Statistical electricity demand modelling from consumer billing data

G.W. Irwin, B.Sc., Ph.D., C.Eng., M.I.E.E., Prof. W. Monteith, B.Sc., Ph.D., C.Eng., M.I.E.E., and W.C. Beattie, B.Sc., Ph.D., C.Eng., F.I.E.E.

Indexing terms: *Modelling, Power utilisation and industrial applications*

Abstract: The paper develops a statistical framework for the modelling of electricity consumption distributions. The method is based directly upon consumer billing data and it is shown that a generalised form of the Weibull distribution model may be successfully applied. Illustrative results are given for a sample of 1653 consumers in the public housing sector. It is concluded that two statistically separate consumption distribution classes are present and the parameters of the appropriate models are estimated. The relation of these models to the problem of demand forecasting is discussed.

1 Introduction

Crude oil price escalation in the past decade has stimulated significant international interest in the modelling of national electricity demand in conjunction with other energy requirements [1]. A primary objective is to permit forecasting of demand for investment decision making. However, the potential range of social, economic and technological factors influencing electricity consumption is extensive and hence much modelling effort has been based upon historical summary data. The consequent forecasting confidence is low and relies heavily on scenario testing which may omit important underlying trends. To address this problem, modelling research has concentrated upon the electricity consumption patterns within identifiable demand sectors. The commonly selected utilisation sectors are residential, industrial and commercial [2, 3] and a number of different types of models are currently under development [4–6].

However, it must be expected that consumer behaviour will differ radically between and indeed within these groups. This implies that forecasting models based upon aggregation at this level cannot be responsive to local variations in parameters of influence. This in turn impedes the proper identification of such parameters and hence degrades the worth of the resulting forecasts.

The new strategy proposed in this paper is to adopt a bottom-up approach in which every consumer consumption pattern is fully utilised in the development of the forecasting model. The most readily available information from which these patterns may be deduced is the monthly or quarterly billing data from the supply authority. Theoretically, such information would enable the consumption pattern of each individual consumer to be modelled. As this is clearly not feasible from a practical viewpoint it is desirable to employ a measure of aggregation by classifying consumers into groups with common consumption characteristics.

This paper proposes a statistical framework which permits the proper identification of appropriate consumer groups. Statistical modelling of consumption is seen as a prerequisite first step in the development of demand models which are responsive to the significant parameters of influence within each group. Demand modelling at a national level is then a logical extension from this.

Paper 4791C (P11), first received 20th November 1985 and in revised form 16th May 1986

Dr. Irwin and Dr. Beattie are, and Prof. Monteith was formerly, with the Department of Electrical and Electronic Engineering, Queen's University, Ashby Building, Belfast BT9 5AH, United Kingdom. Prof. Monteith is now with Massey University, Palmerston North, New Zealand

The statistical modelling of electricity consumption using billing data is examined for a sample of consumers in the public housing sector. The consumption distributions from 11 locations are studied with a view to group identification. From the consumption patterns it is deduced that the Weibull distribution provides an adequate model. Statistical estimation of the Weibull model parameters is considered and it is concluded that there are two distinct groups. It is proposed that these groups be used to examine how the parameters of the distribution models vary with time and external variables of influence.

2 Consumption distributions

2.1 Choice of demand sector

To minimise the number of parameters of influence, attention has been focused on electricity consumption in the public housing sector. In a total consumer population in Northern Ireland of 600 000, 40% of the overall consumption is residential and of this 40% is in the public domain. The stock of dwellings is predominantly brick cavity-wall construction with an increasing number of timber-frame type. Electricity is used primarily for lighting and cooking and some supplementary heating. Housing estates range in size from 100 to 500 dwellings usually arranged in terrace or semidetached form.

2.2 Spatial distribution of sample

For this modelling study, annual consumption data for a total of 1653 public sector dwellings was employed. This was achieved by sampling in eleven locations geographically distributed throughout the province of Northern Ireland and broadly divided into the two arbitrary categories, urban and rural, depending on proximity to the nearest central place.

2.3 Distributions

Fig. 1 shows the relative frequency densities for annual electricity consumption in the 11 samples on a consumption scale normalised, in each case, to the individual sample average. A preliminary study of this sample [7] suggested the classification of the histograms into two groups, one reflecting a high-skew (HS) appearance and the other skew-normal (SN). Visually, it is apparent that a HS histogram can be modelled by the simple exponential form

