# Transformers

# Introduction

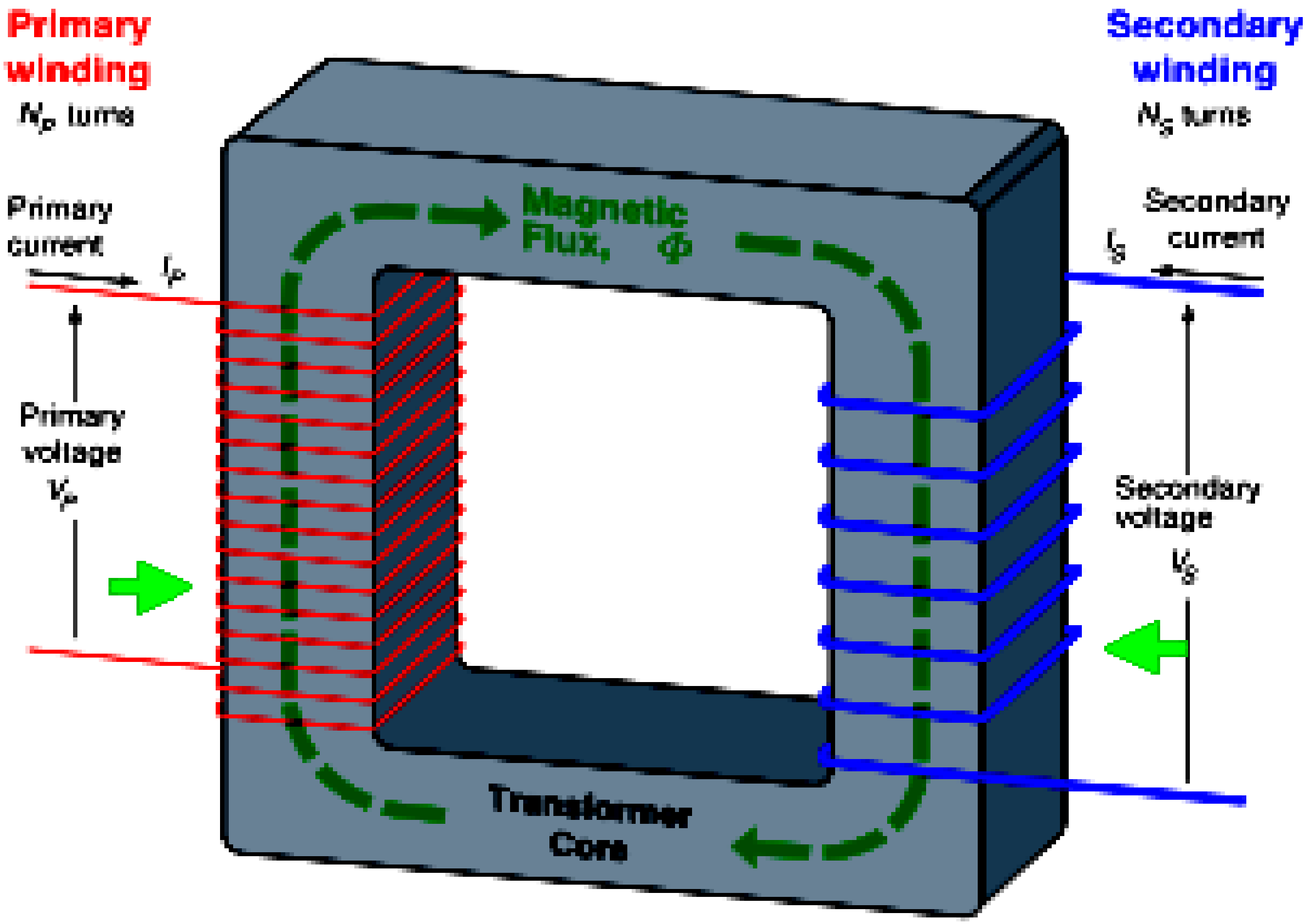
- A transformer is a highly efficient (about 99.5 %) static (non-moving) device.
- It transfers electrical energy form one circuit to another (usually from one ac voltage level to another), without any change in its frequency.
- There exists no simple device that can accomplish such changes in dc voltages.
- Transformation of voltage is necessary at different stages of the electrical network consisting of generation, transmission and distribution.
- Small-size transformers are used in communication circuits, radio and TV circuits, telephone circuits, instrumentation and control systems.

# Principle of Operation

- It operates on the principle of ***mutual induction*** between *two coils.*
    - When two coils are inductively coupled and if current in one coil is changed uniformly, then an EMF gets induced in the other coil.
    - This EMF can drive a current, when a closed path is provided to it.

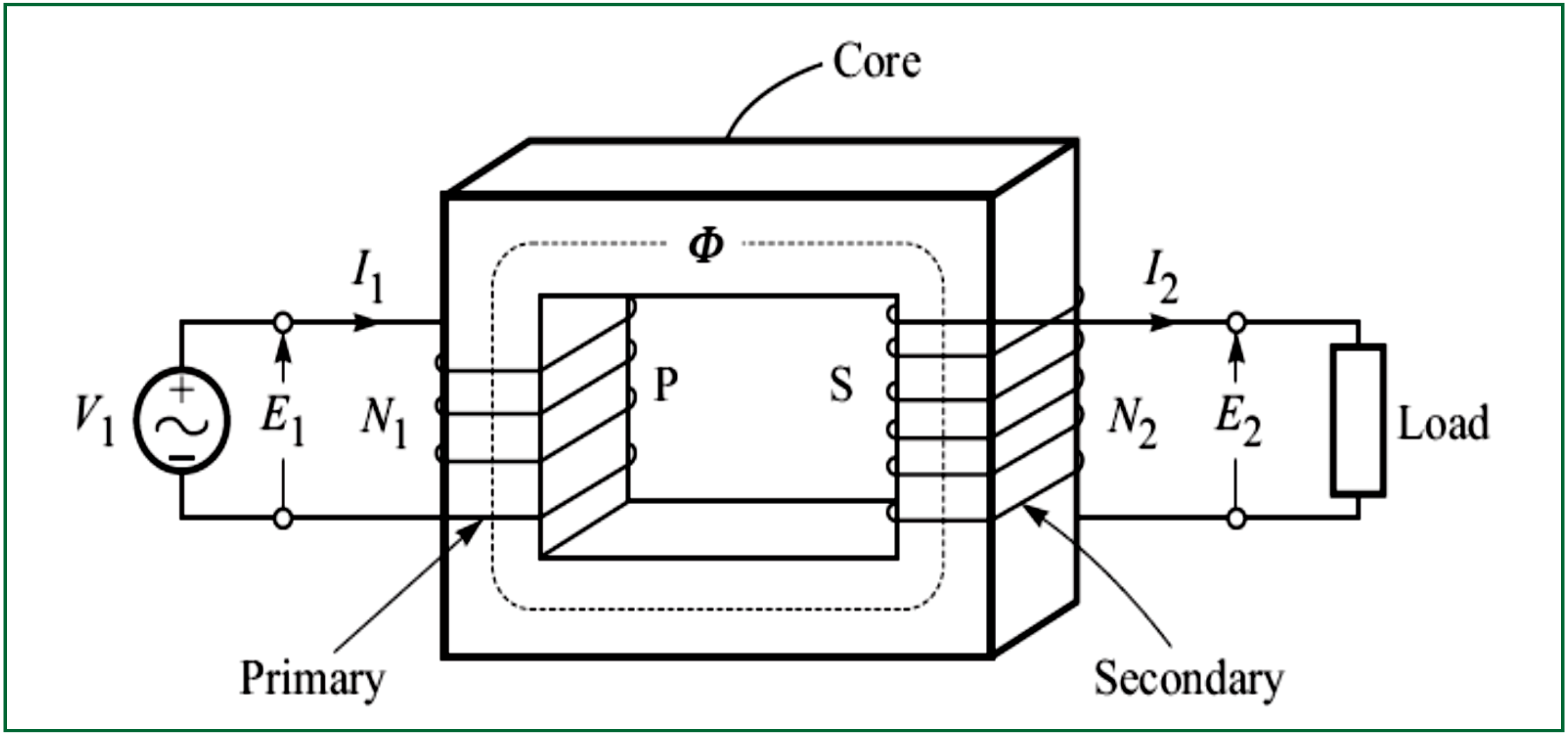
Next

- It consists of two inductive coils electrically separated but magnetically linked through a common magnetic circuit.
- Coil in which electrical energy is fed is Primary Winding.
- Coil in which other load is connected is called as Secondary Winding.



Primary winding
$N_p$ turns
Primary current
$I_p$
Primary voltage
$V_p$
Magnetic Flux, $\Phi$
Transformer Core
Secondary winding
$N_S$ turns
Secondary current
$I_S$
Secondary voltage
$V_S$


Next




(*a*) Construction.

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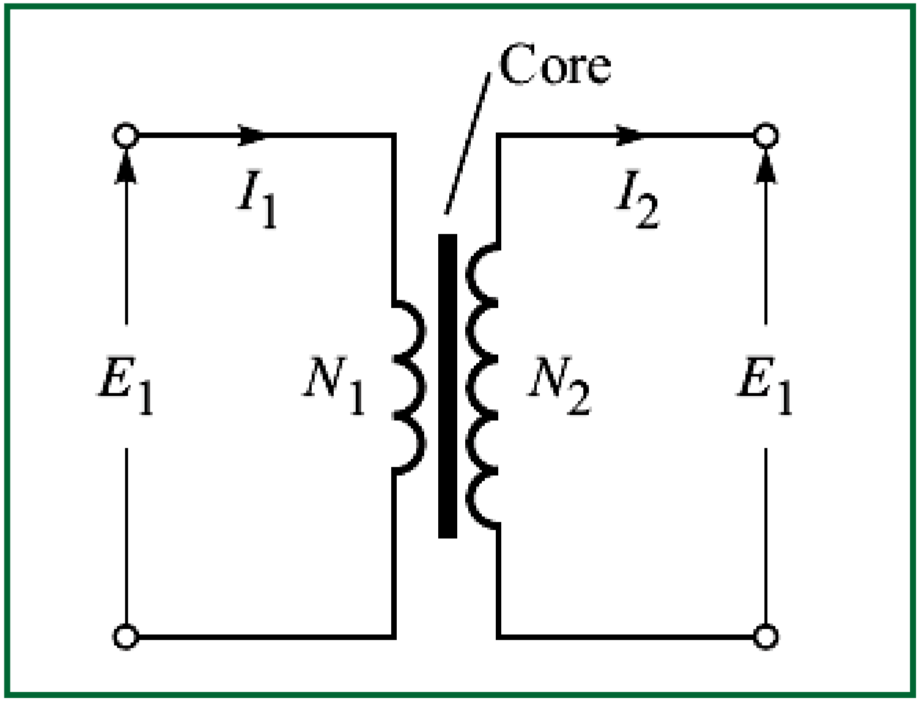


(*b*) Symbol.

- $N_1$ : Number of turns in the Primary
- $N_2$ : Number of turns in the Secondary
- $E_1$ : EMF Induced in the Primary
- $E_2$ : EMF Induced in the Secondary

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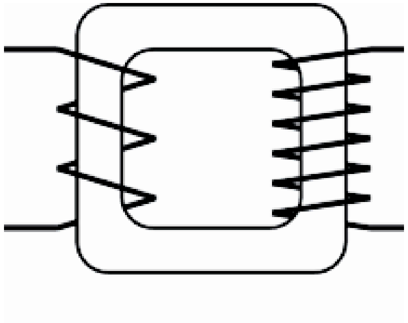
# Step-Up and Step-Down Transformer

If $N_1 < N_2$ ⇨ $E_1 < E_2$ 〉 Step up



If $N_1 > N_2$ ⇨ $E_1 > E_2$ 〉 Step down

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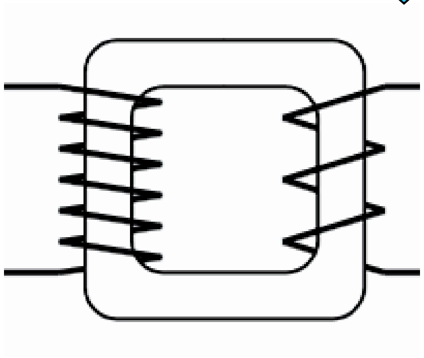
The transformation ratio,

$$\boxed{K = \frac{N_2}{N_1} = \frac{E_2}{E_1}}$$



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# EMF Equation

Due to the sinusoidally varying voltage $V_1$ applied to the primary voltage, the flux set up in the core,

$$\Phi = \Phi_{\mathrm{m}} \sin \omega t = \Phi_{\mathrm{m}} \sin 2\pi f t$$

The resulting induced emf in a winding of $N$ turns,

$$e = -N \frac{d\Phi}{dt} = -N \frac{d}{dt}(\Phi_{\mathrm{m}} \sin \omega t)$$

$$= -N\omega\Phi_{\mathrm{m}} \cos \omega t = \omega N \Phi_{\mathrm{m}} \sin(\omega t - \pi / 2)$$

Thus, the peak value of the induced emf, $E_m = \omega N \Phi m$.

