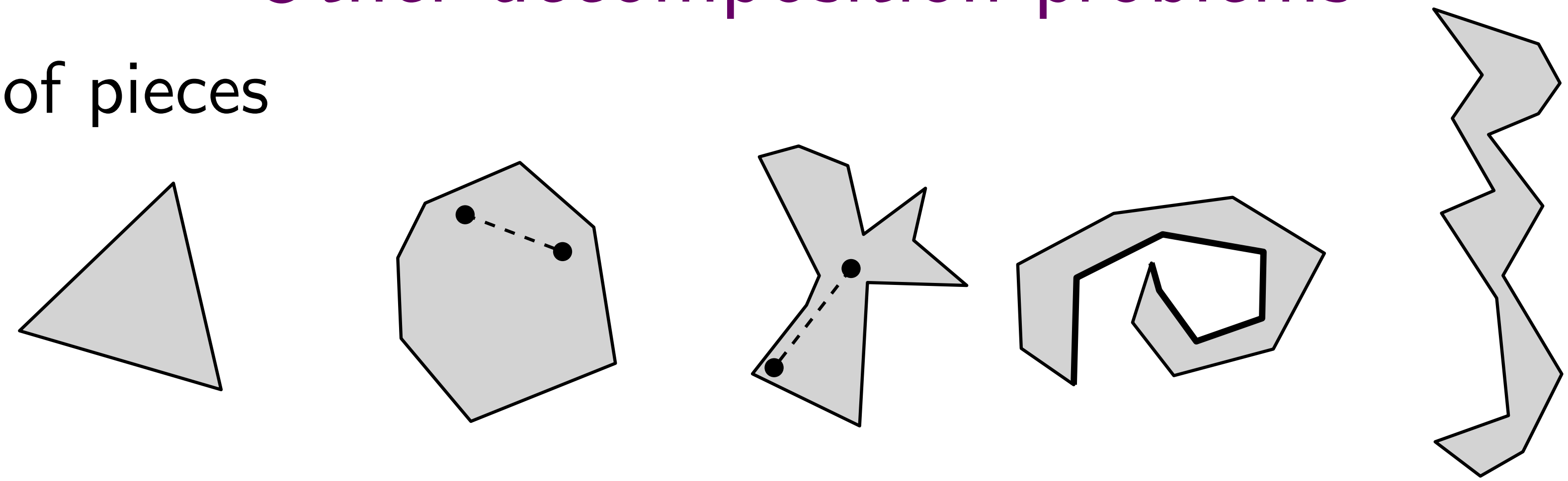


With or without holes

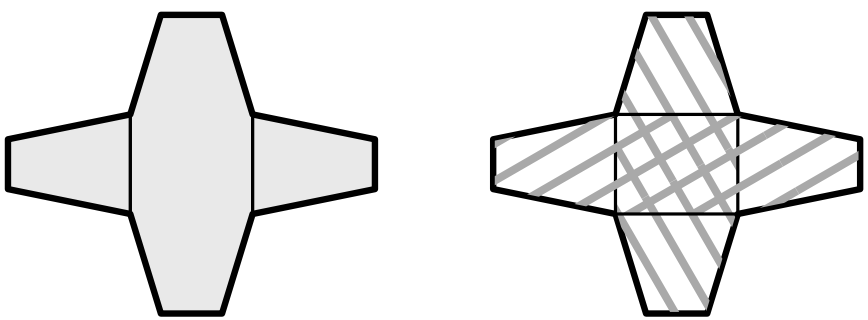


Other decomposition problems

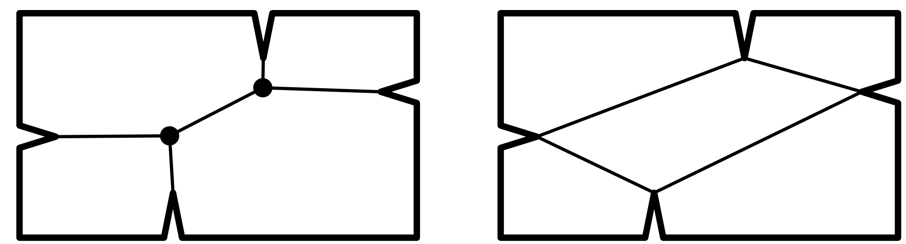
Type of pieces



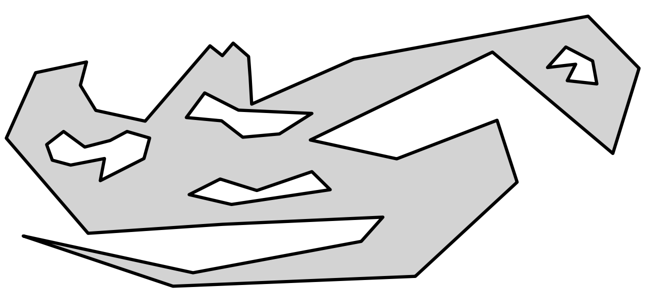
Partition vs. covering



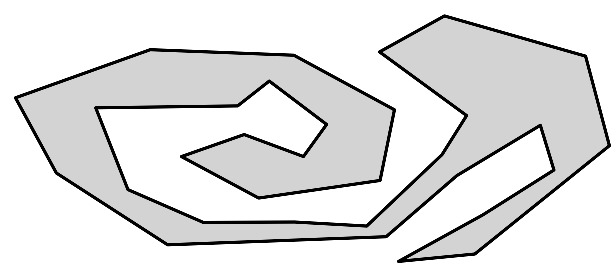
With or without Steiner points



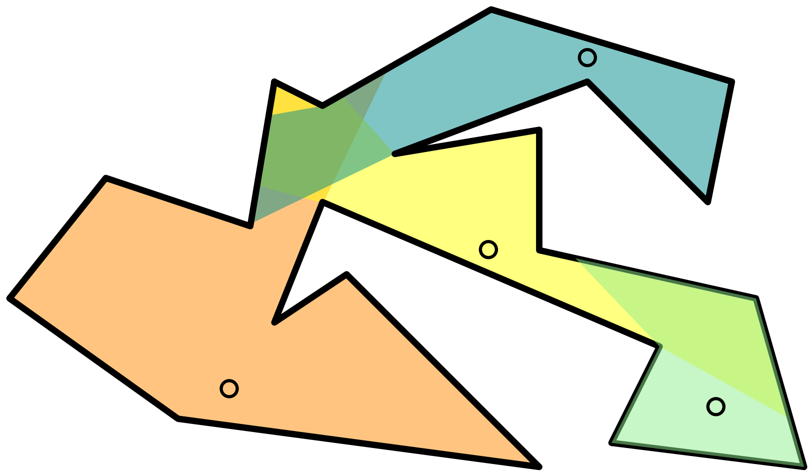
With or without holes

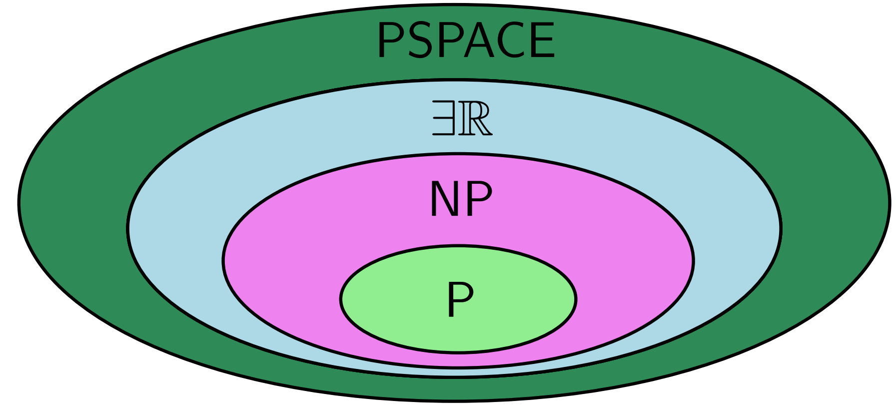


$5 \cdot 2 \cdot 2 \cdot 2 = 40$ problems!



Some of my own work





The Art Gallery Problem Is ∃ℝ-Complete*

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ABSTRACT

We prove that the *art gallery problem* is equivalent under polynomial time reductions to deciding whether a system of polynomial equations over the real numbers has a solution. The art gallery problem is a classic problem in computational geometry, introduced in 1973 by Victor Klee. Given a simple polygon $\mathcal{P}$ and an integer k, the goal is to decide if there exists a set G of k *guards* within $\mathcal{P}$ such that every point $p \in \mathcal{P}$ is seen by at least one guard $g \in G$. Each guard corresponds to a point in the polygon $\mathcal{P}$, and we say that a guard g *sees* a point p if the line segment pg is contained in $\mathcal{P}$.

The art gallery problem has stimulated extensive research in geometry and in algorithms. However, the complexity status of the art gallery problem has not been resolved. It has long been known that the problem is NP-hard, but no one has been able to show that it lies in NP. Recently, the computational geometry community became more aware of the complexity class ∃ℝ, which has been studied earlier by other communities. The class ∃ℝ consists of problems that can be reduced in polynomial time to the problem of deciding whether a system of polynomial equations with integer coefficients and any number of real variables has a solution. It can be easily seen that NP ⊆ ∃ℝ. We prove that the art gallery problem is ∃ℝ-complete, implying that (1) any system of polynomial equations over the real numbers can be encoded as an instance of the art gallery problem, and (2) the art gallery problem is not in the complexity class NP unless NP = ∃ℝ. As a corollary of our construction, we prove that for any real algebraic number α, there is an instance of the art gallery problem where one of the coordinates of the guards equals α in any guard set of minimum cardinality. That rules out many natural geometric approaches to the problem, as it shows that any approach based on constructing a finite set of candidate points for placing guards has to include points with coordinates being roots of polynomials with arbitrary degree. As problem is of independent interest, as it can be used to obtain ∃ℝ-hardness proofs for other problems. In particular, ETR-INV has been used very recently to prove ∃ℝ-hardness of other geometric problems.

