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Module Leader	Anton Bondarev

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General feedback on the work

Just outstanding project. Analysis is original, applying some classical model to an actual problem. Formal statements and solution are all correct with illustrations. The numerical application is a nice addition, meaningfully enhancing the formal analysis. All steps are clearly described and stated.
There are very few things needing improvement in this work. Perhaps some technical parts could be hidden in the Appendix. Some language improvements could be done. Otherwise it reads as a FYP project, not CW and even could be published in a ABS2/3 level journal.

Component	**Weight**	**Mark**
The statement of the problem	20%	90
Mathematical formulation	30%	87
Solution of the mathematical problem	35%	91
Economic interpretation of solution	15%	88
Total	100%	89.1

1st Marker Anton Bondarev **Date** 17.06.2021 **Mark** 89.1

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(if applicable)

Students: *Please start your assignment on the next page.*

Talents incentive mechanism in Chinese high-tech enterprises: Dynamic analysis based on optimal resource extraction model

1. Introduction and Problem Statement

Talent capital is the first resource of an enterprise and the key for an organization to gain competitive advantage (Talent & Modeling Agencies, 2021). Recent years in China, the advancement of science and technology, as well as the increasing degree of the labor division, a career requires intensive training and deep understanding of a particular field. This is especially the case for high-tech enterprises like Tencent, Huawei, and Alibaba etc. who require talent pools to work on highly mental-demanded projects such as cloud computing and software engineering. In those project teams, replacing talent is a long, painful and expensive process, and the fact that short-handed teams impact development schedules and business results only makes matters worse. Therefore, it is imperative for high-tech firms to establish an appropriate talent retention mechanism to motivate and retain core employees. By doing so, talents in return are willing to exert their greatest efforts, make more contributions to the enterprise, at the same time their recognition and loyalty to the enterprise can be enhanced, reducing companies' loses caused by their quits or job-hopping.

Factors that affect people's work enthusiasm can be divided into two categories: health care factors and incentive factors (Herzberg, 1967). For a long time, many enterprises always regard physical incentives including bonus, employee stock ownership plan and other welfare policies as main approach to attract talents. However, those physical incentives are often easy to be satisfied by employees and its effect cannot be lasting. Therefore, it is necessary to strengthen spiritual incentives involving paid leaves, trainings, praises, and promotions.

Nevertheless, carrying out a complete incentive mechanism, while enhancing the core competence, bring high costs to a firm. Also, employees' needs may change in different environmental and psychological context. It is therefore indispensable for enterprises to balance such costs and benefits to pursue profit maximization dynamically.

Hence, the specific problem this essay is going to solve is that:
For high-tech enterprises in China, how to decide the optimal amount of investment in the spiritual incentive mechanism to reduce talents draining so the profit maximization can be pursued with respect to different times. The overall dynamic analysis is based on an optimal resource extraction model constructed by Dasgupta et al. (1978) through optimal control theory and will further focus on problem analysis targeted on the overall A-share listed high-tech companies in Shanghai and Shenzhen stock exchange markets. Solving this problem can help high-tech firms to establish and adjust their private retention strategies and compare their current performance with the average situation of A-share firms.

2. Model Construction and Formulation

2.1. Construct the general model

The goal of talent incentives is to pursue profit maximization under dynamic environment. Hence, the optimal control problem can be expressed as

$$Max \int_0^{\infty} e^{-rt} \pi_i \, dt$$
$$\pi_i = R_i(D,t) - Z_i(P,S,t)$$
$$s.t. \begin{cases} \dot{B(t)} = B(U(P,S,t)) - Z(D,t) \\ R(0) = 0; B(0) = 0; Z(0) = 0 \end{cases}$$

Here, π_i represents the profit function for enterprise i. D is the employees' degree of effort or effort level devoted in a period of time. $R_i(D,t)$ explains the profit brought to the enterprise by employee's degree of effort. The physical motivation provided by the firm including wages, bonus, employee stock ownership plan etc. is expressed as P while the spiritual motivation such as paid leave, training, praise, and promotion is denoted as S. It is worth mentioning that D, P, and S are all varied in terms of time. Thus, $Z_i(P,S,t)$ represents a company's expenditures caused by both physical and spiritual incentive costs, which is a function of P, S and time t. Therefore, the profit function of the enterprise in this case is $R_i(D,t) - Z_i(P,S,t)$.

