# ELECTRICITY AND MAGNETISM

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# Electricity & Magnetism

## 1 Electrostatics

### 1.1 Introduction

Electrostatics is the branch of electricity that deals with charges at rest. Because moving charge gives rise to electric current and electric current is the source of magnetism, the study of electricity and magnetism begins with electrostatics.

An atom consists of a nucleus made up of particles called **protons** and **neutrons**, around which exists a diffuse cloud of particles called **electrons**. Electric charge, like mass, is an intrinsic property of protons and electrons. Experiments have shown the following facts:

1. There exist **two types of electric charge** which we now call positive and negative.

2. **Like charges repel and unlike charges attract**.

3. Charge is **quantized**: Experiments show that electric charge occurs in nature as integral multiples of $1.602\,177\,33(49) \times 10^{-19}\,\mathrm{C} \approx 1.60 \times 10^{-19}\,\mathrm{C}$. The SI unit of charge is the **coulomb (C)**.*

   The magnitude of the charge on the proton or electron is denoted by the symbol $e$. The charge on the proton is exactly **equal and opposite** to the charge on the electron. By convention, the charge of the proton is $+e$ and the charge of the electron is $-e$. The charge $e$ is the **smallest amount of free charge** that has been discovered. Any charge $q$ is therefore an integer multiple of $e$. Thus if $N$ is an integer.

   $$q = Ne. \tag{1}$$

   Atoms normally have an equal number of protons and electrons and therefore have no net charge since the algebraic sum of the charges of all the protons and electrons is zero. An atom that carries no net charge is said to be **electrically neutral**. An atom that has a net electrical charge (because it has lost or gained one or more electrons) is called an **ion**.

4. **Conservation of charge**: The algebraic sum of all electric charges in an isolated system remains constant.

   It is possible to transfer charge from one object to another. For example when hard rubber is rubbed against animal fur, electrons are transferred from atoms in the fur to the rubber. As a result, the rubber gains a net negative charge and the fur gains a net positive charge. No electrons or protons are created or destroyed when charge is transferred. Experiments have verified that during any process the law of conservation of electric charge is obeyed.

*The coulomb is defined in Section 4.6

Electric charge can not only be transferred to and from different objects, it can also move through materials. An **electrical conductor** is a material through which an electric charge is readily transferred. Most metals are good conductors. An **electrical insulator** is a material through which an electric charge is not readily transferred. Good insulators are such poor conductors that we consider them to be nonconductors. Glass, rubbers, plastics, etc. are good insulators.

5. **Invariance of charge**: The charge on a particle is independent of its speed. This is not an obvious property. For instance mass is not invariant, as Einstein's special relativity predicted, and experiment has confirmed.

**Example 1.1:** The number of electrons in a charge

Determine the number of electrons required to produce a charge of 1 coulomb.

**Solution:**

The charge of an electron is $e = 1.60 \times 10^{-19}$ C. Hence the number of electrons required to produce a charge of 1 C may be found using Equation (1):

$$N = \frac{q}{e} = \frac{1}{1.60 \times 10^{-19}} = 6.25 \times 10^{18} \text{ electrons.}$$

## 1.2 Coulomb's law

The electrostatic force that two charges exert on each other depends on the amount of charge on each object and the distance between the two objects. For two stationary charges $q_1$ and $q_2$ separated by a distance $r$ the **magnitude** of this force is given by **Coulomb's law**:

$$F = \frac{kq_1q_2}{r^2} \tag{2}$$

The constant of proportionality $k$ in Coulomb's law is determined experimentally and has a value of $9.0 \times 10^9\,\mathrm{N\,m^2\,C^{-2}}$ if the charges are in vacuum (and also in air to a good approximation). It is common practice to express the constant $k$ as

$$k = \frac{1}{4\pi\varepsilon_0}$$

where $\varepsilon_0$ is the **permittivity of free space**.

The force in Equation (2) between two point charges at rest **acts along the line joining the charges** and is attractive if the charges have opposite signs and repulsive if the charges have the same sign (see figure 1). Thus

$$\mathbf{F}_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2}\hat{\mathbf{r}}, \tag{3}$$

where $\mathbf{F}_{12}$ is the force exerted by $q_1$ on $q_2$ and $\hat{\mathbf{r}}$ is a **unit vector** which points from $q_1$ in the direction of $q_2$.

### 1.2.1 Principle of superposition

The principle of superposition applied to Coulomb's law states that the total force on a charge due to other charges in vacuum is the vector sum of the forces due to each charge on its own, as if only it were present. Experiment shows that such superposition applies.

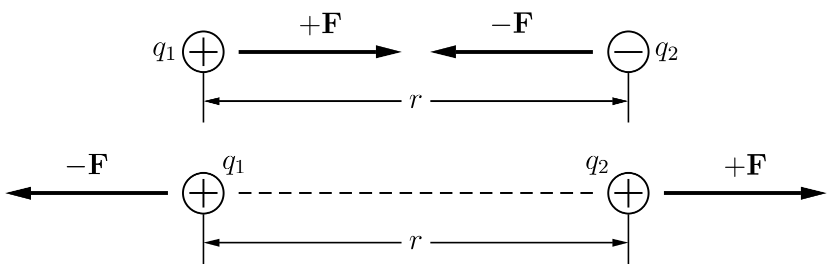



Figure 1: The force between two charges at rest acts along the line joining them. The force is attractive for unlike charges and repulsive for like charges.

### 1.2.2 Applications of Coulomb's law

In calculating a Coulombic force from Equation (3) obtain its

- **magnitude** from $|\mathbf{F}| = \dfrac{1}{4\pi\varepsilon_0}\dfrac{|q_1||q_2|}{r^2}$
- **direction** from: Likes repel, unlikes attract.
- **units** in newtons using $q$ in coulombs and $r$ in metres.

**Example 1.2:** Coulomb's law

Point charges of 2 µC and −3 µC are at rest 4 cm apart in vacuum. Calculate the force on the 2 µC charge.

**Solution:**

The magnitude of the force is

$$F = \frac{kq_1q_2}{r^2} = \frac{\left(9.0\times 10^{9}\right)\times\left(2\times 10^{-6}\right)\times\left(3\times 10^{-6}\right)}{\left(4\times 10^{-2}\right)^2} = 33.75\,\mathrm{N}.$$

The charges have opposite signs so the force is attractive (the force is towards the −3 µC charge).

**Example 1.3:** Coulomb's law

Two stationary electrons lie in a vacuum at points on the $x$ axis with coordinates −2 nm and 3 nm. What is the force on the electron at −2 nm?

**Solution:**

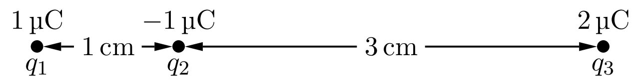
The distance between the electrons is $r = |-2-3| = 5\,\mathrm{nm}$. The magnitude of the force between the electrons is

$$\begin{aligned} F &= \frac{kq_1q_2}{r^2} = \frac{ke^2}{r^2} \\ &= \frac{\left(9.0\times 10^{9}\right)\times\left(1.60\times 10^{-19}\right)^2}{\left(5\times 10^{-9}\right)^2} = 9.2\times 10^{-12}\,\mathrm{N}. \end{aligned}$$

The charges have the same sign so the force is repulsive (the force on the electron at −2 nm is in the negative-$x$ direction).

**Example 1.4:** Superposition and Coulomb's law: one dimension

Charges lie in a vacuum on a line, as in the figure below. Find the force on $q_1$.




**Solution:**

Use the superposition principle (choose positive-$x$ direction to the right):

$$\mathbf{F} = \mathbf{F}_{21} + \mathbf{F}_{31}.$$

The magnitudes of $\mathbf{F}_{21}$ and $\mathbf{F}_{31}$ are

$$F_{21} = \frac{kq_1q_2}{r_{21}^2} = \frac{9 \times 10^9 \times 1 \times 10^{-6} \times 1 \times 10^{-6}}{\left(1 \times 10^{-2}\right)^2} = 90\,\mathrm{N}$$

and

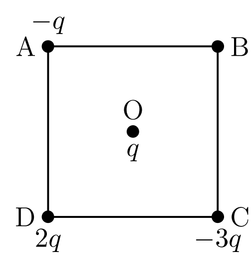
$$F_{31} = \frac{kq_1q_3}{r_{31}^2} = \frac{9 \times 10^9 \times 1 \times 10^{-6} \times 2 \times 10^{-6}}{\left(4 \times 10^{-2}\right)^2} = 11.25\,\mathrm{N}.$$

The force $F_{21}$ exerted by $q_2$ on $q_1$ is attractive and therefore to the right. $F_{31}$ is repulsive and therefore to the left. Hence

$$\mathbf{F} = \mathbf{F}_{21} + \mathbf{F}_{31} = \big(+\,90 - 11\big)\hat{\mathbf{x}} = 79\,\hat{\mathbf{x}}\,\mathrm{N}.$$

**Example 1.5:** Superposition and Coulomb's law: two dimensions

In the figure ABCD is a square of side 1 cm with centre O. Charges are placed in vacuum as shown. Calculate the resultant force on $-3q$, where $q = 10^{-6}$ C.




**Solution:**

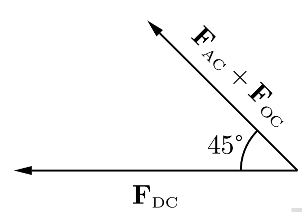
The forces due to the charges at A and O act at C along the diagonal AC. The force due to the charge at D on the charge at C is along DC. The force on the charge at C is the resultant of these forces. The length of the diagonal is $\sqrt{2}$ cm.

$$F_{\mathrm{AC}} = \frac{kq_{\mathrm{A}}q_{\mathrm{C}}}{r_{\mathrm{AC}}} = \frac{\left(9 \times 10^9\right) \times \left(10^{-6}\right) \times \left(3 \times 10^{-6}\right)}{\left(\sqrt{2} \times 10^{-2}\right)^2} = 135\,\mathrm{N}$$

$$F_{\mathrm{OC}} = 4 \times F_{\mathrm{AC}} = 540\,\mathrm{N}$$

$$F_{\mathrm{DC}} = \frac{kq_{\mathrm{D}}q_{\mathrm{C}}}{r_{\mathrm{DC}}} = \frac{\left(9 \times 10^9\right) \times \left(2 \times 10^{-6}\right) \times \left(3 \times 10^{-6}\right)}{\left(10^{-2}\right)^2} = 540\,\mathrm{N}$$

The resultant of $F_{\mathrm{AC}}$ and $F_{\mathrm{OC}}$ is $540 - 135 = 405\,\mathrm{N}$ from C to A, and the direction of $F_{\mathrm{DC}}$ is from C to D.




The resultant force on the $-3q$ charge at C is 875 N at an angle of 19.1° above CD.

**Example 1.6:** A perspective on the magnitude of the force given by Coulomb's law

Estimate the electrostatic force of attraction between the electrons and protons in a grain of salt if these are separated by a distance of 100 m.

**Data**:

Assume a grain of salt is a cube with sides of length $\ell = 0.4$ mm.
The average distance between the nuclei in an NaCl crystal is $d = 2.8 \times 10^{-10}$ m.
The chemical symbols of sodium and chlorine are $^{23}_{11}\mathrm{Na}$ and $^{35.5}_{17}\mathrm{Cl}$ respectively.
The charge of a proton (or electron) is $q = \pm 1.6 \times 10^{-19}$ C.

**Solution:**

From the above data, a grain of salt contains about

$$N = \left(\frac{\ell}{d}\right)^3 = \left(\frac{0.4 \times 10^{-3}}{2.8 \times 10^{-10}}\right)^3 = 2.9 \times 10^{18} \text{ atoms.}$$

The number of electrons (or protons) is

$$N_\mathrm{e} = \tfrac{1}{2}(11 + 17) \times 2.9 \times 10^{18} = 4.1 \times 10^{19}.$$

The force between charges $q_1$ and $q_2$ separated by a distance $r$ is given by Coulomb's law

$$\begin{aligned} F &= \frac{kq_1q_2}{r^2} \\ &= \frac{9 \times 10^9 \times (4.1 \times 10^{19} \times 1.6 \times 10^{-19})^2}{100^2} \\ &= 3.9 \times 10^7 \text{ N.} \end{aligned}$$

To put this number into perspective, the mass of an elephant is about 5 tonne (5000 kg). Thus the electrostatic force of attraction between all the electrons and protons in a grain of salt, when separated by a distance of 100 m, is sufficient to lift about 780 elephants!

## 1.3 The electric field

The region of space surrounding a charged body is affected by the presence of the charge. A second charge brought into this region experiences a force according to Coulomb's law. Because electrical forces involve interactions over a distance it is helpful to introduce the concept of electric field $\mathbf{E}$.

**Electric field**

We define the electric field $\mathbf{E}$ at a point as the force per unit charge exerted on a stationary, positive, test charge placed at that point.

$$\mathbf{E} = \frac{\mathbf{F}}{q} \tag{4}$$

Note that $\mathbf{E}$ is a **vector quantity** and therefore has direction as well as magnitude. The SI unit of electric field is newton per coulomb (N C$^{-1}$).

- The charge used to measure the force (called the test charge) *must be stationary*, because a moving charge might also experience a magnetic force in addition to the electric force on it, as we shall see later.

- To state in the definition of **E** that it is the force *on* a unit charge is *incorrect*. Force per unit charge (units $\mathrm{N\,C^{-1}}$) is not the same as force (unit N), even if it is a force on a unit charge.

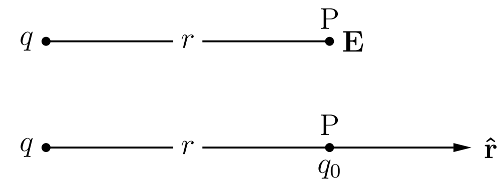
- From Equation (4) the force on a charge $q$ at a point in a vacuum where the field is **E** is

$$\mathbf{F} = q\mathbf{E}. \tag{5}$$

- Since **E** is a vector *its direction must be stated*. If $q$ is a positive charge, then from Equation (5) the vectors **F** and **E** are parallel. Thus to find the direction of **E** at a point, one must first determine the direction of the force on a *positive* test charge at the point, using the result: likes repel, unlikes attract.

#### 1.3.1 E due to a stationary point charge in vacuum

Let $q$ be a stationary point charge in vacuum. To find its field **E** at a point P distance $r$ from $q$, place a stationary test charge $q_0$ at P. The force which $q$ exerts on $q_0$ at P is the Coulomb force. From Equation (2)




$$\mathbf{F} = \frac{1}{4\pi\varepsilon_0}\frac{qq_0}{r^2}\,\hat{\mathbf{r}},$$

where $\hat{\mathbf{r}}$ is the unit vector from $q$ to $q_0$, i.e. from $q$ to P. From the definition in Equation (4) the electric field at P is

$$\mathbf{E} = \frac{\mathbf{F}}{q_0} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\,\hat{\mathbf{r}}. \tag{6}$$

- Since the Coulomb force is inverse square, so also is the electric field (or Coulomb field) of a point charge in vacuum.

- Since the principle of superposition applies to Coulomb forces, it also applies to Coulomb fields.

#### 1.3.2 Field lines

To visualise the electric and magnetic fields, Michael Faraday introduced the idea of field lines (or lines of force). Field lines are imaginary lines, not necessarily straight and are drawn according to the following conventions:

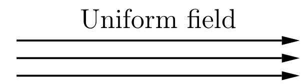
1. The **direction** of the field at any point is the **tangent** to the field line at that point.

2. The magnitude of the field strength is represented by the **number of lines per unit area passing perpendicularly** through a small area at that point. If at any point the net field is zero, then no lines will pass through it. Such a point is called a **neutral point**.

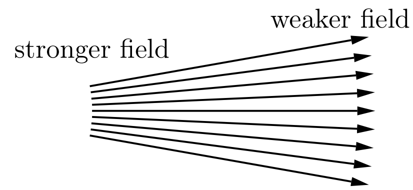
Some properties of field lines are:

- At any point the electric field may have only one direction. Hence field lines never intersect.

