Logarithms:

The idea of using logarithms is to reduce multiplication and division into addition and subtraction respectively which are far more easier to deal with and to simplify the operations. Today methods of logarithms has become a significant tool for handling complex calculations involving 4 digit numbers. Theory of indices is essential to understand logarithms.

Meaning: If a is any +ve real number other than 1 such that $a^x = n$ then x is called log (n) to base a.

$\therefore \log_a(n) = x$ — log n to base a = x

Eg.

$b^x = n$, log n to base b = x

$2^4 = 16 \rightarrow \log_2(16) = 4$

$2^5 = 32 \rightarrow \log_2(32) = 5$

$4^{3/2} = 8 \rightarrow \log_4(8) = 3/2$

$3^{5/6} = 6 \rightarrow \log_3 6 = 5/6$

Thus logarithm of a number to a given base is defined as index to which the base must be raised to obtain the number.

Note → 1) Logarithms to base 10 → Common logarithms.
Logarithms to base e → Natural logarithms.
2) Logarithms

1) It is noted that, Logarithm of any qty ~~to~~ to same base = 1

ie $10^1 = 10 \rightarrow \log_{10} 10 = 1$

$2^1 = 2 \rightarrow \log_2 2 = 1$

$8^1 = 8 \rightarrow \log_8 8 = 1$

In general $a^1 = a$ $\boxed{\log_a a = 1}$

1) Logarithm of 1 to any base is 0.

Eg $10^0 = 1 \rightarrow \log_{10} 1 = 0$

$3^0 = 1 \rightarrow \log_3 1 = 0$

in general $\boxed{a^0 = 1 \rightarrow \log_a 1 = 0}$

(3) Base is not taken as 0 or 1. $\because$ 0^x is meaningless. $1^x = 1$

(4) When Base is not mentioned it is implied that base is 10.

Laws of logarithms:

I law: $\boxed{\log_a(mn) = \log_a m + \log_a n}$

II law: $\boxed{\log_a(m/n) = \log_a m - \log_a n}$

$\log_a 1/n = \log_a 1 - \log_a n$

$= 0 - \log_a n = -\log_a n$

III law: $\boxed{\log_a(m^n) = n \log_a m}$

IV law: $\log_a m = \dfrac{\log_b m}{\log_b a}$

Problems:

State the following in logarithmic form:

1) $2^4 = 16$. $\log_2 16 = 4$

2) $2^5 = 32$ $\log_2 32 = 5$

3) $3^{-2} = 1/9$ $\log_3 1/9 = -2$

4) $5^{-2} = 0.04$ $\log_5 0.04 = -2$

(5) $3^4 = 81$. $\log_3 81 = 4$

(6) $3^{-5} = \frac{1}{243}$ $\log_3 1/243 = -5$

(7) $(125)^{-2/3} = 0.04$ $\log_{125} 0.04 = -2/3$

(8) $(5\sqrt{5})^2 = 125$ $\log_{5\sqrt{5}} 125 = 2$

(9) $3^3 = 27$ $\log_3 27 = 3$

(10) $5^{-1} = 0.2$ $\log_5 0.2 = -1$

Express the following in index (exponential) form. ③

1) $\log_9 81 = 2$. $\quad 9^2 = 81$

2) $\log_{10} 0.01 = -2$. $\quad 10^{-2} = 0.01$

3) $\log_2 1/4 = -2$ $\quad 2^{-2} = 1/4$

4) $\log_4 1024 = 5$ $\quad 4^5 = 1024$

5) $\log_2 1/256 = -8$ $\quad 2^{-8} = 1/256$

6) $\log_{10} 0.0001 = 1/4$ $\quad 10^{1/4} = 0.0001$

7) $\log_5 125 = 3$ $\quad 5^3 = 125$

8) $\log_2 1/2 = -1$ $\quad 2^{-1} = 1/2$

III Find the value of $\log_2 1024$

Express in index form & simplify.

$\log_2 1024 = x$

$2^x = 1024$

$2^x = 2^{10}$

$\therefore \boxed{x = 10}$

2) $\log_{10} 0.00001$

Let $\log_{10} 0.00001 = x$

$10^x = 0.00001$

$10^x = \frac{1}{100000}$

$10^x = 10^{-5}$

$\underline{\underline{x = -5}}$.

⑤ $\log_7 x = 2$.

$x = 7^2$

$\underline{\underline{x = 49}}$

③ Evaluate $\log_{5\sqrt{5}} 125 = x$

$(5\sqrt{5})^x = 125$

$(5\sqrt{5})^x = 5^3$

$(5 \times 5^{1/2})^x = 5^3$

$(5^{1+1/2})^x = 5^3$

$5^{3/2 x} = 5^3$

$\frac{3}{2}x = 3$

$3x = 6$

$\underline{\underline{x = 2}}$.