$$\frac{f}{N} = a \exp(-ac_R) \tag{1}$$

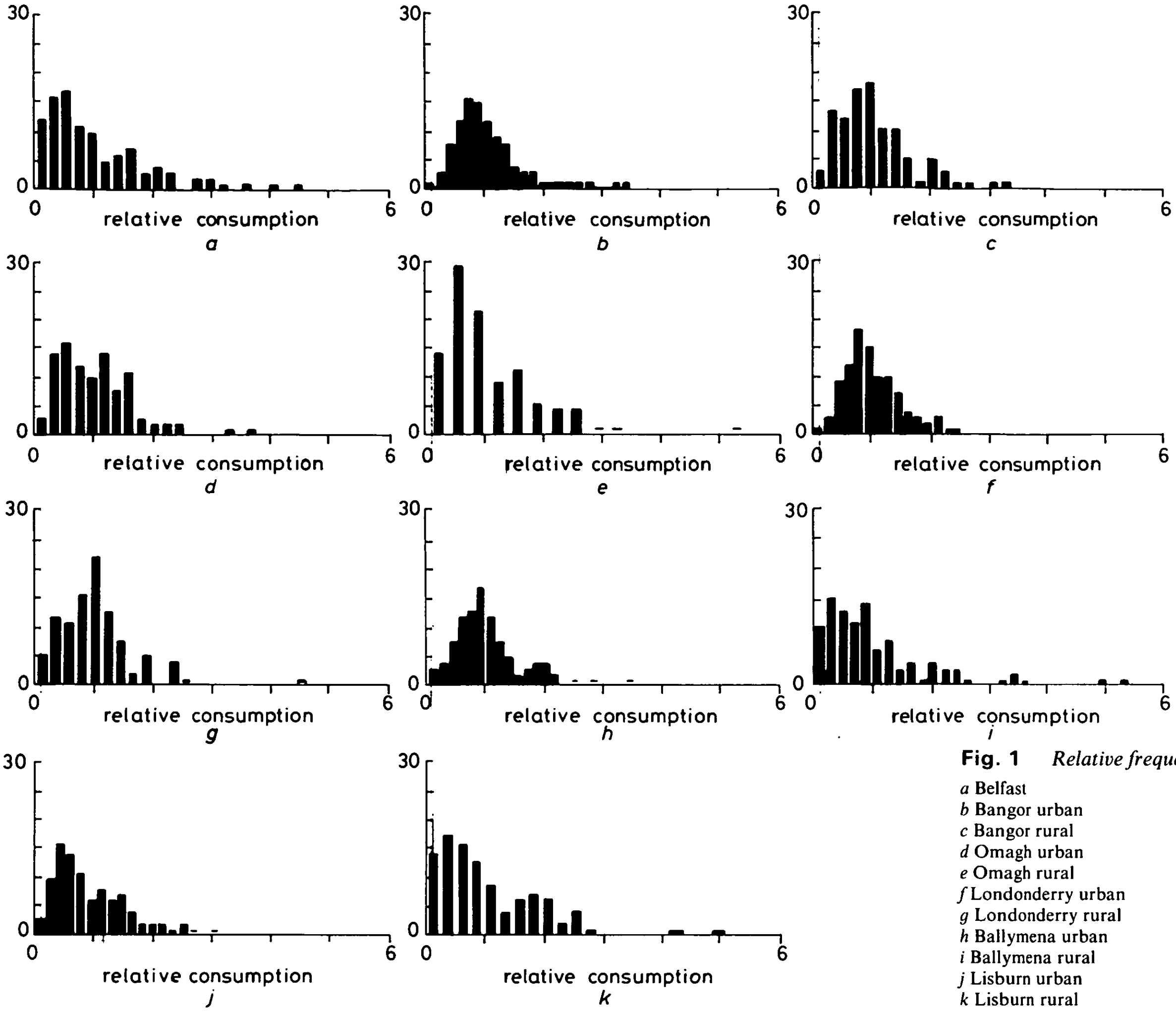

Fig. 1 *Relative frequency density histograms*
a Belfast
b Bangor urban
c Bangor rural
d Omagh urban
e Omagh rural
f Londonderry urban
g Londonderry rural
h Ballymena urban
i Ballymena rural
j Lisburn urban
k Lisburn rural

where

f = frequency density
N = sample size
c_R = relative consumption (consumption/sample average consumption)
a = shape parameter

Similarly, a SN histogram has a close visual commonality with the Rayleigh distribution model

$$\frac{f}{N} = 2ac_R \exp(-ac_R^2) \qquad (2)$$

On this basis it was suggested that the classification of the eleven samples shown in Table 1, based on the models of eqns. 1 and 2, was reasonable.

Table 1 : Preliminary sample classification[7]

High-skew (HS) samples	Skew-normal (SN) samples
Belfast urban	Bangor urban
Omagh rural	Bangor rural
Ballymena rural	Omagh urban
Lisburn rural	Londonderry urban
	Londonderry rural
	Lisburn urban
	Ballymena urban

It is now essential to establish an acceptable statistical framework for the development of accurate forms of the models so that a methodology is available when sample data is difficult or expensive to obtain.

3 Confidence limits for distribution models

A quantitative method for assessing the quality of fit of a candidate model for a measured distribution is now presented. This will be used to show that the HS and SN models, for the distribution of electricity consumption in the public housing sector, need to be refined.

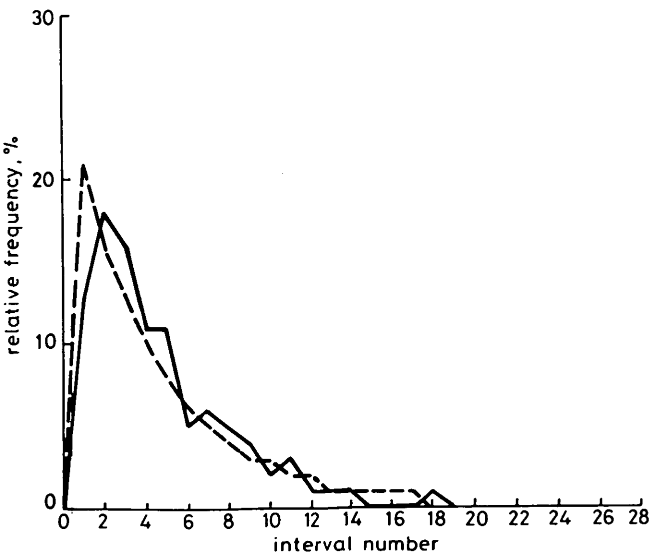

Fig. 2 *Observed and calculated HS frequency polygons of annual electricity consumption for HS sample at intervals of 500 kWhr*
——— observed
- - - - calculated

Suppose that

f_i^o = observed frequency, in the ith interval, from a sample consumption distribution histogram

f_i^c = calculated frequency, in the ith interval, from a candidate distribution model

