

# Computation with Absolutely No Space Overhead

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Developments in Language Theory Conference, 2003



## The Model of Overhead-Free Computation

The Standard Model of Linear Space

Our Model of Absolutely No Space Overhead

## The Power of Overhead-Free Computation

Palindromes

Linear Languages

Context-Free Languages with a Forbidden Subword

Languages Complete for Polynomial Space

## Limitations of Overhead-Free Computation

Linear Space is Strictly More Powerful



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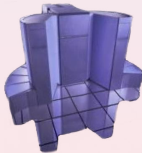
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# The Standard Model of Linear Space

tape

|   |   |   |   |   |   |   |   |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 |
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Turing machine

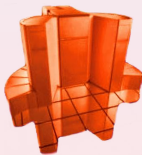
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- Input fills **fixed-size** tape
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- Tape alphabet **is larger than** input alphabet

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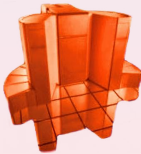
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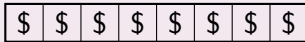
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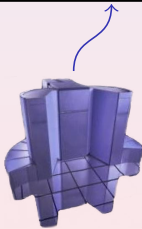
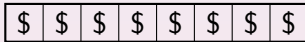
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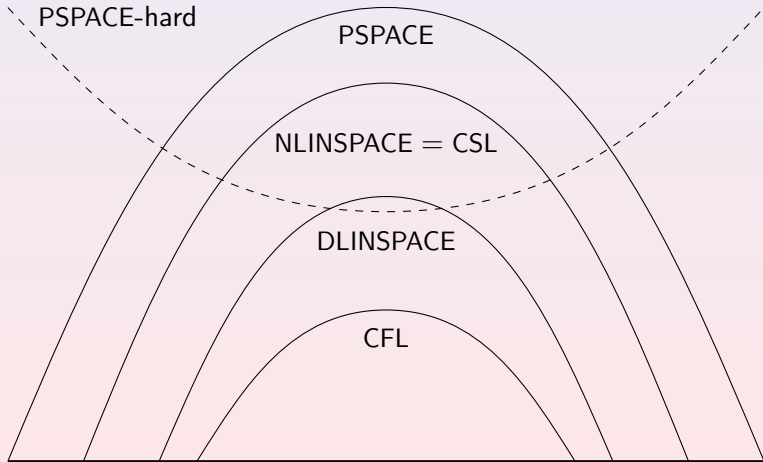


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# Linear Space is a Powerful Model





# Our Model of “Absolutely No Space Overhead”

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Turing machine

## Intuition

- Tape is used like a RAM module.

# Definition of Overhead-Free Computations

## Definition

A Turing machine is **overhead-free** if

- it has only a single tape,
- writes only on input cells,
- writes only symbols drawn from the input alphabet.



# Overhead-Free Computation Complexity Classes

## Definition

A language  $L \subseteq \Sigma^*$  is in

**DOF** if  $L$  is accepted by a deterministic overhead-free machine with input alphabet  $\Sigma$ ,

**DOF<sub>poly</sub>** if  $L$  is accepted by a deterministic overhead-free machine with input alphabet  $\Sigma$  in polynomial time.

is the nondeterministic version of DOF,

**NOF<sub>poly</sub>** is the nondeterministic version of DOF<sub>poly</sub>.



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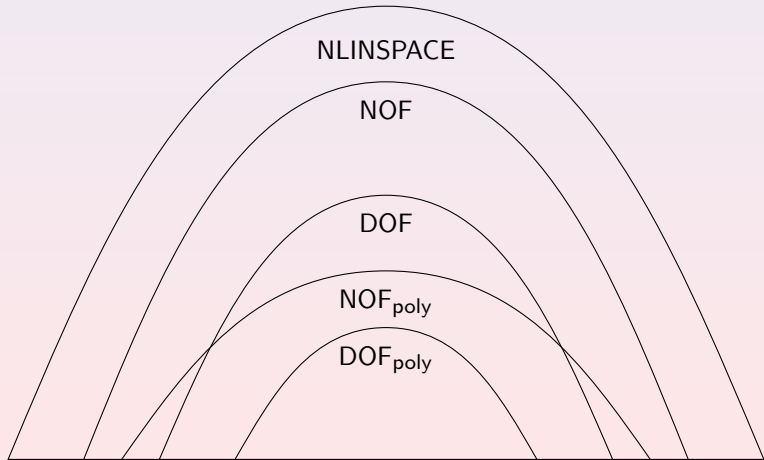
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# Simple Relationships among Overhead-Free Computation Classes



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overhead-free machine

## Algorithm

Phase 1:

Compare first and last bit

Place left end marker

Place right end marker

Phase 2:

Compare bits next to end markers

Find left end marker

Advance left end marker

Find right end marker

Advance right end marker



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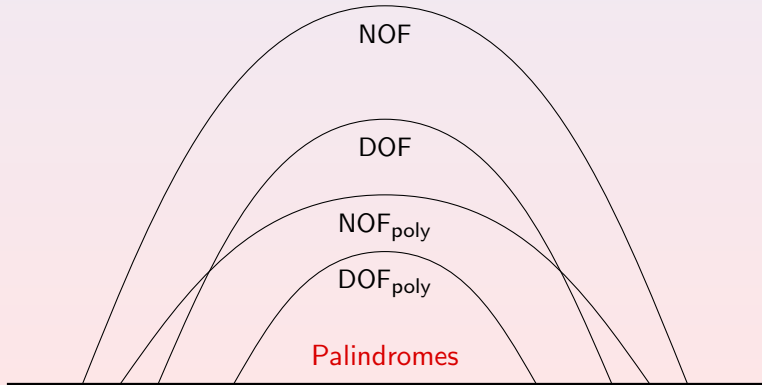
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# Relationships among Overhead-Free Computation Classes



# A Review of Linear Grammars

## Definition

A grammar is **linear** if it is context-free and there is only one nonterminal per right-hand side.

## Example

$G_1: S \rightarrow 00S0 \mid 1.$

$G_2: S \rightarrow 0S10 \mid 0.$

## Definition

A grammar is **deterministic** if  
“there is always only one rule that can be applied.”

## Example

$G_1$  is deterministic.

$G_2$  is **not** deterministic.





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# Deterministic Linear Languages Can Be Accepted in an Overhead-Free Way

## Theorem

Every deterministic linear language is in  $\text{DOF}_{\text{poly}}$ .



# Metalinear Languages

## Can Be Accepted in an Overhead-Free Way

### Definition

A language is **metalinear** if it is the concatenation of linear languages.

### Example

TRIPLE-PALINDROME =  $\{uvw \mid u, v, \text{ and } w \text{ are palindromes}\}$ .

### Theorem

Every metalinear language is in  $\text{NOF}_{\text{poly}}$ .



# Metalinear Languages

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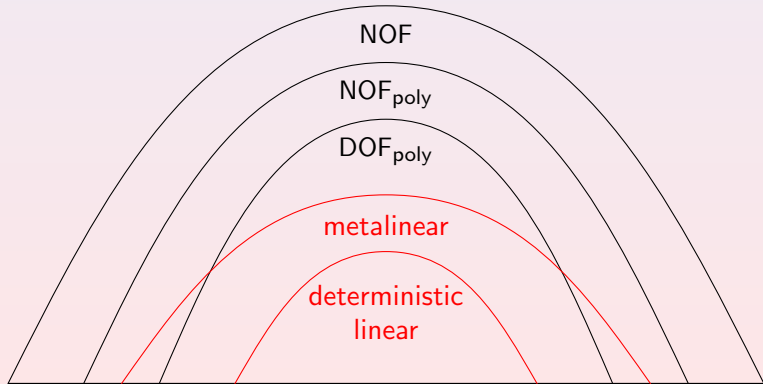
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# Relationships among Overhead-Free Computation Classes



# Definition of Almost-Overhead-Free Computations

## Definition

A Turing machine is **almost-overhead-free** if

- it has only a single tape,
  - writes only on input cells,
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# Context-Free Languages with a Forbidden Subword Can Be Accepted in an Overhead-Free Way

## Theorem

Let  $L$  be a context-free language with a forbidden word.  
Then  $L \in \text{NOF}_{\text{poly}}$ .

► Skip proof



# Context-Free Languages with a Forbidden Subword Can Be Accepted in an Overhead-Free Way

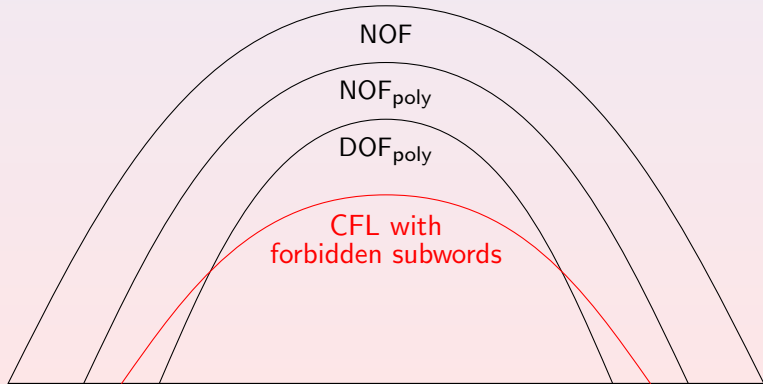
## Theorem

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## Proof.

Every context-free language can be accepted by a nondeterministic almost-overhead-free machine in polynomial time.