CCS CONCEPTS

• **Theory of computation** → **Problems, reductions and completeness**; **Computational geometry**; *Complexity classes*;

KEYWORDS

Art Gallery Problem, Existential Theory of the Reals

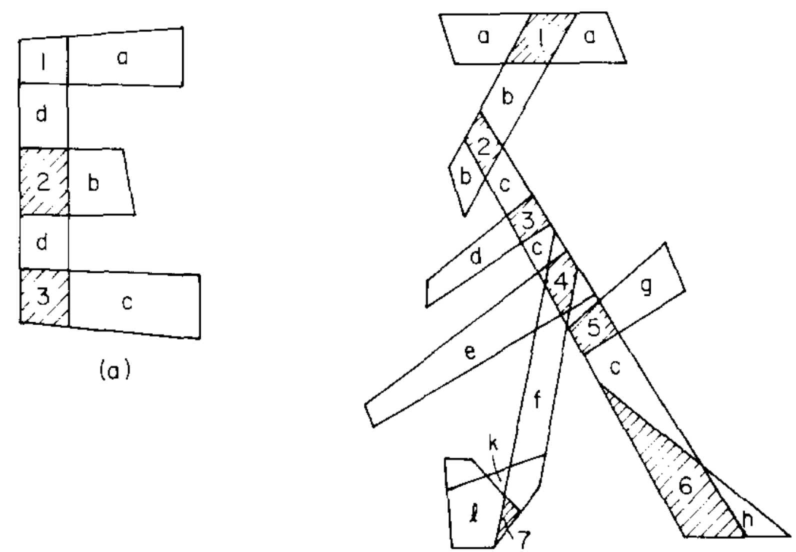
ACM Reference Format:

Mikkel Abrahamsen, Anna Adamaszek, and Tillmann Miltzow. 2018. The Art Gallery Problem Is ∃ℝ-Complete. In *Proceedings of 50th Annual ACM SIGACT Symposium on the Theory of Computing (STOC'18).* ACM, New York, NY, USA, 9 pages. https://doi.org/10.1145/3188745.3188868

1 INTRODUCTION

The Art Gallery Problem. Given a simple polygon $\mathcal{P}$, we say that two points $p, q \in \mathcal{P}$ *see each other* if the line segment pq is contained in $\mathcal{P}$. A set of points $G \subseteq \mathcal{P}$ is said to *guard* the polygon $\mathcal{P}$ if every point $p \in \mathcal{P}$ is seen by at least one guard $g \in G$. Such a set G is called a *guard set* of $\mathcal{P}$, and the points of G are called *guards*. A guard set of $\mathcal{P}$ is *optimal* if it is a minimum cardinality guard set of $\mathcal{P}$.

In the *art gallery problem* we are given an integer g and a polygon $\mathcal{P}$ with corners at rational coordinates, and the goal is to decide if $\mathcal{P}$ has a guard set of cardinality g. We consider a polygon as a Jordan curve consisting of finitely many line segments *and* the region that it encloses. The art gallery problem has been introduced in 1973 by Victor Klee, and it has stimulated extensive research in geometry



(a) (b)

Covering Polygons is Even Harder*

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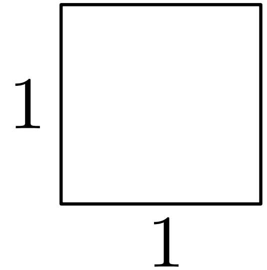
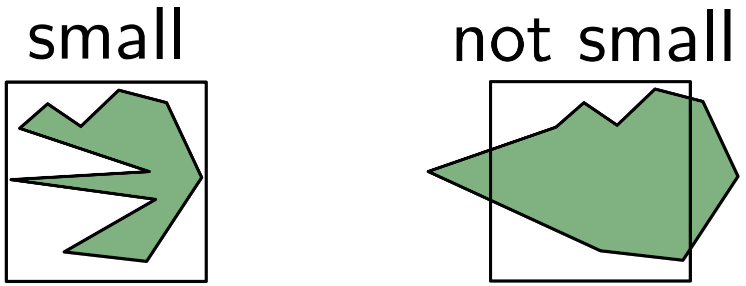
***Abstract*—In the Minimum Convex Cover (MCC) problem, we are given a simple polygon $\mathcal{P}$ and an integer k, and the question is if there exist k convex polygons whose union is $\mathcal{P}$. It is known that MCC is NP-hard [Culberson & Reckhow: Covering polygons is hard, FOCS 1988/Journal of Algorithms 1994] and in ∃ℝ [O'Rourke: The complexity of computing minimum convex covers for polygons, Allerton 1982]. We prove that MCC is ∃ℝ-hard, and the problem is thus ∃ℝ-complete.**