Figs. 2 and 3 show the observed frequency polygons for the HS and SN samples together with those calculated

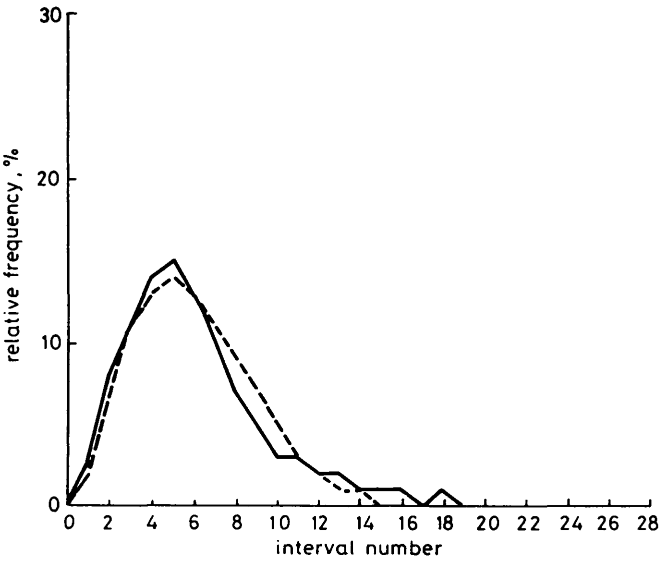

Fig. 3 *Observed and calculated SN frequency polygons of annual electricity consumption for SN sample at intervals of 500 kWhr*

——— observed
– – – – calculated

from the HS and SN models of eqns. 1 and 2. It must be established whether the differences between f_i^o and f_i^c are due to sampling or whether they are statistically significant. In the first case the models are accepted, in the second they are rejected.

For a sample of N consumers let

$$p = \frac{f_i^o}{N} = \text{probability that the consumption of a single consumer lies in the } i\text{th interval} \tag{3}$$

and

$$q = 1 - p = \text{probability that the consumption of a single consumer lies outside the } i\text{th interval} \tag{4}$$

Then, the probability P_r that r consumers out of N sampled all have a consumption in the ith interval follows a Binomial distribution

$$P_r = {}^N C_r p^r q^{n-r} \tag{5}$$

The mean μ and standard deviation σ are given by

$$\mu = Np, \sigma = \sqrt{Npq} \tag{6}$$

Thus, the 95% confidence interval for the model frequency f_i^c, in the ith interval, is

$$\mu \pm 1.96\sigma \tag{7}$$

The confidence interval may then be stated as

$$f_i^o \pm 1.96\sqrt{f_i^o(1 - f_i^o/N)} \tag{8}$$

A 95% confidence interval for all f_i^c, $i = 1, 2, \ldots, 28$ can now be calculated.

Figs. 4 and 5 show the results of applying eqn. 8 to the observed frequency polygons in Figs. 2 and 3. The upper-limit polygons were obtained by joining the upper 95%

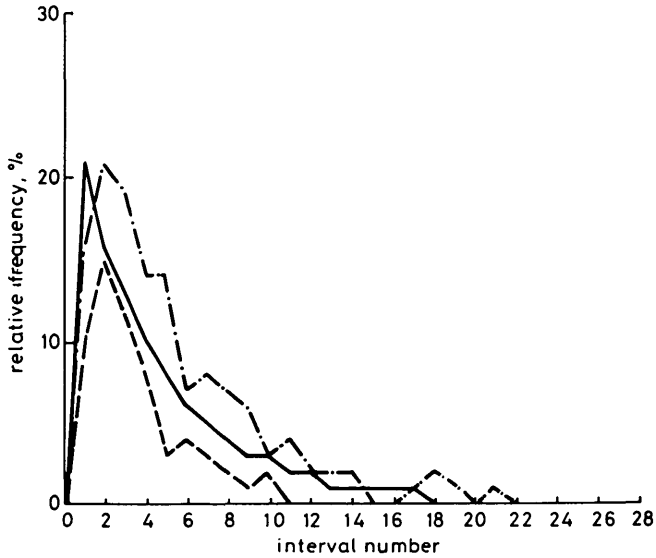

Fig. 4 *Frequency polygon from HS electricity consumption distribution model and upper- and lower-limit polygons for HS sample*

——— HS model
–·–·– upper limit
– – – – lower limit

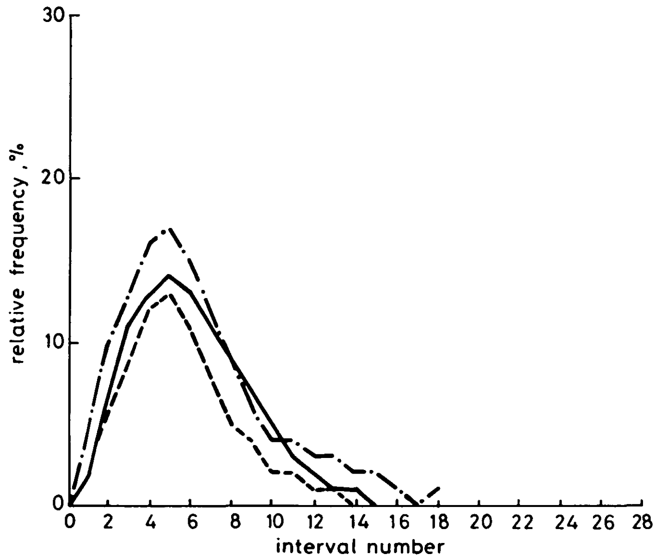

Fig. 5 *Frequency polygon from SN electricity consumption distribution model and upper- and lower-limit polygons for SN sample*

——— SN model
–·–·– upper limit
– – – – lower limit

confidence limits, the lower-limit polygons by joining the lower limits. A candidate model is judged to be acceptable if it produces a frequency distribution lying between these limits. Greater weight should be placed on the earlier part of the distribution where the observed consumption frequencies are greater. The HS and SN models provide a reasonable fit for their respective samples. However, neither constitutes an appropriate model for the complete sample of public sector housing, as shown in Fig. 6.

