Computation with Absolutely No Space Overhead

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Developments in Language Theory Conference, 2003



- 1 The Model of Overhead-Free Computation
 - The Standard Model of Linear Space
 - Our Model of Absolutely No Space Overhead
- 2 The Power of Overhead-Free Computation
 - Palindromes
 - Linear Languages
 - Context-Free Languages with a Forbidden Subword
 - Languages Complete for Polynomial Space
- 3 Limitations of Overhead-Free Computation
 - Linear Space is Strictly More Powerful





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- Input fills fixed-size tape
- Input may be modified
- Tape alphabet is larger than input alphabet







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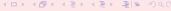






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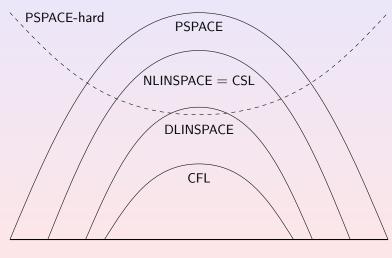


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Linear Space is a Powerful Model



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- Input fills fixed-size tape
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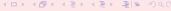
tape



Turing machine

- Input fills fixed-size tape
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tape 1 0 1 0 0 1 0 1

Turing machine

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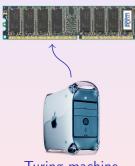




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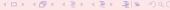


Turing machine

Intuition

 Tape is used like a RAM module.





Definition of Overhead-Free Computations

Definition

A Turing machine is overhead-free if

- it has only a single tape,
- writes only on input cells,
- writes only symbols drawn from the input alphabet.





Definition

A language $L \subseteq \Sigma^*$ is in

DOF if L is accepted by a deterministic overhead-free machine with input alphabet Σ ,

DOF_{poly} if L is accepted by a deterministic overhead-free machine with input alphabet Σ in polynomial time.

NOF is the nondeterministic version of DOF,

NOF_{polv} is the nondeterministic version of DOF_{polv}





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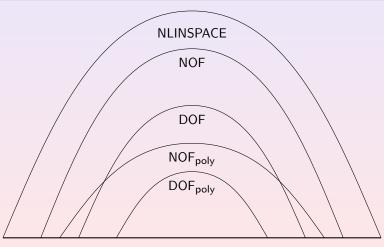
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Simple Relationships among Overhead-Free Computation Classes





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Algorithm

Phase 1:

Compare first and last bit Place left end marker Place right end marker

Phase 2:



Algorithm

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Phase 2:



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Compare first and last bit Place left end marker Place right end marker

Phase 2:



Algorithm

Phase 1:

Compare first and last bit Place left end marker Place right end marker

Phase 2:

Compare bits next to end markers

Find left end marker Advance left end marker Find right end marker Advance right end marker



Algorithm

Phase 1:

Compare first and last bit Place left end marker Place right end marker

Phase 2:



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Phase 2:

Compare bits next to end markers

Find left end marker

Advance left end marker

Find right end marker

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Algorithm

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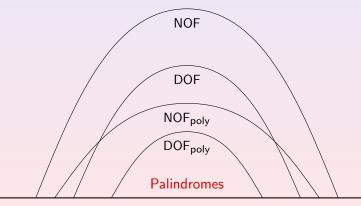
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A Review of Linear Grammars

Definition

A grammar is linear if it is context-free and there is only one nonterminal per right-hand side.

Example

 $\textit{G}_1 \colon \textit{S} \rightarrow 00S0 \mid 1 \text{ and } \textit{G}_2 \colon \textit{S} \rightarrow 0S10 \mid 0.$

Definition

A grammar is deterministic if "there is always only one rule that can be applied."

Example

 $G_1: S \rightarrow 00S0 \mid 1$ is deterministic.

 $G_2: S \rightarrow 0S10 \mid 0$ is **not** deterministic.



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Deterministic Linear Languages Can Be Accepted in an Overhead-Free Way

Theorem

Every deterministic linear language is in DOF_{poly}.





Metalinear Languages Can Be Accepted in an Overhead-Free Way

Definition

A language is metalinear if it is the concatenation of linear languages.

Example

TRIPLE-PALINDROME = $\{uvw \mid u, v, \text{ and } w \text{ are palindromes}\}$

$\mathsf{Theorem}$

Every metalinear language is in NOF_{poly}.