# Relationships among Overhead-Free Computation Classes



# Some PSPACE-complete Languages Can Be Accepted in an Overhead-Free Way

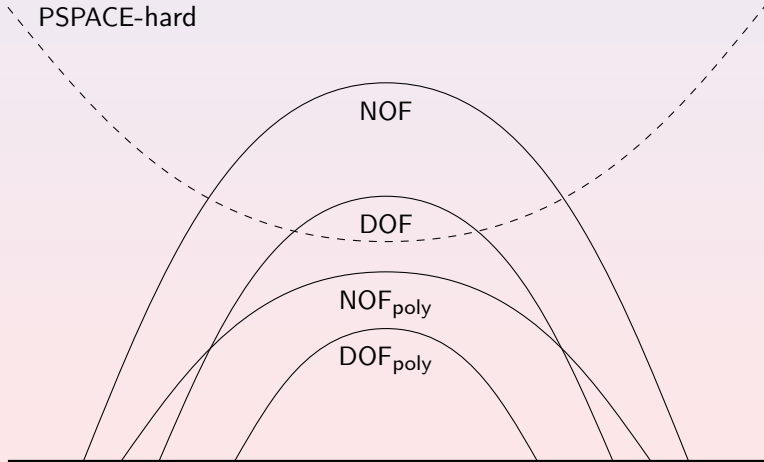
## Theorem

DOF contains languages that are complete for PSPACE.

► [Proof details](#)



# Relationships among Overhead-Free Computation Classes



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# Some Context-Sensitive Languages Cannot be Accepted in an Overhead-Free Way

## Theorem

$\text{DOF} \subsetneq \text{DLINSPACE}$ .

## Theorem

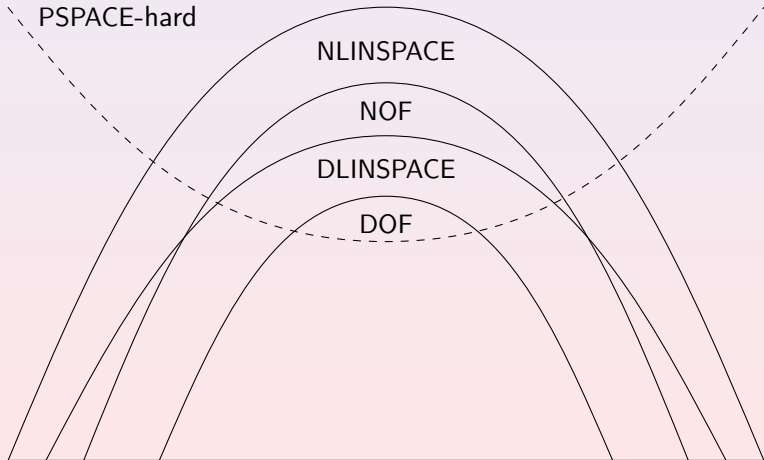
$\text{NOF} \subsetneq \text{NLINSPACE}$ .

The proofs are based on old diagonalisations due to Feldman, Owings, and Seiferas.





# Relationships among Overhead-Free Computation Classes



# Candidates for Languages that Cannot be Accepted in an Overhead-Free Way

## Conjecture

DOUBLE-PALINDROMES  $\notin$  DOF.

## Conjecture

$\{ww \mid w \in \{0,1\}^*\} \notin$  NOF.

Proving the first conjecture would show  $\text{DOF} \subsetneq \text{NOF}$ .



# Summary

- Overhead-free computation is a more faithful **model of fixed-size memory**.
- Overhead-free computation is **less powerful** than linear space.
- **Many** context-free languages can be accepted by overhead-free machines.
- We conjecture that **all** context-free languages are in  $\text{NOF}_{\text{poly}}$ .
- Our results can be seen as new results on the power of **linear bounded automata with fixed alphabet size**.



## For Further Reading



A. Salomaa.

*Formal Languages.*

Academic Press, 1973.



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## Appendix

Overhead Freeness and Completeness  
Improvements for Context-Free Languages  
Abbreviations





# Overhead-Free Languages can be PSPACE-Complete

## Theorem

DOF contains languages that are complete for PSPACE.

## Proof.

- Let  $A \in \text{DLINSPACE}$  be PSPACE-complete.  
Such languages are known to exist.
- Let  $M$  be a linear space machine that accepts  $A \subseteq \{0, 1\}^*$  with tape alphabet  $\Gamma$ .
- Let  $h: \Gamma \rightarrow \{0, 1\}^*$  be an isometric, injective homomorphism.
- Then  $h(L)$  is in DOF and it is PSPACE-complete.

[Return](#)

# Improvements

## Theorem

1.  $\text{DCFL} \subseteq \text{DOF}_{\text{poly}}$ .
2.  $\text{CFL} \subseteq \text{NOF}_{\text{poly}}$ .

# Explanation of Different Abbreviations

|                     |                                                  |
|---------------------|--------------------------------------------------|
| DOF                 | Deterministic Overhead-Free.                     |
| NOF                 | Nondeterministic Overhead-Free.                  |
| DOF <sub>poly</sub> | Deterministic Overhead-Free, polynomial time.    |
| DOF <sub>poly</sub> | Nondeterministic Overhead-Free, polynomial time. |

Table: Explanation of what different abbreviations mean.

