

作業 1. Cobb-Douglas 生產函數: $Q = f(L, K) = L^\alpha K^\beta$, $\alpha, \beta > 0$

① AP_L (平均勞動產量)

$$AP_L = \frac{Q}{L} = \frac{L^\alpha K^\beta}{L} = L^{\alpha-1} K^\beta$$

② AP_K (平均資本產量)

$$AP_K = \frac{Q}{K} = \frac{L^\alpha K^\beta}{K} = L^\alpha K^{\beta-1}$$

③ MP_L (勞動邊際產量)

$$MP_L = \frac{dQ}{dL} = \alpha L^{\alpha-1} K^\beta$$

④ MP_K (資本邊際產量)

$$MP_K = \frac{dQ}{dK} = \beta L^\alpha K^{\beta-1}$$

⑤ $MRTS$ (邊際替代率)

$$MRTS = \frac{-dK}{dL} = \frac{MP_L}{MP_K} = \frac{\alpha L^{\alpha-1} K^\beta}{\beta L^\alpha K^{\beta-1}} = \frac{\alpha K}{\beta L}$$

⑥ ε^L (勞動產量彈性)

$$\varepsilon^L = \frac{\frac{dQ}{Q}}{\frac{dL}{L}} = \frac{\frac{dQ}{dL}}{\frac{Q}{L}} = \frac{MP_L}{AP_L} = \frac{\alpha L^{\alpha-1} K^\beta}{L^{\alpha-1} K^\beta} = \alpha$$

⑦ ε^K (資本產量彈性)

$$\varepsilon^K = \frac{\frac{dQ}{Q}}{\frac{dK}{K}} = \frac{\frac{dQ}{dK}}{\frac{Q}{K}} = \frac{MP_K}{AP_K} = \frac{\beta L^\alpha K^{\beta-1}}{L^\alpha K^{\beta-1}} = \beta$$

⑧ ε^Q (生產力彈性)

$$\varepsilon^Q = \frac{\frac{dQ}{Q}}{\frac{dK}{K}} = \frac{\frac{dQ}{dK}}{\frac{Q}{K}} = \frac{MP_K}{AP_K} = \beta$$

⑨ $\varepsilon^{L,K}$ (替代彈性)

$$\varepsilon^{L,K} = \varepsilon^L + \varepsilon^K = \alpha + \beta$$

作業 2. 隨堂

假設生產函數的型式為 $Q = 3K + 2L$ 。其中, K 為資本, L 為勞動, Q 為產出。考慮生產函數 3 個敘述, 請選正確選項

(1) 函數呈現固定規模報酬 (正確)

當 L & K 增加 n 倍 $\rightarrow nK$ & nL , 生產函數為 $F(nL, nK) = 2(nL) + 3(nK) = n(2L + 3K) = nQ$, 故呈現固定規模報酬。

(2) 函數呈向下遞減 (不正確)

$MP_L = \frac{dQ}{dL} = 2$, $MP_K = \frac{dQ}{dK} = 3$, MP_K & MP_L 皆固定, 故無遞減生產力 \downarrow 。

(3) 函數呈現固定的技術替代率 (正確)

$MRTS = \frac{MP_L}{MP_K} = \frac{2}{3}$, 技術替代率 ($MRTS$) 成固定值 ($\frac{2}{3}$)