Next

Therefore, the rms value of the induced emf *E*,

$$E = \frac{E_{\mathrm{m}}}{\sqrt{2}} = \frac{\omega N \Phi_{\mathrm{m}}}{\sqrt{2}} = \frac{2\pi f N \Phi_{\mathrm{m}}}{\sqrt{2}} = 4.44\, f N \Phi_{\mathrm{m}}$$

or $$\boxed{E = 4.44\, f N \Phi_{\mathrm{m}}}$$

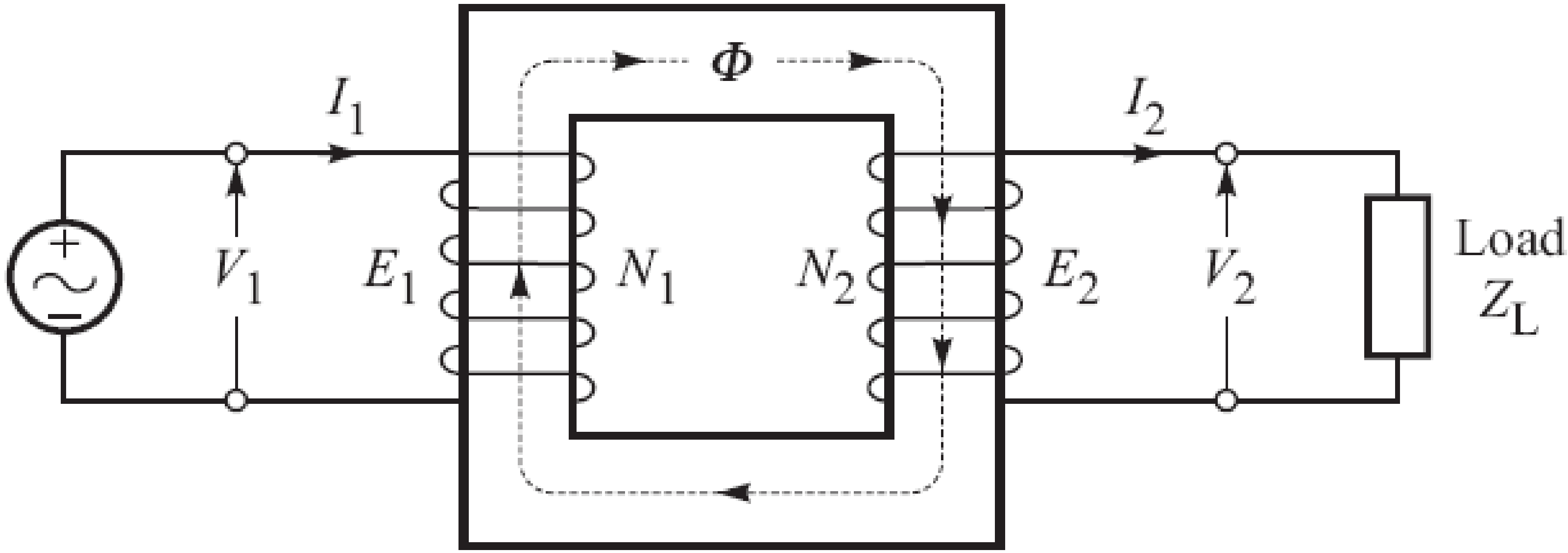
This equation, known as **emf equation** of transformer.

Next

# Conditions for Ideal Transformer :

1. The permeability ($\mu$) of the core is infinite, (i.e., the magnetic circuit has zero reluctance so that no mmf is needed to set up the flux in the core).
2. The core of the transformer has no losses.
3. The resistance of its windings is zero, hence no $I^2R$ losses in the windings.
4. Entire flux in the core links both the windings, i.e., there is no *leakage flux*.

# Ideal transformer




(*a*) The circuit.

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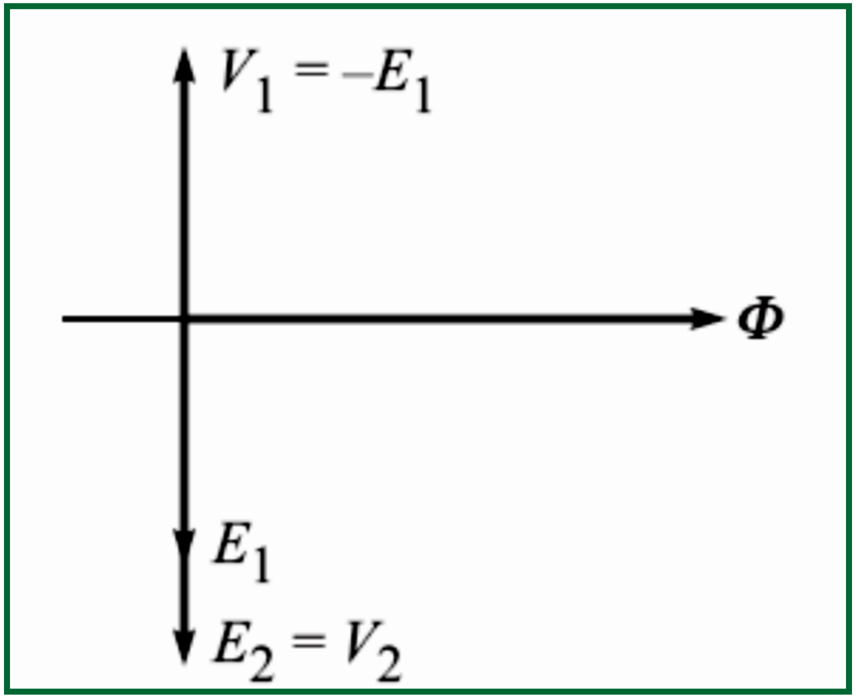


(*b*) The phasor diagram.

- We take flux $\Phi$ as reference phasor, as it is common to both the primary and secondary.
- EMF $E_1$ and $E_2$ lag flux $\Phi$ by 90°.
- The emf $E_1$ in the primary exactly counter balances the applied voltage $V_1$. Hence, $E_1$ is called ***counter emf*** or ***back emf*** .

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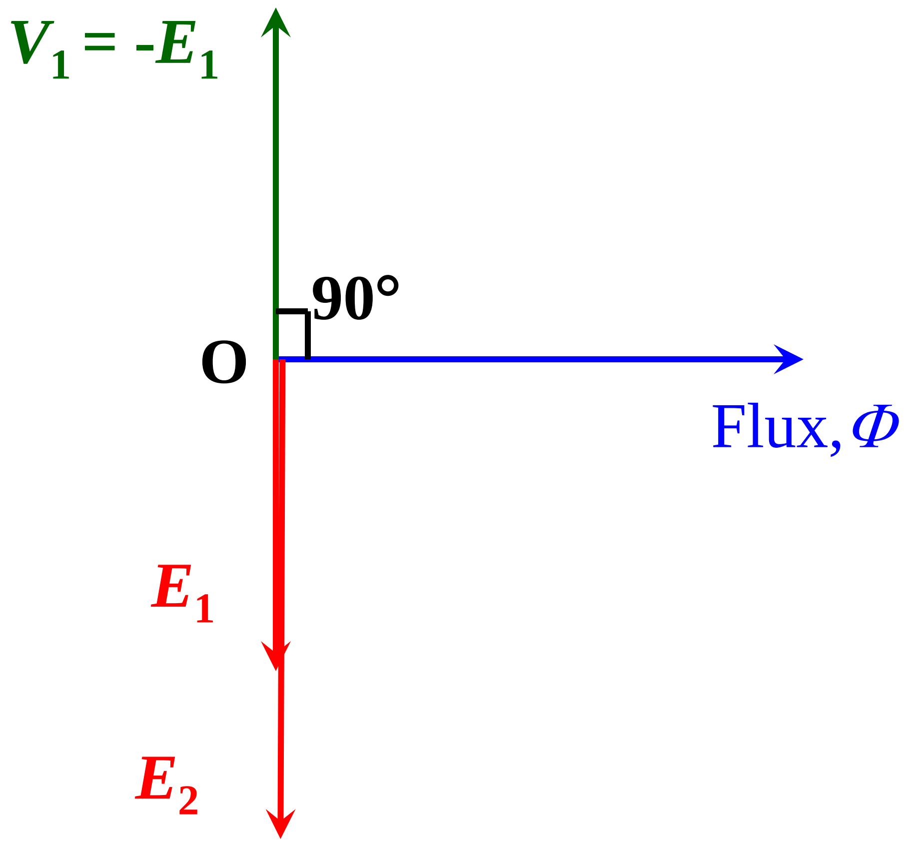
# Drawing the Phasor Diagram

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# Volt-Amperes (in Ideal Transformer)

The current $I_1$ in the primary is just sufficient to provide mmf $I_1N_1$ to overcome the demagnetizing effect of the secondary mmf $I_2N_2$. Hence,

$$I_1N_1 = I_2N_2 \qquad \text{or} \qquad \frac{I_2}{I_1} = \frac{N_1}{N_2} = \frac{1}{K}$$

Note that *the current is transformed in the reverse ratio of the voltage*. If $V_2 > V_1$, then $I_2 < I_1$. Also, we have

$$E_1I_1 = E_2I_2$$

Hence, *in an ideal transformer the input* VA *and output* VA *are identical*.

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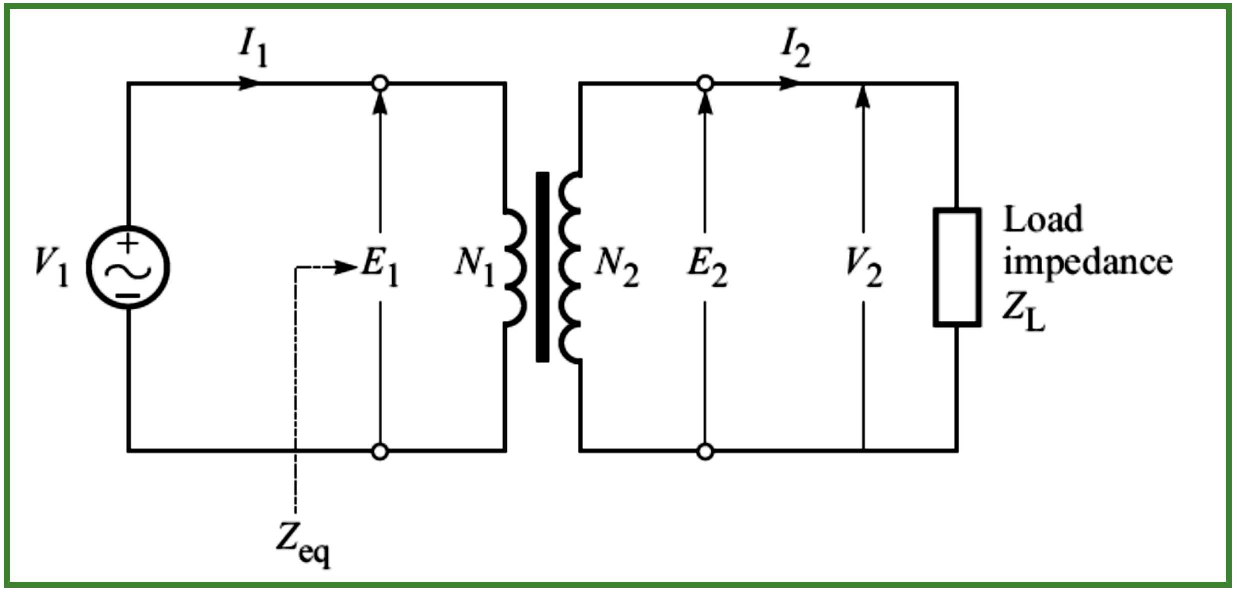
# Impedance Transformation




$$Z_{eq} = \frac{V_1}{I_1} = \frac{V_1 \times (V_2 I_2)}{I_1 \times (V_2 I_2)} = \left(\frac{V_1}{V_2}\right) \times \left(\frac{I_2}{I_1}\right) \times \left(\frac{V_2}{I_2}\right) = \left(\frac{1}{K}\right) \times \left(\frac{1}{K}\right) \times Z_L$$

or $$\boxed{Z_{eq} = Z_L / K^2}$$

The concept of impedance transformation is used for impedance matching.

Next

# Practical Transformer at no Load

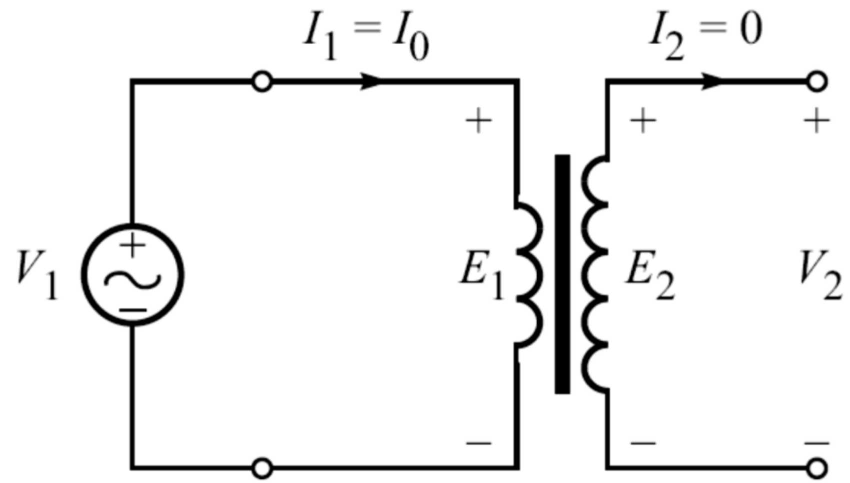
- There are following two reasons why the ***no-load current*** (also called ***exciting current***) $I_0$ is not zero :

**1. Effect of Magnetization :**

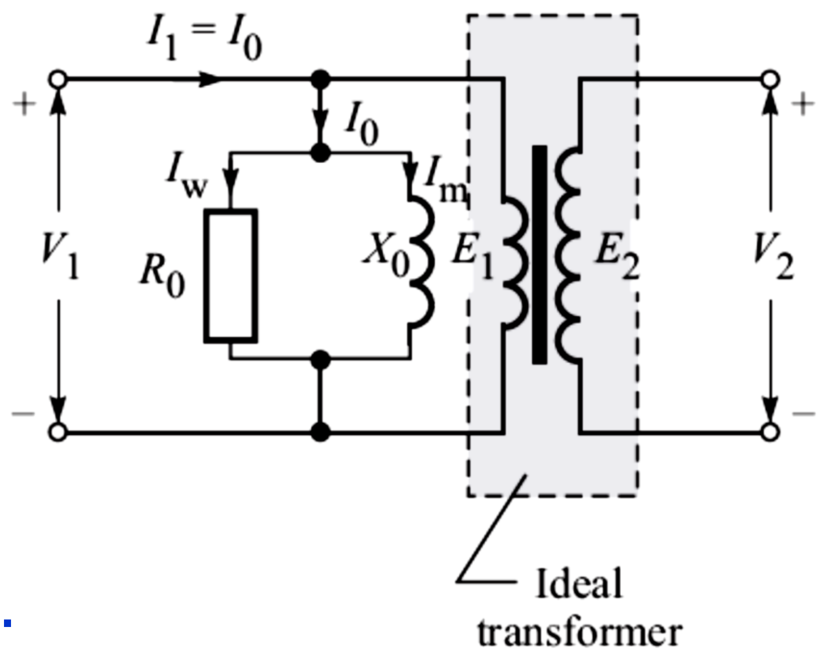
  - No magnetic material can have infinite permeability.
  - A finite mmf is needed to establish magnetic flux in the core.
  - An in-phase ***magnetizing current*** $I_m$ in the primary is needed.
  - $I_m$ is purely reactive (current $I_m$ lags voltage $V_1$ by 90°).
  - This effect is modelled by putting $X_0$ in parallel with the ideal transformer.

