Computation with Absolutely No Space Overhead

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The Model of Overhead-Free Computation
- The Standard Model of Linear Space
- Our Model of Absolutely No Space Overhead

The Power of Overhead-Free Computation
- Palindromes
- Linear Languages
- Context-Free Languages with a Forbidden Subword
- Languages Complete for Polynomial Space

Limitations of Overhead-Free Computation
- Linear Space is Strictly More Powerful

The Standard Model of Linear Space

tape

0	0	1	0	0	1	0	0

Turing machine

Characteristics

- Input fills fixed-size tape
- Input may be modified
- Tape alphabet is larger than input alphabet

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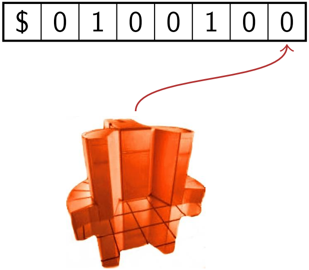

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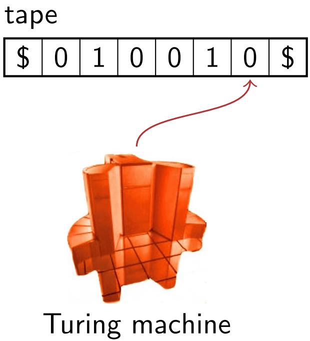

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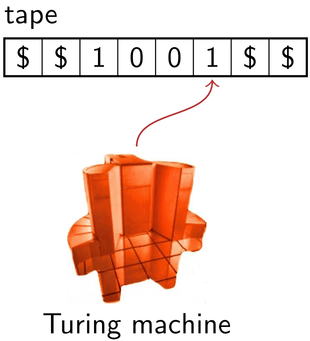

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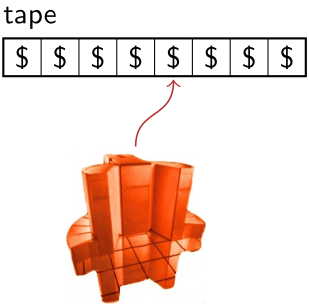

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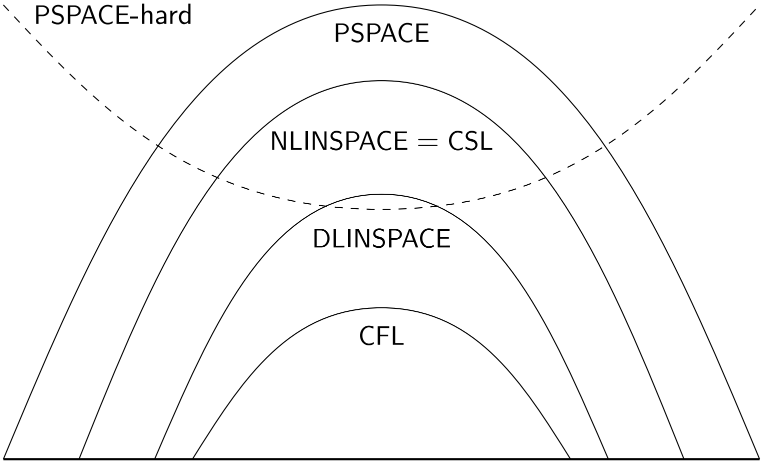
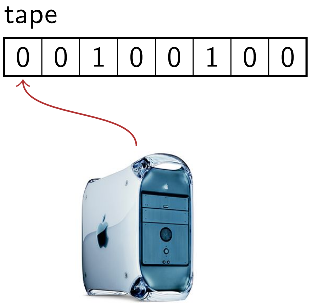
Linear Space is a Powerful Model



Our Model of “Absolutely No Space Overhead”



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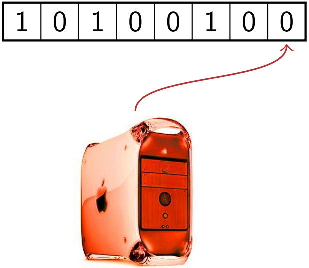

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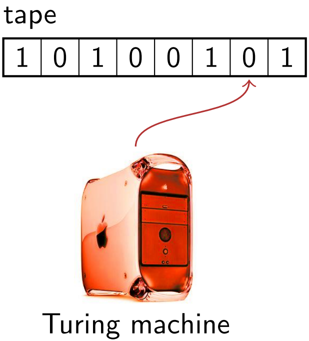

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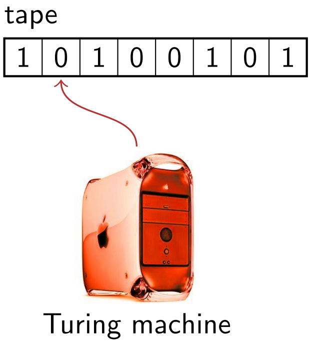

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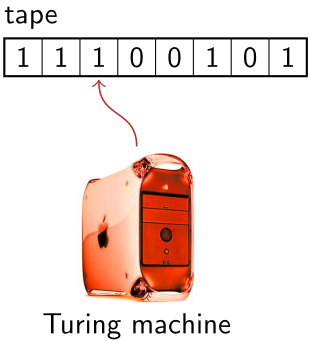

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Our Model of “Absolutely No Space Overhead”

Turing machine

Intuition

- Tape is used like a RAM module.

