$$y = m_1 x_1 + m_2 x_2 + c$$

$$\text{error} = \sum_{i}^{N} \left(y - (m_1 x_1 + m_2 x_2 + c)\right)^2$$

$$\frac{\partial\, \text{error}}{\partial m_1} = \sum_{i}^{N} \left(y^i - (m_1 x_1^i + m_2 x_2^i + c)\right) x_1^i$$

$$= \sum_{i}^{N} y \cdot x_1 - (m_1 x_1^2 + m_2 x_2 x_1 + c x_1)$$

$$= \sum_{i}^{N} \frac{y \cdot x_1}{N} - \left(\frac{m_1 x_1^2}{N} + \frac{m_2 x_2}{N} + \frac{c x_1}{N}\right)$$

$$= y.x_1.\text{mean}() - (m_1 x_1^2.\text{mean}() + m_2 x_2.\text{mean}() + c.x_1.\text{mean}())$$

$$= y.x_1.\bar{m} - (m_1 x_1^2.\bar{m}) - m_2 x_2.\bar{m} - c.x_1.\bar{m}$$

$$\frac{\partial\, \text{error}}{\partial m_2} = y x_2.\bar{m} - (m_1 x_1 x_2.\bar{m}) - m_2 x_2^2.\bar{m} - c x_2.\bar{m}$$

$$\frac{\partial\, \text{error}}{\partial c} = \sum_{i}^{N} \left(y - (m_1 x_1 + m_2 x_2 + c)\right)$$

$$\Rightarrow \sum_{i}^{N} y - m_1 x_1 - m_2 x_2 - c = 0$$

$$\Rightarrow \sum_{i}^{N} \frac{y}{N} - \frac{m_1 x_1}{N} - \frac{m_2 x_2}{N} = c$$

$$\Rightarrow y.\text{mean}() - m_1 x_1.\text{mean}() - m_2 x_2.\text{mean}() = c$$

$$\Rightarrow y x_1.\bar{m} - (m_1 x_1^2.\bar{m}) - (m_2 x_2 x_1.\bar{m}) + c.x_1\bar{m} = 0 \quad \ldots (i)$$

$$\Rightarrow y x_2.\bar{m} - (m_1 x_1 x_2.\bar{m}) - (m_2 x_2^2.\bar{m}) + c x_2.\bar{m} = 0 \quad \ldots (ii)$$

$$\Rightarrow c = y.\bar{m} - m_1 x_1.\bar{m} - m_2 x_2.\bar{m} \quad \ldots (iii)$$

putting (iii) in (ii)

$$y x_2.\bar{m} - (m_1 x_1 x_2.\bar{m}) - (m_2 x_2^2.\bar{m}) + (y.\bar{m} - m_1 x_1.\bar{m} - m_2 x_2.\bar{m}) x_2.\bar{m} = 0$$

$$y x_2.\bar{m} - (m_1 x_1 x_2.\bar{m}) - (m_2 x_2^2.\bar{m}) + ((y.\bar{m})(x_2.\bar{m})) - m_1 (x_1.\bar{m})(x_2.\bar{m}) - m_2 (x_2.\bar{m})^2 = 0$$

$$y x_2.\bar{m} - (m_1 x_1 x_2.\bar{m}) + ((y.\bar{m})(x_2.\bar{m})) - m_1 (x_1.\bar{m})(x_2.\bar{m}) = m_2 (x_2.\bar{m})^2 + m_2 (x_2^2.\bar{m})$$

$$\frac{y x_2.\bar{m} - (m_1 x_1 x_2.\bar{m}) + ((y.\bar{m})(x_2.\bar{m})) - m_1 (x_1.\bar{m})(x_2.\bar{m})}{(x_2.\bar{m})^2 + (x_2^2.\bar{m})} = m_2$$

$$\downarrow \quad \ldots (iv)$$

putting (iii) in (i)

$$y x_1.\bar{m} - (m_1 x_1^2.\bar{m}) - m_2 (x_2 x_1.\bar{m}) + (y.\bar{m} - m_1 x_1.\bar{m} - m_2 x_2.\bar{m}) x_1.\bar{m} = 0$$

$$y x_1.\bar{m} - (m_1 x_1^2.\bar{m}) - m_2 (x_2 x_1.\bar{m}) + (y.\bar{m})(x_1.\bar{m}) - m_1 (x_1.\bar{m})^2 - m_2 ((x_2.\bar{m})(x_1.\bar{m})) = 0$$

$$\ldots (v)$$

putting m_2 from (iv). in (v)

