

# Computation with Absolutely No Space Overhead

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# Outline

- 1 The Model of Overhead-Free Computation
  - The Standard Model of Linear Space
  - Our Model of Absolutely No Space Overhead
- 2 The Power of Overhead-Free Computation
  - Palindromes
  - Linear Languages
  - Context-Free Languages with a Forbidden Subword
  - Languages Complete for Polynomial Space
- 3 Limitations of Overhead-Free Computation
  - Linear Space is Strictly More Powerful

▶ Second slide



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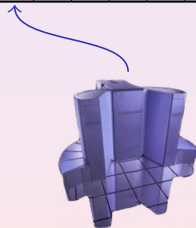
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# The Standard Model of Linear Space

tape

0	0	1	0	0	1	0	0
---	---	---	---	---	---	---	---



Turing machine

## Characteristics

- Input fills **fixed-size** tape
- Input may be **modified**
- Tape alphabet **is larger than** input alphabet

# The Standard Model of Linear Space

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Turing machine

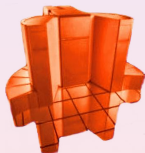
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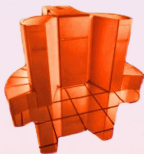
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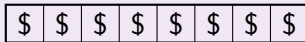
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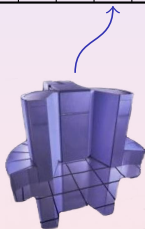
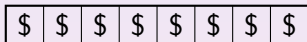
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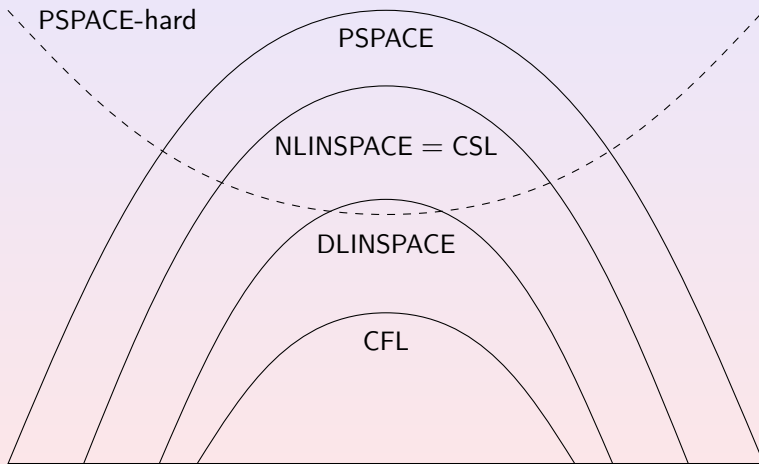


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# Linear Space is a Powerful Model





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# Our Model of “Absolutely No Space Overhead”

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# Our Model of “Absolutely No Space Overhead”



Turing machine

## Intuition

- Tape is used like a RAM module.



# Definition of Overhead-Free Computations

## Definition

A Turing machine is **overhead-free** if

- it has only a single tape,
- writes only on input cells,
- writes only symbols drawn from the input alphabet.

# Overhead-Free Computation Complexity Classes

## Definition

A language  $L \subseteq \Sigma^*$  is in

**DOF** if  $L$  is accepted by a deterministic overhead-free machine with input alphabet  $\Sigma$ ,

$\text{DOF}_{\text{poly}}$  if  $L$  is accepted by a deterministic overhead-free machine with input alphabet  $\Sigma$  in polynomial time.

$\text{NOF}$  is the nondeterministic version of  $\text{DOF}$ ,

$\text{NOF}_{\text{poly}}$  is the nondeterministic version of  $\text{DOF}_{\text{poly}}$ .



