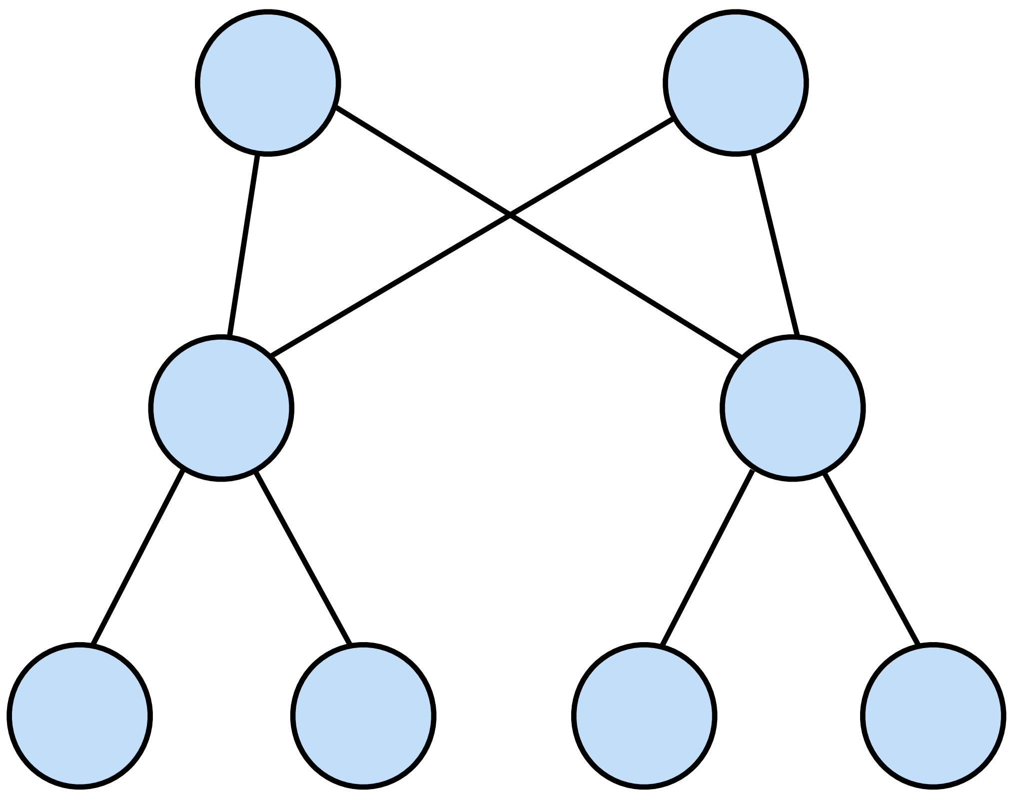
Canonical Graph Shapes

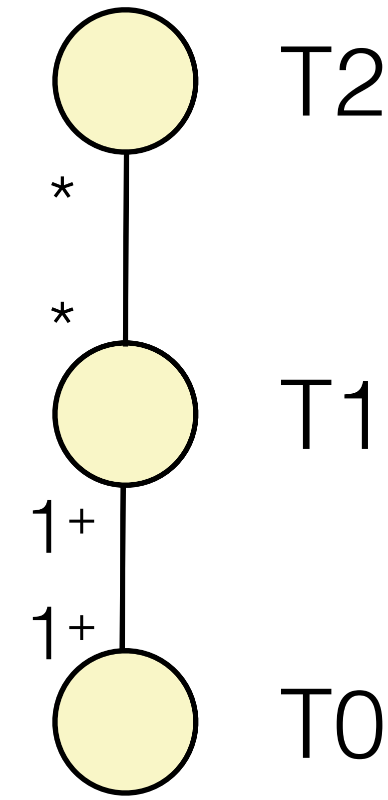
Local Shape Logic

$$\xi ::= v \ \mid\ \xrightarrow{a} v \ \mid\ \xleftarrow{a} v \ \mid\ \overset{a}{\text{—}} \ .$$
$$\phi ::= \mathbf{tt} \ \mid\ {}^{\mu}[\xi] \ \mid\ \neg\phi \ \mid\ \phi \vee \phi \ \mid\ \forall_v \phi \ .$$

- Only ever talk about 2 nodes at a time
- Constraints given as multiplicities

Reachability

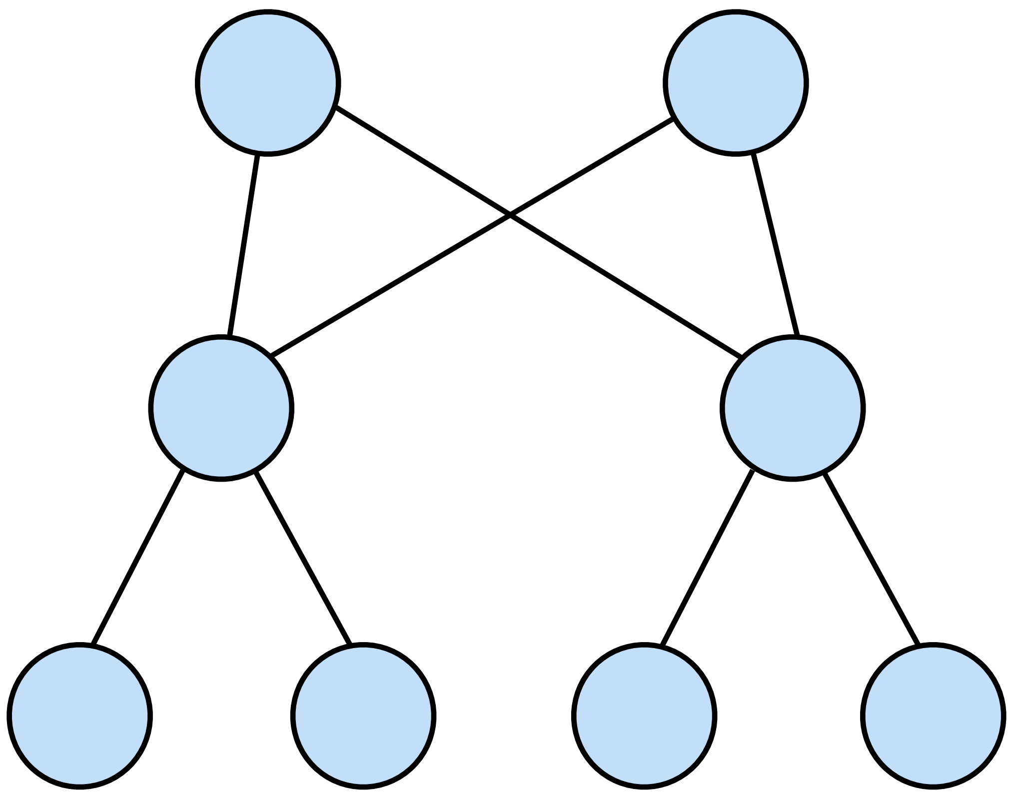


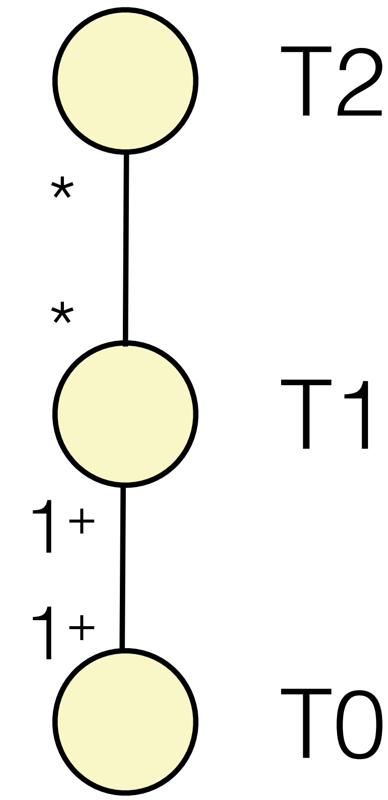


Reachability Query

If I start from some node in T0, which/how many nodes are reachable in T2, T1, T0?

Reachability

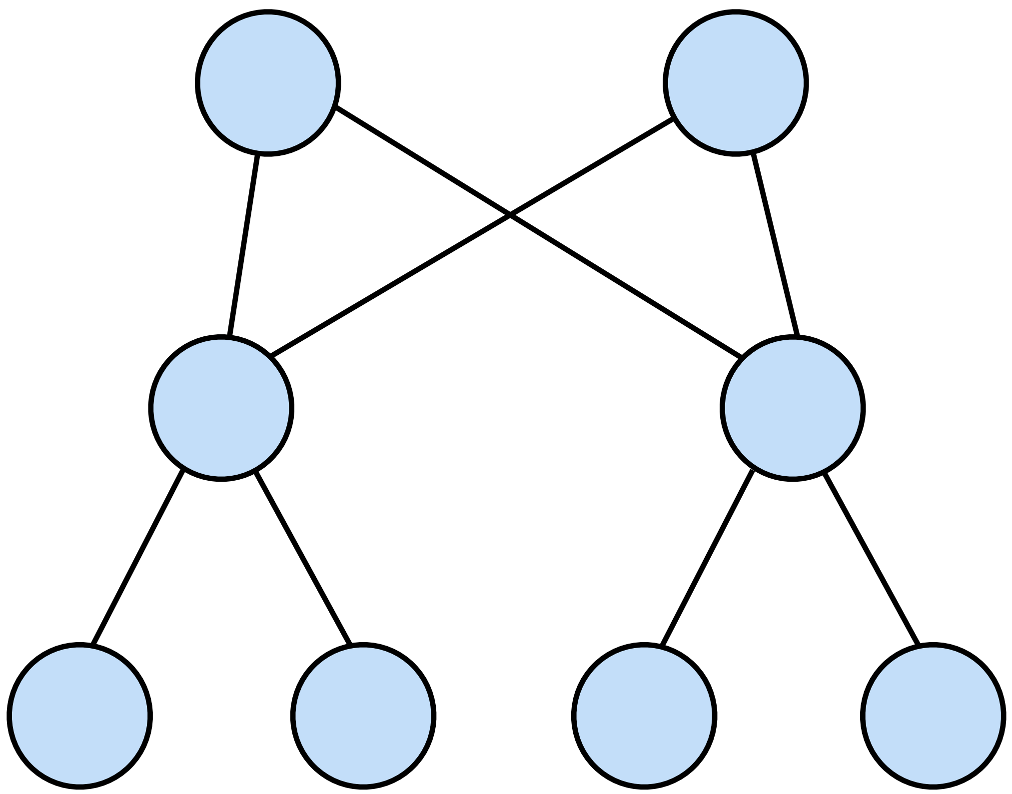


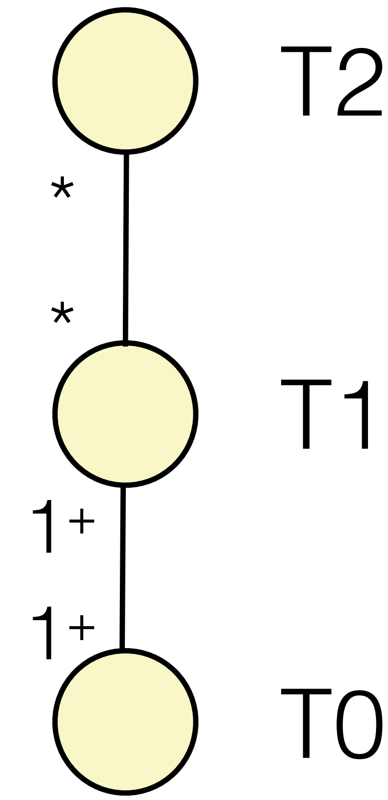


Idea

Abstract the reachable nodes as (None | Some | All)