⑥ $\log_{\sqrt{x}} 4 = 2$

$(\sqrt{x})^2 = 4$

$(x^{1/2})^2 = 4$

$\underline{\underline{x = 4}}$

④ Find value of $\log_{\sqrt{3}} 27$.

let $\log_{\sqrt{3}} 27 = x$

$(\sqrt{3})^x = 27$

$(\sqrt{3})^x = 3^3$

$(3^{1/2})^x = 3^3$

$3^{x/2} = 3^3$

$\frac{x}{2} = 3$

$\underline{\underline{x = 6}}$.

III Solve for x

$\log_2 \sqrt{32} = x$

$2^x = \sqrt{32}$

$2^x = \sqrt{2^5}$

$2^x = (2^5)^{1/2}$

$x = 5/2$

$\log_{0.1} 10 = x$

$(0.1)^x = 10$

$\left(\frac{1}{10}\right)^x = 10$

$10^{-x} = 10^1$

$\therefore x = -1$

$\log_{\sqrt[3]{5}} x = 6$

$(\sqrt[3]{5})^6 = x$

$(5^{1/3})^6 = x$

$5^{6/3} = x$

$5^2 = x$

$x = 25$

$\log_x 625 = 4$

$x^4 = 625$

$x^4 = 5^4$

$x = 5$

$\log_x 343 = 3$

$x^3 = 343$

$x^3 = 7^3$

$x = 7$

$\log_x 243 = 5$

$x^5 = 243$

$x^5 = 3^5$

$x = 3$

$\log_{10} x = 2$

$10^2 = x$

$x = 100$

$\log_3 x = 6$

$3^6 = x$

$x = 729$

Find the value of.

$\log 2 + \log 3 + \log 4$.

$\log 2 + \log 3 + \log 4$

$= \log(2 \times 3 \times 4)$

$= \log 24$.

Find m if $\log_3 m = 3$

$\log_3 m = 3$

$3^3 = m$

$27 = m$

$m = 27$

2. Simplify: $\log 0.1 + \log 0.01 + \log 0.5$

I Law: $\log(0.1 \times 0.01 \times 0.5)$

$\log 0.0005$.

3. $\log 5 + \log 8 + \log 6$

I Law: $\log(5 \times 8 \times 6)$

$\log 240$.

Show that $\log(1+2+3) = \log 1 + \log 2 + \log 3$ (5)

w.k.t $\log m + \log n = \log(mn)$

so that $\log(1) + \log(2) + \log(3) = \log(1\times 2\times 3)$

$= \log(1) + \log(2) + \log(3) = \log(6)$.

$= \log(1+2+3) \cdot \log(3+3) = \log(6)$

$\therefore \log(1+2+3) = \log 1 + \log 2 + \log 3$

$\log 6 = \log 6$

Hence proved.

) simplify $\frac{1}{2}\log_{10} 25 - 2\log_{10} 3 + \log_{10} 18$

$= \log 25^{1/2} - 2\log 3 + \log 18$

$\log (5^2)^{1/2} - 2\log 3 + \log 18$

$= \log 5 - \log 3^2 + \log 18$

$\log 5 - \log 9 + \log 18$

$= \log 5/9 + \log 18$

$= \log 5/9 \times \overset{2}{\cancel{18}}$

$= \log_{10} 10$

$= \underline{\underline{1}}$

7) Find value of $\log_b a, \log_c b, \log_d c, \log_e d, \log_a e$ change of base 10.

$$\frac{\log a}{\log b} \times \frac{\log b}{\log c} \times \frac{\log c}{\log d} \times \frac{\log d}{\log e} \times \frac{\log e}{\log a} = \underline{\underline{1}}$$

(8) $\log \frac{26}{33} - \log \frac{65}{69} + \log \frac{55}{46} = 0$

$\log m/n = \log m - \log n$.

$(\log 26 - \log 33) - (\log 65 - \log 69) + (\log 55 - \log 46)$

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$(\log 13\times 2 - \log 11\times 3) - (\log 13\times 5 - \log 23\times 3) + (\log 11\times 5 + \log 23\times 2)$

$= (\log 13 + \log 2) - (\log 11 + \log 3) - [(\log 13 + \log 5) - (\log 23 + \log 3)]$
$\quad + (\log 11 + \log 5) + (\log 23 + \log 2)$

$\cancel{\log 13} + \cancel{\log 2} - \cancel{\log 11} - \cancel{\log 3} - \cancel{\log 13} - \cancel{\log 5} + \log 23 + \cancel{\log 3}$
$\quad + \cancel{\log 11} + \cancel{\log 5} + \cancel{\log 23} - \cancel{\log 2}$
$= \underline{\underline{0}}$