## 2. Effect of Core Losses :

- There exist hysteresis and eddy current losses for the energy loss in the core.
- The source must supply enough power to the primary to meet the core losses.
- These iron losses can be represented by putting a resistance $R_0$ in parallel.
- The **core-loss current** $I_w$ flowing through $R_0$ is in phase with the applied voltage $V_1$,




(a) The circuit.




(*b*) The equivalent circuit.

- The $R_0$-$X_0$ circuit is called ***exciting circuit*.**

$$I_0 = \sqrt{I_w^2 + I_m^2}; \qquad \phi_0 = \tan^{-1}(I_m / I_w);$$

$$\text{and} \quad \text{Input power} = \text{Iron loss}$$

$$= V_1 I_w = V_1 I_0 \cos\phi_0$$

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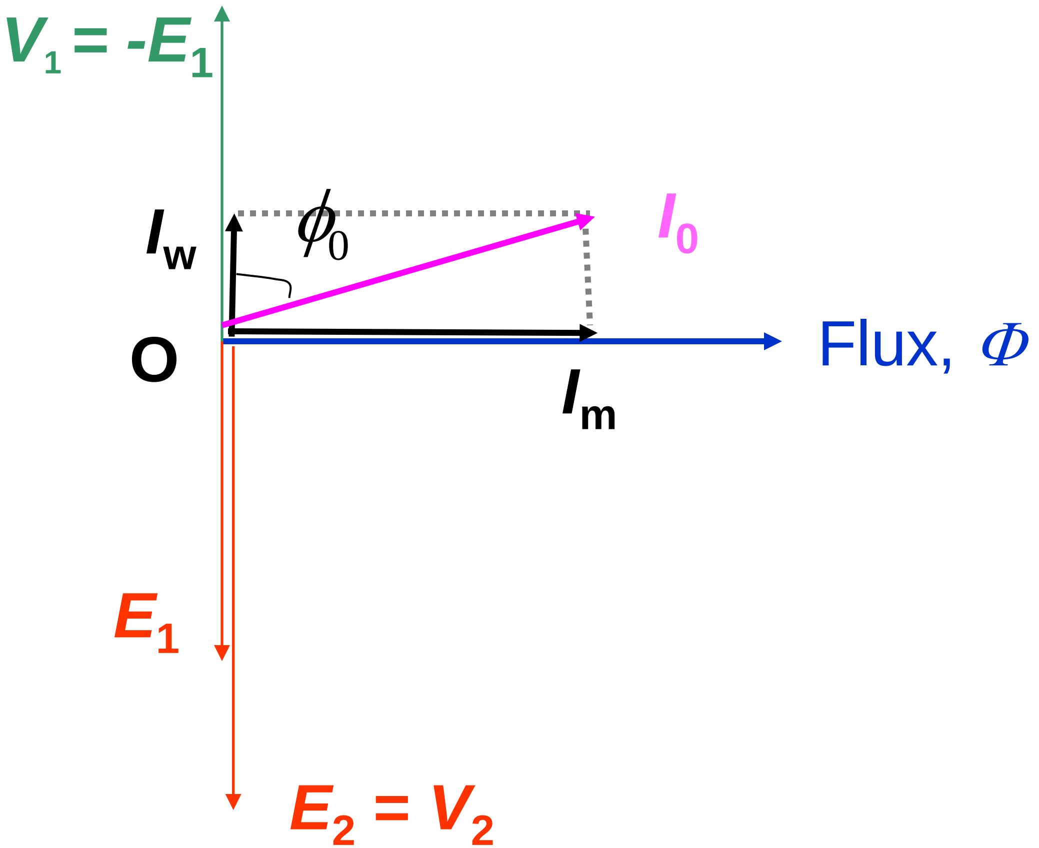
# Modified phasor diagram

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# Iron Losses

- The core losses occur in iron core, hence these are also called iron losses.
- There are two reasons for these losses:

1. Hysteresis Loss .
2. Eddy current loss.

Next

- When alternating current flows through the windings, the core material undergoes cyclic process of magnetization and demagnetization.

$$\boxed{P_{\mathrm{h}} = K_{\mathrm{h}} B_{\mathrm{m}}^{n} f V}$$

$K_{\mathrm{h}}$ = hysteresis coefficient

whose value depends upon the material

($K_{\mathrm{h}}$ = 0.025 for cast steel, $K_{\mathrm{h}}$ = 0.001 for silicon steel)

$B_{\mathrm{m}}$ = maximum flux density (in tesla)

$n$ = a constant, depending upon the material

$f$ = frequency (in hertz)

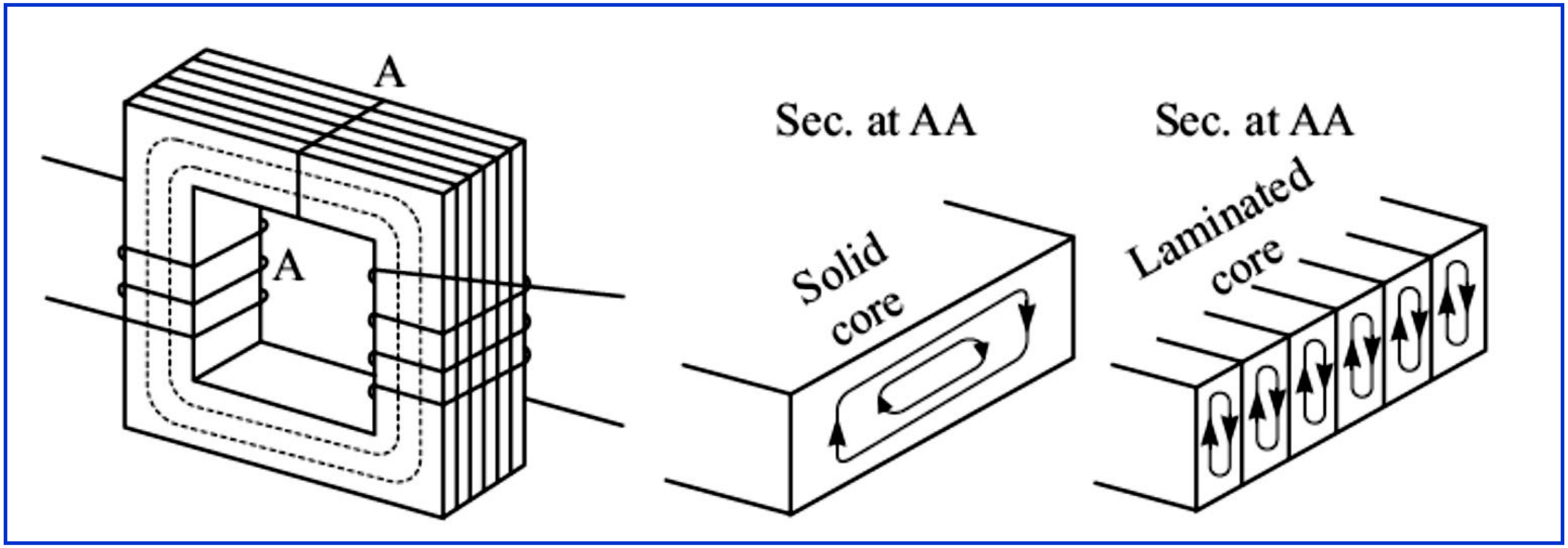
$V$ = volume of the core material (in $\mathrm{m}^3$)

# Eddy current Loss

$$\boxed{P_e = K_e B_m^2 f^2 t^2 V}$$

where $K_e$ = a constant dependent upon the material

$t$ = thickness of laminations (in metre)

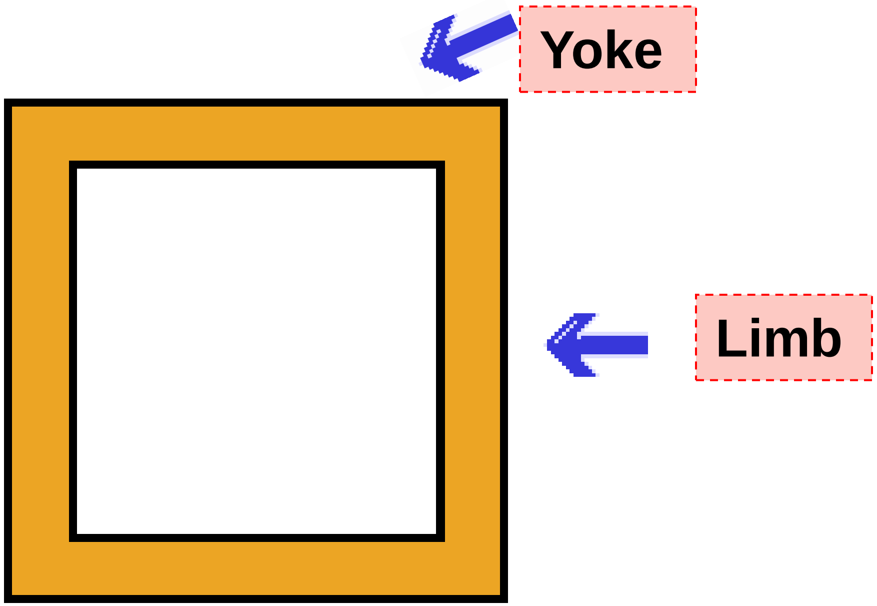



$$\boxed{P_i = P_h + P_e}$$

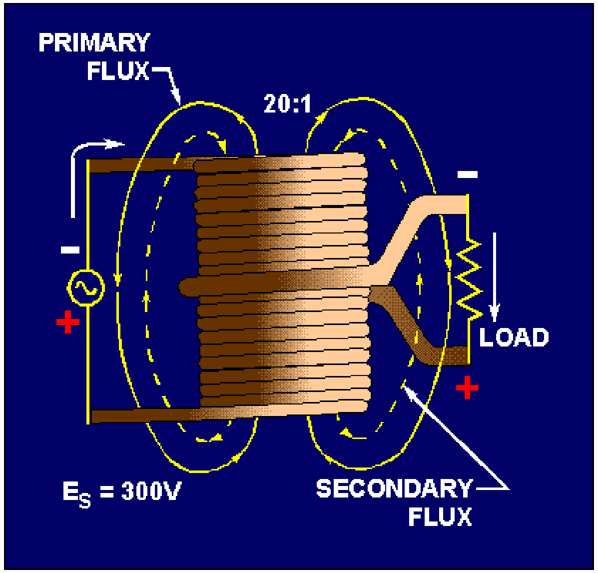
# Construction of a Transformer

- There are two basic parts of a transformer :
  - Magnetic Core
  - Winding or Coils

## Magnetic core




## Winding or coil



# Construction of the Core

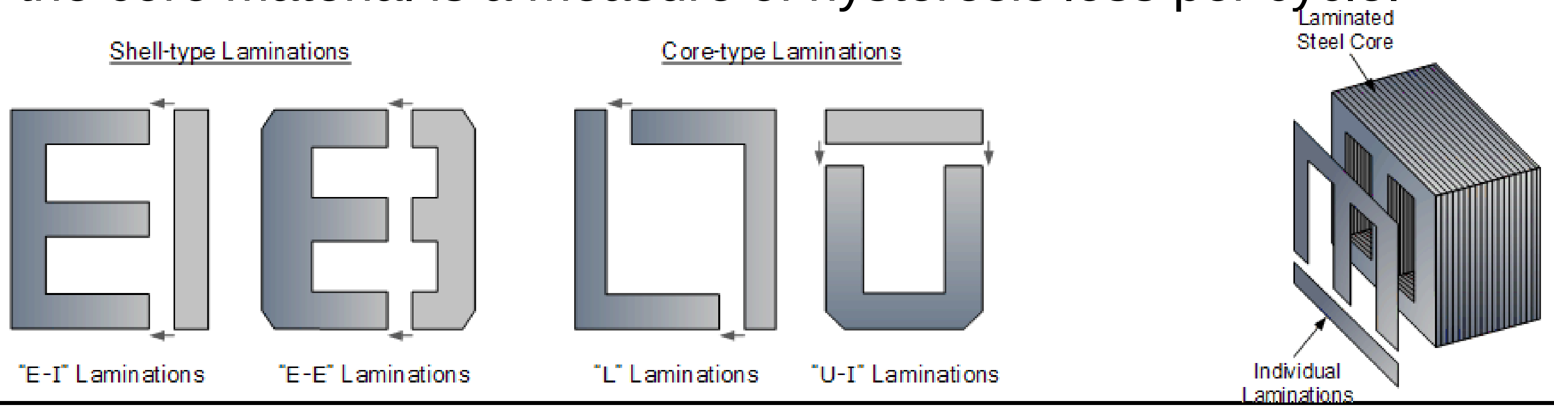
Each winding is wound on a separate limb or core of the soft iron form which provides the necessary magnetic circuit.

This magnetic circuit, know as "transformer core" is designed to provide a path for the magnetic field to flow around, which is necessary for induction of the voltage between the two windings.

The core of the material is constructed using thin plates called lamination to reduce eddy current loss in a material. Each plate is given a varnish coating for providing necessary insulation between the plates.