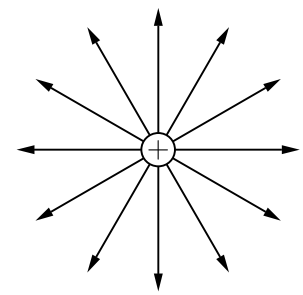
- In a uniform field the lines are straight, parallel and equispaced.



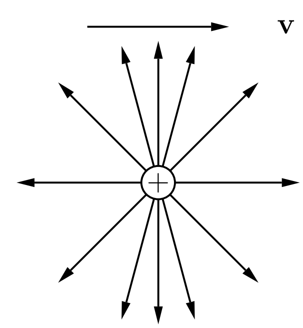

- The stronger the field at any point the more closely spaced are the field lines.




- As the field from a stationary positive charge acts outwards from the charge in all directions because of Equation (6), its field lines are straight lines radiating outwards. For a stationary negative charge they radiate inwards to the charge. Hence in an electrostatic field the field lines necessarily **begin on positive charge** and **end on negative charge** (if necessary assumed at infinity). They cannot start or stop in free space.




- It can be shown that the field due to a point charge moving with constant velocity is still radial but is now bunched up perpendicular to its velocity. The number of field lines for a given charge is the same as when it is at rest.




Figures 2(a) and 2(b) show the electric field due to two equal charges of opposite sign and two equal charges of the same sign, respectively.

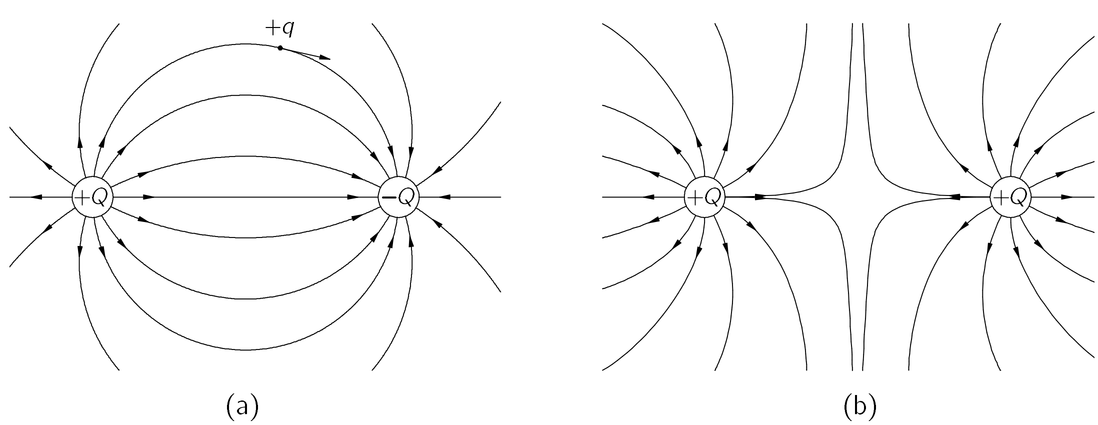



Figure 2: Electric field lines of (a) two unlike charges and (b) two like charges.

### 1.3.3 Examples of calculating Coulomb fields

In using Equation (6) to calculate the electric field at a point due to one or more point charges:

- Find the **magnitude** of the field due to each charge at the point from

$$|\mathbf{E}| = \frac{1}{4\pi\varepsilon_0}\frac{|q|}{r^2}.$$

- The **direction** of the field due to each charge at the point is the direction of the force on a positive test charge at the point.

- The **principle of superposition** is then used to find the resultant field at the point due to more than one charge. Since $\mathbf{E}$ is a vector, either resolve in two perpendicular directions, followed by the use of Pythagoras, or use a vector polygon.

**Example 1.7:** Electric field of an electron

Calculate the field of an electron at a point $5 \times 10^{-11}$ m away.

**Solution:**

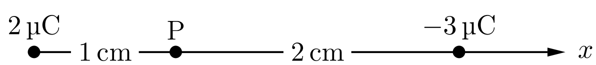
The magnitude of the field is

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} = \frac{\left(9 \times 10^{9}\right) \times \left(1.60 \times 10^{-19}\right)}{\left(5 \times 10^{-11}\right)^2} = 5.76 \times 10^{11}\,\mathrm{N\,C^{-1}}.$$

A positive test charge at the point will experience a force radially in towards the electron. The field therefore also points radially in towards the electron.

**Example 1.8:** Electric field at a point due to two charges

Stationary charges lie in a vacuum, as in the figure below.




Find the field at P.

**Solution:**

The magnitude of the field at P due to the 2 µC charge is

$$E_1 = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} = \frac{\left(9 \times 10^{9}\right) \times \left(2 \times 10^{-6}\right)}{\left(1 \times 10^{-2}\right)^2} = 1.8 \times 10^{8}\,\mathrm{N\,C^{-1}}$$

in the positive-$x$ direction. The magnitude of the field at P due to the −3 µC charge is

$$E_1 = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} = \frac{\left(9 \times 10^{9}\right) \times \left(3 \times 10^{-6}\right)}{\left(2 \times 10^{-2}\right)^2} = 6.75 \times 10^{7}\,\mathrm{N\,C^{-1}}$$

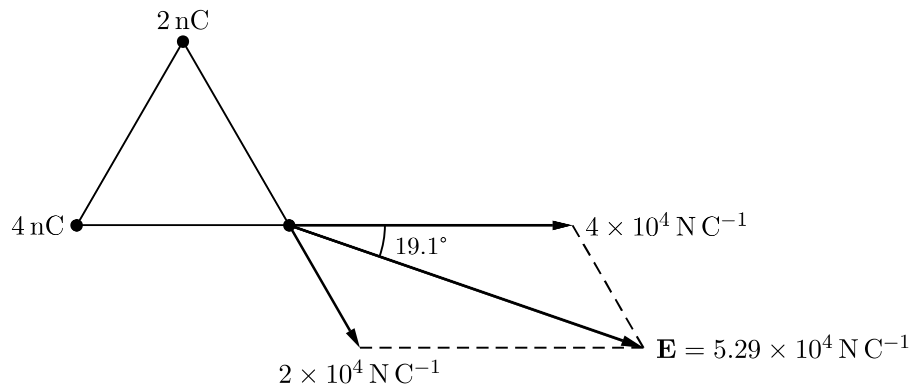
also in the positive-$x$ direction. The resultant field is therefore $\mathbf{E} = 2.475 \times 10^{8}\,\hat{\mathbf{x}}\,\mathrm{N\,C^{-1}}$.

**Example 1.9:** Electric field due to an arrangement of charges in two dimensions

Charges of 2 nC and 4 nC lie in vacuum at two corners of an equilateral triangle of side 3 cm. Find the field at the third corner.

**Solution:**

The calculation is left as an exercise.




**Example 1.10:** Neutral point in a field due to two point charges

Charges of +5 nC and +20 nC are located 30 cm apart in air. Calculate

(a) the force between them, and

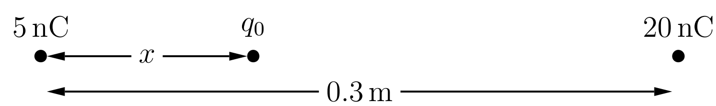
(b) the position of the neutral point in the resulting electric field.

**Solution:**

(a) Since both charges are positive, they repel each other. The force is given by Equation (2):

$$F = \frac{kq_1q_2}{r^2} = \frac{9 \times 10^9 \times \left(5 \times 10^{-9}\right)\left(20 \times 10^{-9}\right)}{0.3^2} = 1.0 \times 10^{-5}\,\mathrm{N}.$$

(b) The neutral point will be where the net force exerted on a test charge $q_0$ is zero. This has to be between the charges, on the line joining the charges and closer to the smaller charge. Let $x$ be the unknown distance.




Use Equation (2) to determine the force on a test charge $q_0$ for each charge. Thus

$$\frac{k \times 5 \times 10^{-9} \times q_0}{x^2} = \frac{k \times 20 \times 10^{-9} \times q_0}{(0.3 - x)^2},$$

which gives

$$\frac{5}{x^2} = \frac{20}{(0.3 - x)^2}.$$

Solving the above equation gives $x = -0.3\,\mathrm{m}$ or $x = 0.1\,\mathrm{m}$. We reject the negative solution as unphysical.

## 1.4 Electric potential and potential difference

A charge in an electric field experiences a force $\mathbf{F} = q\mathbf{E}$. If the charge is moved between two points A and B in the field, then work is done because of this force. Let $W_{\mathrm{AB}}$ be the work done. Then

- if $W_{\mathrm{AB}} = 0$, points A and B are said to be at the same electric potential.
- if $W_{\mathrm{AB}} \neq 0$, points A and B are at different electric potential; that is there is a potential difference (pd) between the points.

The **potential difference** $V_{\mathrm{AB}}$ between points A and B in an electrostatic field is the work per unit charge done on any charge in moving it slowly from A to B.

If the potential difference is denoted $V_{\mathrm{AB}}$, then in symbols,

$$V_{\mathrm{AB}} = \frac{W_{\mathrm{AB}}}{q}, \tag{7}$$

where $W_{\mathrm{AB}}$ is the work done in slowly moving $q$ from A to B. The unit of potential difference in the SI system is the volt ($1\,\mathrm{V} = 1\,\mathrm{J\,C^{-1}}$).

One **volt** is the pd between two points in an electric field if 1 joule of work moves a charge of 1 coulomb between these points.

The pd $V_{\mathrm{AB}}$ between points A and B is the difference between the electric potential at A and the electric potential at B. If we also denote electric potential by the symbol $V$ then

$$V_{\mathrm{AB}} = V_{\mathrm{B}} - B_{\mathrm{A}}, \tag{8}$$

which is the potential at B relative to A.

One often specifies an electric potential relative to some convenient reference point A, whose potential $V_{\mathrm{A}}$ is arbitrarily set to zero. Then Equations (7) and (8) become

$$V_{\mathrm{B}} - V_{\mathrm{A}} = V_{\mathrm{B}} = \frac{W_{\mathrm{AB}}}{q} \qquad \text{when } V_{\mathrm{A}} = 0,$$

where A is the reference point. This leads to the definition of electric potential.

The **electric potential** $V$ at a point P is the work per unit charge that must be done on any charge to take it slowly from a chosen reference point to the point P.

Thus

$$V = \frac{W}{q}. \tag{9}$$

### Note

- Potential and potential difference are **scalar quantities**.
- If the work $W$ done on a positive test charge is greater than zero, then the potential at B is higher than the potential at A. Conversely, if $W < 0$ then $V_{\mathrm{B}} < V_{\mathrm{A}}$.
- In the above definitions, $q$ is moved slowly (quasi-statically) so that its kinetic energy is zero. The work done on the charge, $W = qV$, is then the **potential energy** of $q$ relative to the reference point.

## 1.5 An important connection between $V$ and $\mathbf{E}$

It is possible to prove an important connection between the electric field $\mathbf{E}$ and the electric potential $V$. Instead of working through a formal derivation, we quote the result:

$$V_{\mathrm{AB}} = V_{\mathrm{B}} - V_{\mathrm{A}} = -\int_{\mathrm{A}}^{\mathrm{B}} E dr, \tag{10}$$

where $dr$ is the magnitude of an infinitesimal displacement in the direction of the field. Here $dr = |d\mathbf{r}|$ and $E = |\mathbf{E}|$.

### 1.5.1 Electric field between two uniformly charged parallel plates

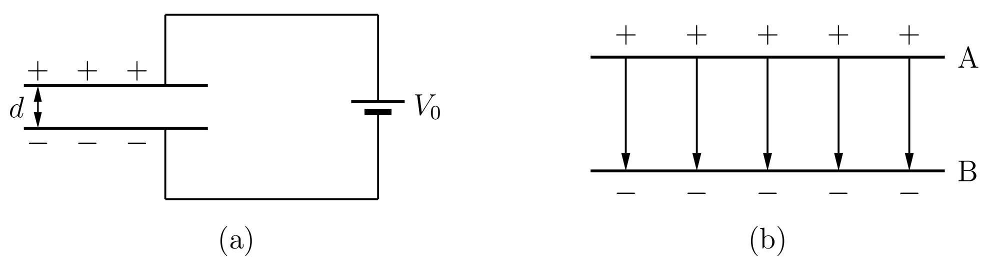



Figure 3: The electric field between parallel plates.

Consider two parallel metal plates separated by a distance $d$ and connected to a battery which maintains a constant pd $V_0$ between the plates. Suppose the top plate is positively charged with respect to the bottom plate which is grounded (i.e. at zero potential) as shown in Figure 3. Then Equation (10) gives

$$V_{\mathrm{B}} - V_{\mathrm{A}} = -E\int_{\mathrm{A}}^{\mathrm{B}} dr = -E(r_{\mathrm{B}} - r_{\mathrm{A}}) = -Ed,$$

since $r_{\mathrm{B}} - r_{\mathrm{A}} = d$. Also $V_{\mathrm{B}} = 0$ and $V_{\mathrm{A}} = V_0$, hence

$$0 - V_0 = -Ed$$

or

$$E = \frac{V_0}{d}. \tag{11}$$

Note that Equation (11) gives as units for electric field $\mathrm{V\,m^{-1}}$. ($1\,\mathrm{V\,m^{-1}}$ is the same as $1\,\mathrm{N\,C^{-1}}$ — these units are equivalent.)

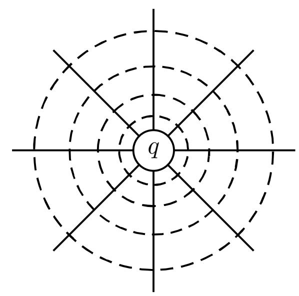
### 1.5.2 The potential of a stationary point charge in vacuum

The defining equations: $\mathbf{E} = \mathbf{F}/q$ and $V = W/q$ are general definitions of electric field and electric potential. They can be used to find $\mathbf{E}$ and $V$ for specific cases. The simplest case that one can treat is an **isolated**, stationary point charge $q$. The electric field at a point P from a point charge (Equation (6)) was determined in Section 1.3.1, here we derive its potential $V$ at P.

As reference point we choose A to be at infinity with $V_\mathrm{A} = 0$. (For practical purposes the surface of the earth may be regarded as at zero potential.) Then Equation (10) gives

$$\begin{aligned}
V_\mathrm{B} - V_\mathrm{A} &= -\int_\mathrm{A}^\mathrm{B} E dr \\
V(r) &= -\int_\infty^r E dr \\
&= -\int_\infty^r \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} dr \\
&= -\frac{1}{4\pi\varepsilon_0} q \int_\infty^r \frac{dr}{r^2} \\
&= -\frac{1}{4\pi\varepsilon_0} q \left[-\frac{1}{r}\right]_\infty^r \\
&= -\frac{1}{4\pi\varepsilon_0} q \left[-\frac{1}{r} + \frac{1}{\infty}\right] \\
V(r) &= \frac{1}{4\pi\varepsilon_0}\frac{q}{r}. \qquad (12)
\end{aligned}$$

Equation (12) shows that for a fixed distance $r$ from $q$ the potential $V$ remains constant. Consider an imaginary sphere centred on $q$. The surface of this sphere connects all points at the same potential; it is called an *equipotential surface* . The diagram alongside shows that $\mathbf{E}$ is everywhere perpendicular to the equipotential surface.




### 1.5.3 Two (or more) point charges in vacuum

We use the principle of superposition to find the net potential at a point in vacuum if there are two (or more) charges present.

Suppose three point charges $q_1$, $q_2$ and $q_3$ are situated at distances $r_1$, $r_2$ and $r_3$ from a point P. Then Equation (12) becomes

$$V_\mathrm{P} = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_1} + \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_2} + \frac{1}{4\pi\varepsilon_0}\frac{q_3}{r_3}.$$

In general, for $n$ charges

$$V_\mathrm{P} = \frac{1}{4\pi\varepsilon_0}\sum_n \frac{q_n}{r_n}, \qquad (13)$$

where $q_n$ is the $n$th charge and $r_n$ its distance from the point P.

