Computation with Absolutely No Space Overhead

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Developments in Language Theory Conference, 2003

The Standard Model of Linear Space

tape

0	0	1	0	0	1	0	0

Turing machine

Characteristics

- Input fills fixed-size tape
- Input may be modified
- Tape alphabet is larger than input alphabet

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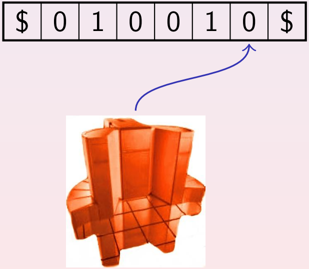

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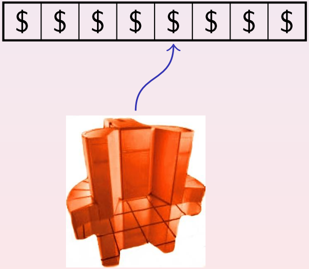

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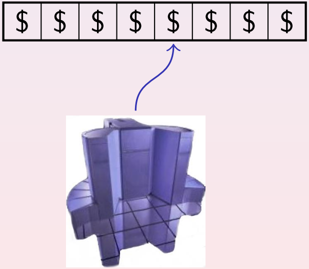

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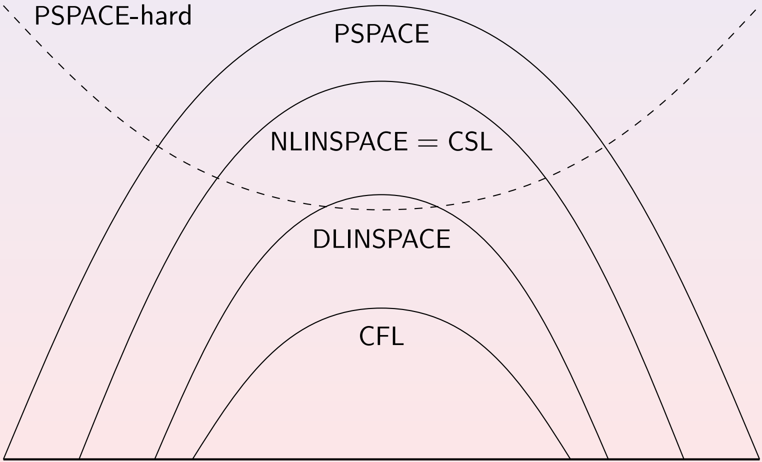
Linear Space is a Powerful Model



Our Model of “Absolutely No Space Overhead”

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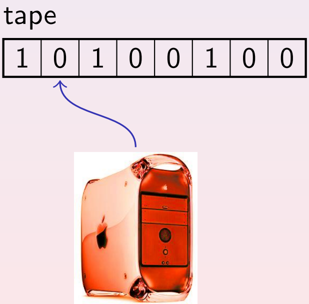

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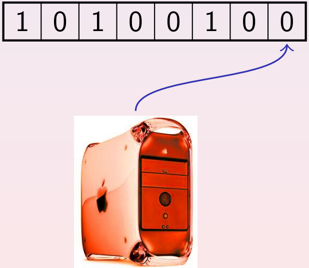

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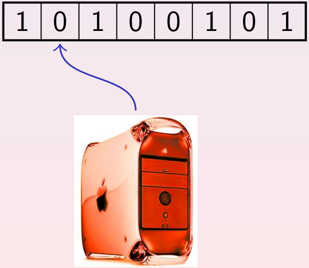

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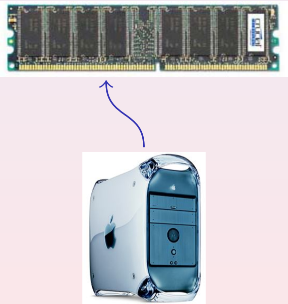

Turing machine

Intuition

- Tape is used like a RAM module.

Definition of Overhead-Free Computations

Definition

A Turing machine is overhead-free if

- it has only a single tape,
- writes only on input cells,
- writes only symbols drawn from the input alphabet.