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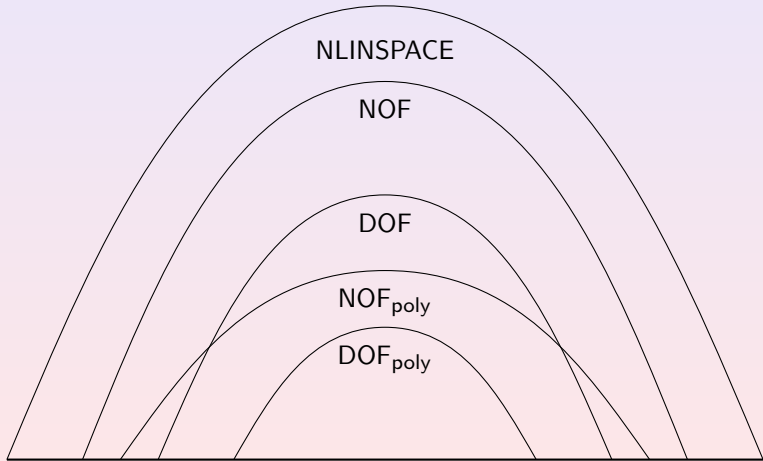
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# Simple Relationships among Overhead-Free Computation Classes



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# Palindromes Can be Accepted in an Overhead-Free Way

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overhead-free machine

## Algorithm

Phase 1:

Compare first and last bit

Place left end marker

Place right end marker

Phase 2:

Compare bits next to end markers

Find left end marker

Advance left end marker

Find right end marker

Advance right end marker





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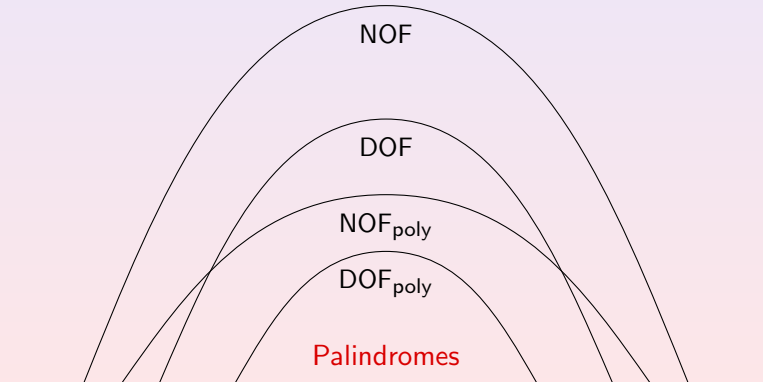
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# Relationships among Overhead-Free Computation Classes





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# A Review of Linear Grammars

## Definition

A grammar is **linear** if it is context-free and there is only one nonterminal per right-hand side.

## Example

$G_1: S \rightarrow 00S0 \mid 1$  and  $G_2: S \rightarrow 0S10 \mid 0$ .

## Definition

A grammar is **deterministic** if  
“there is always only one rule that can be applied.”

## Example

$G_1: S \rightarrow 00S0 \mid 1$  is deterministic.  
 $G_2: S \rightarrow 0S10 \mid 0$  is **not** deterministic.



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# Deterministic Linear Languages Can Be Accepted in an Overhead-Free Way

## Theorem

Every deterministic linear language is in  $\text{DOF}_{\text{poly}}$ .

# Metalinear Languages Can Be Accepted in an Overhead-Free Way

## Definition

A language is **metalinear** if it is the concatenation of linear languages.

## Example

$\text{TRIPLE-PALINDROME} = \{uvw \mid u, v, \text{ and } w \text{ are palindromes}\}.$

## Theorem

Every metalinear language is in  $\text{NOF}_{\text{poly}}$ .



# Metalinear Languages

## Can Be Accepted in an Overhead-Free Way

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A language is **metalinear** if it is the concatenation of linear languages.

