Tutorial 1

September 11, 2019

1. Prove the Bonferroni's inequality.

   (a) Special Case:
   $$P(A_1 \cap A_2) \geq P(A_1) + P(A_2) - 1$$

   (b) General Case:
   $$P(A_1 \cap A_2 \cap \cdots \cap A_n) \geq P(A_1) + P(A_2) + \cdots + P(A_n) - (n-1)$$

2. Prove the following statements:

   (a) If $P(A|B) > P(A|B^c)$, then
   $$P(B|A) > P(B|A^c)$$

   (b) Suppose $0 < P(B) < 1$. A and B are independent if and only if
   $$P(A|B) = P(A|B^c)$$

3. Generally, n ECNUers need to share a dorm. A female student wonders the probability that at least two of the her roommates and herself have a common constellation.

   (a) Suppose that she is an undergraduate. ($n = 4$);

   (b) Suppose that she is a graduate. ($n = 2$);

4. Suppose that the data in Table 1 represent approximate distribution of blood type frequency in percentage of total population.

   Assume that the blood types are distributed the same in both male and female populations. Also, assume that the blood types are independent of marriage.

   (a) What is the probability that is a randomly chosen couple the wife has type B blood and the husband has type O blood?

   (b) It is known that a person with B blood can safely receive transfusions only from people with type B or type O blood. What is the probability a husband has type B or type O blood? It is given that a woman has type B blood, what is the probability that her husband is an acceptable donor for her?

Table 1: Blood Type Frequency

Blood Type	O	A	B	AB
Frequency(%)	45	40	10	5

5. This problem introduces some aspects of a simple genetic model. Assume that genes in an organism occur in pairs and that each member of the pair can be either of the types a or A. The possible genotypes of an organism are then AA, Aa, and aa (Aa and aA are equivalent). When two organisms mate, each independently contributed one of its two genes; either one of the pair is transmitted with probability 0.5.

   (a) Suppose that the genotypes of the parents are AA and Aa. Find the possible genotypes of their offspring and the corresponding probabilities.

   (b) Suppose that the probabilities of the genotypes AA, Aa and aa are $p, 2q$ and r, respectively, in the first generation. Find the probabilities in the second and third generations, and show that these are the same. The result is called the Hardy-Weinberg Law.

6. (**Bonus**) If A is independent of B and B is independent of C, then A is independent of C. Prove this statement or give a counterexample if it is false.