Computation with Absolutely No Space Overhead

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The Standard Model of Linear Space

tape

0	0	1	0	0	1	0	0

Turing machine

Characteristics

- Input fills fixed-size tape
- Input may be modified
- Tape alphabet is larger than input alphabet

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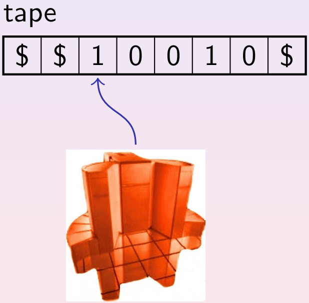

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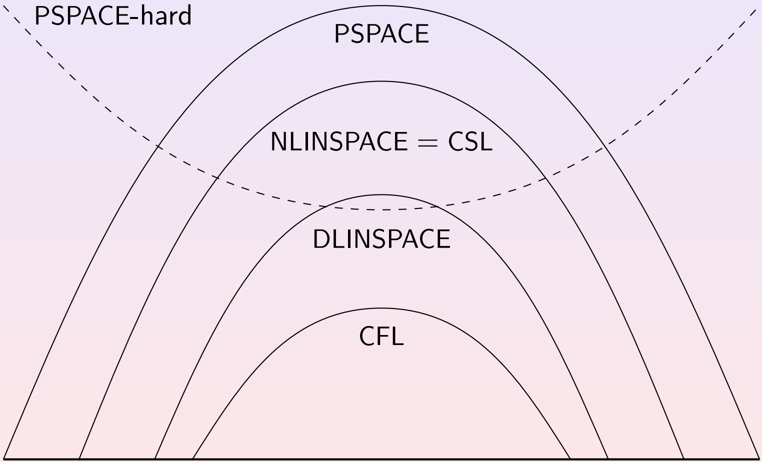
Linear Space is a Powerful Model



Our Model of “Absolutely No Space Overhead”

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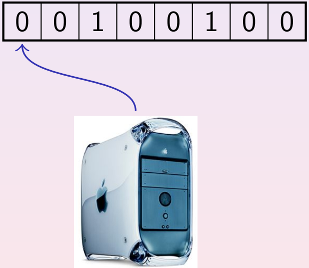

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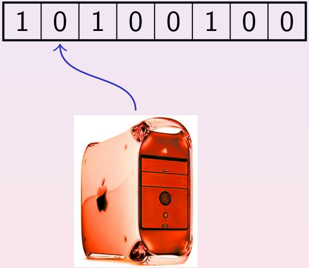

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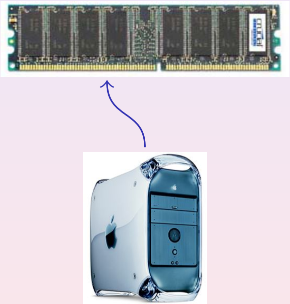

Turing machine

Intuition

- Tape is used like a RAM module.

Definition of Overhead-Free Computations

Definition

A Turing machine is overhead-free if

- it has only a single tape,
- writes only on input cells,
- writes only symbols drawn from the input alphabet.

Overhead-Free Computation Complexity Classes

Definition

A language $L \subseteq \Sigma^*$ is in

- DOF if L is accepted by a deterministic overhead-free machine with input alphabet Σ,

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- $\mathrm{DOF}_{\mathrm{poly}}$ if L is accepted by a deterministic overhead-free machine with input alphabet Σ in polynomial time.

