# Tutorial 2

## September 22, 2019

1. Show that each of the following family is an exponential family:
   (a) Binary distribution $bin(n, \theta)$, where $n$ is fixed and $\theta \in (0, 1)$;
   (b) Poisson distribution $P(\theta)$, $\theta > 0$;
   (c) Negative binomial distribution $NB(r, \theta)$, where $r$is fixed and $\theta \in (0, 1)$;
   (d) Exponential distribution $Exp(\theta)$, $\theta > 0$;
   (e) Gamma distribution $Ga(\alpha, \gamma)$ and $\theta = (\alpha, \gamma)$, where $\alpha > 0$ and $\gamma > 0$;
   (f) Beta distribution $Be(\alpha, \beta)$ and $\theta = (\alpha, \beta)$, where $\alpha \in (0, 1)$ and $\beta \in (0, 1)$;

2. Suppose $X$ is a Cauchy random variable, that is, the p.d.f. of $X$ is

$$f(x) = \frac{1}{\pi}\frac{1}{1+x^2}, -\infty < x < \infty$$

   (a) Prove the expectation does not exist.
   (b) Find the median.

3. If $X \sim Exp(\lambda)$, then $Y = X^{1/r}$. $Y$ is said to have the Weibull distribution with two parameter $\gamma$ and $\lambda$. Find the p.d.f and calculate the expectation, variance and $\alpha$th quantiles, where $\alpha$ is given.

4. Suppose $X$ is a discrete random variable and the possible values of $X$ are non-negative integers. If $E(x)$ exists, prove the following statements:
   (a) $E(X) = \sum_{k=1}^{\infty} P(X \geq k)$;
   (b) $\sum_{k=0}^{\infty} kP(X > k) = \frac{1}{2}(E(X^2) - E(X))$;

5. Suppose $X$ is a continuous random variable on the interval $[a, b]$. Prove that
   (a) $a \leq E(X) \leq b$;
   (b) $Var(X) \leq \left(\frac{b-a}{2}\right)^2$

6. Let $f(x)$ be a p.d.f. and let $c$ be a real number such that, for all $\epsilon$, $f(c+\epsilon) = f(c-\epsilon)$. Such a p.d.f. is said to be symmetric about the point $c$.
   (a) Show that if $f(x)$ is symmetric, then the median of $X$ is the number $c$;
   (b) Show that if $f(x)$ is symmetric and $E(X)$ exists, then $E(X) = c$;
   (c) If $c = 0$, then $x_\alpha = -x_{1-\alpha}$.

7. Suppose $X$ is a random variable and $Y = a + bX$ for every real numbers $a, b (b \neq 0)$. Prove that they have the same coefficient of skewness and the same coefficient of kurtosis.