Problem 12–4

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Section A

Scenario	$level_1^s$	$level_2^s$	$level_3^s$	$level_4^s$	Probability
1	100	100	150	150	0.0625
2	100	100	150	50	0.03125
3	100	100	150	350	0.03125
4	100	100	50	150	0.03125
5	100	100	50	50	0.015625
6	100	100	50	350	0.015625
7	100	100	350	150	0.03125
8	100	100	350	50	0.015625
9	100	100	350	350	0.015625
10	100	50	150	150	0.03125
11	100	50	150	50	0.015625
12	100	50	150	350	0.015625
13	100	50	50	150	0.015625
14	100	50	50	50	0.0078125
15	100	50	50	350	0.0078125
16	100	50	350	150	0.015625
17	100	50	350	50	0.0078125
18	100	50	350	350	0.0078125
19	100	250	150	150	0.03125
20	100	250	150	50	0.015625
21	100	250	150	350	0.015625
22	100	250	50	150	0.015625
23	100	250	50	50	0.0078125
24	100	250	50	350	0.0078125
25	100	250	350	150	0.015625
26	100	250	350	50	0.0078125
27	100	250	350	350	0.0078125

Scenario	$level_1^s$	$level_2^s$	$level_3^s$	$level_4^s$	Probability
28	50	100	150	150	0.03125
29	50	100	150	50	0.015625
30	50	100	150	350	0.015625
31	50	100	50	150	0.015625
32	50	100	50	50	0.0078125
33	50	100	50	350	0.0078125
34	50	100	350	150	0.015625
35	50	100	350	50	0.0078125
36	50	100	350	350	0.0078125
37	50	50	150	150	0.015625
38	50	50	150	50	0.0078125
39	50	50	150	350	0.0078125
40	50	50	50	150	0.0078125
41	50	50	50	50	0.00390625
42	50	50	50	350	0.00390625
43	50	50	350	150	0.0078125
44	50	50	350	50	0.00390625
45	50	50	350	350	0.00390625
46	50	250	150	150	0.015625
47	50	250	150	50	0.0078125
48	50	250	150	350	0.0078125
49	50	250	50	150	0.0078125
50	50	250	50	50	0.00390625
51	50	250	50	350	0.00390625
52	50	250	350	150	0.0078125
53	50	250	350	50	0.00390625
54	50	250	350	350	0.00390625

Scenario	$level_1^s$	$level_2^s$	$level_3^s$	$level_4^s$	Probability
55	250	100	150	150	0.03125
56	250	100	150	50	0.015625
57	250	100	150	350	0.015625
58	250	100	50	150	0.015625
59	250	100	50	50	0.0078125
60	250	100	50	350	0.0078125
61	250	100	350	150	0.015625
62	250	100	350	50	0.0078125
63	250	100	350	350	0.0078125
64	250	50	150	150	0.015625
65	250	50	150	50	0.0078125
66	250	50	150	350	0.0078125
67	250	50	50	150	0.0078125
68	250	50	50	50	0.00390625
69	250	50	50	350	0.00390625
70	250	50	350	150	0.0078125
71	250	50	350	50	0.00390625
72	250	50	350	350	0.00390625
73	250	250	150	150	0.015625
74	250	250	150	50	0.0078125
75	250	250	150	350	0.0078125
76	250	250	50	150	0.0078125
77	250	250	50	50	0.00390625
78	250	250	50	350	0.00390625
79	250	250	350	150	0.0078125
80	250	250	350	50	0.00390625
81	250	250	350	350	0.00390625

Section B

- e_t^s: The flood level in period t at scenario s
- s_t^s: The required water to be imported in period t at scenario s

$$\min \quad \sum_{s=1}^{81} p_s \sum_{t=1}^{4} (5000 s_t^s + 10000 e_t^s)$$

$$x_0^s = -150 \quad \forall s \in [1, 2, \ldots, 81]$$

$$x_t^s = x_{t-1}^s + level_t^s - y_t^s \quad \forall s \in [1, 2, \ldots, 81], \forall t \in [1, 2, 3, 4]$$

$$x_t^s - e_t^s \leq 0 \quad \forall s \in [1, 2, \ldots, 81], \forall t \in [1, 2, 3, 4]$$

$$x_t^s + s_t^s \geq -250 \quad \forall s \in [1, 2, \ldots, 81], \forall t \in [1, 2, 3, 4]$$

Nonanticipativity Constraints (t = 1):

$$\begin{aligned} x_1^s = x_1^r, y_1^s = y_1^r \quad & \forall s, r \in [1, 2, \ldots, 27] \\ x_1^s = x_1^r, y_1^s = y_1^r \quad & \forall s, r \in [28, 29, \ldots, 54] \\ x_1^s = x_1^r, y_1^s = y_1^r \quad & \forall s, r \in [55, 56, \ldots, 81] \end{aligned}$$

Nonanticipativity Constraints (t = 2):

$$\begin{aligned} x_2^s = x_2^r, y_2^s = y_2^r \quad & \forall s, r \in [1, 2, \ldots, 9] \\ x_2^s = x_2^r, y_2^s = y_2^r \quad & \forall s, r \in [10, 11, \ldots, 18] \\ \vdots & \\ x_2^s = x_2^r, y_2^s = y_2^r \quad & \forall s, r \in [73, 74, \ldots, 81] \end{aligned}$$

Nonanticipativity Constraints (t = 3):

$$\begin{aligned} x_3^s = x_3^r, y_3^s = y_3^r \quad & \forall s, r \in [1, 2, 3] \\ x_3^s = x_3^r, y_3^s = y_3^r \quad & \forall s, r \in [4, 5, 6] \\ \vdots & \\ x_3^s = x_3^r, y_3^s = y_3^r \quad & \forall s, r \in [79, 80, 81] \end{aligned}$$

$$\begin{aligned} x_t^s urs \quad & \forall t \in [1, 2, 3, 4], \forall s \in [1, 2, \ldots, 81] \\ y_t^s \geq 0 \quad & \forall t \in [1, 2, 3, 4], \forall s \in [1, 2, \ldots, 81] \\ s_t^s \geq 0 \quad & \forall t \in [1, 2, 3, 4], \forall s \in [1, 2, \ldots, 81] \\ e_t^s \geq 0 \quad & \forall t \in [1, 2, 3, 4], \forall s \in [1, 2, \ldots, 81] \end{aligned}$$

Section C

We have the following chance constraint:

$$P(e_t^s + s_t^s \leq 0) \geq 1 - \alpha$$

After converting it into normal form:

$$e_t^s + s_t^s \leq \delta_t^s M \quad \forall s \in [1, 2, \ldots, 81], \forall t \in [1, 2, 3, 4]$$

$$\sum_{s=1}^{81} p^s \delta_t^s \leq \alpha \quad \forall t \in [1, 2, 3, 4]$$

$$\delta_t^s binary \quad \forall t \in [1, 2, 3, 4], \forall s \in [1, 2, \ldots, 81]$$