Reachability

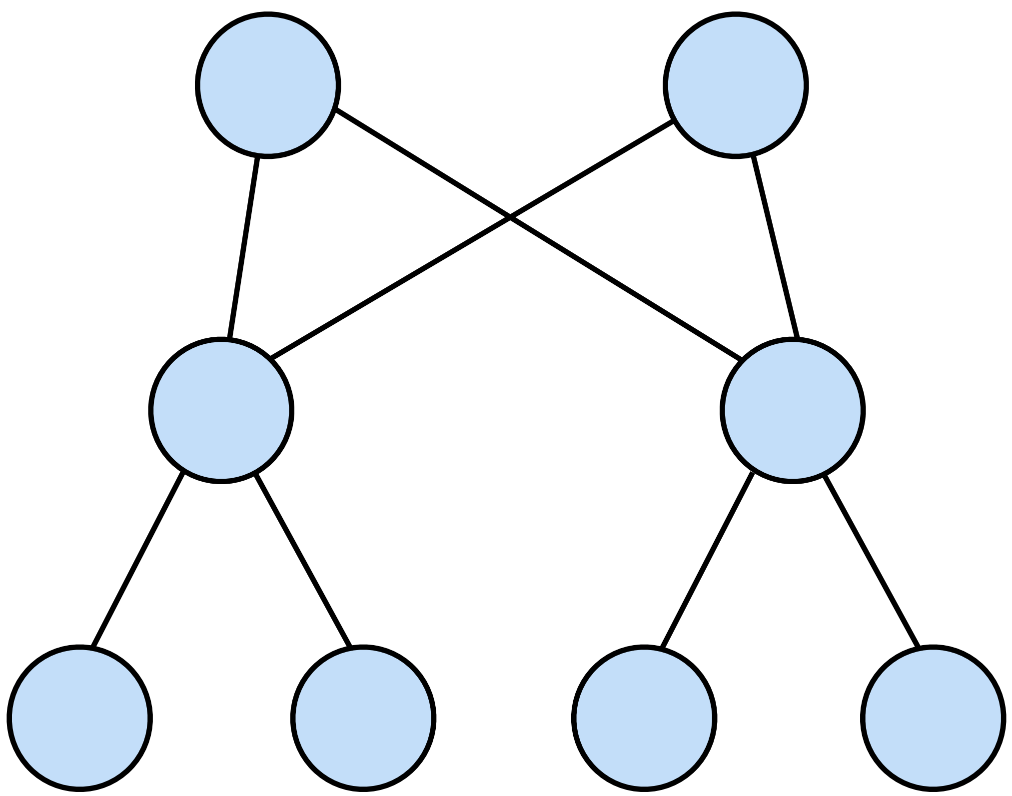


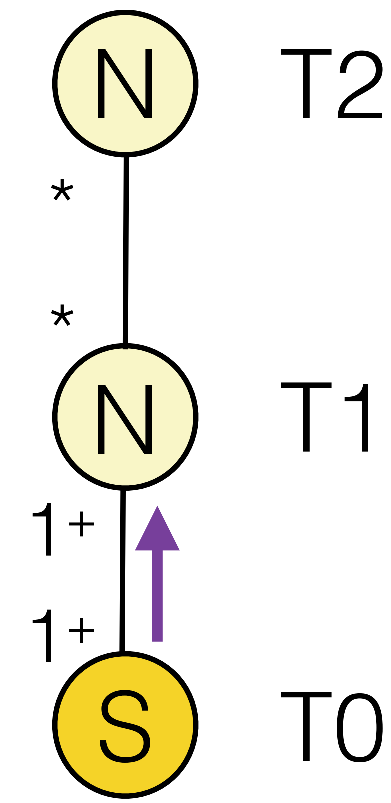


Define max operator

	N	S	A
N	N	S	A
S	S	S	A
A	A	A	A

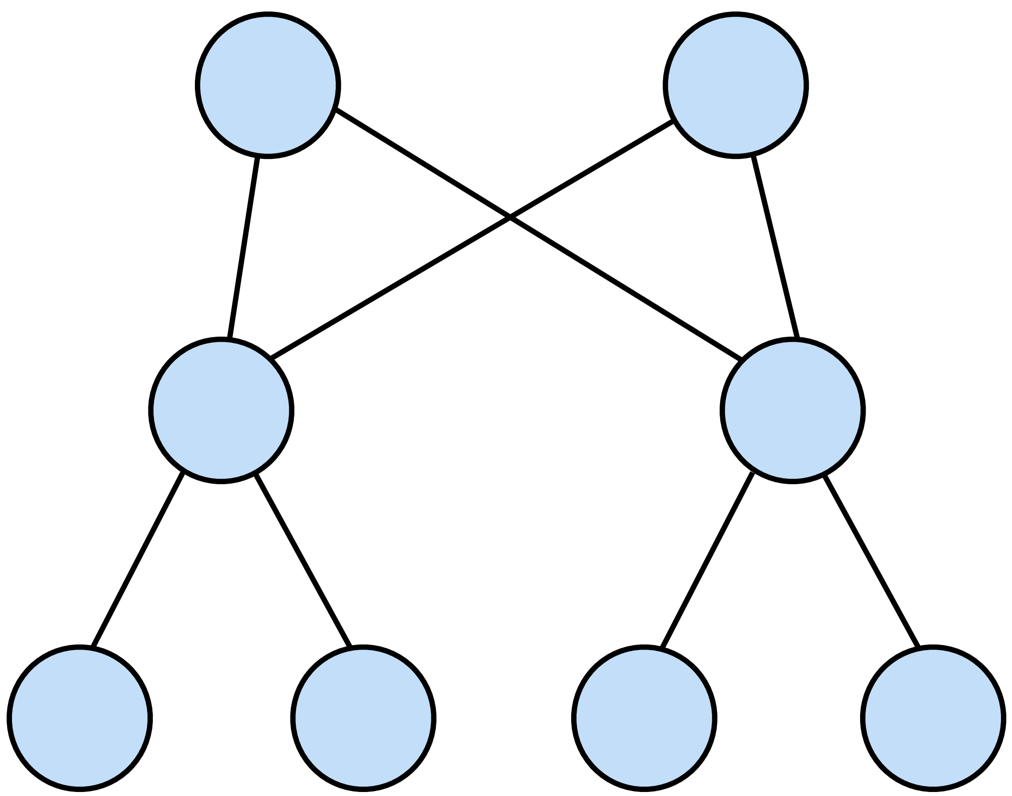
Reachability - Example 1

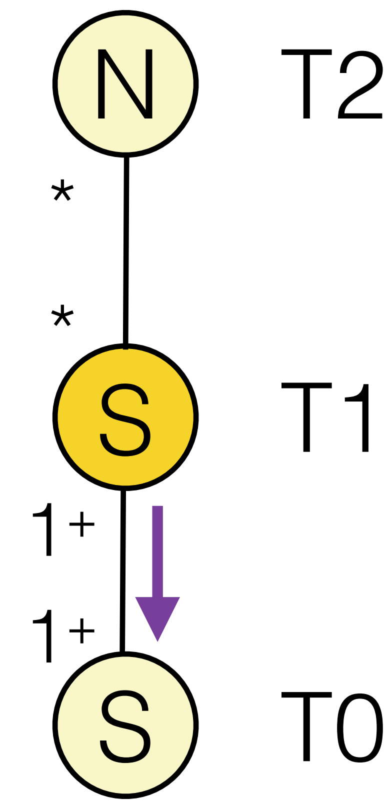




Start from *some* arbitrary node in T0

Reachability - Example 1





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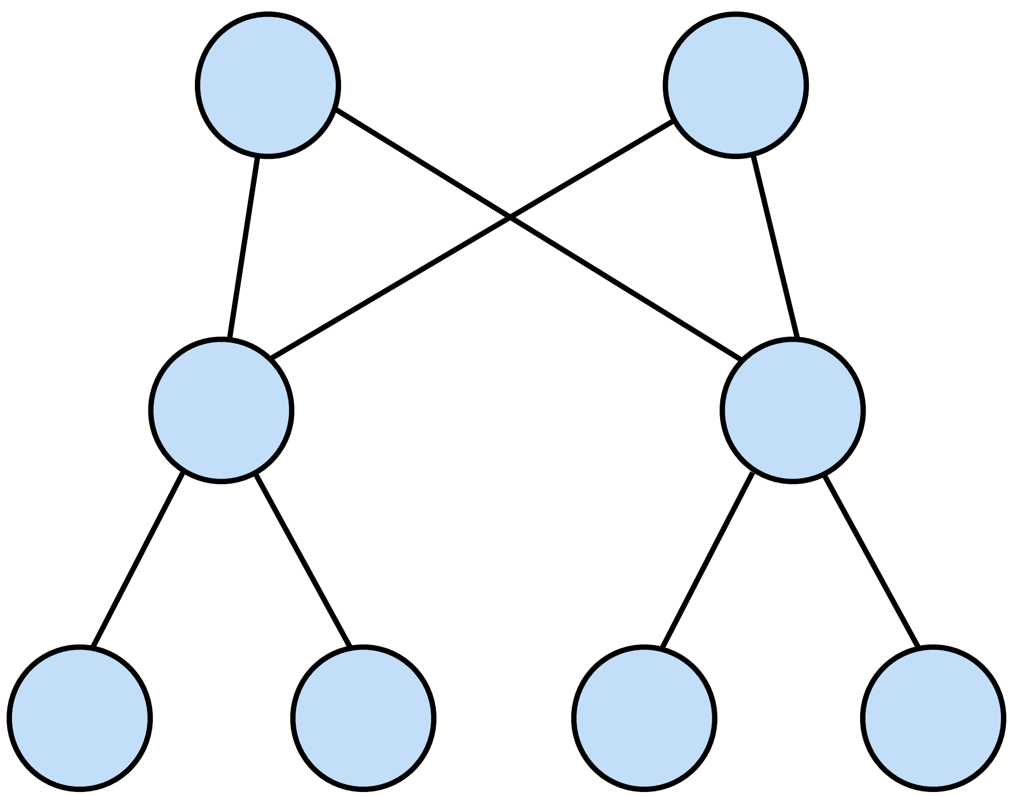
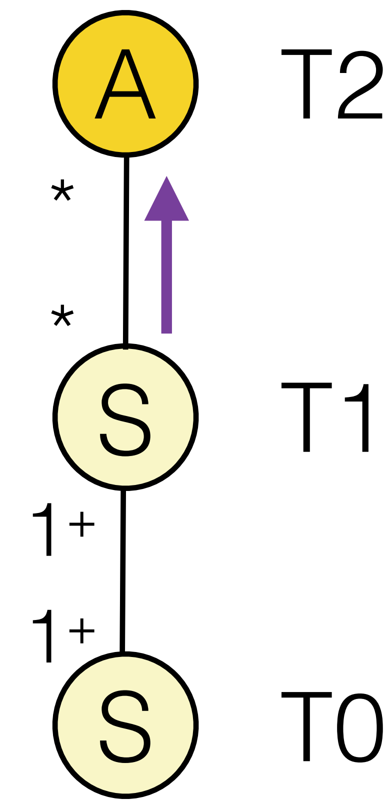
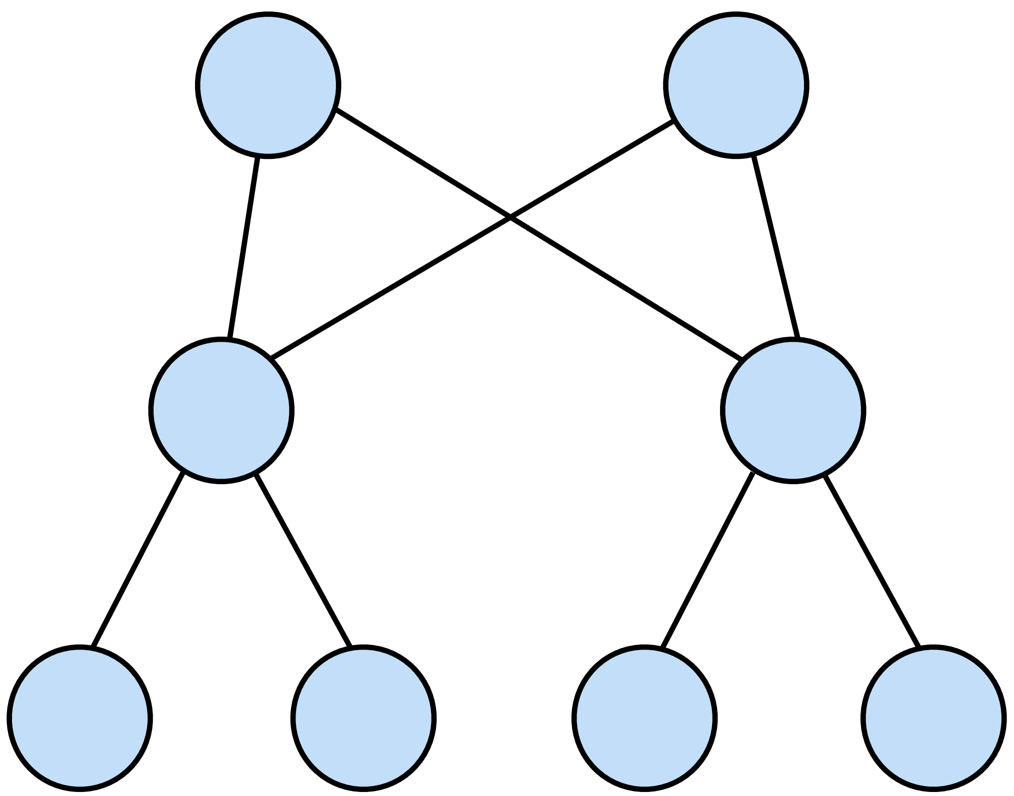
Reachability - Example 1

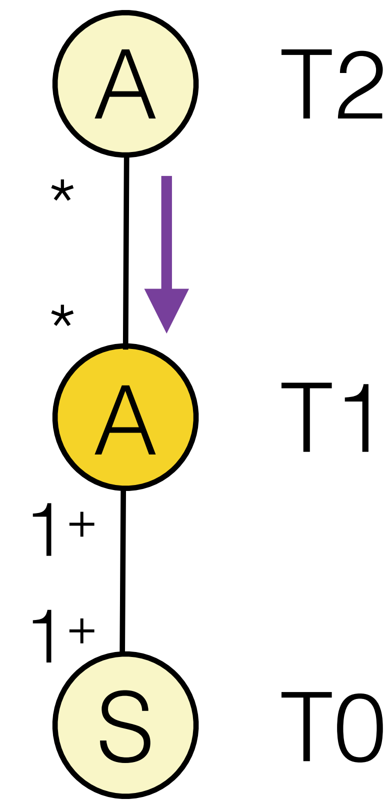




Start from *some* arbitrary node in T0