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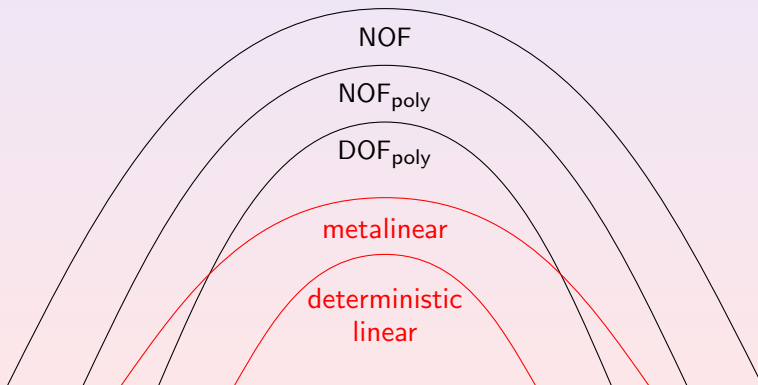
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# Definition of Almost-Overhead-Free Computations

## Definition

A Turing machine is **almost-overhead-free** if

- it has only a single tape,
- writes only on input cells,
- writes only symbols drawn from the input alphabet  
plus one special symbol.

# Definition of Almost-Overhead-Free Computations

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# Context-Free Languages with a Forbidden Subword Can Be Accepted in an Overhead-Free Way

## Theorem

Let  $L$  be a context-free language with a forbidden word.  
Then  $L \in \text{NOF}_{\text{poly}}$ .

► Skip proof



# Context-Free Languages with a Forbidden Subword Can Be Accepted in an Overhead-Free Way

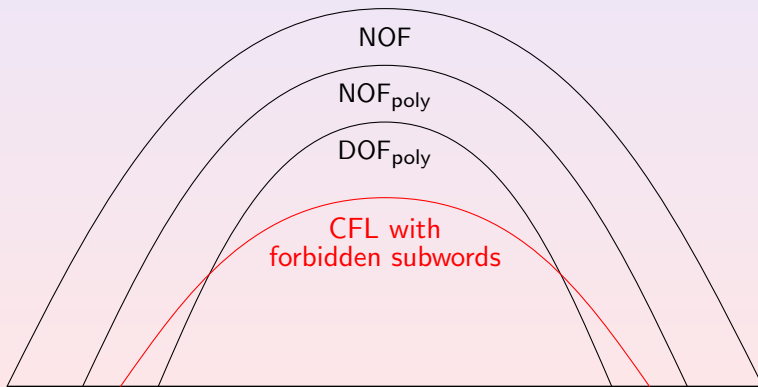
## Theorem

Let  $L$  be a context-free language with a forbidden word.  
Then  $L \in \text{NOF}_{\text{poly}}$ .

## Proof.

Every context-free language can be accepted by a nondeterministic almost-overhead-free machine in polynomial time.

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# Overhead-Free Languages can be PSPACE-Complete

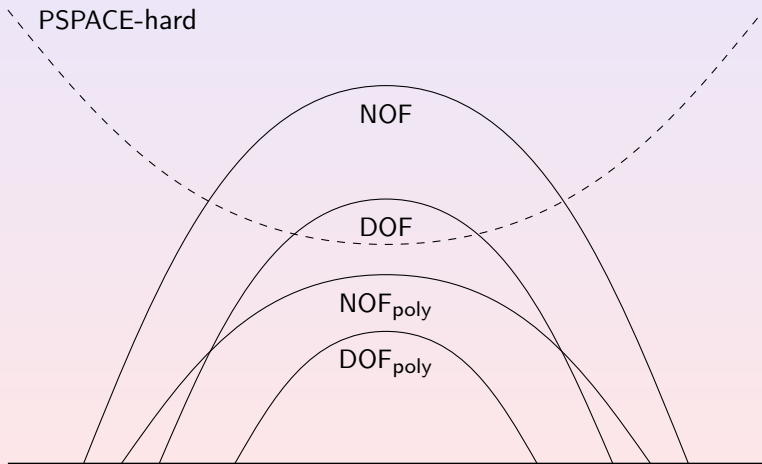
## Theorem

DOF contains languages that are complete for PSPACE.

