Computation with Absolutely No Space Overhead

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Technical University of Berlin

Developments in Language Theory Conference, 2003

The Standard Model of Linear Space

tape

0	0	1	0	0	1	0	0

Turing machine

Characteristics

- Input fills fixed-size tape
- Input may be modified
- Tape alphabet is larger than input alphabet Go to End

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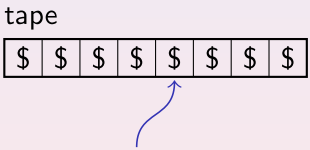

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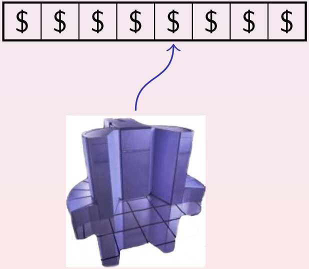

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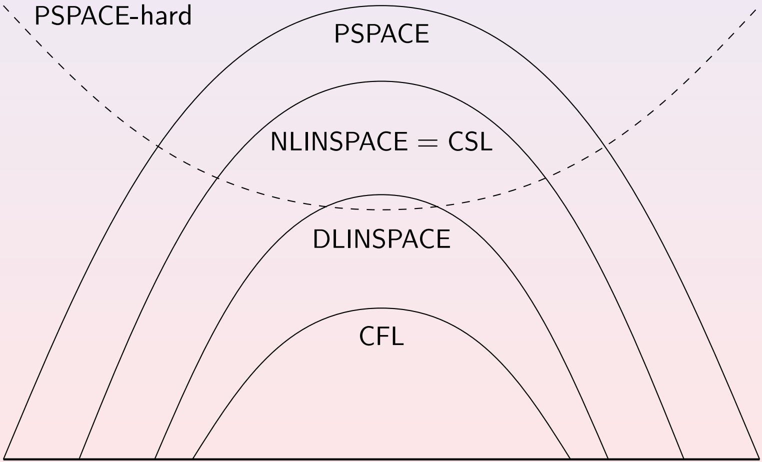
Linear Space is a Powerful Model



Our Model of “Absolutely No Space Overhead”

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Characteristics

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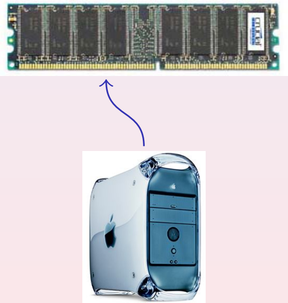

Turing machine

Intuition

- Tape is used like a RAM module.

Definition of Overhead-Free Computations

Definition

A Turing machine is overhead-free if

- it has only a single tape,
- writes only on input cells,
- writes only symbols drawn from the input alphabet.

Overhead-Free Computation Complexity Classes

Definition

A language $L \subseteq \Sigma^*$ is in

- DOF if L is accepted by a deterministic overhead-free machine with input alphabet Σ,
- $\mathrm{DOF}_{\mathrm{poly}}$ if L is accepted by a deterministic overhead-free machine with input alphabet Σ in polynomial time.
- NOF is the nondeterministic version of DOF,
- $\mathrm{NOF}_{\mathrm{poly}}$ is the nondeterministic version of $\mathrm{DOF}_{\mathrm{poly}}$.

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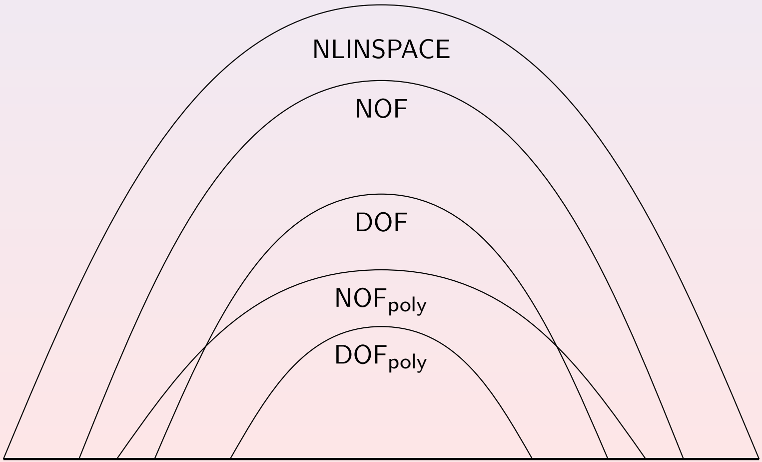
Simple Relationships among Overhead-Free Computation Classes