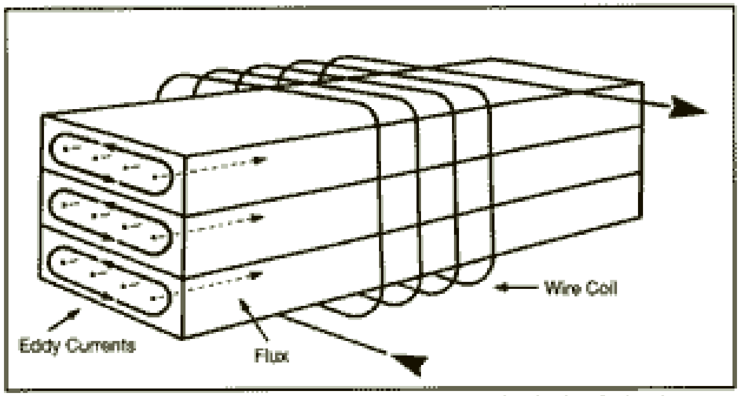
Cold Rolled Grain Oriented, in short CRGO sheets are used to make transformer core.

The area enclosed by the hysteresis loop involving B-H characteristic of the core material is a measure of hysteresis loss per cycle.



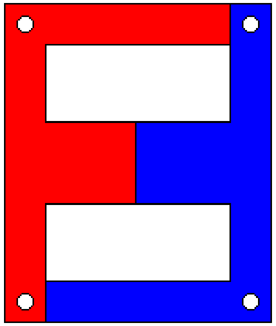
# Laminations

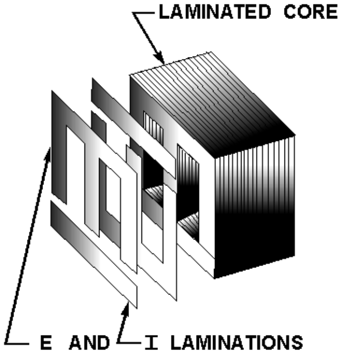
- The core of a transformer is usually laminated to reduce the eddy currents.

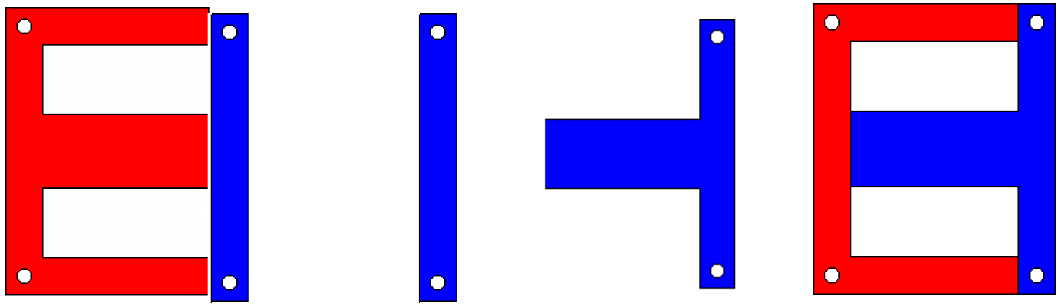


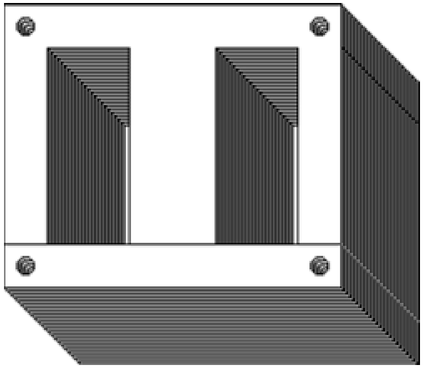

*Alloying with manganese, silicon and aluminum increases iron's electrical resistance to eddy currents*

- These laminations may be different sections of E,I,T,F.
- They are stacked finally to get the complete core of the transformer.







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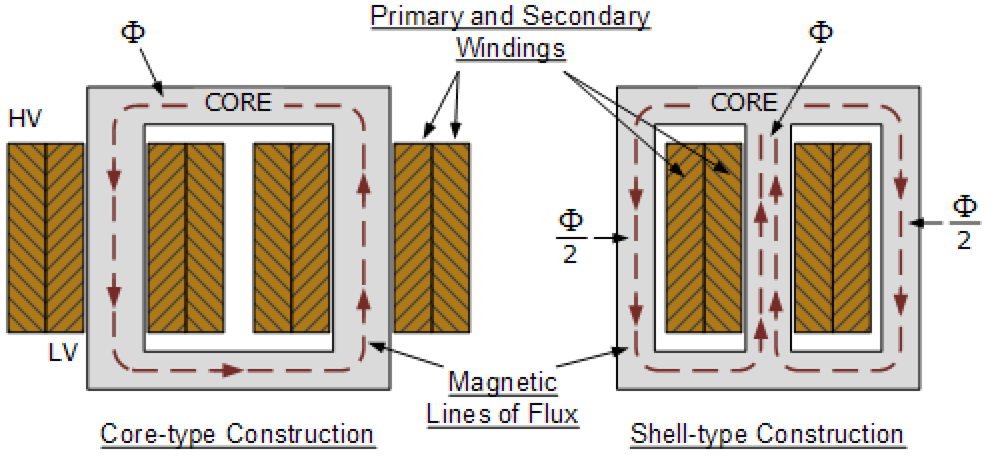
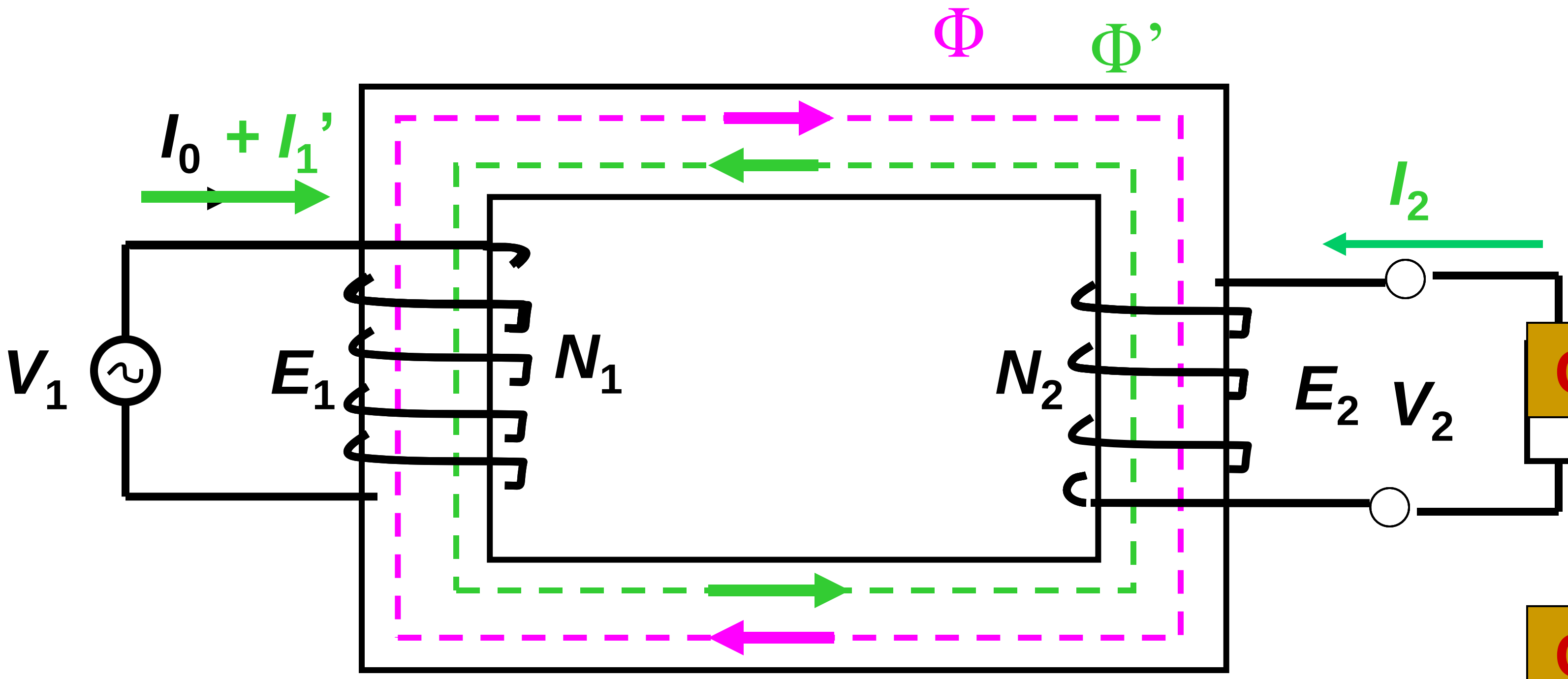
In the core type transformer, the primary and secondary windings are wound outside and surround the core ring.
In the shell type transformer, the primary and secondary windings pass inside the steel magnetic circuit (core) which forms a shell around the windings as shown below.



# How $I_0$ changes on Loading ?

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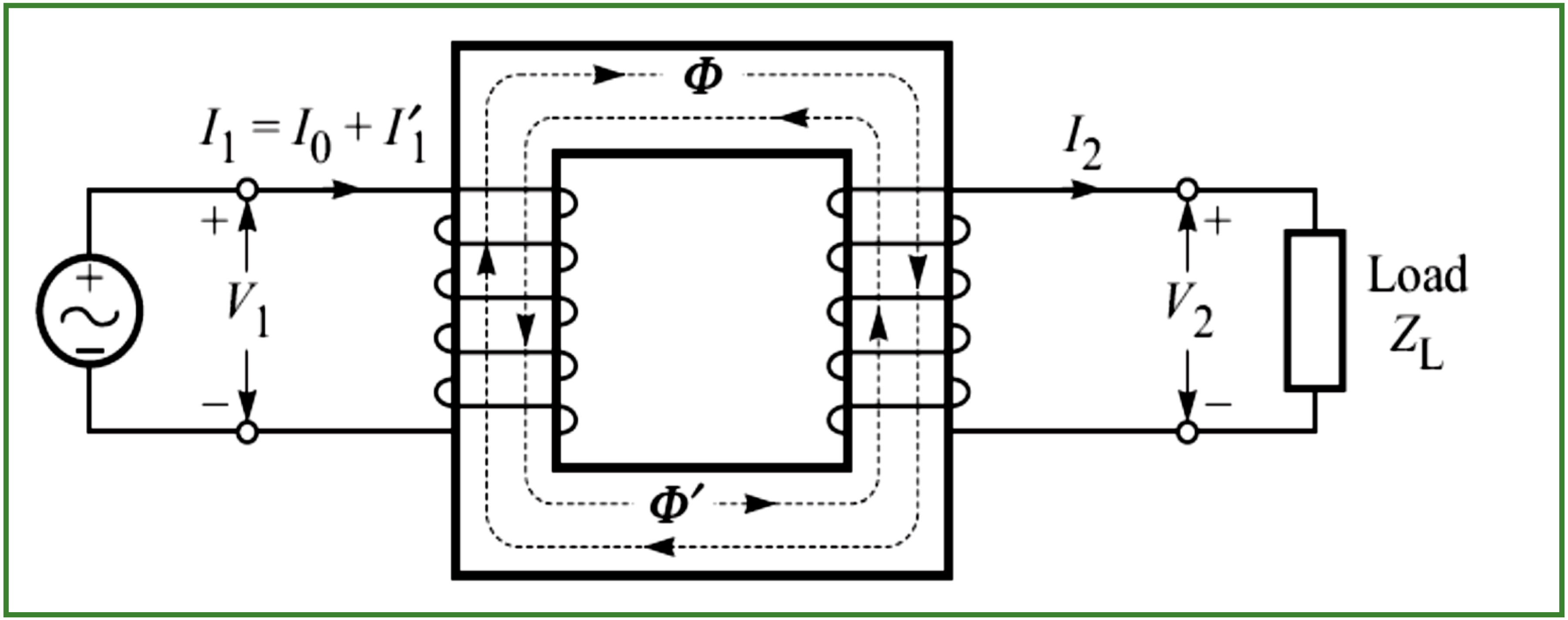


Next

- Before connecting the load, there exists a flux $\Phi$ requiring current $I_0$ in the primary.
- On connecting the load, a current $I_2$ flows in the secondary.
- The magnitude and phase of $I_2$ with respect to $V_2$ depends upon the nature of the load.
- The current $I_2$ sets up a flux $\Phi$', which opposes the main flux $\Phi$. Hence, it is called demagnetizing flux.
- This momentarily weakens $\Phi$, and back emf $E_1$ gets reduced.
- As a result, $V_1$ - $E_1$ increases and more current is drawn from the supply.
- This again increases $E_1$ to balance the applied voltage $V_1$.

- In this process, the primary current increases by $I_1$'.
- This current is known as ***primary balancing current***, or ***load component of primary current***.
- Under such a condition, the secondary ampere-turns must be counterbalanced by the primary ampere-turns.

$$N_1 I_1^{'} = N_2 I_2$$

$$\Rightarrow \quad I_1^{'} = \left( \frac{N_2}{N_1} \right) I_2 = K I_2 \quad \text{and} \quad \mathbf{I}_1 = \mathbf{I}_0 + \mathbf{I}_1^{'}$$

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# Volt Ampere Rating of a Transformer

- *Output power depends on* $\cos\phi_2$ ( power factor of secondary).
- As *pf* can change depending on the load, the rating is not specified in watts or kilowatts.
- But is indicated as a product of voltage and current called VA RATING.

