Tutorial 8

December 22, 2019

1. A factory produced a kind of components and the average lifetime was 1200 hours. With the technical development, the lifetime of the components is thought to be improved. Now 8 components are randomly selected and used for a lifetime test. The data are shown as follows:

$$2686 \quad 2001 \quad 2082 \quad 792 \quad 1660 \quad 4105 \quad 1416 \quad 2089.$$

   Suppose that the lifetime of a component is distributed as an exponential distribution. After the technical development, the average lifetime of the components has been improved at the significance level $\alpha = 0.05$?

2. Suppose that $x_1, x_2, \cdots, x_n$ is a sample from a normal distribution $N(\mu, \sigma^2)$. Construct a likelihood ratio test for $H_0 : \sigma^2 = \sigma_0^2$ vs $H_1 : \sigma^2 \neq \sigma_0^2$.

3. Consider that the significance level $\alpha = 0.05$.

   (a) In 1965, a newspaper carried a story about a high school student who reported getting 9207 heads and 8743 tails in 17950 coin tosses. Is this a significant discrepancy from the null hypothesis $H_0 : p = \frac{1}{2}$?

   (b) Jack Youden, a statistician at the National Bureau of Standards, contacted the student and asked him exactly how he had performed the experiment (Youden 1974). To save time, the student had tossed groups of five coins at a time, and a younger brother had recoded the results, shown in the following tables: Are

Number of Heads	Frequency
0	100
1	524
2	1080
3	1126
4	655
5	105

   the data consistent with the hypothesis that all the coins were fair $p = \frac{1}{2}$?

   (c) Are the data consistent with the hypothesis that all five coins had the sample probability of heads but that this probability was not necessarily $\frac{1}{2}$?

4. Suppose that there are n products with two attributes A and B. This is a contingency table: where $n = a + b + c + d$. Prove the test statistic can be written as

$$\chi^2 = \frac{n(ad - bc)^2}{(a+b)(c+d)(a+c)(b+d)}.$$

	B	$\overline{B}$	Sum
A	a	b	$a+b$
$\overline{A}$	c	d	$c+d$
Sum	$a+c$	$b+d$	n

5. Dowdall (1974) [also discussed in Haberman (1978)] studied the effect of ethnic background on role attitude of women of ages 15-64 in Rhode Island. Respondents were asked whether they thought it was all right for a woman to have a job instead of taking care of the house and children while her husband worked. The following table breaks down the responses by ethnic origin of the respondent. Is there a relationship between response and ethnic group? If so, describe it.

Ethnic Origin	Yes	No
Italian	78	47
Northern European	56	29
Other European	43	29
English	53	32
Irish	43	30
French Canadian	36	22
French	42	23
Portuguese	29	7

6. Suppose that $X \sim N(\mu_1, \sigma_1^2)$ and $Y \sim N(\mu_2, \sigma_2^2)$. There are two independent samples with $n_1 = 10$ and $n_2 = 15$. It is calculated that $\bar{x} = 82, s_x^2 = 56.5, \bar{y} = 76, s_y^2 = 52.4$.

   (a) Given $\sigma_1^2 = 64$ and $\sigma_2^2 = 49$, find a 95% confidence interval for $\mu_1 - \mu_2$;

   (b) If $\sigma_1^2 = \sigma_2^2$, find a 95% confidence interval for $\mu_1 - \mu_2$;

   (c) If we know nothing about σ_1^2 and σ_2^2, find a 95% confidence interval for $\mu_1 - \mu_2$;

   (d) Find a 95% confidence interval for σ_1^2/σ_2^2;

7. Suppose that $x_1, x_2, \cdots, x_n$ is a sample from $N(\mu, 16)$. We would like to find a $1-\alpha$ confidence interval of μ and the length of CI is not larger than a given constant L. How many samples do you think we need?