## Differentiation - Product Rule

Differentiate each function with respect to x.

1) 
$$y = -x^3(3x^4 - 2)$$

2) 
$$f(x) = x^2(-3x^2 - 2)$$

3) 
$$y = (-2x^4 - 3)(-2x^2 + 1)$$

4) 
$$f(x) = (2x^4 - 3)(x^2 + 1)$$

5) 
$$f(x) = (5x^5 + 5)(-2x^5 - 3)$$

6) 
$$f(x) = (-3 + x^{-3})(-4x^3 + 3)$$

7) 
$$y = (-2x^4 + 5x^2 + 4)(-3x^2 + 2)$$

8) 
$$y = (x^4 + 3)(-4x^5 + 5x^4 + 5)$$

9) 
$$y = (5x^4 - 3x^2 - 1)(-5x^2 + 3)$$

10) 
$$f(x) = (-10x^2 - 7\sqrt[5]{x^2} + 9)(2x^3 + 4)$$

11) 
$$y = (5 + 3x^{-2})(4x^5 + 6x^3 + 10)$$

12) 
$$y = (-6x^4 + 2 + 6x^{-4})(6x^4 + 7)$$

13) 
$$f(x) = \left(-7x^4 + 10x^{\frac{2}{5}} + 8\right)(x^2 + 10)$$

## **Critical thinking question:**

14) A classmate claims that  $(f \cdot g)' = f' \cdot g'$  for any functions f and g. Show an example that proves your classmate wrong.

## Differentiation - Product Rule

Differentiate each function with respect to x.

1) 
$$y = -x^3(3x^4 - 2)$$
  

$$\frac{dy}{dx} = -x^3 \cdot 12x^3 + (3x^4 - 2) \cdot -3x^2$$

$$= -21x^6 + 6x^2$$

2) 
$$f(x) = x^2(-3x^2 - 2)$$
  
 $f'(x) = x^2 \cdot -6x + (-3x^2 - 2) \cdot 2x$   
 $= -12x^3 - 4x$ 

3) 
$$y = (-2x^4 - 3)(-2x^2 + 1)$$
  

$$\frac{dy}{dx} = (-2x^4 - 3) \cdot -4x + (-2x^2 + 1) \cdot -8x^3$$

$$= 24x^5 - 8x^3 + 12x$$

4) 
$$f(x) = (2x^4 - 3)(x^2 + 1)$$
  
 $f'(x) = (2x^4 - 3) \cdot 2x + (x^2 + 1) \cdot 8x^3$   
 $= 12x^5 + 8x^3 - 6x$ 

5) 
$$f(x) = (5x^5 + 5)(-2x^5 - 3)$$
  
 $f'(x) = (5x^5 + 5) \cdot -10x^4 + (-2x^5 - 3) \cdot 25x^4$   
 $= -100x^9 - 125x^4$ 

6) 
$$f(x) = (-3 + x^{-3})(-4x^3 + 3)$$
  
 $f'(x) = (-3 + x^{-3}) \cdot -12x^2 + (-4x^3 + 3) \cdot -3x^{-4}$   
 $= 36x^2 - \frac{9}{x^4}$ 

7) 
$$y = (-2x^4 + 5x^2 + 4)(-3x^2 + 2)$$
  

$$\frac{dy}{dx} = (-2x^4 + 5x^2 + 4) \cdot -6x + (-3x^2 + 2)(-8x^3 + 10x)$$

$$= 36x^5 - 76x^3 - 4x$$

8) 
$$y = (x^4 + 3)(-4x^5 + 5x^4 + 5)$$
  

$$\frac{dy}{dx} = (x^4 + 3)(-20x^4 + 20x^3) + (-4x^5 + 5x^4 + 5) \cdot 4x^3$$

$$= -36x^8 + 40x^7 - 60x^4 + 80x^3$$

9) 
$$y = (5x^4 - 3x^2 - 1)(-5x^2 + 3)$$
  

$$\frac{dy}{dx} = (5x^4 - 3x^2 - 1) \cdot -10x + (-5x^2 + 3)(20x^3 - 6x)$$

$$= -150x^5 + 120x^3 - 8x$$

10) 
$$f(x) = (-10x^{2} - 7\sqrt[5]{x^{2}} + 9)(2x^{3} + 4)$$
$$f'(x) = (-10x^{2} - 7x^{\frac{2}{5}} + 9) \cdot 6x^{2} + (2x^{3} + 4)(-20x - \frac{14}{5}x^{-\frac{3}{5}})$$
$$= -100x^{4} - \frac{238x^{\frac{12}{5}}}{5} + 54x^{2} - 80x - \frac{56}{5x^{\frac{3}{5}}}$$

11) 
$$y = (5 + 3x^{-2})(4x^5 + 6x^3 + 10)$$
  

$$\frac{dy}{dx} = (5 + 3x^{-2})(20x^4 + 18x^2) + (4x^5 + 6x^3 + 10) \cdot -6x^{-3}$$

$$= 100x^4 + 126x^2 + 18 - \frac{60}{x^3}$$

12) 
$$y = (-6x^4 + 2 + 6x^{-4})(6x^4 + 7)$$
  

$$\frac{dy}{dx} = (-6x^4 + 2 + 6x^{-4}) \cdot 24x^3 + (6x^4 + 7)(-24x^3 - 24x^{-5})$$

$$= -288x^7 - 120x^3 - \frac{168}{x^5}$$

13) 
$$f(x) = \left(-7x^4 + 10x^{\frac{2}{5}} + 8\right)(x^2 + 10)$$
$$f'(x) = \left(-7x^4 + 10x^{\frac{2}{5}} + 8\right) \cdot 2x + (x^2 + 10)\left(-28x^3 + 4x^{-\frac{3}{5}}\right)$$
$$= -42x^5 - 280x^3 + 24x^{\frac{7}{5}} + 16x + \frac{40}{x^{\frac{3}{5}}}$$

## **Critical thinking question:**

14) A classmate claims that  $(f \cdot g)' = f' \cdot g'$  for any functions f and g. Show an example that proves your classmate wrong.

Many answers. Ex: f = 2x, g = 4,  $8 \ne 0$