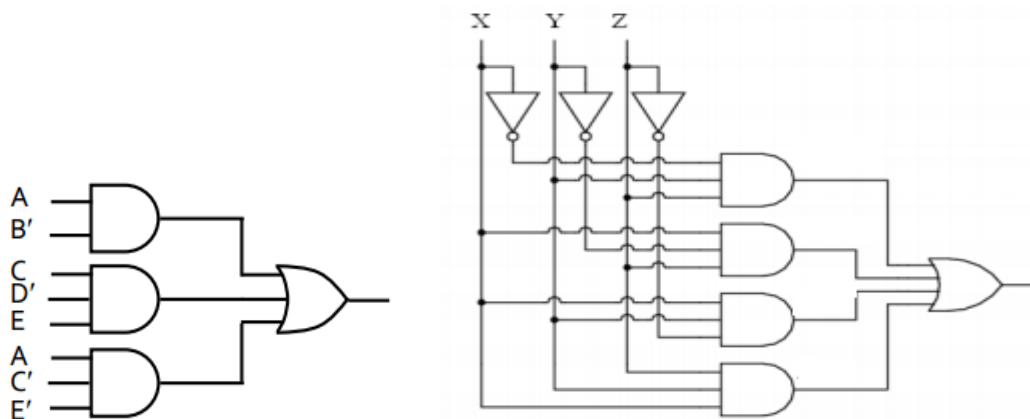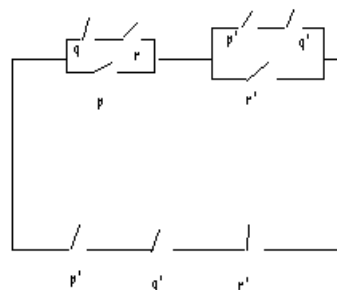| Assignment No. 3 |
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| Title of Course : Discrete Structures |
| Course Code : AML4209 |
| Topic Name : Logics , Lattice and Boolean Algebra |

1. Write the Boolean expression for the following circuits, write the logic table and also write the output of each gate symbolically

2. Use Boolean expression to simplify the switch circuit

3. The following hasse diagrams represent the various focal points in Chandigarh. Help a person to design his tour by finding maximal, minimal, greatest(Last/Unit) element and least(First/Zero) element in the following posets:
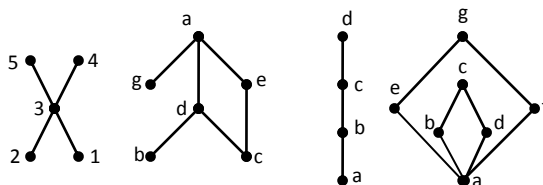
4. A student has used following commands in his program as an arguments.
   Rhombus R is a square or a parallelogram.
   Rhombus R is a parallelogram.
   ∴ Rhombus R is not a Square.
   Determine whether the argument is valid. Justify your answer.

5. A COE of Cipla company faces the problem of wages in his company after listening following argument he reached to a conclusion. Check whether his argument is valid or not?
   "If the labour market is perfect then the wages of all persons in a particular employment will be equal. But it is always the case that wages for such persons are not equal therefore the labour market is not perfect."

6. The weather forecast has announced that "If the sky is cloudy then it will rain and it will not rain", show logically that this statement is not a contradiction.

7. Consider the following open propositions over the universe

   $$U = \{-4, -2, 0, 1, 3, 5, 6, 8, 10\}$$

   $P(x): x \geq 4$, $Q(x): x^2 = 25$, $R(x)$:is a multiple of 2.
   Find the truth values of
   (i) $P(x) \wedge R(x)$ (ii) $P(x) \wedge [\sim Q(x)]$

8. Let A $= \{1, 2, 4, 8, 16\}$ and relation R1 be partial order of divisibility on A. Let
   A'$= \{0, 1, 2, 3, 4\}$ and R2 be the relation "less than or equal to" on integers. Show that (A, R_1) and (A', R_2) are isomorphic lattices.

9. Prove or disprove the following equivalence,

   $$\sim (p \leftrightarrow q) \equiv ((p \wedge \sim q) \vee (q \wedge \sim p))$$

10. Let the relation "divides" be written as a /b iff a divides b or b = ac for some integer c. Draw the Hasse diagram and determine whether D_{12} is a lattice.

11. Show that $(P \vee Q) \wedge (P \rightarrow R) \wedge (Q \rightarrow S)$ is tautology .

12. Construct truth tables for the following:

   $$([p \vee q) \wedge (\sim r)] \leftrightarrow q$$

13. Prove the following Boolean expression:

   $$(x \vee y) \wedge (x \vee \sim y) \wedge (\sim x \vee z) = x \wedge z$$

14. Minimize the Boolean expression (by algebraic method)
   $F = \bar{A}C + \bar{A}B + A\bar{B}C + BC$ and then draw the circuit diagram using only NAND gate.

15. X={2, 3, 6, 12, 24, 36}, R on X ={(x , y) belongs to R, x divides y}
(a) Construct Hasse diagram
(b) Maximal and minimal element?
(c) Is poset a lattice?

16. Consider the lattice D_{60}={1,2,3,4,5,6,10,12,15,20,30,60}, the divisors of 60 ordered by divisibility.
(a) Draw diagram of D_{60}.
(b) Find complements of 2 and 10, if they exist.

17. Find the complement of each element in $D_{105.}$

18. Find the dual of following expressions:
   (X+0)*(Y*1) + 0
   (0*1)+(1+0) *Y

19. Find the atoms of D_{30}, $D_{42.,}$ Boolean Algebra B={(0,0),(1,0),(0,1),(1,1)}.

20. Find the sum-of-products expression for following function, $f(x, y, z) = y + \bar{z}$

21. Consider the following lattice L
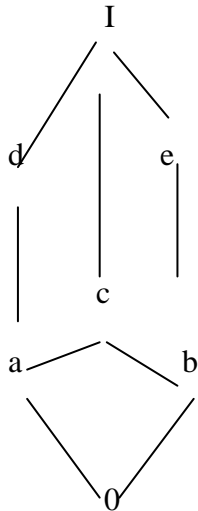
(i) Which of the following are sub lattices of L:
$L_1 = \{0, a, b, I\}$ $L_2 = \{0, a, e, I\}$
$L_3 = \{a, c, d, I\}$ $L_4 = \{0, c, d, I\}$
(ii) Is L distributive?
(iii) Find complements, if they exist, for the elements a,b and c.
(iv) Is L complemented lattice?
(v) Which elements are atoms?

22. Simplify the given expression
$AB + (AC)' + AB'C(AB + C)$

23. Test the validity of argument:
"If it rains tomorrow, I will carry my umbrella, if its cloth is mended. It will rain tomorrow and the cloth will not be mended. Therefore I will not carry my umbrella"

24. Find the converse , inverse and contrapositive of the statement "If 4x-2=10 then x=3"

25. Use switching algebra to simplify the following expressions as much as possible.

(a). $xyz' + xy'z' + x'y$

(b). $(wx')'(w + y)(x'y'z')'$

(c). $x'(y + wy'z') + x'y'(w'z' + z)$

(d). $(w + x)(w' + x + yz')(w + y')$