**Example 1.11:** Electric field and electric potential due to a point charge

Calculate (a) the electric field, and (b) the electric potential at a distance of 15 cm from an isolated point charge of 5 µC.

**Solution:**

(a) The electric field due to a point charge is given by Equation (6). The magnitude is

$$E = \frac{kq}{r^2} = \frac{9\times 10^9 \times \left(5\times 10^{-6}\right)}{\left(15\times 10^{-2}\right)^2} = 2\times 10^6\,\mathrm{V\,m^{-1}}.$$

Hence $\mathbf{E} = 2\times 10^6\,\hat{\mathbf{r}}\,\mathrm{V\,m^{-1}}$ (radially outwards).

(b) The electric potential due to a point charge is found from Equation (12):

$$V = \frac{kq}{r} = \frac{9\times 10^9 \times \left(5\times 10^{-6}\right)}{15\times 10^{-2}} = 3\times 10^5\,\mathrm{V}.$$

## 1.6 Electric potential energy

Consider two charges $q_1$ and $q_2$ a distance $r$ apart. In order to increase the separation between them, an external agent must do work that will be positive if $q_1$ and $q_2$ are opposite in sign and negative otherwise. The energy represented by this work can be thought of as stored electric potential energy. This energy, like all forms of potential energy, can be transformed into other types. For example, if $q_1$ and $q_2$ are released they will either accelerate towards or away from each other converting potential energy into kinetic energy.

The **electric potential energy** of a system of point charges is the work required to assemble this system of charges by bringing them in slowly from an infinite distance, assuming that the charges have no initial kinetic energy.

Imagine a charge $q_2$ infinitely far away from a charge $q_1$ and that both are at rest. The work done in bringing the charges to a distance $r$ apart is given by the potential difference $\times$ charge (Equation (9)). Then

$$\begin{aligned} W &= V_1 q_2 &&= V_2 q_1 \\ &= \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r}\,q_2 &&= \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r}\,q_1. \end{aligned}$$

Hence the electric potential energy $U$ stored in the system is

$$U = W = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}. \tag{14}$$

By the principle of superposition this result may obviously be extended to any number of charges.

**Example 1.12:** Potential due to a point charge

Calculate the potential at the electron due to the proton in the hydrogen atom, a distance $5\times 10^{-11}$ m away.

**Solution:**

Treat the proton as a point charge, then Equation (12) applies.

$$\begin{aligned} V(r) &= \frac{1}{4\pi\varepsilon_0}\frac{q}{r} \\ &= \frac{9\times 10^9 \times 1.60\times 10^{-19}}{5\times 10^{-11}} \\ &= 28.8\,\mathrm{V} \end{aligned}$$

**Example 1.13:** Speed of a particle accelerated across a potential difference

A particle (mass $m$; charge $q$) is accelerated from rest through a pd of $V_0$ volts. (a) Calculate its final speed $v$. (b) Suppose the particle is an electron in a TV tube and that $V_0 = 12\,000\,\mathrm{V}$. Find $v$ (the mass of en electron is $m_e = 9.1 \times 10^{-31}\,\mathrm{kg}$).

**Solution:**

(a) Since the particle moves freely (i.e. no force on it other than that due to the field), then the work done on it increases its kinetic energy $E_k$ by $qV$. By the work-energy theorem of mechanics

$$W = qV = \Delta E_k = \tfrac{1}{2}mv^2 - 0$$

or

$$v = \sqrt{\frac{2qV}{m}}.$$

(b) From the equation derived in (a)

$$\begin{aligned} v &= \sqrt{2qV/m} \\ &= \sqrt{2 \times 1.6 \times 10^{-19} \times 12 \times 10^3/9.1 \times 10^{-31}} \\ &= 6.5 \times 10^7\,\mathrm{m\,s^{-1}}. \end{aligned}$$

This speed is about 20 % of the speed of light. This calculation should take into account relativistic effects.

### 1.6.1 The electron volt — a unit of energy

The joule is a very large unit for dealing with energies of electrons, atoms or molecules. For this purpose, the **electron volt** is used (abbreviated eV).

**Electron volt**

One electron volt is defined as the energy acquired by an electron when moving through a potential difference of 1 V.

From Equation (7):

$$W = qV = \left(1.6 \times 10^{-19}\,\mathrm{C}\right) \times (1\,\mathrm{V}),$$

hence $1\,\mathrm{eV} = 1.6 \times 10^{-19}\,\mathrm{J}$.

**Example 1.14:** Charged particle in the field of parallel plates

A 1 kV pd is connected across two parallel metal plates which are 10 cm apart. Calculate (a) the magnitude of the electric field between them; (b) the energy gained in eV by (i) an electron, and (ii) an $O^{2-}$ ion when travelling freely from one plate to the other; (c) the speed reached by the electron in (b)(i) above, if it starts from rest. (The mass of an electron $m_e = 9.1 \times 10^{-31}\,\mathrm{kg}$.)

**Solution:**

(a) From Equation (11):

$$E = \frac{V}{d} = \frac{1000}{0.1} = 10\,000\,\mathrm{V\,m^{-1}}.$$

(b) (i) 1 eV is the energy gained by the electron if it moves freely through a pd of 1 volt.

$$\therefore \quad \text{energy gained } 1\,\text{kV} \times e = 1000\,\text{eV}.$$

(ii) The oxygen ion has charge $2e$.

$$\therefore \quad \text{energy gained} = 1000 \times 2 = 2000\,\text{eV}.$$

(c) The work done by the field in accelerating the electron (Equation (7)) equals the change in kinetic energy of the electron. Hence

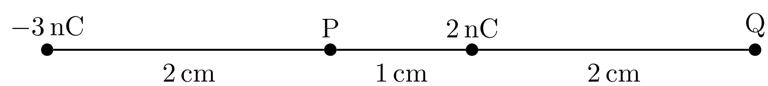
$$W = eV = \Delta\text{KE} = \tfrac{1}{2}m\left(v_f^2 - v_i^2\right).$$

The electron starts from rest, hence $v_i = 0$ and

$$v_f = \sqrt{\frac{2eV}{m}} = \sqrt{\frac{2 \times 1.60 \times 10^{-19} \times 1000}{9.1 \times 10^{-31}}} = 1.88 \times 10^7\,\text{m}\,\text{s}^{-1}.$$

**Example 1.15:** Potential difference and work

Charges lie on a line as shown in the figure below.




Calculate
(a) the potential at P and Q.
(b) the potential difference between Q and P.
(c) the work done to take a charge of 3 µC from Q to P.
(d) the gain in energy of an electron in moving freely from Q to P in joules and eV.

**Solution:**

(a) By the principle of superposition, the potentials at P and Q are the sums of the potentials due to the −3 µC and 2 µC charges. Hence

$$V_\text{P} = \frac{1}{4\pi\varepsilon_0}\left(\frac{-3 \times 10^{-9}}{2 \times 10^{-2}} + \frac{2 \times 10^{-9}}{1 \times 10^{-2}}\right) = 4.5 \times 10^2\,\text{V}$$
$$V_\text{Q} = \frac{1}{4\pi\varepsilon_0}\left(\frac{-3 \times 10^{-9}}{5 \times 10^{-2}} + \frac{2 \times 10^{-9}}{2 \times 10^{-2}}\right) = 3.6 \times 10^2\,\text{V}.$$

(b) $V_\text{QP} = V_\text{P} - V_\text{Q} = 4.5 \times 10^2 - 3.6 \times 10^2 = 90\,\text{V}$.

(c) $W = qV = 3 \times 10^{-6} \times 90 = 2.7 \times 10^{-4}\,\text{J}$.

(d) $\Delta E_k = W = qV = 90\,\text{eV} = 1.6 \times 10^{-19} \times 90 = 1.44 \times 10^{-17}\,\text{J}$.

**Example 1.16:** Potential of a system of charges

Three charges $q_1 = -4\,\mu\text{C}$, $q_2 = 1\,\mu\text{C}$ and $q_3 = 2\,\mu\text{C}$ are located at the vertices of an equilateral triangle of side 5 cm. Calculate the potential energy of the system.

**Solution:**

The potential energy of the system is equal to the sum of the potential energy of each pair of charges in the final configuration. Thus

$$
\begin{aligned}
U &= \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1q_2}{r_{12}} + \frac{q_1q_3}{r_{13}} + \frac{q_2q_3}{r_{23}}\right) \\
&= 9\times 10^9 \times \left(\frac{-4\times 10^{-6}\times 1\times 10^{-6}}{5\times 10^{-2}} + \frac{-4\times 10^{-6}\times 2\times 10^{-6}}{5\times 10^{-2}} + \frac{1\times 10^{-6}\times 2\times 10^{-6}}{5\times 10^{-2}}\right) \\
&= -9\,\mathrm{J}.
\end{aligned}
$$

## 1.7 Capacitance

Capacitors are devices which store electric charge. They consist of two conductors of any shape, placed near, but not touching one another. Often the space between the conductors is filled with an electrically insulating material. According to this definition, the parallel plate arrangement discussed earlier is a capacitor. Each plate carries a charge of the **same magnitude**, one being positive, while the other is negative. The charge that is stored on the plates of a capacitor is proportional to the potential difference across the plates. Thus

$$q = CV, \tag{15}$$

where $C$ is the **capacitance**. The unit of capacitance is the farad: $1\,\mathrm{F} = 1\,\mathrm{C\,V^{-1}}$.

One farad is a very large unit and in practice, capacitances are normally much less than this. Some common capacitances are:

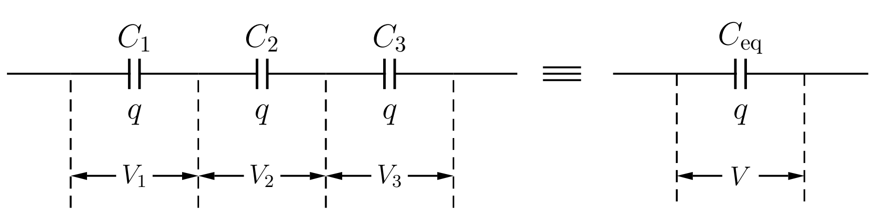
$$
\begin{aligned}
1\,\mu\mathrm{F} &= 10^{-6}\,\mathrm{F} \quad \text{(micro)}\\
1\,\mathrm{nF} &= 10^{-9}\,\mathrm{F} \quad \text{(nano)}\\
1\,\mathrm{pF} &= 10^{-12}\,\mathrm{F} \quad \text{(pico)}.
\end{aligned}
$$

In electric circuits, the symbol for an ordinary capacitor is $\dashv\vdash$. For these capacitors, it does not matter which way round they are connected in a circuit. However, there are special capacitors called **electrolytic** capacitors denoted by the symbol $\overset{+}{\dashv}\!($, which have to be connected with the 'positive' plate (denoted by the straight line in the symbol) being connected to the point of higher positive potential in the circuit.

Capacitors can be connected together in different ways. Two important combinations are considered below.

### 1.7.1 Capacitors in series

Capacitors connected in series can be replaced by a single equivalent capacitor $C_{eq}$.



Capacitors in series **all carry the same charge** which is equal to the charge on $C_{\text{eq}}$. The sum of the potential difference across each capacitor is equal to the potential difference across $C_{\text{eq}}$:

$$V = V_1 + V_2 + V_3.$$

Use of Equation (15) gives

$$\frac{q}{C_{\text{eq}}} = \frac{q}{C_1} + \frac{q}{C_2} + \frac{q}{C_3}$$

or

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}, \tag{16}$$

which can be generalized to any number of capacitors in series.

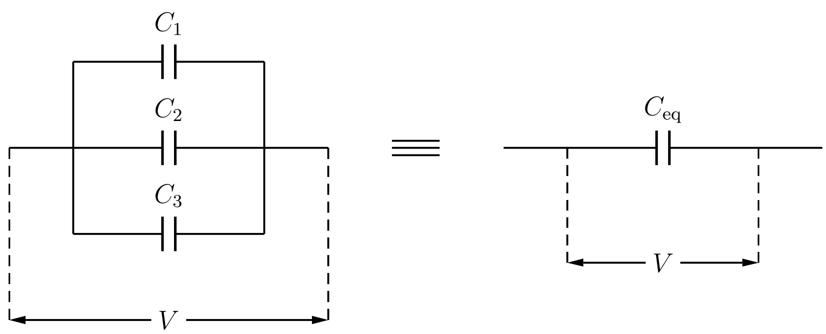
For **two** capacitors in series (common case)

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2}$$

or

$$C_{\text{eq}} = \frac{C_1 C_2}{C_1 + C_2}. \tag{17}$$

### 1.7.2 Capacitors in parallel




Capacitors connected in parallel all experience the **same potential difference** across them, and

$$q = q_1 + q_2 + q_3.$$

Therefore

$$C_{\text{eq}}V = C_1 V + C_2 V + C_3 V$$

or

$$C_{\text{eq}} = C_1 + C_2 + C_3, \tag{18}$$

which can be generalized to any number of capacitors in parallel.

## 1.8 Energy storage in a Capacitor

Charging a capacitor requires energy. The work done in completely charging a capacitor $C$ is given by $W = q\overline{V}$ where $\overline{V}$ is the average voltage across the plates during charging. If the final voltage is $V$ then $\overline{V} = \frac{1}{2}V$ and the work done is

$$W = q\overline{V} = \tfrac{1}{2}qV,$$

which is stored as electric potential energy in the capacitor. Since $q = CV$ the energy stored becomes

$$\text{Energy} = \tfrac{1}{2}qV = \tfrac{1}{2}CV^2 = \frac{q^2}{2C}. \tag{19}$$

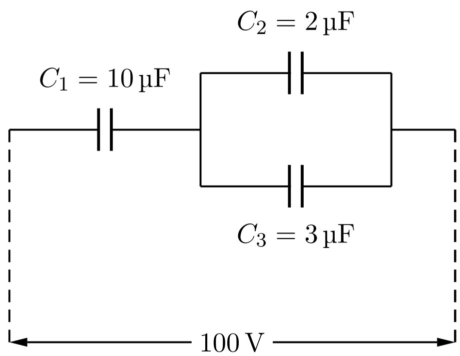
**Example 1.17:** The equivalent capacitance of a combination of capacitors

For the arrangement shown alongside, calculate

(a) the single equivalent capacitance,

(b) the pd across the 10 µF capacitor,

(c) the energy stored in the 2 µF capacitor.




**Solution:**

(a) For the 2 µF and 3 µF capacitors in parallel:

$$C = C_2 + C_3 = 5\,\mu\text{F}.$$

For the (equivalent) 5 µF and 10 µF capacitors in series:

$$\frac{1}{C_{\text{tot}}} = \frac{1}{5} + \frac{1}{10},$$

which gives $C_{\text{tot}} = 3.3\,\mu\text{F}$.

(b) The charge $Q$ that would exist on $C_{\text{tot}}$ is

$$Q = C_{\text{tot}} \times V = 3.33 \times 100 = 333\,\mu\text{C},$$

which is also the charge on the 10 µF capacitor. Hence the pd across the 10 µF capacitor

$$V_{10\,\mu\text{F}} = \frac{Q}{C} = \frac{333}{10} = 33.3\,\text{V}.$$

(c) The pd across the combination $C_2$ and $C_3$, $V = 100 - 33.3 = 66.7\,\text{V}$. Therefore the energy stored in the 2 µF capacitor

$$E = \tfrac{1}{2}CV^2 = \tfrac{1}{2} \times 2 \times 10^{-6} \times 66.7^2 = 4.45 \times 10^{-3}\,\text{J}.$$

## 1.9 Electric flux and Gauss' law

Suppose an electric field **E** makes an angle $\theta$ with the outward normal to a small area $\Delta A$ as shown in Figure 4. Since $\Delta A$ is small, then **E** has effectively the same value at each point on the surface.

