

Computation with Absolutely No Space Overhead

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The Model of Overhead-Free Computation

The Standard Model of Linear Space

Our Model of Absolutely No Space Overhead

The Power of Overhead-Free Computation

Palindromes

Linear Languages

Context-Free Languages with a Forbidden Subword

Languages Complete for Polynomial Space

Limitations of Overhead-Free Computation

Linear Space is Strictly More Powerful

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tape

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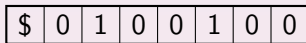
Turing machine

Characteristics

- Input fills **fixed-size tape**
- Input may be **modified**
- Tape alphabet **is larger than** input alphabet

The Standard Model of Linear Space

tape



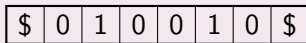
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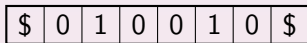
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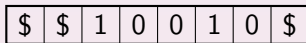
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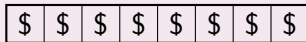
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Our Model of “Absolutely No Space Overhead”



Turing machine

Intuition

- Tape is used like a RAM module.

Definition of Overhead-Free Computations

Definition

A Turing machine is **overhead-free** if

- it has only a single tape,
- writes only on input cells,
- writes only symbols drawn from the input alphabet.

Overhead-Free Computation Complexity Classes

Definition

A language $L \subseteq \Sigma^*$ is in

DOF if L is accepted by a deterministic overhead-free machine with input alphabet Σ ,

DOF_{poly} if L is accepted by a deterministic overhead-free machine with input alphabet Σ in polynomial time.

NOF is the nondeterministic version of DOF,

NOF_{poly} is the nondeterministic version of DOF_{poly}.

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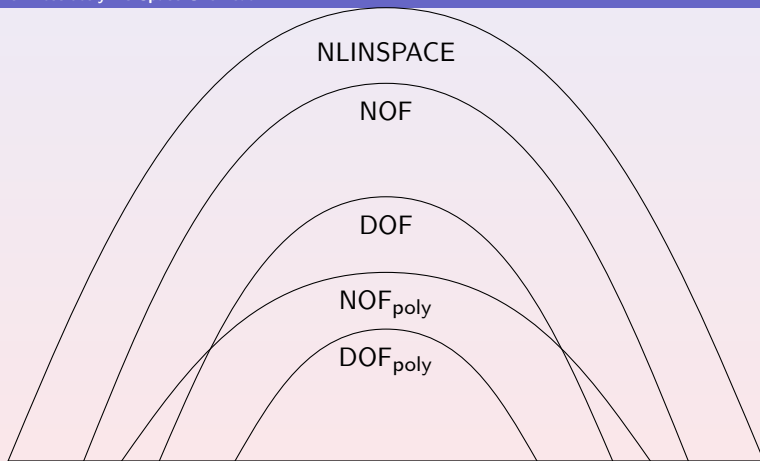
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Simple Relationships among Overhead-Free Computation Classes



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Algorithm

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overhead-free machine

Phase 1:

Compare first and last bit

Place left end marker

Place right end marker

Phase 2:

Compare bits next to end markers

Find left end marker

Advance left end marker

Find right end marker

Advance right end marker

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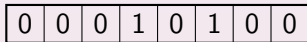
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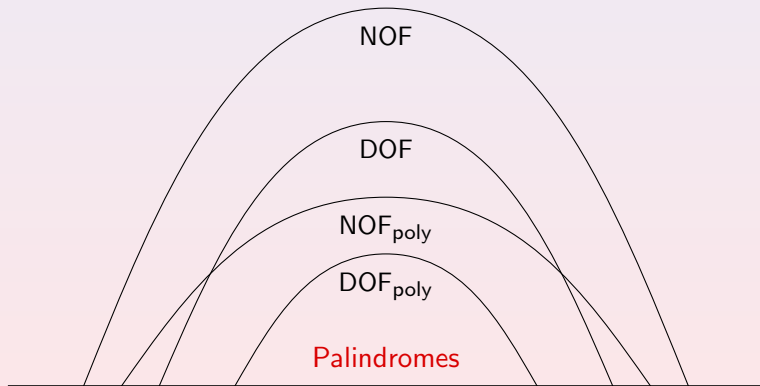
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Relationships among Overhead-Free Computation Classes



A Review of Linear Grammars

Definition

A grammar is **linear** if it is context-free and there is only one nonterminal per right-hand side.

Example

$$G_1: S \rightarrow 00S0 \mid 1.$$

$$G_2: S \rightarrow 0S10 \mid 0.$$

Definition

A grammar is **deterministic** if
“there is always only one rule that can be applied.”

Example

Linear Languages

G_1 is deterministic.

G_2 is not deterministic.

A Review of Linear Grammars

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Definition

A grammar is **deterministic** if
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Example

Linear Languages

G_1 is deterministic.

G_2 is not deterministic.

Deterministic Linear Languages Can Be Accepted in an Overhead-Free Way

Theorem

Every deterministic linear language is in DOF_{poly} .

Metalinear Languages

Can Be Accepted in an Overhead-Free Way

Definition

A language is **metalinear** if it is the concatenation of linear languages.

