# Computation with Absolutely No Space Overhead

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# The Standard Model of Linear Space

tape

| 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 |
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Turing machine

## Characteristics

- Input fills fixed-size tape
- Input may be modified
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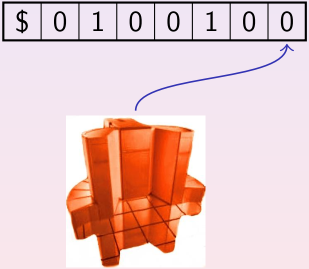


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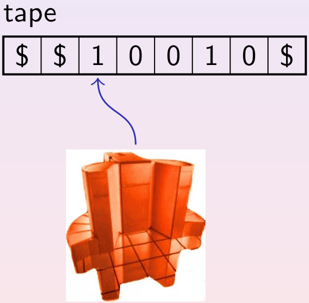


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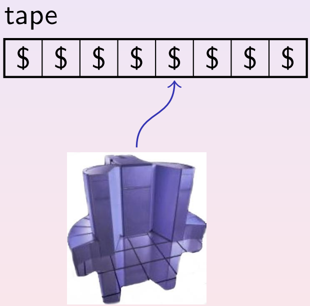


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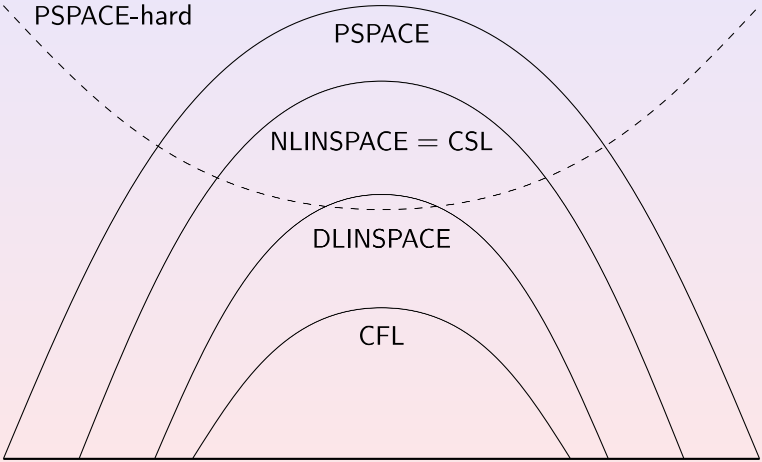
# Linear Space is a Powerful Model



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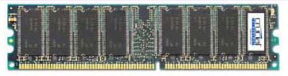

Turing machine

## Intuition

- Tape is used like a RAM module.

# Definition of Overhead-Free Computations

### Definition

A Turing machine is overhead-free if

- it has only a single tape,
- writes only on input cells,
- writes only symbols drawn from the input alphabet.

# Overhead-Free Computation Complexity Classes

### Definition

A language $L \subseteq \Sigma^*$ is in

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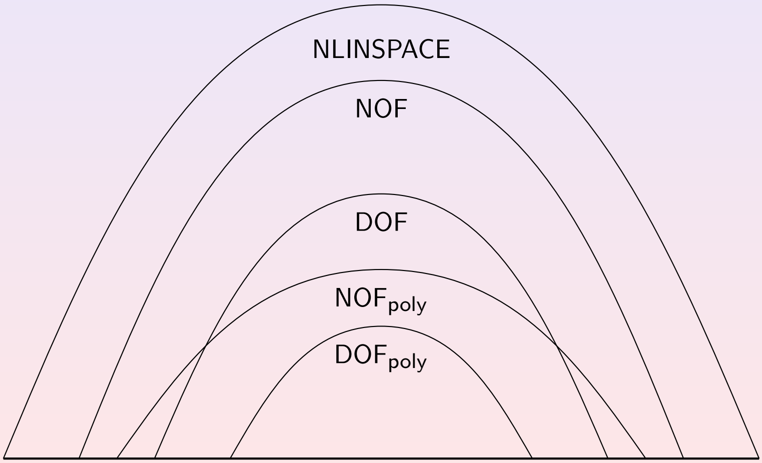
- NOF is the nondeterministic version of DOF,
- $\text{NOF}_{\text{poly}}$ is the nondeterministic version of $\text{DOF}_{\text{poly}}$.

