Computation with Absolutely No Space Overhead

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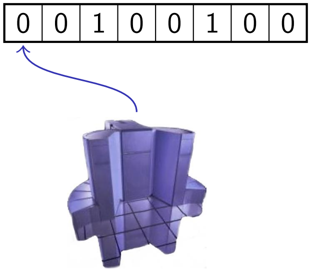
The Standard Model of Linear Space

tape



Turing machine

Characteristics

- Input fills fixed-size tape
- Input may be modified
- Tape alphabet is larger than input alphabet

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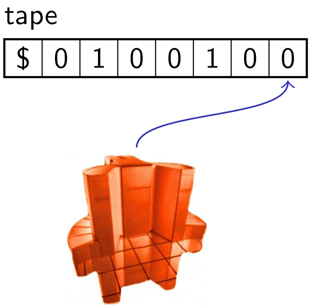

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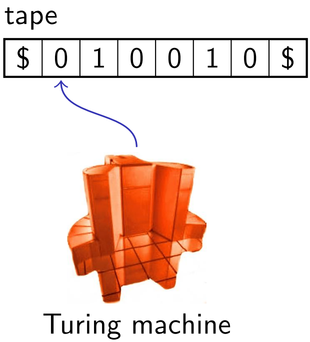

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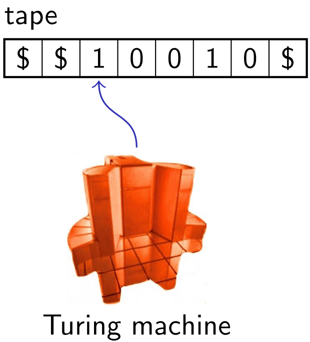

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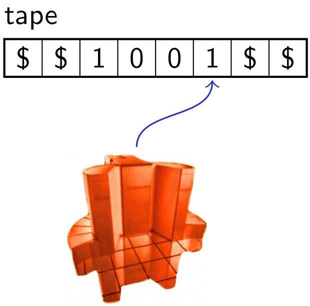

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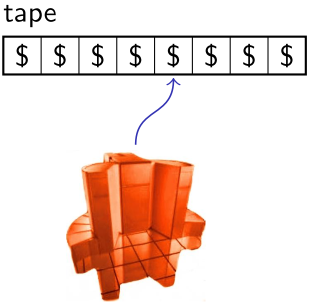

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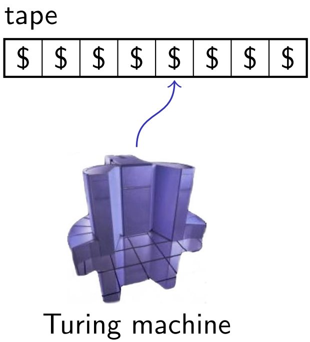

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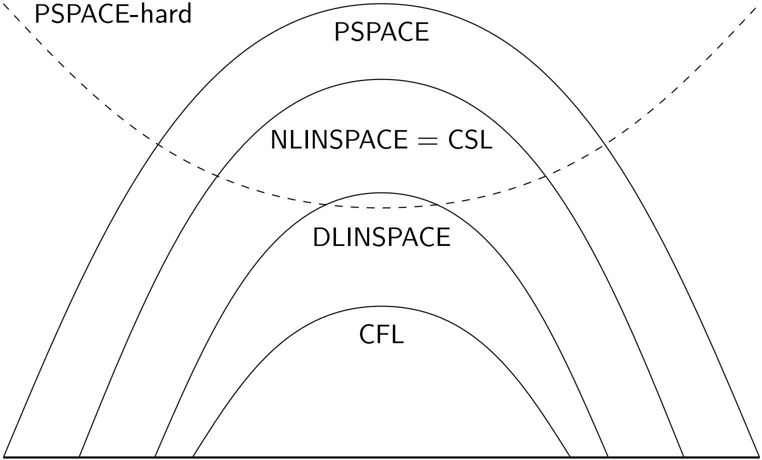
Linear Space is a Powerful Model



