# Computation with Absolutely No Space Overhead

Lane Hemaspaandra[1] Proshanto Mukherji[1] Till Tantau[2]

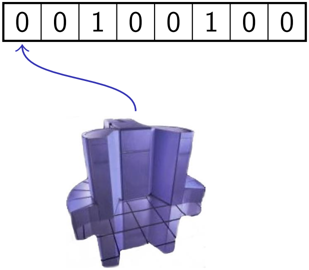
[1]Department of Computer Science
University of Rochester

[2]Fakultät für Elektrotechnik und Informatik
Technical University of Berlin

Developments in Language Theory Conference, 2003

# The Standard Model of Linear Space

tape




Turing machine

## Characteristics

- Input fills fixed-size tape
- Input may be modified
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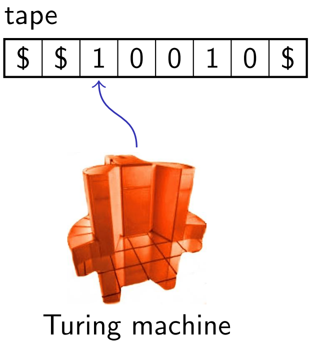


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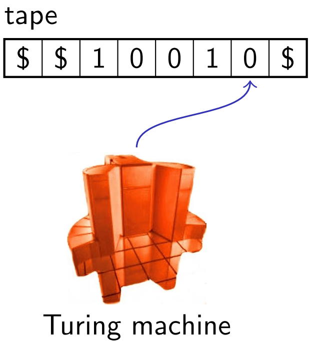


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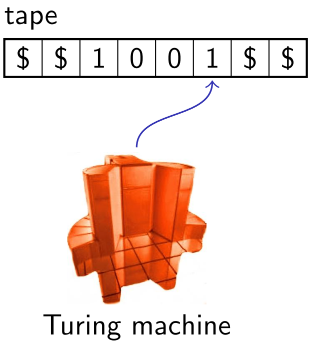


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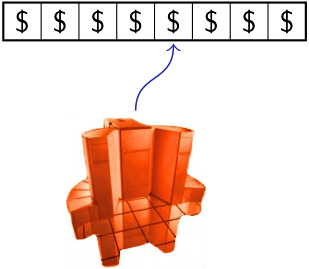

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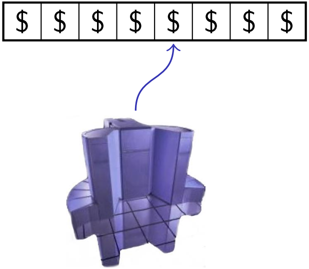


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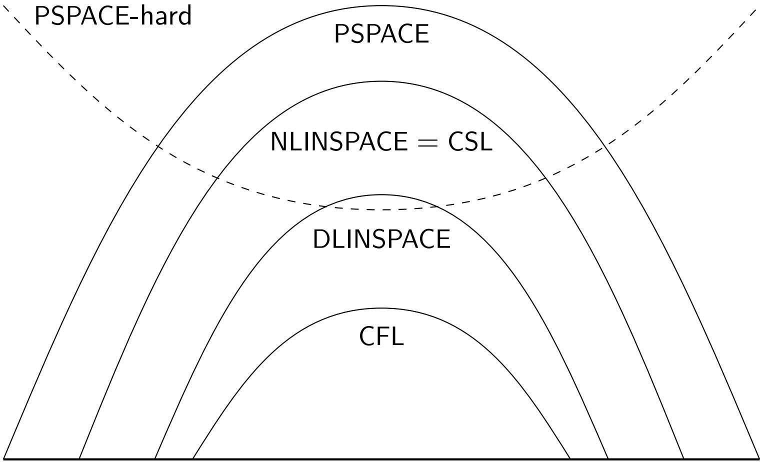
# Linear Space is a Powerful Model