* $\log 9/5 + \log 15/9 - \log 3/2 = \log 2$

$\log 9/5 + \log 15/9 - \log 3/2$
$(\log 9 - \log 5) + (\log 15 - \log 9) - (\log 3 - \log 2)$
$(\log 9 - \log 5) + (\log 3\times 5) - (\log 9) - (\log 3 - \log 2)$
$(\log 9 - \log 5) + [\log 3 + \log 5 - \log 9] - (\log 3 - \log 2)$
$\log 9 - \log 5 + [\log 3 + \log 5 - \log 9 - \log 3 + \log 2$
$= \cancel{\log 9} - \cancel{\log 5} + \cancel{\log 3} + \cancel{\log 5} - \cancel{\log 9} - \cancel{\log 3} + \log 2$
$= \underline{\underline{\log 2}}$

Prove that
$2\log 3/7 + \log \frac{49}{9} = 0.$

$2\log 3/7 + \log 49/9.$
$\log (3/7)^2 + \log 49/9.$
$= \log \frac{9}{49} + \log 49/9.$
$= \log [\cancel{9}/\cancel{49} \times \cancel{49}/\cancel{9}]$
$= \log 1 = \underline{\underline{0}}.$

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Prove that $\log_b a \times \log_c b \times \log_a c = 1$

Changing to base 10

$$\frac{\log b}{\log a} \times \frac{\log k}{\log c} \times \frac{\log c}{\log c}$$

$$\frac{\log a}{\log b} \times \frac{\log b}{\log c} \times \frac{\log c}{\log a} = \underline{1}$$

Show that $\log\left(\frac{a^2}{bc}\right) + \log\left(\frac{b^2}{ca}\right) + \log\left(\frac{c^2}{ab}\right) = 0$

$(2\log a - (\log bc) + (2\log b - (\log ca) + 2\log c - (\log ab)$

$= 2\log a + 2\log b + 2\log c\,[-(\log b + \log c) - (\log c + \log a) - (\log a + \log b)]$

$2\log a + 2\log b + 2\log c - \log b - \log c - \log c - \log a - \log a - \log b.$

$= 2\log a + 2\log b + 2\log c - 2\log b - 2\log c - 2\log a.$

$= \underline{0}$

~~Show~~ Simplify

$\log\left(\frac{81}{16}\right) - \log\frac{8}{9} + \log\frac{128}{243}$

$= (\log 81 - \log 16) - [\log 8 - \log 9] + [\log[128 - \log 243]$

$= (\log 3^4 - \log 2^4) - [\log 2^3 - \log 3^2] + [\log 2^7 - \log 3^5]$

$= (4\log 3 - 4\log 2) - [3\log 2 - 2\log 3] + [7\log 2] - 5\log 3]$

$= 4\log 3 - 4\log 2 - 3\log 2 + 2\log 3 + 7\log 2 - 5\log 3.$

$\underline{\log 3}.$

Simplify

$\log 75/16 - 2\log 5/9 + \log 32/243.$

$(\log 75 - \log 16) - [2\log 5 - 2\log 9] + [\log 32 - \log 243]$

$= [\log(3\times5^2) - \log 2^4] - [2\log 5 - 2\log 3^2] + [\log 2^5 - \log 3^5]$

$= [\log 3 + 2\log 5 - 4\log 2] - [2\log 5 - 4\log 3] + (5\log 2 - 5\log 3)$

$= \log 3 + 2\log 5 - 4\log 2 - 2\log 5 + 4\log 3 + 5\log 2 - 5\log 3$

$5\log 2 - 4\log 2$

$= \underline{\log 2}.$

xithout using log tables

simplify $\log_5 \frac{(125)(625)}{25}$

$: \log_5 \frac{(5^3)(5^4)}{5^2} = \log_5 . 5^{3+4-2}$

$= \log_5 . 5^5$

$= 5 \log_5 5 = \underline{\underline{5}}$

$\underline{5(1)}$.

Given $\log 2 = 0.3010$, $\log 3 = 0.4771$, find $\log 6$

$\log(6) = \log(2\times3)$

$= \log 2 + \log 3.$

$= 0.3010 + 0.4771 = \underline{\underline{0.7781}}$.

=

.3010
.4771
.7781

$\log \sqrt[3]{36}. = \log(36)^{1/3}$

$= 1/3 \log(36)$

$= \frac{1}{3} \log 6^2$

$= \frac{1}{3} \times 2 \log 6$

$= \frac{2}{3} \log(2\times3) = \frac{2}{3}[\log 2 + \log 3]$

$= \frac{2}{3}[0.3010] + \frac{2}{3}[0.4771]$

$= 0.2006 + 0.3181 = \underline{\underline{0.5187}}$

$\log(0.125)$

$= \log(0.5)^3$

$= 3\log(0.5)$

$3\log 1 - 3\log 2$

$3(0) - 3\log 2$

$0 - 3\log 3$

$0 - 3(0.3010)$

$= -0.9030$.

(matissa part should be >1.

To make it +ve, take whole no > integer part of martissa

$1 - 1 - 0.9030$

-1

$1.$

$-0.9030.$

$\overline{0.0970} - 1$

$= \underline{\underline{\bar{1}.0970}}$.