Our Model of “Absolutely No Space Overhead”

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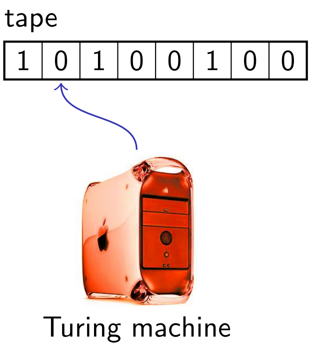

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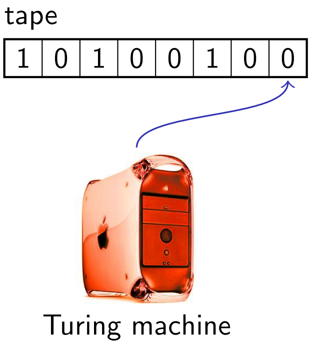

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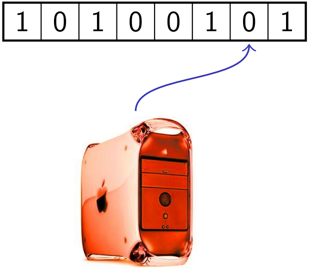

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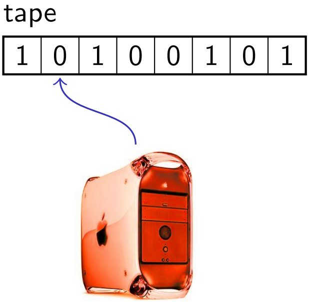

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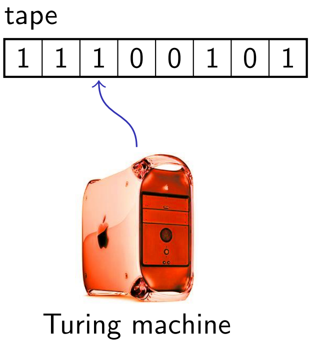

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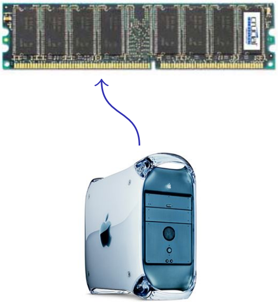

Turing machine

Intuition

- Tape is used like a RAM module.

Definition of Overhead-Free Computations

Definition

A Turing machine is overhead-free if

- it has only a single tape,
- writes only on input cells,
- writes only symbols drawn from the input alphabet.

Overhead-Free Computation Complexity Classes

Definition

A language $L \subseteq \Sigma^*$ is in

DOF if L is accepted by a deterministic overhead-free machine with input alphabet Σ,

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- **DOF** if L is accepted by a deterministic overhead-free machine with input alphabet Σ,
- **$\mathrm{DOF}_{\mathrm{poly}}$** if L is accepted by a deterministic overhead-free machine with input alphabet Σ in polynomial time.

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- **NOF** is the nondeterministic version of DOF,

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- **DOF** if L is accepted by a deterministic overhead-free machine with input alphabet Σ,
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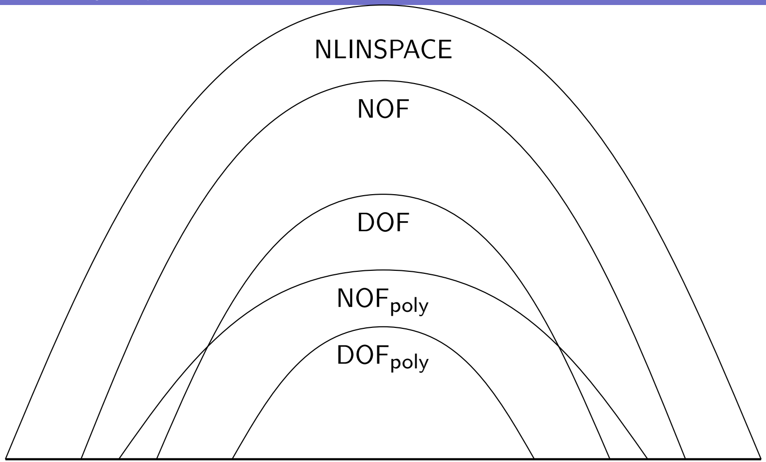
- **NOF** is the nondeterministic version of DOF,
- **$\mathrm{NOF}_{\mathrm{poly}}$** is the nondeterministic version of $\mathrm{DOF}_{\mathrm{poly}}$.