Fore ideal transformer :

$$\boxed{V_1 I_1 = V_2 I_2}$$

$$kVA\,rating\,of\;\;a\,transformer = \frac{V_1 I_1}{1000} = \frac{V_2 I_2}{1000}$$

$$\boxed{I_{1\ \text{(full load)}} = \frac{\text{kVA rating} \times 1000}{V_1}}$$

$$\boxed{I_{2\ \text{(full load)}} = \frac{\text{kVA rating} \times 1000}{V_2}}$$

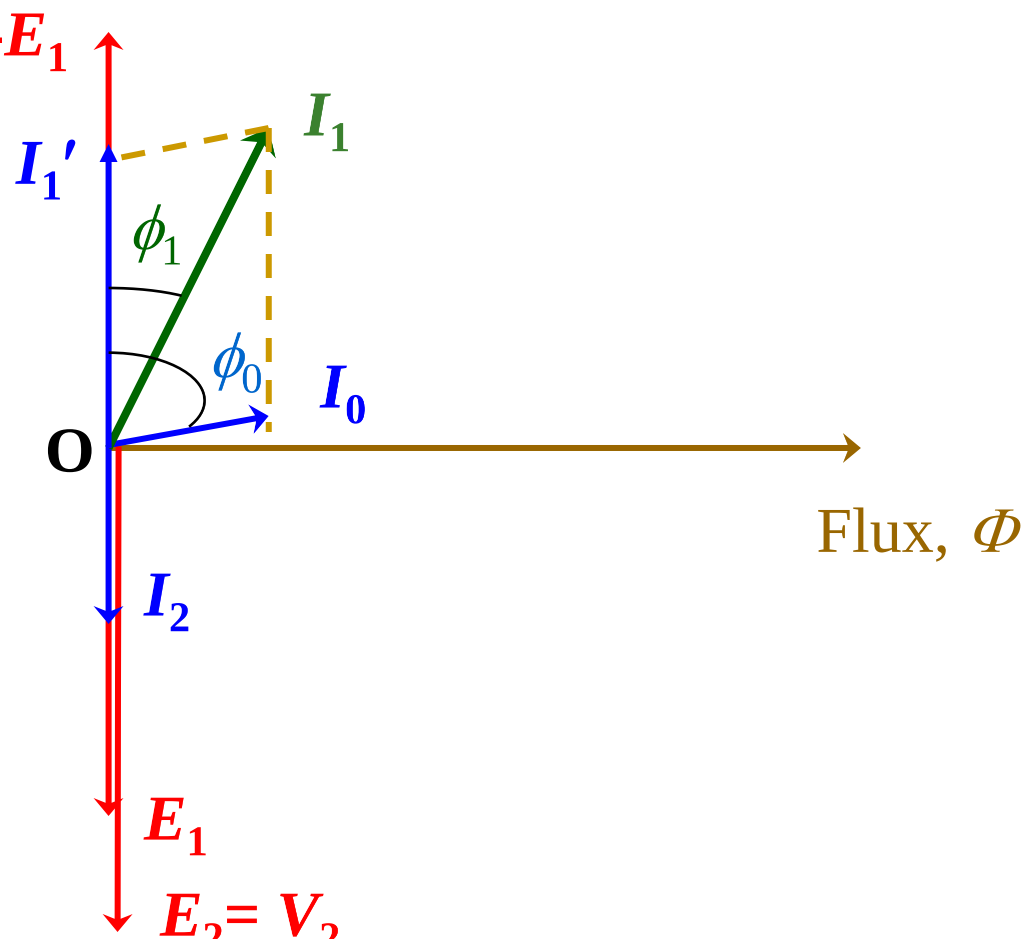
Next

# Transformers rating in kVA ?

- Transformers are rated in VA, because the manufacturer does not know the power factor of the load which you are going to connect.
- So the customer should not exceed the VA rating of the transformer.
- In case of motors, the manufacturer knows exactly the power factor at full load.
- That is why motors are rated in kW.

Next

# Phasor Diagram for **Resistive Load**

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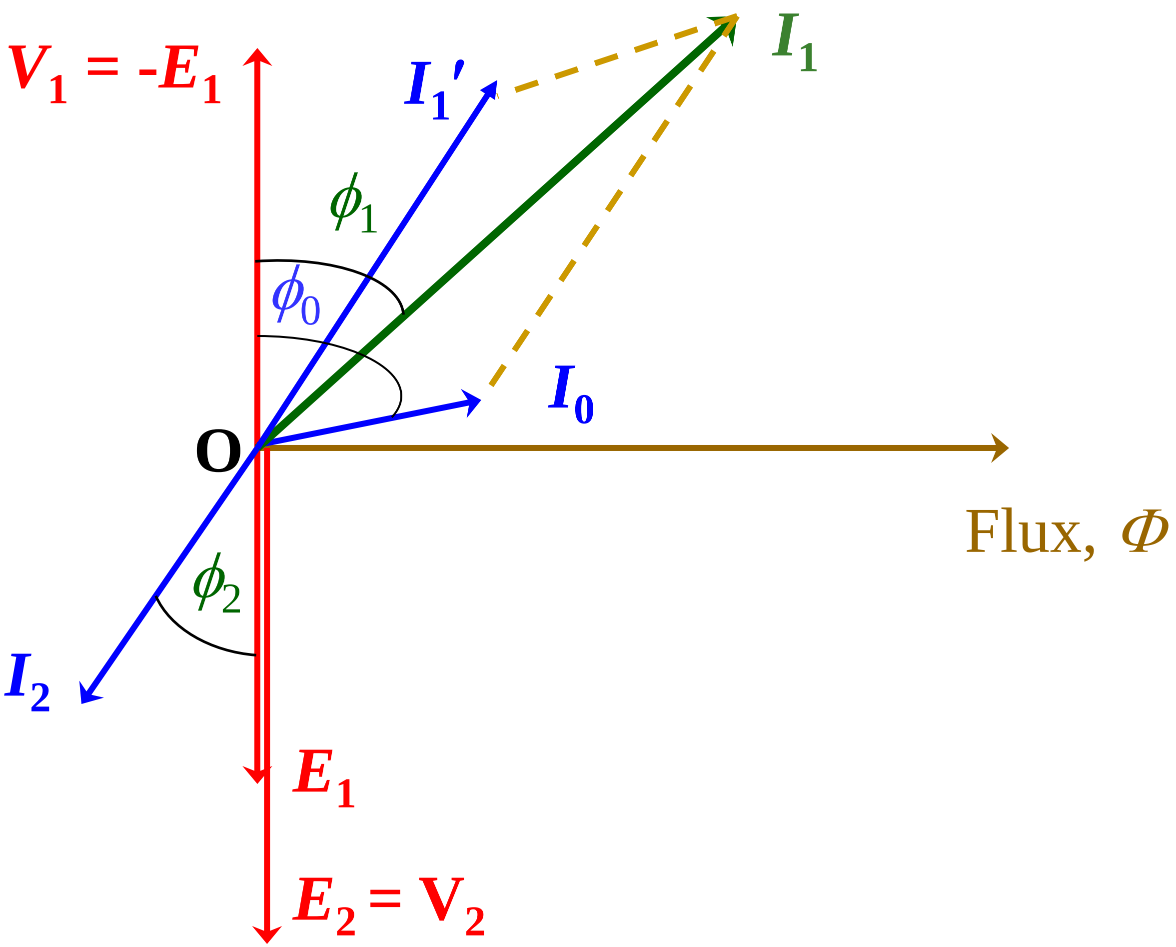
# Phasor Diagram for **Inductive Load**

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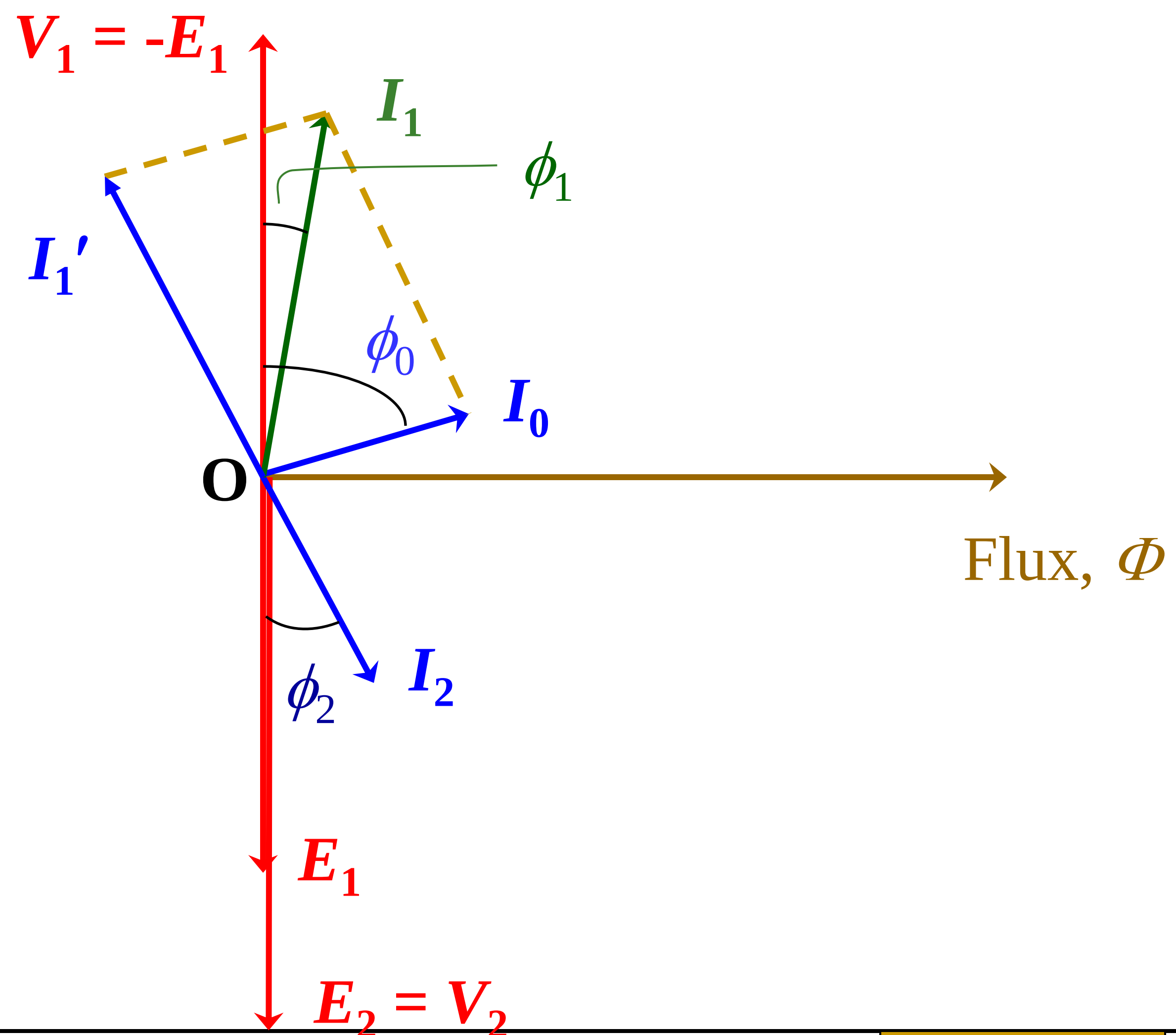
# Phasor Diagram for CAPACITIVE Load

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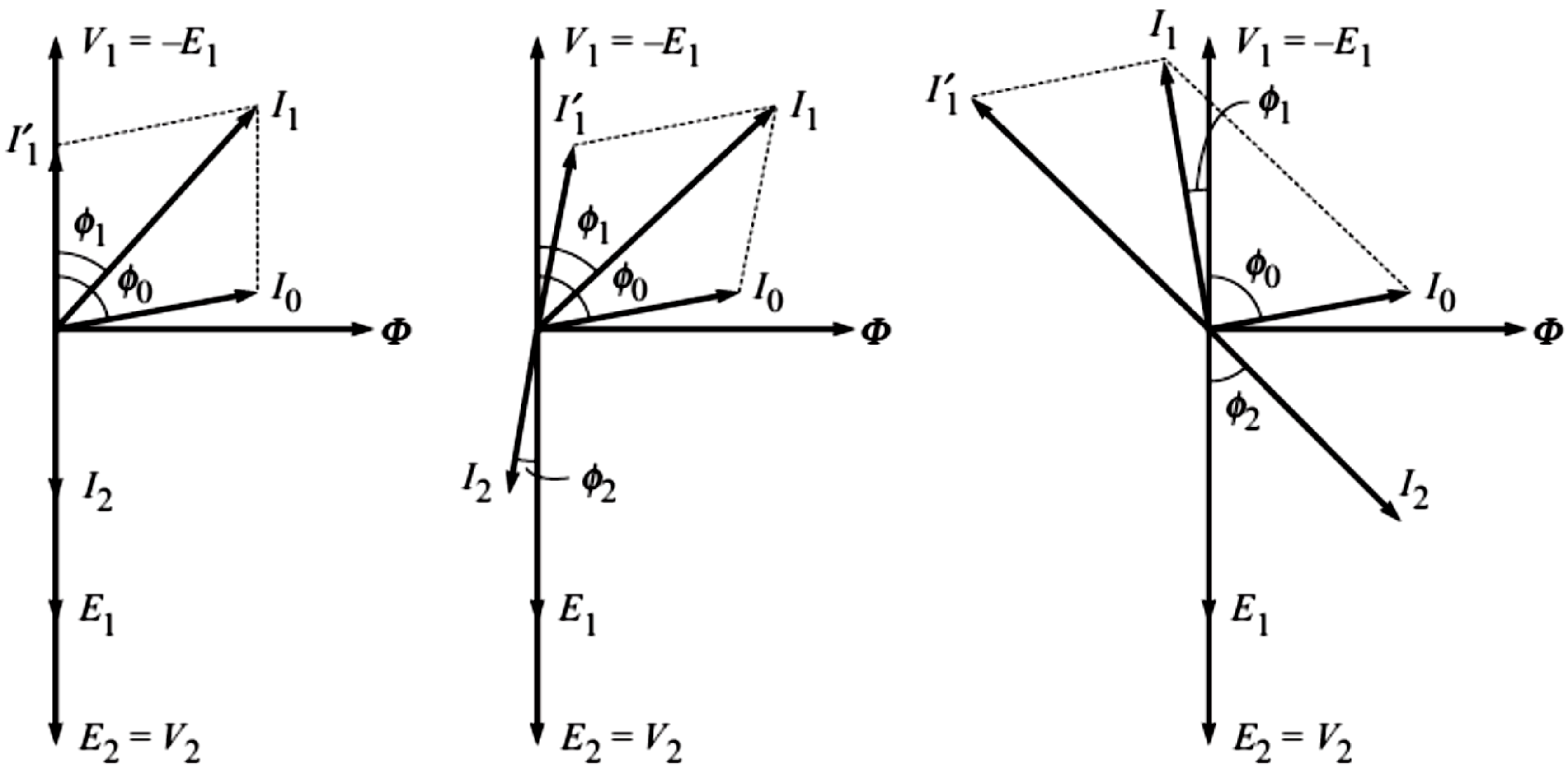
# Phasor Diagrams for Different Types of Loads




(*a*) Resistive. (*b*) Inductive. (*c*) Capacitive.