Palindromes Can be Accepted in an Overhead-Free Way

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overhead-free machine

Algorithm

Phase 1:

Compare first and last bit

- Place left end marker
- Place right end marker

Phase 2:

Compare bits next to end markers

- Find left end marker
- Advance left end marker
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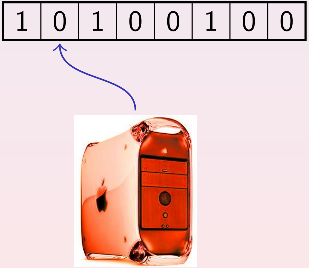

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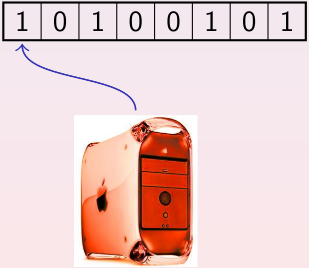

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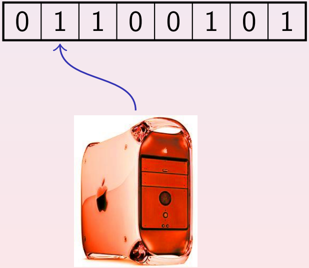

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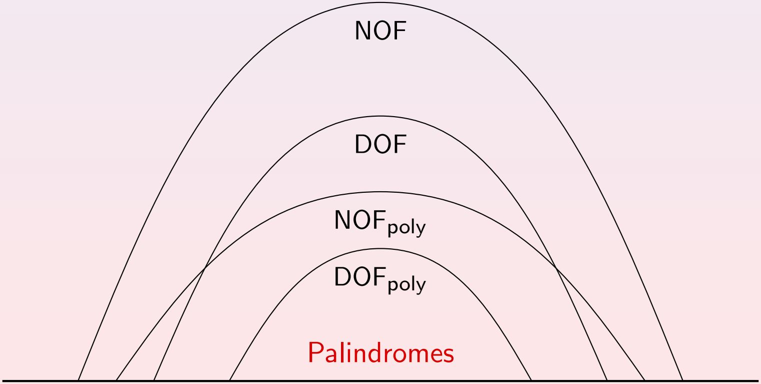
Relationships among Overhead-Free Computation Classes



A Review of Linear Grammars

Definition

A grammar is linear if it is context-free and there is only one nonterminal per right-hand side.

Example

$G_1\colon S \to 00S0 \mid 1.$
$G_2\colon S \to 0S10 \mid 0.$

Definition

A grammar is deterministic if "there is always only one rule that can be applied."

Example

G_1 is deterministic.
G_2 is not deterministic.

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A grammar is linear if it is context-free and
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Definition

A grammar is deterministic if
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Example

G_1 is deterministic.
G_2 is not deterministic.

Deterministic Linear Languages Can Be Accepted in an Overhead-Free Way

Theorem

Every deterministic linear language is in $\mathsf{DOF}_{\text{poly}}$.

Metalinear Languages Can Be Accepted in an Overhead-Free Way

Definition

A language is metalinear if it is the concatenation of linear languages.

Example

$\text{TRIPLE-PALINDROME} = \{uvw \mid u, v, \text{ and } w \text{ are palindromes}\}$.

Theorem

Every metalinear language is in $\mathrm{NOF}_{\mathrm{poly}}$.

