Computation with Absolutely No Space Overhead

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The Standard Model of Linear Space

tape

0	0	1	0	0	1	0	0

Turing machine

Characteristics

- Input fills fixed-size tape
- Input may be modified
- Tape alphabet is larger than input alphabet

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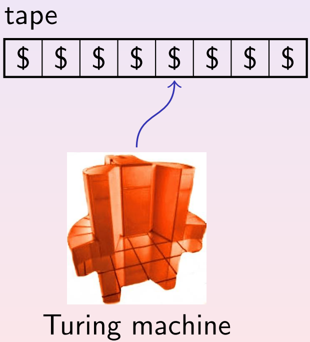

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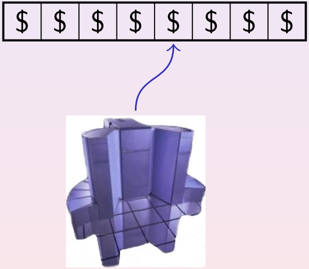

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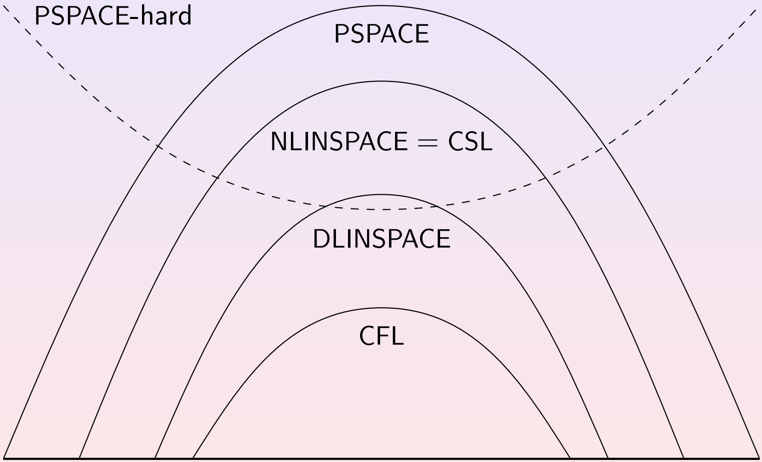
Linear Space is a Powerful Model



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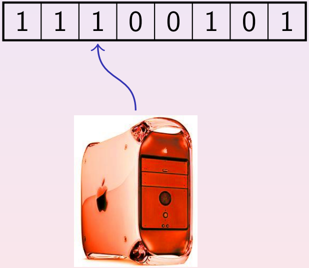

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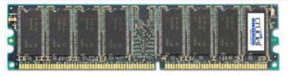

Turing machine

Intuition

- Tape is used like a RAM module.

Definition of Overhead-Free Computations

Definition

A Turing machine is overhead-free if

- it has only a single tape,
- writes only on input cells,
- writes only symbols drawn from the input alphabet.

Overhead-Free Computation Complexity Classes

Definition

A language $L \subseteq \Sigma^*$ is in

DOF if L is accepted by a deterministic overhead-free machine with input alphabet Σ,

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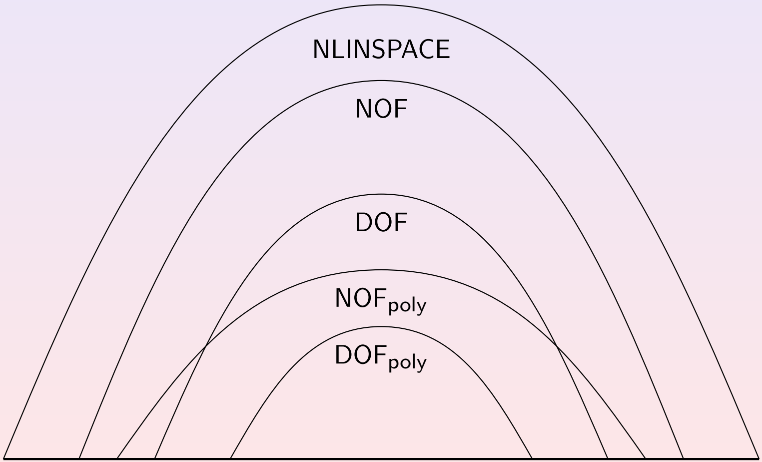
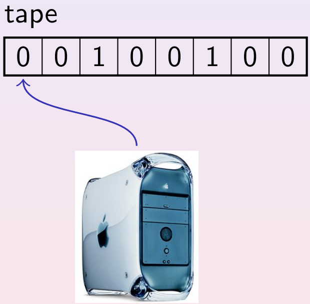
Simple Relationships among Overhead-Free Computation Classes