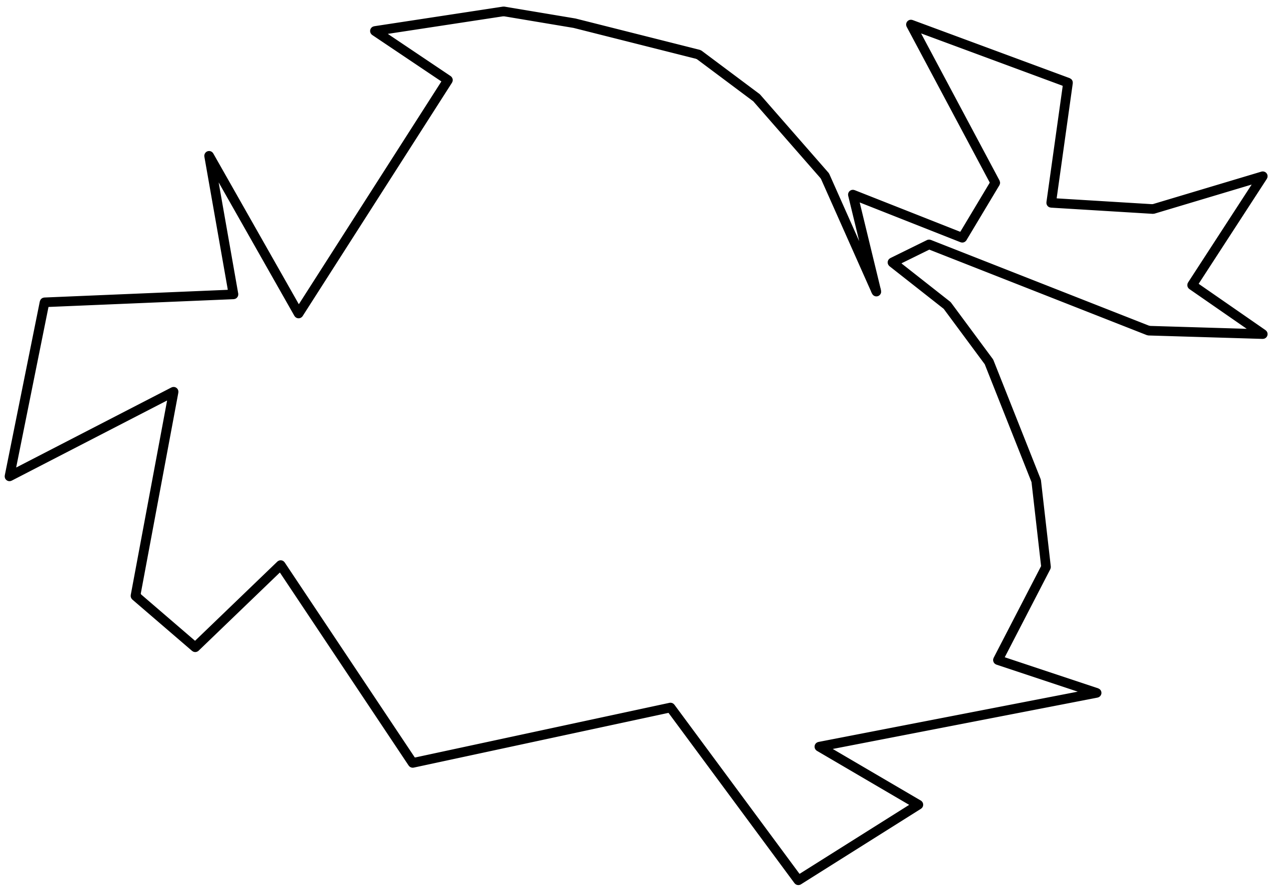
to a more restricted class of polygons. Ideally, we find a decomposition of the polygon $\mathcal{P}$ into the minimum number of pieces.

Many different decomposition problems arise this way, depending on restrictions on the polygon $\mathcal{P}$, the type of basic pieces, and whether the decomposition is a cover or a partition. In covering problems, we just require the union of the pieces to equal $\mathcal{P}$, whereas in partition problems

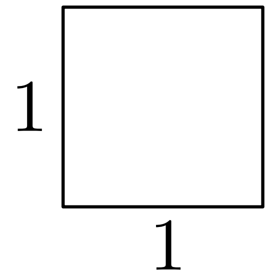
More recent work

small: contained in 





More recent work

small: contained in 

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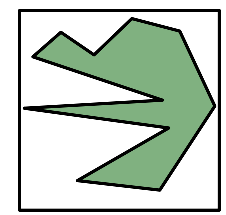

