

# Computation with Absolutely No Space Overhead

Lane Hemaspaandra<sup>1</sup>   Proshanto Mukherji<sup>1</sup>   Till Tantau<sup>2</sup>

<sup>1</sup>Department of Computer Science  
University of Rochester

<sup>2</sup>Fakultät für Elektrotechnik und Informatik  
Technical University of Berlin

Developments in Language Theory Conference, 2003

## The Model of Overhead-Free Computation

The Standard Model of Linear Space

Our Model of Absolutely No Space Overhead

## The Power of Overhead-Free Computation

Palindromes

Linear Languages

Context-Free Languages with a Forbidden Subword

Languages Complete for Polynomial Space

## Limitations of Overhead-Free Computation

Linear Space is Strictly More Powerful

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Turing machine

## Characteristics

- ▶ Input fills **fixed-size** tape
- ▶ Input may be **modified**
- ▶ Tape alphabet **is larger than** input alphabet

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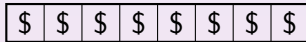
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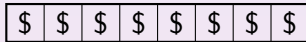
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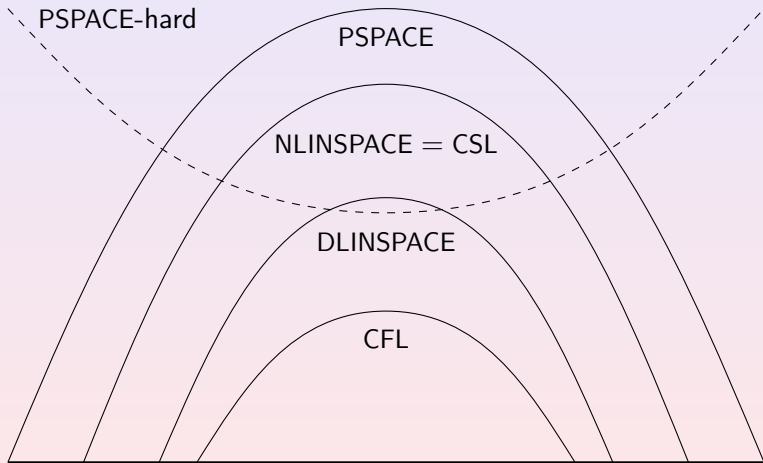


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# Linear Space is a Powerful Model



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# Our Model of “Absolutely No Space Overhead”



Turing machine

## Intuition

- ▶ Tape is used like a RAM module.

# Definition of Overhead-Free Computations

## Definition

A Turing machine is **overhead-free** if

- ▶ it has only a single tape,
- ▶ writes only on input cells,
- ▶ writes only symbols drawn from the input alphabet.

# Overhead-Free Computation Complexity Classes

## Definition

A language  $L \subseteq \Sigma^*$  is in

**DOF** if  $L$  is accepted by a deterministic overhead-free machine with input alphabet  $\Sigma$ ,

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**NOF** is the nondeterministic version of DOF,

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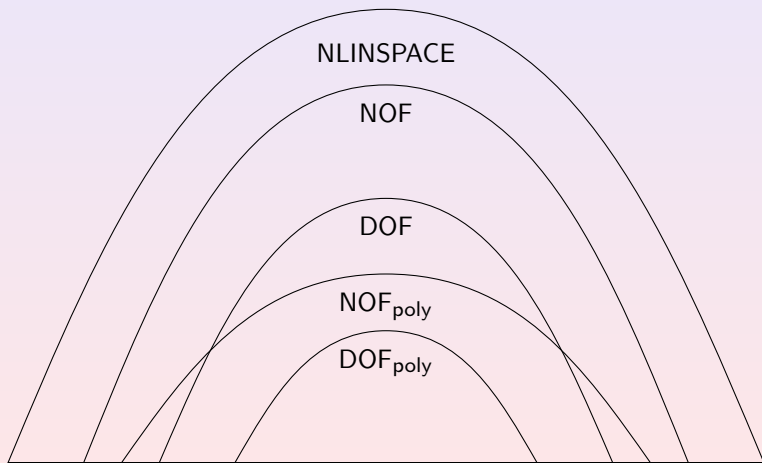
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# Simple Relationships among Overhead-Free Computation Classes



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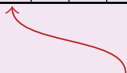
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# Palindromes Can be Accepted in an Overhead-Free Way

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overhead-free machine

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Compare first and last bit

Place left end marker

Place right end marker

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Compare bits next to end markers