$$y x_1 \cdot \bar{m} - (m_1 x_1^2 \cdot \bar{m}) + (y \cdot \bar{m})(x_1 \cdot \bar{m}) - m_1 (x_1 \cdot \bar{m})^2$$
$$- m_2 (x_2 x_1 \cdot \bar{m} + (x_2 \cdot \bar{m})(x_1 \cdot \bar{m})) = 0$$

$$y x_1 \cdot \bar{m} - (m_1 x_1^2 \cdot \bar{m}) + (y \cdot \bar{m})(x_1 \cdot \bar{m}) - m_1 (x_1 \cdot \bar{m})^2$$

$$- (y x_2 \cdot \bar{m} - (m_1 x_1 x_2 \cdot \bar{m}) + ((y \cdot \bar{m})(x_2 \cdot \bar{m}))$$
$$- m_1 (x_1 \cdot \bar{m})(x_2 \cdot \bar{m}))$$
$$\frac{(x_2 x_1 \cdot \bar{m} + (x_2 \cdot \bar{m})(x_1 \cdot \bar{m}))}{(x_2 \cdot \bar{m})^2 + (x_2^2 \cdot \bar{m})} = 0$$

$$\text{Let} \quad \frac{x_2 x_1 \cdot \bar{m} + (x_2 \cdot \bar{m})(x_1 \cdot \bar{m})}{(x_2 \cdot \bar{m})^2 + (x_2^2 \cdot \bar{m})} = \alpha$$

$$y x_1 \cdot \bar{m} - (m_1 x_1^2 \cdot \bar{m}) + (y \cdot \bar{m})(x_1 \cdot \bar{m}) - m_1 (x_1 \cdot \bar{m})^2$$

$$- \alpha (y x_2 \cdot \bar{m} - (m_1 x_1 x_2 \cdot \bar{m}) + ((y \cdot \bar{m})(x_2 \cdot \bar{m}))$$
$$- m_1 (x_1 \cdot \bar{m})(x_2 \cdot \bar{m}) = 0$$

$$y x_1 \cdot \bar{m} + (y \cdot \bar{m})(x_1 \cdot \bar{m}) - \alpha y x_2 \cdot \bar{m} - \alpha ((y \cdot \bar{m})(x_2 \cdot \bar{m}))$$
$$- m_1 x_1^2 \cdot \bar{m} - m_1 (x_1 \cdot \bar{m})^2 + \alpha (m_1 x_1 x_2 \cdot \bar{m})$$
$$+ \alpha m_1 (x_1 \cdot \bar{m})(x_2 \cdot \bar{m}) = 0$$

$$y x_1 \cdot \bar{m} + (y \cdot \bar{m})(x_1 \cdot \bar{m}) - \alpha (y x_2 \cdot \bar{m}) - \alpha ((y \cdot \bar{m})(x_2 \cdot \bar{m}))$$
$$= m_1 x_1^2 \cdot \bar{m} + m_1 (x_1 \cdot \bar{m})^2 - \alpha (m_1 x_1 x_2 \cdot \bar{m})$$
$$- \alpha m_1 (x_1 \cdot \bar{m})(x_2 \cdot \bar{m}) = 0$$

$$y x_1 \cdot \bar{m} + (y \cdot \bar{m})(x_1 \cdot \bar{m}) - \alpha(y x_2 \cdot \bar{m}) - \alpha((y \cdot \bar{m})(x_2 \cdot \bar{m}))$$

$$= m_1 x_1^2 \cdot \bar{m} + m_1 (x_1 \cdot \bar{m})^2 - \alpha(m_1 x_1 x_2 \cdot \bar{m})$$

$$- \alpha(m_1)(x_1 \cdot \bar{m})(x_2 \cdot \bar{m}) = 0$$

$$y x_1 \cdot \bar{m} + (y \cdot \bar{m})(x_1 \cdot \bar{m}) - \alpha(y x_2 \cdot \bar{m}) - \alpha((y \cdot \bar{m})(x_2 \cdot \bar{m}))$$

$$= m_1 (x_1^2 \cdot \bar{m} + (x_1 \cdot \bar{m})^2 - \alpha x_1 x_2 \cdot \bar{m}$$

$$- \alpha(x_1 \cdot \bar{m})(x_2 \cdot \bar{m})) = 0$$

$$\frac{y x_1 \cdot \bar{m} + (y \cdot \bar{m})(x_1 \cdot \bar{m}) - \alpha(y x_2 \cdot \bar{m}) - \alpha((y \cdot \bar{m})(x_2 \cdot \bar{m}))}{x_1^2 \cdot \bar{m} + (x_1 \cdot \bar{m})^2 - \alpha x_1 x_2 \cdot \bar{m} - \alpha(x_1 \cdot \bar{m})(x_2 \cdot \bar{m})} = m_1$$