From the perspective of employees, their net benefit remaining at time t is $B(U(P,S,t)) - Z(D,t)$, where $B(U(P,S,t))$ is the revenue function based on physical and spiritual welfare they obtained, and $Z(D,t)$ is the cost function measuring how much they sacrifice for their work. Since individual's benefit is associated with welfare, the utility function is applied in B to better express the relationship between incentives and talents' benefits. Finally, the boundary conditions for the initial values of R, B, Z at time $t = 0$ are assumed to be 0.

2.2. Assumptions and justifications

- According to the law of diminishing marginal benefits, the greater the incentive is, the greater the employee benefits will be, but the growth rate of such incentives to benefits is decreasing, which is

$$B_S'\big(U(P,S,t)\big) > 0; \;\; B_{SS}''\big(U(P,S,t)\big) < 0$$

$$B_P'\big(U(P,S,t)\big) > 0; \;\; B_{PP}''\big(U(P,S,t)\big) < 0$$

- When the employee's effort level is high, the cost is large. In this situation, increasing one more unit of effort degree requires more motivation and more sacrifices, which is

$$Z'(D) > 0; \;\; Z''(D) > 0$$

- We assume a high-tech company's expenditure on talents' physical and spiritual incentives is the total of how much the company has spent in carrying out related projects, initiatives, activities, and trials.

- The discount rate r used to generate the maximization profit is constant and we assume it equals the free-risk interest rate. In China, the Ten-year Treasury yields is most frequently used to approximate this indicator.

2.3. Settle function forms

To analyze further, we adopt specific forms appropriately to reflect various functional relationships encountered based on assumptions above, which improves the possibility to reach explicit solutions.

According to the first assumption of $B(U(P,S,t))$, the Cobb-Douglas utility function form can be employed to reflect this diminishing feature. Denote the parameters involved in the exponent part as β_1 for physical incentives spending and β_2 for spiritual incentives spending. Thus, it is appropriate to define

$$B(U(P,S,t)) = \delta P(t)^{\beta_1} S(t)^{\beta_2}$$
$$(\beta_1 > 0;\ \beta_2 > 0;\ \delta > 0)$$

In terms of the second assumption, for employees, the cost of their effort suffers from an accelerating rate, thus, we take exponential function form into consideration and assume that

$$Z(D,t) = \frac{1}{\theta-1} D(t)^{\theta-1}$$
$$(\theta > 1)$$

When it comes to company expenditures on incentives mechanism $Z_i(P,S,t)$, a linear function form is applied here according to the third assumption above. In this function, α_1 is assigned to be associated with physical incentives spending while α_2 is with the spiritual one.

$$Z_i(P,S,t) = \alpha_1 P(t) + \alpha_2 S(t)$$
$$(\alpha_1 > 0;\ \alpha_2 > 0)$$

As for $R_i(D,t)$, we follow a normal revenue function form, which means a firm's revenue is calculated through price times quantity. However, in our case, it is impractical to define an 'effort price'. We thus simply define a parameter ω to somehow play a role of price.

$$R_i(D,t) = \omega D(t)$$
$$(\omega > 0)$$

Therefore, we can construct the dynamic model with more specific function forms and our problem is transformed to solving the following dynamic optimization question with the state variable $S(t)$:

$$Max \int_0^{\infty} e^{-rt} \pi_i \, dt$$

$$\pi_i = \omega D(t) - \left(\alpha_1 P(t) + \alpha_2 S(t)\right)$$

$$s.t.\begin{cases} B\dot{(t)} = \delta P(t)^{\beta_1} S(t)^{\beta_2} - \dfrac{1}{\theta - 1} D(t)^{\theta-1} \\ D(0) = 0; P(0) = 0; S(0) = 0 \end{cases}$$

