

# Computation with Absolutely No Space Overhead

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## Abstract

We study Turing machines that are allowed absolutely no space overhead. The only work space the machines have, beyond the fixed amount of memory implicit in their finite-state control, is that which they can create by cannibalizing the input bits' own space. This model more closely reflects the fixed-sized memory of real computers than does the standard complexity-theoretic model of linear space. Though some context-sensitive languages cannot be accepted by such machines, we show that all deterministic context-free languages can be accepted deterministically in polynomial time with absolutely no space overhead. Similarly, all context-free languages can be accepted nondeterministically without space overhead.

*Keywords.* Space overhead, space reuse, overhead-free computation, context-sensitive languages, context-free languages, linear space, in-place algorithms.

## 1 Introduction

Complexity theory studies which problems can be solved *realistically* using a computer. Since it depends on context what resources are deemed “realistic,” different resource bounds on various models have been studied. For example, deterministic linear space is a possible formalization of the limited memory of computers. Unfortunately, the standard complexity-theoretic formalizations may be too “rough” in realistic contexts, as most have hidden constants tucked away inside their definitions. Polynomial-time algorithms with a time bound of  $n^{100}$  and linear-space algorithms that need one megabyte of extra memory per input bit will typically be unhelpful from a practical point of view.

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Outline

Part I: Review of Last Lecture  
Part II: Today's Lecture

## Part I: Review of Last Lecture

Results of Last Lecture

- First Result
- Second Result

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Figure 1: The outline.

Models  
Power of the Model  
Limitations of the Model  
Summary

Palindromes  
Linear Languages  
Forbidden Subword  
Complete Languages

## Some PSPACE-complete Languages Can Be Accepted in an Overhead-Free Way

Theorem

DOF contains languages that are complete for PSPACE.

▶ Proof details

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Figure 2: A jump back.