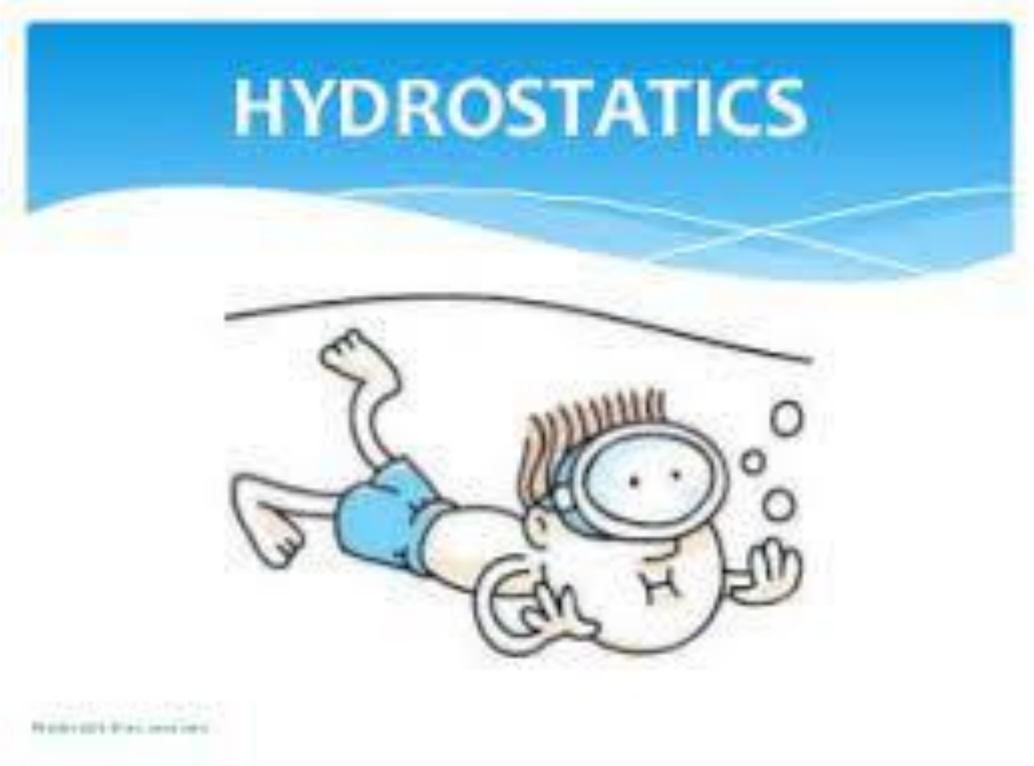

PHY 101

GENERAL PHYSICS I

HYDROSTATICS

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INTRODUCTION

- **Hydrostatics** deals with the equilibrium of fluids and with the forces acting on them when at rest.
- It is based on Newton's 1^{st} and 3^{rd} laws.

- **A fluid** is any state of matter that flows when there is an applied shear stress.

- There exist three distinct states of matter, viz:
  - Solid
  - Liquid
  - Gases

- An example of fluid being water can exist in the three distinct states of matter.
- When frozen, it takes the form of a solid (ice); at ordinary temperatures it is a liquid (water); and when boiled by heating, it becomes a vapour or gas (steam).

CHARACTERISTICS OF SOLIDS

- **Solids** are substances which tends to keep the same shape for an indefinite length of time, and whose various parts cannot move freely among themselves.
- True solids are not incompressible but can be molded from one shape into another by applying considerable forces or pressure to them. They do not yield to "the slightest" force.
- In some cases, the force between molecules can cause the molecules to organize into a lattice as shown in the Fig. 1.
- The structure of this three-dimensional lattice is represented as molecules connected by rigid bonds (modeled as stiff springs), which allow limited freedom for movement.
- Rigidity is the property of which a body tends to retain the same shape permanently. Hence, a solid is distinguished from a fluid by being rigid.
- Solids also resist shearing forces.

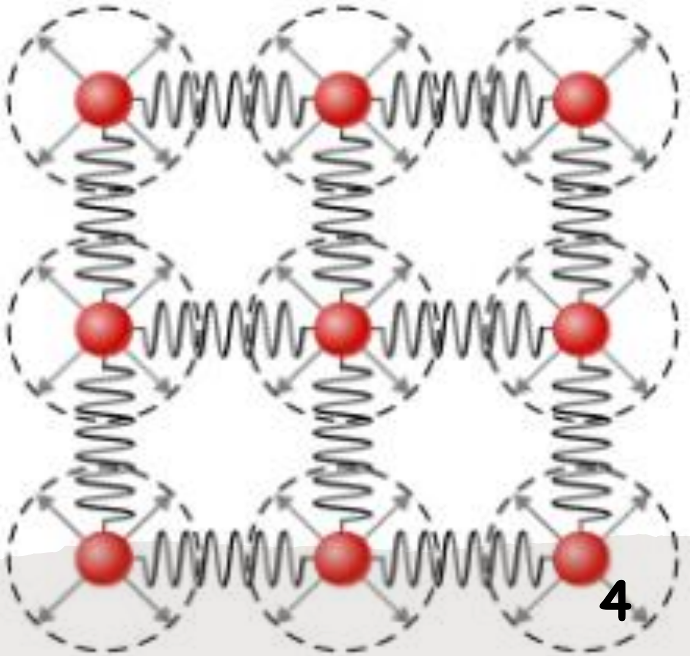


Fig. 1

CHARACTERISTICS OF FLUIDS

- **Fluids** are substances which yields to any force, however small, tending to change its shape or to produce movement of its parts among themselves.
- Both **liquids and gases** (e.g., water and gases) are fluids.
- The molecules in a liquid are bonded to neighboring molecules but possess many fewer of these bonds.
- The molecules in a liquid are not locked in place and can move with respect to each other.
- The distance between molecules is similar to the distances in a solid, and so liquids have definite volumes, but the shape of a liquid changes, depending on the shape of its container.
- Atoms in a liquid are also in close contact but can slide over one another.
- Forces between the atoms strongly resist attempts to compress the atoms.

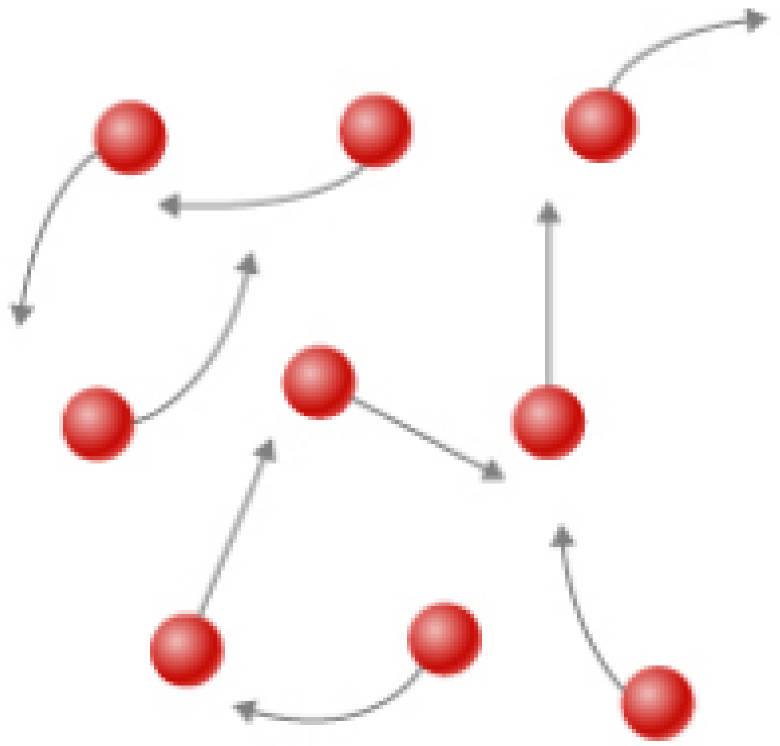


Fig. 2

- Liquids deform easily when stressed and do not spring back to their original shape once a force is removed.
- This occurs because the atoms or molecules in a liquid are free to slide about and change neighbours. That is, liquids flow (so they are a type of fluid), with the molecules held together by mutual attraction.
- When a liquid is placed in a container with no lid, it remains in the container.
- Because the atoms are closely packed, liquids, like solids, resist compression; an extremely large force is necessary to change the volume of a liquid.

- Gases are not bonded to neighboring atoms and can have large separations between molecules.
- Gases have neither specific shapes nor definite volumes, since their molecules move to fill the container in which they are held (**Fig. 3**).
- Atoms in a gas move about freely and are separated by large distances.
- A gas must be held in a closed container to prevent it from expanding freely and escaping.
- In contrast, atoms in gases are separated by large distances, and the forces between atoms in a gas are therefore, very weak, except when the atoms collide with one another.
- This makes gases relatively easy to compress and allows them to flow (which makes them fluids).
- When placed in an open container, gases, unlike liquids, will escape.

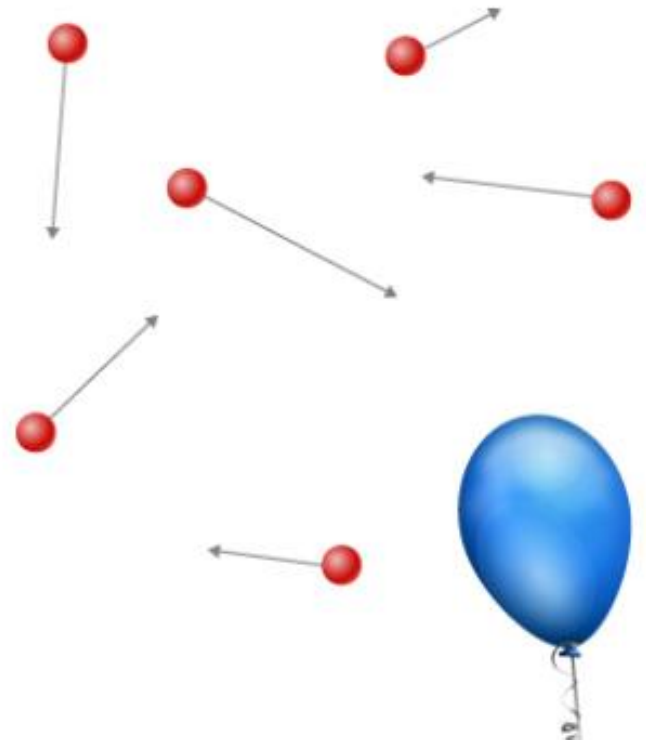


Fig. 3

The Difference

- **A liquid** is a fluid whose volume will not increase beyond a certain limit, and which offers a very great resistance to any decrease of volume,
- **A gas** is a fluid which always tends to occupy as large a volume as possible, but which may be readily forced to occupy any space, however small.
- If a bottle is half full of water, the water cannot be made to occupy either more or less than half of the bottle. If the bottle is full, we cannot get any more water in by squeezing, nor can we squeeze the water into a smaller space by pushing a cork in or otherwise.
- On the other hand, any amount of air can be compressed into a bottle, or, again, part of the air in a bottle may be sucked out (by means of an air pump) and then the remainder will still continue to occupy the whole of the bottle.