# Simple Relationships among Overhead-Free Computation Classes



# Palindromes Can be Accepted in an Overhead-Free Way

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overhead-free machine

## Algorithm

Phase 1:
Compare first and last bit
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Phase 2:
Compare bits next to end markers
- Find left end marker
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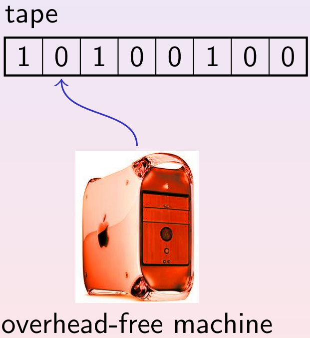


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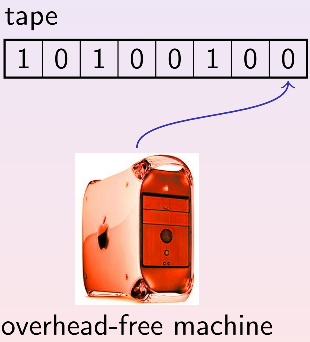


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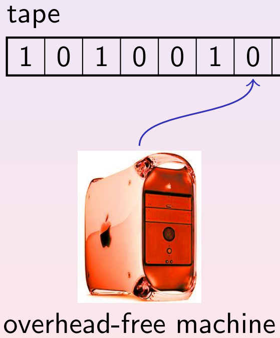


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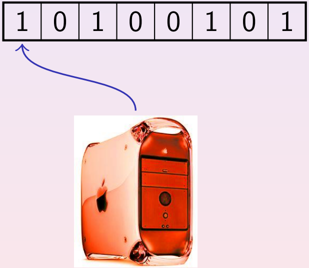


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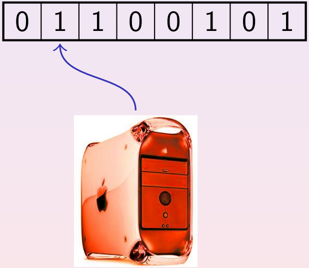


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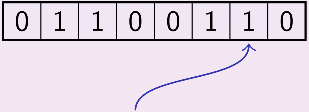


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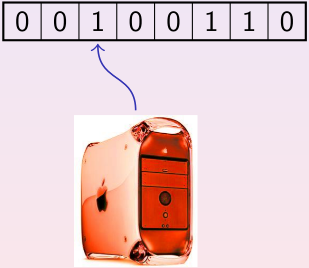


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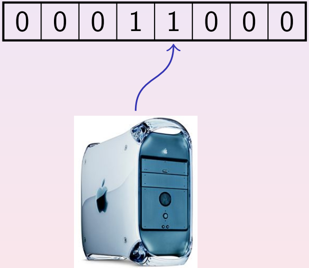


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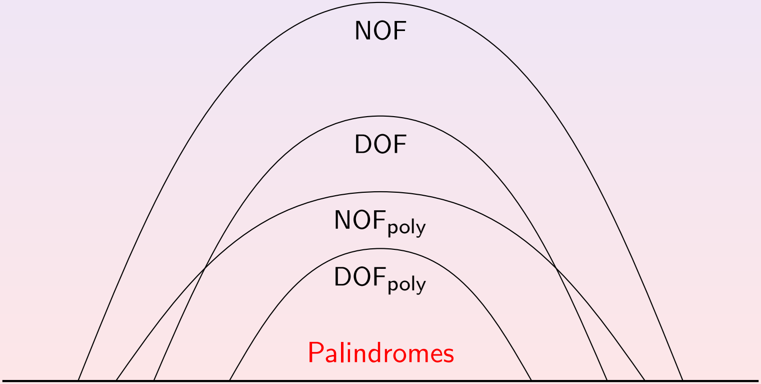
# Relationships among Overhead-Free Computation Classes



# A Review of Linear Grammars

## Definition

A grammar is linear if it is context-free and
there is only one nonterminal per right-hand side.