2.4. Solve for optimality condition

Let $\lambda(t)$ represent the shadow price of converting core employee's benefit into the enterprise's wealth or profit, then the current-value Hamilton function of the above optimization problem is:

$$H^{CV} = \pi_i + \lambda^{CV}(\delta P(t)^{\beta_1} S(t)^{\beta_2} - \frac{1}{\theta - 1} D(t)^{\theta-1}),\ \lambda^{CV} := e^{rt}\lambda(t)$$

$$H^{CV} = \omega D(t) - \big(\alpha_1 P(t) + \alpha_2 S(t)\big) + \lambda^{CV}[\delta P(t)^{\beta_1} S(t)^{\beta_2} - \frac{1}{\theta - 1} D(t)^{\theta-1}],\ \lambda^{CV} := e^{rt}\lambda(t)$$

where $S(t)$ is the state variable, $\lambda^{CV}(t)$ is the co-state variable, and $P(t)$, $D(t)$ are two control variables. The transversality conditions is

$$\lim_{t\to\infty} e^{-rt}\lambda^{CV}(t) = 0$$

Based on the Hamilton functions constructed above, we derive the first-order condition of optimality.

$$\frac{\partial H^{CV}}{\partial D(t)} = R'(D) - \lambda^{CV} Z'(D) = \omega - \lambda^{CV} D(t)^{\theta-2} = 0$$

$$\frac{\partial H^{CV}}{\partial P(t)} = -Z_P'(P, S, t) + \lambda^{CV} B_P'\big(U(P, S, t)\big)$$

$$= -\alpha_1 + \lambda^{CV}\beta_1 \delta P(t)^{\beta_1 - 1} S(t)^{\beta_2} = 0$$

$$\begin{cases} D(t)^* = \left(\dfrac{\omega}{\lambda^{CV}}\right)^{1/\theta-2} \\ P(t)^* = \left(\dfrac{\alpha_1}{\lambda^{CV}\beta_1 \delta S(t)^{\beta_2}}\right)^{1/\beta_1 - 1} \end{cases}$$

Substitute $D(t)^*$ and $P(t)^*$ back into Hamiltonian and derive state equation, we have

$$S\dot{(t)} = \frac{\partial H^{CV}}{\partial \lambda^{CV}} = \delta P(t)^{*\beta_1} S(t)^{\beta_2} - \frac{1}{\theta - 1} D(t)^{*\theta-1}$$

Co-state equation

$$\lambda^{C\dot{V}}(t) = r\lambda^{CV} - \frac{\partial H^{CV}}{\partial S(t)} = r\lambda^{CV} + \alpha_2 - \lambda^{CV}\delta\beta_2 P(t)^{*\beta_1} S(t)^{\beta_2 - 1}$$

Where $D(t)^* = \left(\frac{\omega}{\lambda^{CV}}\right)^{1/\theta-2}$ and $P(t)^* = \left(\frac{\alpha_1}{\lambda^{CV}\beta_1\delta S(t)^{\beta_2}}\right)^{1/\beta_1-1}$

Thus, the canonical system can be established as

$$\begin{cases} \dot{S(t)} = \delta P(t)^{*\beta_1} S(t)^{\beta_2} - \dfrac{1}{\theta - 1} D(t)^{*\theta - 1} \\ \dot{\lambda^{CV}}(t) = r\lambda^{CV} + \alpha_2 - \lambda^{CV} \delta \beta_2 P(t)^{*\beta_1} S(t)^{\beta_2 - 1} \end{cases}$$

With transversality condition

$$\lim_{t \to \infty} e^{-rt} \lambda^{CV}(t) = 0$$

To further derive canonical steady states, we build

$$\begin{cases} \dot{S(t)} = \delta P(t)^{*\beta_1} S(t)^{\beta_2} - \dfrac{1}{\theta - 1} D(t)^{*\theta - 1} = 0 \\ \dot{\lambda^{CV}}(t) = r\lambda^{CV} + \alpha_2 - \lambda^{CV} \delta \beta_2 P(t)^{*\beta_1} S(t)^{\beta_2 - 1} = 0 \end{cases}$$

The canonical steady state can therefore be obtained as follows,

$$\overline{S(t)} = \left(\frac{1}{\theta - 1} D(t)^{*\theta - 1} / \delta P(t)^{*\beta_1} \right)^{1/\beta_2}$$

$$\overline{\lambda^{CV}(t)} = \alpha_2 / \left(\delta \beta_2 P(t)^{*\beta_1} \overline{S(t)}^{\beta_2 - 1} - r \right)$$