Next

- Is it ever possible that the load connected to the secondary is capacitive but the overall power factor is inductive ?

Click

- Ans. : Yes. See the phasor diagram for capacitive load.
- Is it ever possible that the load connected to the secondary is inductive but the overall power factor is capacitive?

Click

- Ans. : No. Not possible.

Next

# Practical Transformer on Load

- We now consider the deviations from the last two *ideality conditions* :

1. The resistance of its windings is zero.
2. There is no *leakage flux*.

- The effects of these deviations become more prominent when a practical transformer is put on load.

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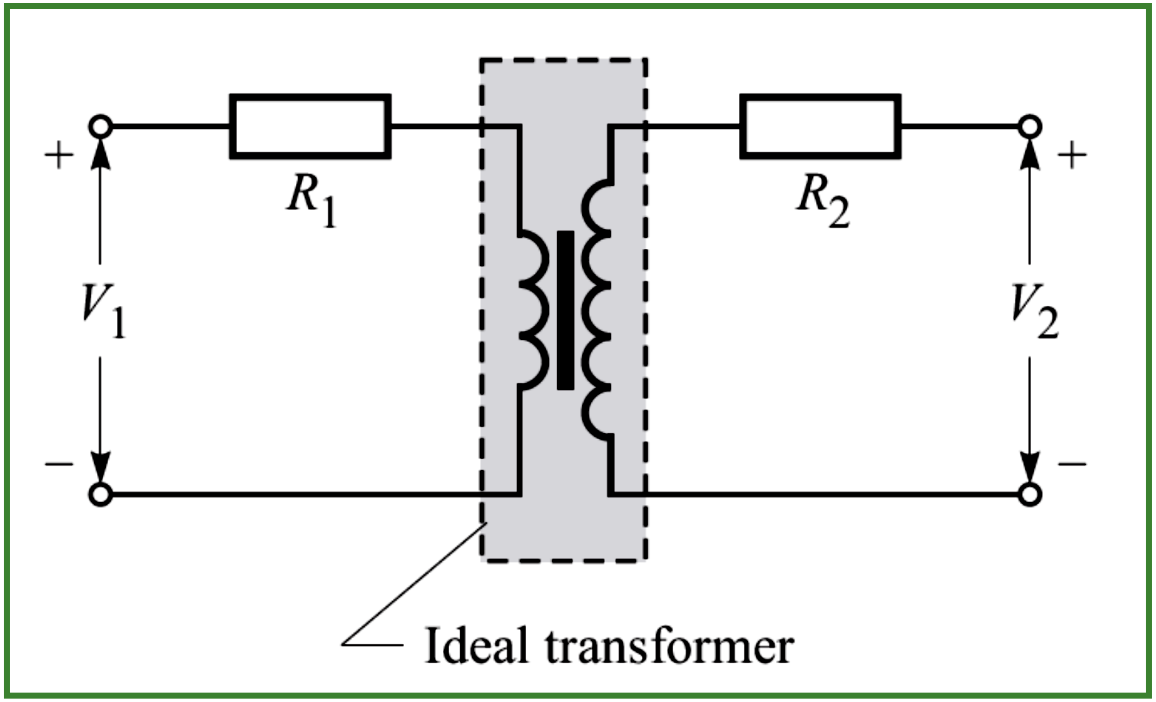
# (1) Effect of Winding Resistance

- Current flow through the windings causes a power loss called *$I^2R$ loss* or *copper loss*.
- This effect is accounted for by including a resistance $R_1$ in the primary and resistance $R_2$ in the secondary

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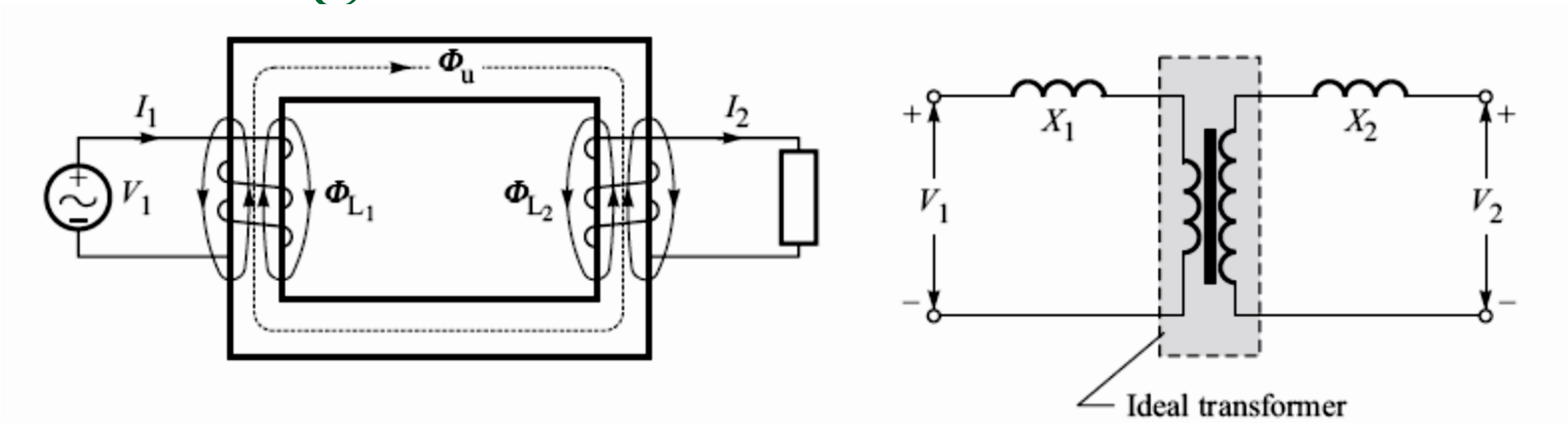
# Leakage flux in a transformer




(*a*) Its definition. (*b*) Its effect accounted for.

- The useful mutual flux $\Phi_u$ is responsible for the transformer action.
- The leakage flux $\Phi_{L1}$ induces an emf $E_{L1}$ in the primary winding.

Next

- Similarly, flux $\Phi_{L2}$ induces an emf $E_{L2}$ in the secondary.
- Hence, we include reactances $X_1$ and $X_2$ in the primary and secondary windings, in the equivalent circuit.
- The paths of leakage fluxes $\Phi_{L1}$ and $\Phi_{L2}$ are almost entirely due to the long air paths and are therefore practically constant.
- The reluctance of the paths being very high, $X_1$ and $X_2$ are relatively small even on full load.
- However, the useful flux $\Phi_u$ remains almost independent of the load.

Next

# Equivalent Circuit of Transformers

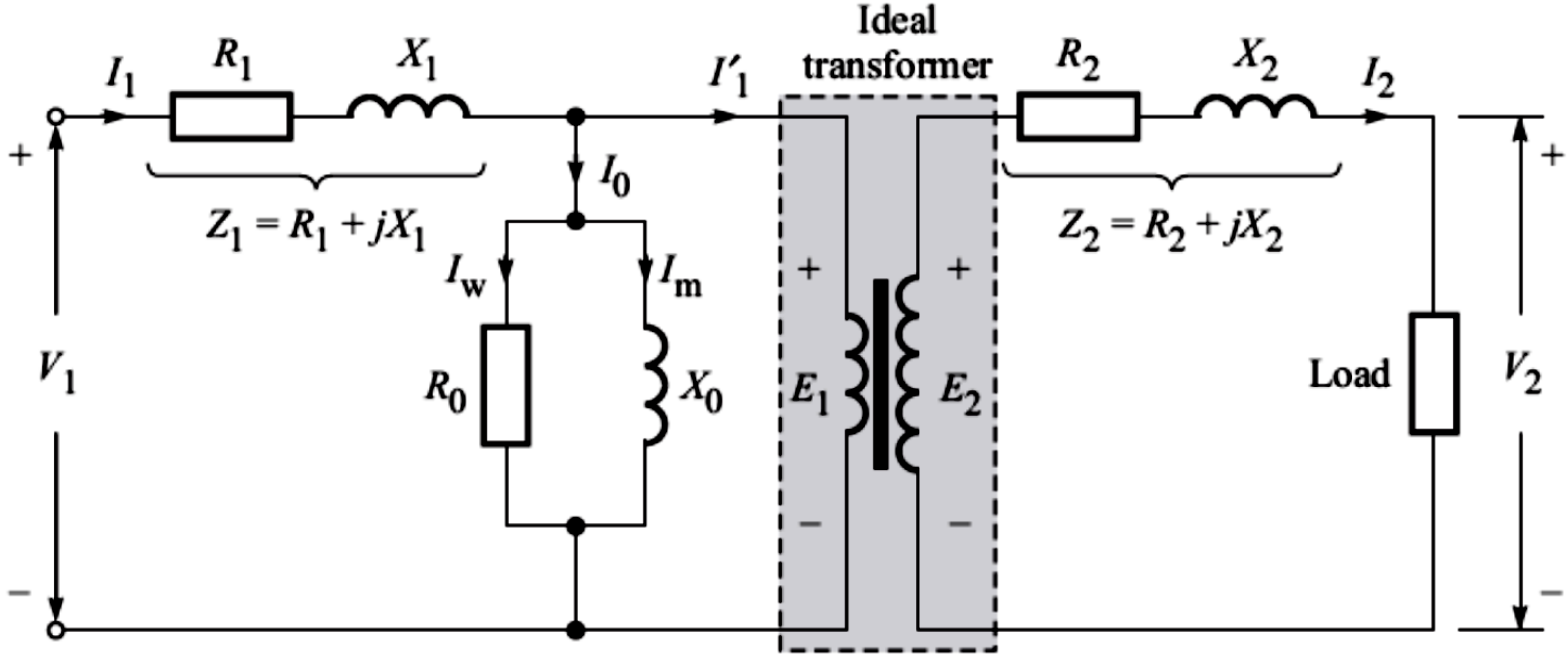
Next

# Equivalent Circuit of a Transformer

It is merely a representation of the following KVL equations :

$$\mathbf{V}_1 = I_1 R_1 + jI_1 X_1 - \mathbf{E}_1 = I_1 (R_1 + jX_1) - \mathbf{E}_1$$

$$\mathbf{E}_2 = I_2 R_2 + jI_2 X_2 + \mathbf{V}_2 = I_2 (R_2 + jX_2) + \mathbf{V}_2$$