Simple Relationships among Overhead-Free Computation Classes

Our Model of Absolutely No Space Overhead



Palindromes Can be Accepted in an Overhead-Free Way

tape

0	0	1	0	0	1	0	0

overhead-free machine

Algorithm

Phase 1:
Compare first and last bit
- Place left end marker
- Place right end marker

Phase 2:
Compare bits next to end markers
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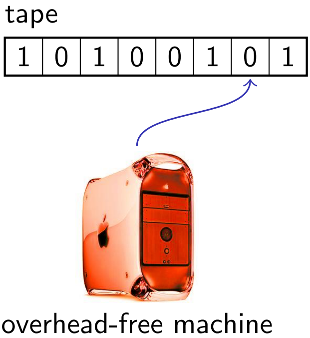

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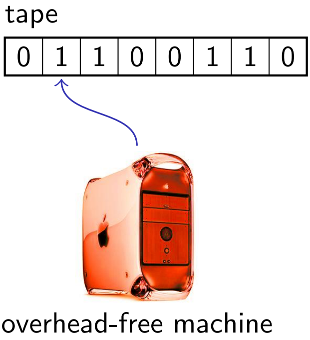

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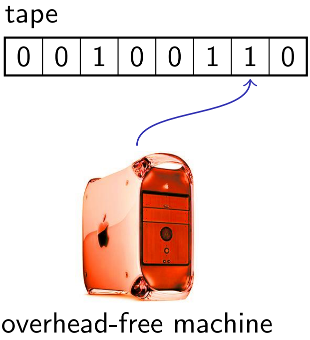

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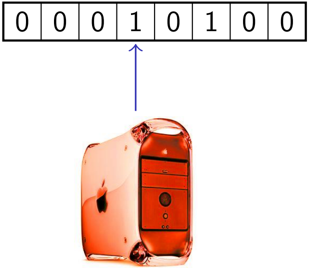

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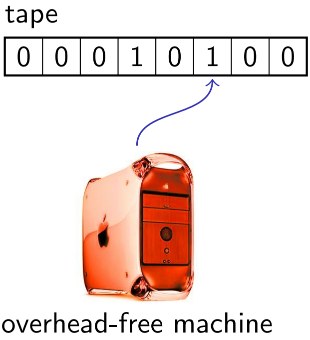

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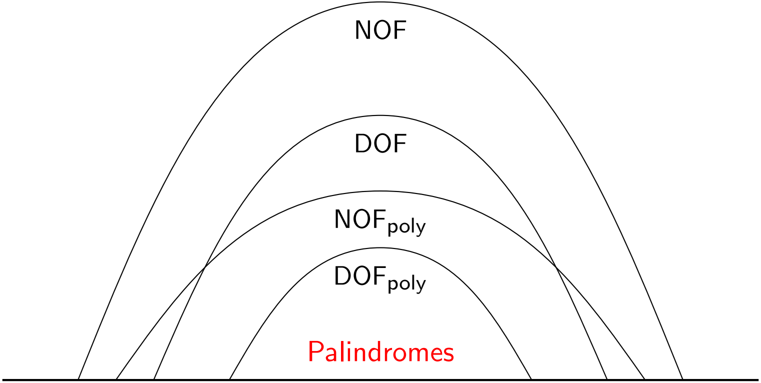
Relationships among Overhead-Free Computation Classes

Palindromes



A Review of Linear Grammars

Definition

A grammar is linear if it is context-free and
there is only one nonterminal per right-hand side.