## Example

$G_1\colon S \longrightarrow 00S0 \mid 1.$

$G_2\colon S \longrightarrow 0S10 \mid 0.$

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## Example

$G_1$: $S \to 00S0 \mid 1.$
$G_2$: $S \to 0S10 \mid 0.$

## Definition

A grammar is deterministic if
"there is always only one rule that can be applied."

## Example

$G_1$ is deterministic.
$G_2$ is not deterministic.

# Deterministic Linear Languages Can Be Accepted in an Overhead-Free Way

### Theorem

Every deterministic linear language is in $\mathsf{DOF}_{\mathsf{poly}}$.

# Continued Review of Linear Grammars

### Definition

A language is metalinear if it is the concatenation of linear languages.

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### Definition

A language is metalinear if it is the concatenation of linear languages.

### Example

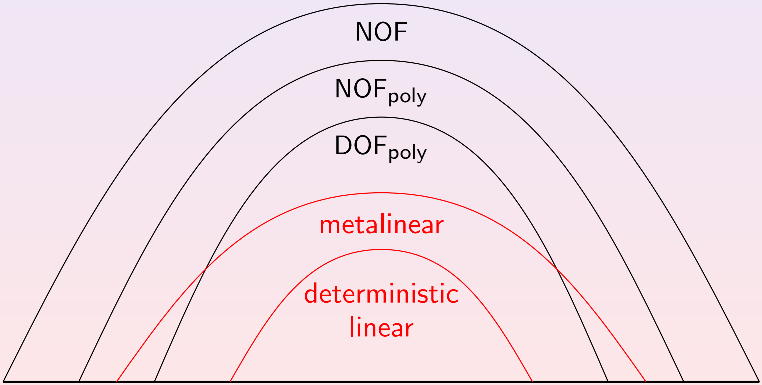
$\textsc{triple-palindrome} = \{uvw \mid u, v, \text{ and } w \text{ are palindromes}\}$.

# Metalinear Languages Can Be Accepted in an Overhead-Free Way

### Theorem

Every metalinear language is in $\mathrm{NOF}_{\mathrm{poly}}$.

# Relationships among Overhead-Free Computation Classes



# Definition of Almost-Overhead-Free Computations

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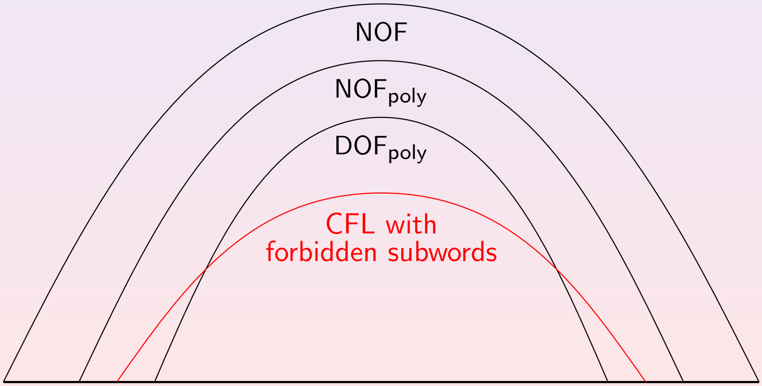
# Context-Free Languages with a Forbidden Subword Can Be Accepted in an Overhead-Free Way

**Theorem**

Let $L$ be a context-free language with a forbidden word.
Then $L \in \mathrm{NOF}_{\mathrm{poly}}$.

The proof is based on the fact that every context-free language can be accepted by a nondeterministic almost-overhead-free machine in polynomial time.