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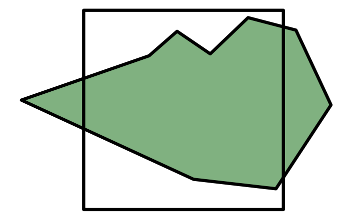
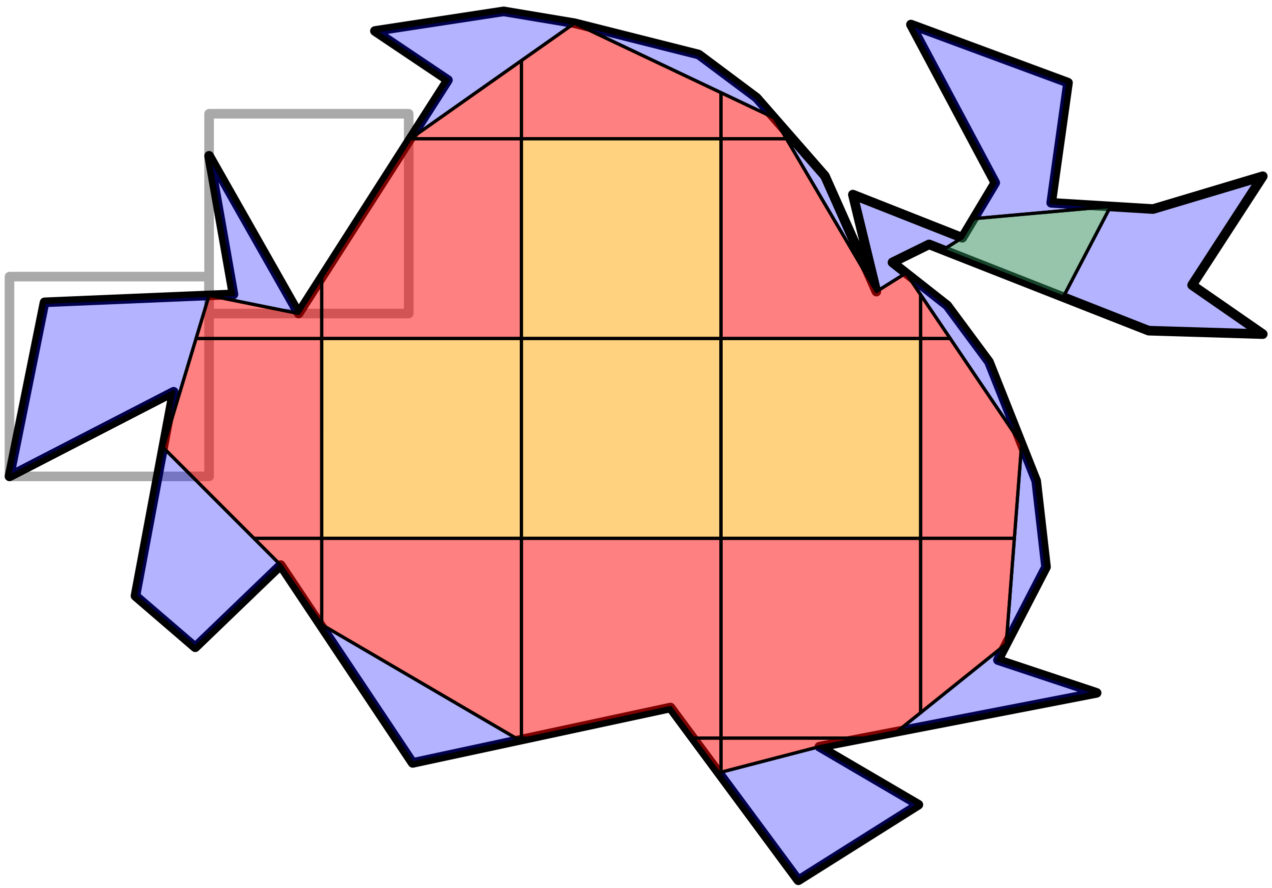
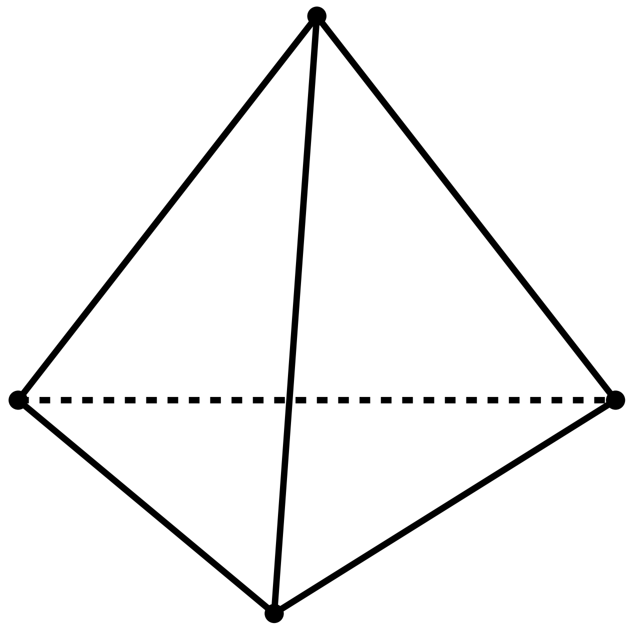
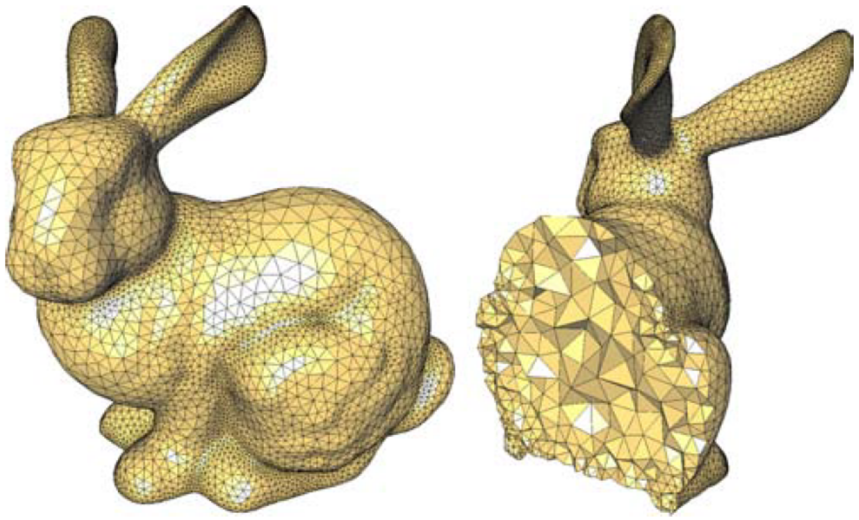




ALG ≤ 13 OPT

3D





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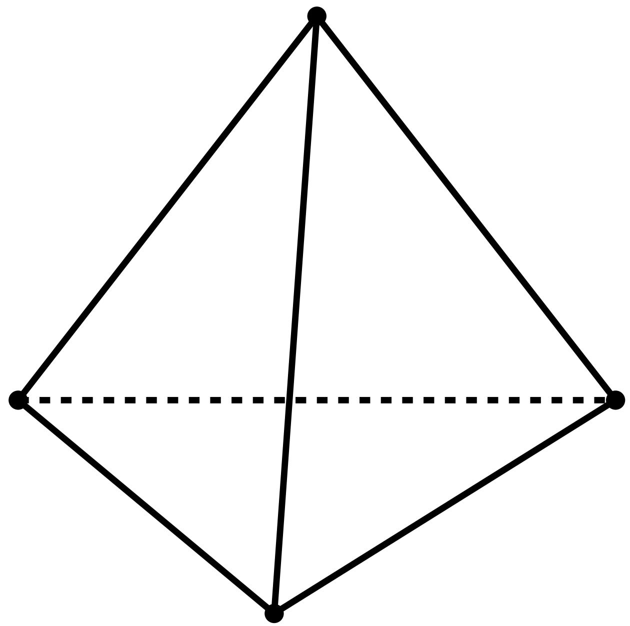

