Subject:
Date

Eq (11) $\rightarrow$ Eq (13)

$$H = \frac{e_1^{\xi\varepsilon} - e_1^{-\xi\varepsilon}}{e_1^{\xi\varepsilon} + e_1^{-\xi\varepsilon}}$$

$$e_1^{\xi\varepsilon} = T \Rightarrow H = \frac{T - \frac{1}{T}}{T + \frac{1}{T}} \Rightarrow HT^2 + H = T^2 - 1$$

$$\Rightarrow T^2(1-H) = H+1 \Rightarrow T = \sqrt{\frac{H+1}{1-H}} \Rightarrow \varepsilon = \frac{1}{\xi} \ln \sqrt{\frac{H+1}{1-H}}$$

$$\dot{\varepsilon} = \frac{1}{\xi} \frac{\left(\sqrt{\frac{H+1}{1-H}}\right)'}{\sqrt{\frac{H+1}{1-H}}} = \frac{1}{\xi} \sqrt{\frac{1-H}{H+1}} \left(\sqrt{\frac{H+1}{1-H}}\right)'$$

$$= \frac{1}{\xi} \sqrt{\frac{1-H}{H+1}} \frac{\dot{H}(1-H) + \dot{H}(H+1)}{(1-H)^2} \frac{\sqrt{1-H}}{2\sqrt{H+1}}$$

$$= \frac{1}{\xi} \frac{2\dot{H}}{2(1-H)(H+1)} = \frac{1}{\xi} \frac{\dot{H}}{1-H^2}$$

PAPCO

Scanned by CamScanner

Subject:
Date

$x_2 - \dot{x}_{1d}$ $\quad -\tau(\delta_0 - \delta_\infty) e_1^{-\tau t}$

$= -\tau\delta + \tau\delta_\infty$

$$\dot{\mathcal{E}} = \frac{1}{\xi}\frac{1}{1-H^2}\frac{\dot{e}_1 l\delta - l\dot{\delta}e_1}{l^2\delta^2}$$

$$= \frac{1}{\xi}\frac{1}{1-H^2}\left(x_2 - \dot{x}_{1d} - \tau e_1 + \tau\frac{\delta_\infty e_1}{\delta}\right)\frac{1}{l\delta}$$

$$= \frac{1}{\xi}\frac{1}{l\delta(1-H^2)}\left(x_2 - \dot{x}_{1d} + \tau e_1\left(\frac{\delta_\infty}{\delta} - 1\right)\right)$$

Eq 18 to Eq 21

The input : $pu + \phi$

$$pu = \frac{p}{\hat{p}}\bar{u} = \frac{\hat{p} - \tilde{p}}{\hat{p}}\bar{u} = \bar{u} - \frac{\tilde{p}}{\hat{p}}\bar{u}$$

P4PCO

$$V = V_2 + \frac{1}{2}\left(\tilde{\Gamma}^2 + \tilde{\phi}^2 + \tilde{d}^2 + \tilde{\rho}^2\right)$$

$$\Rightarrow \dot{V} = \dot{V}_2 + \tilde{\Gamma}\dot{\hat{\Gamma}} + \tilde{\phi}\dot{\hat{\phi}} + \tilde{d}\dot{\hat{d}} + \tilde{\rho}\dot{\hat{\rho}}$$

design $$\dot{\hat{\Gamma}} = e_2 - \frac{\lambda}{\eta_\Gamma}\hat{\Gamma} - \frac{\bar{\lambda}}{\eta_\Gamma^2}\hat{\Gamma}^3$$

$$\dot{\hat{\phi}} = e_2 - \frac{\psi}{\eta_\phi}\hat{\phi} - \frac{\bar{\psi}}{\eta_\phi^2}\hat{\phi}^3$$

$$\dot{\hat{d}} = e_2 - \frac{\varphi}{\eta_d}\hat{d} - \frac{\bar{\varphi}}{\eta_d^2}\hat{d}^3$$

$$\dot{\hat{\rho}} = e_2\frac{\bar{u}}{\hat{\rho}} - \frac{\sigma}{\eta_\rho}\hat{\rho} - \frac{\bar{\sigma}}{\eta_\rho^2}\hat{\rho}^3$$

where $\lambda, \bar{\lambda}, \psi, \bar{\psi}, \varphi, \bar{\varphi}, \sigma, \bar{\sigma}, \eta_\Gamma, \eta_\phi, \eta_d, \eta_\rho > 0$

$$\Rightarrow \dot{V}_2 = -\left(\frac{1}{2}\right)^{\frac{3}{4}}K_1\varepsilon^{\frac{3}{2}} - \left(\frac{1}{2}\right)^2 K_2\varepsilon^4 - \left(\frac{1}{2}\right)^{\frac{3}{4}}K_3 e_2^{\frac{3}{2}}$$