The **electric flux** $\psi$ of **E** through $A$ is defined by the equation

$$\psi = (E\cos\theta)(\Delta A). \tag{20}$$

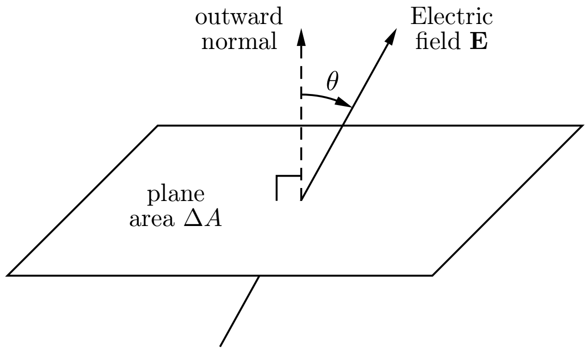



Figure 4: The electric field **E** through a small area $\Delta A$

The unit of electric flux is $\mathrm{N\,m^2\,C^{-1}}$.

- We can visualize $\psi$ as being proportional to the number of electric field lines that are drawn through the area $\Delta A$.
- If **E** is constant over the area then Equation (20) becomes

$$\psi = \begin{pmatrix} \text{Component of } \mathbf{E} \\ \text{perpendicular to the area} \end{pmatrix} \times (\text{the area}). \tag{21}$$

- A *closed* surface can frequently be divided conveniently into several elementary areas such as $\Delta A$. The *total* flux through the closed surface is found by the scalar addition of the flux through the elementary areas.
- For a closed surface, the value of $\psi$ is positive if the field lines point out from the surface ($\theta < 90°$) and negative if they point into it.
- If **E** is non-uniform or the area non-planar then we need calculus

$$\psi = \int \mathbf{E} \cdot d\mathbf{a}.$$

**Gauss' law**

Gauss' law states that for any closed surface the outward electric flux is proportional to the algebraic sum of the charges enclosed by the surface.

In SI units, Gauss' law is

$$\psi = \frac{1}{\varepsilon_0} Q_{\text{net}}, \tag{22}$$

where $Q_{\text{net}}$ is the net enclosed charge. Gauss' law is one of the fundamental equations of electromagnetism. It is sometimes used to calculate electric fields as we see below.

### Rules for applying Gauss' law

- Draw the situation
- Imagine a closed surface to be constructed which is **appropriate to the symmetry**. This imaginary surface is usually called a Gaussian surface (G).
- Apply Equation (22).

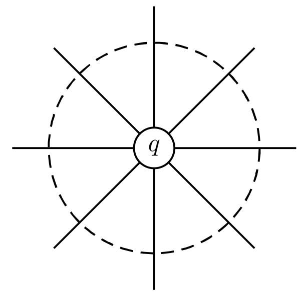
### 1.9.1 Applications of Gauss' law

**Example 1.18:** Electric field of a point charge

Use Gauss' law to obtain the electric field of a stationary point charge $q$.

**Solution:**

By symmetry the field is radial and spherically symmetric. Choose a spherical Gaussian surface centred on $q$. Then $\mathbf{E}$ has the same magnitude at all points on G. By Gauss' law and Equation (21)



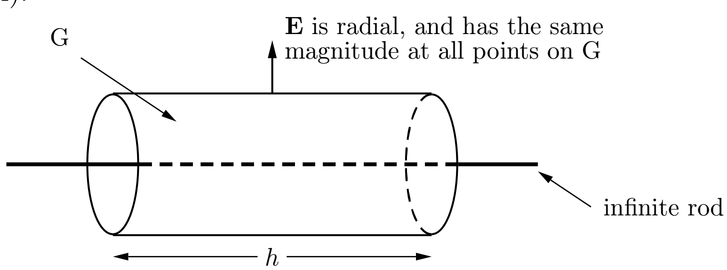

$$\begin{aligned}\psi = \frac{1}{\varepsilon_0} &= (E) \times (\text{surface of a sphere}) \\ &= E \times 4\pi r^2\end{aligned}$$

Hence $E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{r^2}$ and is radial in direction, which is Equation (6).

**Example 1.19:** Electric field of an infinite rod

Use Gauss' law to obtain $\mathbf{E}$ due to an infinite rod of linear charge density $\lambda$ (i.e. charge/length and assumed uniform).

**Solution:**




By symmetry, $\mathbf{E}$ is radial. Hence choose a cylindrical Gaussian surface G coaxial with the line charge. Then $\mathbf{E}$ has the same magnitude at all points on G, and the outward flux is

- zero from the plane ends ($\theta = 90°$), and
- $(E)(2\pi rh)$ over the curved surface ($\theta = 0°$).

By Gauss' law

$$(E)(2\pi rh) = \frac{1}{\varepsilon_0}Q_{\text{net}} = \frac{1}{\varepsilon_0}\lambda h$$

or

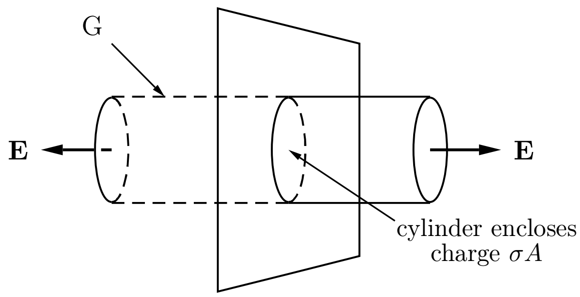
$$E = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{r}$$

and is radial in direction.

**Example 1.20:** Electric field of an infinite sheet

Use Gauss' law to obtain $\mathbf{E}$ due to an infinite conducting sheet of surface charge density $\sigma$ (i.e. charge/area and assumed uniform).

**Solution:**




Since the sheet is infinite, there are no 'edge effects' and the field lines are everywhere normal to the sheet. Hence choose a cylindrical Gaussian surface as shown. The flux through G is then

- zero over the curved surface ($\theta = 90°$),
- $EA$ over the left plane end; $EA$ over the right plane end.

By Gauss' law $2EA = \dfrac{1}{\varepsilon_0} Q_{\text{net}} = \frac{1}{\varepsilon_0}(\sigma A)$, hence

$$E = \frac{\sigma}{2\varepsilon_0} \tag{23}$$

and is normal to the surface.

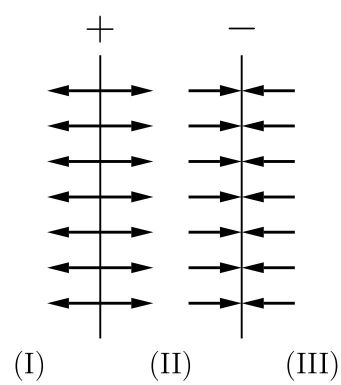
**Example 1.21:** Electric field of two parallel planes

Find the electric field of two parallel planes having equal and opposite uniform charge. Use Equation (23) and superposition.

**Solution:**

Consider the diagram alongside. Directions of the field lines are as shown (away from a positive; towards a negative). In regions (I) and (III) the fields oppose each other. The net electric field is $\sigma/2\varepsilon_0 - \sigma/2\varepsilon_0 = 0$. In region (II) the fields add and the net field is $\sigma/2\varepsilon_0 + \sigma/2\varepsilon_0 = \sigma/\varepsilon_0$. Hence

$$E = \frac{\sigma}{\varepsilon_0}.$$




**Example 1.22:** Capacitance of an ideal capacitor

Suppose two plates each of area $A$ are separated by a constant distance $d$. The plates are connected to a battery which maintains a constant pd $V_0$ across them.

(a) Use the equation derived in Example 1.21 to derive an expression for the capacitance $C$ in terms of $\varepsilon_0$, $A$ and $d$.

(b) Calculate $A$ for $d = 1.0\,\text{mm}$ and $C = 1.0\,\text{F}$.

**Solution:**

(a) If $d \ll \sqrt{A}$, then $\mathbf{E}$ is uniform (to a good approximation) and $E = \sigma/\varepsilon_0$. Now for parallel plates $V_0 = Ed$ (see Equation (11)) and so $V_0 = \sigma d/\varepsilon_0$. If each plate carries a charge $q$, then by definition $\sigma = q/A$ and $V_0 = qd/\varepsilon_0 A$. This last equation in $C = q/V_0$ (definition of capacitance) gives

$$C = \varepsilon_0 A/d,$$

for an ideal, parallel plate capacitor.

(b) From the equation above,

$$A = \frac{dC}{\varepsilon_0} = \frac{4\pi dC}{4\pi\varepsilon_0} = 9\times 10^9 \times 4\pi \times 1 \times 10^{-3} \times 1 = 1.13\times 10^8\,\mathrm{m}^2. \tag{24}$$

The area calculated above corresponds to a square 10.6 km on edge; 1 farad is a very large unit.

# 2 Current Electricity

## 2.1 Electric Current

An electric current is caused by moving charge. Charge may be transported by various carriers — by electrons in the vacuum of an x-ray tube; by ionised atoms and molecules in a gas discharge; by ions in an electrolytic solution; and by electrons in a metal conductor. Some of these charge carriers have positive charge and some negative.

A solid metal is basically a crystal, with its atoms in regular positions in the crystal lattice. However, some of the valence electrons of each atom are delocalized, that is, free to move through the lattice. These are the conduction electrons. In the absence of an applied field the motion of these electrons is random and undirected. When a field is applied to a system containing charge carriers, the positive charges move in the direction of the field and the negatives in the opposite direction, thereby giving rise to current

**Current**

If charge $dq$ passes through an area in a time $dt$, then an electric current $i$ flows equal to the rate at which charge passes through that area.

$$i = \frac{dq}{dt}, \tag{25}$$

The SI unit of charge is the **ampere*** ($1\,\mathrm{A} = 1\,\mathrm{C\,s^{-1}}$).

We shall adopt the convention that the direction of a current is the direction in which positive charge carriers move. In metals the charge carriers are electrons and these move in the opposite direction to the conventional current. In gases and ionic solutions the charge carriers are both positive and negative ions. For most of our purposes we will not be concerned with the nature of the charge carriers.

*The ampere is defined in Section 4.6.

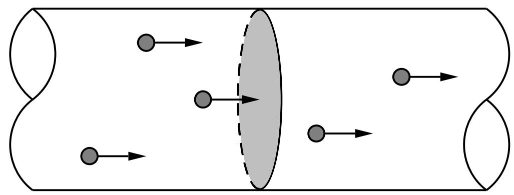


Figure 5: Electric current is the amount of charge per unit time that passes through a surface perpendicular to the motion of the charges.

### 2.1.1 Current density

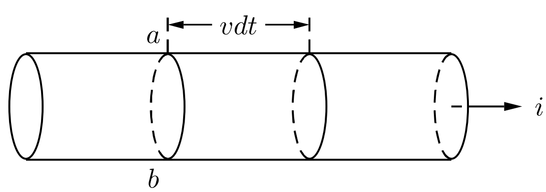
Let a current $i$ flow in a wire of uniform cross-sectional area $A$. Let $ab$ be an imaginary plane perpendicular to the current. Then the **current density** $j$ is defined as the current per unit charge through an area **perpendicular** to the current. Thus

$$j = \frac{i}{A}. \tag{26}$$

The unit of current density is $\mathrm{A\,m^{-2}}$.

### 2.1.2 Drift velocity

A field $\mathbf{E}$ exerts a force $\mathbf{F} = q\mathbf{E}$ on a charge $q$, causing it to accelerate. If the charge moves in a medium, for example a conduction electron moving in a metal, the charge interacts with the particles in the medium, thereby losing some of its kinetic energy to its surroundings which then heat up. Instead of accelerating continuously the moving charge travels with a velocity which , when averaged over successive paths, remains constant. This is called its **drift velocity**. Electrical resistance is due to the "collisions" that a moving charge undergoes with its surroundings, and is accompanied by a rise in temperature of the surroundings.




Let there be $n$ charge carriers per unit volume in a conductor, each with charge $e$ and drift velocity $\mathbf{v}$. Then in a time $dt$ all the charges in a cylinder of length $vdt$ and cross-sectional area $A$ pass through a perpendicular plane $ab$. The volume of this cylinder is $Avdt$ and it contains $nAvdt$ charge carriers. Thus the total charge $dq$ passing through the plane $ab$ in time $dt$ is

$$dq = nevAdt.$$

Hence

$$i = \frac{dq}{dt} = nevA \tag{27}$$

and

$$j = \frac{i}{A} = nev. \tag{28}$$

**Example 2.1:** Drift velocity in a copper wire

A copper wire 1 mm in diameter carries a current of 5 A. The density of copper is $9000\,\mathrm{kg\,m^{-3}}$ and there is one conduction electron per atom. Calculate he drift velocity $v$. (Copper has a relative atomic mass of 63.5.)

**Solution:**

We must first find $A$ and $n$.

$$A = \pi r^2 = \pi \times \left(0.5 \times 10^{-3}\right)^2 = 7.9 \times 10^{-7}\,\mathrm{m}^2.$$

Each $\mathrm{m}^3$ of copper has a mass of 9000 kg and contains

$$N_{\mathrm{A}}\frac{m}{M} = 6.02 \times 10^{23} \times \frac{9000 \times 10^3}{63.5} = 8.5 \times 10^{28}\,\text{atoms}.$$

Since each atom contributes one electron,

$$n = 8.5 \times 10^{28} \times 1 = 8.5 \times 10^{28}\,\mathrm{m}^{-3}.$$

Then

$$v = \frac{i}{neA} = 5/\left(8.5 \times 10^{28} \times 1.6 \times 10^{-19} \times 7.9 \times 10^{-7}\right) = 4.7 \times 10^{-4}\,\mathrm{m\,s^{-1}}.$$

This corresponds to a human-friendly velocity of $168\,\mathrm{cm\,h^{-1}}$.

## 2.2 Resistance and Resistivity

If the same potential difference is applied between the ends of a rod of copper and of a rod of wood, very different currents result. The characteristic of the conductor that enters here is its resistance.

**Resistance**

The **resistance** ($R$) of a material (resistor) is defined as the ratio of the potential difference $V$ applied across a piece of the material to the current $I$ through the material.

$$R = \frac{V}{I}. \tag{29}$$

The unit of resistance is the ohm: $1\,\Omega = 1\,\mathrm{V\,A^{-1}}$.

For a wide range of materials, the resistance of a piece of material of length $\ell$ and cross-sectional area $A$ is

$$R = \rho\frac{\ell}{A}, \tag{30}$$

where $\rho$ is a proportionality constant known as the **resistivity** of the material. For most metals near room temperature $\rho \approx 10^{-8}\,\Omega\,\mathrm{m}$ whereas for a good insulator $\rho$ might be $\approx 10^{19}\,\Omega\,\mathrm{m}$.

Resistance and resistivity change with temperature. For many conductors, one finds an approximately linear dependence of resistance on temperature over a considerable range.

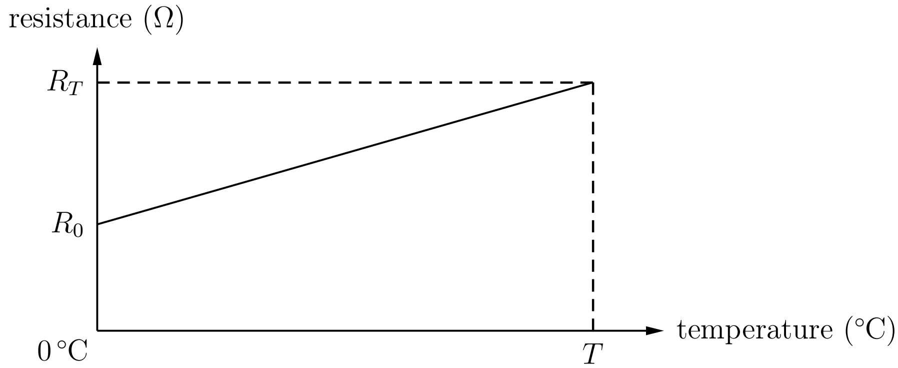



Figure 6: The relation between resistance $R$ and temperature $T$ for many conductors.