Metalinear Languages Can Be Accepted in an Overhead-Free Way

Definition

A language is metalinear if it is the concatenation of linear languages.

Example

$\textsc{triple-palindrome} = \{uvw \mid u, v, \text{ and } w \text{ are palindromes}\}$.

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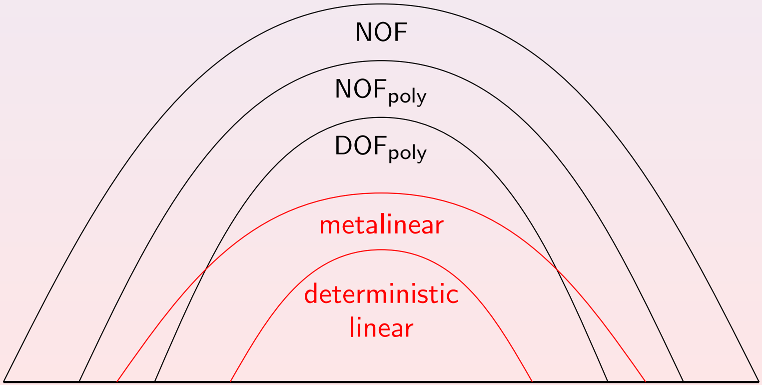

Definition of Almost-Overhead-Free Computations

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A Turing machine is almost-overhead-free if

- it has only a single tape,
- writes only on input cells,
- writes only symbols drawn from the input alphabet plus one special symbol.

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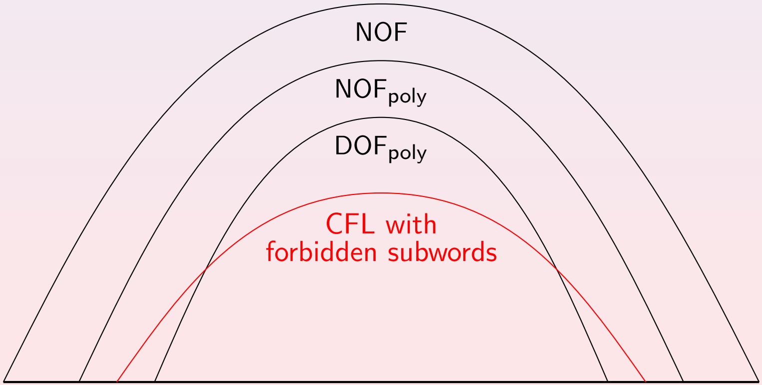
Context-Free Languages with a Forbidden Subword Can Be Accepted in an Overhead-Free Way

Theorem

Let L be a context-free language with a forbidden word.
Then $L \in \mathsf{NOF}_{\mathsf{poly}}$.

The proof is based on the fact that every context-free language can be accepted by a nondeterministic almost-overhead-free machine in polynomial time.

Relationships among Overhead-Free Computation Classes



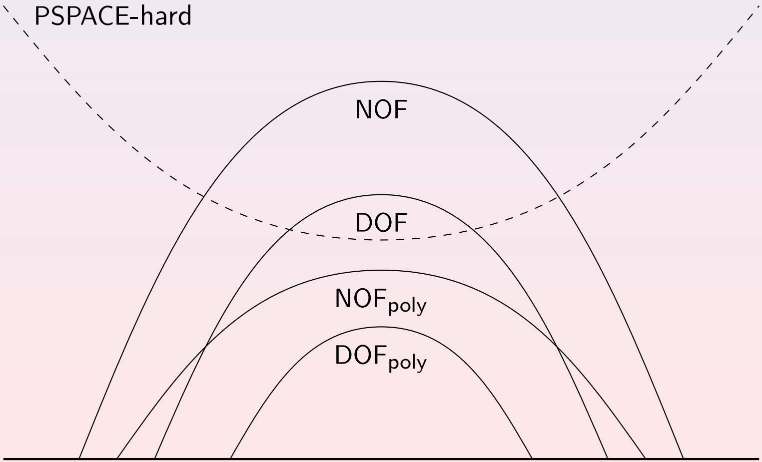
Some PSPACE-complete Languages Can Be Accepted in an Overhead-Free Way

Theorem

DOF contains languages that are complete for PSPACE.