Palindromes Can be Accepted in an Overhead-Free Way



overhead-free machine

Algorithm

Phase 1:
Compare first and last bit
- Place left end marker
- Place right end marker

Phase 2:
Compare bits next to end markers
- Find left end marker
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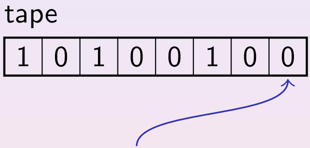

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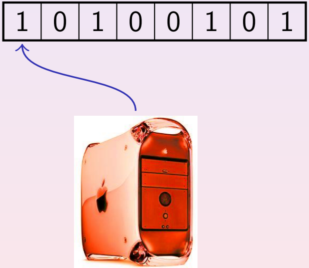

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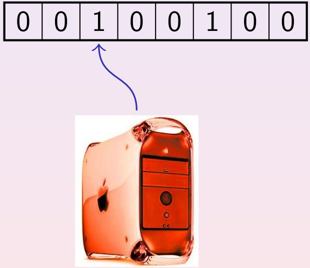

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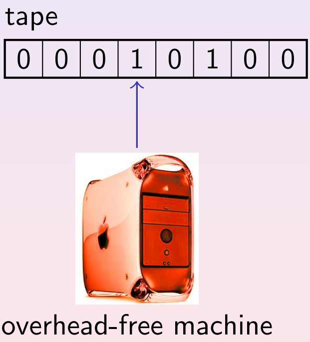

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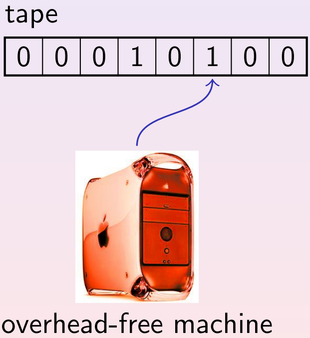

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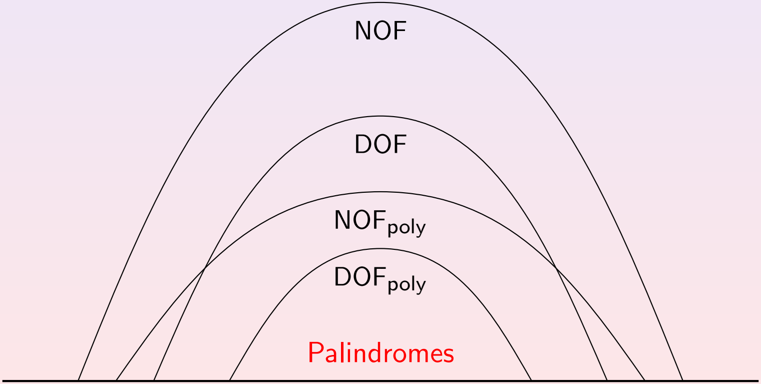
Relationships among Overhead-Free Computation Classes



A Review of Linear Grammars

Definition

A grammar is linear if it is context-free and there is only one nonterminal per right-hand side.

Example

$G_1\colon S \longrightarrow 00S0 \mid 1.$

$G_2\colon S \longrightarrow 0S10 \mid 0.$

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Definition

A grammar is deterministic if
"there is always only one rule that can be applied."