Compressibility and Elasticity

- A liquid is called incompressible when it cannot be forced to occupy a smaller volume.
- No liquid is perfectly incompressible; by means of great pressure, water may be forced to occupy a slightly smaller bulk, but in hydrostatics, liquids may be treated as incompressible.
- Again, liquids are called inelastic, because they have no tendency to expand and increase in bulk.
- A gas is always compressible, because it can be easily compressed into any volume.
- Gases are called elastic, because they tend to expand to occupy a large space as possible.

Perfect and Viscous Fluids

- Although all fluids eventually yield to changes of shape or to stirring, different fluids behave differently while changing their shape or being stirred.
- Some seem to yield very readily, others only with apparent reluctance.
- Water may be stirred up easily and quickly, and little resistance will be experienced.
- But honey can only be stirred with difficulty, and the faster we try to stir it the more resistance we encounter.
- If, however, we were to stir it sufficiently slowly, we should feel hardly any resistance, showing that honey is not solid.
- But the resistance always tends to retard the passage of the spoon through the honey.

- **A perfect fluid** is one whose parts can move among themselves without retardation.

- **A viscous fluid** is one which continually retards the motion of its parts among themselves.

- Practically, there is no such thing as a perfect fluid. If water were a perfect fluid, a ship when once set in motion would continue to move through it without ever stopping.
- Air and some gases much more closely resemble the ideal perfect fluid, but a bullet experiences considerable resistance from the air. Hence air is not a perfect fluid.
- At the same time, some fluids are much more viscous than others.
- Viscosity of fluids does not affect their equilibrium but only their motion.

DENSITY

Density

- Density is an important property/characteristics of any material/substance.
- **The density of a small amount of matter is defined to be the amount of mass ΔM divided by the volume ΔV of that element of matter.**

$$\rho = \Delta M / \Delta V \qquad (1)$$

- The SI unit for density is the kilogram per cubic meter, $kg \cdot m^{-3}$.

If the density of a material is the same at all points, then the density is given by

$$\rho = M / V \qquad (2)$$

Where M is the mass and V is the volume of the materials, respectively.

- A material with constant density is called **homogeneous.**
- For a homogeneous material, density is an intrinsic property.
- If the material is divided in two parts, the density will be the same in both halves.

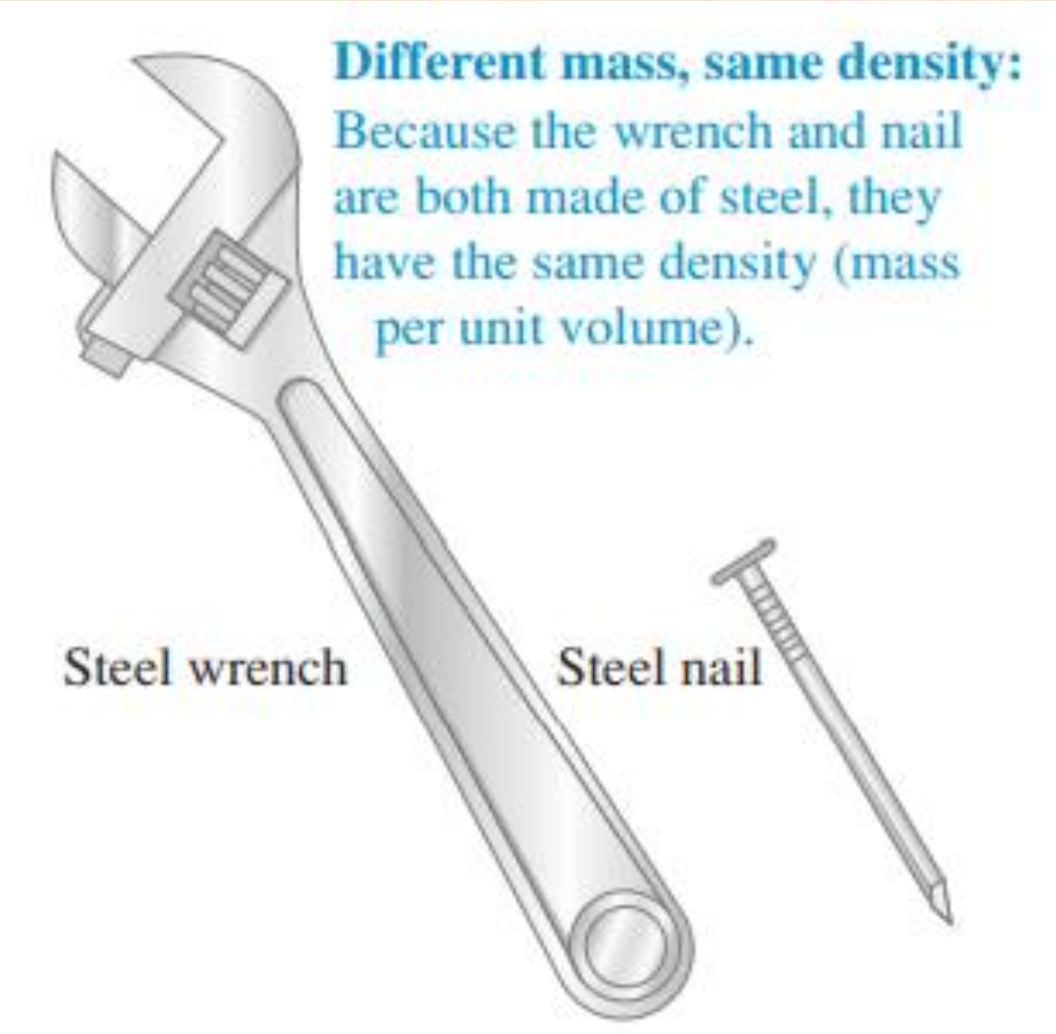


Fig. 4

- Two objects (Fig. 4) made of the same material will have the same density even though they may have different masses and different volumes. That’s because the ratio of mass to volume is the same for both objects

$$\rho = \rho_1 = \rho_2 \qquad (3)$$

- However, mass and volume are extrinsic properties of the material.
- If we divide the material into two parts, the mass is the sum of the individual masses

$$M = M_1 + M_2 \qquad (4)$$

- So, also the volume

$$V = V_1 + V_2 \qquad (5)$$

- Suppose a block of brass and a block of wood have the same weight and mass **(Fig. 5a)**.
- If both blocks are dropped in a tank of water, why does the wood float and the brass sink **(Fig, 5b)**?
- This occurs because the brass has a greater density than water, whereas the wood has a lower density than water.