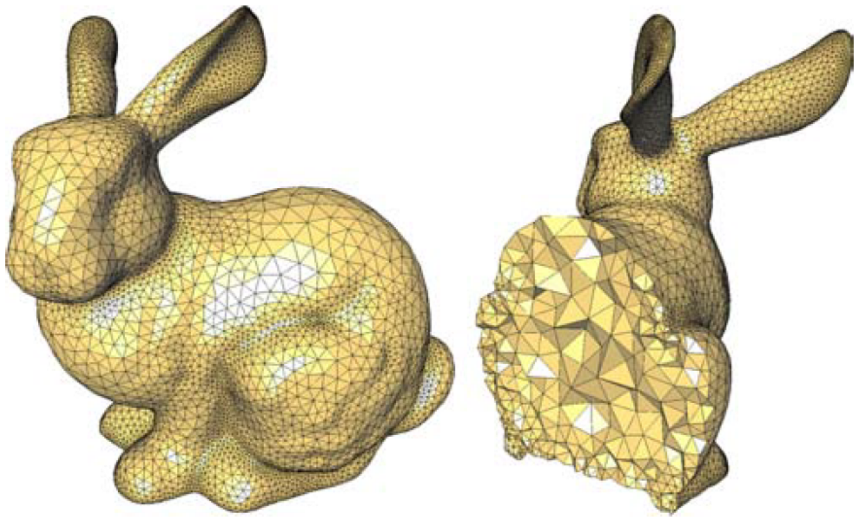

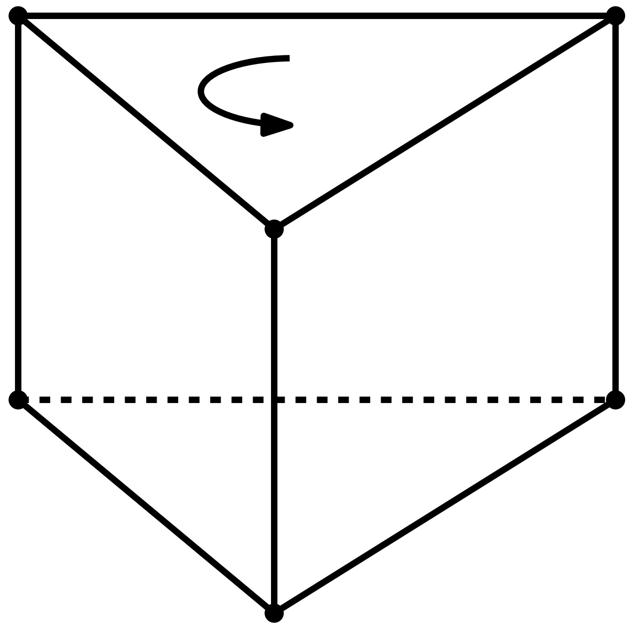

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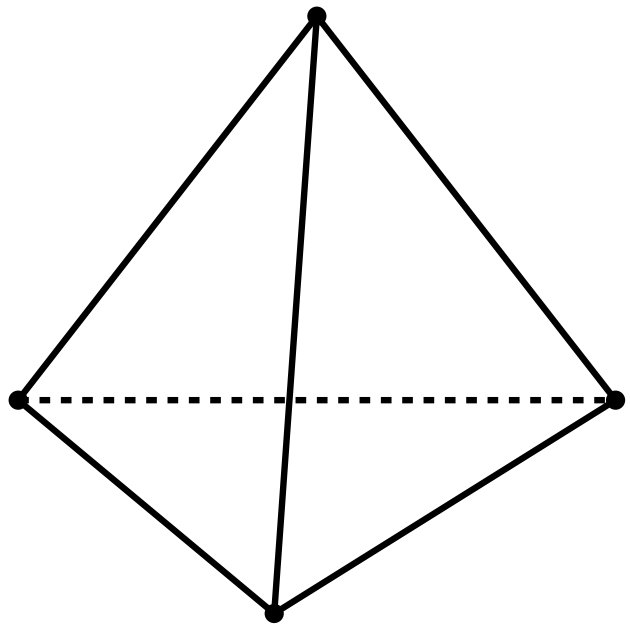

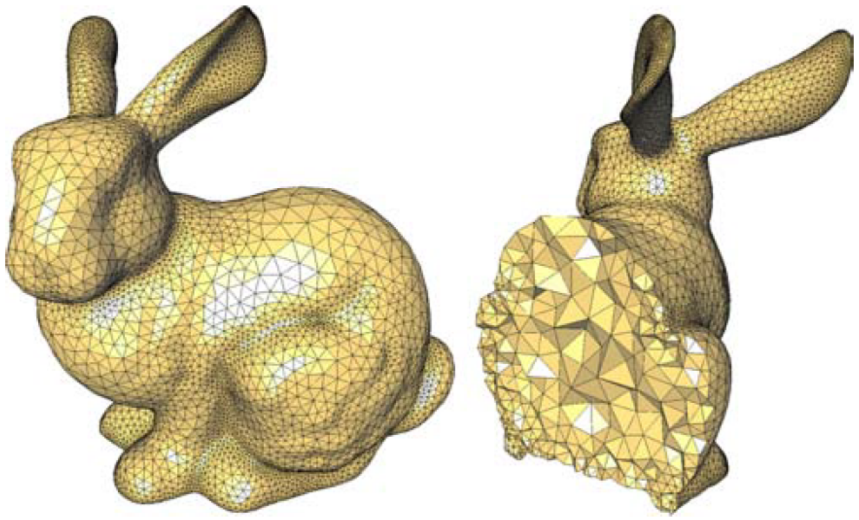

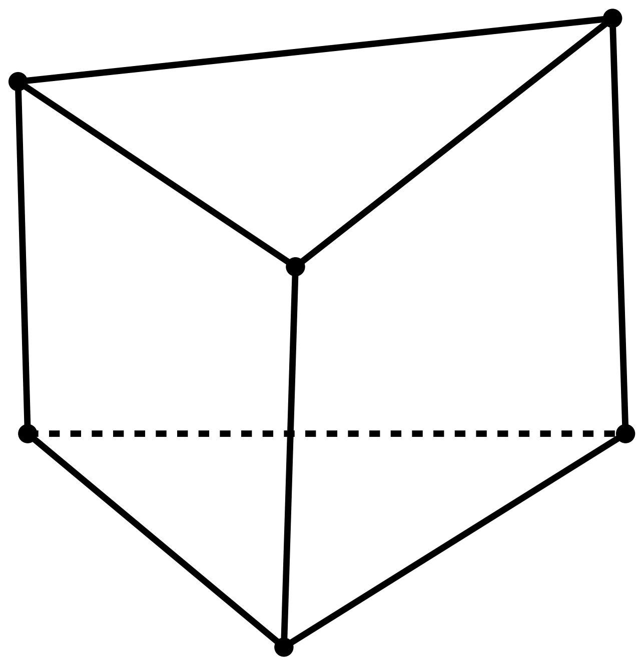

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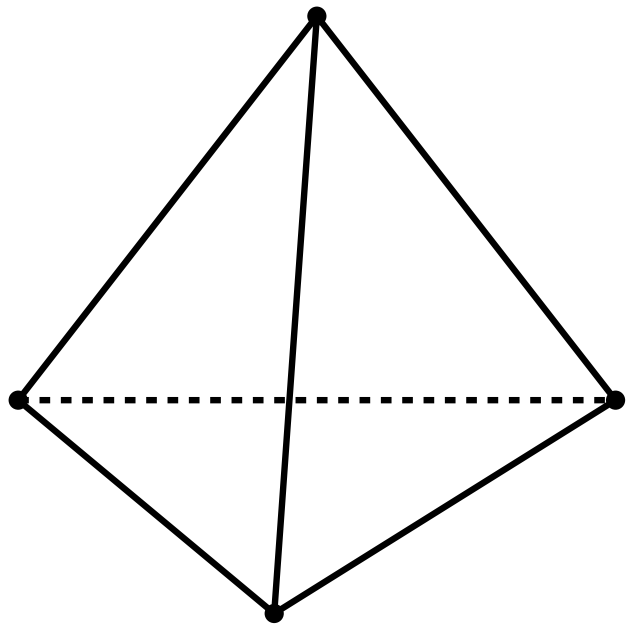