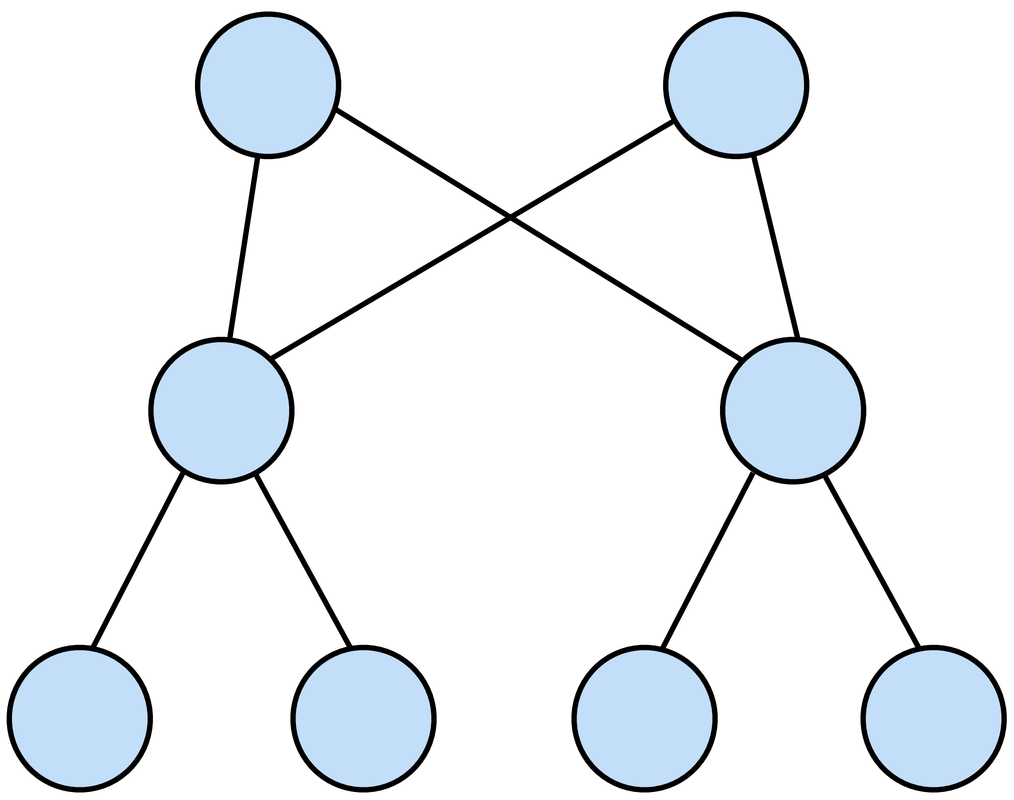
Reachability - Example 1

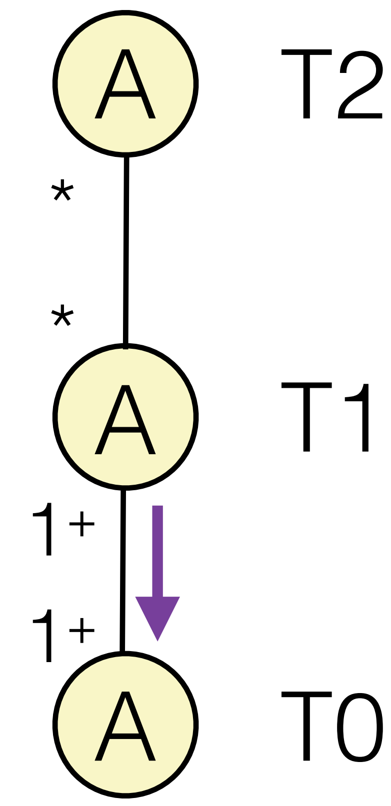




Start from *some* arbitrary node in T0

Reachability - Example 1





Start from *some* arbitrary node in T0

Abstract Topology

	0	1	0^+	1^+	*
N	N	N	N	N	N
S	N	S	N	S	A
A	N	S	N	S	A

	0	1	0^+	1^+	*
N	N	N	N	N	N
S	N	S	N	S	A
A	N	**A**	N	**A**	A

Outgoing production

↑

Max → Current Value

↓

Incoming production

Overview

$$
\begin{tikzcd}
& \text{Graph Morphism} \arrow[dr] & & \text{Multiplicity Constraints} \arrow[dl] \\
& & \text{Abstract Graph} \arrow[d] & \\
\text{Graph Property} \arrow[rr] & & \text{Conservative Algorithm} &
\end{tikzcd}
$$