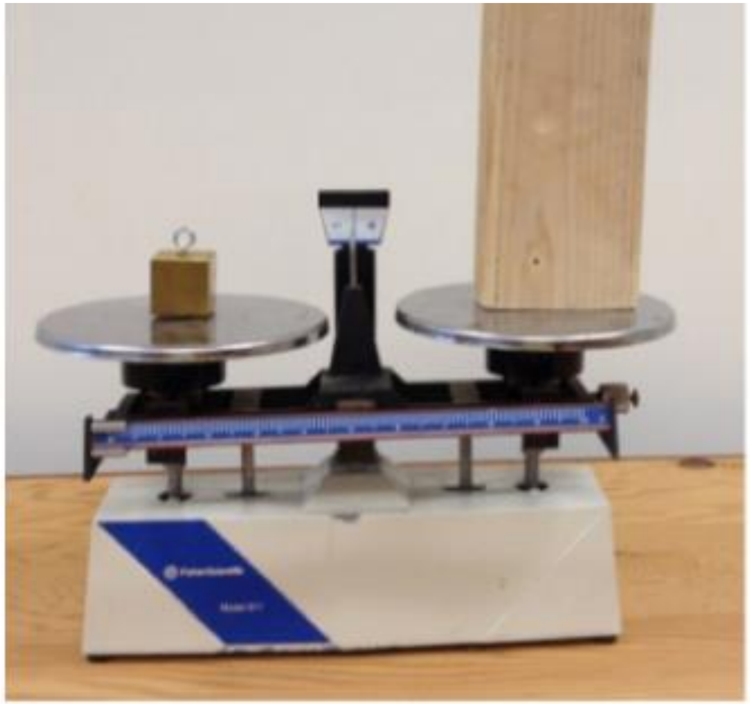


Fig. 5a

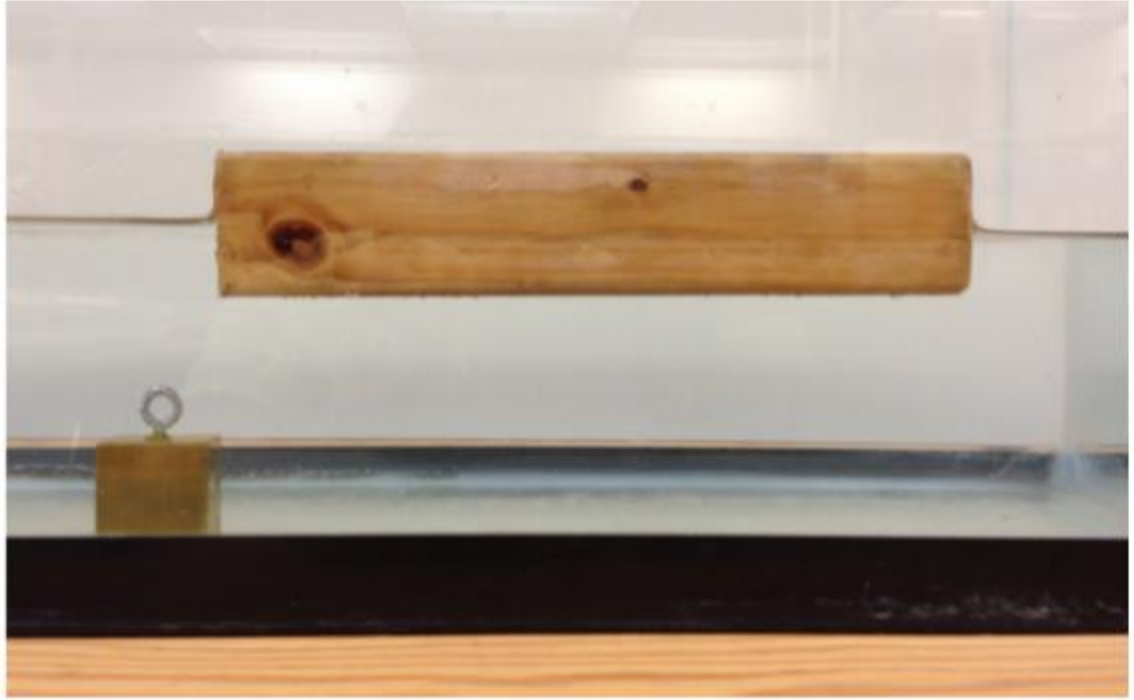


Fig. 5b

Density for Various Materials (Unless otherwise noted, all densities given are at standard conditions for temperature and pressure, that is, 273.15 K (0.00 °C) and 100 kPa (0.987 atm)

Table 1: Densities of some common materials

Material	Density (kg/m^3)*	Material	Density (kg/m^3)*
Air (1 atm, 20°C)	1.20	Iron, steel	7.8×10^3
Ethanol	0.81×10^3	Brass	8.6×10^3
Benzene	0.90×10^3	Copper	8.9×10^3
Ice	0.92×10^3	Silver	10.5×10^3
Water	1.00×10^3	Lead	11.3×10^3
Seawater	1.03×10^3	Mercury	13.6×10^3
Blood	1.06×10^3	Gold	19.3×10^3
Glycerine	1.26×10^3	Platinum	21.4×10^3
Concrete	2×10^3	White dwarf star	10^{10}
Aluminum	2.7×10^3	Neutron star	10^{18}

Relative Density/Specific Gravity

- The relative density of a material is the ratio of its density to the density of water at $4.0℃, 1000\ kg/m^3$ and one atmosphere of pressure, which is $1000\ kg/m^3$;

  it is a pure number without units.

$$Special\ gravity = \frac{Density\ of\ material}{Density\ of\ water}$$

- For example, the relative density of aluminum is 2.7.
- The density of some materials varies from point to point within the material (See Fig. 6)

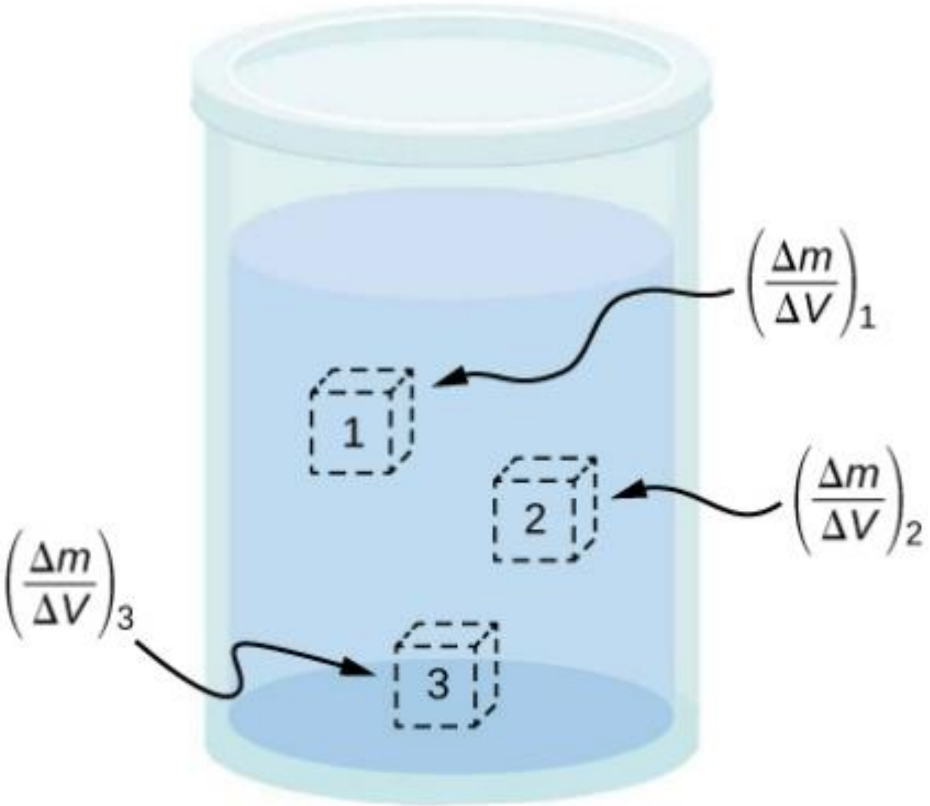


Fig. 6: Density may vary throughout a heterogeneous mixture. Local density at a point is obtained from dividing mass by volume in a small volume around a given point.

- Such materials are called **heterogeneous materials/substance,** e.g., the human body, which includes low-density fat (about $940\, kg/m^3$) and high-density bone (from $1700\ to\ 2500\, kg/m^3$).

- Two other examples are:
  - earth's atmosphere (which is less dense at high altitudes) and
  - oceans (which are denser at greater depths).
- In general, the density of a material depends on environmental factors.

Example 1

- Find the mass and weight of the air at $20°C$ in a living room with a $4.0\ m \times 5.0\ m$ floor and a ceiling $3.0\ m$ high, and the mass and weight of an equal volume of water.

Solution

- $V = (l)(b)(h)$
- $V = (4.0\ m)(5.0\ m)(3.0\ m) = 60\ m^3$
- $\therefore\ m_{air} = \rho_{air} V$
- $m_{air} = (1.20\ kg/m^3)(60\ m^3)$

$$= 72\ kg$$

- $w_{air} = m_{air} g$

$$w_{air} = (72\ kg)(9.8\ m/s^2) = 700\ N$$

The mass and weight of an equal volume of water are:

- $m_{water} = \rho_{water} V$

$$m_{water} = (1000\ kg/m^3)(60\ m^3)$$

$$= 6.0 \times 10^4\ kg$$

- $w_{water} = m_{water} g$

$$w_{water} = (6.0 \times 10^4\ kg\ )(9.8\ m/s^2)$$

$$= 5.9 \times 10^5\ \mathrm{N}$$

Question

- Rank the following objects in order from highest to lowest average density:
- (i) mass $4.00\ kg$, volume $1.6 \times 10^{-3}\ m^3$
- (ii) mass $8.00\ kg$, volume $1.6 \times 10^{-3}\ m^3$
- (iii) mass $8.00\ kg$, volume $3.20 \times 10^{-3}\ m^3$
- (iv) mass $2560\ kg$, volume $0.640\ m^3$
- (v) mass $2560\ kg$, volume $1.28\ m^3$

PRESSURE

Pressure in a Fluid

- Hydrostatic Pressure is experienced by a fluid (such as liquid) at REST.
- Two important points:
  - A fluid will exert a pressure in all direction.
  - A fluid will exert a pressure perpendicular to any surface it compacts.

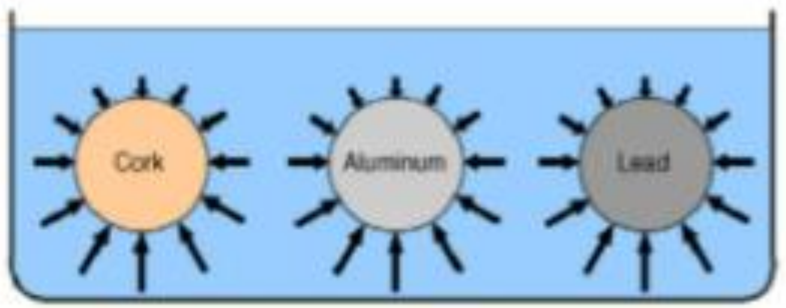


Fig. 7

- Notice that the arrows on TOP of the objects are smaller than at the BOTTOM. This is because pressure is greatly affected by the **DEPTH** of the object. Since the bottom of each object is deeper than the top, the pressure is greater at the bottom.

- When a fluid (either liquid or gas) is at rest, it exerts a force perpendicular to any surface in contact with it, such as a container wall or a body immersed in the fluid.
- This is the force that is felt when a leg is dangled in a swimming pool.
- While the fluid as a whole is at rest, the molecules that make up the fluid are in motion; the force exerted by the fluid is therefore due to the molecules colliding with their surroundings.
- If an imaginary surface is placed within the fluid, the fluid on the two sides of the surface will exerts equal and opposite forces on the surface. Otherwise, the surface would accelerate, and the fluid would not remain at rest.
- For a static fluid, these forces must sum to zero.

- Considering a small surface of area dA centered on a point in the fluid; the normal force exerted by the fluid on each side is $dF_{\perp}$(Fig. 8a)

Pressure p at that point is defined as the normal force per unit area (Fig. 8b) i.e.,

$$P = \frac{dF_{\perp}}{dA} \qquad (1)$$

- If the pressure is the same at all points of a finite plane surface with area A, then

$$P = \frac{F_{\perp}}{A} \qquad (2)$$

where $F_{\perp}$ is the net normal force on one side of the surface.

