Tutorial 4 Solutions

1. The results are shown in the table below: And the R code is

Table 1: Results of Two Methods with Different Sample Size.

	Method 1			Method 2		
	100	500	1000	100	500	1000
a)	0.410	0.412	0.426	0.4095157	0.4444560	0.4258007
b)	1.652947	1.697622	1.689031	1.651872	1.743808	1.716597

```
set.seed(1001)
n <- c(100, 500, 1000)
J_1_a <- NULL
J_1_b <- NULL
J_2_a <- NULL
J_2_b <- NULL
for (i in 1:3){
  x_1_a <- runif(n[i], 0, 1)
  y_1_a <- runif(n[i], 0, 1)
  x_1_b <- runif(n[i], 0, 1)
  x_2_a <- runif(n[i], 0, 1)
  y_2_a <- runif(n[i], 0, 1)
  x_2_b <- runif(n[i], 0, 1)
  count_1 <- 0
  sum_1 <- 0
  count_2 <- 0
  sum_2 <- 0
  for(j in 1:n[i]){
    if (y_1_a[j] < (exp(1)^x_1_a[j] - 1)/(exp(1) - 1)){
      count_1 <- count_1 + 1
    }
    sum_1 <- sum_1 + (exp(1)^x_1_b[j] - 1)/(exp(1) - 1)

    if (y_2_a[j] < (exp(1)^(x_2_a[j]) - 1)/(exp(1) - 1)){
      count_2 <- count_2 + 1
    }
    sum_2 <- sum_2 +  exp(1)^x_2_b[j]
```

```
  }
  J_1_a <- c(J_1_a, count_1/n[i])
  J_1_b <- c(J_1_b, sum_1/n[i])
  J_2_a <- c(J_2_a, count_2/n[i] * (exp(1) - 1) + 1)
  J_2_b <- c(J_2_b, sum_2/n[i])
}
print(J_1_a)
print(J_1_b)
print(J_2_a)
print(J_2_b)
```

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2. Since U_i is a uniform distribution $U(0,10)$, $E(U_i) = 5, Var(U_i) = \frac{100}{12}$. By using Central Limit Theorem, we have $\frac{\sum_{i=1}^{50} U_i - 50\times 5}{\sqrt{50}*\sqrt{\frac{100}{12}}} \sim N(0,1)$. Thus,

$$
\begin{aligned}
P(\sum_{i=1}^{50} U_i > 300) =& P(\frac{\sum_{i=1}^{50} U_i - 50\times 5}{\sqrt{50}*\sqrt{\frac{100}{12}}} > \frac{300 - 50\times 5}{\sqrt{50}*\sqrt{\frac{100}{12}}}) \\
\approx& P(Z_n > \sqrt{6}) \\
=& 1 - \Phi(\sqrt{6}) \approx 0.0072,
\end{aligned}
$$

where Z_n is standard normal distribution and $\Phi(\cdot)$ is the cdf of standard normal distribution.

3. Suppose X_i are i.i.d. $Exp(1)$(exponential distribution with parameter 1), we have $E(X_i) = 1, Var(X_i) = 1$,thus $X = \sum_{i=1}^{n} X_i \sim Ga(n,1)$. By using Central Limit Theorem, we have $\frac{X-n}{\sqrt{n}} \sim N(0,1)$. Thus,

$$
\begin{aligned}
P(|\frac{X}{n} - 1| > 0.01) =& P(|\frac{X-n}{\sqrt{n}}| > 0.01\sqrt{n}) \\
&\approx 2(1 - \Phi(0.01\sqrt{n}) < 0.01,
\end{aligned}
$$

where $\Phi(\cdot)$ is the cdf of standard normal distribution.Therefore $\Phi(0.01\sqrt{n}) > 0.995$ and $n > 66348.9660$.