# Relationships among Overhead-Free Computation Classes



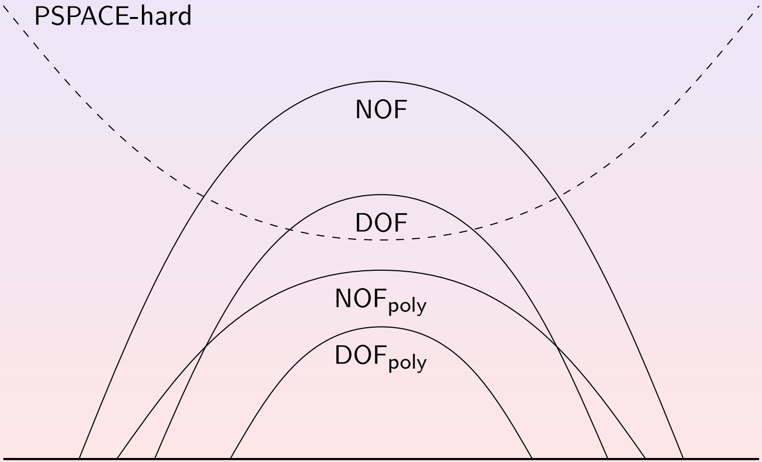
# Some PSPACE-complete Languages Can Be Accepted in an Overhead-Free Way

### Theorem

DOF contains languages that are complete for PSPACE.

The proof is based on the fact that for every $L \in \text{DLINSPACE}$ there exists an isometric homomorphism $h$ such that $h(L) \in \text{DOF}$.

# Relationships among Overhead-Free Computation Classes



# Some Context-Sensitive Languages Cannot be Accepted in an Overhead-Free Way

**Theorem**

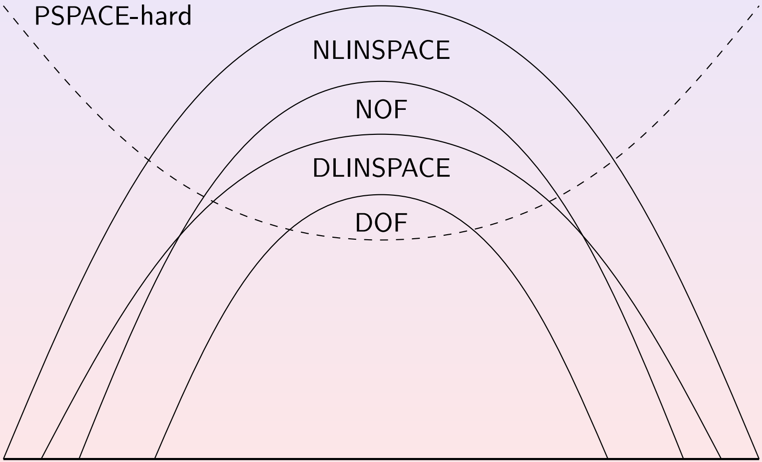
DOF $\subsetneq$ DLINSPACE.

**Theorem**

NOF $\subsetneq$ NLINSPACE.

The proofs are based on old diagonalisations due to Feldman, Owings, and Seiferas.

# Relationships among Overhead-Free Computation Classes



# Candidates for Languages that Cannot be Accepted in an Overhead-Free Way

**Conjecture**

$\textsc{double-palindromes} \notin \mathsf{DOF}$.

**Conjecture**

$\{ww \mid w \in \{0,1\}^*\} \notin \mathsf{NOF}$.

Proving the first conjecture would show $\mathsf{DOF} \subsetneq \mathsf{NOF}$.

# Summary

- Overhead-free computation is a more faithful model of fixed-size memory.
- Overhead-free computation is less powerful than linear space.
- Many context-free languages can be accepted by overhead-free machines.
- We conjecture that all context-free languages are in $\mathrm{NOF}_{\mathrm{poly}}$.
- Our results can be seen as new results on the power of linear bounded automata with fixed alphabet size.

# For Further Reading

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