Overhead-Free Computation Complexity Classes

Definition

A language $L \subseteq \Sigma^*$ is in

- DOF if L is accepted by a deterministic overhead-free machine with input alphabet Σ,
- $\mathrm{DOF}_{\mathrm{poly}}$ if L is accepted by a deterministic overhead-free machine with input alphabet Σ in polynomial time.
- NOF is the nondeterministic version of DOF,
- $\mathrm{NOF}_{\mathrm{poly}}$ is the nondeterministic version of $\mathrm{DOF}_{\mathrm{poly}}$.

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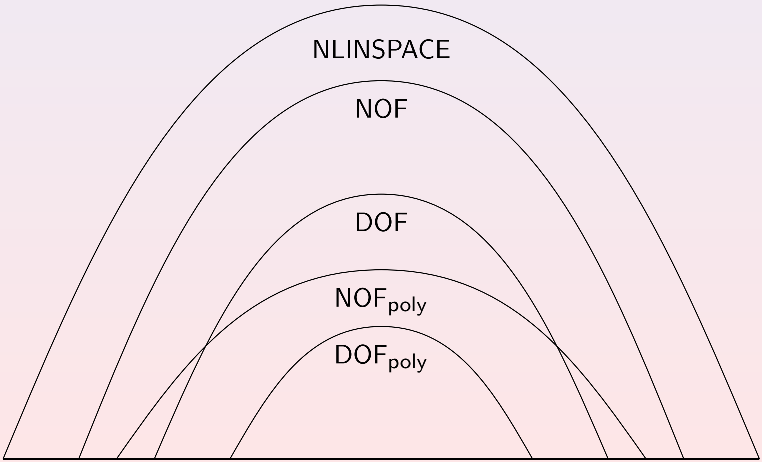
Simple Relationships among Overhead-Free Computation Classes



Palindromes Can be Accepted in an Overhead-Free Way

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overhead-free machine

Algorithm

Phase 1:
Compare first and last bit
- Place left end marker
- Place right end marker

Phase 2:
Compare bits next to end markers
- Find left end marker
- Advance left end marker
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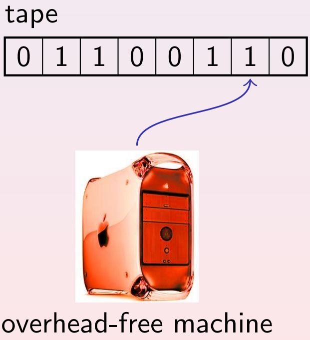

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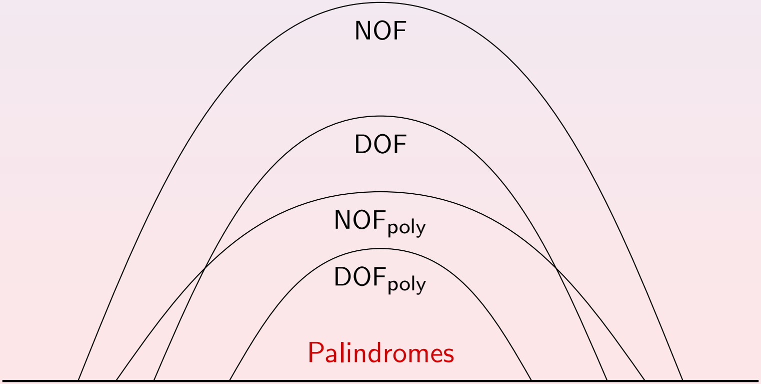
Relationships among Overhead-Free Computation Classes



A Review of Linear Grammars

Definition

A grammar is linear if it is context-free and there is only one nonterminal per right-hand side.

Example

$G_1\colon S \to 00S0 \mid 1.$
$G_2\colon S \to 0S10 \mid 0.$

Definition

A grammar is deterministic if "there is always only one rule that can be applied."

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G_1 is deterministic.
G_2 is not deterministic.

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Example

G_1 is deterministic.
G_2 is not deterministic.

Deterministic Linear Languages Can Be Accepted in an Overhead-Free Way

Theorem

Every deterministic linear language is in $\mathrm{DOF}_{\mathrm{poly}}$.