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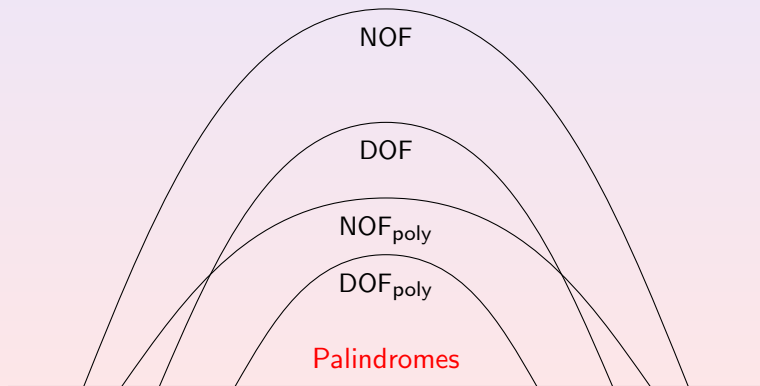
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# Relationships among Overhead-Free Computation Classes



# A Review of Linear Grammars

## Definition

A grammar is **linear** if it is context-free and there is only one nonterminal per right-hand side.

## Example

$$G_1: S \rightarrow 00S0 \mid 1.$$

$$G_2: S \rightarrow 0S10 \mid 0.$$

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## Example

$$G_1: S \rightarrow 00S0 \mid 1.$$

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## Definition

A grammar is **deterministic** if  
“there is always only one rule that can be applied.”

## Example

$G_1$  is deterministic.

$G_2$  is not deterministic.

# Deterministic Linear Languages Can Be Accepted in an Overhead-Free Way

## Theorem

Every deterministic linear language is in  $\text{DOF}_{\text{poly}}$ .



# Continued Review of Linear Grammars

## Definition

A language is **metalinear** if it is the concatenation of linear languages.

# Continued Review of Linear Grammars

## Definition

A language is **metilinear** if it is the concatenation of linear languages.

## Example

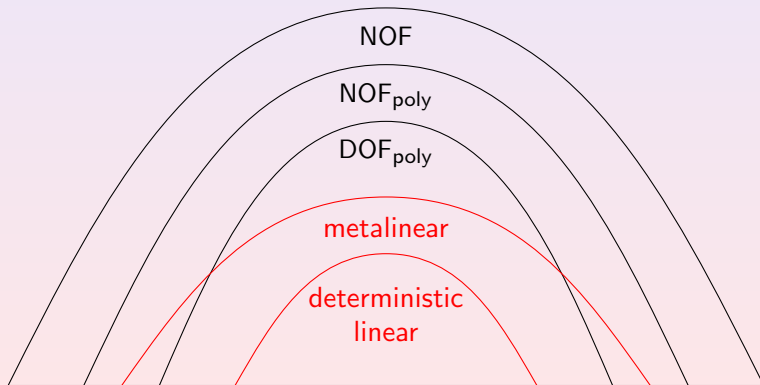
$\text{TRIPLE-PALINDROME} = \{uvw \mid u, v, \text{ and } w \text{ are palindromes}\}.$

# Metalinear Languages Can Be Accepted in an Overhead-Free Way

## Theorem

Every metalinear language is in  $\text{NOF}_{\text{poly}}$ .

# Relationships among Overhead-Free Computation Classes



# Definition of Almost-Overhead-Free Computations

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plus one special symbol.

# Context-Free Languages with a Forbidden Subword Can Be Accepted in an Overhead-Free Way

## Theorem

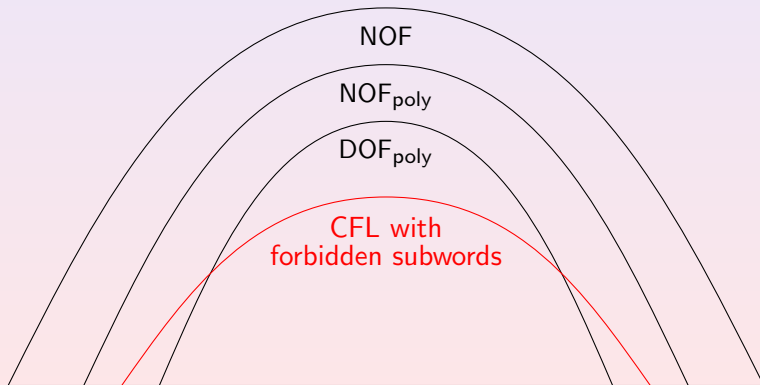
Let  $L$  be a context-free language with a forbidden word.

Then  $L \in \text{NOF}_{\text{poly}}$ .

The proof is based on the fact that every context-free language can be accepted by a nondeterministic almost-overhead-free machine in polynomial time.



