CHITKARA UNIVERSITY

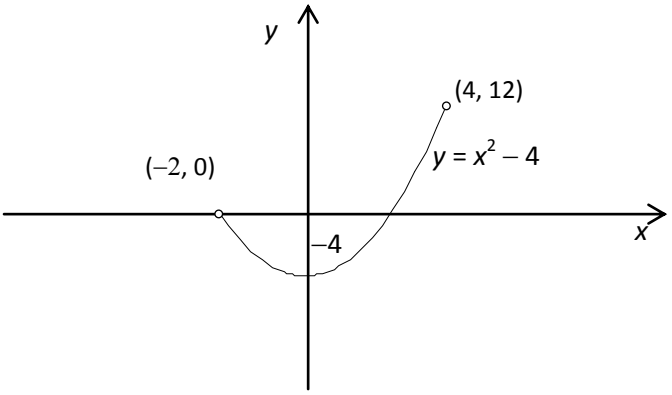
DEPARTMENT OF APPLIED SCIENCES (MATHEMATICS)

Assignment No. 1
Title of Course:Discrete Structures
Course Code: AML4209
Topic Name: Sets, Relations and Functions

1. The loudness of sound measured in decibels (dB) varies inversely as the square of the distance between the listener and the source of the sound. If the loudness of sound is 17.92 dB at a distance of 10 ft from a stereo speaker, what is the decibel level 20 ft from the speaker?
2. A function f is defined on the set of integers as follows

$$f(x) = \begin{cases} 1+x & 1 \le x \le 2 \\ 2x-1 & 2 \le x \le 4 \\ 3x-10 & 4 < x \le 6 \end{cases}$$

   (i) Find the domain and range of the function.
   (ii) Find the value of $f(4)$.
   (iii) State whether f is one-one or many one function.
3. Find the domain and range of the following functions:
   (i) f(x)= $\sqrt{16-x^2}$
   (ii) f(x)= $\frac{1}{(x-1)(x-2)}$
4. The range of the function with graph as shown is:



5. The speed of a racing canoe in still water varies directly as the square root of the length of the canoe.
   (a) If a 16-ft canoe can travel 6.2 mph in still water, find a variation model that relates the speed of a canoe to its length.
   (b) Find the speed of a 25-ft canoe.
6. Write the Sinusoidal function with amplitude 3 and having period 4π. Find its domain and range.
7. What is the sum of all integers from 1 to 100 that are multiples of 2 or 3?
8. A veterinarian surveys 26 of his patrons. He discovers that 14 have dogs, 10 have cats, and 5 have fish. Four have dogs and cats, 3 have dogs and fish, and one has a cat and fish. If no one has all three kinds of pets, how many patrons have none of these pets?

9. Find the period of following

   (a) If 3 complete cycles of the graph will be seen in the standard domain of **0** to 2π in sine or cosine function.
   (b) cos3 x + sin 5 x
   (c) |cos x| + |sin2 x|
   (d) x - |x|.

10. If f(x) = 1 + x; 0 < x < 2
    = 3 - x; 2 < x < 3
    Determine
    (a) g(x) = f(f(x))
    (b) f(f(f(x)))
    (c) f([x])
    (d) [f(x)]
    Where [ ] represents the greatest integer function.

11. In the early morning hours of August 29, 2005, Hurricane Katrina plowed into the Gulf Coast of the United States, bringing unprecedented destruction to southern Louisiana, Mississippi, and Alabama. The kinetic energy of an object varies jointly as the weight of the object at sea level and as the square of its velocity. During a hurricane, a stone traveling at 60 mph has 81 joules (J) of kinetic energy. Suppose the wind speed doubles to 120 mph. Find the kinetic energy.

12. The table gives a relation between a person's age and the person's maximum recommended heart rate.
    (a) What is the domain?
    (b) What is the range?
    (c) The range element 200 corresponds to what element in the domain?
    (d) Complete the ordered pair: (50,).
    (e) Complete the ordered pair: (, 190).