Thus

$$R_T = R_0(1 + \alpha T), \tag{31}$$

where $R_0$ is the resistance at 0 °C, $R_T$ is the resistance at temperature $T$ °C and $\alpha$ is the **mean temperature coefficient of resistance** between 0 °C and $T$ °C. For typical metals $\alpha \approx 10^{-3}\,^{\circ}\mathrm{C}^{-1}$.

## 2.3 Ohm's law

When a current is passed through a conductor, the potential difference $V$ across its ends is proportional to the current $I$ at constant temperature. We write $V \propto I$ or $V/I$ is a constant. Thus

$$R = \text{constant}. \tag{32}$$

Materials for which the resistance is constant at constant temperature obey **Ohm's law** and are called ohmic conductors.

**Example 2.2:** The resistivity of a wire

A current of 0.5 A passes through a copper wire 1.8 m long and 0.1 mm in diameter at 20 °C. If the p.d. across the ends of the wire is 2 V, calculate

(a) the resistance of the wire, and
(b) the resistivity of copper.

**Solution:**

(a) Copper at 20 °C is an ohmic conductor so we may use Equation (29):

$$R = \frac{V}{I} = \frac{2}{0.5} = 4\,\Omega.$$

(b) The resistivity is obtained from Equation (30) with

$$A = \pi r^2 = \pi(0.05 \times 10^{-3})^2 = 7.85 \times 10^{-9}\,\mathrm{m}^2.$$

Hence

$$\rho = \frac{RA}{\ell} = \frac{4 \times 7.85 \times 10^{-9}}{1.8} = 1.74 \times 10^{-8}\,\Omega\,\mathrm{m}.$$

**Example 2.3:** The resistance of a wire at different temperatures

Calculate the resistance of the copper wire in the example above, if its temperature rises from 20 °C to 100 °C. Take $\alpha_{\text{Cu}} = 3.9 \times 10^{-3}\,°\text{C}^{-1}$.

**Solution:**

We use Equation (31). The resistance at 20 °C and 100 °C is

$$\begin{aligned} R_{20} &= R_0(1 + 20\alpha) \\ R_{100} &= R_0(1 + 100\alpha). \end{aligned}$$

The resistance at 0 °C is unknown, hence we divide the above equations to elliminate $R_0$:

$$\frac{R_{100}}{R_{20}} = \frac{R_0(1 + 100\alpha)}{R_0(1 + 20\alpha)} = \frac{1 + 100\alpha}{1 + 20\alpha}.$$

From part (a) of Exercise 2.2 above, $R_{20} = 4\,\Omega$. Using $\alpha = 3.9 \times 10^{-3}\,°\text{C}^{-1}$ gives

$$\frac{R_{100}}{4} = \frac{\left(1 + 100 \times 3.9 \times 10^{-3}\right)}{\left(1 + 20 \times 3.9 \times 10^{-3}\right)} = 1.289.$$

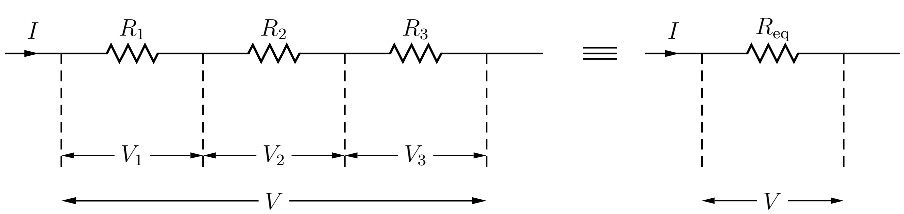
Therefore

$$R_{100} = 4 \times 1.289 = 5.16\,\Omega.$$

## 2.4 Resistors in series

In electric circuits, the symbol used for a resistor is . Some resistances are designed so that they are adjustable, the symbol for a variable resistor is . A **rheostat** is a resistor that is continuously adjustable. The symbol for a rheostat is .

Suppose we connect three resistors $R_1$, $R_2$ and $R_3$ in series as shown below.




The same **current** $I$ passes through all three resistors. From Equation (29) we have $V = IR$, so

$$IR_{\text{eq}} = IR_1 + IR_2 + IR_3$$

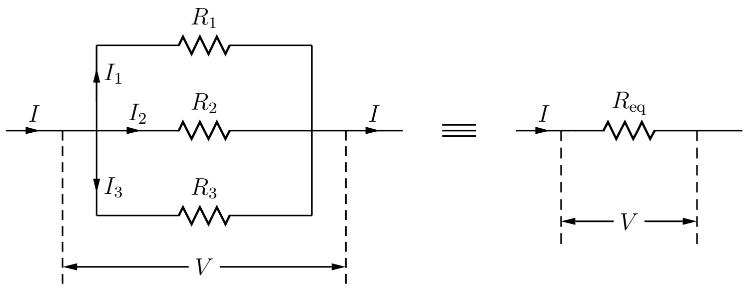
and therefore

$$R_{\text{eq}} = R_1 + R_2 + R_3. \tag{33}$$

Equation (33) can be generalized to any number of resistors connected in series.

## 2.5 Resistors in parallel

Now suppose we connect the above three resistors in parallel.




Here the **voltage** across all three resistors is the same and the current divides into $I_1$ through $R_1$, $I_2$ through $R_2$ and $I_3$ through $R_3$. Then

$$I = I_1 + I_2 + I_3 = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}.$$

If $R_{\text{eq}}$ is the equivalent resistance, then

$$\frac{V}{R_{\text{eq}}} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}.$$

For resistors in parallel therefore

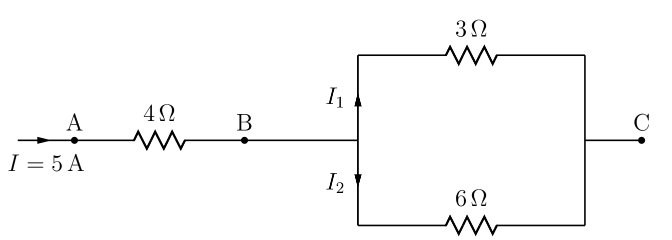
$$\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}. \tag{34}$$

Equation (34) can also be generalized to any number of resistors in parallel. For **two** resistors in parallel (common case)

$$R = \frac{R_1 R_2}{R_1 + R_2}. \tag{35}$$

Note that $\frac{1}{R_{\text{eq}}} > \frac{1}{R_{\text{s}}}$, where $R_{\text{s}}$ is the smallest resistance in the set, so $R_{\text{eq}} < R_{\text{s}}$. Hence the combined resistance of a number of resistances in parallel is always smaller than the smallest resistor in the set.

**Example 2.4:** Equivalent resistance of a resistor combination




Determine the combined resistance $R_{\text{AC}}$ for the circuit above. Calculate the p.d.'s $V_{\text{AB}}$, $V_{\text{BC}}$ and $V_{\text{AC}}$ and the currents $I_1$ and $I_2$.

**Solution:**

We first determine the equivalent resistance of the parallel network BC:

$$\frac{1}{R_{\rm BC}} = \frac{1}{3} + \frac{1}{6} = \frac{1}{2}\Omega^{-1}, \quad \text{hence} \quad R_{\rm BC} = 2\,\Omega.$$

The equivalent resistance $R_{\rm AC}$ is therefore

$$R_{\rm AC} = R_{\rm AB} + R_{\rm BC} = 4 + 2 = 6\,\Omega.$$

To determine the potentials $V_{\rm AB}$, $V_{\rm BC}$ and $V_{\rm AC}$ we use Equation (29). Thus

$$V_{\rm AB} = 4 \times 5 = 20\,\text{V}, \quad V_{\rm BC} = 2 \times 5 = 10\,\text{V}, \quad \text{and} \quad V_{\rm AC} = 6 \times 5 = 30\,\text{V}$$

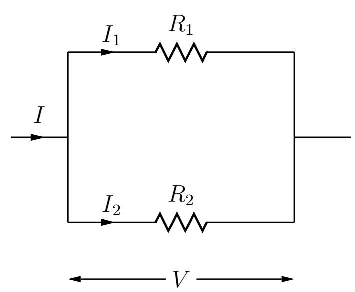
Notice that $V_{\rm AC} = V_{\rm AB} + V_{\rm BC}$ as expected.

The currents $I_1$ and $I_2$ may be found by considering each branch of the parallel network BC separately and again using Equation (29). We find

$$I_1 = \frac{V_{\rm BC}}{3} = \frac{10}{3} = 3.3\,\text{A} \quad \text{and} \quad I_2 = \frac{V_{\rm BC}}{6} = \frac{10}{6} = 1.7\,\text{A}.$$

Notice that $I = I_1 + I_2$ as expected.

### 2.5.1 A current-splitting rule for two resistors in parallel




Two resistors $R_1$ and $R_2$ are connected in parallel as shown opposite. Current divides as shown. From Ohm's law and the equivalent resistance for parallel resistors:

$$V = R_{\rm eq}I = \left(\frac{R_1R_2}{R_1 + R_2}\right)I.$$

Dividing through by $R_1$ and noting that $I_1 = V/R_1$, gives

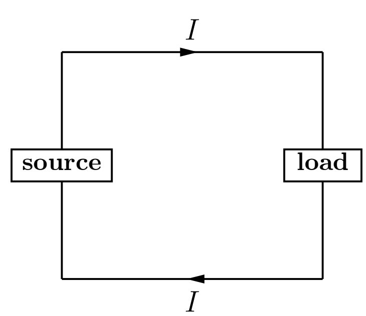
$$\frac{V}{R_1} = I_1 = \left(\frac{R_2}{R_1 + R_2}\right)I.$$

Similarly

$$I_2 = \left(\frac{R_1}{R_1 + R_2}\right)I. \tag{36}$$

The fraction of the current flowing through any one branch of a pair of resistors in parallel, is the resistance **in the other** branch divided by the sum of the branch resistances.

## 2.6 Electric power and energy




In the figure opposite, the **source** could be a battery, power supply, mains outlet etc. The **load** might be a resistor, motor, or a combination of devices.

By definition, whatever the nature of the load, a charge $q$ passing through the load loses energy $W$ given by $W = Vq$ (see Equation (7)). The rate at which energy is transferred from the source to the load is given by the power

$$P = \frac{W}{t} = V\frac{q}{t}$$

or, using Equation (25)

$$P = VI. \tag{37}$$

Equation (37) is a general definition of power dissipation. The SI unit of power is the **watt** ($1\,\mathrm{W} = 1\,\mathrm{V\,A} = 1\,\mathrm{J\,s^{-1}}$).

Equation (37) enables us to calculate the rate at which the load converts electrical energy to other forms of energy. For example, if the load is a resistor, electrical energy is converted to heat, or if the load is an electric motor, to mechanical work, etc.

### 2.6.1 Joule heating

For a resistor carrying a steady current $I$, we obtain, using the definition of resistance (Equation (29)), the equivalent forms:

$$P = I^2R = \frac{V^2}{R}. \tag{38}$$

In a time $t$, the electrical energy converted into heat is given by

$$W = I^2Rt = \frac{V^2}{R}t. \tag{39}$$

Equations (39) are known as Joule's laws.

Note that Equations (38) and (39) apply only to Ohmic conductors.

### 2.6.2 The kilowatt hour — a unit of energy

Most municipalities measure electrical energy in kilowatt hours (kW h). One kilowatt hour is defined as the electrical energy converted to other forms when a power of 1 kW is used for 1 hour. Thus

$$1\,\mathrm{kW\,h} = (1000\,\mathrm{watts}) \times (3600\,\mathrm{seconds}) = 3.6 \times 10^6\,\mathrm{W\,s} = 3.6 \times 10^6\,\mathrm{J}.$$

Note: The kilowatt hour is a unit of energy **not** power.

**Example 2.5:** The cost of using electricity

A domestic electric heater is rated at 230 V; 9 A. If the consumer pays 30 c/kW h for electricity, what is the cost of running this appliance continuously for 11 hours?

**Solution:**

First determine the power rating of the kettle in kW:

$$P = VI = 230 \times 9 \times 10^{-3} = 2.07\,\mathrm{kW}.$$

The kettle is on for 11 hours, therefore the work done in kW h is

$$W = Pt = 2.07 \times 11 = 22.77\,\mathrm{kW\,h}.$$

One kW h costs 30 cents, hence 22.77 kW h costs $22.77 \times 30 = 342\,\mathrm{cents} = \mathrm{R}\,6.83$.

## 2.7 Electromotive force (EMF)

A seat of emf is a device (like a battery) which can supply energy to an electric current. It does this by producing and maintaining a potential difference between two points to which it is attached.

**Electromotive force**
The electromotive force (EMF) $\mathcal{E}$ of a cell may be defined as the energy it will supply per unit charge to drive charge once around a closed path quasistatically.

If we take a charge around a circuit and the work turns out to be

- zero, then only electrostatic forces act on the charge and the emf is zero,
- non-zero, then non-electrostatic forces act and an emf exists in the circuit. Thus emf is the non-Coulomb work per unit charge and might in principle be gravitational or nuclear in nature. For example, the emf of a 1.5 V torchlight battery is due to chemical reactions in which electrons are reactants or products.

Emf in a circuit does work on taking a charge through a potential difference. Emf thus raises charge from low potential (negative side of the emf source) to hight potential (positive side of the emf source). This rise in potential equals the emf. The work done in raising the potential of the charge carrier is derived at the expense of the emf source. For example, a cell 'runs down' as it drives a current around a circuit.

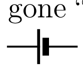
Suppose non-electrical energy $W$ is converted to electrical energy when a charge $q$ is taken through a seat of emf. Then

$$\mathcal{E} = \frac{W}{q}. \tag{40}$$

Note that the name emf is a misnomer because it is not in any way like a force. It is also important to keep in mind that although the units of emf are volts, the emf is not a potential difference.

## 2.8 Chemical cells and batteries

Certain chemical reactions (called redox reactions) take place in an electrochemical cell. The essential components of an electrochemical cell are an electrolyte and two electrodes of unlike materials, one of which reacts with the electrolyte. There are two types of chemical cells called **primary** (or voltaic) cells and **secondary** (or storage) cells. The essential difference is that in the latter type the chemical reactions in a cell may be reversed by passing a "charging" current through it while in the former they cannot. The normal type of cell in a torch is a primary cell — once it has gone "flat" it must be discarded.

The symbol for a cell is : the long line represents the positive terminal and the short line is the negative terminal. When an external resistance is connected across the terminals, charge flows through the complete circuit **including** the cell. We expect and experimentally find that all cells possess some **internal resistance**.

### 2.8.1 Internal resistance

In the circuit depicted in Figure 7, a cell of emf $\mathcal{E}$ having **internal** resistance $r$ is connected to a **load** resistance $R$. Suppose the cell drives a charge $Q$ around the circuit in time $t$.

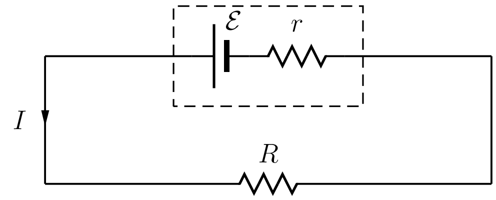



Figure 7: A cell with emf $\mathcal{E}$ connected to a load resistance $R$. The internal resistance of the cell is $r$.