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- NOF is the nondeterministic version of DOF,

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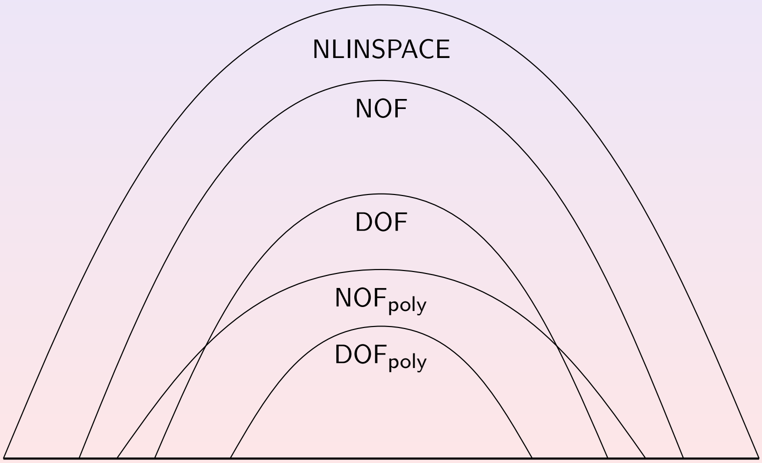
Simple Relationships among Overhead-Free Computation Classes



Palindromes Can be Accepted in an Overhead-Free Way

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overhead-free machine

Algorithm

Phase 1:
Compare first and last bit
- Place left end marker
- Place right end marker

Phase 2:
Compare bits next to end markers
- Find left end marker
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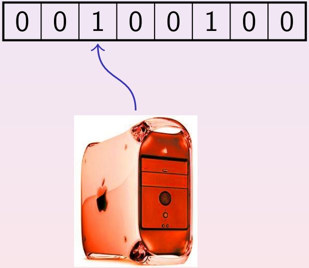

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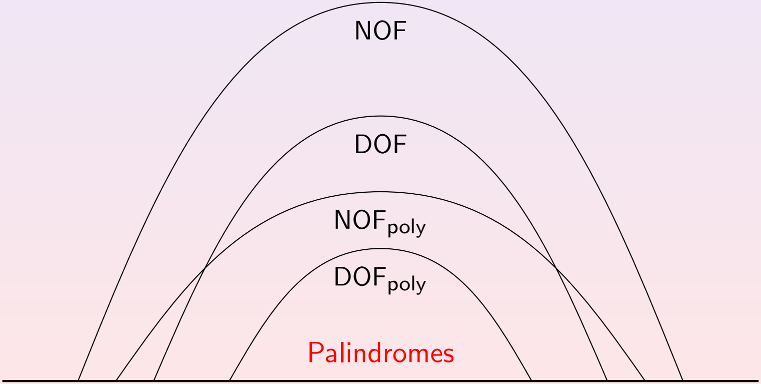
Relationships among Overhead-Free Computation Classes



A Review of Linear Grammars

Definition

A grammar is linear if it is context-free and
there is only one nonterminal per right-hand side.

Example

$G_1 \colon S \longrightarrow 00S0 \mid 1.$
$G_2 \colon S \longrightarrow 0S10 \mid 0.$

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$G_1\colon S \longrightarrow 00S0 \mid 1.$
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Definition

A grammar is deterministic if
"there is always only one rule that can be applied."

Example

G_1 is deterministic.
G_2 is not deterministic.

Deterministic Linear Languages Can Be Accepted in an Overhead-Free Way

Theorem

Every deterministic linear language is in $\mathrm{DOF}_{\mathrm{poly}}$.

Continued Review of Linear Grammars

Definition

A language is metalinear if it is the concatenation of linear languages.

Continued Review of Linear Grammars

Definition

A language is metalinear if it is the concatenation of linear languages.