Definition of Overhead-Free Computations

Definition

A Turing machine is overhead-free if

- it has only a single tape,
- writes only on input cells,
- writes only symbols drawn from the input alphabet.

Overhead-Free Computation Complexity Classes

Definition

A language $L \subseteq \Sigma^*$ is in

DOF if L is accepted by a deterministic overhead-free machine with input alphabet Σ,

Overhead-Free Computation Complexity Classes

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A language $L \subseteq \Sigma^*$ is in

- **DOF** if L is accepted by a deterministic overhead-free machine with input alphabet Σ,
- **$\mathrm{DOF}_{\mathrm{poly}}$** if L is accepted by a deterministic overhead-free machine with input alphabet Σ in polynomial time.

Overhead-Free Computation Complexity Classes

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- DOF if L is accepted by a deterministic overhead-free machine with input alphabet Σ,
- $\mathrm{DOF}_{\mathrm{poly}}$ if L is accepted by a deterministic overhead-free machine with input alphabet Σ in polynomial time.
- NOF is the nondeterministic version of DOF,

Overhead-Free Computation Complexity Classes

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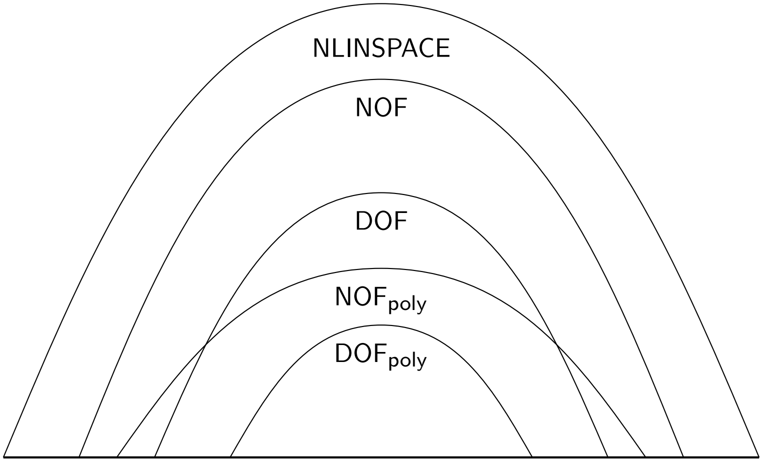
- $\mathrm{NOF}_{\mathrm{poly}}$ is the nondeterministic version of $\mathrm{DOF}_{\mathrm{poly}}$.

Simple Relationships among Overhead-Free Computation Classes



Palindromes Can be Accepted in an Overhead-Free Way

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overhead-free machine

Algorithm

Phase 1:
Compare first and last bit
- Place left end marker
- Place right end marker

Phase 2:
Compare bits next to end markers
- Find left end marker
- Advance left end marker
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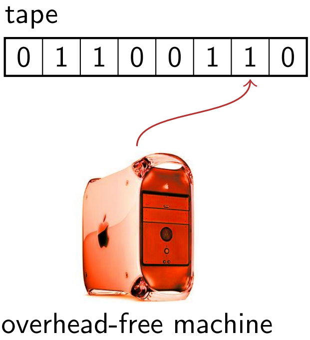

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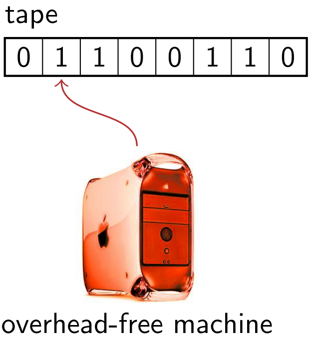

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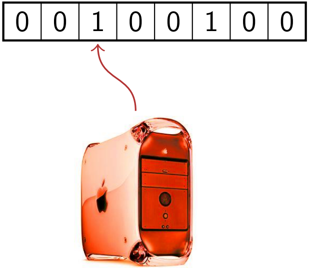

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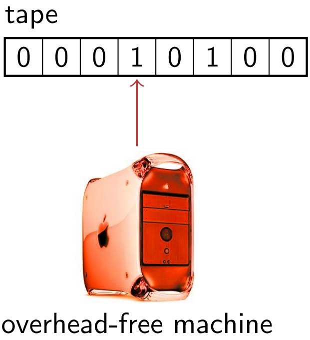