There is therefore a need to broaden the approach to a suitable model for the complete sample as well as refining the distribution models for the HS and SN data.

4 Weibull distribution

The Weibull distribution [8] has proved useful in engineering applications such as failure analysis. Here, the distribution takes the normalised form (Section 2)

$$f/N = amc_R^{m-1} \exp(-ac_R^m) \tag{9}$$

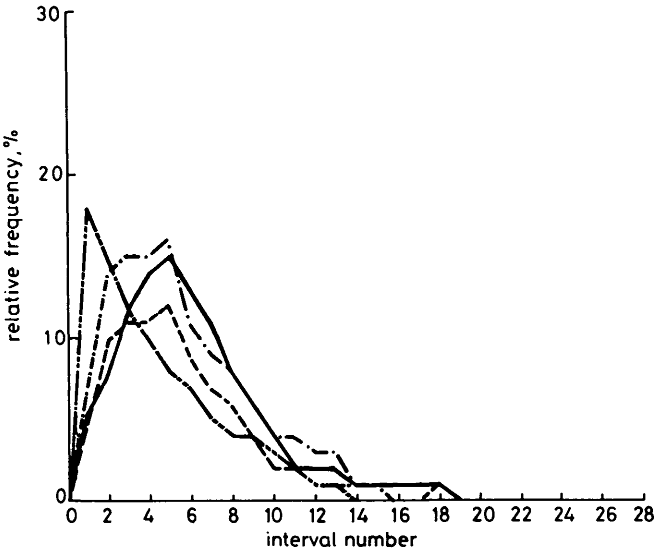

Fig. 6 *Frequency polygons from SN and HS electricity consumption distribution models and upper- and lower-limit polygons for complete sample of public sector housing*

——— SN model
– · · – HS model
–·–·– upper limit
– – – – lower limit

where

f = frequency density
c_R = relative electricity consumption
N = total number of consumers
m, a = model parameters

The HS distribution, eqn. 1 and the SN distribution, eqn. 2, are obtained by inserting $m = 1$ and $m = 2$ in eqn. 9. These, then, are special cases of the Weibull model.

It will now be shown that the required distribution models of electricity consumption in the chosen demand sector can be derived by appropriate selection of parameters a and m in eqn. 9.

5 Parameter estimation

The general form of the 2-parameter Weibull distribution is given in eqn. 9. The parameters a and m control the shape of the distribution and must be estimated from the available sample data.

Two common methods of parameter estimation are available, the method of moments and the method of maximum likelihood. It will be demonstrated that the pursuit of both methods yields useful insight into the numerical behaviour of the final distribution models.

5.1 Method of moments estimation

The method of moments matches theoretical derivations of the first and second moments, based on the general Weibull form of eqn. 9, with the values obtainable from practical samples of consumption levels.

Suppose a consumer population has been examined by recording consumption levels for N consumers. The continuous frequency density is assumed to be Weibull and will be given by

$$f = Namc^{m-1} \exp(-ac^m) \tag{10}$$

The first moment about $c = 0$ is then

$$M_1 = \frac{N\left(\frac{1}{m}\right)!}{a^{1/m}} \tag{11}$$

But

$$M_1 = c_T = \text{total consumption for the sample} \tag{12}$$

Eqns. 11 and 12 may be combined to give

$$\frac{c_T}{N} = \bar{c} = \frac{\left(\frac{1}{m}\right)!}{a^{1/m}} \tag{13}$$

The second moment about $c = 0$ is given by

$$M_2 = \frac{N\left(\frac{2}{m}\right)!}{a^{2/m}} \tag{14}$$

When the population has been sampled practically, the values of M_1 and M_2 may be estimated from the observed frequency distribution. If the histogram has n intervals and observed frequency of f_i in the ith interval,

$$\hat{M}_1 = \sum_{i=1}^{n} f_i c_i \tag{15}$$

and

$$\hat{M}_2 = \sum_{i=1}^{n} f_i c_i^2 \tag{16}$$

Estimation of the model parameters a, m is achieved by matching the theoretical and practical values of these two moments by choice of a and m.

Hence

$$\frac{N\left(\frac{1}{m}\right)!}{a^{1/m}} = \sum_{i=1}^{n} f_i c_i \tag{17}$$

and

$$\frac{N\left(\frac{2}{m}\right)!}{a^{2/m}} = \sum_{i=1}^{n} f_i c_i^2 \tag{18}$$

whence

$$\frac{\left(\frac{2}{m}\right)!}{\left[\left(\frac{1}{m}\right)!\right]^2} = \frac{\sum_{i=1}^{n} f_i c_i^2}{N\left[\sum_{i=1}^{n} f_i c_i\right]^2} \tag{19}$$

Eqn. 19 provides the basis for an estimator for m because the f_i and c_i are known for a given sample. The equation may be solved graphically. Table 2 gives the experimental frequency data for the sample of 1653 residential consumers in the public sector. The estimator for m then becomes

$$\frac{\left(\frac{2}{m}\right)!}{\left[\left(\frac{1}{m}\right)!\right]^2} = \frac{16266.81}{1653 \times 4243.75} = 2.8627 \times 10^{-3}$$

or

$$\left(\frac{2}{m}\right)! = \left[\left(\frac{1}{m}\right)!\right]^2 \times 2.8627 \times 10^{-3} \tag{20}$$