Age (years) x	Maximum Recommended Heart Rate (Beats per Minute) y
20	200
30	190
40	180
50	170
60	160

13. (a) Use a mathematical equation to define a relation whose second component y is 1 less than 2 times the first component x.

    (b) Use a mathematical equation to define a relation whose second component y is 3 more than the first component x.

    (c) Use a mathematical equation to define a relation whose second component is the square of the first component.

14. In a competition, a school awarded medals in different categories. 36 medals in dance, 12 medals in dramatics and 18 medals in music. If these medals went to a total of 45 persons and only 4 persons got medals in all the three categories, how many received medals in exactly two of these categories?

15. Each student in a class of 40 plays at least one indoor game chess, carom and scrabble. 18 play chess, 20 play scrabble and 27 play carom. 7 play chess and scrabble, 12 play scrabble and carom and 4 play chess, carom and scrabble. Find the number of students who play (i) chess and carom. (ii) Chess carom but not scrabble.

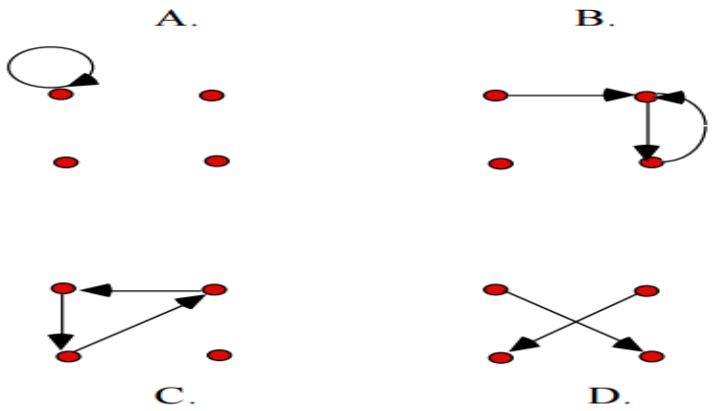
16. Given three sets P, Q and R such that:
    P = {x : x is a natural number between 10 and 16},
    Q = {y : y is a even number between 8 and 20} and
    R = {7, 9, 11, 14, 18, 20}
    (a) Find the difference of two sets P and Q
    (b) Find Q – R
    (c) Find R - P
    (d) Find Q – P
17. Prove that (mod m) is an equivalence relation iff $a \equiv b(\text{mod } m)$ where $a, b \in I$.
18. If R and S be the following relations on A = { 1,2,3 }
    R^{-1} = {(x, y): x + y<3}, S = {(x, y): x-y>0}
    Find (a) RoS
    (b) S^2 = SoS
19. Let R be the relation defined on the set of natural numbers N as
    R = $\{ (x, y) : x \in N, y \in N, 2x + y = 41 \}$
    (a) Find the domain and range of this relation R.
    (b) Write R as a set of ordered pairs.
    (c) Find inverse relation R^{-1} of R.

20. Two finite sets have m and n elements. The total number of subsets of the first set is 56 more than the total number of subsets of the second set. And sum of m and n is 9.Find values of m and n.

21. If $f(x) = x^2 - 1, g(x) = 3x + 1$ then find the following functions:
    (a) gof (b) fog (c) gog

22. Let A={1,2,3,4,5,6,7,8,9,10} .Find all min terms generated by B={4,5,6},C={7,8,9} and D={1,2,3}.

23. Check the following relations A,B,C and D for Reflexive, Symmetric, Antisymmetric and Transitive property?



24. A relation R defined on the set of real numbers as (a, b) R (c, d) iff $a^2 + b^2 = c^2 + d^2$. Show that R is an equivalence relation.
25. Show that the function f and g both of which are from NxN to N given by f(x, y) = x + y and g(x, y) = xy are onto but not one-one.