Multiplicity Algebra

M : set of multiplicities

$\mathbb{N}$: natural numbers

$$\mathbb{N}^{\mu} = \{m \in \mathbb{N} | m : \mu\}$$

$\sqcap : 2^M \to M$

$$m : \sqcap M \iff \forall \mu \in M, m : \mu$$

$0 \in M$

$1 \in M$

$$\lfloor \mu \rfloor = min\ \mathbb{N}^{\mu}$$

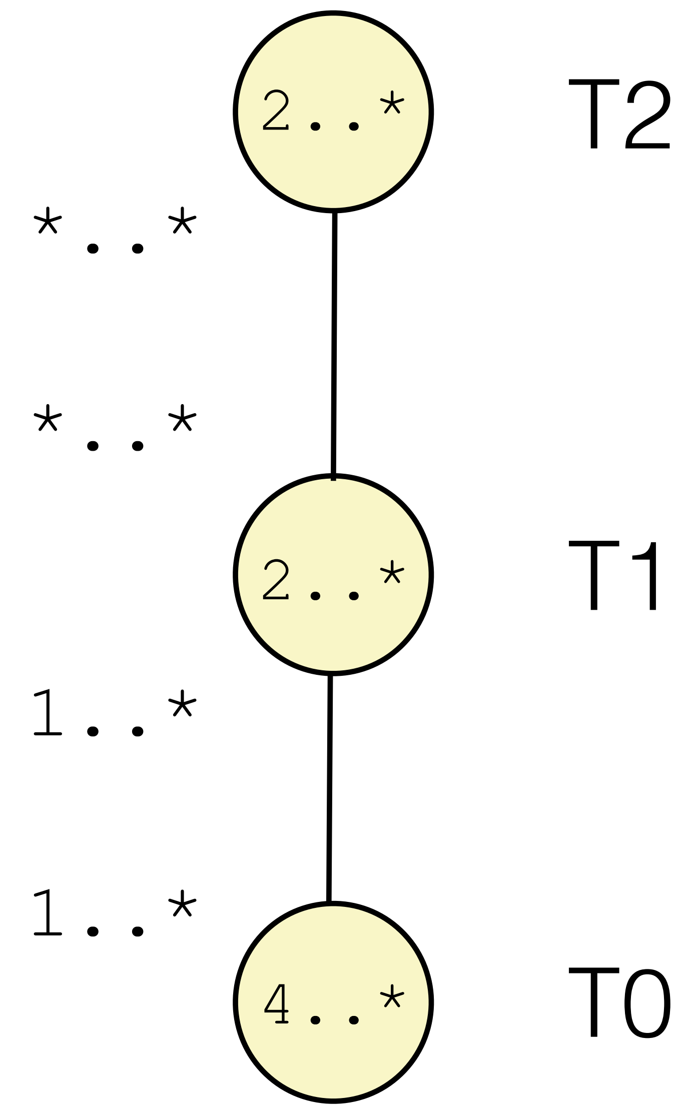
$$\lceil \mu \rceil = max\ \mathbb{N}^{\mu}$$

$$\forall \mu, \mu \leq \omega$$

UML multiplicities

$$i..j \text{ for } i, j \in \mathbb{N} \cup \{*\}$$

$$\omega = * \quad i, j < *$$



Graph Properties

Does each router in T0 have a path to all other routers in T0?

Is every router in X dominated by some router in Y?

Will k failures disconnect some router in X from some router in Y?

Graph Properties

Does each router in T0 have a
path to all other routers in T0?

$$\forall x \in T_0, \forall y \in T_0, \exists p \in \sigma, (x..y)_p$$

Is every router in X dominated by some router in Y?

$$\forall x \in X, \exists y \in Y, \forall_{x,y}\, p \in \sigma, (start..y..x)_p$$

Will k failures disconnect some router in X from some router in Y?

$${}^{k}[\forall x \in X, \forall y \in Y, \exists p \in \sigma, (x..y)_p]$$

Path Logic

$m ::= \; \vdash \mid \dashv$	Start/end
$\sigma ::= \; x <_p y \mid x \leq_p y \mid m$	Path order
$\phi ::= \; \neg\phi \mid \phi \vee \phi \mid \phi \wedge \phi \mid \forall x \in X, \phi \mid \sigma$	Logic operators
$p ::= \; {}^k[\phi]$	Under k failures

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$p ::= \ {}^{k}[\phi]$	Under k failures

$$(\vdash ..x..y.. \dashv)_p \ = \ (y <_p \dashv) \wedge (x <_p y) \wedge \ldots$$

Restrictions for now:

Consider only formulas of the form:

$$(\forall, \exists \text{ node})^* \ (\forall, \exists \text{ path}) \ (x..y..z..)_p$$

High-level Idea

Strategy: Perform the following

- Each quantifier maps to (A | S)
- Generate fixed point computation for formula
- Path quantifier determines neighbor condition
- Base case from substitution of formula

Fixed Point Computation

Family of functions: $f^i : V^i \rightarrow \{A, S\}^i$

Extend a tuple: $(x_1, \ldots, x_n), x \ = \ (x_1, \ldots, x_n, x)$

Tuple Projection: $\Pi_{i..j}(x_1, \ldots, x_i, \ldots x_j, \ldots) \ = \ (x_i, \ldots, x_j)$

$\Pi_i \ = \ \Pi_{i..i}$

Node Multiplicity: μ_x

Edge Multiplicity: $\mu_{x,y}$

Path (∃) Case:

$$f^2(v, v) = \{(A, S)\}$$

$$f^n(v_1, ..., v_{n-1}, v_n) =$$

$$\bigcup_{a \in adj(v_n)}$$

$$\{X, lb(v_{n-1}, \Pi_{n-1}) \mid X \in f^{n-1}(v_1, ..., v_{n-1})\} \qquad \text{when } a = v_{n-1}$$

$$\{\Pi_{1..n-1}X, lb(a, v_n, \Pi_n X) \mid X \in f^n(v_1, ..., v_{n-1}, a)\} \qquad \text{otherwise}$$

Path (∃) Case:

$$f^2(v, v) = \{(A, S)\}$$

Recursively defined

$$f^n(v_1, ..., v_{n-1}, v_n) =$$

$$\bigcup_{a \in adj(v_n)}$$

$$\{X, lb(v_{n-1}, \Pi_{n-1}) \mid X \in f^{n-1}(v_1, ..., v_{n-1})\} \qquad \text{when } a = v_{n-1}$$

$$\{\Pi_{1..n-1}X, lb(a, v_n, \Pi_n X) \mid X \in f^n(v_1, ..., v_{n-1}, a)\} \qquad \text{otherwise}$$

lower bound (A | S) between neighbors

Path (∃) Case:

$$f^2(v, v) = \{(A, S)\}$$

Check (v_1 .. v_{n-1}) for each node and then extend that information

$$f^n(v_1, ..., v_{n-1}, v_n) =$$

$$\bigcup_{a \in adj(v_n)}$$

$$\{X, lb(v_{n-1}, \Pi_{n-1}) \mid X \in f^{n-1}(v_1, ..., v_{n-1})\} \qquad \text{when } a = v_{n-1}$$

$$\{\Pi_{1..n-1}X, lb(a, v_n, \Pi_n X) \mid X \in f^n(v_1, ..., v_{n-1}, a)\} \qquad \text{otherwise}$$

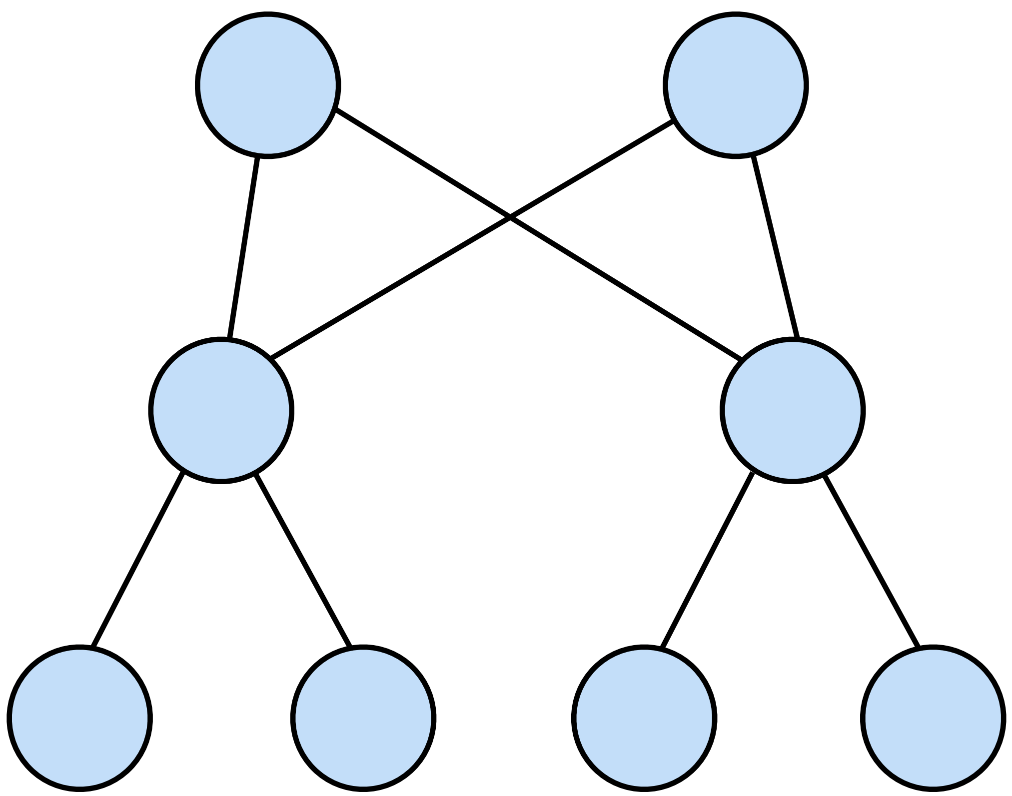
What information can we infer from the neighbors

Tuple: $(\lfloor \mu_{x,y} \rfloor, \lfloor \mu_{y,x} \rfloor, \lceil \mu_y \rceil)$

$\odot$	$(-, \geq 1, -)$	$(n \geq 1, -, m \leq n)$	$(n \geq 1, -, m > n)$	otherwise
N	N	N	N	N
S	S	A	S	N
A	A	A	S	N

$$lb(x, y, L) = max\{L, L \odot (x, y)\}$$

Reachability - Example 1



T0 .. T0

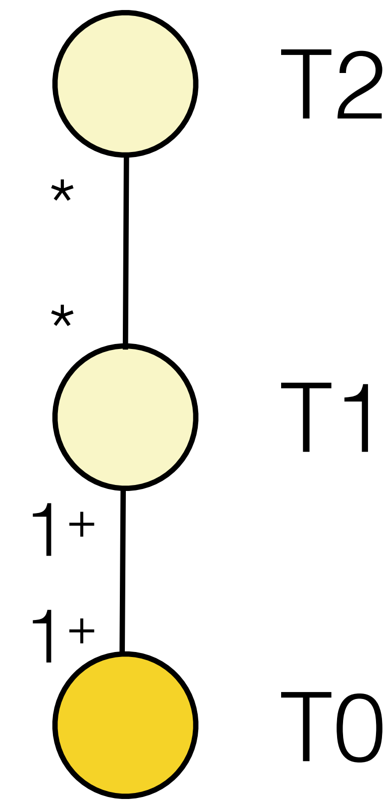
A T0

AS (T0,T1)