Table 2: Consumption frequency distribution data for the sample of public sector houses

Interval number i	Annual consumption c_i, 10^3 KWh	Frequency f_i	Relative annual consumption $c_i/\bar{c}$
1	0.25	104	0.10
2	0.75	195	0.29
3	1.25	218	0.49
4	1.75	210	0.68
5	2.25	228	0.88
6	2.75	163	1.07
7	3.25	140	1.27
8	3.75	111	1.46
9	4.25	80	1.66
10	4.75	45	1.85
11	5.25	48	2.04
12	5.75	30	2.24
13	6.25	25	2.43
14	6.75	13	2.63
15	7.25	10	2.82
16	7.75	7	3.02
17	8.25	3	3.21
18	8.75	10	3.41
19	9.25	3	3.60
20	9.75	0	3.80
21	10.25	5	3.99
22	10.75	3	4.19
23	11.25	0	4.38
24	11.75	0	4.58
25	12.25	0	4.77
26	12.75	1	4.97
27	13.25	0	5.16
28	13.75	1	5.36

The graphical solution of eqn. 20 is shown in Fig. 7. The intersection of the graphs produces an estimated m value of 1.50. From eqn. 13 the estimate for a is 0.21.

5.2 Normalised model form

The method above illustrates the following important properties of the parameter values:

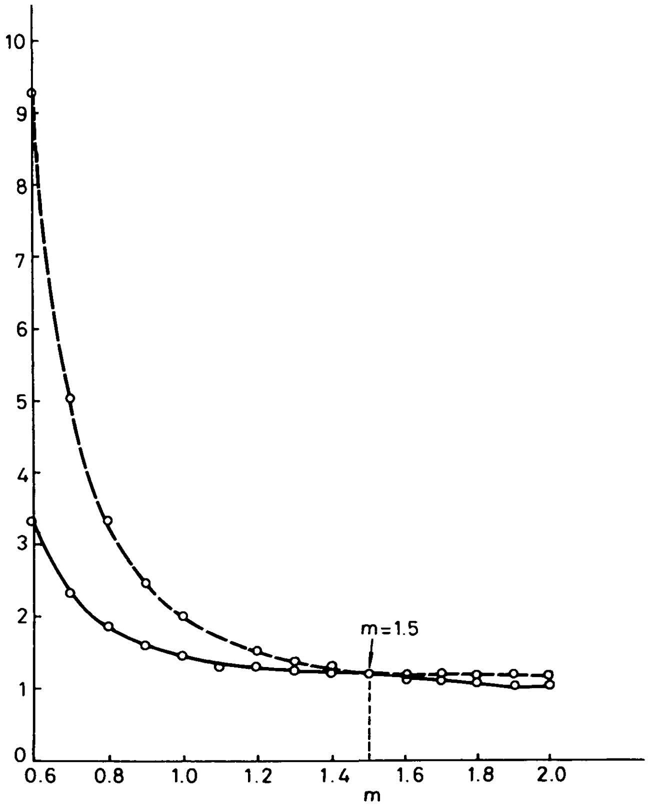

Fig. 7 *Matching moments solution for Weibull parameter m (eqn. 20)*
——— $[(1/m)!]^2 \times 2.8627 \times 10^{-3}$
- - - - $(2/m)!$

(i) From eqn. 19, the estimate of m is independent of the scale chosen for consumption

(ii) From eqn. 13, the value of the a parameter is influenced by the average consumption and hence the scale for consumption affects the value of the a parameter. It is attractive to normalise the model by using the relative consumption, c_R. The a parameter is then given by

$$a = \left[\left(\frac{1}{m}\right)!\right]^m \tag{21}$$

For the sample detailed in Table 2 the normalised model parameters then become $m = 1.50$ and $a = 0.86$.

5.3 Maximum likelihood estimation

If a sample of consumption levels, c_i for $i = 1$ to n, is encountered, the log likelihood function, based on the normalised 2-parameter Weibull model, is

$$\ln (L) = n \ln (am) + \ln \left(\prod_{i=1}^{n} c_{Ri}^{m-i}\right) - a \sum_{i=1}^{n} c_{Ri}^{m}$$

where

$$c_{Ri} = c_i/\bar{c} \tag{22}$$

Performing the partial differentiation for the two variables a and m in the presence of known consumption sample values yields

$$a = \frac{n}{\sum_{i=1}^{n} c_i^m} \tag{23}$$

and

$$\left[\frac{n}{\sum_{i=1}^{n} c_{Ri}^m}\right]\left[\sum_{i=1}^{n} c_{Ri}^m \ln c_{Ri}\right] = \frac{n}{m} + \sum_{i=1}^{n} \ln c_{Ri} \tag{24}$$

These equations may be solved iteratively to find m and a. For the housing sample of Table 2 the estimates for the normalised parameters then become $m = 1.55$ and $a = 0.85$.

The parameters a and m have also been estimated for all 11 public sector housing samples and plotted as shown in Fig. 8. The theoretical relationship between a and m for the normalised Weibull models is given by eqn. 21 and this is shown in Fig. 8 for comparison. Two important deductions may be made:

(i) The a parameter has an average value of 0.845 with a standard deviation of 0.053. The value of a may then be taken at the average value of 0.845 and the models will be characterised by the value of m

(ii) If $m = 1.6$ is taken as a break point, an obvious classification of the eleven samples into two distinct categories may be defined as shown in Table 3.

With the exception of the Lisburn urban sample this classification matches the HS and SN suggested in the preliminary work [7].

