**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. **0.2676**
4. 0.5
5. 0.6987

**Ans:** *μ* = 45 minutes and *σ* = 8 minutes

Distribution is normal

Let x be the time which take for repair the car.

We have only 50 min remaining.

= 50

We want to find P(x > 50)

P( x > 50 ) = 1- P(x<50)

Z-value = ( - μ) / σ

= (50-45)/ 8

= 0.625

P = stats.norm.cdf(0.625)

P

P= 0.7340

1 – P = 0.2660

P( x > 50 ) = 26.6%

**Therefor, the probability that the service manager cannot meet his commitment is 26.6%**

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans: *μ* = 38 and *σ* = 6

Distribution is normal

Let x be the number of the employee

= 44

We want to find P(x > 44)

P( x > 44 ) = 1- P(x<44)

Z-value = ( - μ) / σ

= (44-38)/ 6

= 1

P = stats.norm.cdf(1)

P = 0.8413

1 – P = 0.1586

P( x > 44 ) = 0.1586 = 15.86%

the probability of number of employees between 38-44 years of age = o.5 - P( x > 44 )

= 0.3413

the probability of number of employees between 38-44 years of age is 34.13%

**Therefore, the statement that “More employees at the processing center are older than 44 than between 38 and 44” is TRUE.**

b) Probability of employees less than age of 30 = P(X<30).

Z-value = ( - μ) / σ

= (30-38)/ 6

= -1.33

P = stats.norm.cdf(-1.33)

P = 0.912

P(X ≤ 30) = 9.12%

So, the number of employees with probability 0.912 of them being under age 30 =

0.0912\*400=36.48( or 36 employees).

Therefore, the statement B of the question is also TRUE.

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

**Ans**: As we know that if X  N(µ1, σ1^2 ), and Y  N(µ2, σ2^2 ) are two independent random variables then X + Y  N(µ1 + µ2, σ1^2 + σ2^2 ) , and X − Y  N(µ1 − µ2, σ1^2 + σ2^2 ) .

2X1 ~ N(2 μ ,4 σ2 ) ………………. (1)

X1 +X2 ~ N(µ + µ, σ2+ σ2) ~ N(2 µ, 2σ2) ……………….(2)

Adding 1 & 2, we get

2X1 + (X1 +X2 ) = N(2 μ ,4 σ2 ) + N(2 µ, 2σ2)

= N( 4µ,6 σ2 ) .

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Ans: The Probability of getting value between a and b should be 0.99.

So the Probability of going wrong a and b area is 0.01

The Probability towards left from a = -0.005 (ie. 0.01/2).

The Probability towards right from b = +0.005 (ie. 0.01/2).

So since we have the probabilities of a and b, we need to calculate X, the random variable at a and b which has got these probabilities.

By finding the Standard Normal Variable Z (Z Value), we can calculate the X values.

Z=(X- μ) / σ

For Probability 0.005 the Z Value is -2.57 (from Z Table).

Z \* σ + μ = X

Z(-0.005)\*20+100 = -(-2.57)\*20+100 = 151.5

Z(+0.005)\*20+100 = (-2.57)\*20+100 = 48.5.

**So the option D is correct.**

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

**Ans:** Mean = 5 + 7 = 12

Mean of population in million is 12\*45 = 540 millions

Standard deviation is 32 + 42 = 25

Standard deviation of population in million is (square root of 25)\*45

= 5\*45 = 215 millions

stats.norm.interval(0.95, 540, 215)

range is (99.00810347848784, 980.9918965215122) in Millions

1. Specify the 5th percentile of profit (in Rupees) for the company

**Ans:** To compute 5th Percentile,

we use the formula X=μ + Zσ; where in from z table, 5 percentile = -1.645

X= 540+(-1.645)\*(225) = 170 millions

5th percentile of profit (in Million Rupees) is 170.0

1. Which of the two divisions has a larger probability of making a loss in a given year?

**Ans:** stats.norm.cdf(0,5,3)

0.0477903522728147

stats.norm.cdf(0,7,4)

0.040059156863817086

Probability of Division 1 making a loss in a given year is more than Division 2.