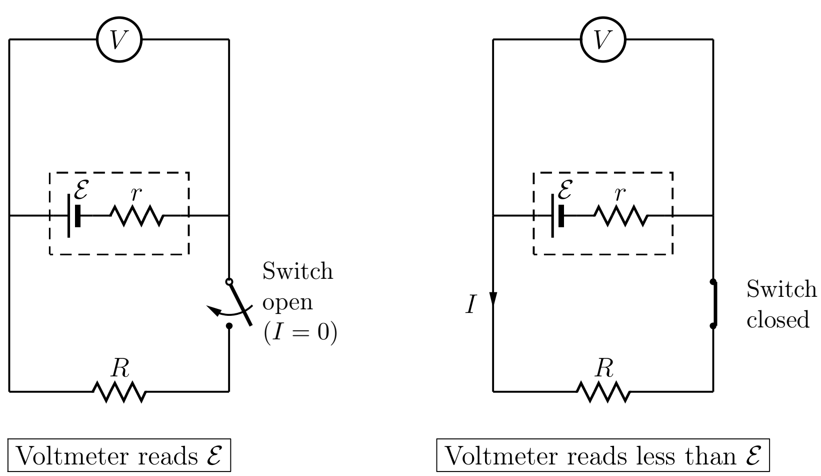
$$\begin{array}{lccccc}
 & \begin{pmatrix}\text{Total energy}\\ \text{supplied}\end{pmatrix} & = & \begin{pmatrix}\text{energy to drive}\\ Q\text{ through the}\\ \text{load resistance } R\end{pmatrix} & + & \begin{pmatrix}\text{energy to drive}\\ Q\text{ through the}\\ \text{internal resistance } r\end{pmatrix}\\
 & \mathcal{E}Q & = & I^2Rt & + & I^2rt\\
\therefore & \mathcal{E}Q & = & QIR & + & QIr\\
\text{or} & \mathcal{E} & = & V & + & Ir
\end{array}$$

Hence the potential difference $V$ measured across a cell connected in a circuit is given by

$$V = \mathcal{E} - Ir. \tag{41}$$

The potential difference $Ir$ is commonly referred to as the "lost volts". Note that:

$$\left.\begin{array}{llllll}\text{(a)} & \text{If} & I = 0 & \text{then} & V = \mathcal{E}\\ \text{(b)} & \text{If} & I > 0 & \text{then} & V < \mathcal{E}\end{array}\right\} \quad \text{See the diagrams below.}$$




**Example 2.6:** Terminal voltage of a battery under load

A battery has an emf of 12.0 V and an internal resistance of 0.15 Ω. What is the terminal voltage when the battery is connected to a 1.50 Ω resistor?

**Solution:**

The total resistance is the sum of the internal resistance and the resistance of the load (see Equation (33)):

$$R_{\text{tot}} = r + R = 0.15 + 1.50 = 1.65\,\Omega.$$

The current in the circuit is given by Ohm's law (Equation (29)):

$$I = \frac{\mathcal{E}}{R_{\text{tot}}} = \frac{12.0}{1.65} = 7.27\,\text{A}.$$

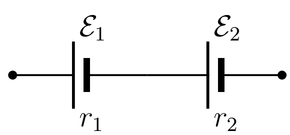
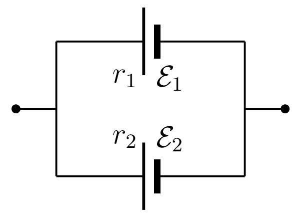
The terminal resistance may now be determined from Equation (41):

$$V = \mathcal{E} - Ir = 12.0 - 7.27 \times 0.15 = 10.9\,\text{V}.$$

### 2.8.2 Batteries

Groups of cells can be connected together in different ways to form a battery. A car battery for instance consists of several cells connected in series.

The table below summarizes two common ways in which cells can be combined. In these diagrams, $r$ represents the internal resistance of the cell.

| Cells in series | Cells in parallel |
|---|---|
| <br><br>effective $r = r_1 + r_2$<br>effective $\mathcal{E} = \mathcal{E}_1 + \mathcal{E}_2$ | <br><br>effective $r$ given by $\frac{1}{r} = \frac{1}{r_1} + \frac{1}{r_2}$<br>effective $\mathcal{E}$:<br>(a) when $\mathcal{E}_1 = \mathcal{E}_2$ then $\mathcal{E} = \mathcal{E}_1$<br>(b) when $\mathcal{E}_1 \neq \mathcal{E}_2$ we don't consider in this course. |

### 2.8.3 Ampere-hour rating of a battery

A useful quantity to look for when buying a battery is the ampere-hour rating. This is best illustrated by means of an example. A 12 V car battery rated at 40 ampere-hour (40 A h) means that this battery can deliver 40 A for 1 hour whilst maintaining a potential difference of 12 V across its terminals. Hence the total available energy is $E = VIt = 12 \times 40 \times 3600 = 1.728 \times 10^6$ J. As a rule of thumb remember that: ampere-hour rating = number of amps delivered × time in hours.

**Example 2.7:** Amp-hour rating of a battery

If the 40 A h car battery above supplies 800 mA at 12 V, how long will it last?

**Solution:**

The time the battery lasts, multiplied by the current supplied by the battery must equal 40 A h. Thus

$$40 \times 1 = 0.8 \times \text{time}$$

which gives

$$\text{time} = 40/0.8 = 50\,\text{hours}.$$

## 2.9 Voltmeters and Ammeters

Voltmeters are used to measure potential differences in a circuit. An ideal voltmeter has **infinite** internal resistance. As such, a voltmeter is always connected **across** a resistor or cell. ('Across' usually means 'in parallel with'.)

Ammeters are used to measure current in a circuit. An ideal ammeter has **zero** internal resistance and is always connected in **series** with a cell or resistor.
Note: Real meters can be made to approximate these ideal requirements rather well.

## 2.10 Multi-loop circuits: Kirchhoff's rules

Not all dc circuits can be analysed in terms of simple series and parallel resistors, particularly when the circuit contains subcircuits or loops with their own emf sources. Kirchhoff's two rules apply to all dc circuits, whether multi loop or not. These rules are:

K1. The *algebraic* sum of all the currents entering any junction point in a circuit is zero.

K2. The *algebraic* sum of all the voltage drops taken in a specified direction around a closed path is zero.

Note that:

- The first rule may be restated, using the definition of current, in the form: The total charge entering a point per unit time equals the total charge leaving that point per unit time. Thus Kirchoff's first rule is a consequence of charge conservation.

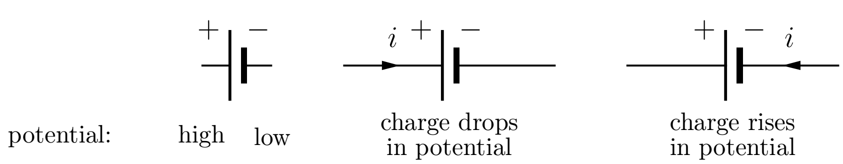
- Based on the definitions of emf and pd, Kirchhoff's second rule may be restated as: The total work per unit charge that is supplied to charge by the sources of emf around a circuit equals the total work per unit charge that is used up in various ways (e.g. as heat in a resistor) in taking charge through the pds around that circuit. Thus this rule is a statement of the conservation of energy.

### 2.10.1 Applications of Kirchhoff's rules

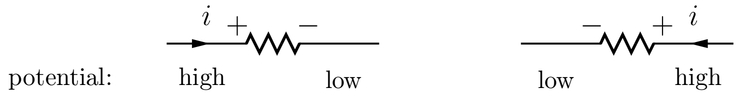
In applying Kirchhoff's second rule great care must be taken with the signs of emf and pds when these are summed algebraically. The system for determining a sign is based on our understanding of the roles of emf and of potential drop across a resistor.

(a) A potential drop is a negative of a potential rise.

(b) An emf source provides the energy to raise the electric potential of charge in a current. By convention, the high potential side of an emf source is positive, the low potential side negative.




(c) Because of the energy loss from "collisions" inside a resistor, the charge in a current *drops* in potential as it passes through a resistor. Thus the side on which current enters a resistor is the high potential side, and the side on which it leaves is the low potential side.




When analyzing a dc circuit using K1 and K2, always:

(a) Decide on the number of unknowns to be found, for this fixes the number of independent equations that must be obtained from the two rules in order to solve for the unknowns.

(b) Make a *quick* choice of direction for each unknown current and then work with it consistently until the problem is solved. It does not matter which direction you choose because if the answer turns out to be negative, then the correct current direction is opposite to that of your initial choice.

(c) Label all branch points and also any others that would help to specify the various circuits.

(d) State the label of each branch point to which K1 is applied; e.g.

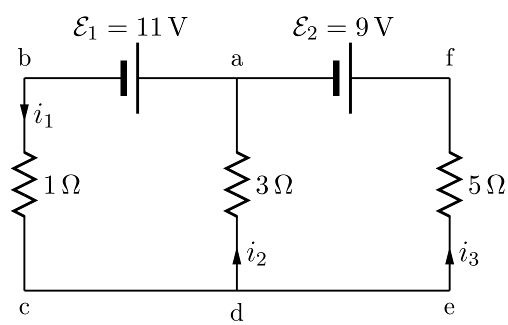
$$\begin{aligned} &\text{K1: b:} \qquad i_2 - i_3 - i_5 = 0 \\ &\text{K1: d:} \qquad i_3 - i_4 - i_1 = 0 \end{aligned}$$

(e) For each closed path to which K2 is applied, state its labels in the order which you choose to go around the path. (It does not matter whether you go clockwise or anticlockwise, as long as it is the *same sense* for the algebraic sum of the potential drops; e.g.

$$\begin{aligned} &\text{K2: abcda:} \qquad \mathcal{E}_1 - \mathcal{E}_2 - 3i_1 + 4i_2 + i_3 = 0 \\ &\text{K2: defcd:} \qquad -\mathcal{E}_3 - \mathcal{E}_4 - 2i_4 + 3i_5 = 0 \end{aligned}$$

**Example 2.8:** Kirchhoff's rules

Consider the circuit shown below. Find $i_1$, $i_2$ and $i_3$.




**Solution:**

| | | |
|---|---|---|
| K1 at junction d gives: | $i_1 - i_2 - i_3 = 0$ | (i) |
| K2 around loop bcdab: | $1 \times i_1 + 3 \times i_2 + 11 = 0$ | (ii) |
| K2 around loop adefa: | $-3 \times i_2 + 5 \times i_3 + 9 = 0$ | (iii) |
| K2 around loop bcdefab: | $1 \times i_1 + 5 \times i_3 + 9 + 11 = 0$ | (iv) |

Notice that of the three K2 equations, only two are independent. We will choose (ii) and (iii). So the equations to be solved are

$$\begin{aligned} i_1 - i_2 - i_3 &= 0 \\ 11 + 3i_2 + i_1 &= 0 \\ 9 + 5i_3 - 3i_2 &= 0, \end{aligned}$$

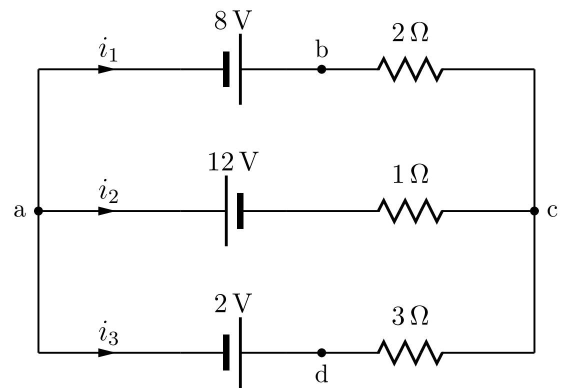
which have the solution

$$i_1 = -5\,\text{A}, \quad i_2 = -2\,\text{A} \quad \text{and} \quad i_3 = -3\,\text{A}.$$

Thus, in all cases, the currents flow in the opposite directions to those indicated above.

**Example 2.9:** Kirchhoff's rules

Find the currents in the three branches if the circuit in the figure.




**Solution:**

Assume the currents and their directions as in the figure. (Obviously at least on of these chosen directions is incorrect.)

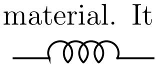
| | |
|---|---|
| K1 at a gives: | $i_1 + i_2 + i_3 = 0$ |
| K2 for loop abca: | $-8 + 2i_1 - i_2 - 12 = 0$ |
| K2 for loop abcda: | $-8 + 2i_1 - 3i_3 + 2 = 0$ |

Solving these three equations simultaneously gives

$$i_1 = 6\,\text{A}, \quad i_2 = -8\,\text{A} \quad \text{and} \quad i_3 = 2\,\text{A}.$$

Hence $i_1$ and $i_3$ flow in the directions indicated but $i_2$ flows from c to a.

## 2.11 Back EMF and inductance

An inductor is a coil of wire with many closely-spaced turns, sometimes with a core of magnetic material. It is used for a variety of functions in electric and electronic circuits. Its symbol is . Although made of wire, an ideal conductor is assumed to have no resistance. An inductor does, however, possess a property called inductance, symbol $L$, which is defined later.

When a changing current $i$ passes through a coil (or solenoid), a changing magnetic flux is produced inside the coil, and this in turn induces an emf. This induced emf opposes the change in flux (Lenz's law) and is sometimes called 'back emf'. For example, if the current through the coil is increasing, the increasing magnetic flux induces an emf that opposes the original current and tends to retard its increase. If the current is decreasing in the coil, the decreasing flux induces an emf in the same direction as the current, thus tending to maintain the original current. In either case, the induced emf $\mathcal{E}$ is proportional to the rate of change of the current. Thus

$$\mathcal{E} = -L\frac{di}{dt}, \tag{42}$$

where the minus sign denotes a back emf and the inductance $L$ is a positive constant.

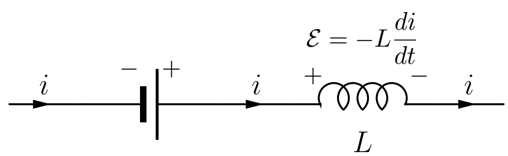
The inductance is a property of the geometry of the coil (number of turns, length, diameter) and of the material inside and surrounding the coil. From Equation (42)

$$L = \left| \frac{\mathcal{E}}{di/dt} \right|, \tag{43}$$

which leads to the definition:

The **inductance** of an inductor is the emf developed across its end per unit rate of change of current through it.

- From Equation (42) the SI units of $L$ are those of $\dfrac{\mathcal{E}}{di/dt}$, namely $\mathrm{V\,s\,A^{-1}}$, called a henry (H). $1\,\mathrm{H} = 1\,\mathrm{V\,s\,A^{-1}}$.
- A positive emf raises the potential of charge entering at the negative and leaving at the positive terminal. A back emf drops the potential for the same charge going through it in the same direction as shown in the figure.