5.4 Parameter confidence limits

The parameter values indicated in Fig. 8 result from point estimation by maximum likelihood. The results of Section 3 have established upper and lower 95% confidence limits for frequency data. If the upper and lower limit frequency values are input to the maximum likelihood method the resulting point estimates of m and a represent the range of parameter values corresponding to the 95% confidence interval on frequencies.

Table 4 shows the resulting range of m values for Class I, Class II and the total sample.

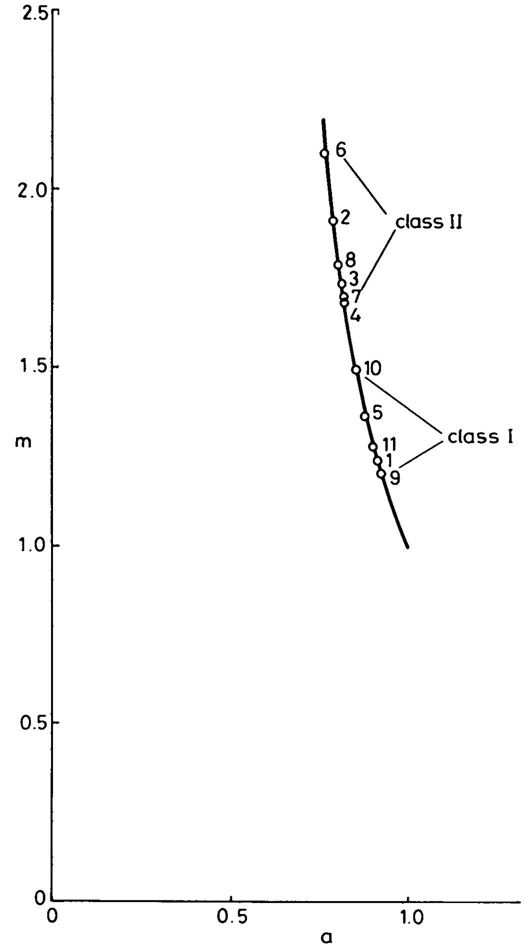

Fig. 8 *Maximum likelihood estimates of normalised Weibull model parameters a, m*

1 Belfast urban
2 Bangor urban
3 Bangor rural
4 Omagh urban
5 Omagh rural
6 Londonderry urban
7 Londonderry rural
8 Ballymena urban
9 Ballymena rural
10 Lisburn urban
11 Lisburn rural
—— theoretical curve

Table 3: Sample classification by parameter m

Class I $m < 1.6$	Class II $m > 1.6$
Belfast urban	Bangor urban
Omagh rural	Bangor rural
Ballymena rural	Omagh urban
Lisburn urban	Londonderry urban
Lisburn rural	Londonderry rural
	Ballymena urban

Table 4: Maximum likelihood estimates of the Weibull model m parameter

Sample	Upper limit frequencies	Observed frequencies	Lower limit frequencies
Total sample	1.52	1.55	1.72
Class I	1.26	1.34	1.43
Class II	1.75	1.89	2.12

5.5 Parameter sensitivity

Maximum likelihood estimation of a and m provides point estimates but does not offer insight into the sensitivity of the resulting model to parameter variations in the vicinity of these estimates. A useful technique is to examine the behaviour of the log likelihood in the neighbourhood of the optimum in the ma plane by plotting contours of constant log likelihood.

It may be concluded from Fig. 9 that the 2-parameter Weibull model is relatively insensitive to variation in m and a in the vicinity of the optimum point estimates. The adoption of the point estimates in Table 4 is therefore justifiable.