Since the canonical system consists of two non-linear equations, it is unable to analyze the stability features of the steady state by constructing the Hessian matrix. Hence, we construct the Jacobian matrix instead. The Jacobian is

$$\boldsymbol{J} = \begin{bmatrix} \beta_2 \delta P(t)^{*\beta_1} S(t)^{\beta_2 - 1} & 0 \\ -\lambda^{CV} \beta_2 (\beta_2 - 1) \delta P(t)^{*\beta_1} S(t)^{\beta_2 - 2} & r - \delta \beta_2 P(t)^{*\beta_1} S(t)^{\beta_2 - 1} \end{bmatrix}$$

In order to obtain the eigenvalues, we construct $|J - \varphi I| = 0$, which is

$$\begin{vmatrix} \beta_2 \delta P(t)^{*\beta_1} S(t)^{\beta_2 - 1} - \varphi_1 & 0 \\ -\lambda^{CV} \beta_2 (\beta_2 - 1) \delta P(t)^{*\beta_1} S(t)^{\beta_2 - 2} & r - \delta \beta_2 P(t)^{*\beta_1} S(t)^{\beta_2 - 1} - \varphi_2 \end{vmatrix} = 0$$

$$\left(\beta_2 \delta P(t)^{*\beta_1} S(t)^{\beta_2 - 1} - \varphi_1 \right) \left(r - \delta \beta_2 P(t)^{*\beta_1} S(t)^{\beta_2 - 1} - \varphi_2 \right) = 0$$

Since the stability is provided by Jacobian at the canonical steady state, which means the eigenvalues evaluated at $\overline{S(t)}$ and $\overline{\lambda^{CV}(t)}$ are

$$\varphi_1 = \beta_2 \delta P(t)^{*\beta_1} \overline{S(t)}^{\beta_2 - 1}$$

$$\varphi_2 = r - \beta_2 \delta P(t)^{*\beta_1} \overline{S(t)}^{\beta_2 - 1}$$

Where

$$\overline{S(t)} = \left(\frac{1}{\theta - 1} D(t)^{*\theta - 1} / \delta P(t)^{*\beta_1}\right)^{1/\beta_2}$$

$$D(t)^* = \left(\frac{\omega}{\lambda^{CV}}\right)^{1/\theta - 2}$$

$$P(t)^* = \left(\frac{\alpha_1}{\lambda^{CV} \beta_1 \delta S(t)^{\beta_2}}\right)^{1/\beta_1 - 1}$$

Since $\delta > 0$, $\beta_1 > 0$, and $\theta > 1$, we know $P(t)^* > 0$ and $\overline{S(t)} > 0$. The first eigenvalue φ_1 should be positive. For the second eigenvalue φ_2, it is really hard to decide the sign of it under the unknown parameters. Different high-tech companies may tend to have different coefficients level for ω, δ, θ, β_1, β_2, α_1, and α_2. Hence, to further determine the stability of the canonical steady state, more efforts are required to estimate the value of the parameters based on Chinese high-tech firms' situation overall.

3. Solution of the Problem

3.1. Formulation settlement

3.1.1. Data specification

In order to obtain significative parameters reflecting real situation of Chinese high-tech industry, the latest disclosed annual data (year 2020) for all A-share listed high-tech companies is collected from WIND financial terminal. After dropping invalid observations, we obtain a sample size of 750. Using STATA, Table 1 summaries information for variables used in the parameter estimation process. Generally, they are divided into two categories, target variables and control variables.

The first 7 rows illustrate information about target variables, which are used to quantify R_i, Z_i, B, Z, $D(t)$, $P(t)$, $S(t)$ in the dynamic model above. For talent's sacrifices Z, we define it as the opportunity cost that a core employee choose to work for this company instead of start up his own businesses. According to Chinese Enterprise Yearbook (2020), core employees in high-tech enterprises are usually equipped with strong personal capability and approximately 70% of them start their own business after quitting. Moreover, there is at least 50% chance for them to obtain first round of at least 5 million angel funding and the average net profit margin for high-tech industry in 2020 is about 0.83%. Thus, we can calculate their gaining of starting up a business and this can be deemed as their opportunity cost (the 4th row in Table 1). For physical incentives $P(t)$, the bonus, welfare funds, and housing funds received by core workers are involved. Investments used in the spiritual incentives mainly contains personnel education funds, labor unition dues, and various insurance such as security pension, unemployment insurance etc.