- The SI unit of pressure is the **pascal**,

Where $1\ pascal = 1\ Pa = 1\ N/m^2$

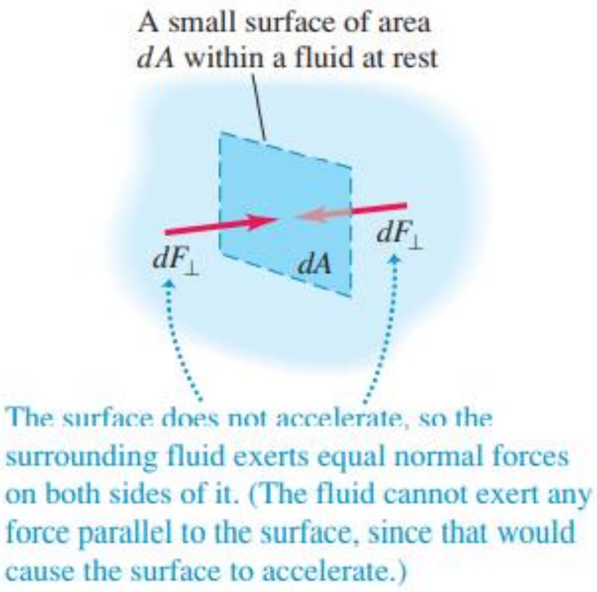


Fig. 8a

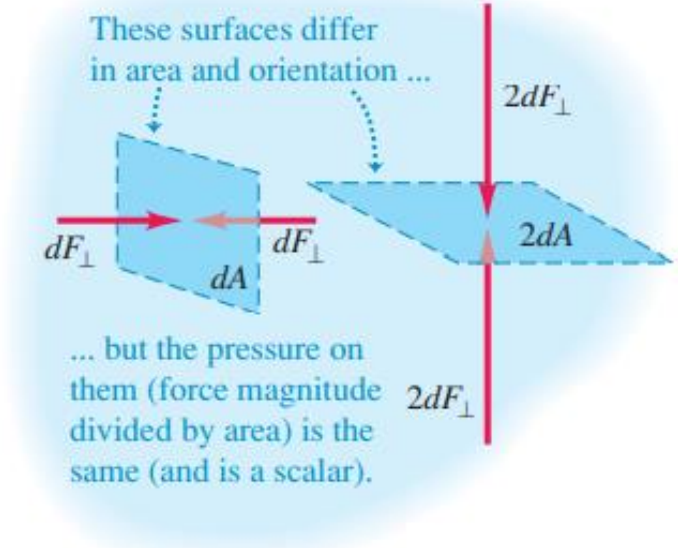


Fig. 8b

Atmospheric pressure

- This is the pressure of the earth's atmosphere, the pressure at the bottom of this sea of air in which we live.
- **Atmospheric pressure at a point is the force per unit area exerted on a small surface containing that point by the weight of air above that surface.**
- This pressure varies with weather changes and with elevation.
- Normal atmospheric pressure at sea level (an average value) is $\boldsymbol{1\ atmosphere\ (atm)}$, defined to be exactly $101{,}325\ Pa$.
- To four significant figures,
- $(p_a)_{av} = 1\ atm = 1.013 \times 10^5\ Pa$

$$= 1.013\ bar = 1013\ millibar = 14.70\ \ lb/in.^2$$

Example 2

In the room described in Example 1, what is the total downward force on the floor due to an air pressure of 1.00 atm?

Answer

- $A = (4.0\ m)(5.0\ m) = 20\ m^2$

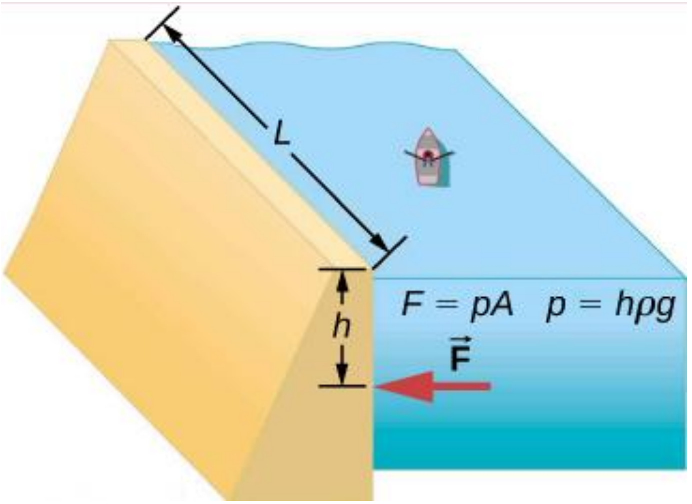
- $F_{\perp} = pA$
- $F_{\perp} = (1.013 \times 10^5\ \ N/m^2)(20\ m^2)$

$$= 2.0 \times 10^6\ N = 4.6 \times 10^5\ lb$$

$$= \mathbf{230\ tons}$$

Example 3

Consider the pressure and force acting on the dam retaining a reservoir of water in the **Figure** below. Suppose the dam is 500 m wide and the water is 80.0 m deep at the dam, as shown below. (a) What is the average pressure on the dam due to the water?
(b) Calculate the force exerted against the dam.



Solution 3

The average pressure p due to the weight of the water is the pressure at the average depth h of $40.0\ m$, since pressure increases linearly with depth

(a) The average pressure due to the weight of a fluid is

$$p = h\rho g$$

Entering the density of water and taking h to be the average depth of $40.0\ m$, we obtain

$$p = (40.0m)(10^3\ kg/m^3)(9.80\ m/s^2)$$
$$= 3.92 \times 105\ \ N/m^2$$
$$= 392kPa$$

(b) We have already found the value for p.

The area of the dam is

$$A = 80.0\ m \times 500\ m$$
$$= 4.00 \times 104m^2,$$

so that

$$F = (3.92 \times 10^5\ N/m^2)(4.00 \times 10^4\ m^2)$$
$$= 1.57 \times 10^{10}\ N.$$

Significance

Although this force seems large, it is small compared with the $1.96 \times 10^{13}\ N$ weight of the water in the reservoir. In fact, it is only 0.0800% of the weight.

Pressure, Depth, and Pascal's Law

- The pressure in a fluid will be the same throughout its volume, if the weight of the fluid can be neglected.
- But often the fluid's weight is not negligible.
- Atmospheric pressure is less at high altitude than at sea level, which is why an airplane cabin has to be pressurized when flying at 35,000 feet.
- When you dive into deep water, your ears tell you that the pressure increases rapidly with increasing depth below the surface.
- We can derive a general relationship between the pressure p at any point in a fluid at rest and the elevation y of the point.
- We'll assume that the density has the same value throughout the fluid (that is, the density is uniform), as does the acceleration due to gravity g.
- If the fluid is in equilibrium, every volume element will also be in equilibrium.

- Considering a thin element of fluid with thickness dy (Fig. 9a).
- The bottom and top surfaces each have area A, and they are at elevations y and $y + dy$ above some reference level where $y = 0$.
- The volume of the fluid element is $dV = Ady$,
- Its mass is $dm = \rho dV = \rho A dy$
- And its weight is $dw = dm\, g = \rho g A\, dy$
- **The other forces on this fluid element are (Fig. 9b)**
- Let's call the pressure at the bottom surface p;

then the total y −component of upward force on this surface is pA.

- The pressure at the top surface is $p + dp$, and

the total y −component of (downward) force on the top surface is $-(p + dp)A$

Fig. 9a

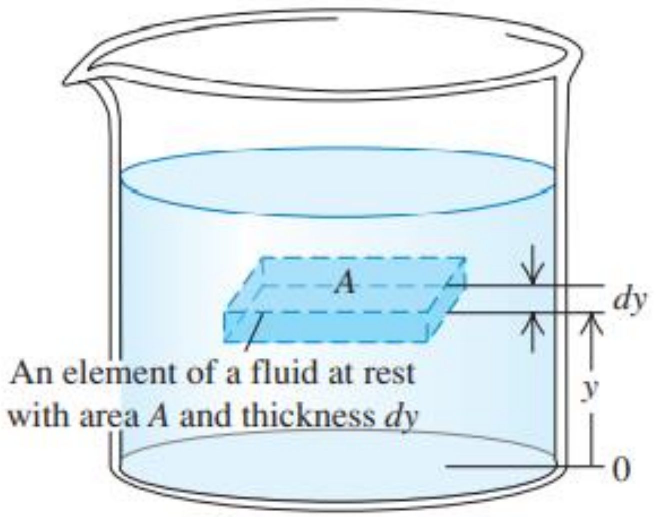
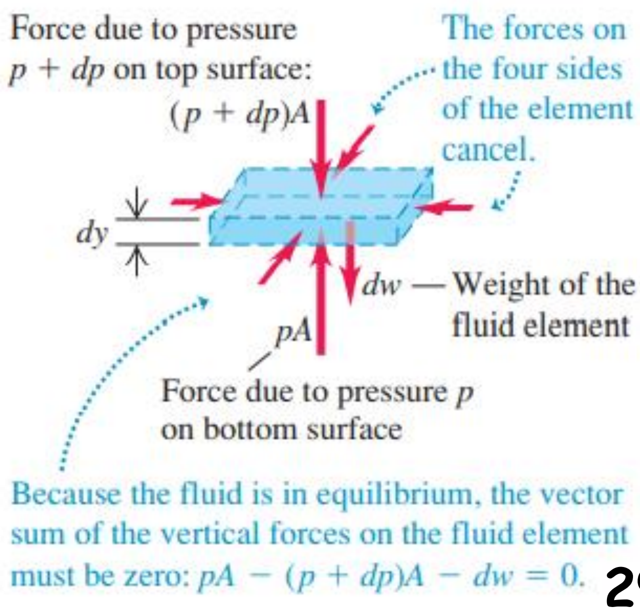


Fig. 9b



- The fluid element is in equilibrium, so the total y −component of force, including the weight and the forces at the bottom and top surfaces, must be zero:

$$\sum F_y = 0$$

- So, $$pA - (p + dp)A - \rho g A\, dy = 0$$
- Factorize A out and rearrange to obtain

$$\frac{dp}{dy} = -\rho g$$

This equation shows that when y increases, p decreases; that is, as we move upward in the fluid, pressure decreases, as we expect.

If p_1 and p_2 are the pressures at elevations y_1 and y_2, respectively, and if ρ and g are constant, then

$$p_2 - p_1 = -\rho g(y_2 - y_1) \qquad (4)$$

This is the pressure in a fluid of uniform density

- It's often convenient to express Eq. (5) in terms of the ***depth*** below the surface of a fluid (Fig. 10).
- Take **point 1** at any level in the fluid and let p represent the pressure at this point.
- Take **point 2** at the $\boldsymbol{surface}$ of the fluid, where

the pressure is p_o (subscript zero for zero depth).