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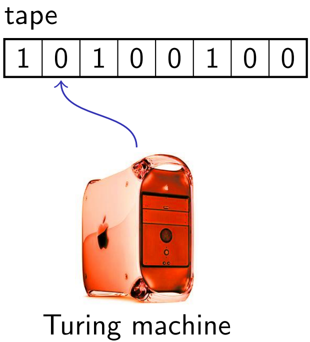


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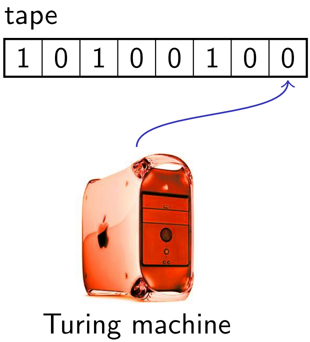


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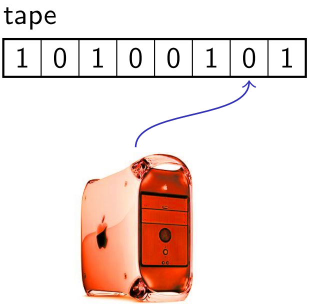

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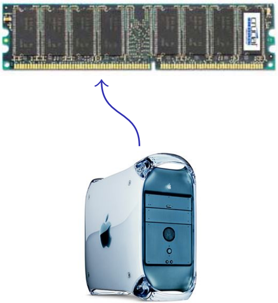

Turing machine

## Intuition

- Tape is used like a RAM module.

# Definition of Overhead-Free Computations

## Definition

A Turing machine is overhead-free if

- it has only a single tape,
- writes only on input cells,
- writes only symbols drawn from the input alphabet.

# Overhead-Free Computation Complexity Classes

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A language $L \subseteq \Sigma^*$ is in

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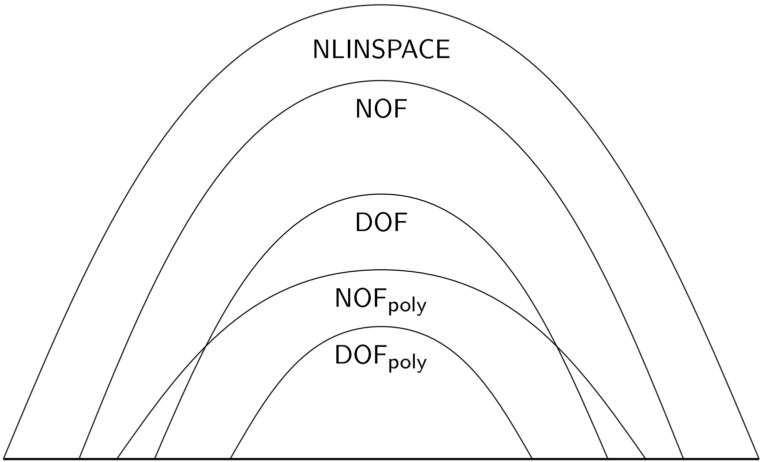
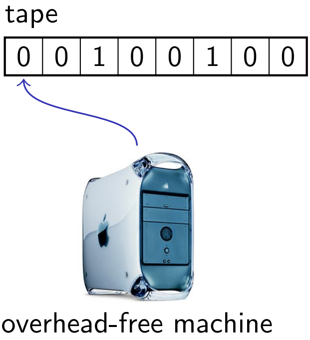
# Simple Relationships among Overhead-Free Computation Classes



# Palindromes Can be Accepted in an Overhead-Free Way

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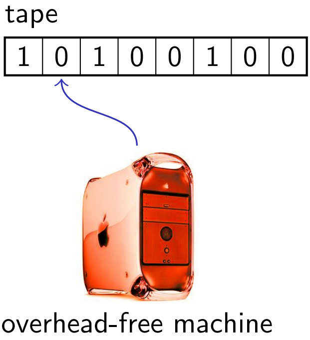