Example

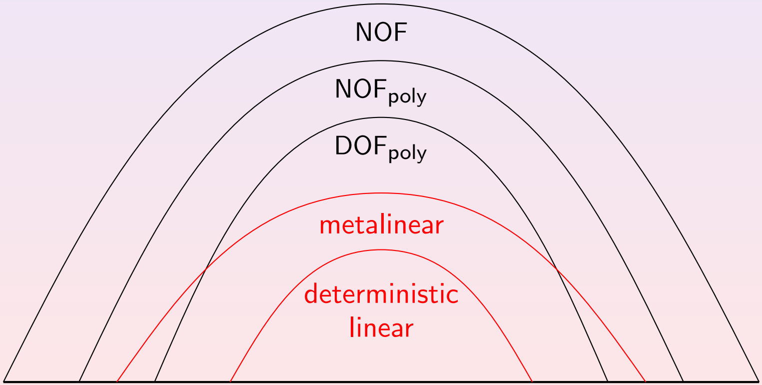
$\textsc{triple-palindrome} = \{uvw \mid u, v, \text{ and } w \text{ are palindromes}\}$.

Metalinear Languages Can Be Accepted in an Overhead-Free Way

Theorem

Every metalinear language is in $\mathrm{NOF}_{\mathrm{poly}}$.

Relationships among Overhead-Free Computation Classes



Definition of Almost-Overhead-Free Computations

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- writes only symbols drawn from the input alphabet plus one special symbol.

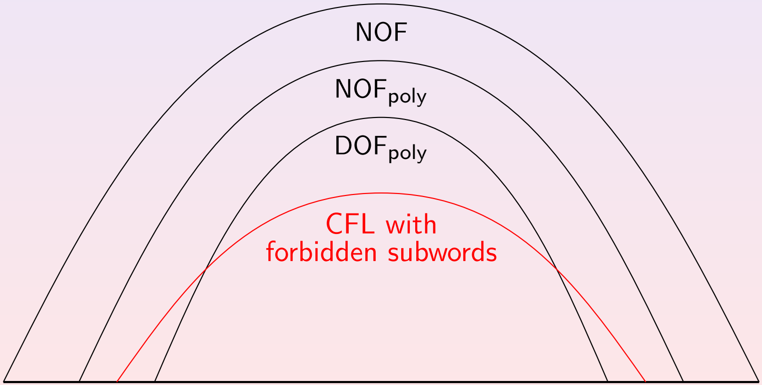
Context-Free Languages with a Forbidden Subword Can Be Accepted in an Overhead-Free Way

Theorem

Let L be a context-free language with a forbidden word.
Then $L \in \mathsf{NOF}_{\mathrm{poly}}$.

The proof is based on the fact that every context-free language can be accepted by a nondeterministic almost-overhead-free machine in polynomial time.

Relationships among Overhead-Free Computation Classes



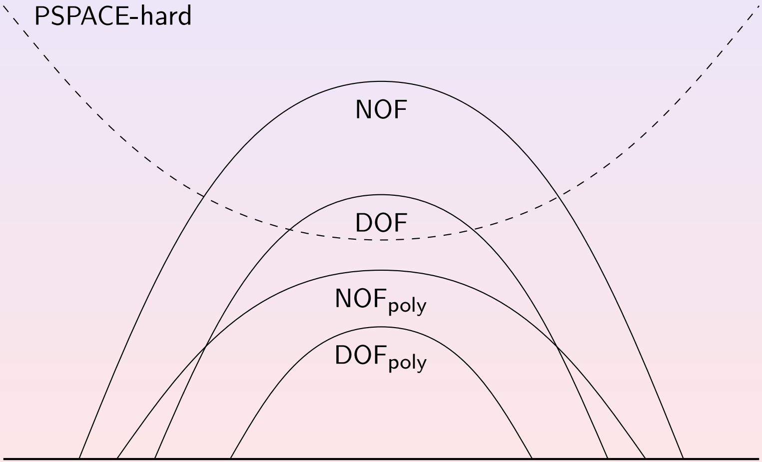
Some PSPACE-complete Languages Can Be Accepted in an Overhead-Free Way

Theorem

DOF contains languages that are complete for PSPACE.

Go to proof details.