Control variables are listed in the last 6 rows. The reason for containing such kind of variable is that in our dynamic model, R_i and Z_i are defined as amount of company revenues and expenditures only from core workers' devotion and incentives. However, in reality, there is no such an accounting term only expressing revenues generated by core workers' efforts. Fortunately, since R_i and Z_i are all linear functions, we thus employ a set of variables to control other variations which influence company's operating revenue and cost. By doing so, we endeavor to analyze partial effects brought by degree of effort and incentives.

3.1.2. Derive company revenue function

After conducting ordinary linear regression using STATA, appropriate value for parameter ω in year 2020 is reached. Form table 2, the significant coefficient for ED is 463.656, hence,

$$\omega = 463.656$$
$$R_i(D,t) = \omega D(t)$$
$$R_i(D,t) = 463.656D(t)$$

3.1.3. Derive company cost function

Similarly, after regress OC in terms of PI, SI, RD, TAX, and FinC, the result in table 2 shows that the estimators at a significance level of 0.01% for PI and SI are all larger than 0, which satisfy our conditions for α_1 and α_2. Thus, it is reasonable to decide

$$\alpha_1 = 13.91$$
$$\alpha_2 = 117.975$$

And the company's expenditures on talents' physical and spiritual incentives can be expressed as

$$Z_i(P,S,t) = \alpha_1 P(t) + \alpha_2 S(t)$$
$$Z_i(P,S,t) = 13.91P(t) + 117.975S(t)$$

3.1.4. Derive talent's benefit function

Since this function involved Cobb-Douglas function form, which is non-linear, conducting ordinary linear estimation is no longer applicable. Therefore, we stratify the 750 observations for B , $P(t)$, and $S(t)$ into three groups and calculate their mean value for each group. Then, three equations with given B , $P(t)$, and $S(t)$ can be obtained to solve parameters out. Using Matlab, the values for δ, β_1, β_2 is therefore

```
>> syms delta beta1 beta2
>> eqns=[337637885.4926==delta*284589443.2818^beta1*36173772.9525^beta2,75850872.59==delta*47317734.62^beta1*32823
>> vars=[delta beta1 beta2];
>> [delta,beta1,beta2]=vpasolve(eqns,vars)

delta =

120.09657355028831085663281060863


beta1 =

0.62280274739351299008791337709176


beta2 =

0.15659542198020152023139323925092
```

Keeping three decimals, the benefit function is

$$\delta = 120.097 \ ; \ \beta_1 = 0.623 \ ; \ \beta_2 = 0.157$$
$$B(U(P,S,t)) = \delta P(t)^{\beta_1} S(t)^{\beta_2}$$
$$B(U(P,S,t)) = 120.097 P(t)^{0.623} S(t)^{0.157}$$

3.1.5. Derive talent's opportunity cost

When it comes to function $Z(D,t)$, a similar procedure is applied to deal with the non-linear form. Sample mean for $Z(D,t)$ and $D(t)$ are calculated and used as the given value to solve θ. Again, the result is presented below with the assistance of Matlab.

```
>> syms sita
>> eqn=1/(sita-1)*5086382.006^(sita-1)==28114059.0062;
>> sol(sita)=solve(eqn,sita)

sol(sita) =

(30*log(2) - log(5461461092683219) + lambertw(0, (2013265920*log(2))/1886702562335051 - (67108864*log

>> (30*log(2) - log(5461461092683219) + lambertw(0, (2013265920*log(2))/1886702562335051 - (67108864

ans =

   15.4421
```

Therefore, the opportunity cost function is

$$\theta = 15.442$$
$$Z(D,t) = \frac{1}{\theta - 1} D(t)^{\theta - 1}$$
$$Z(D,t) = \frac{1}{14.442} D(t)^{14.442}$$

3.1.6. Settle the discount rate

According to the last assumption mentioned in 2.2., the Ten-year Treasury yield is commonly used as the discount rate. The ministry of Finance of China disclosed this figure in Dec 31, 2020 of 3.14%. Thus,

$$r = 3.14\% = 0.0314$$

Table1. Descriptive Statistics Summary (unit: Million)