Thus the back emf $\mathcal{E} = -L\,di/dt$ may also be regarded as a potential drop

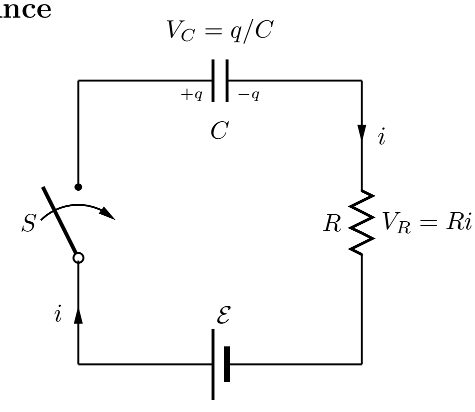
$$V_L = L\frac{di}{dt}. \tag{44}$$

## 2.12 Transient currents: Charging and discharging of capacitance

### 2.12.1 Charging of capacitance through resistance

A circuit contains emf $\mathcal{E}$ resistance $R$, capacitance $C$, and a switch $S$ in series. The capacitor is uncharged initially and at time $t = 0$ the switch is closed. We wish to find how the charge $q$ on the capacitor, the pd $V_C$ across its plates, and the current $i$ through the circuit each depends on time $t$. To do this we use Kirchhoff's second rule:

$$\mathcal{E} = V_C + V_R = q/C + Ri.$$




This contains two time-changing quantities $i$ and $q$. Because they are related by

$$I = \frac{dq}{dt},$$

we can eliminate $i$ and work only with $q$ to obtain

$$\mathcal{E} = \frac{q}{C} + R\frac{dq}{dt}. \tag{45}$$

To integrate this we rearrange:

$$\frac{dq}{dt} = \frac{\mathcal{E} - q/C}{R} = \frac{\mathcal{E}C - q}{RC},$$

and then obtain

$$\int_0^q \frac{dq}{\mathcal{E}C - q} = \int_0^t \frac{dt}{RC}.$$

In this we used the limits: At $t = 0$, $q = 0$; at $t = t$, $q = q$. Thus

$$\begin{aligned}
-\ln(\mathcal{E}C - q)\big|_0^q &= t/RC\big|_0^t \\
\therefore \quad -\ln(\mathcal{E}C - q) + \ln \mathcal{E}C &= t/RC \\
\therefore \quad \ln \frac{\mathcal{E}C - q}{\mathcal{E}C} &= -t/RC \\
\therefore \quad \frac{\mathcal{E}C - q}{\mathcal{E}C} &= e^{-t/RC} \\
\therefore \quad q &= \mathcal{E}C\big(1 - e^{-t/RC}\big).
\end{aligned} \tag{46}$$

As an exponent is a pure number without dimension, $t/RC$ must be dimensionless. Thus $RC$ must have the SI unit of time. It is called the time constant, symbol $\tau$. Thus

$$\tau = RC, \tag{47}$$

is the time constant for $R$ and $C$ in series with $\mathcal{E}$. Then Equation (46) can be written as

$$q = \mathcal{E}C\big(1 - e^{-t/\tau}\big). \tag{48}$$

Then

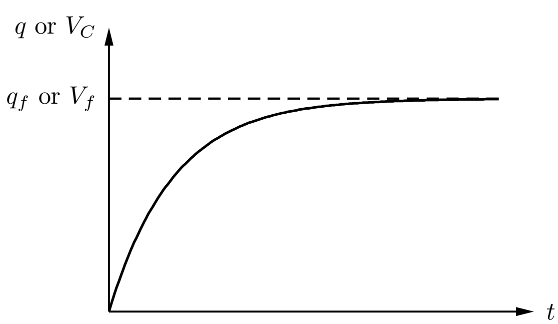
$$i = \frac{dq}{dt} = \frac{\mathcal{E}}{R} e^{-t\tau} \tag{49}$$

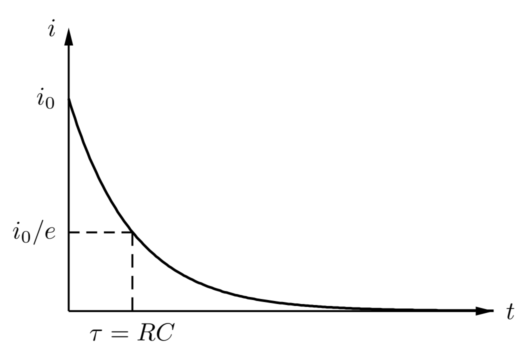
and

$$V_C = \frac{q}{C} = \mathcal{E}\big(1 - e^{-t/\tau}\big). \tag{50}$$

- When $t = 0$, $e^{-t/\tau} = 1$, so that initially $q = 0$, $i = i_0 = \mathcal{E}/R$ and $V_C = 0$.
- When $t = \infty$, $e^{-t\tau} = 0$, so that after a very long time $q = q_f - \mathcal{E}C$, $i = 0$ and $V = V_f = \mathcal{E}$.

The graphs of $q$, $i$ and $V_C$ against time are:




After the switch is closed let a time of one time constant pass, i.e. $t = RC$. Then

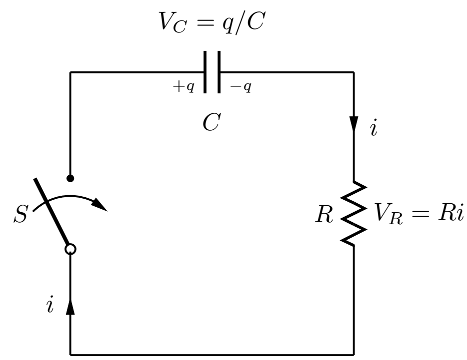
$$e^{-t/RC} = e^{-1} = 1/e = 1/2.718 = 0.3679 \approx 37\,\%.$$

Thus after $t = RC$, $i = i_0/e \approx 37\,\%$ of $i_0$. Also $q \approx 63\,\%$ of $q_f$ and $V \approx 63\,\%$ of $V_f$.

### 2.12.2 Discharging of capacitance through resistance

A circuit contains resistance $R$, capacitance $C$ and a switch $S$ in series. With the switch open the capacitor carries initial charge $q_0$. At time $t = 0$ the switch is closed. We wish to find the charge $q$ on the capacitor, the pd $V_C$ across its plates, and the current $i$ in the circuit at a later time.

To do this we may use the same circuit as that in Section 2.12.1 for charging a capacitor, with the current in the same direction, but now $\mathcal{E} = 0$. Then Equation (45) becomes




$$\frac{q}{C} + R\frac{dq}{dt} = 0.$$

$$\therefore \quad \int_{q_0}^{q} \frac{dq}{q} = -\int_0^t \frac{dt}{RC}$$

$$\therefore \quad \ln q - \ln q_0 = -t/RC = -t/\tau$$

Therefore

$$q = q_0 e^{-t/\tau}. \tag{51}$$

Then

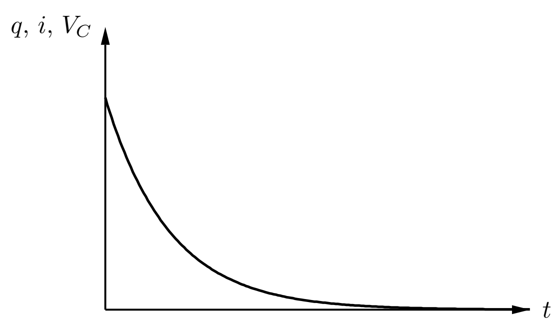
$$i = \frac{dq}{dt} = -\frac{q_0}{RC} e^{-t\tau} = -i_0 e^{-t\tau} \tag{52}$$

and

$$V_C = \frac{q}{C} = \frac{q_0}{C} e^{-t/\tau} = V_0 e^{-t/\tau}, \tag{53}$$

where $i_0 = q_0/RC$ and $V_0 = q_0/C$ are, respectively, the current in the circuit and the pd across the capacitor at $t = 0$. The negative sign in Equation (52) means that the current flows in the opposite direction to that in the figure, as one would expect from the signs of the charges on the plates of the discharging capacitor.

The graphs of $q$, $i$ and $V_C$ vs $t$ all have the same form, as Equations (51)–(53) show. Note that the time constant for $C$ discharging through $R$ is also $RC$.




# 3 Alternating Current

## 3.1 Introduction

When a battery is connected to a circuit, the current flows steadily in one direction. This is called a **direct current** (dc). Electric generators at power stations, however, produce **alternating current** (ac). An alternating current reverses direction many times per second.

Figure 8(a) below shows a steady direct current. Its value does not change with time. Figure 8(b), on the other hand, shows how an alternating current varies with time. The electrons in a wire move first in one direction and then in the other.

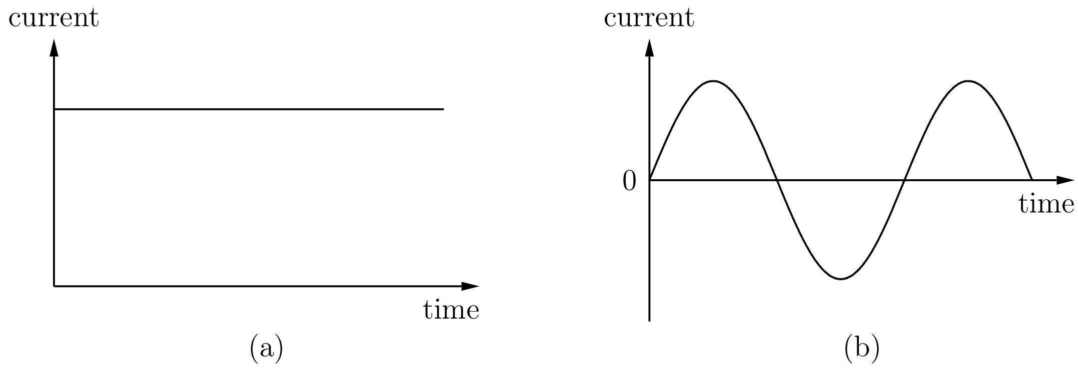
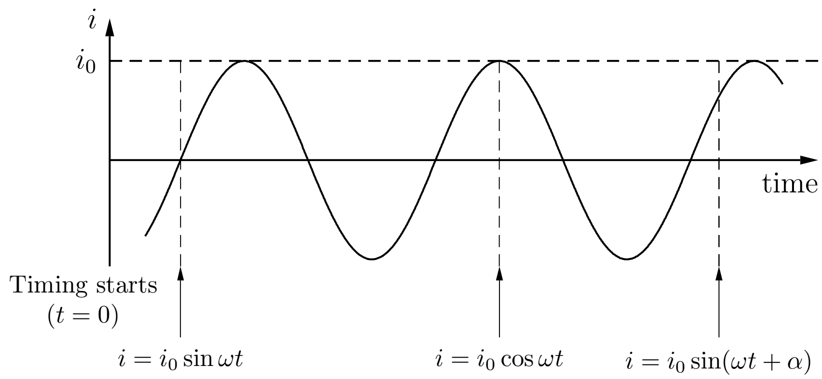



Figure 8: Variation of current with time for (a) direct current and (b) alternating current.

Depending on where one starts timing, an ac current may be expressed in different sinusoidal forms:




Here

$i_0$ is the peak current or current amplitude,
$\omega = 2\pi f$, where $\omega$ and $f$ are the angular and linear frequencies,
$\alpha$ is the initial phase angle.

In all that follows we shall choose

$$i = i_0 \cos \omega t.$$

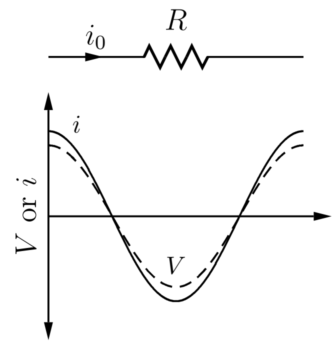
(Physical properties cannot depend on the arbitrary choice of when to start a stopwatch, e.g. the period of a pendulum.)

A very important feature of ac circuits is the phase angle difference between the current in the circuit and the pd across a circuit element such as $C$ or $L$. Because the current may be regarded as constant around the circuit at an instant of time, it is the reference against which the phase of a pd is compared. For example, if

$$i = i_0 \cos \omega t \quad \text{and} \quad V = V_0 \cos(\omega t + 37°),$$

we say that $V$ leads $i$ by 37°.

### 3.2 Resistance in an AC circuit

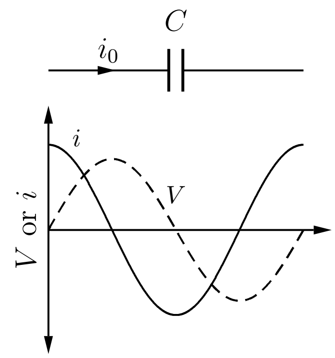



Suppose an alternating current $i = i_0 \cos \omega t$ flows in a resistor $R$ as shown. From Ohm's law, $R = V/i$, we write

$$V_R = Ri_0 \cos \omega t = V_{0R} \cos \omega t,$$

where the peak voltage $V_{0R}$ is $Ri_0$. Note that in this case *the voltage and current are in phase with each other.*

### 3.3 Capacitance in an AC circuit

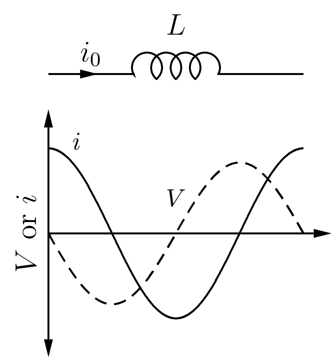



If the same current $i = i_0 \cos \omega t$ now flows in a purely capacitative circuit, then Equation (15) gives

$$\begin{aligned} V_C &= \frac{q}{C}\,\frac{\int i dt}{C} = \frac{i_0}{C}\int \cos \omega t\, dt = \frac{i_0}{\omega C} \sin \omega t \\ &= V_{0C} \cos(\omega t - \pi/2), \end{aligned}$$

where $V_{0C}$ is $i_0/\omega C$ is the peak voltage across $C$. Note that *the voltage across the capacitor is* 90° *behind in phase of the current.* We say that $V$ lags $i$ by 90°.

### 3.4 Inductance in an AC circuit




We again let $i = i_0 \cos \omega t$. If this current flows in a coil having inductance $L$, then from Equation (44)

$$\begin{aligned} V_L &= L\frac{di}{dt} = Li_0 \frac{d}{dt} \cos \omega t = -Li_0 \omega \sin \omega t \\ &= V_{0L} \cos(\omega t + \pi/2), \end{aligned}$$

where $V_{0L} = \omega L i_0$ is the peak voltage across $L$. Note that *the voltage across an inductor is* 90° *ahead in phase of the current.* We say that $V$ leads $i$ by 90°.

### 3.5 Resistance and reactance

We have the following results:

| | |
|---|---|
| for a resistor | $V_{0R} = Ri_0$ |
| for an inductor | $V_{0L} = \omega L i_0$ |
| for a capacitor | $V_{0C} = i_0/\omega C$ |

All the cases above can be expressed in the form $\dfrac{\text{peak voltage}}{\text{peak current}} = \text{constant}$.

For a resistor $\dfrac{V_{0R}}{i_0} = R$ is called the *resistance*

$$\text{For an inductor} \quad \frac{V_{0L}}{i_0} = \omega L = X_L \quad \text{is called the } \textit{inductive reactance} \tag{54}$$

$$\text{For a capacitor} \quad \frac{V_{0C}}{i_0} = \frac{1}{\omega C} = X_C \quad \text{is called the } \textit{capacitative reactance} \tag{55}$$

(The *non-resistive* opposition to current in an ac circuit is called reactance.)

- Unlike resistance, reactance does not apply to instantaneous values as the ratio $V/i$ varies from $-\infty$ to $+\infty$ during a complete cycle.
- Unlike resistance, reactances are frequency dependent. That is, at high frequencies, $X_L$ is large and $C_C$ is small whilst for low frequencies $X_L$ is small and $X_C$ is large.
- Both $X_L$ and $X_C$ have units of resistance.

## 3.6 Root-mean-square current and voltage

In a dc circuit the power dissipated in an ohmic resistor is $P = Ri^2 = V^2/R$. In an ac circuit the power dissipated in an *ohmic resistor* is also

$$P = Ri^2R = R\big(i_0 \cos \omega t)\big)^2 = Ri_0^2 \cos^2 \omega t.$$

Thus ac power varies according to $\cos^2 \omega t$ (or $\sin^2 \omega t$), varying between zero and $Ri_0^2 (= V_0^2/R)$. Unless the frequency is very low this variation is not noticed. It is the *average power* over a period of time comprising many cycles of the ac current or voltage that is usually observed.

To evaluate the average power over one cycle we use the trigonometric identity

$$\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1.$$

Thus $\cos^2 \omega t = \frac{1}{2}(1 + \cos 2\omega t)$ and the average of $\cos^2 \omega t$ is

$$\big\langle \cos^2 \omega t \big\rangle = \big\langle \tfrac{1}{2}(1 + \cos 2\omega t)\big\rangle = \tfrac{1}{2},$$

since over one cycle $\langle \cos\theta \rangle = 0$. Thus the average power

$$\langle P \rangle = Ri_0^2/2 = V_0^2/2R = V_0 i_0/2.$$

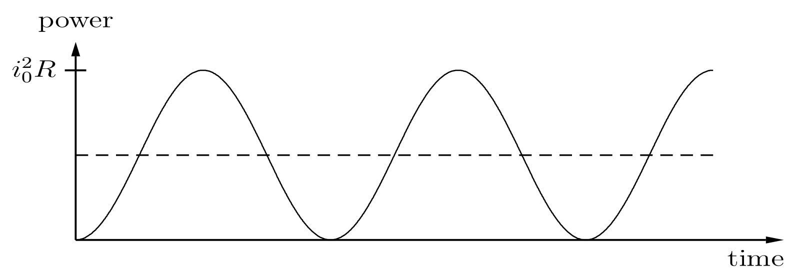



Figure 9: Variation of power with time for a sinusoidally varying current.