Example

G_1 is deterministic.

G_2 is not deterministic.

Deterministic Linear Languages Can Be Accepted in an Overhead-Free Way

Theorem

Every deterministic linear language is in $\mathsf{DOF}_{\mathsf{poly}}$.

Continued Review of Linear Grammars

Definition

A language is metalinear if it is the concatenation of linear languages.

Continued Review of Linear Grammars

Definition

A language is metalinear if it is the concatenation of linear languages.

Example

$\text{TRIPLE-PALINDROME} = \{uvw \mid u, v, \text{ and } w \text{ are palindromes}\}$.

Metalinear Languages Can Be Accepted in an Overhead-Free Way

Theorem

Every metalinear language is in $\mathsf{NOF}_{\mathrm{poly}}$.

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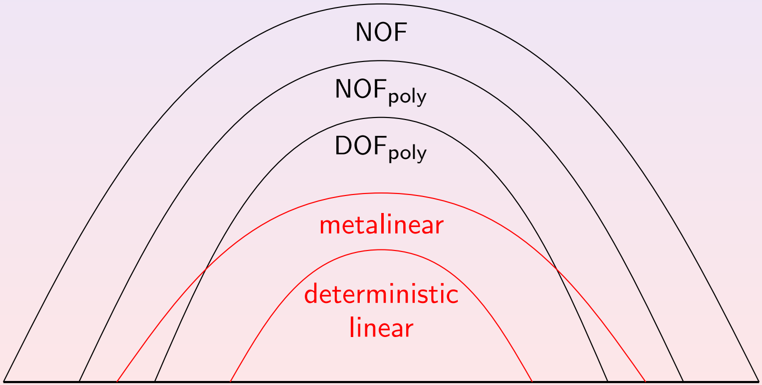

Definition of Almost-Overhead-Free Computations

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- writes only symbols drawn from the input alphabet plus one special symbol.

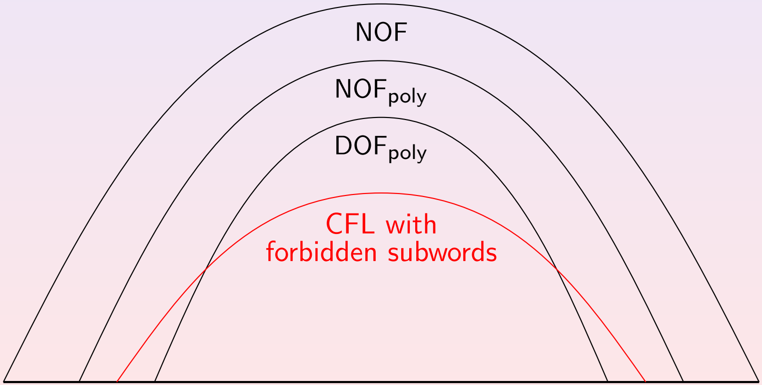
Context-Free Languages with a Forbidden Subword Can Be Accepted in an Overhead-Free Way

Theorem

Let L be a context-free language with a forbidden word.
Then $L \in \mathrm{NOF}_{\mathrm{poly}}$.

The proof is based on the fact that every context-free language can be accepted by a nondeterministic almost-overhead-free machine in polynomial time.

Relationships among Overhead-Free Computation Classes



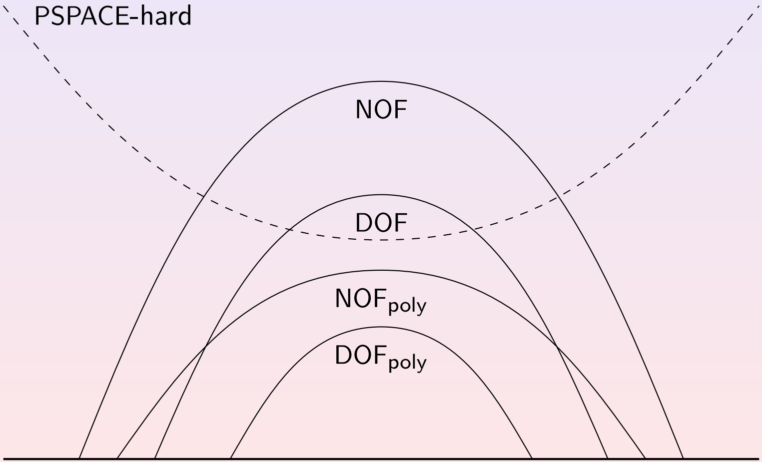
Some PSPACE-complete Languages Can Be Accepted in an Overhead-Free Way