Variable	Definition	Mean	Std. Dev.	Min	Max
OR	Company operating revenue (Ri)	4563.185	12654.165	12.342	135552.57
OC	Company operating cost (Zi)	3511.002	10489.298	1.14	108823.12
SalaryHT	Salary and compensation for high-tech workers (B)	148.103	370.232	0.729	5154.825
Sacrif	Opportunity cost of workers to start up business(Z)	28.114	64.084	0.208	882.788
DE	Working hours of high-tech workers per year D(t)	4955.891	10460.853	60.065	119200
PI	Physical incentives provided by company (P(t))	115.224	422.565	0	8491.622
SI	Spiritual incentives provided by company (S(t))	13.346	85.244	0	2078.99
SE	Sales expenses (Control variable)	214.596	646.801	0.436	8212.937
AE	Administration expenses (Control variable)	195.353	453.538	5.873	6203.601
RD	Research and development expenses (Control var)	285.811	853.559	0.736	14797.025
Lscal	Enterprises scale- Large and middle (Control var)	0.969	0.173	0	1
TAX	Tax and surtax (Control variable)	21.622	70.36	0.3	1078.9
FinC	Financial costs (Control variable)	58.372	233.196	-1260	2650.154

Observations: 750

Table2. Regression results

OR	Coef.	St.Err.	t-value	Sig	OC	Coef.	St.Err.	t-value	Sig
ED	463.656	64.034	7.24	***	PI	13.91	2.236	6.22	***
SE	-2.708	0.904	-3	***	SI	117.975	7.851	15.03	***
AE	14.831	1.892	7.84	***	RD	8.288	1.196	6.93	***
RD	5.996	1.17	5.12	***	TAX	113.397	12.386	9.16	***
R-squared				0.639	R-squared				0.654
F-test				279.073	F-test				296.259
Number of obs				750	Number of obs				750

**** p<.01, ** p<.05, * p<.1* **** p<.01, ** p<.05, * p<.1*

3.2. Outcomes and results

3.2.1. Solve for the canonical steady state

Given estimated coefficients based on an overall situation of Chinese listed high-tech firms illustrated above, we can make efforts to explore the steady state and solve for the explicit solution of this optimal talent incentive mechanism problem. Plugging coefficient values into the model, we make the problem more direct and specific as follows:

$$Max \int_0^{\infty} e^{-rt} \pi_i \, dt$$

$$\pi_i = 463.656D(t) - (13.91P(t) + 117.975S(t))$$

$$s.t.\begin{cases} B\dot{(t)} = 120.097P(t)^{0.623}S(t)^{0.157} - \dfrac{1}{14.442}D(t)^{14.442} \\ P(0) = 0; S(0) = 0; D(0) = 0 \end{cases}$$

The optimal controls are reached:

$$\begin{cases} D(t)^* = \left(\dfrac{\omega}{\lambda^{CV}}\right)^{1/\theta-2} = \left(\dfrac{463.656}{\lambda^{CV}}\right)^{1/15.442-2} = \left(\dfrac{463.656}{\lambda^{CV}}\right)^{1/13.442} \\ P(t)^* = \left(\dfrac{\alpha_1}{\lambda^{CV}\beta_1\delta S(t)^{\beta_2}}\right)^{1/\beta_1-1} = \left(\dfrac{13.91}{\lambda^{CV}0.623 * 120.097S(t)^{0.157}}\right)^{1/0.623-1} = \left(\dfrac{13.91}{74.820\lambda^{CV}S(t)^{0.157}}\right)^{-1/0.377} \end{cases}$$

The canonical system is therefore

$$\begin{cases} S\dot{(t)} = 120.097\left(\dfrac{13.91}{74.820\lambda^{CV}S(t)^{0.157}}\right)^{-0.623/0.377} S(t)^{0.157} - \dfrac{1}{14.442}\left(\dfrac{463.656}{\lambda^{CV}}\right)^{14.442/13.442} \\ \lambda^{CV}\dot{(t)} = 0.0314\lambda^{CV} + 117.975 - 18.855\lambda^{CV}\left(\dfrac{13.91}{74.820\lambda^{CV}S(t)^{0.157}}\right)^{-0.623/0.377} S(t)^{-0.843} \end{cases}$$