Example

$G_1 : S \rightarrow 00S0 \mid 1.$
$G_2 : S \rightarrow 0S10 \mid 0.$

G_1 is deterministic.

G_2 is not deterministic.

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$G_1 \colon S \to 00S0 \mid 1.$
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Definition

A grammar is deterministic if
"there is always only one rule that can be applied."

Example

G_1 is deterministic.

G_2 is not deterministic.

Deterministic Linear Languages Can Be Accepted in an Overhead-Free Way

Theorem

Every deterministic linear language is in $\mathsf{DOF}_{\mathsf{poly}}$.

Continued Review of Linear Grammars

Definition

A language is metalinear if it is the concatenation of linear languages.

Continued Review of Linear Grammars

Definition

A language is metalinear if it is the concatenation of linear languages.

Example

$\textsc{triple-palindrome} = \{uvw \mid u, v, \text{ and } w \text{ are palindromes}\}$.

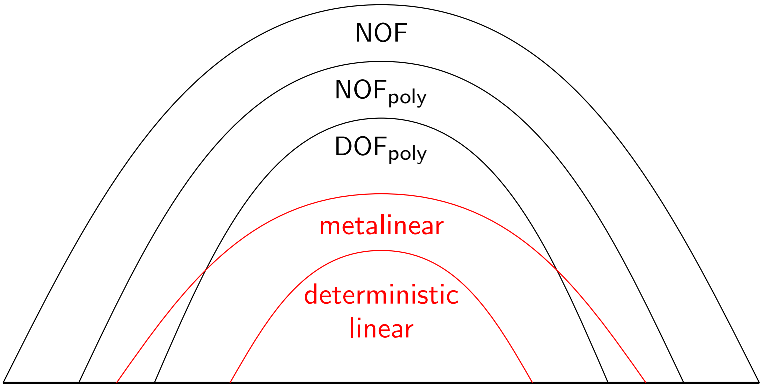
Metalinear Languages Can Be Accepted in an Overhead-Free Way

Theorem

Every metalinear language is in $\mathsf{NOF}_{\mathsf{poly}}$.

Relationships among Overhead-Free Computation Classes

Linear Languages



Definition of Almost-Overhead-Free Computations

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- writes only symbols drawn from the input alphabet plus one special symbol.

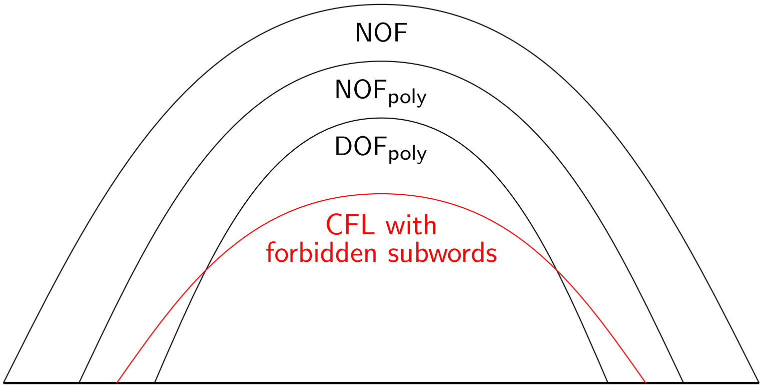
Context-Free Languages with a Forbidden Subword Can Be Accepted in an Overhead-Free Way

Theorem

Let L be a context-free language with a forbidden word.
Then $L \in \mathsf{NOF}_{\text{poly}}$.

The proof is based on the fact that every context-free language can be accepted by a nondeterministic almost-overhead-free machine in polynomial time.