Relationships among Overhead-Free Computation Classes



The Model of Overhead-Free Computation
- The Standard Model of Linear Space
- Our Model of Absolutely No Space Overhead

The Power of Overhead-Free Computation
- Palindromes
- Linear Languages
- Context-Free Languages with a Forbidden Subword
- Languages Complete for Polynomial Space

Limitations of Overhead-Free Computation
- Linear Space is Strictly More Powerful

Some Context-Sensitive Languages Cannot be Accepted in an Overhead-Free Way

Theorem

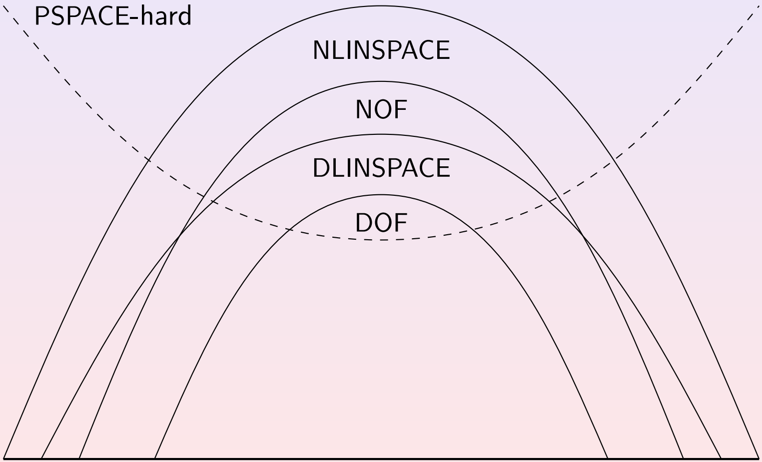
DOF $\subsetneq$ DLINSPACE.

Theorem

NOF $\subsetneq$ NLINSPACE.

The proofs are based on old diagonalisations due to Feldman, Owings, and Seiferas.

Relationships among Overhead-Free Computation Classes



Candidates for Languages that Cannot be Accepted in an Overhead-Free Way

Conjecture

DOUBLE-PALINDROMES $\notin$ DOF.

Conjecture

$\{ww \mid w \in \{0,1\}^*\} \notin$ NOF.

Proving the first conjecture would show DOF $\subsetneq$ NOF.

Summary

- Overhead-free computation is a more faithful model of fixed-size memory.
- Overhead-free computation is less powerful than linear space.
- Many context-free languages can be accepted by overhead-free machines.
- We conjecture that all context-free languages are in $\mathrm{NOF}_{\mathrm{poly}}$.
- Our results can be seen as new results on the power of linear bounded automata with fixed alphabet size.

For Further Reading

A. Salomaa.
Formal Languages.
Academic Press, 1973.

E. Dijkstra.
Smoothsort, an alternative for sorting in situ.
Science of Computer Programming, 1(3):223–233, 1982.

E. Feldman and J. Owings, Jr.
A class of universal linear bounded automata.
Information Sciences, 6:187–190, 1973.

P. Jančar, F. Mráz, M. Plátek, and J. Vogel.
Restarting automata.
FCT Conference 1995, LNCS 985, pages 282–292. 1995.

Appendix

Overhead Freeness and Completeness

Improvements for Context-Free Languages

Overhead-Free Languages can be PSPACE-Complete

Theorem

DOF contains languages that are complete for PSPACE.

Proof.

- Let $A \in \text{DLINSPACE}$ be PSPACE-complete.
  Such languages are known to exist.
- Let M be a linear space machine that accepts $A \subseteq \{0,1\}^*$ with tape alphabet Γ.
- Let $h\colon \Gamma \to \{0,1\}^*$ be an isometric, injective homomorphism.
- Then $h(L)$ is in DOF and it is PSPACE-complete.

return

Improvements

Theorem

1. $\mathsf{DCFL} \subseteq \mathsf{DOF}_{\mathrm{poly}}$.
2. $\mathsf{CFL} \subseteq \mathsf{NOF}_{\mathrm{poly}}$.