Theorem

DOF contains languages that are complete for PSPACE.

Go to proof details.

Relationships among Overhead-Free Computation Classes



Some Context-Sensitive Languages Cannot be Accepted in an Overhead-Free Way

Theorem

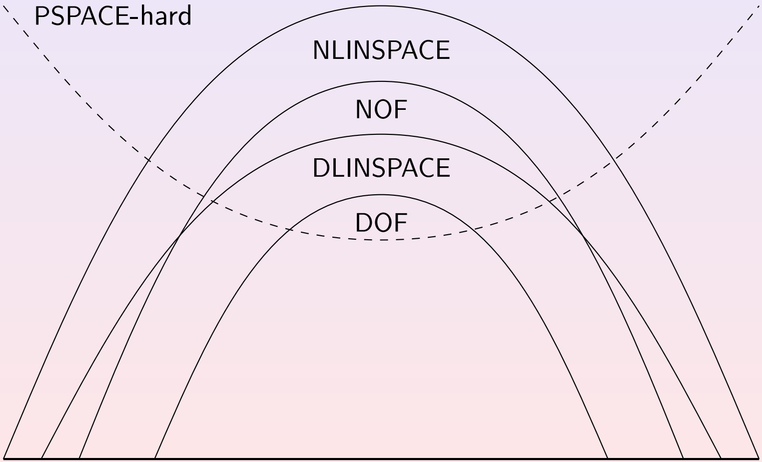
DOF $\subsetneq$ DLINSPACE.

Theorem

NOF $\subsetneq$ NLINSPACE.

The proofs are based on old diagonalisations due to Feldman, Owings, and Seiferas.

Relationships among Overhead-Free Computation Classes



Candidates for Languages that Cannot be Accepted in an Overhead-Free Way

Conjecture

$\text{DOUBLE-PALINDROMES} \notin \mathrm{DOF}$.

Conjecture

$\{ww \mid w \in \{0,1\}^*\} \notin \mathrm{NOF}$.

Proving the first conjecture would show $\mathrm{DOF} \subsetneq \mathrm{NOF}$.

Summary

- Overhead-free computation is a more faithful model of fixed-size memory.
- Overhead-free computation is less powerful than linear space.
- Many context-free languages can be accepted by overhead-free machines.
- We conjecture that all context-free languages are in $\mathrm{NOF}_{\mathrm{poly}}$.
- Our results can be seen as new results on the power of linear bounded automata with fixed alphabet size.

For Further Reading

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Restarting automata.
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Appendix

Overhead Freeness and Completeness

Improvements for Context-Free Languages

Overhead-Free Languages can be PSPACE-Complete

Theorem

DOF contains languages that are complete for PSPACE.

Proof.

- Let $A \in$ DLINSPACE be PSPACE-complete.
  Such languages are known to exist.
- Let M be a linear space machine that accepts $A \subseteq \{0,1\}^*$ with tape alphabet Γ.
- Let $h\colon \Gamma \to \{0,1\}^*$ be an isometric, injective homomorphism.
- Then $h(L)$ is in DOF and it is PSPACE-complete.

return

Improvements

Theorem

1. $\mathsf{DCFL} \subseteq \mathsf{DOF}_{\mathrm{poly}}$.
2. $\mathsf{CFL} \subseteq \mathsf{NOF}_{\mathrm{poly}}$.