$$-\left(\frac{1}{2}\right)^2 K_4 e_2^4 - \tilde{\Gamma}\left(\frac{\lambda}{\eta_\Gamma}\hat{\Gamma} + \frac{\bar{\lambda}}{\eta_\Gamma^2}\hat{\Gamma}^3\right) - \tilde{\phi}\left(\frac{\psi}{\eta_\phi}\hat{\phi} + \frac{\bar{\psi}}{\eta_\phi^2}\hat{\phi}^3\right)$$

$$-\tilde{d}\left(\frac{\varphi}{\eta_d}\hat{d} + \frac{\bar{\varphi}}{\eta_d^2}\hat{d}^3\right) - \tilde{\rho}\left(\frac{\sigma}{\eta_\rho}\hat{\rho} + \frac{\bar{\sigma}}{\eta^2}\hat{\rho}^3\right)$$

$$-\frac{\lambda}{\eta_r}\tilde{r}\hat{r} = -\frac{\lambda}{\eta_r}\tilde{r}(\tilde{r}+r) = -\frac{\lambda}{\eta_r}\tilde{r}^2 - \underbrace{\frac{\lambda}{\eta_r}\tilde{r}r}_{\le \frac{\lambda}{2\eta_r}(\tilde{r}^2+r^2)}$$ 5

$$\le \frac{-\lambda}{2\eta_r}\tilde{r}^2 + \frac{\lambda}{2\eta_r}r^2$$

$$-\frac{\lambda}{2\eta_r}\tilde{r}^2 = \frac{-\lambda}{4\eta_r}\tilde{r}^2 - \frac{\lambda}{4\eta_r}\tilde{r}^2 + \frac{\vartheta_r^2}{2\sqrt{2\eta_r}\,\lambda}|\tilde{r}|$$

$$-\frac{\vartheta_r^2}{2\sqrt{2\eta_r}\,\lambda}|\tilde{r}| + \vartheta_r^2\left(\frac{1}{2\eta_r}\tilde{r}^2\right)^{\frac{3}{4}} - \vartheta_r\left(\frac{1}{2\eta_r}\tilde{r}^2\right)^{\frac{3}{4}}$$

$$= -\frac{\lambda}{4\eta_r}\tilde{r}^2 + \frac{\vartheta_r^2}{2\sqrt{2\eta_r}\,\lambda}|\tilde{r}| - \vartheta_r\left(\frac{1}{2\eta_r}\tilde{r}^2\right)^{\frac{3}{4}}$$

$$-\frac{1}{4\eta_r}\left(\sqrt{\lambda}\,|\tilde{r}| - (2\eta_r)^{\frac{1}{4}}\frac{\vartheta_r}{\sqrt{\lambda}}\sqrt{|\tilde{r}|}\right)^2, \quad \vartheta_r \ge 0$$

$$\frac{\vartheta_r^2}{2\sqrt{2\eta_r}\,\lambda}|\tilde{r}| \le \frac{\lambda}{8\eta_r}|\tilde{r}|^2 + \frac{\vartheta_r^4}{4\lambda^3}$$

$$-\frac{\lambda}{\eta_r}\tilde{r}\hat{r} \le \frac{\lambda}{2\eta_r}r^2 - \frac{\lambda}{8\eta_r}\tilde{r}^2 + \frac{\vartheta_r^4}{4\lambda^3} - \vartheta_r\left(\frac{1}{2\eta_r}\tilde{r}^2\right)^{\frac{3}{4}}$$

$$\le C_{1r} - \vartheta_r\left(\frac{1}{2}\tilde{r}^2\right)^{\frac{3}{4}}, \quad C_{1r} = \frac{\lambda}{2\eta_r}r^2 + \frac{\vartheta_r^4}{4\lambda^3}$$

$$-\frac{\bar{\lambda}}{\eta_r^2}\tilde{r}\hat{r}^3 = -\frac{\bar{\lambda}}{\eta_r^2}\tilde{r}(\tilde{r}+r)^3 = \frac{-\bar{\lambda}}{\eta_r^2}(\tilde{r}^4 + 3\tilde{r}^3 r$$ 6

$$+ 3\tilde{r}^2 r^2 + \tilde{r} r^3)$$

using Lemma 4 : $ab \leqslant \frac{\varepsilon^p}{p}|a|^p + \frac{1}{q\varepsilon^q}|b|^q$, $\varepsilon \geqslant 0$, $p>1$,

$q>1$, $\frac{1}{p}+\frac{1}{q}=1$

$$-3\frac{\bar{\lambda}}{\eta_r^2}\tilde{r}^3 r \leqslant \frac{3\bar{\lambda}}{\eta_r^2}\left(\frac{3}{4}h_r^{\frac{4}{3}}|\tilde{r}^3|^{\frac{4}{3}} + \frac{1}{4h_r^4}r^4\right)$$