- The depth of point 1 below the surface is

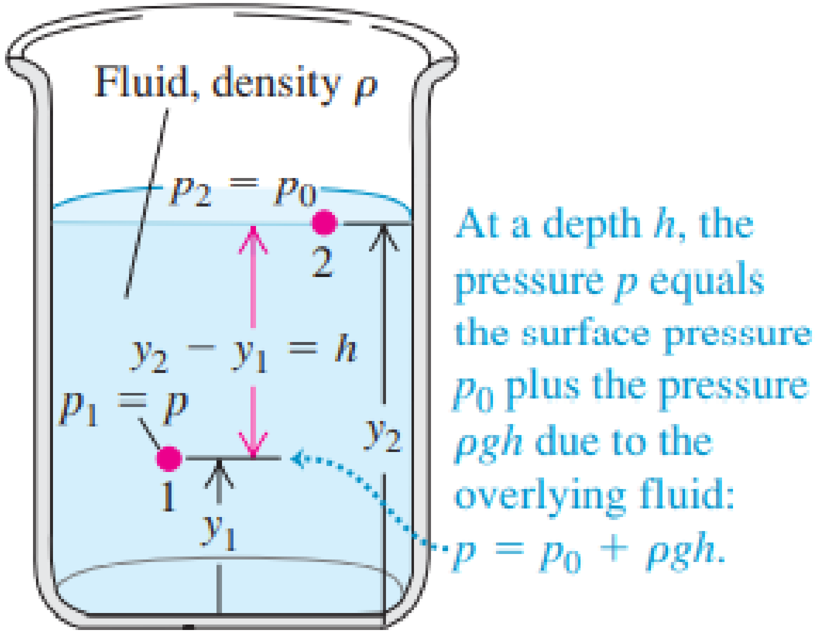
$$h = y_2 - y_1 \qquad (5)$$

and Eq. (5) becomes

$$p_o - p = -\rho g(y_2 - y_1) = -\rho g h$$

$$p = p_o + \rho g h \qquad (6)$$

This is how pressure varies with depth in a fluid with uniform density.



Pressure difference between levels 1 and 2:
$p_2 - p_1 = -\rho g(y_2 - y_1)$
The pressure is greater at the lower level.

Fig. 10

- The pressure p at a depth h is greater than the pressure p_0 at the surface by an amount $\rho g h$.
- Note that the pressure is the same at any two points at the same level in the fluid.
- The *shape* of the container does not matter (Fig. 11).
- Equation (6) shows that if the pressure p_o at the top surface is increase, possibly by using a piston that fits tightly inside the container to push down on the fluid surface, the pressure p at any depth increases by exactly the same amount.

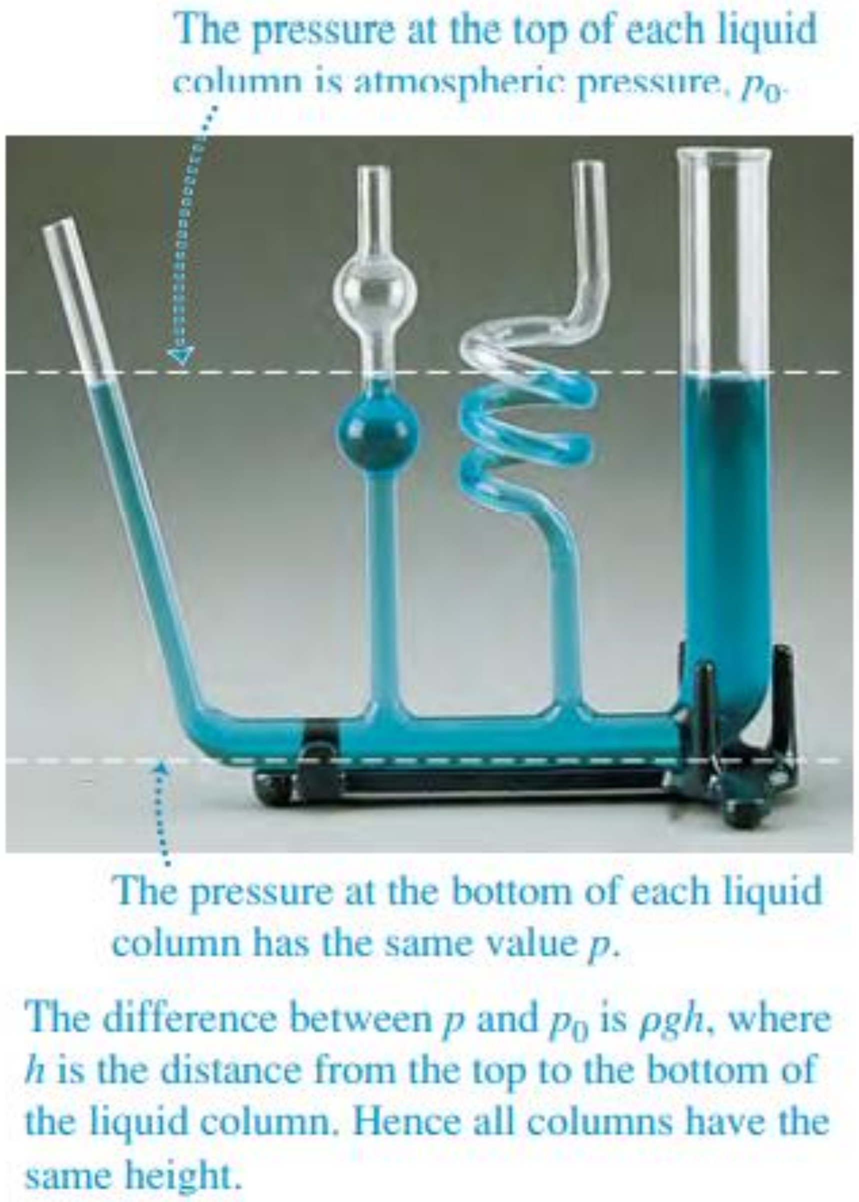


Fig. 11

- This fact was recognized in 1653 by the French scientist Blaise Pascal (1623–1662) and is called Pascal’s law.

- ***Pascal’s law states that Pressure applied to an enclosed fluid is transmitted undiminished to every portion of the fluid and the walls of the containing vessel.***

Example 4

(a) Calculate the absolute pressure at an ocean depth $1000\ m$. Assume that the density of water is $1000\ kg/m^3$ and that $P_o = 1.01 \times 10^5\ Pa\ (N/m^2)$

(b) Calculate the total force exerted on the outside of a $30.0\ cm$ radius circular submarine window at this depth.

Solution

(a) $$P = P_o + \rho g h$$

$$P = 1.01 \times 10^5 + (1000)(9.8)(1000)$$

$$P = 9.9 \times 10^6\ N/m^2$$

(b) $P = \frac{F}{A}$, $F = PA = P\pi r^2$

$$F = 9.9 \times 10^6 \times 3.142 \times (0.30)^2$$

$$F = 2.80 \times 10^6\ N$$

Pascal's law Illustrated

- The hydraulic lift is an application of Pascal's law (Fig. 12).
- A piston with small cross-sectional area A_1 exerts a force F_1 on the surface of a liquid such as oil.
- The applied pressure $p = F_1/A_1$ is transmitted through the connecting pipe to a larger piston of area A_2.
- The applied pressure is the same in both cylinders, so

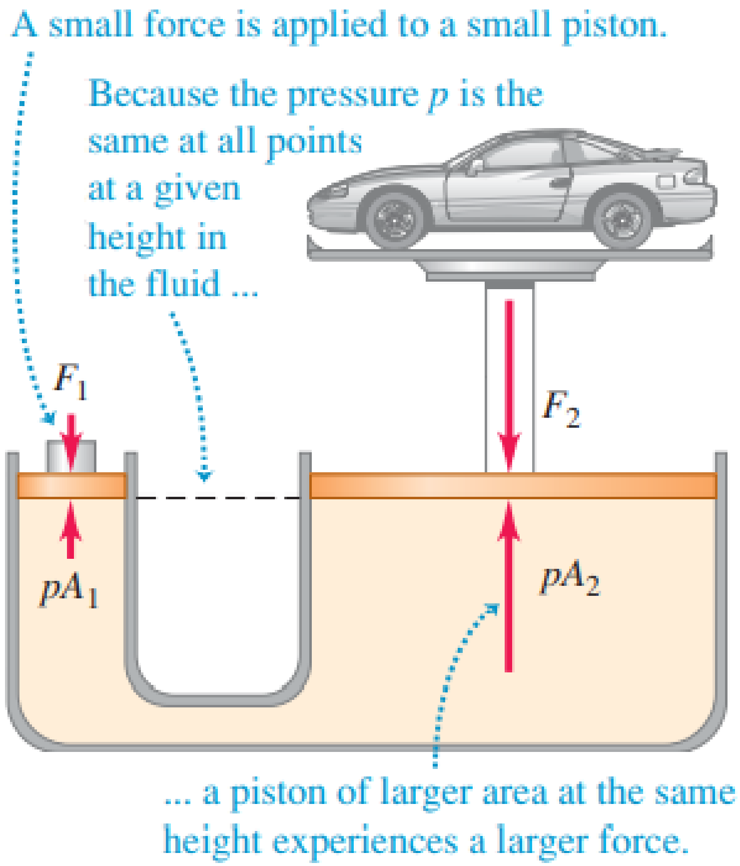


Fig. 12

$$P = \frac{F_1}{A_1} = \frac{F_2}{A_2} \quad \text{and} \quad F_2 = \frac{A_2}{A_1}F_1 \qquad (7)$$

- The hydraulic lift is a force-multiplying device with a multiplication factor equal to the ratio of the areas of the two pistons.
- Dentist's chairs, car lifts and jacks, many elevators, and hydraulic brakes all use this principle.
- For gases, the assumption that the density is uniform is realistic only over short vertical distances. For instance; in a room with a ceiling height of $3.0\,m$ filled with air of uniform density $1.2\ kg/m^3$, the difference in pressure between floor and ceiling, is given by

$$p - p_o = \rho g h$$

$$= (1.2\ kg/m^3)(9.8\ m/s^2)(3.9\,m) = 35\,Pa = about\ 0.000035\,atm$$

This is a very small difference.