► Proof details



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# Some Context-Sensitive Languages Cannot be Accepted in an Overhead-Free Way

## Theorem

$\text{DOF} \subsetneq \text{DLINSPACE}$ .

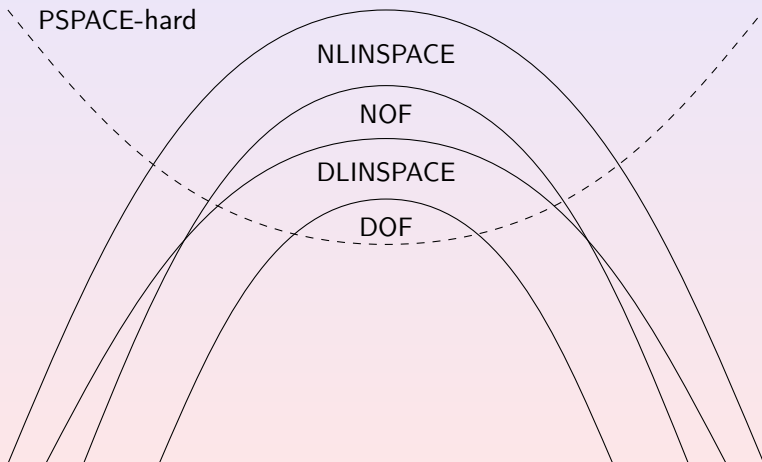
## Theorem

$\text{NOF} \subsetneq \text{NLINSPACE}$ .

The proofs are based on old diagonalisations due to Feldman, Owings, and Seiferas.



# Relationships among Overhead-Free Computation Classes



# Candidates for Languages that Cannot be Accepted in an Overhead-Free Way

## Conjecture

DOUBLE-PALINDROMES  $\notin$  DOF.

## Conjecture

$\{ww \mid w \in \{0,1\}^*\} \notin$  NOF.

Proving the first conjecture would show  $\text{DOF} \subsetneq \text{NOF}$ .



# Candidates for Languages that Cannot be Accepted in an Overhead-Free Way

## Theorem

$\text{DOUBLE-PALINDROMES} \in \text{DOF}.$

## Conjecture

$\{ww \mid w \in \{0,1\}^*\} \notin \text{NOF}.$

Proving the first conjecture would show  $\text{DOF} \subsetneq \text{NOF}.$



# Summary

- Overhead-free computation is a more faithful **model of fixed-size memory**.
- Overhead-free computation is **less powerful** than linear space.
- **Many** context-free languages can be accepted by overhead-free machines.
- We conjecture that **all** context-free languages are in  $\text{NOF}_{\text{poly}}$ .
- Our results can be seen as new results on the power of **linear bounded automata with fixed alphabet** size.





# For Further Reading



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Restarting automata.

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# Appendix Outline

## 4 Appendix

- Complete Languages
- Improvements for Context-Free Languages
- Abbreviations



# Overhead-Free Languages can be PSPACE-Complete

## Theorem

DOF contains languages that are complete for PSPACE.

Proof.

- Let  $A \in \text{DLINSPACE}$  be PSPACE-complete.  
Such languages are known to exist.
- Let  $M$  be a linear space machine that accepts  $A \subseteq \{0, 1\}^*$  with tape alphabet  $\Gamma$ .
- Let  $h: \Gamma \rightarrow \{0, 1\}^*$  be an isometric, injective homomorphism.
- Then  $h(L)$  is in DOF and it is PSPACE-complete.

# Improvements

## Theorem

1.  $\text{DCFL} \subseteq \text{DOF}_{\text{poly}}$ .
2.  $\text{CFL} \subseteq \text{NOF}_{\text{poly}}$ .

# Explanation of Different Abbreviations

DOF	Deterministic Overhead-Free.
NOF	Nondeterministic Overhead-Free.
DOF <sub>poly</sub>	Deterministic Overhead-Free, polynomial time.
DOF <sub>poly</sub>	Nondeterministic Overhead-Free, polynomial time.

Table: Explanation of what different abbreviations mean.