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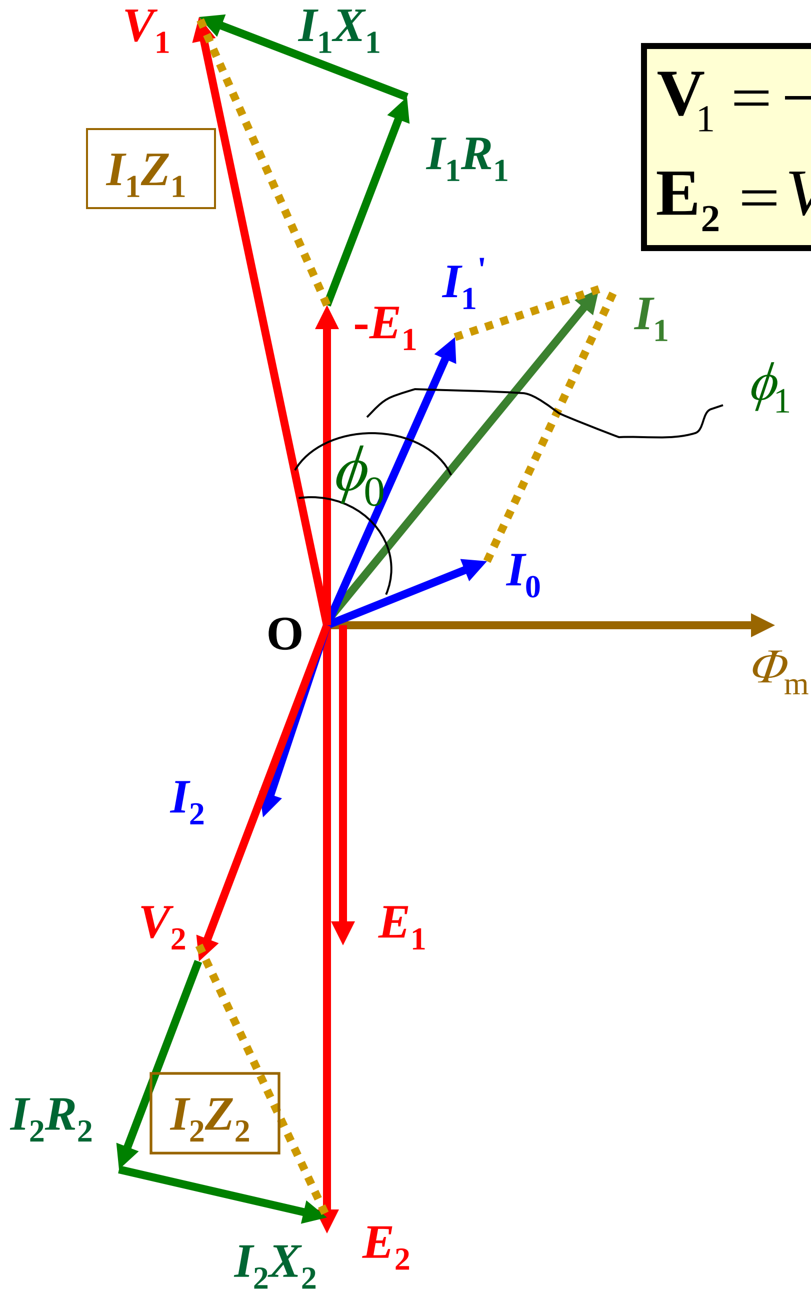
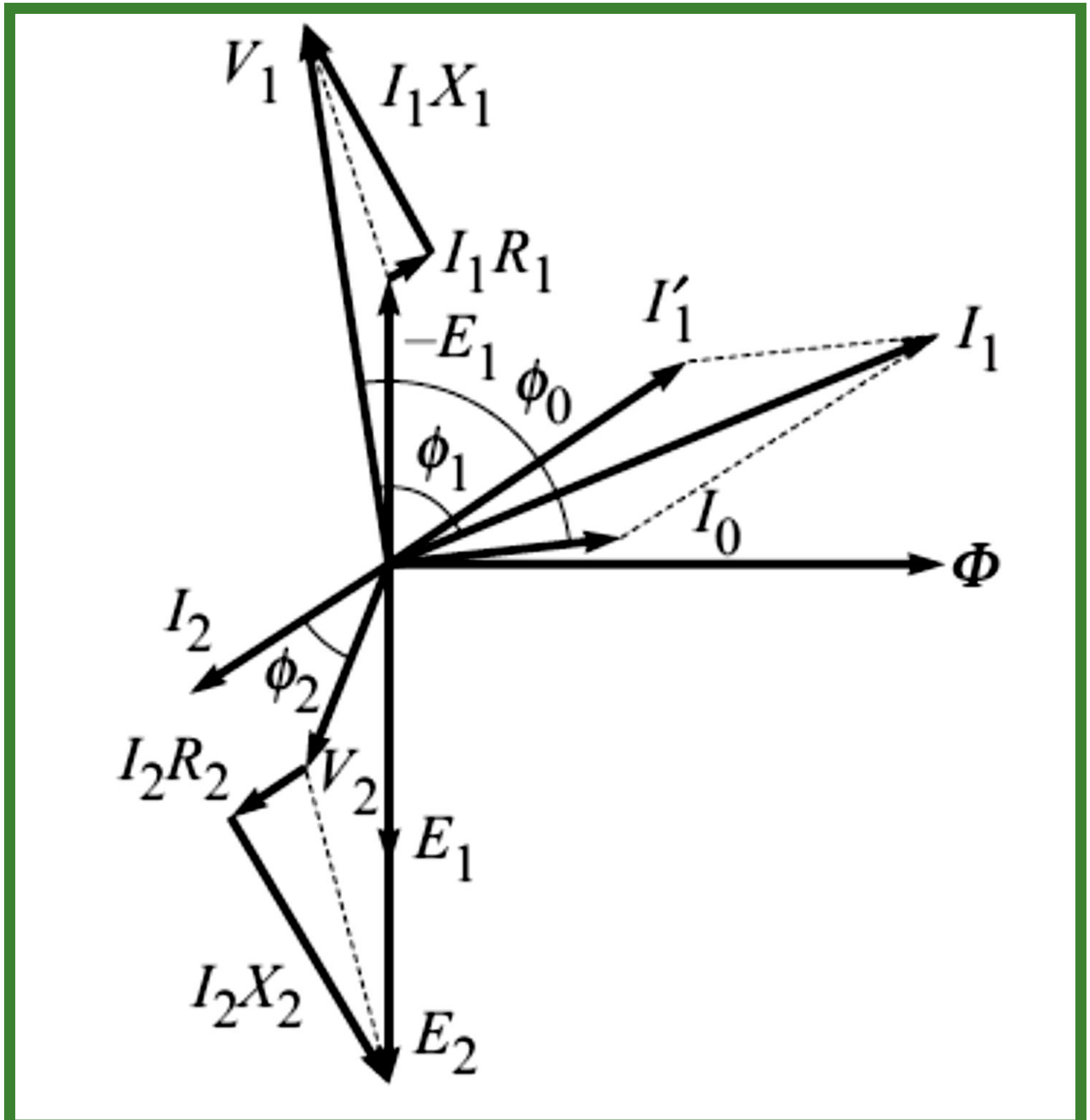
# Phasor Diagram for Practical Transformer on **Resistive Load**



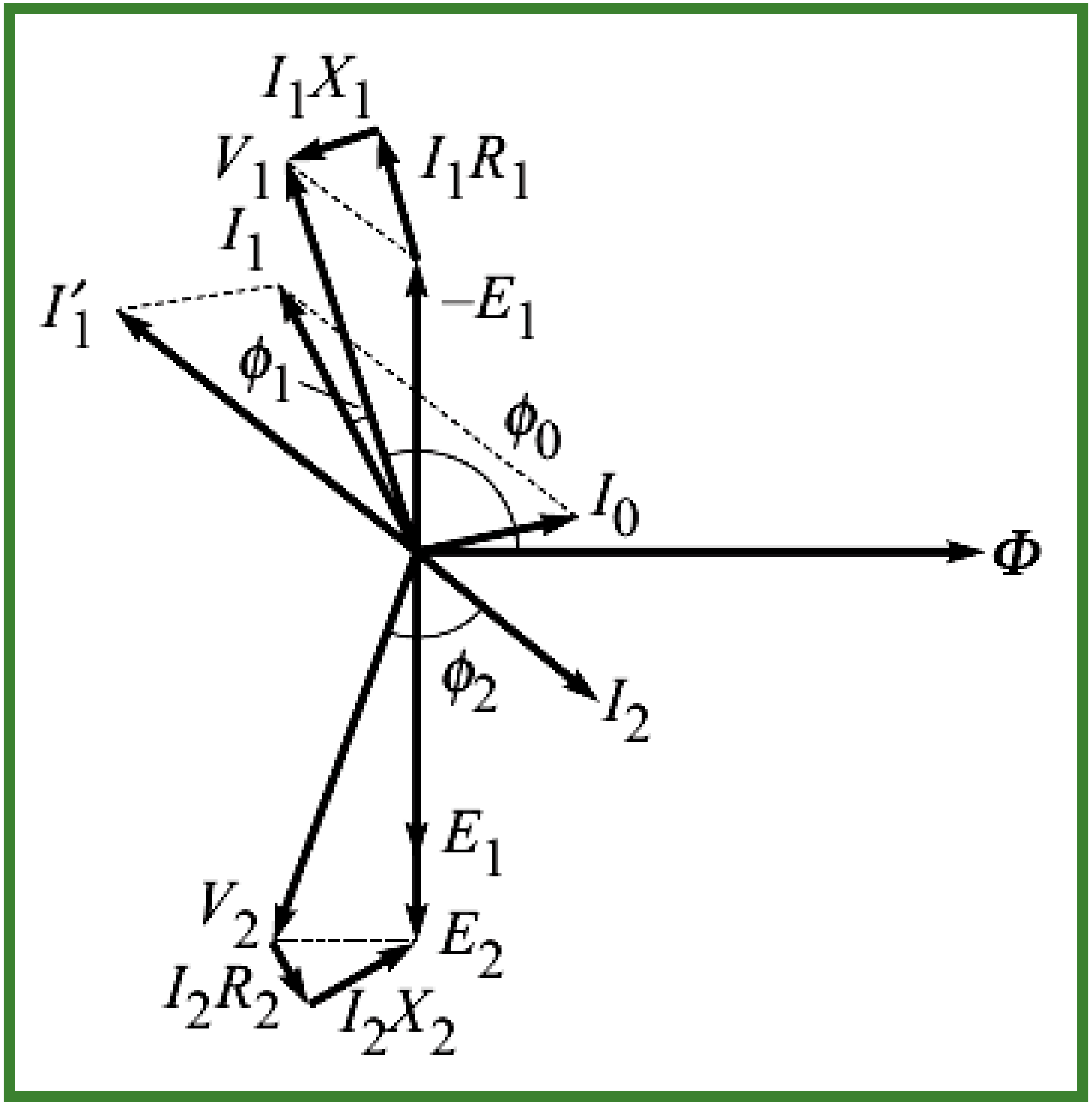

$$\mathbf{V}_1 = -\mathbf{E}_1 + I_1(R_1 + jX_1)$$
$$\mathbf{E}_2 = V_2 + I_2(R_2 + jX_2)$$

Next

Practical Transformer on **Inductive Load**



## Practical Transformer on **Capacitive Load**

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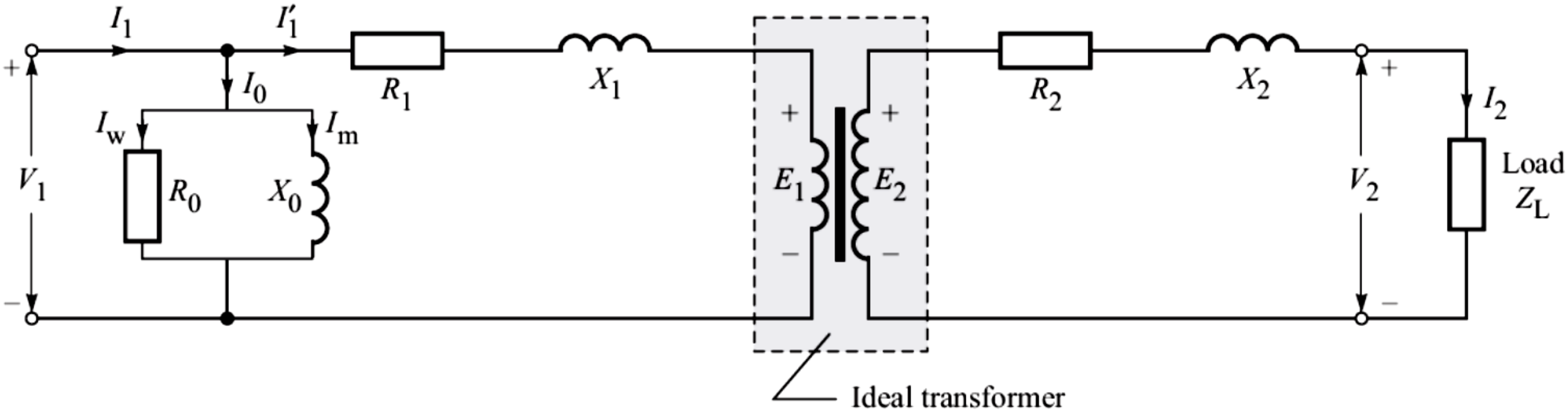
# Simplified Equivalent Circuit

- The no-load current $I_0$ is only about 3-5 % percent of the full-load current.
- The exciting circuit $R_0$-$X_0$ in is shifted to the left of impedance $R_1$-$X_1$.




Transforming the impedances from the secondary to the primary side.

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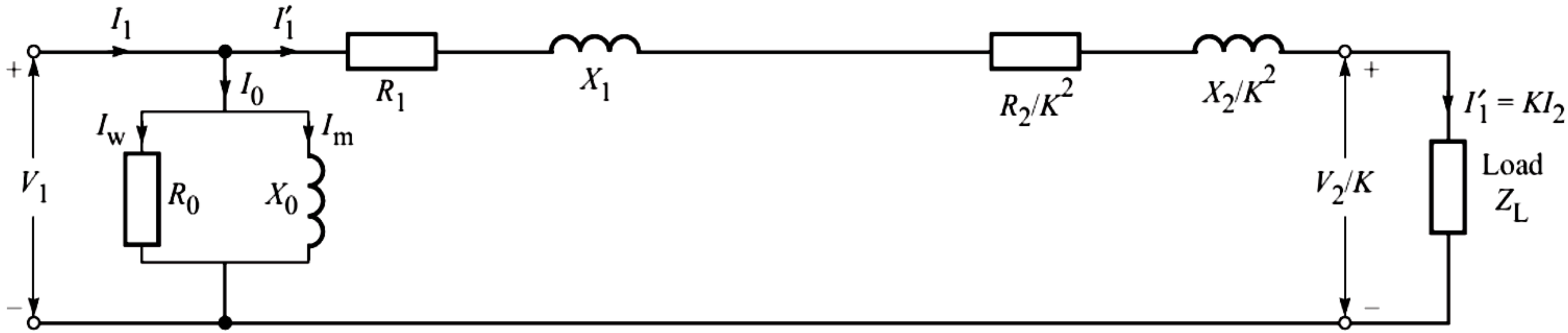
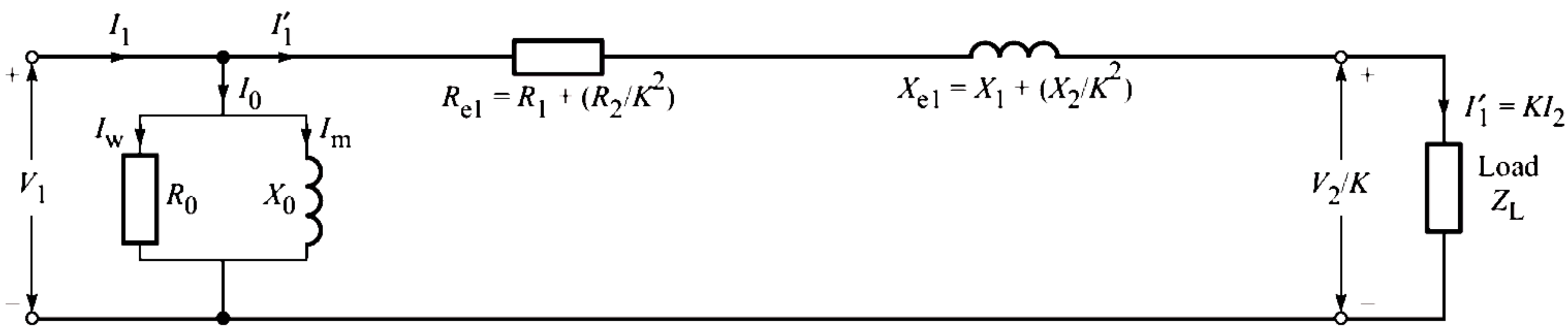



Equivalent resistance and reactance referred to the primary side




$$R_{e1} = R_1 + (R_2 / K^2) \qquad \text{and} \qquad X_{e1} = X_1 + (X_2 / K^2)$$

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# Approximate Equivalent Circuit

As referred to primary side.