With the assistant of Matlab, we can solve the canonical steady state after substituting each estimated coefficient values. The outcomes, keeping three decimals, are shown below,

$$\overline{S(t)} = \left(\frac{1}{15.442-1}D(t)^{*15.442-1}/120.097P(t)^{*0.623}\right)^{1/0.157} = 0.071$$

$$\overline{\lambda^{CV}(t)} = 117.975/\left(120.097 * 0.157P(t)^{*0.623}\overline{S(t)}^{0.157-1} - 0.0314\right) = 0.271$$

```
>> syms lumda S
eqns=[1936.47*(lumda^1.6525)*(S^0.4165)-50.689*lumda^(-1.074)==0,0.0314*lu
vars=[lumda S];
[lumda,S]=vpasolve(eqns,vars)

lumda =

- 0.27051555266533379521997521998218 + 0.28459026811573930556613234477718i


S =

0.071238253316722426811902891211791 - 0.012414965652257744747971082422257i
```

3.2.2. Steady state analysis

To analyze this steady state, we start with calculating eigenvalues. The Jacobian matrix mentioned in section 2 is obtained below:

$$
\boldsymbol{J} = \begin{bmatrix} 304.024\left(\frac{1}{\lambda^{CV}S(t)^{0.157}}\right)^{-1.653} S(t)^{-0.843} & 0 \\ \lambda^{CV}256.296\left(\frac{1}{\lambda^{CV}S(t)^{0.157}}\right)^{-1.653} S(t)^{-1.843} & 0.0314 - 304.024\left(\frac{1}{\lambda^{CV}S(t)^{0.157}}\right)^{-1.653} S(t)^{-0.843} \end{bmatrix}
$$

Then, constructing $|J - \varphi I| = 0$, we have

$$
\left(304.024\left(\frac{1}{\lambda^{CV}S(t)^{0.157}}\right)^{-1.653} S(t)^{-0.843} - \varphi_1\right)\left(0.0314 - 304.024\left(\frac{1}{\lambda^{CV}S(t)^{0.157}}\right)^{-1.653} S(t)^{-0.843} - \varphi_2\right) = 0
$$

Replacing λ^{CV} and $S(t)$ with the canonical steady state level $\overline{\lambda^{CV}(t)}$ and $\overline{S(t)}$, two eigenvalues can be carried out:

$$
\varphi_1 = 304.024\left(\frac{1}{\overline{\lambda^{CV}(t)S(t)}^{0.157}}\right)^{-1.653} \overline{S(t)}^{-0.843} = 164.389
$$

$$
\varphi_2 = 0.0314 - 304.024\left(\frac{1}{\overline{\lambda^{CV}(t)S(t)}^{0.157}}\right)^{-1.653} \overline{S(t)}^{-0.843} = -164.358
$$

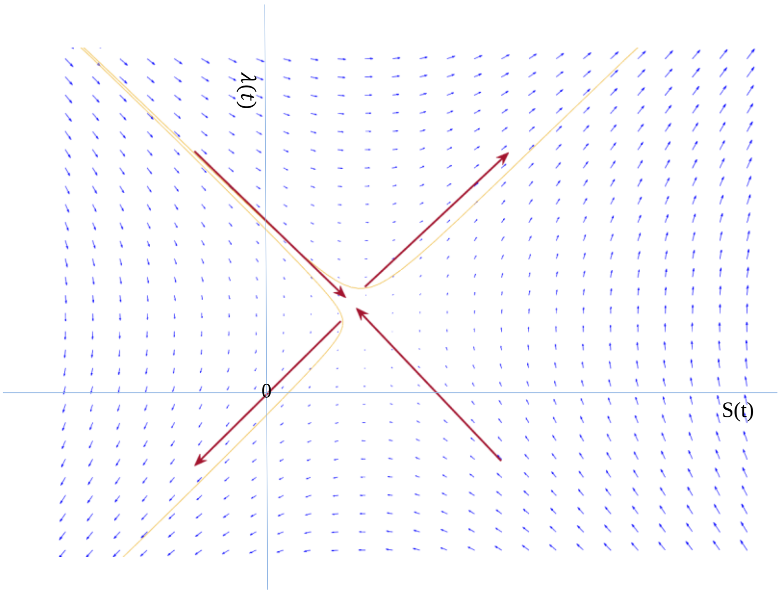
Hence, it is illustrated clearly that the first eigenvalue is indeed positive, which proved the result analyzed in section 2.4. As for the second eigenvalue, with specific coefficient levels, it is a negative number. Therefore, two eigenvalues are two real numbers and are of different signs, implying that this unique canonical steady state is a saddle with all trajectories converge to equilibrium from one direction, but diverge from it in the other.