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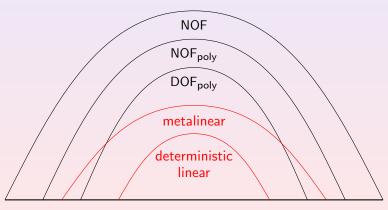
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Definition of Almost-Overhead-Free Computations

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- A Turing machine is almost-overhead-free if
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Palindromes Linear Languages Forbidden Subword Complete Languages

Context-Free Languages with a Forbidden Subword Can Be Accepted in an Overhead-Free Way

Theorem

Let L be a context-free language with a forbidden word. Then $L \in NOF_{poly}$.

→ Skip proof





Context-Free Languages with a Forbidden Subword Can Be Accepted in an Overhead-Free Way

Theorem

Let L be a context-free language with a forbidden word. Then $L \in NOF_{poly}$.

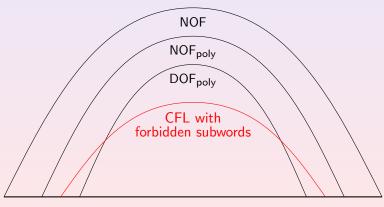
Proof.

Every context-free language can be accepted by a nondeterministic almost-overhead-free machine in polynomial time.





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Some PSPACE-complete Languages Can Be Accepted in an Overhead-Free Way

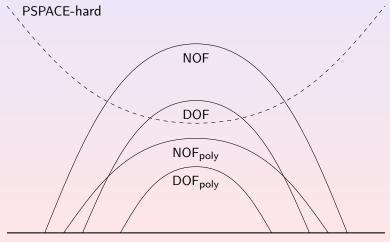
Theorem

DOF contains languages that are complete for PSPACE.

▶ Proof details



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Some Context-Sensitive Languages Cannot be Accepted in an Overhead-Free Way

Theorem

 $DOF \subseteq DLINSPACE$.

Theorem

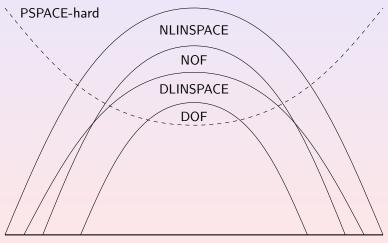
 $NOF \subseteq NLINSPACE$.

The proofs are based on old diagonalisations due to Feldman, Owings, and Seiferas.





Relationships among Overhead-Free Computation Classes





Candidates for Languages that Cannot be Accepted in an Overhead-Free Way

Conjecture

Double-palindromes \notin DOF.

Conjecture

 $\{ww \mid w \in \{0,1\}^*\} \notin NOF.$

Proving the first conjecture would show DOF \subsetneq NOF.



Candidates for Languages that Cannot be Accepted in an Overhead-Free Way

Theorem

DOUBLE-PALINDROMES \in DOF.

Conjecture

 $\{ww \mid w \in \{0,1\}^*\} \notin NOF.$

Proving the first conjecture would show DOF ⊊ NOF



Summary

- Overhead-free computation is a more faithful model of fixed-size memory.
- Overhead-free computation is less powerful than linear space.
- Many context-free languages can be accepted by overhead-free machines.
- We conjecture that all context-free languages are in NOF_{poly}.
- Our results can be seen as new results on the power of linear bounded automata with fixed alphabet size.







Formal Languages.

Academic Press, 1973.







A. Salomaa.

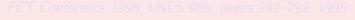
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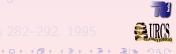
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FCT Conference 1995, LNCS 985, pages 282–292. 1995.



Appendix Outline

- 4 Appendix
 - Overhead Freeness and Completeness
 - Improvements for Context-Free Languages
 - Abbreviations





Overhead-Free Languages can be PSPACE-Complete

Theorem

DOF contains languages that are complete for PSPACE.

Proof.

- Let $A \in \mathsf{DLINSPACE}$ be PSPACE-complete. Such languages are known to exist.
- Let M be a linear space machine that accepts $A \subseteq \{0,1\}^*$ with tape alphabet Γ .
- Let $h: \Gamma \to \{0,1\}^*$ be an isometric, injective homomorphism.
- Then h(L) is in DOF and it is PSPACE-complete.





Improvements

Theorem

- 1. $\mathsf{DCFL} \subseteq \mathsf{DOF}_{\mathsf{poly}}$.
- 2. $CFL \subseteq NOF_{poly}$.





Explanation of Different Abbreviations

DOF	Deterministic Overhead-Free.
NOF	Nondeterministic Overhead-Free.
DOF _{poly}	Deterministic Overhead-Free, polynomial time.
DOF _{poly}	Nondeterministic Overhead-Free, polynomial time.

Table: Explanation of what different abbreviations mean.