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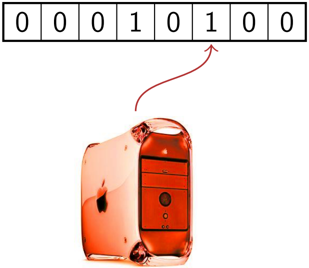

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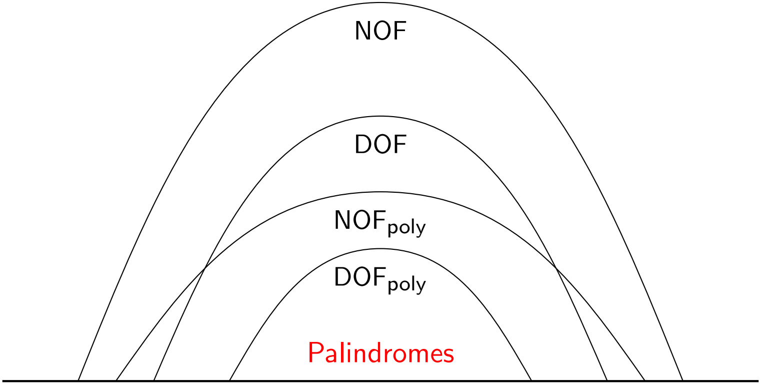
Relationships among Overhead-Free Computation Classes



A Review of Linear Grammars

Definition

A grammar is linear if it is context-free and there is only one nonterminal per right-hand side.

Example

G_1: $S \longrightarrow 00S0 \mid 1$.

G_2: $S \longrightarrow 0S10 \mid 0$.

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$G_1\colon S \longrightarrow 00S0 \mid 1.$
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Definition

A grammar is deterministic if
"there is always only one rule that can be applied."

Example

G_1 is deterministic.
G_2 is not deterministic.

Deterministic Linear Languages Can Be Accepted in an Overhead-Free Way

Theorem

Every deterministic linear language is in $\mathrm{DOF}_{\mathrm{poly}}$.

Continued Review of Linear Grammars

Definition

A language is metalinear if it is the concatenation of linear languages.

Continued Review of Linear Grammars

Definition

A language is metalinear if it is the concatenation of linear languages.

Example

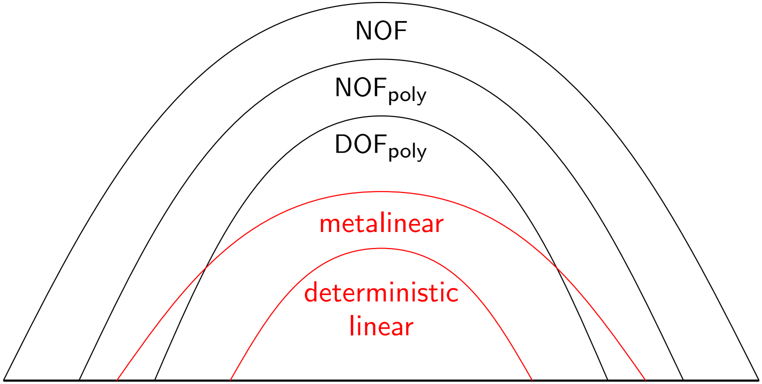
$\textsc{triple-palindrome} = \{uvw \mid u, v, \text{ and } w \text{ are palindromes}\}$.

Metalinear Languages Can Be Accepted in an Overhead-Free Way

Theorem

Every metalinear language is in $\mathrm{NOF}_{\mathrm{poly}}$.

Relationships among Overhead-Free Computation Classes



Definition of Almost-Overhead-Free Computations

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- writes only symbols drawn from the input alphabet plus one special symbol.

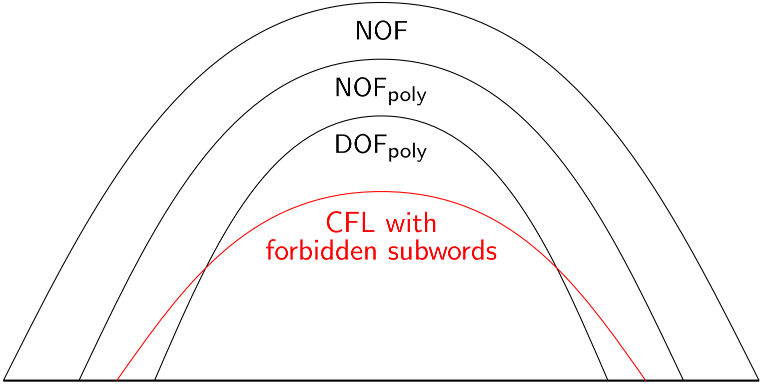
Context-Free Languages with a Forbidden Subword Can Be Accepted in an Overhead-Free Way

Theorem

Let L be a context-free language with a forbidden word.
Then $L \in \mathrm{NOF}_{\mathrm{poly}}$.