Example

$\text{TRIPLE-PALINDROME} = \{uvw \mid u, v, \text{ and } w \text{ are palindromes}\}.$

Theorem

Every metalinear language is in NOF_{poly} .

Metalinear Languages

Can Be Accepted in an Overhead-Free Way

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Example

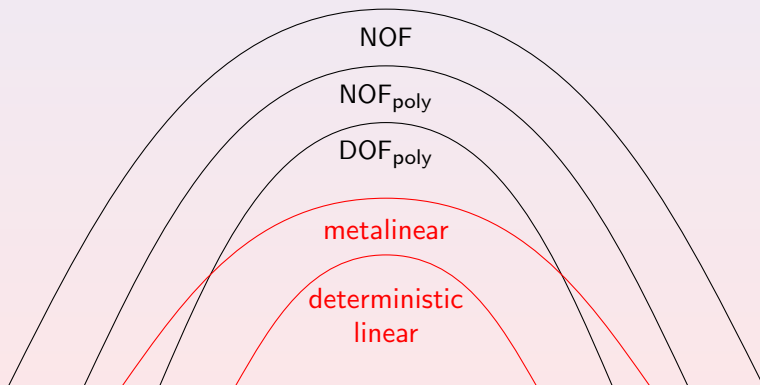
$\text{TRIPLE-PALINDROME} = \{uvw \mid u, v, \text{ and } w \text{ are palindromes}\}.$

Theorem

Every metalinear language is in NOF_{poly} .

Relationships among Overhead-Free Computation Classes

Linear Languages



Definition of Almost-Overhead-Free Computations

Definition

A Turing machine is **almost-overhead-free** if

- it has only a single tape,
- writes only on input cells,
- writes only symbols drawn from the input alphabet
plus one special symbol.

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Context-Free Languages with a Forbidden Subword Can Be Accepted in an Overhead-Free Way

Theorem

Let L be a context-free language with a forbidden word.

Then $L \in \text{NOF}_{\text{poly}}$.

[» Skip proof](#)

Context-Free Languages with a Forbidden Subword Can Be Accepted in an Overhead-Free Way

Theorem

Let L be a context-free language with a forbidden word.

Then $L \in \text{NOF}_{\text{poly}}$.

Proof.

Every context-free language can be accepted by a nondeterministic almost-overhead-free machine in polynomial time.

Relationships among Overhead-Free Computation Classes

Some PSPACE-complete Languages Can Be Accepted in an Overhead-Free Way

Theorem

DOF contains languages that are complete for PSPACE.

► Proof details

Relationships among Overhead-Free Computation Classes

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Some Context-Sensitive Languages Cannot be Accepted in an Overhead-Free Way

Theorem

$\text{DOF} \subsetneq \text{DLINSPACE}.$

Theorem

$\text{NOF} \subsetneq \text{NLINSPACE}.$

The proofs are based on old diagonalisations due to Feldman, Owings, and Seiferas.

Relationships among Overhead-Free Computation Classes

Candidates for Languages that Cannot be Accepted in an Overhead-Free Way

Conjecture

DOUBLE-PALINDROMES \notin DOF.

Conjecture

$\{ww \mid w \in \{0, 1\}^*\} \notin$ NOF.

Proving the first conjecture would show $\text{DOF} \subsetneq \text{NOF}$.

Summary

- Overhead-free computation is a more faithful **model of fixed-size memory**.
- Overhead-free computation is **less powerful** than linear space.
- **Many** context-free languages can be accepted by overhead-free machines.
- We conjecture that **all** context-free languages are in NOF_{poly} .
- Our results can be seen as new results on the power of **linear bounded automata with fixed alphabet** size.

For Further Reading



A. Salomaa.

Formal Languages.

Academic Press, 1973.



E. Dijkstra.

Smoothsort, an alternative for sorting in situ.

Science of Computer Programming, 1(3):223–233, 1982.



E. Feldman and J. Owings, Jr.

A class of universal linear bounded automata.

Information Sciences, 6:187–190, 1973.

Further Reading



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Restarting automata.

FCT Conference 1995, LNCS 985, pages 282–292. 1995.

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Appendix

Overhead Freeness and Completeness
Improvements for Context-Free Languages
Abbreviations

Overhead-Free Languages can be PSPACE-Complete

Theorem

DOF contains languages that are complete for PSPACE.

Proof.

- Let $A \in \text{DLINSPACE}$ be PSPACE-complete.
Such languages are known to exist.
- Let M be a linear space machine that accepts $A \subseteq \{0, 1\}^*$ with tape alphabet Γ .
- Let $h: \Gamma \rightarrow \{0, 1\}^*$ be an isometric, injective homomorphism.
- Then $h(L)$ is in DOF and it is PSPACE-complete.

Improvements

Theorem

1. $\text{DCFL} \subseteq \text{DOF}_{\text{poly}}$.
2. $\text{CFL} \subseteq \text{NOF}_{\text{poly}}$.

Explanation of Different Abbreviations

DOF	Deterministic Overhead-Free.
NOF	Nondeterministic Overhead-Free.
DOF _{poly}	Deterministic Overhead-Free, polynomial time.
DOF _{poly}	Nondeterministic Overhead-Free, polynomial time.

Table: Explanation of what different abbreviations mean.