AA (T0,T2)

AA (T0,T1)

AA (T0,T0)



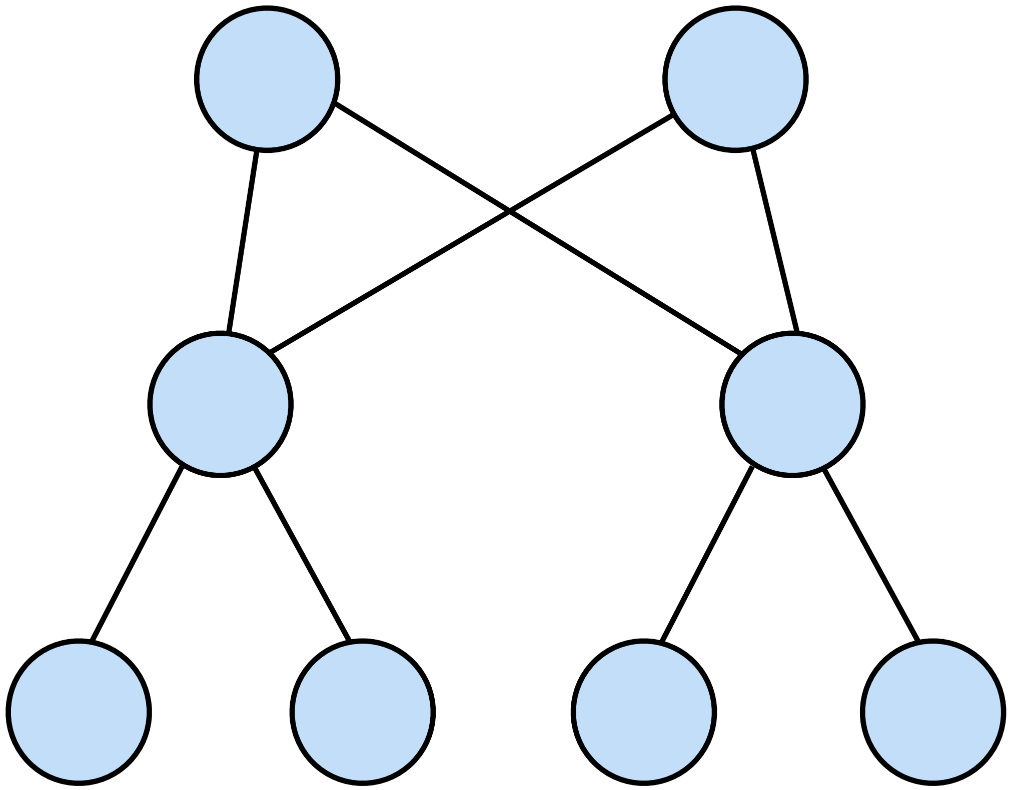
Start from *some* arbitrary node in T0

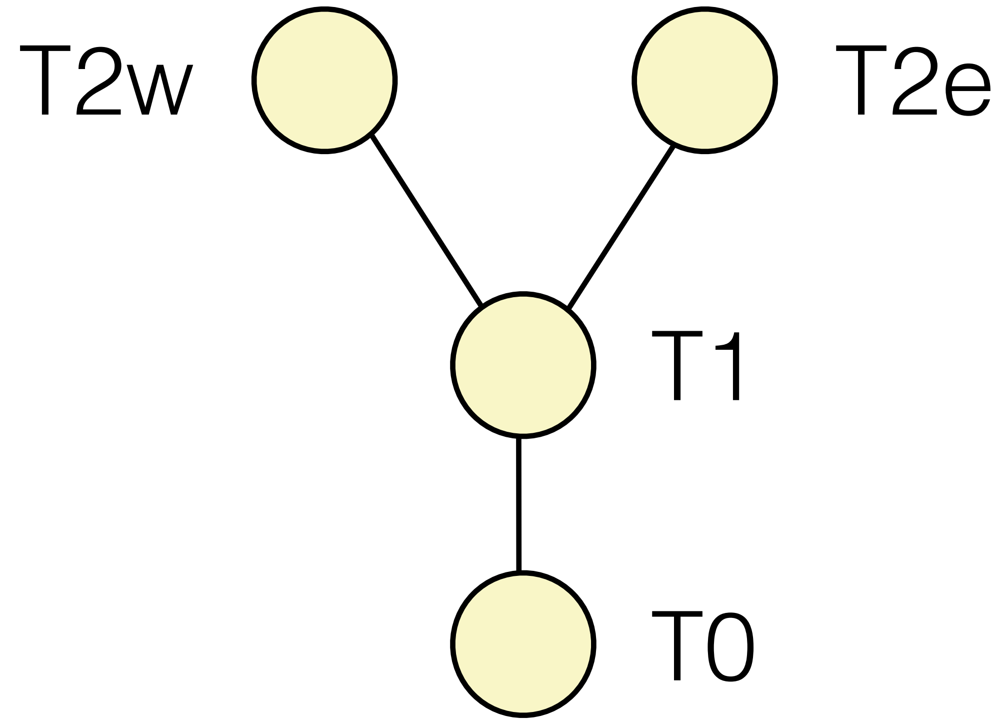
Path (∀) Case

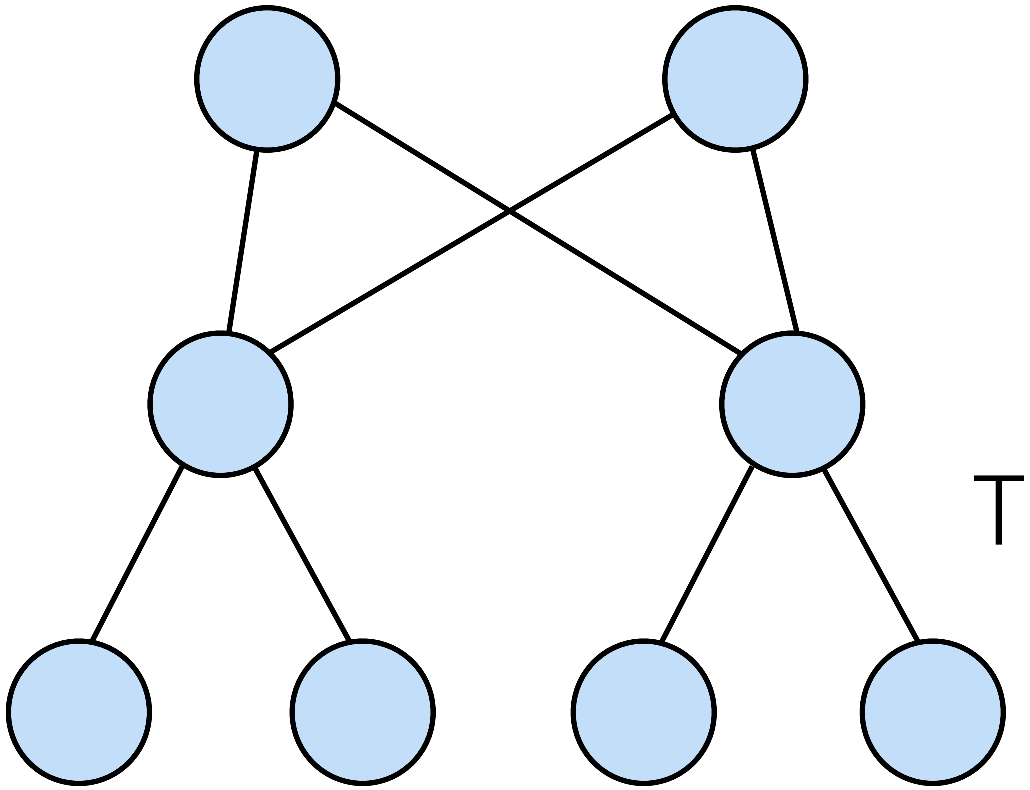
Changes:

- Define a new lower bound operator
- Set union changes to set intersection

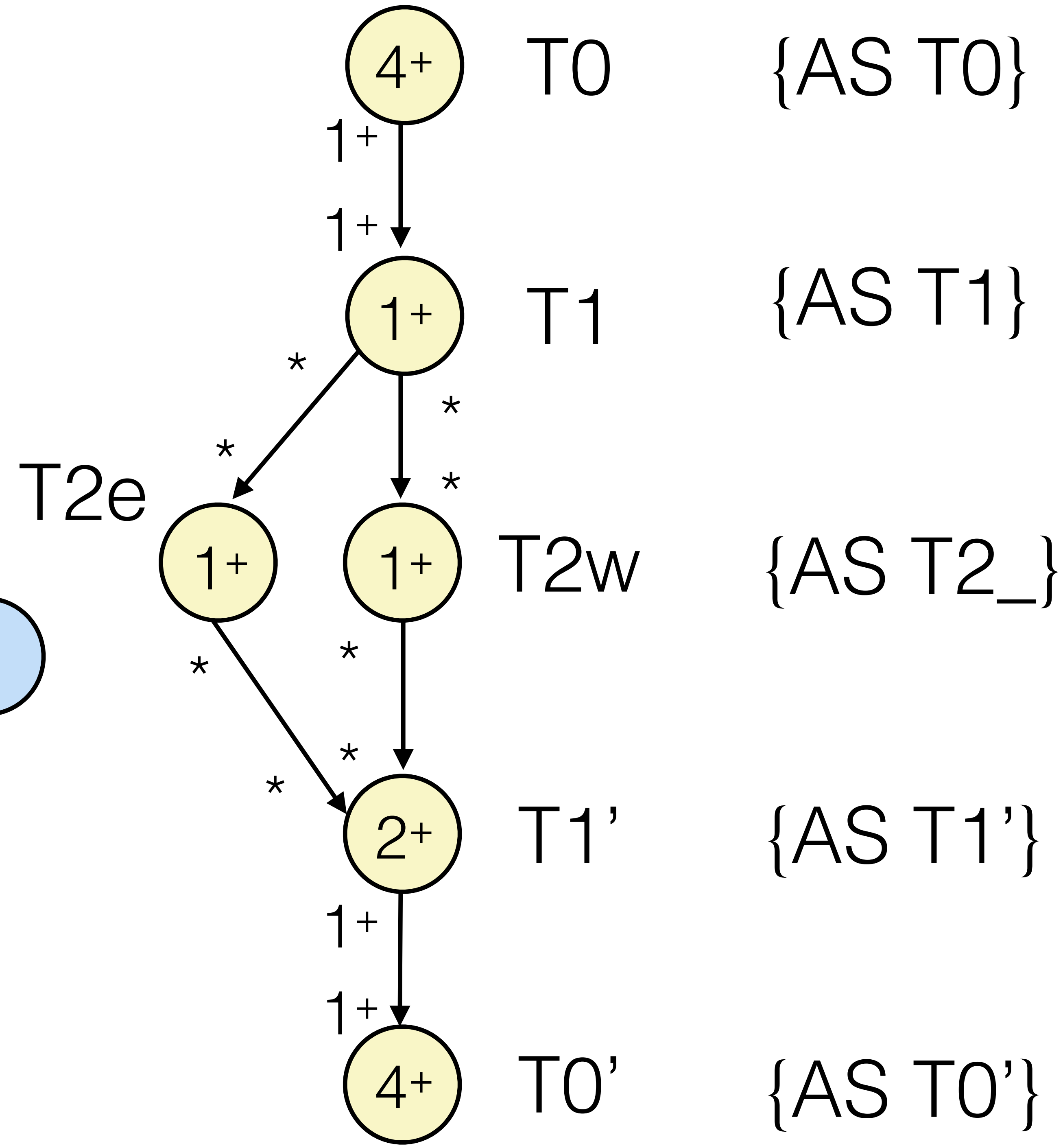
Dominators



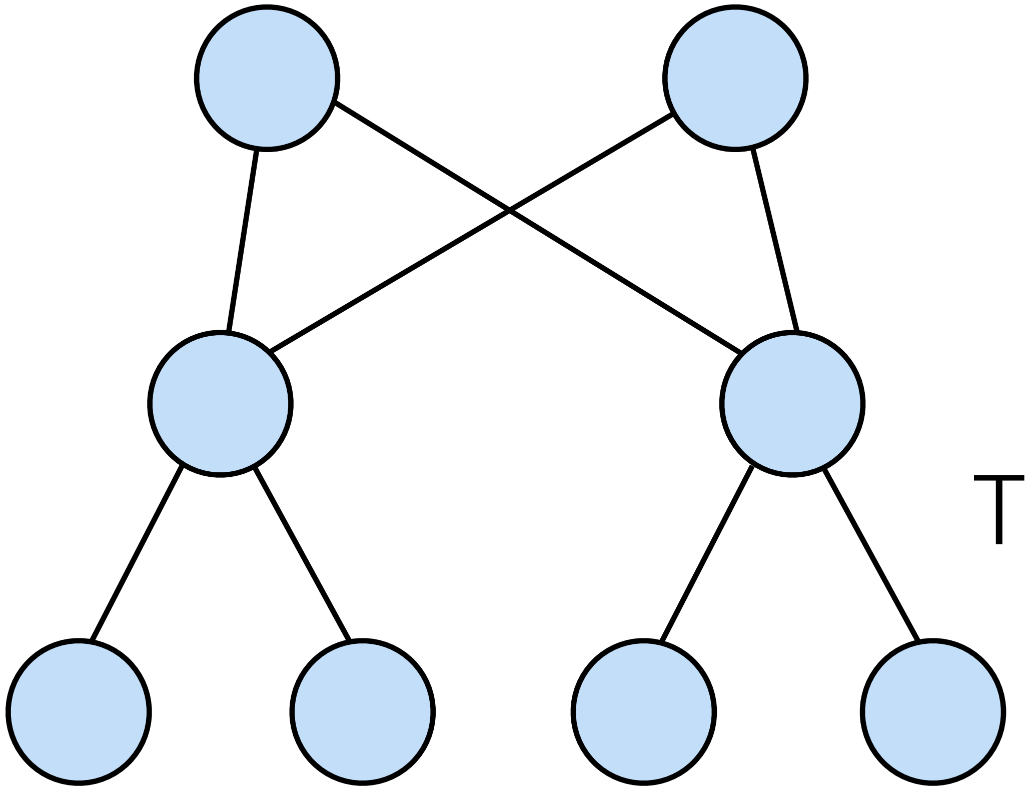




waypoint(T2e | T2w)



Is there a concrete topology under which,
some node in T0’ is dominated by some node in another?



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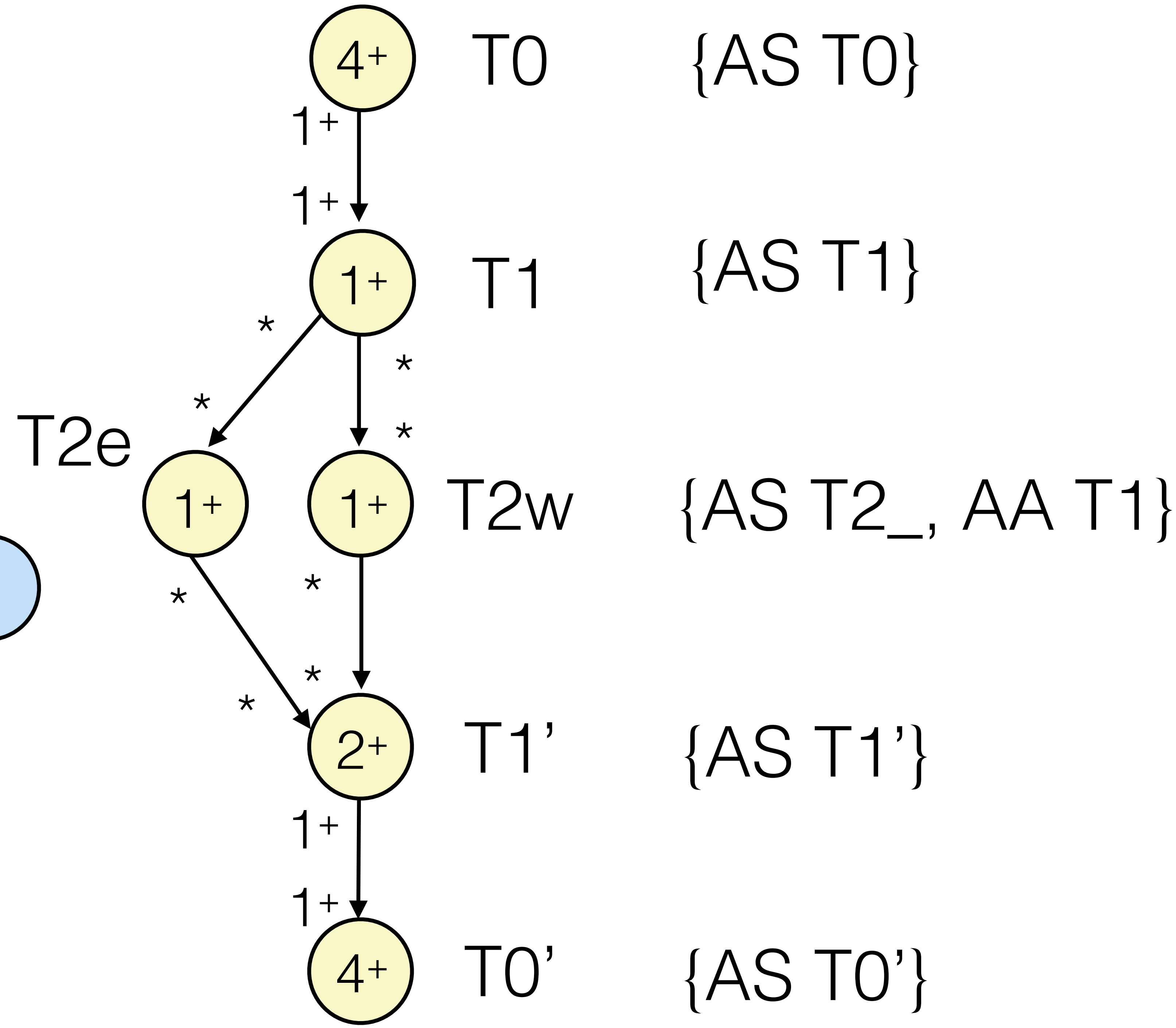

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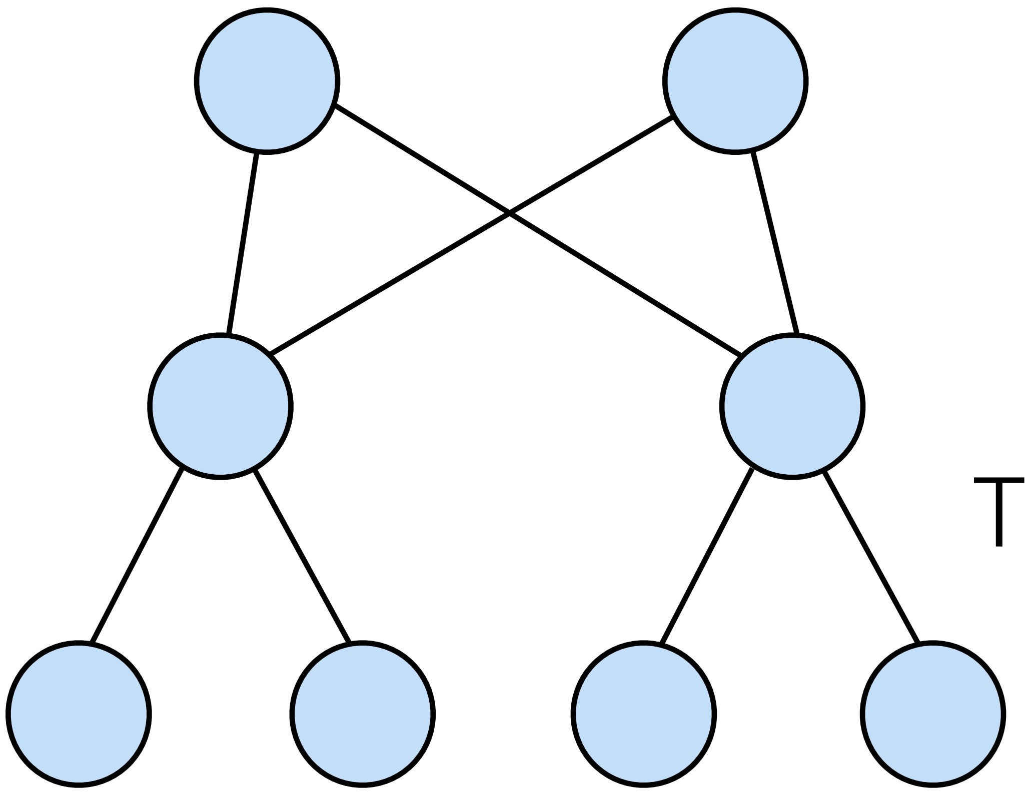

