Progressions

Progressions (or Sequences and Series) are numbers arranged in a particular order such that they form a predictable order. By predictable order, we mean that given some numbers, we can find next numbers in the series.

Types of progressions:

Arithmetic Progression (AP)
A sequence of numbers is called an arithmetic progression if the difference between any two consecutive terms is always same. In simple terms, it means that next number in the series is calculated by adding a fixed number to the previous number in the series. This fixed number is called the common difference.
For example, 2,4,6,8,10 is an AP because difference between any two consecutive terms in the series (common difference) is same (4 – 2 = 6 – 4 = 8 – 6 = 10 – 8 = 2).

If ‘a’ is the first term and‘d’ is the common difference,

- nth term of an AP = a + (n-1) d
- Arithmetic Mean = Sum of all terms in the AP / Number of terms in the AP
- Sum of ‘n’ terms of an AP = 0.5 n (first term + last term) = 0.5 n [2a + (n-1) d]

Geometric Progression (GP)
A sequence of numbers is called a geometric progression if the ratio of any two consecutive terms is always same. In simple terms, it means that next number in the series is calculated by multiplying a fixed number to the previous number in the series. This fixed number is called the common ratio.
For example, 2,4,8,16 is a GP because ratio of any two consecutive terms in the series (common difference) is same (4 / 2 = 8 / 4 = 16 / 8 = 2).

If ‘a’ is the first term and ‘r’ is the common ratio,

- nth term of a GP = a r^{n-1}
- Geometric Mean = nth root of product of n terms in the GP
- Sum of ‘n’ terms of a GP (r < 1) = [a (1 – r^n)] / [1 – r]
- Sum of ‘n’ terms of a GP (r > 1) = [a (r^n – 1)] / [r – 1]
- Sum of infinite terms of a GP (r < 1) = (a) / (1 – r)

Harmonic Progression (HP)
A sequence of numbers is called a harmonic progression if the reciprocal of the terms are in AP. In simple terms, a,b,c,d,e,f are in HP if 1/a, 1/b, 1/c, 1/d, 1/e, 1/f are in AP.

For two terms ‘a’ and ‘b’,

- Harmonic Mean = (2 a b) / (a + b)

For two numbers, if A, G and H are respectively the arithmetic, geometric and harmonic means, then

- $A \geq G \geq H$
- $A H = G^2$, i.e., A, G, H are in GP

Problems in Arithmetic Progression:

Q.1: What is the common difference of the arithmetic progression 10, 5, 0, -5?
Solution:
The common difference is -5

Q.2: Find the 10th term of the arithmetic progression 1, 3.5, 6, 8.5,...
Solution:
d = 3.5 - 1 = 6 - 3.5 = 2.5
n = 10
a is the first term
10th term = a +(n-1)d = 1 + (10-1)2.5 = 1 + 9 × 2.5 = 1 + 22.5 = 23.5

Q.3: Write first four terms of the A.P. when the first term a and the common difference d are given as follows:

(i) a = 10, d = 10
(ii) a = -2, d = 0
(iii) a = 4, d = – 3

Solution:

(i) a = 10, d = 10

Let us consider, the Arithmetic Progression series be a_1, a_2, a_3, a_4, a_5 …
$a_1 = a = 10$
$a_2 = a_1 + d = 10 + 10 = 20$
$a_3 = a_2 + d = 20 + 10 = 30$
$a_4 = a_3 + d = 30 + 10 = 40$
$a_5 = a_4 + d = 40 + 10 = 50$
And so on…

Therefore, the A.P. series will be 10, 20, 30, 40, 50 …
And First four terms of this A.P. will be 10, 20, 30, and 40.

(ii) a = – 2, d = 0

Let us consider, the Arithmetic Progression series be $a_1, a_2, a_3, a_4, a_5 \ldots$

$a_1 = a = -2$
$a_2 = a_1 + d = -2 + 0 = -2$
$a_3 = a_2 + d = -2 + 0 = -2$
$a_4 = a_3 + d = -2 + 0 = -2$
Therefore, the A.P. series will be – 2, – 2, – 2, – 2 …
And, First four terms of this A.P. will be – 2, – 2, – 2 and – 2.

(iii) a = 4, d = – 3

Let us consider, the Arithmetic Progression series be $a_1, a_2, a_3, a_4, a_5 \ldots$
$a_1 = a = 4$
$a_2 = a_1 + d = 4 - 3 = 1$
$a_3 = a_2 + d = 1 - 3 = -2$
$a_4 = a_3 + d = -2 - 3 = -5$

Therefore, the A.P. series will be 4, 1,- 2, – 5 …
And, First four terms of this A.P. will be 4, 1, – 2 and – 5.

Q.4: Which term of the AP: 21, 18, 15, . . . is – 81? Also, is any term 0? Give reason for your answer.

Solution : Here, a = 21,

d = 18 – 21 = – 3

and

$a_n = -81$, and we have to find n.
As $a_n = a + (n - 1) d$,
we have;

$-81 = 21 + (n - 1)(-3)$
$-81 = 24 - 3n$
$-105 = -3n$
So, n = 35
Therefore, the 35th term of the given AP is – 81.
Next, we want to know if there is any n for which $a_n = 0$.

If such an n is there, then;
$21 + (n - 1)(-3) = 0$,
$3(n - 1) = 21$
$n = 8$

Therefore, the eighth term is 0.

Q.5: Check whether – 150 is a term of the AP: 11, 8, 5, 2 . . .

Solution:

For the given, A.P. 11, 8, 5, 2, …
First term, a = 11
Common difference, $d = a_2 - a_1 = 8 - 11 = -3$

Let −150 be the nth term of this A.P.
As we know, for an A.P.,
$a_n = a + (n - 1) d$

$-150 = 11 + (n - 1)(-3)$
$-150 = 11 - 3n + 3$
$-164 = -3n$
$n = 164/3$

Clearly, n is not an integer but a fraction.
Therefore, – 150 is not a term of this A.P.

Q.6: Which term of the A.P. 3, 15, 27, 39, … will be 132 more than its 54th term?

Solution: Given A.P. is 3, 15, 27, 39, …
first term, a = 3
common difference, $d = a_2 - a_1 = 15 - 3 = 12$

We know that,
$a_n = a + (n - 1) d$

Therefore,

$a_{54} = a + (54 - 1) d$
$\Rightarrow 3 + (53) (12)$
$\Rightarrow 3 + 636 = 639$
$a_{54} = 639$We have to find the term of this A.P. which is 132 more than a_{54}, i.e. 771.
Let nth term be 771.
$a_n = a + (n - 1) d$
$771 = 3 + (n - 1) 12$
$768 = (n - 1) 12$

$(n - 1) = 64$
$n = 65$

Therefore, 65th term was 132 more than 54th term.