✯ Minimum Jerk Trajectories.

- Start from the standard quintic normalized displacement function :

$$s(\tau) = a_0 + a_1\tau + a_2\tau^2 + a_3\tau^3 + a_4\tau^4 + a_5\tau^5, \quad \tau \in [0,1].$$

For each reaching, we impose the six common boudaries constraints (start/end position, velocity, and acceleration):

$s(0) = 0$
$s(1) = 1$
$s'(1) = 0$
$s'(0) = 0$
$s''(0) = 0$
$s''(\phi) = 0$

1. Apply three constraints $\tau = 0$

$s(0) = a_0 = 0$

$s'(\tau) = a_1 + 2a_2\tau + 3a_3\tau^2 + 4a_4\tau^3 + 5a_5\tau^4 \Rightarrow s'(0) = a_1 = 0$

$s''(\tau) = \cdots\cdots\cdots \Rightarrow s''(0) = a_2 = 0$

Thus, simplification is

$$s(\tau) = a_3\tau^3 + a_4\tau^4 + a_5\tau^5$$

2. Apply $\tau = 1$

$s(1) = a_3 + a_4 + a_5 = 1$
$s'(1) = 3a_3 + 4a_4 + 5a_5 = 0$
$s''(1) = 6a_3 + 12a_4 + 20a_5 = 0$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 3 & 4 & 5 \\ 6 & 12 & 20 \end{bmatrix} \begin{bmatrix} a_3 \\ a_4 \\ a_5 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

Solving → $a_3 = 10$, $a_4 = -15$, $a_5 = 6$

Thus, $s(\tau) = 10\tau^3 - 15\tau^4 + 6\tau^5$.