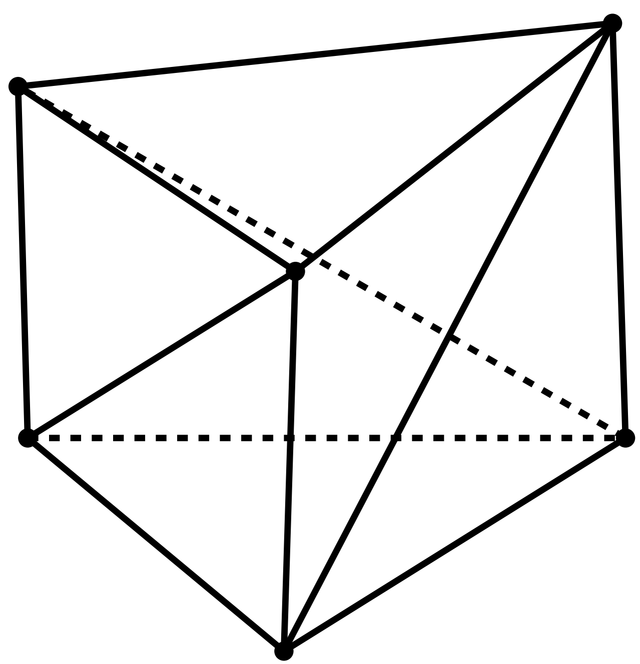

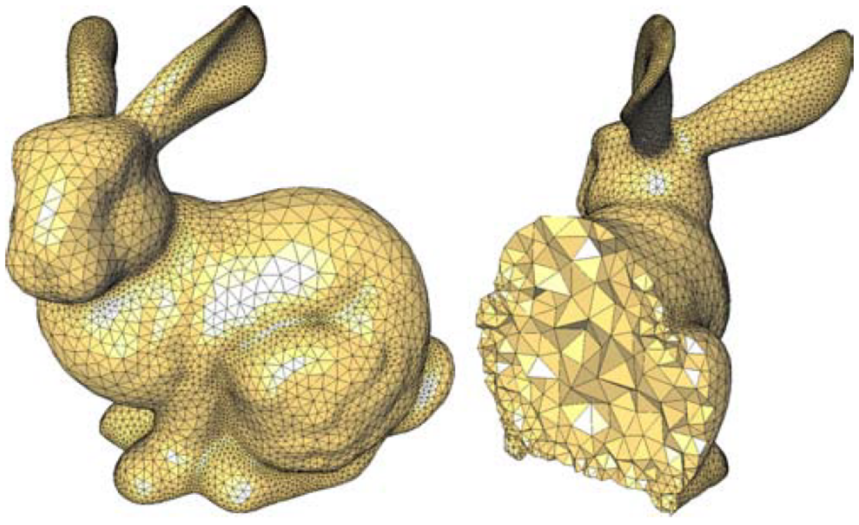

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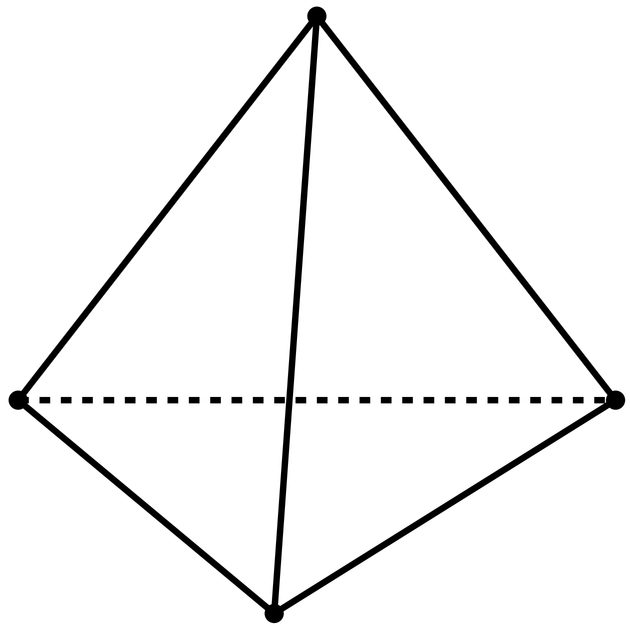