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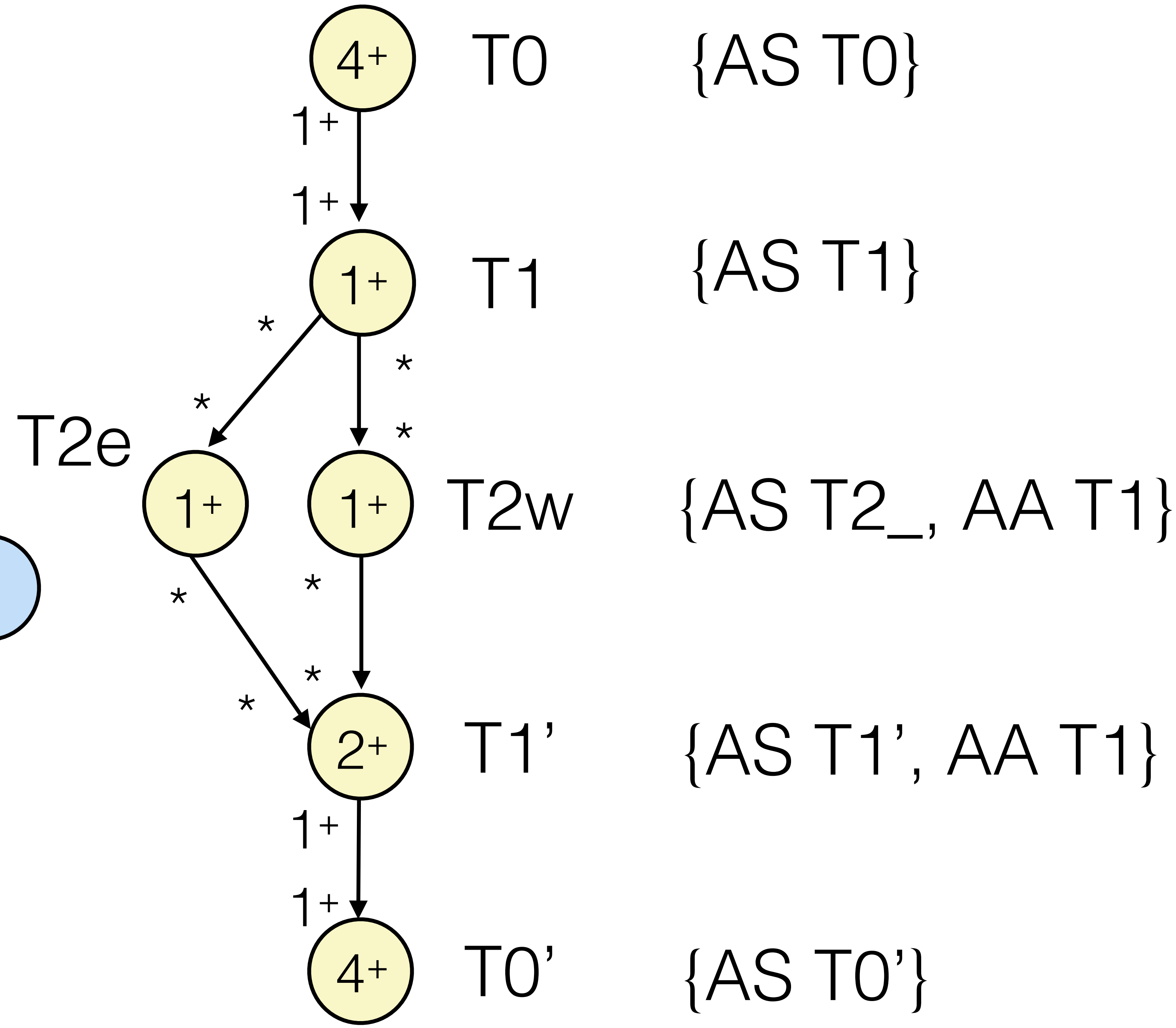

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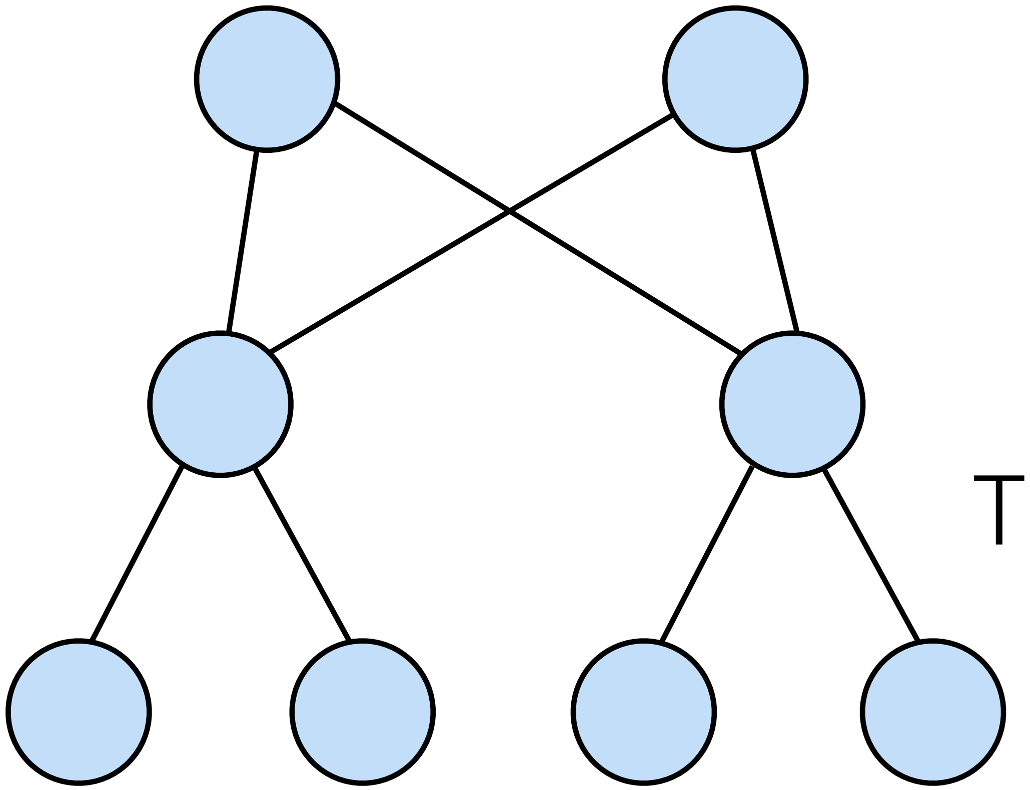

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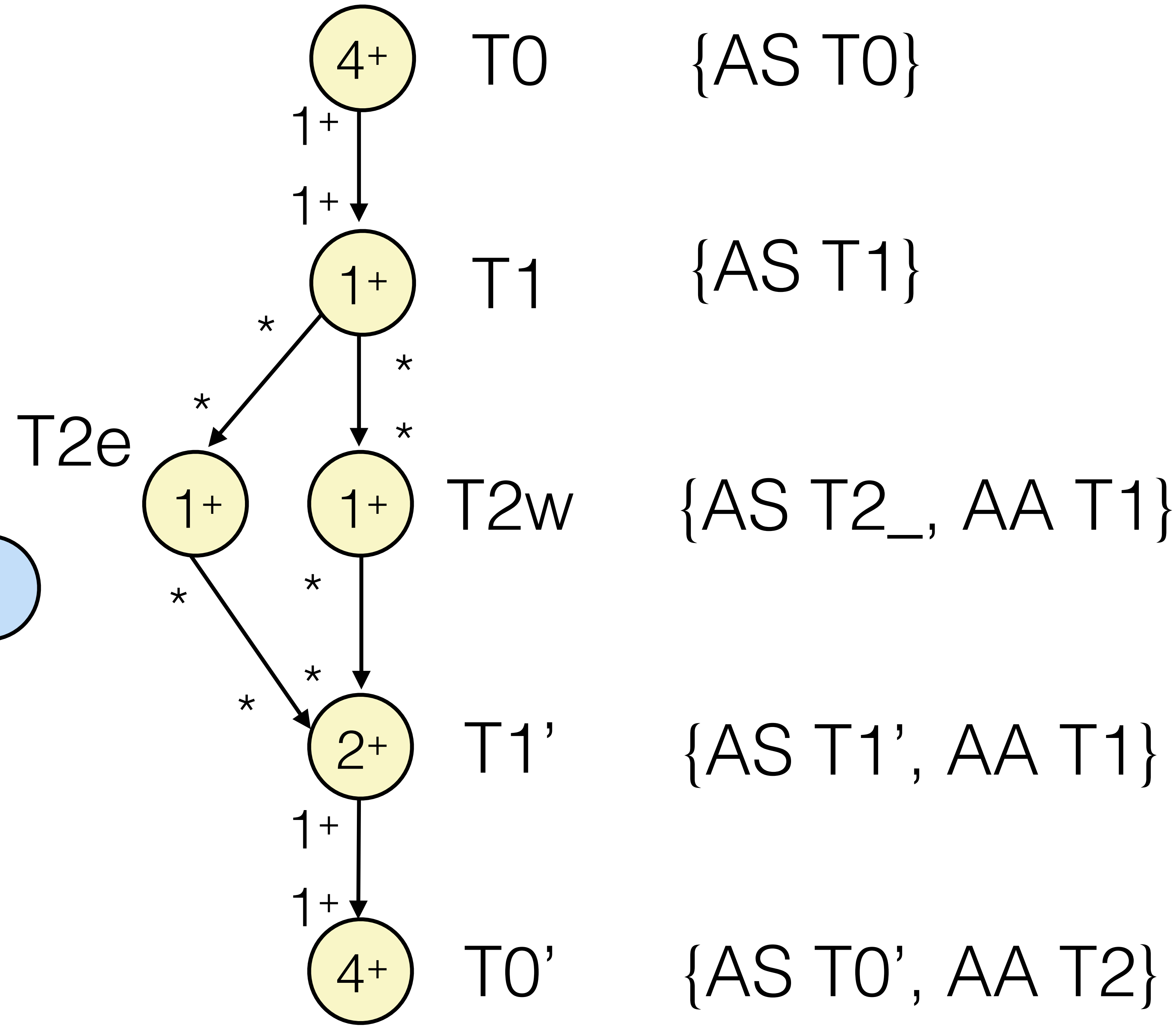