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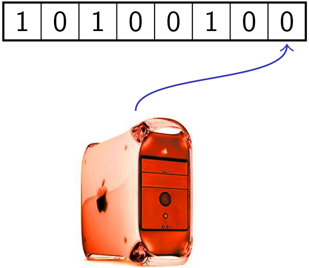


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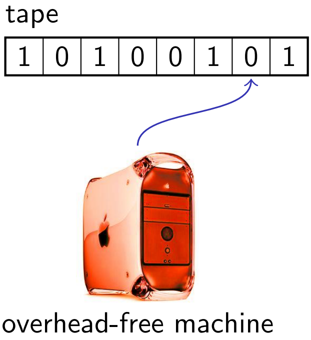


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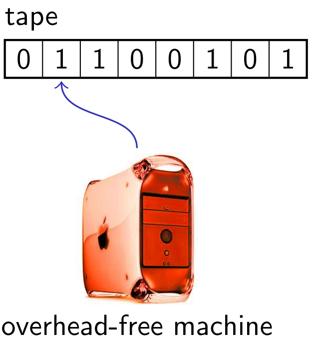


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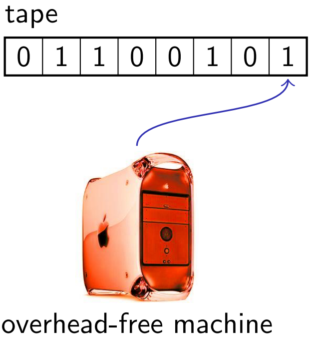


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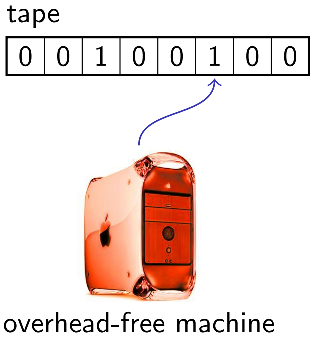


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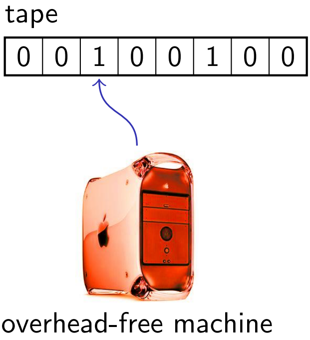


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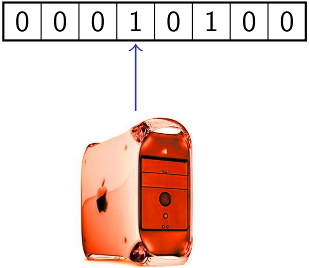


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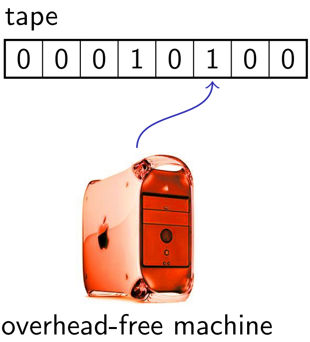


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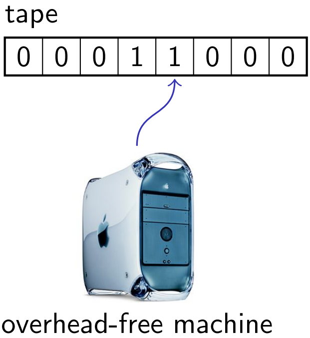


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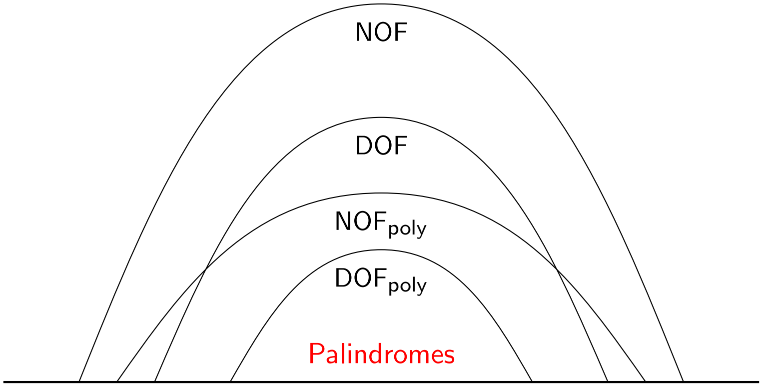
# Relationships among Overhead-Free Computation Classes



# A Review of Linear Grammars

## Definition

A grammar is linear if it is context-free and
there is only one nonterminal per right-hand side.