$h_r>0$, $h_r \leqslant \left(\frac{2}{3}\right)^{\frac{3}{2}}$

$$-\frac{\bar{\lambda}}{\eta_r^2}\tilde{r}\hat{r}^3 \leqslant \left(\sqrt{3\frac{\bar{\lambda}}{\eta_r^2}}\,\tilde{r}r + \frac{1}{2}\sqrt{\frac{\bar{\lambda}}{3\eta_r^2}}\right)^2 = \frac{3\bar{\lambda}}{\eta_r^2}\tilde{r}^2 r^2 + \frac{\bar{\lambda}}{12\eta_r^2}r^2$$

$$\Rightarrow -\frac{\bar{\lambda}}{\eta_r^2}\tilde{r}\hat{r} \leqslant -\frac{\bar{\lambda}}{\eta_r^2}\tilde{r}^4 + \frac{9}{4}\frac{\bar{\lambda}}{\eta_r^2}h_r^{\frac{4}{3}}|\tilde{r}|^4 + C_{2r}$$

$$C_{2r} = \frac{3}{4}\frac{\bar{\lambda}}{\eta_r^2 h_r^4}r^4 + \frac{\bar{\lambda}}{12\eta_r^2}r^2$$

The same process for $\phi$, $\alpha$, $p$

$$\Rightarrow \dot{V} \leq -K_1\left(\frac{1}{2}\varepsilon^2\right)^{\frac{3}{4}} - K_2\left(\frac{1}{2}\varepsilon^2\right)^2 - K_3\left(\frac{1}{2}e_2^2\right)^{\frac{3}{4}}$$

$$-K_4\left(\frac{1}{2}e_2^2\right)^2 - \frac{\lambda}{\eta_\Gamma}\left(\frac{1}{2}\tilde{\Gamma}^2\right)^{\frac{3}{4}} - \frac{\bar{\lambda}}{\eta_\Gamma^2}\left(4-9\ell_\Gamma^{\frac{4}{3}}\right)\left(\frac{1}{2}\tilde{\Gamma}^2\right)^2$$

$$-\frac{\Psi}{\eta_\phi}\left(\frac{1}{2}\tilde{\phi}^2\right) - \frac{\bar{\Psi}}{\eta_\phi^2}\left(4-9\ell_\phi^{\frac{4}{3}}\right)\left(\frac{1}{2}\tilde{\phi}^2\right)^2 - \frac{\varphi}{\eta_d}\left(\frac{1}{2}\tilde{d}^2\right)^{\frac{3}{4}}$$

$$-\frac{\bar{\varphi}}{\eta_d^2}\left(4-9\ell_d^{\frac{4}{3}}\right)\left(\frac{1}{2}\tilde{d}^2\right)^2 - \frac{\sigma}{\eta_p}\left(\frac{1}{2}\tilde{p}^2\right)^{\frac{3}{4}}$$

$$-\frac{\bar{\sigma}}{\eta_p^2}\left(4-9\ell_p^{\frac{4}{3}}\right)\left(\frac{1}{2}\tilde{p}^2\right)^2 + C$$

$$C = C_{1\Gamma} + C_{2\Gamma} + C_{1\phi} + C_{2\phi} + C_{1d} + C_{2d} + C_{1p} + C_{2p}$$

$$\mu_1 = \min\left(K_1, K_3, \frac{\lambda}{\eta_\Gamma}, \frac{\Psi}{\eta_\phi}, \frac{\varphi}{\eta_d}, \frac{\sigma}{\eta_p}\right)$$

$$\mu_2 = \min\left(K_2, K_4, \frac{\bar{\lambda}}{\eta_\Gamma^2}\left(4-9\ell_\Gamma^{\frac{4}{3}}\right), \frac{\bar{\Psi}}{\eta_\phi^2}\left(4-9\ell_\phi^{\frac{4}{3}}\right),\right.$$

$$\left.\frac{\bar{\varphi}}{\eta_d^2}\left(4-9\ell_d^{\frac{4}{3}}\right), \frac{\bar{\sigma}}{\eta_p^2}\left(4-9\ell_p^{\frac{4}{3}}\right)\right)$$

using Lemma 1–3

$$\Rightarrow \dot{V} < -\mu_1 V^{\frac{3}{4}} - \frac{\mu_2}{6} V^2 + C$$

for $V^2 \geqslant \frac{6C}{\varsigma \mu_2}$ , $0 < \varsigma < 1$

$$\Rightarrow C \leqslant \frac{\varsigma \mu_2}{6} V^2$$

$$\Rightarrow \dot{V} \leqslant -\mu_1 V^{\frac{3}{4}} - (1-\varsigma) \frac{\mu_2}{6} V^2$$

$\Rightarrow V$ Converges to $\left\{ V : V < \sqrt{\frac{6C}{\varsigma \mu_2}} \right\}$

using lemma 5

, $T_{max} = \dfrac{6}{(1-\varsigma)\mu_2} + \dfrac{4}{\mu_1}$

$\frac{1}{2}\varepsilon^2 < V \Rightarrow$ $\varepsilon$ Converges to $\left\{ \varepsilon : |\varepsilon| < \sqrt{2\sqrt{\frac{6C}{\varsigma \mu_2}}} \right\}$