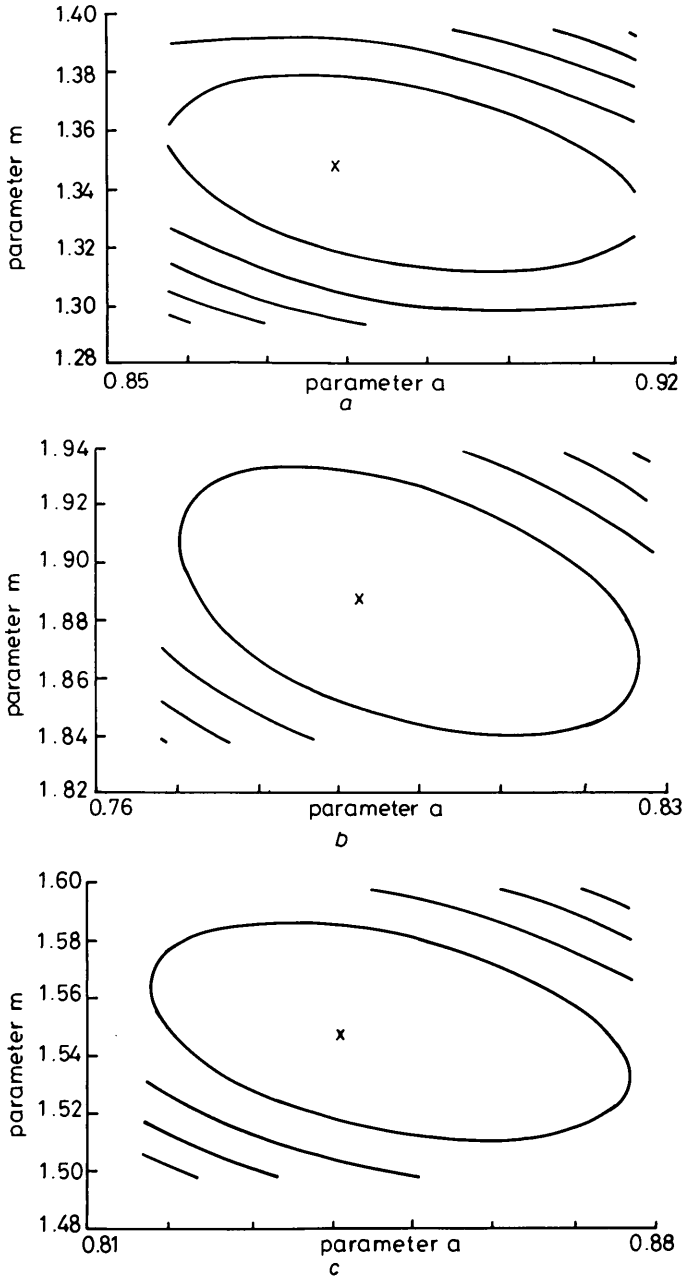

Fig. 9 *5% log likelihood contours*

a Class I (normalised)
b Class II (normalised)
c Total sample (normalised)

6 Verification

Fig. 6 showed that the HS ($m = 1$) and SN ($m = 2$) distribution models were unsuitable for describing the electricity consumption in the complete sample from the chosen demand sector. Maximum likelihood parameter estimation produced an m value of 1.55 for the Weibull distribution. The distribution model follows by substituting $m = 1.55$ and $a = 0.845$ in eqn. 9:

$$f/N = 1.26c_R^{0.55} \exp(-0.845c_R^{1.55}) \quad (25)$$

Fig. 10 shows that the frequency polygon obtained from eqn. 25 gives a better fit as required.

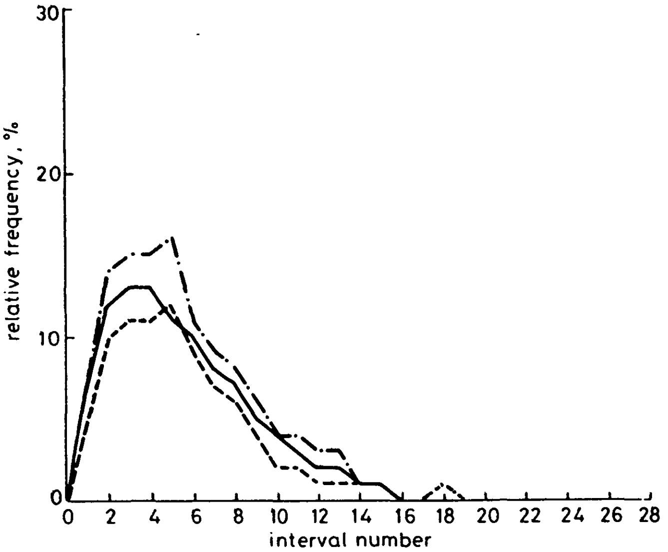

Fig. 10 *Frequency polygon from Weibull distribution model ($m = 1.55$, $a = 0.845$) and upper- and lower-limit polygons for complete sample of public sector housing*

——— Weibull model
–·–·– upper limit
– – – – lower limit

The Weibull distribution models for the class I and class II data (Table 3) are given by

$$f/N = 1.13c_R^{0.34} \exp(-0.845c_R^{1.34}) \quad (26)$$

$$f/N = 1.60c_R^{0.89} \exp(-0.845c_R^{1.89}) \quad (27)$$

6.1 Significance between class I and class II

The statistical approach to assessing candidate distribution models can also be used to test whether the observed frequency polygons for the class I and class II samples are significantly different.

Fig. 11 gives the upper and lower frequency polygons obtained from the class II data. The observed frequency

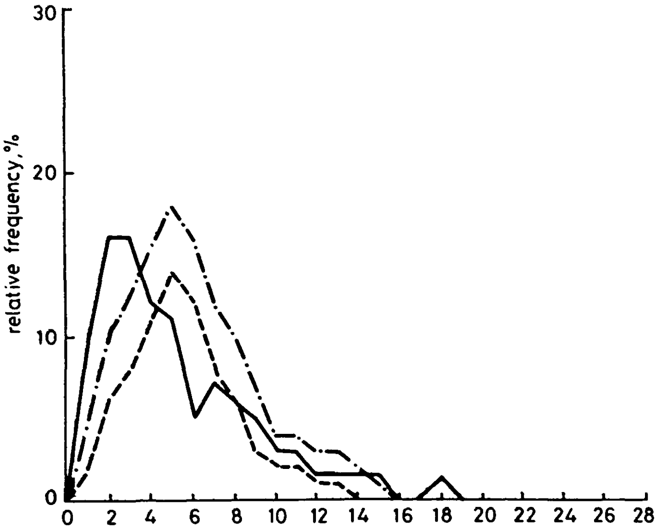

Fig. 11 *Upper- and lower-limit polygons for Class II data and observed frequency polygon of class I data*

——— observed
–·–·– upper limit
– – – – lower limit

polygon from the class I data can be seen to lie outside these confidence limits. This substantiates further the conclusion that two underlying consumption distributions

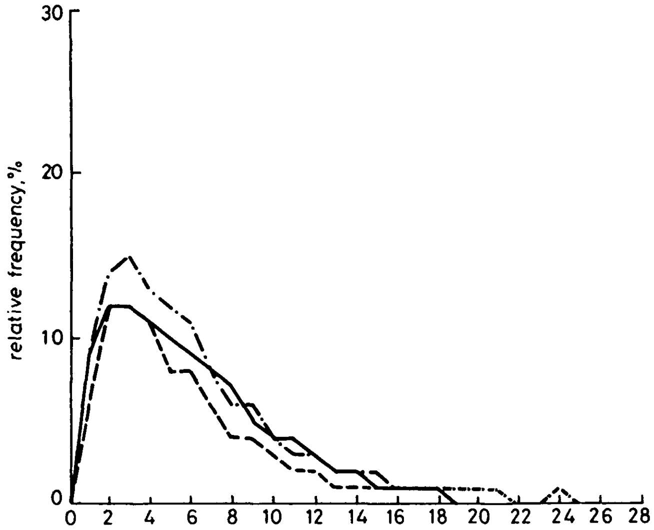

Fig. 12 *Frequency polygon from Class I Weibull model and upper- and lower-limit polygons for an independent sample of 2704 consumers from Belfast*

——— calculated
–·–·– upper limit
– – – – lower limit

exist within this demand sector, for which the models are given in eqns. 26 and 27.