Metalinear Languages Can Be Accepted in an Overhead-Free Way

Definition

A language is metalinear if it is the concatenation of linear languages.

Example

$\text{TRIPLE-PALINDROME} = \{uvw \mid u, v, \text{ and } w \text{ are palindromes}\}$.

Theorem

Every metalinear language is in NOF_{poly}.

Metalinear Languages Can Be Accepted in an Overhead-Free Way

Definition

A language is metalinear if it is the concatenation of linear languages.

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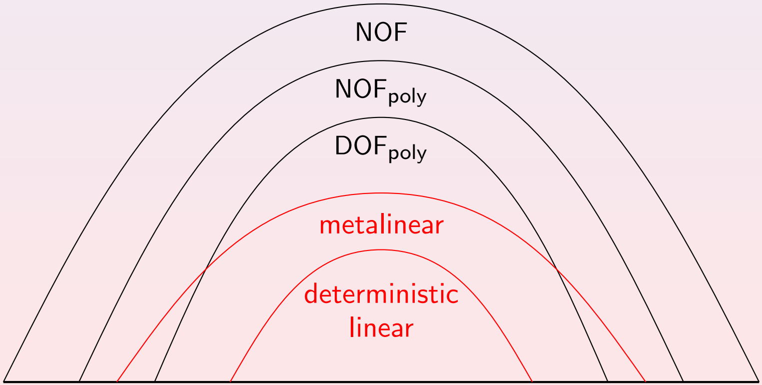

Definition of Almost-Overhead-Free Computations

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Context-Free Languages with a Forbidden Subword Can Be Accepted in an Overhead-Free Way

Theorem

Let L be a context-free language with a forbidden word. Then $L \in \mathrm{NOF}_{\mathrm{poly}}$.

Skip proof

Context-Free Languages with a Forbidden Subword Can Be Accepted in an Overhead-Free Way

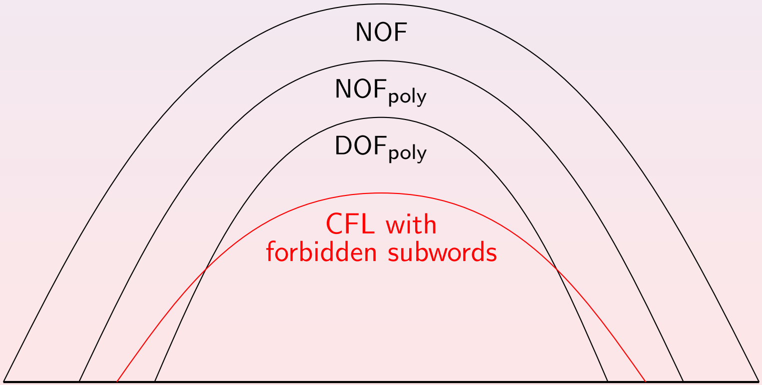
Theorem

Let L be a context-free language with a forbidden word.
Then $L \in \mathrm{NOF}_{\mathrm{poly}}$.

Proof.

Every context-free language can be accepted by a nondeterministic almost-overhead-free machine in polynomial time.

Relationships among Overhead-Free Computation Classes



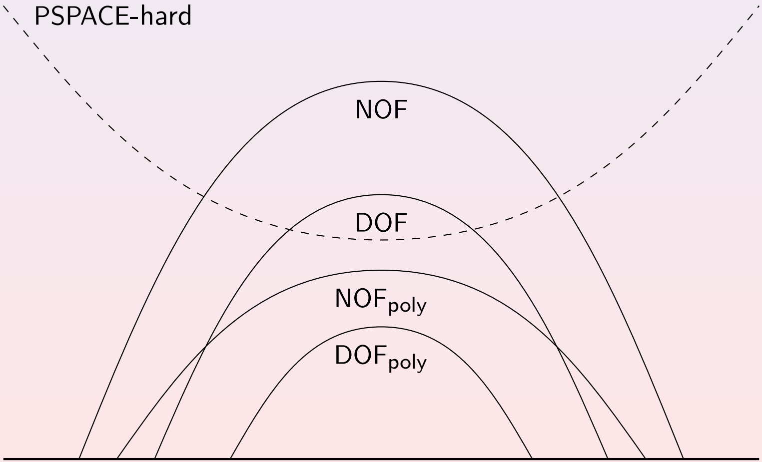
Some PSPACE-complete Languages Can Be Accepted in an Overhead-Free Way

Theorem

DOF contains languages that are complete for PSPACE.