## Example

$G_1$: $S \longrightarrow 00S0 \mid 1.$
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$G_1$: $S \longrightarrow 00S0 \mid 1.$
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### Definition

A grammar is deterministic if "there is always only one rule that can be applied."

### Example

$G_1$ is deterministic.
$G_2$ is not deterministic.

# Deterministic Linear Languages Can Be Accepted in an Overhead-Free Way

### Theorem

Every deterministic linear language is in $\mathrm{DOF}_{\mathrm{poly}}$.

# Continued Review of Linear Grammars

### Definition

A language is metalinear if it is the concatenation of linear languages.

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### Definition

A language is metalinear if it is the concatenation of linear languages.

### Example

$\text{TRIPLE-PALINDROME} = \{uvw \mid u, v, \text{ and } w \text{ are palindromes}\}$.

# Metalinear Languages Can Be Accepted in an Overhead-Free Way

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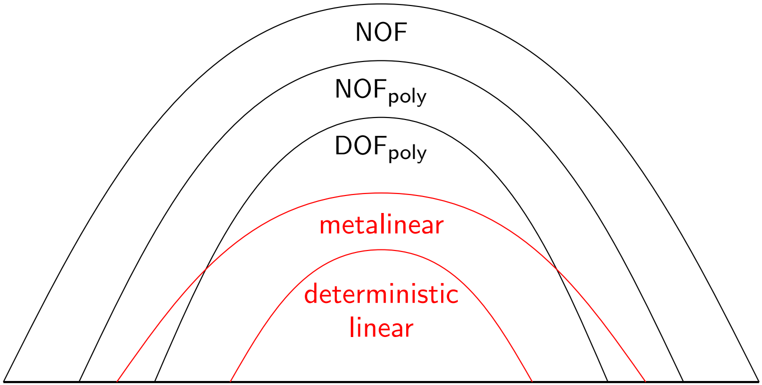
Every metalinear language is in $\mathrm{NOF}_{\mathrm{poly}}$.

# Relationships among Overhead-Free Computation Classes



# Definition of Almost-Overhead-Free Computations

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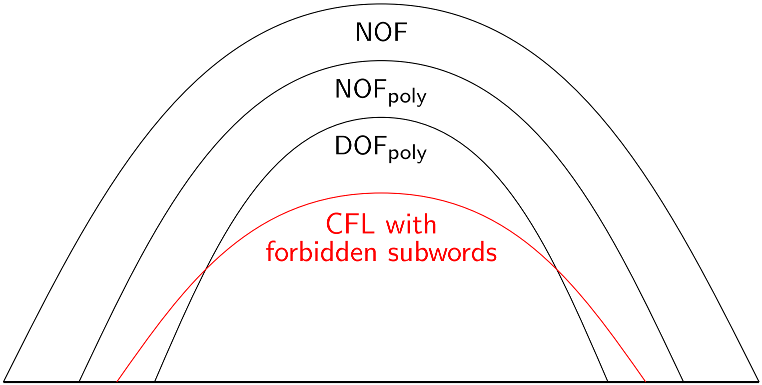
# Context-Free Languages with a Forbidden Subword Can Be Accepted in an Overhead-Free Way

### Theorem

Let $L$ be a context-free language with a forbidden word.
Then $L \in \mathrm{NOF}_{\mathrm{poly}}$.

The proof is based on the fact that every context-free language can be accepted by a nondeterministic almost-overhead-free machine in polynomial time.