6.2 Validation of the class I model

The electricity consumption of a further 2704 households in Belfast has been obtained from billing data. This new sample has been used to validate the class I model. Fig. 12 gives the upper and lower limit polygons calculated from this data together with the frequency polygon calculated from eqn. 27. The good fit obtained provides independent verification of this model.

7 Utilisation of distribution models

The aim of this Section is to outline the planned approach to using the electricity distribution models within a forecasting context.

The public housing sector, which provided the data for this study, constitutes a homogeneous group in social and economic terms, yet the statistical evidence indicates that in a particular year two models are required to describe the electricity demand patterns. The approach therefore allows categorisation of groups within a particular demand sector.

To explain the existence of two distinct groups, an examination must be made of likely parameters of influence. These include tariff structures, rateable value and local employment as well as scientific measures such as house structure. Note that the analysis discussed here is based on annual consumption data for a particular year and work is progressing to discover how the m and a parameters vary with time. Preliminary results indicate that the parameters do exhibit a small seasonal trend within a particular year but the group classification is maintained. The stability of the Weibull parameters and associated group classification is now being studied on an annual basis. It is anticipated that m and a will vary with time and the degree of correlation with dynamic parameters of influence may be used to decide which parameters are significant. For example, local levels of employment might explain the existence of two groups within the public housing sector and the classification of an individual estate. A sudden increase in unemployment, caused by the closure of a factory, would then be expected to result in a reclassification of that estate in a subsequent year. In this way the electricity consumption for the entire sector would be characterised by two distribution models with *m* and *a* parameters and relative weights between the two groups varying dynamically with employment levels. This would provide the supply authority with a useful tool to use for forecasting purposes.

8 Discussion

The 2-parameter Weibull model has been shown to have sufficient flexibility to model electricity consumption distributions. This is achieved by variation of the single parameter *m* with the second Weibull parameter sensibly constant.

The analysis has shown that there are two groups within the public housing demand sector and illustrates the viability of the technique for group identification to a selected confidence level. The existence of two groups permits an examination to be made of static factors which might influence electricity consumption, e.g. location and rateable value. In addition, the variation of the Weibull parameters with time, for a group, provides a systematic way of testing dynamic factors such as oil price and disposable income.

Extension to private housing and other demand sectors such as commercial and industrial is straightforward. The end result is a complete set of groups, described by Weibull distribution models, together with associated parameters of influence. The more general problem of demand forecasting may then be approached on the basis of contributions from these disaggregate groups. The work reported in the present paper is therefore seen as an important preliminary step towards demand forecasting from consumer billing data.

9 Conclusions

The following conclusions follow from the work:

(i) the generalised Weibull model in two parameters may be employed to model demand patterns over a wide range of underlying distributions

(ii) in practical terms, the normalised form of this model may be characterised by the single parameter *m*

(iii) maximum likelihood methods provide a convenient means of estimating the Weibull parameters

(iv) likelihood contouring permits an assessment of parameter sensitivity

(v) the consumption frequency confidence limits may be used to estimate parameter ranges

(vi) the existence of two statistically separated subclasses, within the public housing sector, has been established

(vii) additional data has permitted independent validation of the Weibull model for an identified class.

10 Acknowledgments

The authors wish to acknowledge the co-operation and support of the Northern Ireland Electricity Service.

11 References

1 FAZZOLARE, R.A., and SMITH, C.B.: 'Beyond the energy crises'. Proceedings of 3rd International Conference on Energy Use Management, Berlin, West Germany, October 1981

2 WARD, R.K.: 'An econometrical model to forecast electricity demand for Zimbabwe Rhodesia', *Trans. S. Afr. Inst. Electr. Eng.*, 1980, **71**, Pt. 1, pp. 2–10

3 BAXTER, R.E., and REES, R.: 'Analysis of the industrial demand for electricity', *Econ. J.*, June 1968, pp. 278–298

4 BROEHL, J.H., and GRIFFIN, J.M.: 'A forecasting model for annual electrical energy consumption and peak demand'. Proceedings of University of Pittsburgh Modelling and Simulation Conference, Pittsburgh, PA, USA, April 1978, Vol. 9, Pt. 1, pp. 249–254

5 ALEXANDER, S.M.: 'A dynamic simulation model for forecasting and policy making in the regulated electricity utility', *Comput. & Ind. Eng.*, 1979, **3**, pp. 269–287

6 URI, N.D.: 'Forecasting peak system load using a combined time-series and econometric model', *Appl. Energy*, 1978, **4**, pp. 219–227

7 MONTEITH, W., IRWIN, G.W., BEATTIE, W.C., and MORRELL, D.: 'Disaggregate approach to residential electricity demand modelling'. Proceedings of Summer Computer Simulation Conference, Boston, MA, USA, July 1984, Vol. 2, pp. 736–742

8 SHAPIRO, S., and GROSS, A.J.: 'Statistical modelling techniques' (Marcel Dekker, New York and Basel, 1981)