Go to proof details.

Relationships among Overhead-Free Computation Classes



Some Context-Sensitive Languages Cannot be Accepted in an Overhead-Free Way

Theorem

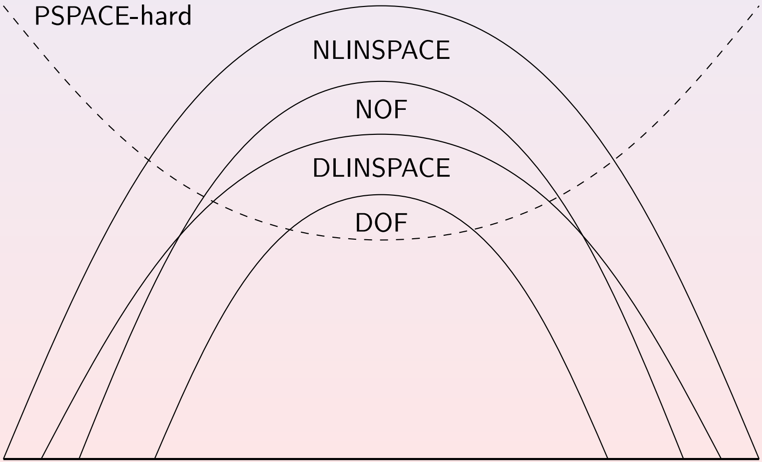
DOF $\subsetneq$ DLINSPACE.

Theorem

NOF $\subsetneq$ NLINSPACE.

The proofs are based on old diagonalisations due to Feldman, Owings, and Seiferas.

Relationships among Overhead-Free Computation Classes



Candidates for Languages that Cannot be Accepted in an Overhead-Free Way

Conjecture

DOUBLE-PALINDROMES $\notin$ DOF.

Conjecture

$\{ww \mid w \in \{0,1\}^*\} \notin$ NOF.

Proving the first conjecture would show DOF $\subsetneq$ NOF.

Summary

- Overhead-free computation is a more faithful model of fixed-size memory.
- Overhead-free computation is less powerful than linear space.
- Many context-free languages can be accepted by overhead-free machines.
- We conjecture that all context-free languages are in $\mathrm{NOF}_{\mathrm{poly}}$.
- Our results can be seen as new results on the power of linear bounded automata with fixed alphabet size.

For Further Reading

A. Salomaa.
Formal Languages.
Academic Press, 1973.

E. Dijkstra.
Smoothsort, an alternative for sorting in situ.
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Appendix

- Overhead Freeness and Completeness
- Improvements for Context-Free Languages
- Abbreviations

Overhead-Free Languages can be PSPACE-Complete

Theorem

DOF contains languages that are complete for PSPACE.

Proof.

- Let $A \in$ DLINSPACE be PSPACE-complete.
  Such languages are known to exist.
- Let M be a linear space machine that accepts $A \subseteq \{0, 1\}^*$ with tape alphabet Γ.
- Let $h\colon \Gamma \to \{0, 1\}^*$ be an isometric, injective homomorphism.
- Then $h(L)$ is in DOF and it is PSPACE-complete.

return

Improvements

Theorem

1. $\mathrm{DCFL} \subseteq \mathrm{DOF}_{\mathrm{poly}}$.
2. $\mathrm{CFL} \subseteq \mathrm{NOF}_{\mathrm{poly}}$.

Explanation of Different Abbreviations

DOF	Deterministic Overhead-Free.
NOF	Nondeterministic Overhead-Free.
$\mathrm{DOF}_{\mathrm{poly}}$	Deterministic Overhead-Free, polynomial time.
$\mathrm{DOF}_{\mathrm{poly}}$	Nondeterministic Overhead-Free, polynomial time.

Table: Explanation of what different abbreviations mean.