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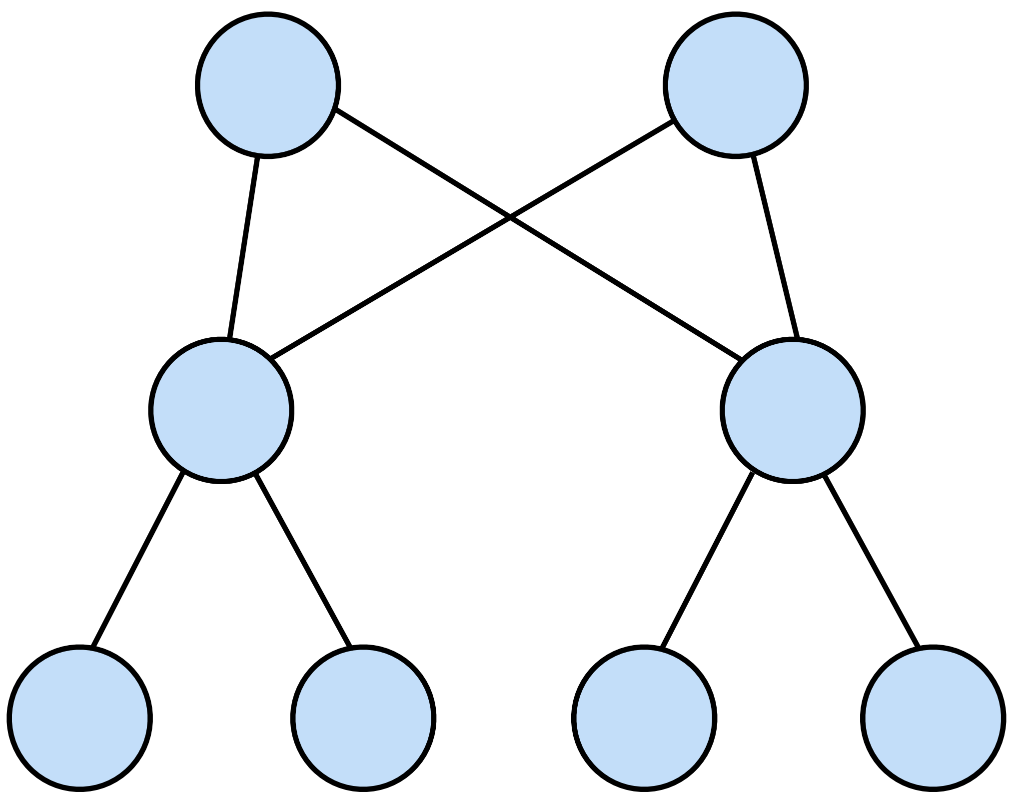
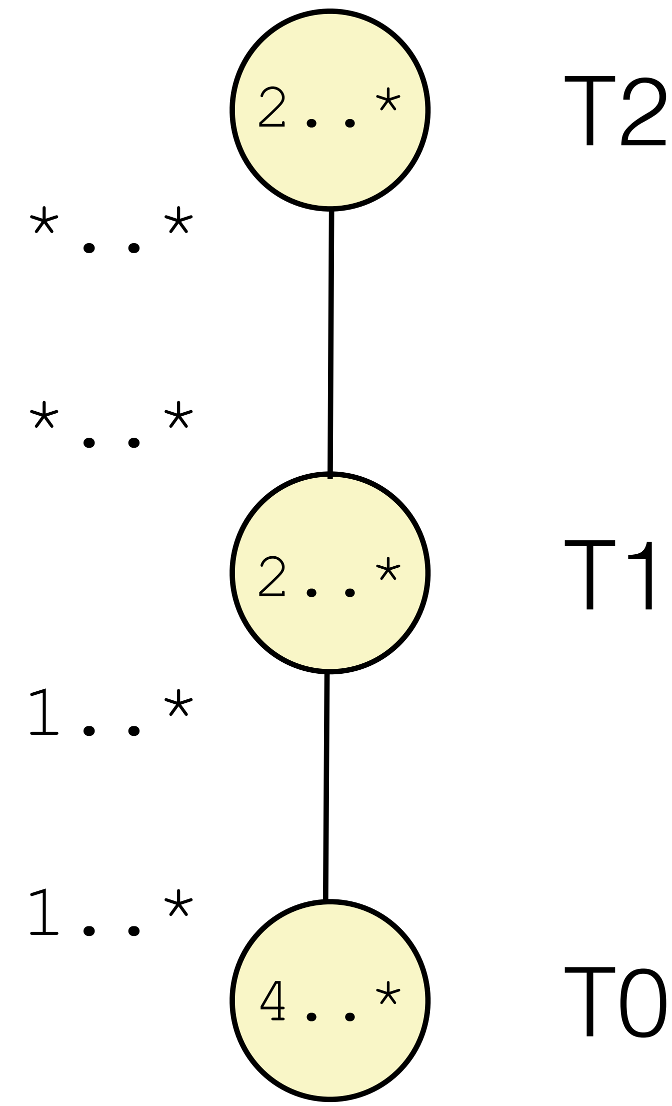
Failures





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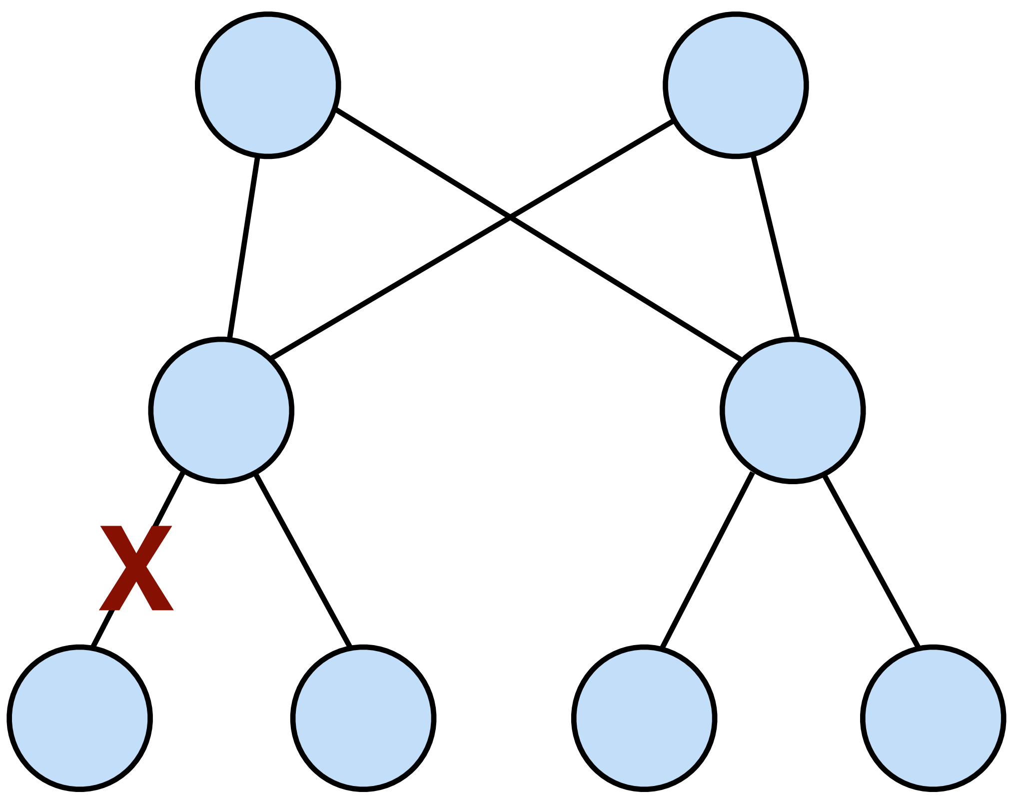

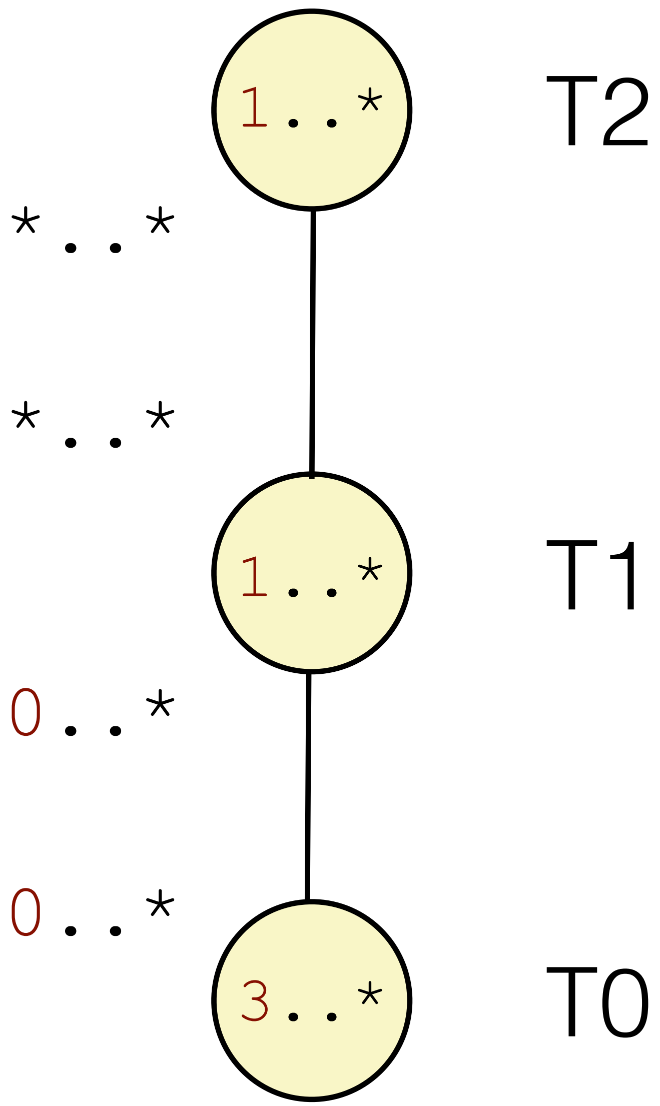

Worst case under 1 failure