# Relationships among Overhead-Free Computation Classes



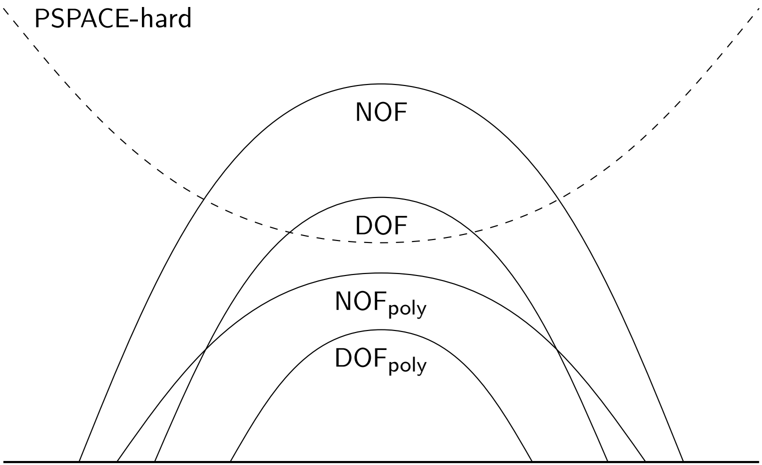
# Some PSPACE-complete Languages Can Be Accepted in an Overhead-Free Way

### Theorem

DOF contains languages that are complete for PSPACE.

The proof is based on the fact that for every $L \in$ DLINSPACE there exists an isometric homomorphism $h$ such that $h(L) \in$ DOF.

# Relationships among Overhead-Free Computation Classes



# Some Context-Sensitive Languages Cannot be Accepted in an Overhead-Free Way

**Theorem**

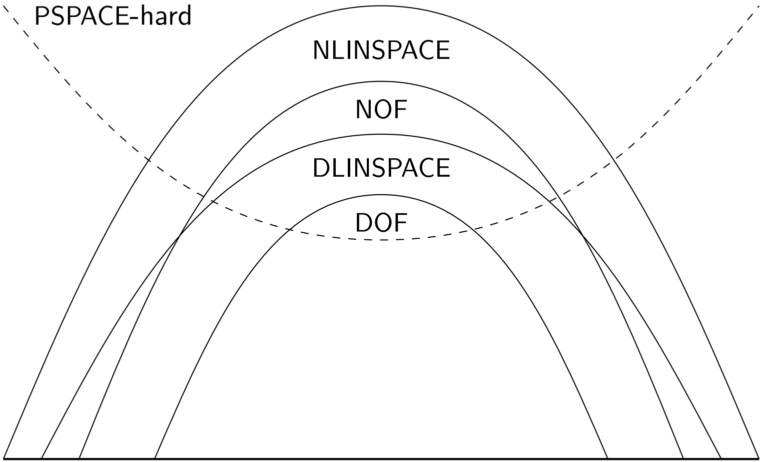
DOF $\subsetneq$ DLINSPACE.

**Theorem**

NOF $\subsetneq$ NLINSPACE.

The proofs are based on old diagonalisations due to Feldman, Owings, and Seiferas.

# Relationships among Overhead-Free Computation Classes



# Candidates for Languages that Cannot be Accepted in an Overhead-Free Way

## Conjecture

DOUBLE-PALINDROMES $\notin$ DOF.

## Conjecture

$\{ww \mid w \in \{0,1\}^*\} \notin$ NOF.

Proving the first conjecture would show DOF $\subsetneq$ NOF.

# Summary

- Overhead-free computation is a more faithful model of fixed-size memory.
- Overhead-free computation is less powerful than linear space.
- Many context-free languages can be accepted by overhead-free machines.
- We conjecture that all context-free languages are in $\mathrm{NOF}_{\mathrm{poly}}$.
- Our results can be seen as new results on the power of linear bounded automata with fixed alphabet size.

# For Further Reading

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