Relationships among Overhead-Free Computation Classes


NOF
$\mathrm{NOF}_{\mathrm{poly}}$
$\mathrm{DOF}_{\mathrm{poly}}$
CFL with
forbidden subwords

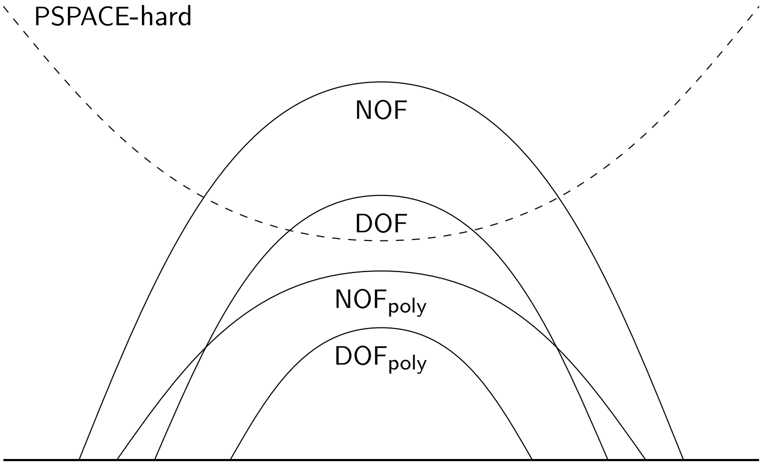
Some PSPACE-complete Languages Can Be Accepted in an Overhead-Free Way

Theorem

DOF contains languages that are complete for PSPACE.

The proof is based on the fact that for every $L \in \mathrm{DLINSPACE}$ there exists an isometric homomorphism h such that $h(L) \in \mathrm{DOF}$.

Relationships among Overhead-Free Computation Classes


PSPACE-hard
NOF
DOF
$\mathrm{NOF}_{\mathrm{poly}}$
$\mathrm{DOF}_{\mathrm{poly}}$

Some Context-Sensitive Languages Cannot be Accepted in an Overhead-Free Way

Theorem

$\mathrm{DOF} \subsetneq \mathrm{DLINSPACE}$.

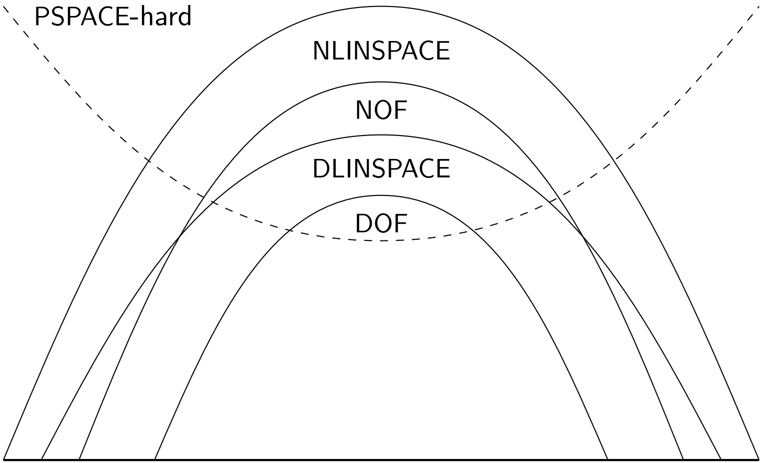
Theorem

$\mathrm{NOF} \subsetneq \mathrm{NLINSPACE}$.

The proofs are based on old diagonalisations due to Feldman, Owings, and Seiferas.

Relationships among Overhead-Free Computation Classes

Linear Space is Strictly More Powerful



Candidates for Languages that Cannot be Accepted in an Overhead-Free Way

Conjecture
$\textsc{double-palindromes} \notin \mathrm{DOF}$.

Conjecture
$\{ww \mid w \in \{0,1\}^*\} \notin \mathrm{NOF}$.

Proving the first conjecture would show $\mathrm{DOF} \subsetneq \mathrm{NOF}$.

Summary

- Overhead-free computation is a more faithful model of fixed-size memory.
- Overhead-free computation is less powerful than linear space.
- Many context-free languages can be accepted by overhead-free machines.
- We conjecture that all context-free languages are in $\mathrm{NOF}_{\mathrm{poly}}$.
- Our results can be seen as new results on the power of linear bounded automata with fixed alphabet size.

For Further Reading

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