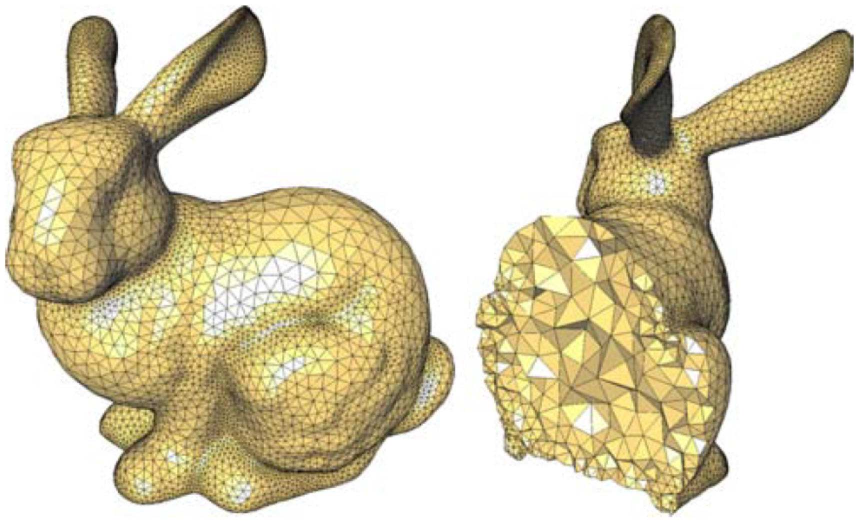

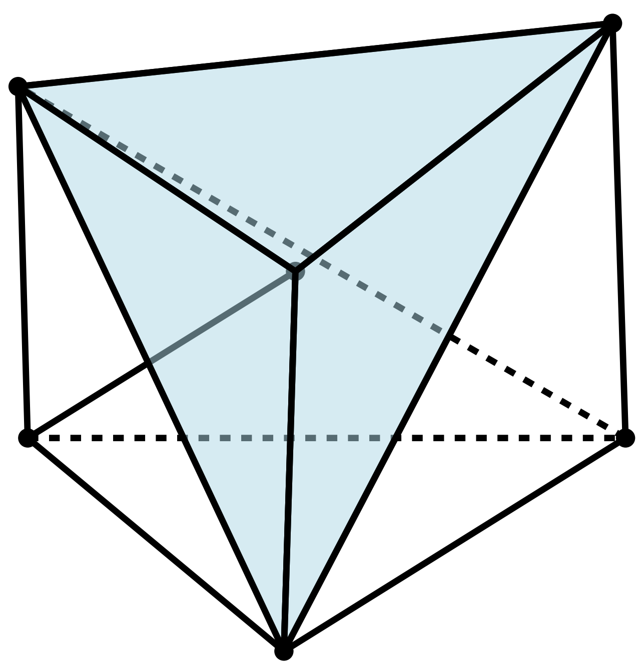

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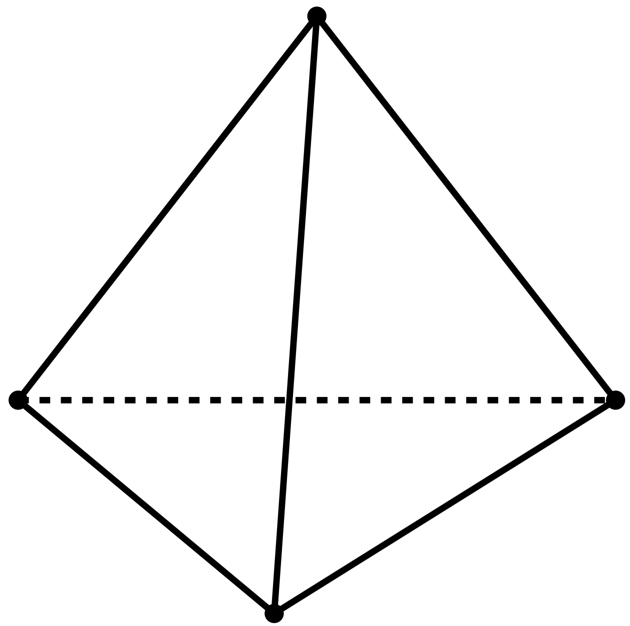

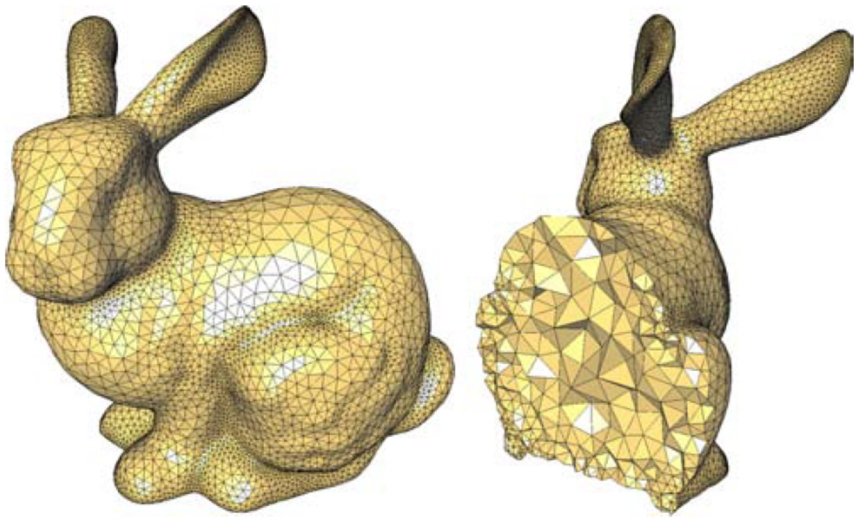

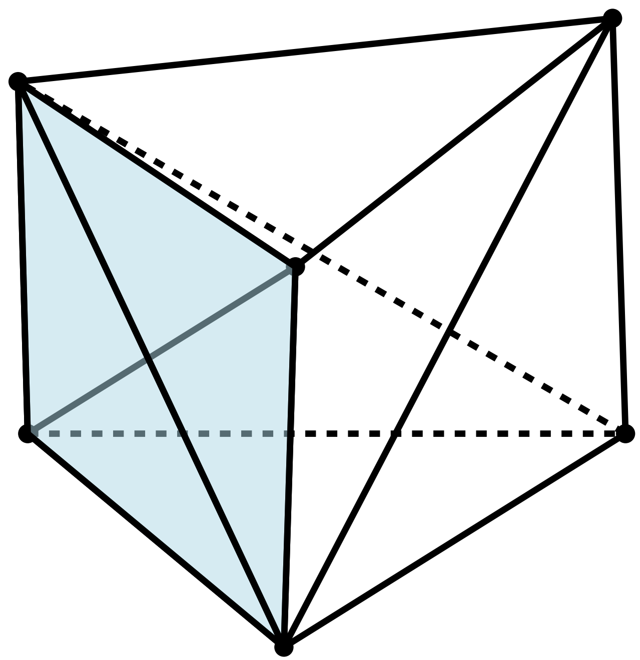

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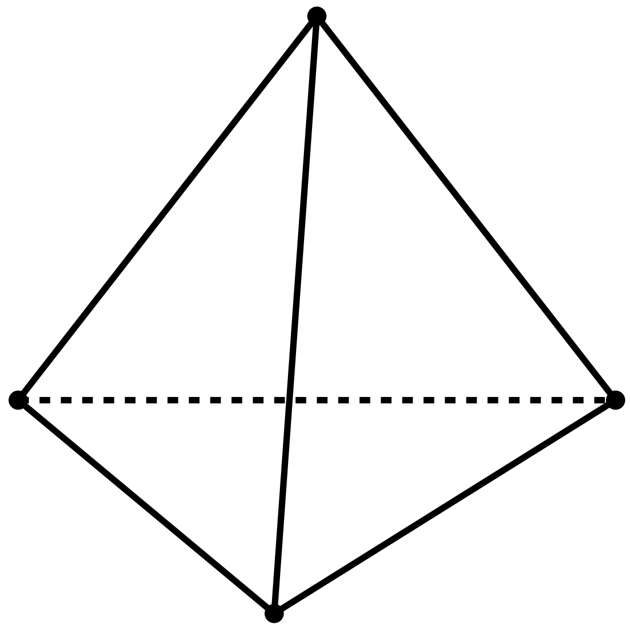