▸ Proof details

Relationships among Overhead-Free Computation Classes



Some Context-Sensitive Languages Cannot be Accepted in an Overhead-Free Way

Theorem

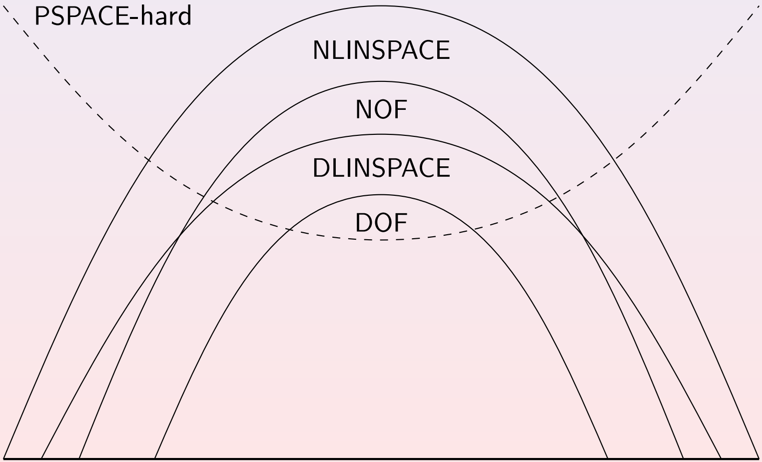
DOF $\subsetneq$ DLINSPACE.

Theorem

NOF $\subsetneq$ NLINSPACE.

The proofs are based on old diagonalisations due to Feldman, Owings, and Seiferas.

Relationships among Overhead-Free Computation Classes



Candidates for Languages that Cannot be Accepted in an Overhead-Free Way

Conjecture

DOUBLE-PALINDROMES $\notin$ DOF.

Conjecture

$\{ww \mid w \in \{0,1\}^*\} \notin$ NOF.

Proving the first conjecture would show DOF $\subsetneq$ NOF.

Summary

- Overhead-free computation is a more faithful model of fixed-size memory.
- Overhead-free computation is less powerful than linear space.
- Many context-free languages can be accepted by overhead-free machines.
- We conjecture that all context-free languages are in $\mathrm{NOF}_{\mathrm{poly}}$.
- Our results can be seen as new results on the power of linear bounded automata with fixed alphabet size.

For Further Reading

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Formal Languages.
Academic Press, 1973.

E. Dijkstra.
Smoothsort, an alternative for sorting in situ.
Science of Computer Programming, 1(3):223–233, 1982.

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Overhead-Free Languages can be PSPACE-Complete

Theorem

DOF contains languages that are complete for PSPACE.

Proof.

- Let $A \in$ DLINSPACE be PSPACE-complete. Such languages are known to exist.
- Let M be a linear space machine that accepts $A \subseteq \{0,1\}^*$ with tape alphabet Γ.
- Let $h\colon \Gamma \to \{0,1\}^*$ be an isometric, injective homomorphism.
- Then $h(L)$ is in DOF and it is PSPACE-complete.

Return

Improvements

Theorem

1. $\mathsf{DCFL} \subseteq \mathsf{DOF}_{\mathsf{poly}}$.
2. $\mathsf{CFL} \subseteq \mathsf{NOF}_{\mathsf{poly}}$.

Explanation of Different Abbreviations

DOF	Deterministic Overhead-Free.
NOF	Nondeterministic Overhead-Free.
$\mathrm{DOF}_{\mathrm{poly}}$	Deterministic Overhead-Free, polynomial time.
$\mathrm{DOF}_{\mathrm{poly}}$	Nondeterministic Overhead-Free, polynomial time.

Table: Explanation of what different abbreviations mean.