- But between sea level and the summit of Mount Everest (8882 m) the density of air changes by nearly a factor of 3, and in this case, Eq. (6) cannot be used.
- Liquids, by contrast, are nearly incompressible, and it is usually a very good approximation to regard their density as independent of pressure.
- A pressure of several hundred atmospheres will cause only a few percent increase in the density of most liquids.

Absolute Pressure and Gauge Pressure

- If the pressure inside a car tire is equal to atmospheric pressure, that indicates that the tire is flat.
- The pressure must be greater than atmospheric to support the car, so the significant quantity is the difference between the inside and outside pressures.
- When we say that the pressure in a car tire is "32 pounds" (actually $32\ lb/in.^2$, equal to $220\ kPa$ or $2.2 \times 10^5\ Pa$), we mean that it is ***greater*** than atmospheric pressure ($14.7\ lb/in.^2$ or $1.01 \times 10^5\ Pa$) by this amount.
- The total pressure in the tire is then $47\ lb/in.^2$ or $320\ kPa$.
- ***The excess pressure above atmospheric pressure is usually called gauge pressure, and the total pressure is called absolute pressure.***
- Engineers use the abbreviations psig and psia for "pounds per square inch gauge" and "pounds per square inch absolute," respectively.
- If the pressure is less than atmospheric, as in a partial vacuum, the gauge pressure is negative.

Example 5

Water stands 12.0 m deep in a storage tank whose top is open to the atmosphere. What are the absolute and gauge pressures at the bottom of the tank?

Hint:

Water is nearly incompressible, so we can treat it as having uniform density. The level of the top of the tank corresponds to point 2, and the level of the bottom of the tank corresponds to point 1.

Solution

- The absolute pressure:

$$p = p_0 + \rho g h$$

$$= (1.01 \times 10^5\ Pa) + (1.000\ kg/m^3)(9.8\ m/s^2)(12.0\ m)$$

$$= 2.19 \times 10^5 Pa = 2.16\, atm$$

$$= 31.8\, lb/in.^2$$

- The gauge pressure:

$$p - p_0$$

$$= (2.19 \times 10^5\ Pa) - (1.01 \times 10^5\ Pa)$$

$$= 1.18 \times 10^5\ Pa = 1.16\, atm$$

$$= 17.1\, lb/in.^2$$

A pressure gauge at the bottom of such a tank would probably be calibrated to read gauge pressure rather than absolute pressure.

Pressure Gauges

- The simplest pressure gauge is the open-tube manometer (Fig. 13)
- The U-shaped tube contains a liquid of density ρ, often mercury or water.
- The left end of the tube is connected to the container where the pressure p is to be measured, and the right end is open to the atmosphere at pressure $p_0 = p_{atm}$.
- The pressure at the bottom of the tube due to the fluid in the left column is $\boldsymbol{p + \rho h y_1}$, and the pressure at the bottom due to the fluid in the right column is $\boldsymbol{p + \rho h y_2}$

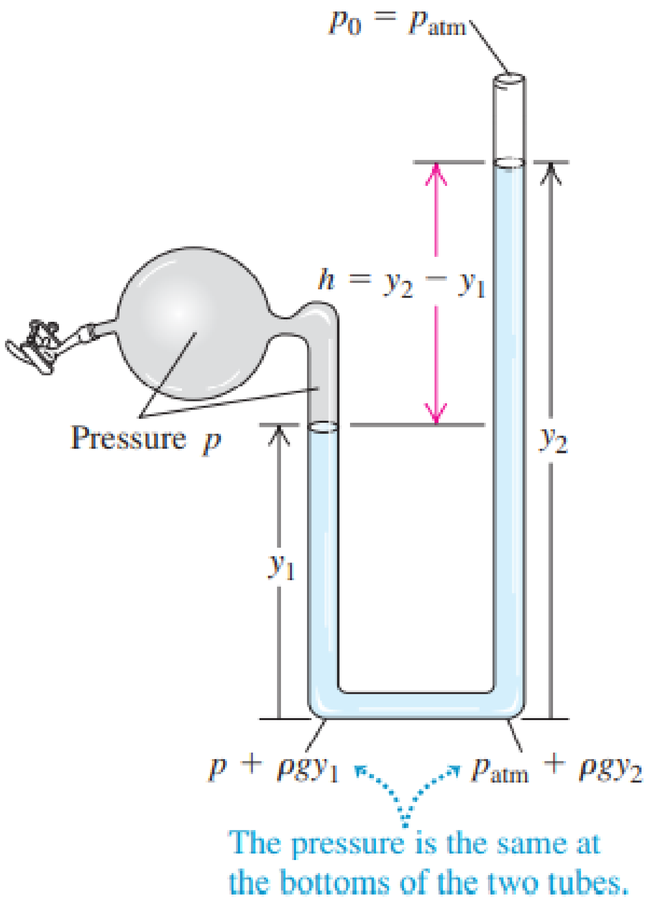


Fig. 13

- These pressures are measured at the same level, so they must be equal:

$$p + \rho g y_1 = p_{atm} + \rho h y_2$$

$$p - p_{atm} = \rho g(y_2 - y_1) = \rho g h \qquad (8)$$

where p is the absolute pressure, and the difference between absolute and atmospheric pressure is the gauge pressure.

- Thus, the gauge pressure is proportional to the difference in height $h = y_2 - y_1$ of the liquid columns.

- Another common pressure gauge is the **mercury barometer.**
- It consists of a long glass tube, closed at one end, that has been filled with mercury and then inverted in a dish of mercury (Fig. 14).
- The space above the mercury column contains only mercury vapor; its pressure is negligibly small, so the pressure p_0 at the top of the mercury column is practically zero.
- Hence;

$$p_{atm} = p = 0 + \rho g(y_2 - y_1) = \rho g \qquad (9)$$

- Thus, the mercury barometer reads the atmospheric pressure directly from the height of the mercury column.
- Pressures are often described in terms of the height of the corresponding mercury column, as so many “inches of mercury” or “millimeters of mercury” (abbreviated mm Hg).

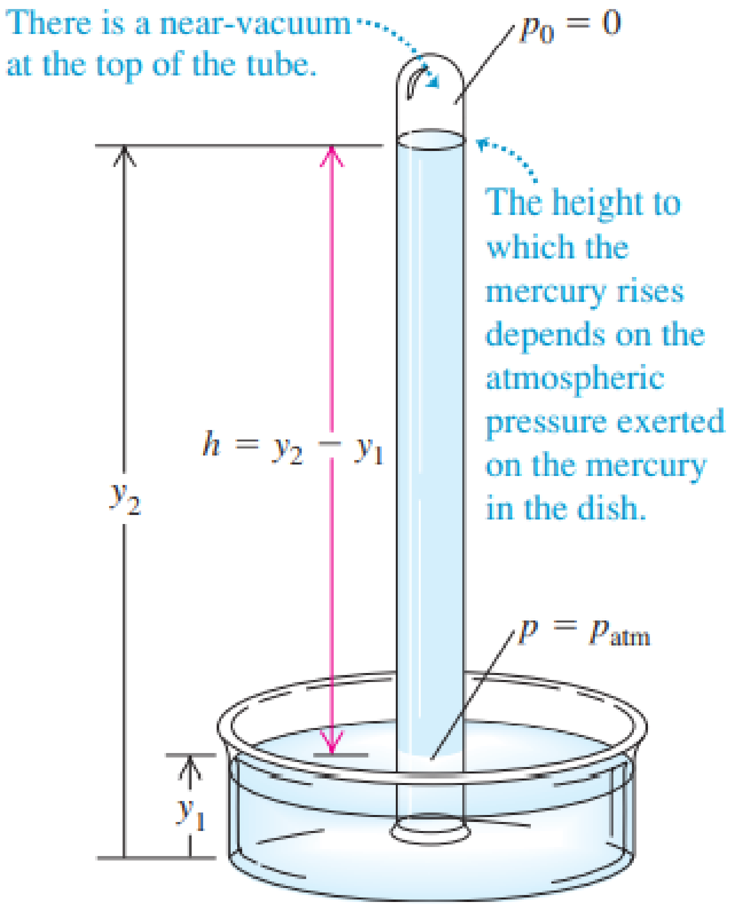


Fig. 14

- A pressure of 1 mm Hg is called 1 torr, after Evangelista Torricelli, inventor of the mercury barometer.
- But these units depend on the density of mercury, which varies with temperature, and on the value of g, which varies with location, so the pascal is the preferred unit of pressure.
- Many types of pressure gauges use a flexible sealed tube (Fig. 15).
- A change in the pressure either inside or outside the tube causes a change in its dimensions.
- This change is detected optically, electrically, or mechanically.

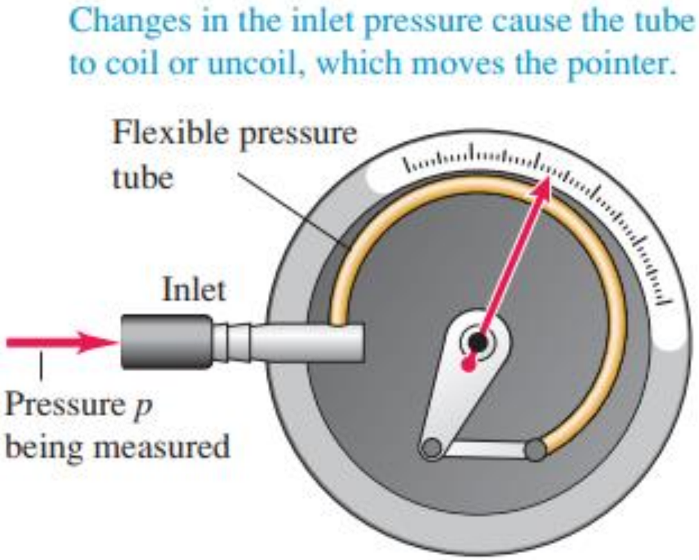
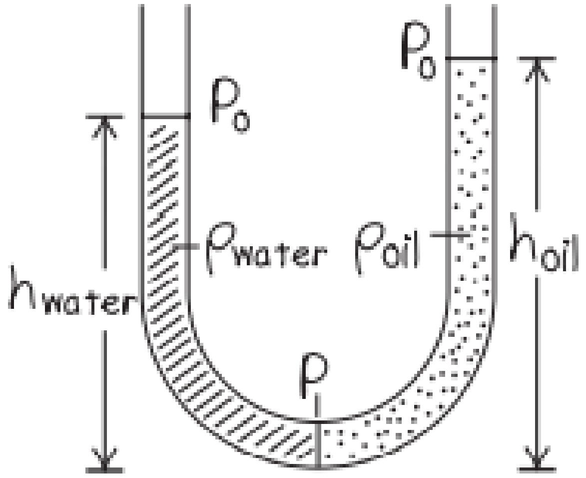


Fig. 15: A Bourdon pressure gauge

Example 6

A manometer tube is partially filled with water. Oil (which does not mix with water) is poured into the right arm of the tube until the oil–water interface is at the midpoint of the tube as shown. Both arms of the tube are open to the air. Find a relationship between the heights h_{oil} and h_{water}.



