

Tutorial 7

December 8, 2019

1. Suppose that x_1, x_2, \dots, x_{16} is a sample from a normal distribution $N(\mu, 4)$. Consider the hypothesis testing problem

$$H_0 : \mu = 6 \text{ vs } H_1 : \mu \neq 6.$$

The rejection region is $W = \{|\bar{x} - 6| \geq c\}$. Suppose that it is a significance test of level $\alpha = 0.05$.

- (a) Find c ;
- (b) Calculate the Type II error when $\mu = 6.5$;
- (c) Find the power function $g(\mu)$ and plot $g(\mu)$, $\mu \in \mathbb{R}$.

In the following hypothesis testing problems, you need to use two different methods answer the question: (1) derive the rejection region; (2) calculate the p value.

2. Empirically, the weight of a machine part X is distributed as a normal distribution with the mean 15 and the known variance σ^2 , that is $X \sim N(15, \sigma^2)$. The technology is reformed, six parts are randomly selected. Their weights are respectively,

$$14.7, 15.1, 14.8, 15.0, 15.2, 14.6$$

Suppose that the variance is constant. The average weight is still 15? (Let $\alpha = 0.05$)

3. It is reported that junior high school students spend eight hours peering into a screen (TV, computer or cell phone) per week. A junior high school headmaster thinks that students at her school spend less time peering into a screen. Then, she randomly surveys 100 students and the average time is $\bar{x} = 6.5$ h and the standard deviation $s = 2$ h. Is the headmaster right? (Let $\alpha = 0.05$)
4. There are two batches of electron devices. The resistances of these electron devices are measured as follows:

Batch $A(x)$: 0.140, 0.138, 0.143, 0.142, 0.144, 0.137;

Batch $B(y)$: 0.135, 0.140, 0.142, 0.136, 0.138, 0.140;

Suppose that the resistances of two batches are respectively distributed as normal distributions $N(\mu_1, \sigma_1^2)$ and $N(\mu_2, \sigma_2^2)$. Two samples are independent. Let $\alpha = 0.05$.

- (a) Are two population variances are equal?
- (b) Are two population means are equal?

5. Proteinuria, the presence of excess protein in urine, is a symptom of renal (kidney) distress among diabetics. Taguma et al. (1985) studied the effects of captopril for treating proteinuria in diabetics. Urinary protein was measured for 12 patients before and after eight weeks of captopril therapy. The amounts of urinary protein (in g/24 hrs) before and after therapy are shown in the following table. What can you conclude about the effect of captopril?

Before	After
24.6	10.1
17.0	5.7
16.0	5.6
10.4	3.4
8.2	6.5
7.9	0.7
8.2	6.5
7.9	0.7
5.8	6.1
5.4	4.7
5.1	2.0
4.7	2.9

6. A pharmaceutical factory produces a new pain reliever and it is interested in the onset of action - the duration of time it takes for a drug's effects to come to prominence upon administration. The factory wants to verify that the onset of action mean of the new pain reliever is a half or less, comparing with an old one. Then, the proposed hypotheses are

$$H_0 : \mu_1 = 2\mu_2 \text{ vs } H_1 : \mu_1 > 2\mu_2$$

where μ_1 and μ_2 are the mean of onset of action of the old and new pain reliever. Suppose that two populations are two normal distributions with given σ_1^2 and σ_2^2 . Suppose that x_1, x_2, \dots, x_m and y_1, y_2, \dots, y_n are two samples and two samples are independent.

- Find the test statistic for this testing problem;
- Derive the rejection region;