# Relationships among Overhead-Free Computation Classes



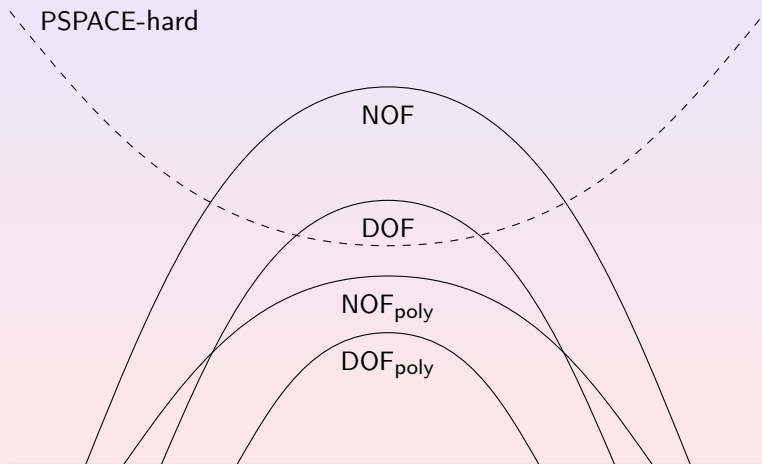
# Some PSPACE-complete Languages Can Be Accepted in an Overhead-Free Way

## Theorem

DOF contains languages that are complete for PSPACE.

Go to proof details.

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# Some Context-Sensitive Languages Cannot be Accepted in an Overhead-Free Way

## Theorem

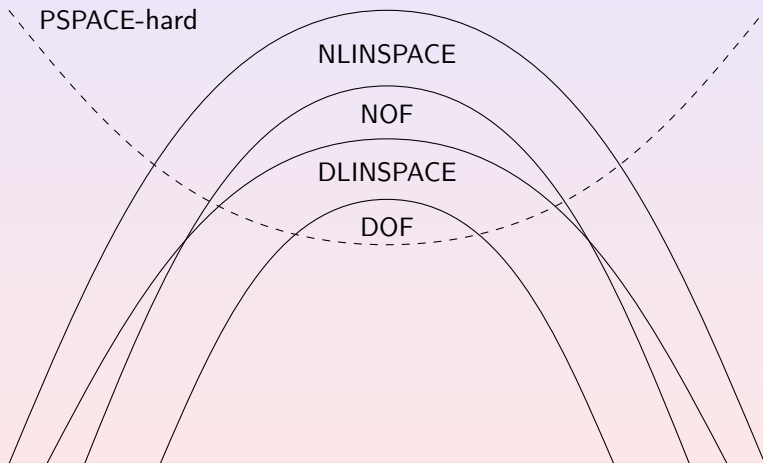
$\text{DOF} \subsetneq \text{DLINSPACE}$ .

## Theorem

$\text{NOF} \subsetneq \text{NLINSPACE}$ .

The proofs are based on old diagonalisations due to Feldman, Owings, and Seiferas.

# Relationships among Overhead-Free Computation Classes



# Candidates for Languages that Cannot be Accepted in an Overhead-Free Way

## Conjecture

DOUBLE-PALINDROMES  $\notin$  DOF.

## Conjecture

$\{ww \mid w \in \{0, 1\}^*\} \notin$  NOF.

Proving the first conjecture would show  $\text{DOF} \subsetneq \text{NOF}$ .

# Summary

- ▶ Overhead-free computation is a more faithful **model of fixed-size memory**.
- ▶ Overhead-free computation is **less powerful** than linear space.
- ▶ **Many** context-free languages can be accepted by overhead-free machines.
- ▶ We conjecture that **all** context-free languages are in  $\text{NOF}_{\text{poly}}$ .
- ▶ Our results can be seen as new results on the power of **linear bounded automata with fixed alphabet** size.



## For Further Reading



A. Salomaa.  
*Formal Languages.*  
Academic Press, 1973.



E. Dijkstra.  
Smoothsort, an alternative for sorting in situ.  
*Science of Computer Programming*, 1(3):223–233, 1982.



E. Feldman and J. Owings, Jr.  
A class of universal linear bounded automata.  
*Information Sciences*, 6:187–190, 1973.



P. Jančar, F. Mráz, M. Plátek, and J. Vogel.  
Restarting automata.  
*FCT Conference 1995*, LNCS 985, pages 282–292. 1995.

## Appendix

### Overhead Freeness and Completeness Improvements for Context-Free Languages

# Overhead-Free Languages can be PSPACE-Complete

## Theorem

DOF contains languages that are complete for PSPACE.

## Proof.

- ▶ Let  $A \in \text{DLINSPACE}$  be PSPACE-complete.  
Such languages are known to exist.
- ▶ Let  $M$  be a linear space machine that accepts  $A \subseteq \{0, 1\}^*$   
with tape alphabet  $\Gamma$ .
- ▶ Let  $h: \Gamma \rightarrow \{0, 1\}^*$  be an isometric, injective homomorphism.
- ▶ Then  $h(L)$  is in DOF and it is PSPACE-complete.

return

# Improvements

## Theorem

1.  $\text{DCFL} \subseteq \text{DOF}_{\text{poly}}$ .
2.  $\text{CFL} \subseteq \text{NOF}_{\text{poly}}$ .