Hint:

Here, there are two fluids of different densities, so we must write a separate pressure–depth relationship for each.

Both fluid columns have pressure p at the bottom (where they are in contact and in equilibrium) and are both at atmospheric pressure at the top (where both are in contact with and in equilibrium with the air).

Solution 6

- The equation for each fluid gives:

$$p = p_0 + \rho_{water} g h_{water}$$

$$p = p_0 + \rho_{oil} g h_{oil}$$

Since the pressure p at the bottom of the tube is the same for both fluids, we set these two expressions equal to each other and solve for h_{oil} in terms of h_{water}.

You can show that the result is

$$h_{oil} = \frac{\rho_{water}}{\rho_{oil}} h_{water}$$

EVALUATE:

Water ($\rho_{water} = 1{,}000\ kg/m^3$) is denser than oil ($\rho_{oil} = 850\ kg/m^3$), so h_{oil} is greater than h_{water} as shown in the Fig.

It takes a greater height of low-density oil to produce the same pressure p at the bottom of the tube.

Buoyancy
&
Archimedes’ Principle

Buoyancy

- A body immersed in water seems to weigh less than when it is in air.
- When the body is less dense than the fluid, it floats.
- The human body usually floats in water, and a helium-filled balloon floats in air.
- If the buoyant force is greater than the object's weight, the object will rise to the surface and float.
- If the buoyant force is less than the object's weight, the object will sink.
- If the buoyant force equals the object's weight, the object will remain suspended at that depth.

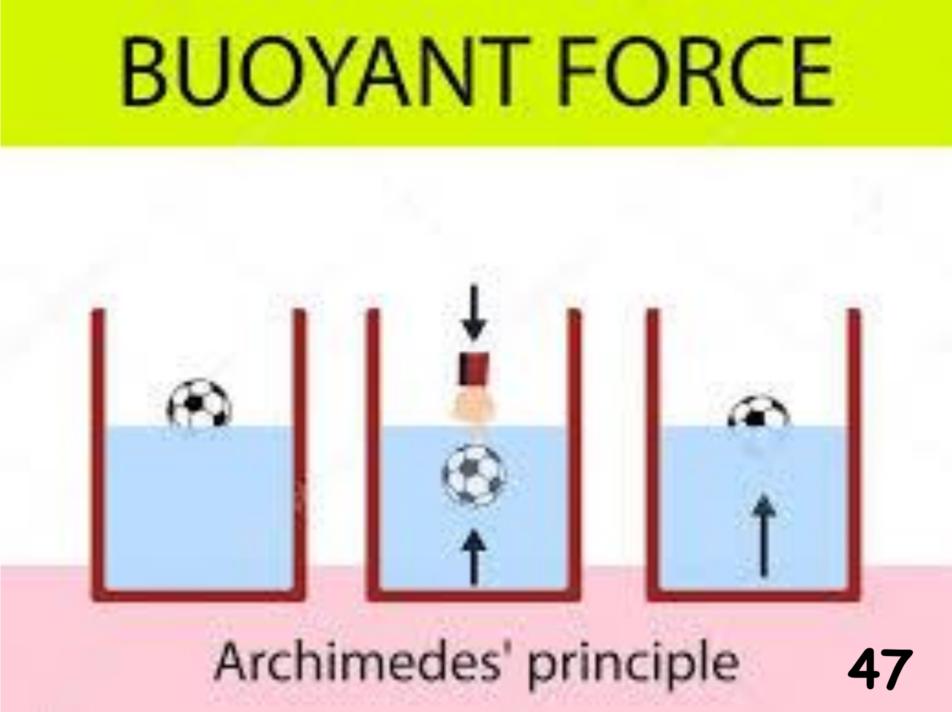


Fig. 16

- The buoyant force is always present whether the object floats, sinks, or is suspended in a fluid.
- What therefore is the buoyant force?
- ***The buoyant force F_B is the net upward force on any object in any fluid.***
- From the fig., the pressure due to the weight of a fluid increases with depth since $p = h\rho g.$
- This pressure and associated upward force on the bottom of the cylinder are greater than the downward force on the top of the cylinder.

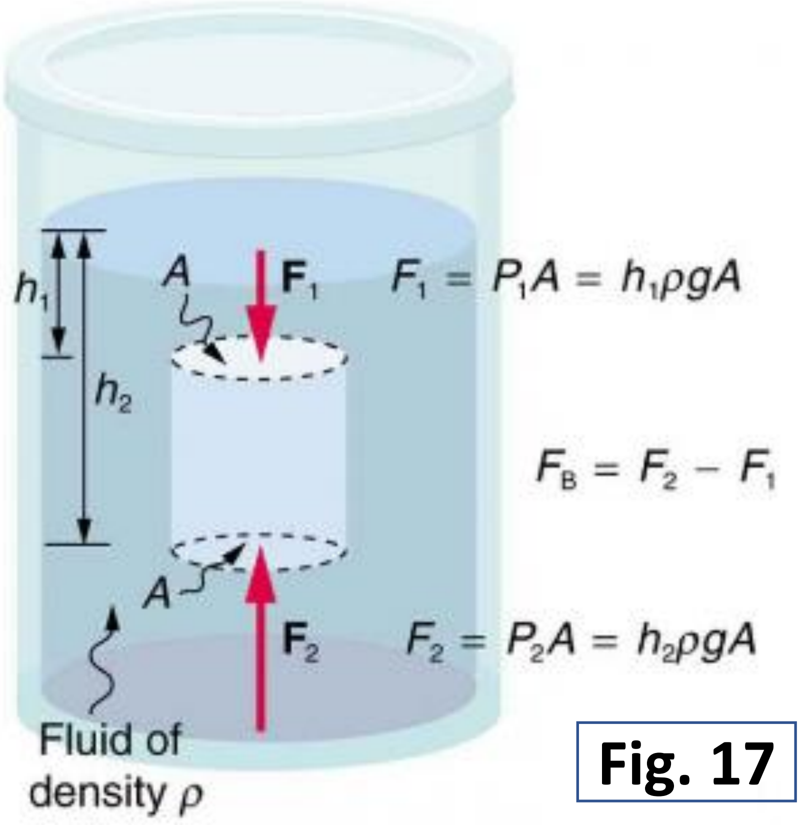


Fig. 17

- Their difference is the buoyant force FB. (Horizontal forces cancel.)

Practical Example of Buoyancy

- A practical example of buoyancy is the hydrometer, used to measure the density of liquids (Fig. 18).
- The calibrated float sinks into the fluid until the weight of the fluid it displaces is exactly equal to its own weight.
- The hydrometer floats higher in denser liquids than in less dense liquids, and a scale in the top stem permits direct density readings.
- Fig. b shows a type of hydrometer that is commonly used to measure the density of battery acid or antifreeze.
- The bottom of the large tube is immersed in the liquid; the bulb is squeezed to expel air and is then released, like a giant medicine dropper. The liquid rises into the outer tube, and the hydrometer floats in this sample of the liquid.

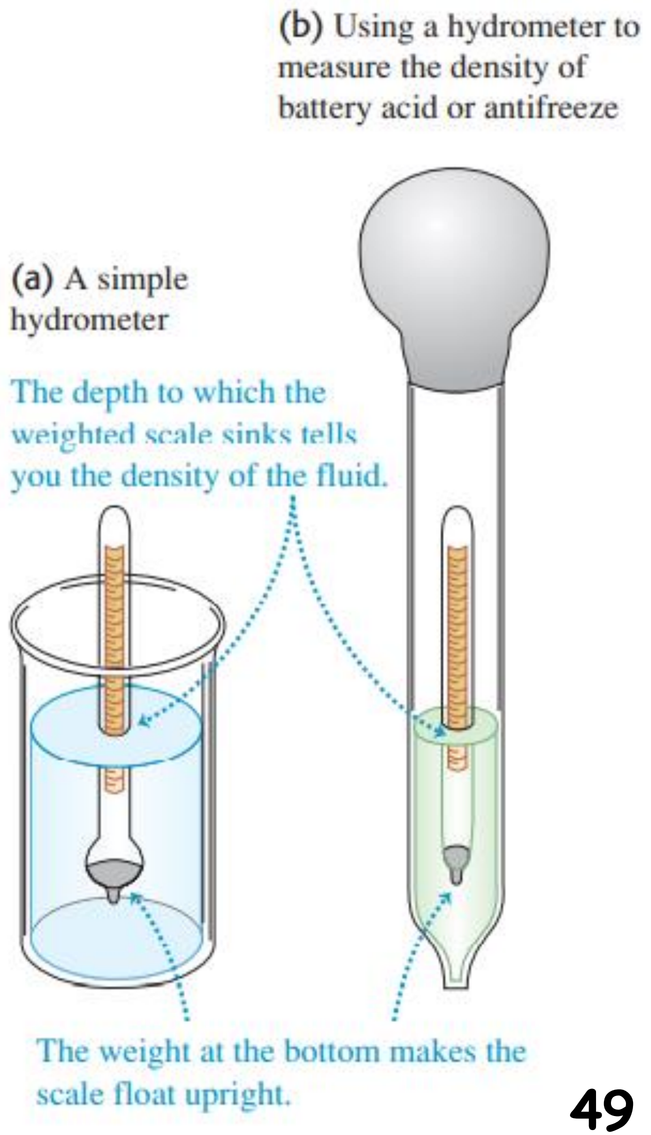


Fig. 18

Archimedes’ Principle

- **This principle states that when a body is completely or partially immersed in a fluid, the fluid exerts an upward force on the body equal to the weight of the fluid displaced by the body (Fig. 19).**

OR

- **This principle states that the buoyant force on an object equals the weight of the fluid it displaces.**

- Mathematically; $$F_B = w_f \qquad (10)$$

where F_B is the buoyant force and w_f is the weight of the fluid displaced by the object.