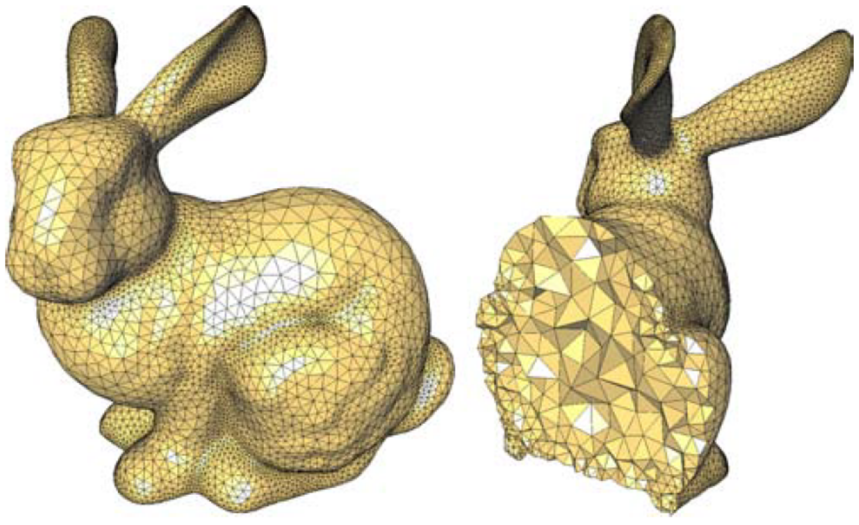

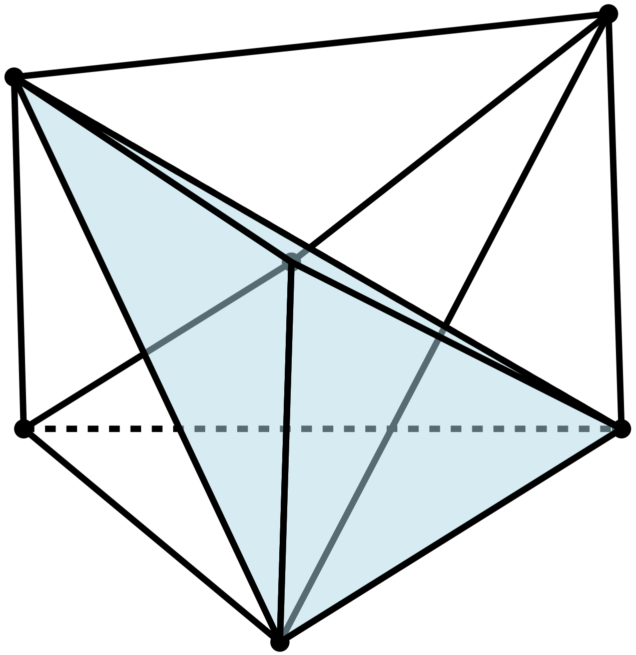

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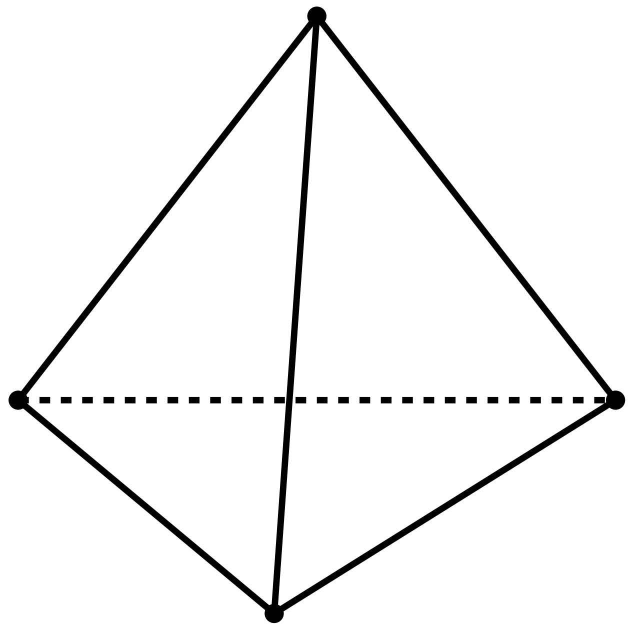

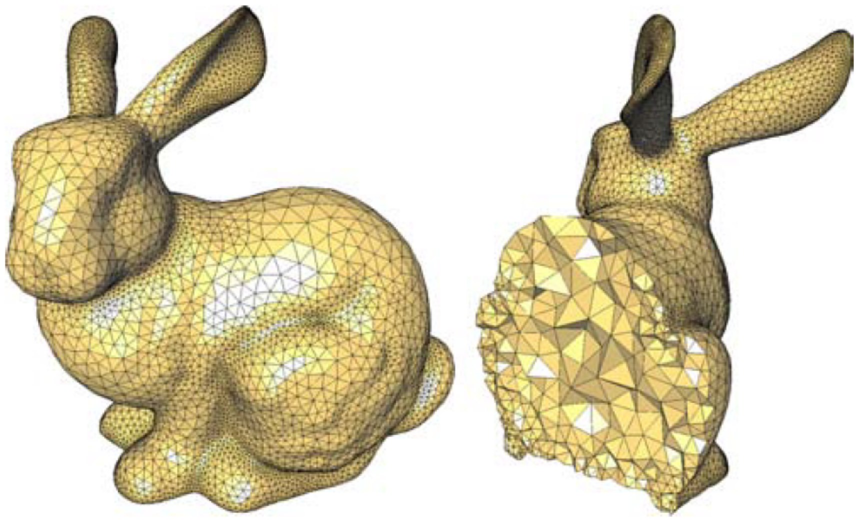

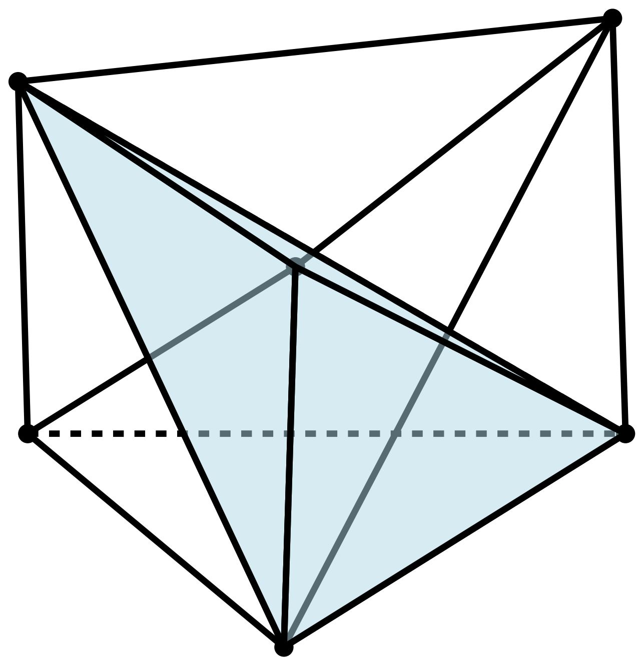

Sometimes $\Theta(n^2)$ Steiner points are needed. NP-complete to decide if possible without Steiner points.

Even when possible without Steiner points, it differs how many tetrahedra are needed.

For some convex polyhedra with n corners, there exist different tetrahedralizations without Steiner points of sizes $\Theta(n)$ and $\Theta(n^2)$.

Take the Computational Geometry Course!

Block 3
Lecturers: Pawel Winter and MA
Exercises, assignments, seminars with student presentations and guest lectures from industry.
One exam question is polygon triangulation.
Many cool topics!