The proof is based on the fact that every context-free language can be accepted by a nondeterministic almost-overhead-free machine in polynomial time.

Relationships among Overhead-Free Computation Classes



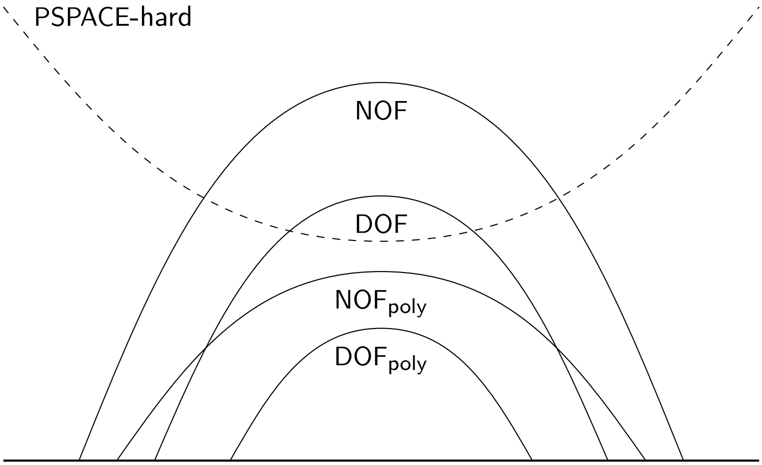
Some PSPACE-complete Languages Can Be Accepted in an Overhead-Free Way

Theorem

DOF contains languages that are complete for PSPACE.

Go to proof details.

Relationships among Overhead-Free Computation Classes



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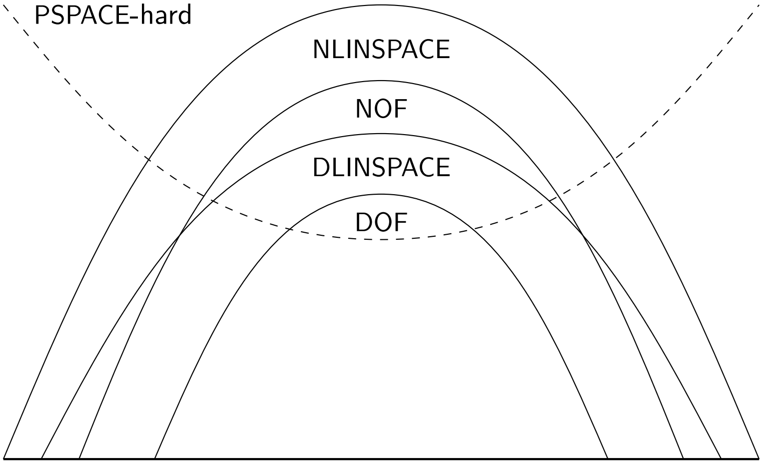
Some Context-Sensitive Languages Cannot be Accepted in an Overhead-Free Way

Theorem
DOF $\subsetneq$ DLINSPACE.

Theorem
NOF $\subsetneq$ NLINSPACE.

The proofs are based on old diagonalisations due to Feldman, Owings, and Seiferas.

Relationships among Overhead-Free Computation Classes



Candidates for Languages that Cannot be Accepted in an Overhead-Free Way

Conjecture

DOUBLE-PALINDROMES $\notin$ DOF.

Conjecture

$\{ww \mid w \in \{0,1\}^*\} \notin$ NOF.

Proving the first conjecture would show DOF $\subsetneq$ NOF.

Summary

- Overhead-free computation is a more faithful model of fixed-size memory.
- Overhead-free computation is less powerful than linear space.
- Many context-free languages can be accepted by overhead-free machines.
- We conjecture that all context-free languages are in $\mathrm{NOF}_{\mathrm{poly}}$.
- Our results can be seen as new results on the power of linear bounded automata with fixed alphabet size.

For Further Reading

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Appendix

Overhead-Free Languages can be PSPACE-Complete

Theorem

DOF contains languages that are complete for PSPACE.

Proof.

- Let $A \in$ DLINSPACE be PSPACE-complete. Such languages are known to exist.
- Let M be a linear space machine that accepts $A \subseteq \{0,1\}^*$ with tape alphabet Γ.
- Let $h\colon \Gamma \to \{0,1\}^*$ be an isometric, injective homomorphism.
- Then $h(L)$ is in DOF and it is PSPACE-complete.

return

Improvements

Theorem

1. $\mathrm{DCFL} \subseteq \mathrm{DOF}_{\mathrm{poly}}$.
2. $\mathrm{CFL} \subseteq \mathrm{NOF}_{\mathrm{poly}}$.