- Archimedes’ principle refers to the force of buoyancy that results when a body is submerged in a fluid, whether partially or wholly.

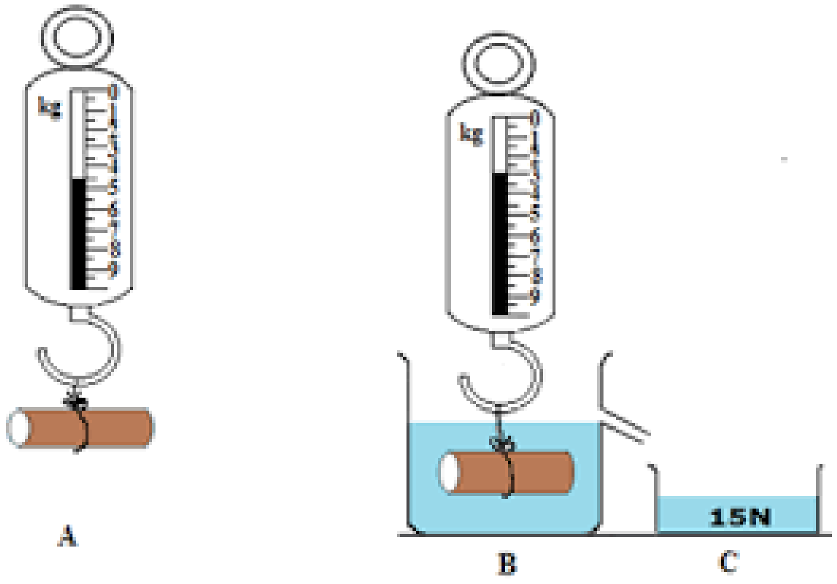


Fig. 19

Density and Archimedes' Principle

- The extent to which a floating object is submerged depends on how the object's density compares to the density of the fluid.
- In the Fig. (20a), for example, the unloaded ship has a lower density and less of it is submerged compared with the same ship when loaded (Fig. 20b).

- We can derive a quantitative expression for the fraction submerged by considering density.

Fig. 20: An unloaded ship (a) floats higher in the water than a loaded ship (b).

- The fraction submerged is the ratio of the volume submerged to the volume of the object, or

$$\boldsymbol{fraction\ submerged} = \frac{\boldsymbol{V_{sub}}}{\boldsymbol{V_{obj}}} = \frac{\boldsymbol{V_{fl}}}{\boldsymbol{V_{obj}}} \qquad (11)$$

- The volume submerged equals the volume of fluid displaced.
- Now we can obtain the relationship between the densities:

$$\frac{\boldsymbol{V_{fl}}}{\boldsymbol{V_{obj}}} = \frac{\boldsymbol{m_{fl}/\rho_{fl}}}{\boldsymbol{m_{obj}/\rho_{obj}}} \qquad (12)$$

- where ρ_{obj} is the average density of the object and ρ_{fl} is the density of the fluid

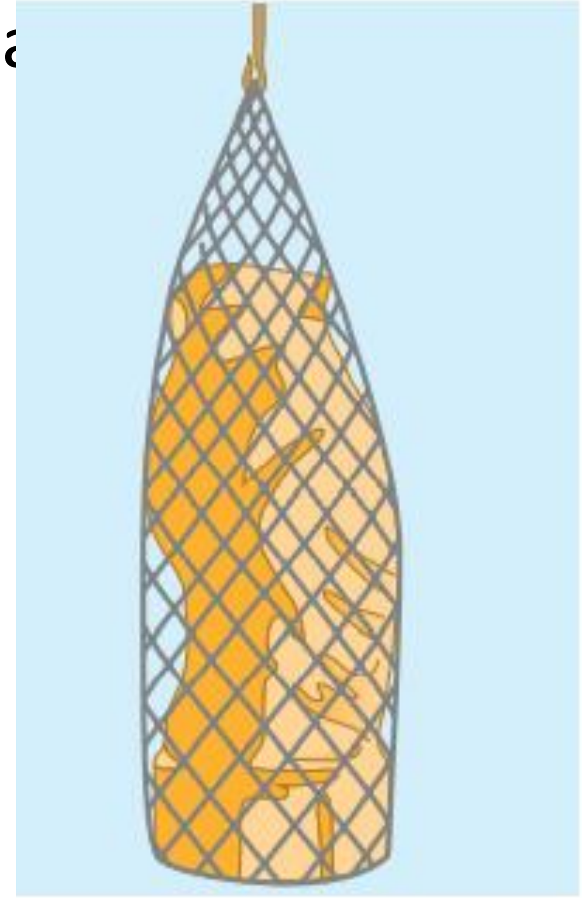
- Since the object floats, its mass and that of the displaced fluid are equal, so they cancel from the equation, leaving

$$\boldsymbol{fraction\ submerged} = \frac{\boldsymbol{\rho_{obj}}}{\boldsymbol{\rho_{fl}}} \qquad (13)$$

- This relationship is used to measure densities.

Example 7

A 15.0 kg solid gold statue is raised from the sea bottom (See Fig.). What is the tension in the hoisting cable (assumed ma



statue is

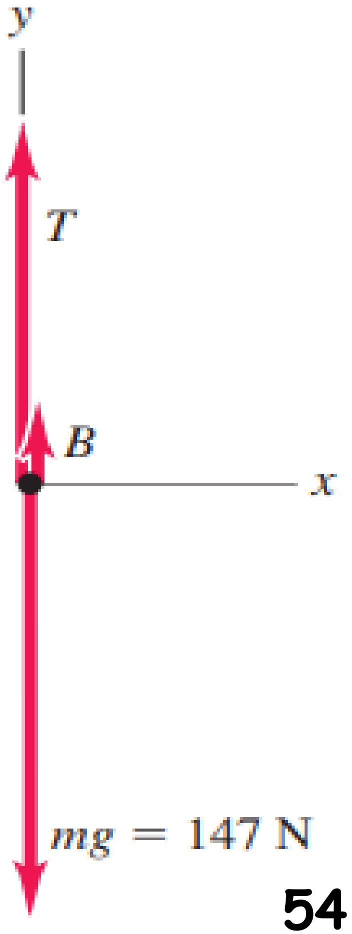
(a) at rest and completely underwater and

(b) at rest and completely out of the water?

Solution

Hint

In both cases the statue is in equilibrium and experiences three forces: its weight, the cable tension, and a buoyant force equal in magnitude to the weight of the fluid displaced by the statue (seawater in part (a), air in part (b)). The Fig. shows the free-body diagram for the statue.



Our target variables are the values of the tension in seawater (T_{sw}) and in air (T_{air}).

We are given the mass m_{statue}, and we can calculate the buoyant force in seawater (B_{sw}) and in air (B_{air}) using Archimedes' principle.

(a) To find B_{sw}, we first find the statue's volume V using the density of gold:

$$V = \frac{m_{gold}}{\rho_{gold}} = \frac{15.0\ kg}{19.3 \times 10^{3}\ kg/m^{3}}$$
$$= 7.77 \times 10^{-4}\ m^{3}$$

- The buoyant force equals the weight of this same volume of seawater.

$$\boldsymbol{B_{sw} = w_{sw} = m_{sw} g = \rho_{sw} V g}$$

$$= (1.03 \times 10^{3}\ kg/m^{3})(7.77 \times 10^{-4}\ m^{3})(9.80\ m/s^{2}) = \mathbf{7.84\ N}$$

The statue is at rest, so the net external force acting on it is zero. From Fig. b,

$$\boldsymbol{\sum F_y = B_{sw} + T_{sw} + (-m_{statue} g) = 0}$$

$$\boldsymbol{T_{sw} = m_{statue} g - B_{sw}}$$

$$= (15.0\ kg)(9.80\ m/s^{2}) - 7.84\ N$$
$$= 147\ N - 7.84\ N = \mathbf{139.16\ N}$$

- A spring scale attached to the upper end of the cable will indicate a tension of 7.84 N less than the statue's actual weight $\boldsymbol{m_{statue}g} = \mathbf{147\ N}$.
- (b) The density of air is about 1.2 kg/m^3, so the buoyant force of air on the statue is

$$B_{air} = \rho_{air}Vg = (1.2\ kg/m^3)(7.77 \times 10^{-4}\ m^3)(9.80\ m/s^2)$$
$$= 9.1 \times 10^{-3}\ N$$

- This is negligible compared to the statue's actual weight $\boldsymbol{m_{statue}g} = \mathbf{147\ N}$.
- So, within the precision of our data, the tension in the cable with the statue in air is

$$T_{air} = m_{statue}g = 147\ N$$

Note

- that the buoyant force is proportional to the density of the fluid in which the statue is immersed, not the density of the statue.
- The denser the fluid, the greater the buoyant force and the smaller the cable tension.
- If the fluid had the same density as the statue, the buoyant force would be equal to the statue’s weight and the tension would be zero (the cable would go slack).
- If the fluid were denser than the statue, the tension would be negative: The buoyant force would be greater than the statue’s weight, and a downward force would be required to keep the statue from rising upward.

Gravity vs Buoyancy

Which one wins?

Think of **gravity** & **buoyancy** as 2 opposing forces.

Gravity pulls **down**, **buoyancy** pushes **up**.

If weight (gravity) is greater than the displaced water (buoyancy), the object will sink.

G ↓ B ↑ , object sinks

If the displaced water (buoyancy) is greater than the weight (gravity), the object will float.

G ↓ B ↑ , object floats

Question

Suppose a $60.0\,kg$ woman floats in fresh water with 97.0% of her volume submerged when her lungs are full of air. What is her average density?

- Ans: $970\ kg/m^3$