Since our model contains non-linear parts, finding an explicit solution is hard even with the help of mathematical software.

```
>> syms S(t) lumda(t)
>> eqns=[diff(S,t)==1936.47*(1/(lumda(t)*S(t)^0.157))^(-1.653)*S(t)^0.157-50.689*(1/lumda(t))^(1.074),
>> cond=[S(0)==0,lumda(0)==0];
>> [S(t),lumda(t)]=dsolve(eqns)
Warning: Unable to find explicit solution.
```

Therefore, phase space analysis is what we pursued. For an optimal control problem with single state, the phase space will be a plane and in our case, it is a plane of $S(t)$ and λ^{CV}. Usually, we adopt direction fields and solution curves to study the equilibrium. Involving boundary and transversality conditions, the following trajectory graph depicts our equilibrium:

Figure 1



4. Economic Interpretation of Solution

4.1. The co-state variable

The co-state variable $\lambda(t)$ in the present-value Hamiltonian function measures the sensitivity of H to one unit shift of the dynamic constraint in one time period. In the economics field, this co-state variable can also be defined as shadow price. When the social economy is in a certain optimal state, the marginal contribution of each different resource is defined as its shadow price.

In our model, a current-value Hamiltonian is applied instead of present-value one because our objective function contains a discount factor to reflect the philosophy that future is valued less than today in the economics and finance fields. Therefore, $\lambda(t)$ is transformed to $\lambda^{CV}(t)$ with $\lambda^{CV} := e^{rt}\lambda(t)$, which interprets the effect of current one unit shift of constraint on future profit level. Our result of $\overline{\lambda^{CV}(t)} = 0.271$ shows that, at steady state, if there is a high-tech firm in China who shares all the features with the averaged level of overall listed high-tech companies in terms of annual revenue, talents number, expenditures on incentives, operating scales and so on, the extra one unit of employee benefit increase will contribute to 0.271 unit of the future profit to this firm.

Additionally, it is interesting to notice that $\overline{\lambda^{CV}(t)}$ calculated in our model is a positive value. Before, we may intuitively consider a negative relationship between the company's gain and employee's gain, because for high-tech firms, workers' wages and bonuses may cost a fortune, which holdback firms profit margin. Now we notice that investing in talents incentives mechanism and provide them with higher benefit is actually a win-win strategy in China. Indeed, firms sacrifice more at the moment, but once employees receive benefits, they tend to devote more to their jobs, creating greater revenue for their company in the future.

4.2. The steady state

After filling the model with numerical coefficients based on the real situation of Chinese high-tech firms in terms of their talents incentives investment and financial data in year 2020, the canonical steady state level is reached.

$\overline{\lambda^{CV}(t)} = 0.271$ is the shadow price interpreted above. As for $\overline{S(t)} = 0.071$, it implies that at the equilibrium, when the profit growth is constant and the growth rate is zero, the equilibrium level of companies' investment on talents' spiritual incentives is 0.071 million yuan. Compared with those A-shares' 2020 average operating revenue of 4563.185 million yuan, this amount of investment not even made up to 0.01% of the revenue. Technical workers in Chinese IT industry sacrifices too much at the current steady state level. This result confirms a common phenomenon recently in China called 'nine nine six', referring to working from 9am to 9pm, 6 days a week, with low overtime compensation. This result may also help to explain why the current talent turnover rate is so high in Chinese IT industry.

5. Conclusion and Suggestion

Talents retention plays an increasingly imperative role in China to enhance a company's competitive advantages, especially for those highly mental-demanded industry like high-tech industry. However, how to balance the investment in talents and company's profit margin, and handle the high talent turnover rate is tricky. This paper first construct appropriate models and generate steady state analysis for individual firm's use based on their own parameters. Secondly, based on the real situation of A-share listed high-tech firms, the average industry situation is analyzed through financial data collection and coefficients estimation. Finally, a long run equilibrium has been solved out, implying Chinese high-tech firms may invest too little, with only 0.071 million yuan (less than 0.01% of the average operation revenue) per year in talent's benefits at current steady state. Since the co-state variable is positive, larger investment in talents' welfare returns positively on firm's profit margin, it is suggested that when adjusting or establishing the incentive mechanism, more proportional of investments is preferred, especially the spiritual incentive part.

(word account: 2966)

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