Solving a split feasibility problem

This repository contains the Matlab code used in our paper [1]. We investigated a new algorithm for the solving of so-called monotone inclusions and applied it to the split feasibility problem

$$x \in C \text{ and } Lx \in Q, \tag{1}$$

where

- C $:= \left\{ x \in L^2([0,2\pi]) : \int_0^{2\pi} x(t)dt \leq 1 \right\}$ and $Q := \left\{ x \in L^2([0,2\pi]) : \int_0^{2\pi} |x(t) - \sin(t)|^2 dt \leq 16 \right\}$ subsets of $L^2([0,2\pi]) := \left\{ f : [0,2\pi] \to \mathbb{R} : \int_0^{2\pi} |f(t)|^2 dt < +\infty \right\}$.
- $L : L^2([0,2\pi]) \to L^2([0,2\pi])$, $(Lx)(t) := \int_0^{2\pi} x(s)ds \cdot t$ a linear operator.

This can be equivalently rewritten as

$$\min_{x \in \mathcal{H}} \left\{ \delta_C(x) + \delta_Q(Lx) \right\},$$

where

$$\delta_S(x) := \begin{cases} 0, & \text{if } x \in S \\ +\infty, & \text{else} \end{cases}$$

denotes the indicator function of a subset S of a Hilbert space. Another alternative way to write problem (1) is as the minimization problem

$$\min_{x \in \mathcal{H}} \left\{ \frac{1}{2} d_C^2(x) + \delta_Q(Lx) \right\},$$

where $d_C(x) = \inf_{u \in \mathcal{C}} \|x - u\|$ denotes the distance of the point $x \in \mathcal{H}$ to the set C. Our developed primal-dual algorithms are applied to both of the above minimization problems. The Matlab code consists of parts solving both problems and is commented accordingly.

Remark 1. *The code is based on symbolic computation, i.e. no discretization of the involved functions is performed. This is due to the fact that we want to fully exploit the Tikhonov regularization in our algorithm which guarantees strong convergence. For more details we refer the reader to [1, Section 5].*

References

[1] R. Boţ, R. Csetnek, D. Meier *Inducing strong convergence into the asymptotic behaviour of proximal splitting algorithms in Hilbert spaces,* Optimization Methods and Software 34(3), 489-514, 2019.