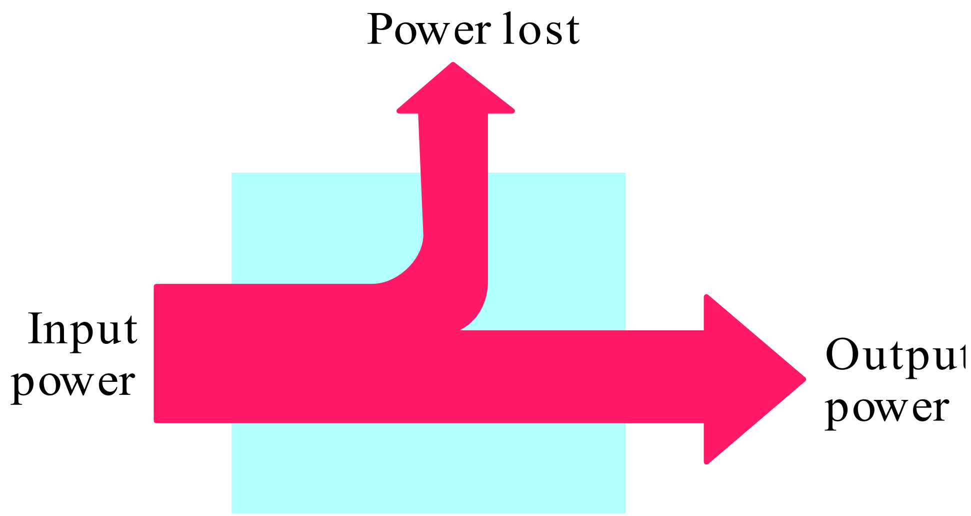
As referred to secondary side.

Next

# Efficiency of a Transformer

Like any other machine, the efficiency of a transformer is defined as

$$\eta = \frac{\text{Power output}}{\text{Power input}} = \frac{\text{Power output}}{\text{Power output} + \text{Power losses}} = \frac{P_o}{P_o + P_l}$$




- **Large-size transformers** are designed to be more efficient ($\eta$ > 98 %)
- But, the efficiency of **small transformers** (used in power adapters for charging mobile phones) is not more than 85 %.

# Power Losses in Transformers

(*i*) ***Copper losses*** or ***$I^2R$ losses*** :

$$P_c = I_1^2 R_1 + I_2^2 R_2 = I_1^2 R_{e1} = I_2^2 R_{e2}$$

*The copper losses are variable* with current. Assuming the voltage to remain constant, the current is proportional to the VA. Therefore, the copper losses for a given load (and hence for given VA) is given as

$$\boxed{P_c = \left(\frac{\text{VA}}{\text{VA}_{\text{FL}}}\right)^2 P_{c(\text{FL})}}$$

## (*ii*) ***Iron losses*** or ***core losses*** :

Due to hysteresis and eddy-currents. $P_\mathrm{i} = P_\mathrm{h} + P_\mathrm{e}$.

Since the flux $\Phi_\mathrm{m}$ does not vary more than about 2 % between no load and full load, it is usual to assume ***the core losses constant at all loads*.**

In general, the efficiency,

$$\eta = \frac{P_\mathrm{o}}{P_\mathrm{o} + P_\mathrm{l}} = \frac{P_\mathrm{o}}{P_\mathrm{o} + P_\mathrm{c} + \mathrm{P}_i} = \frac{V_2 I_2 \cos\phi_2}{V_2 I_2 \cos\phi_2 + I_2^2 R_{e2} + P_\mathrm{i}}$$

# Condition for Maximum Efficiency

Assuming the operation at a constant voltage and a constant power factor, for what load (i.e., what value of $I_2$) the efficiency becomes maximum ?

Let us first divide the numerator and denominator by $I_2$, to get

$$\eta = \frac{V_2 \cos\phi_2}{V_2 \cos\phi_2 + I_2 R_{\mathrm{e}2} + P_{\mathrm{i}} / I_2}$$

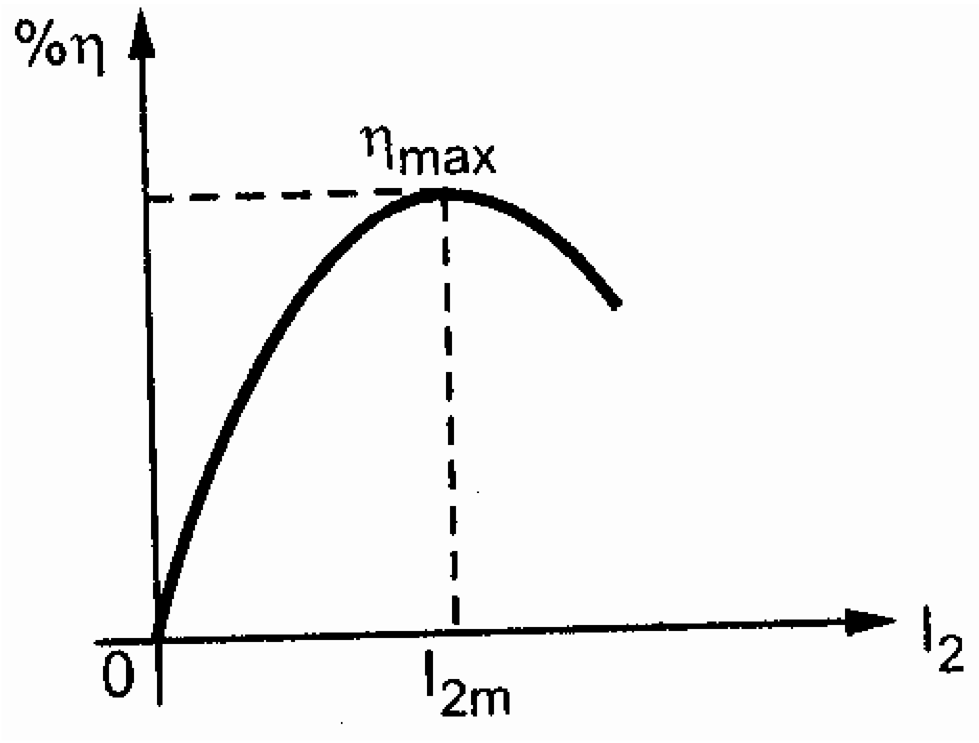
The efficiency will be maximum when the denominator of the above equation is minimum,

$$\frac{d}{dI_2}(V_2 \cos\phi_2 + I_2 R_{\mathrm{e}2} + P_i / I_2) = 0 \qquad \text{or} \qquad R_{\mathrm{e}2} - \frac{P_{\mathrm{i}}}{I_2^2} = 0$$

$$\text{or} \qquad I_2^2 R_{\mathrm{e}2} = P_i \qquad \text{or} \qquad P_{\mathrm{c}} = P_{\mathrm{i}}$$

**Condition :**

**Copper loss = Iron loss**



# All-day Efficiency

- The efficiency defined above is called ***commercial efficiency.***
- In a distribution transformer, the primary remains energized all the time. But the load on the secondary is intermittent and variable during the day.
- The core losses occur throughout the day, but the copper losses occur only when the transformer is loaded.
- Such transformers, therefore, are designed to have minimum core losses. This gives them better ***all-day efficiency***, defined below.

$$\boxed{\eta_{\text{all-day}} = \frac{\text{Output energy (in kW h) in a cycle of 24 hours}}{\text{Total input energy (in kW h)}}}$$

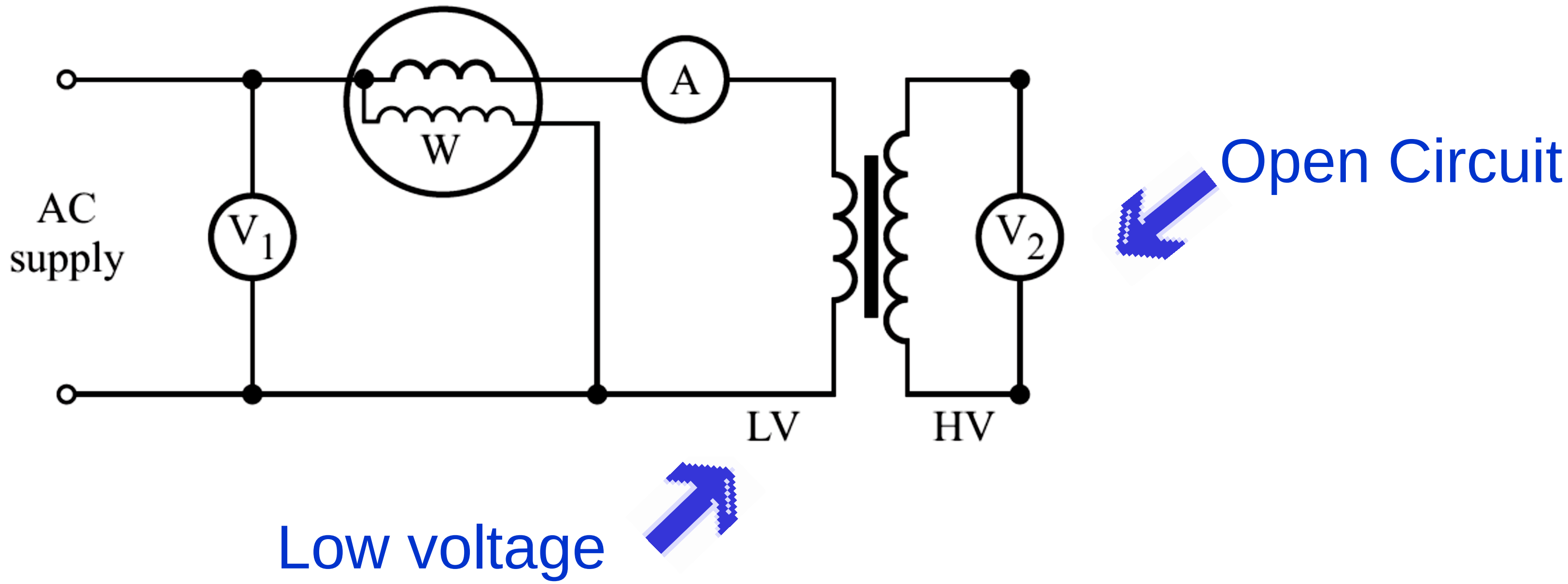
# Transformer Testing

- There are two simple tests to determine the equivalent-circuit parameters and its efficiency and regulation:
  - Open-circuit test (OC Test)
  - Short-circuit test (SC Test)

- *Advantage* of these tests is without actually loading the transformers, we can determine the Losses and Regulation, for full-load.

# (1) Open-Circuit Test

- This test determines the no-load current and the parameters of the exciting circuit of the transformer.
- Generally, the low voltage (LV) side is supplied rated voltage through a ***variac***.
- The high voltage (HV) side is left **open.**



- The $I^2R$ loss on no load is negligibly small compared with the core loss.
- Hence the wattmeter reading, $W_o$, can be assumed to give the core loss of the transformer.

## Calculations :

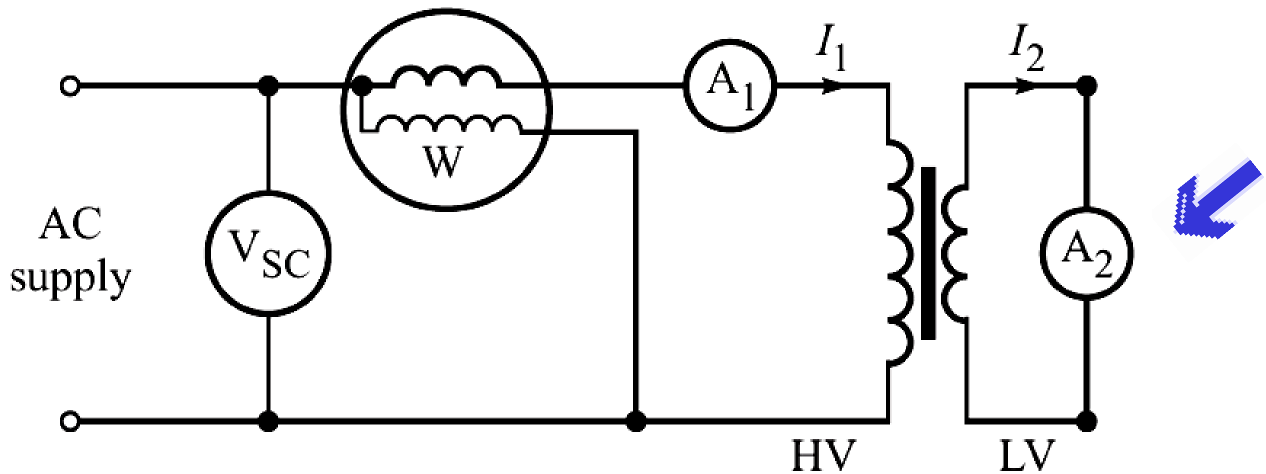
$$P_i = W_o; \qquad I_0 = I_o; \qquad K = \frac{V_2}{V_1}$$

$$I_w = \frac{W_o}{V_1}; \qquad I_m = \sqrt{I_0^2 - I_w^2};$$

$$R_0 = \frac{V_1}{I_w}; \qquad X_0 = \frac{V_1}{I_m}$$

# (2) Short-Circuit Test

- This test determines the equivalent resistance and leakage reactance.
- Generally, the LV side of the transformer is short-circuited through a suitable ammeter $A_2$.
- A *low* voltage is applied to the primary (HV) side.
- This voltage is adjusted with the help of a variac so as to circulate full-load currents in the primary and secondary circuits.




Short Circuit

- The reading of ammeter $A_1$, $I_{sc}$, gives the full-load current in the primary winding.
- Since the applied voltage (and hence the flux) is small, the core loss is negligibly small.
- Hence, the wattmeter reading, $W_{sc}$, gives the copper loss ($P_c$).

## Calculations :

$$R_{e1} = \frac{W_{sc}}{I_{sc}^2}; \qquad Z_{e1} = \frac{V_{sc}}{I_{sc}}; \qquad X_{e1} = \sqrt{Z_{e1}^2 - R_{e1}^2}$$