The average power may be re-written in terms of the root-mean-square current (or voltage) defined by

$$i_{\text{rms}} = \sqrt{\big\langle i^2 \big\rangle} = \sqrt{\big\langle i_0^2 \cos^2 \omega t \big\rangle} = \sqrt{\tfrac{1}{2} i_0^2}.$$

Thus

$$i_{\text{rms}} = \frac{i_0}{\sqrt{2}} = \frac{\text{current amplitude}}{\sqrt{2}}. \tag{56}$$

Furthermore, $V_{\text{rms}} = \sqrt{\big\langle V^2 \big\rangle} = \sqrt{\big\langle V_0^2 \cos^2 \omega t \big\rangle} = \sqrt{\tfrac{1}{2} V_0^2}$, so

$$V_{\text{rms}} = \frac{V_0}{\sqrt{2}} = \frac{\text{voltage amplitude}}{\sqrt{2}}. \tag{57}$$

Hence

$$\langle P\rangle = V_{\text{rms}}i_{\text{rms}} = Ri_{\text{rms}}^2 = V_{\text{rms}}^2/R. \tag{58}$$

Note that the effective value of an alternating current (or voltage) is equal to the direct current (or voltage) which produces the same joule heating in a resistor. By comparing Equation (58) with $P = Ri^2 = V^2/R$ we see that the *effective value* of an alternating current (or voltage) is simply its peak value divided by $\sqrt{2}$. For example, our mains supply is quoted as 50 Hz, 240 V. This means that the effective or rms voltage of the supply is 240 V. Hence $V_0 = 240\sqrt{2} = 339$ V.

## 3.7 Power dissipated in AC circuits containing both resistance and reactance

Let us now consider an ac circuit in which there is a phase difference $\alpha$ between voltage and current (i.e. the circuit need not be purely resistive as in Section 3.6). Suppose we write $i - i_0\cos\omega t$ and $V = V_0\cos(\omega t + \alpha)$.

The *instantaneous power* $P = Vi$ becomes

$$\begin{aligned} P &= V_0 i_0 \cos\omega t\cos(\omega t + \alpha) \\ &= V_0 i_0[\cos\omega t(\cos\omega t\cos\alpha - \sin\omega t\sin\alpha)] \\ &= V_0 i_0\left[\cos^2\omega t\cos\alpha - \cos\omega t\sin\omega t\sin\alpha\right] \\ &= V_0 i_0\left[\cos^2\omega t\cos\alpha - \tfrac{1}{2}\sin 2\omega t\sin\alpha\right]. \end{aligned}$$

The *average power* $\langle P\rangle$ is then

$$\begin{aligned} \langle P\rangle &= V_0 i_0\left[\left\langle\cos^2\omega t\right\rangle\cos\alpha - \tfrac{1}{2}\langle\sin 2\omega t\rangle\sin\alpha\right] \\ &= V_0 i_0\left[\tfrac{1}{2}\cos\alpha - 0\right] \\ &= V_0 i_0\cos\alpha/2, \end{aligned}$$

or

$$\langle P\rangle = V_{\text{rms}}i_{\text{rms}}\cos\alpha. \tag{59}$$

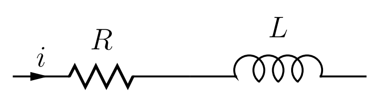
The term $\cos\alpha$ is called the power factor.

- For a purely resistive circuit $\alpha = 0$ and $\cos\alpha = 1$.
- For pure capacitative and inductive circuits, $\alpha = \pi/2$ and $\cos\alpha = 0$. Thus for such circuits, the *average power* dissipated is zero, although the *instantaneous power* need not be zero.

## 3.8 Resultant voltage across combinations of $R$, $L$ and $C$ in series

### 3.8.1 Voltage across $R$ and $L$ in series

Suppose the current $i = i_0\cos\omega t$ flows through a circuit containing $R$ and $L$, which could for example be a coil. We would like to know $V_{RL}$, the resultant voltage across *both* $R$ and $L$. Since $V_R$ and $V_L$ are out of phase, $V_{RL} \neq V_R + V_L$. We can however write $V_{RL}$ in the form

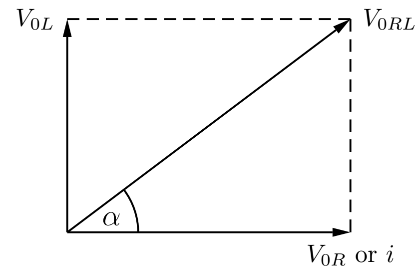



$i = i_0\cos\omega t$

$$V_{RL} = V_{0RL}\cos(\omega t + \alpha),$$

where $V_{0RL}$ is the peak voltage across both $R$ and $L$ and $\alpha$ the phase difference between the current and the resultant voltage. For this circuit the phase angle $\alpha$ must lie in the interval $0° < \alpha < 90°$. Our immediate task is to find $V_{0RL}$ and $\alpha$.

To treat problems of this kind we draw *phasor diagrams* observing the following conventions:



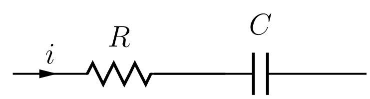

- Plot $V_{0R}$ along the $+x$ axis. Remember, $V_{0R}$ is in phase with $i$.
- Plot $V_{0L}$ along the $+y$ axis. Recall that $V_L$ leads $i$ by 90°.
- The peak resultant voltage across the coil is given by

$$V_{0RL} = \sqrt{V_{0R}^2 + V_{0L}^2} = \sqrt{i_0^2 R^2 + i_0^2 X_L^2} = i_0\sqrt{R^2 + X_L^2}\,.$$

  Note that $V_{0RL}/i_0 = \sqrt{R^2 + \omega^2 L^2}$, which is a constant at a fixed frequency. We call $\sqrt{R^2 + \omega^2 L^2}$ the **impedance** $Z$ of the circuit. Note that $Z$ has units of ohms and like $X$, $Z$ is frequency dependent.
- The phase angle $\alpha$ is given by

$$\tan\alpha = \frac{V_{0L}}{V_{0R}} = \frac{i_0 X_L}{i_0 R} = \frac{\omega L}{R}\,.$$

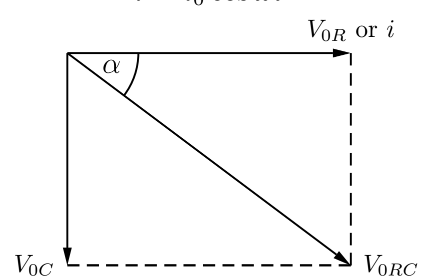
### 3.8.2 Voltage across $R$ and $C$ in series




$i = i_0 \cos\omega t$

Consider the circuit shown opposite. In this case $\alpha$ will lag $i$ and $-90° < \alpha < 0°$. Once again we write the resultant voltage as

$$V_{RC} = V_{0RC}\cos(\omega t - \alpha),$$




Note that we use $-\alpha$ here to indicate that $V_{RC}$ lags $i$. From the phasor diagram we obtain the following:

$$\begin{aligned} V_{0RC} &= \sqrt{V_{0R}^2 + V_{0C}^2} = \sqrt{i_0^2 R^2 + i_0^2 X_C^2} \\ &= i_0\sqrt{R^2 + X_C^2}. \end{aligned}$$

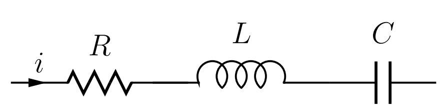
Hence

$$Z = \frac{V_{0RC}}{i_0} = \sqrt{R^2 + X_C^2} = \sqrt{R^2 + \frac{1}{\omega^2 C^2}}$$

and

$$\tan\alpha = \frac{V_{0C}}{V_{0R}} = \frac{i_0 X_C}{i_0 R} = \frac{1}{\omega RC}\,.$$

### 3.8.3 Voltage across $R$, $L$ and $C$ in series




$i = i_0 \cos\omega t$

We now consider a combination of $R$, $L$ and $C$. The voltage across $V_{RLC}$ across the combination is

$$V_{RLS} = V_{0RLC}\cos(\omega t + \alpha)$$

If $X_L > X_C$, the circuit is inductive and $V$ leads $i$ by $\alpha$. However, if $X_L < X_C$, the circuit is capacitative and $V$ lags $i$ by $\alpha$.

As an exercise using the phasor-diagram method, show that

$$Z = \sqrt{R^2 + \left(X_L - X_C\right)^2} \tag{60}$$

and

$$\tan\alpha = \frac{X_L - X_C}{R} \,. \tag{61}$$

## 3.9 Resonance in an $RLC$ circuit

We have discovered that the impedance $Z$ of an $RLC$ circuit is frequency dependent. Let us consider what happens as $\omega$ is increased from zero. It is clear from Equations (54) and (55) that $X_L$ increases and $X_C$ decreases. At some particular frequency $\omega_0$, $X_L = X_C$, then

$$\omega_0 L = \frac{1}{\omega_0 C}.$$

Thus $\omega_0^2 = \dfrac{1}{LC}$ or

$$\omega_0 = \frac{1}{\sqrt{LC}} \,. \tag{62}$$

The frequency $\omega_0$ is called the **resonance frequency**. When the circuit is at resonance

- $Z$ is a minimum,
- $Z$ is purely resistive,
- $i$ is a maximum,
- $V_L$ and $V_C$ are equal in magnitude but 180° out of phase (i.e. they cancel each other exactly).
- The voltage $V_{RLC}$ is in phase with $i$, that is $\alpha = 0$.

**Example 3.1:** Series $RL$

The current in a series $RL$ circuit ($R = 40\,\Omega$, $L = 30\,\text{mH}$) is

$$i = 8\cos 1000t \,\text{amps} \quad (t \text{ in seconds}).$$

Calculate the reactance, the impedance, the peak and instantaneous voltage across the $R$ and $L$ combination, and the rms voltage across $L$.

**Solution:**

From the given current, $i_0 = 8\,\text{A}$ and $\omega = 1000\,\text{rad}\,\text{s}^{-1}$. The reactance is due to the inductor alone. Thus

$$X_L = \omega L = 1000 \times 30 \times 10^{-3} = 30\,\Omega.$$

The impedance

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{40^2 + 30^2} = 50\,\Omega.$$

The peak voltage

$$V_{0RL} = Zi_0 = 50 \times 8 = 400\,\text{V}.$$

The instantaneous voltage

$$V_{RL} = V_{0RL}\cos(\omega t + \alpha),$$

where

$$\alpha = \tan^{-1}\frac{X_L}{R} = \tan^{-1}\frac{30}{40} = 0.64\,\text{rad}.$$

Therefore

$$V_{RL} = 400\cos(1000t + 0.64).$$

The voltage across the inductor

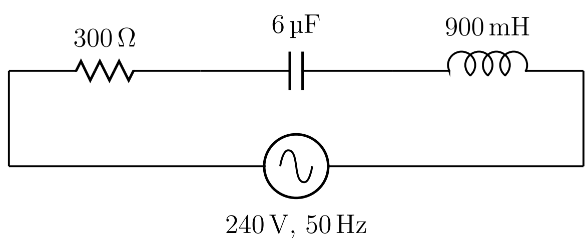
$$V_{0L} = X_L i_0 = 30 \times 8 = 240\,\text{V},$$

hence rms voltage across the inductor

$$V_{\text{rms}} = \frac{V}{\sqrt{2}} = \frac{240}{\sqrt{2}} = 170\,\text{V}.$$

**Example 3.2:** Series $RLC$

A series $RLC$ circuit is connected to the mains supply as shown. Calculate (a) $X_L$, $X_C$, $Z$; (b) $i_{\text{rms}}$, $i_0$; (c) $V_{0R}$, $V_{0L}$, $V_{0C}$, $V_{0RLC}$; (d) the phase angle $\alpha$ between $V_{RLC}$ and $i$; (e) the average power supplied by the mains; (f) the average power consumed in each of $R$, $L$, $C$; (g) the resonant frequency $\omega_0$.




**Solution:**

(a) $X_L = \omega L = 2\pi \times 50 \times 0.9 = 283\,\Omega$

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi \times 50\,6 \times 10^{-6}} = 531\,\Omega$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{300^2 + (283 - 531)^2} = 389\,\Omega.$$

(b) $i_{\text{rms}} = \dfrac{V_{\text{rms}}}{Z} = \dfrac{240}{389} = 0.617\,\text{A}$

$$i_0 = \sqrt{2}\,i_{\text{rms}} = \sqrt{2} \times 0.617 = 0.872\,\text{A}.$$

(c) $V_{0R} = i_0 R = 0.872 \times 300 = 262\,\text{V}$

$$V_{0L} = i_0 X_L = 0.872 \times 283 = 247\,\text{V}$$

$$V_{0C} = i_0 X_C = 0.872 \times 531 = 463\,\text{V}$$

(d) $\alpha = \tan^{-1}\left(\dfrac{X_L - X_C}{R}\right) = \tan^{-1}\left(\dfrac{283 - 531}{300}\right) = -39.6°.$

(e) $\langle P \rangle = V_{\text{rms}} i_{\text{rms}} \cos\alpha = 240 \times 0.617 \times \cos 39.6° = 114\,\text{W}.$

(f) $\langle P_R \rangle = \langle P \rangle = 114\,\text{W}$

$$\langle P_L \rangle = \langle P_C \rangle = 0.$$

(g) $\omega_0 = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{0.9 \times 6 \times 10^{-6}}} = 430\,\text{rad}\,\text{s}^{-1}.$

# 4 Magnetism

## 4.1 Introduction

Consider an experiment with two bar magnets, close to, but not touching one another. We find that the magnets exert a force on each other. This force can be either attractive or repulsive depending on how we orient the magnets with respect to each other. Now perhaps this reminds us of a similar experiment in electrostatics. Namely, that like charges repel and unlike charges attract. It turns out that magnetism is due to the **motion of electric charges**. Clearly then, there must be a link between magnetism and electricity; but they are not the same phenomenon.

At an atomic level, two kinds of electron motions are important in our modern concept of magnetism:

1. An electron **revolving** about the nucleus of an atom imparts a magnetic property to the atom structure.

2. The second kind of electron motion is due to the **spinning** of the electron about its own axis.

### 4.1.1 Magnetic poles

In magnetism there are **no magnetic charges**; we speak instead about **magnetic poles**. We distinguish the two ends of our bar magnet using the labels: **north pole** and **south pole**, and we say that *like poles repel and unlike poles attract*.

Magnets usually have two well-defined poles, one N and one S. Long bar magnets may sometimes acquire more than two poles, and an iron ring may have no poles at all when magnetized. We believe that **a single isolated pole is not a physical possibility**. Scientists have tried to isolate a magnetic "monopole", but none have succeeded.

Note: Do not confuse magnetic poles with electric charge; they are not the same thing.

## 4.2 Magnetic fields

We found it useful to speak of an electric field surrounding an electric charge. In the same way we can imagine a magnetic field surrounding a magnet. Figure 10 shows the magnetic field lines of a bar magnet.

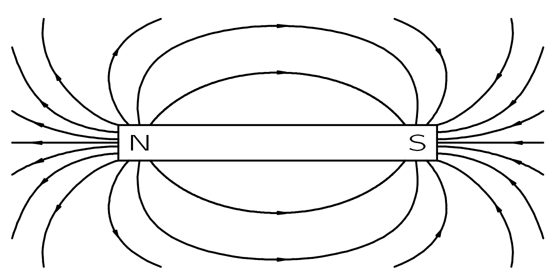



Figure 10: Magnetic field lines of a bar magnet.

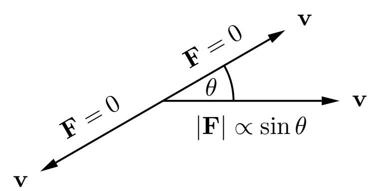
By convention, the magnetic field lines always point away from a north pole and towards a south pole. As in the case of the electric field, the magnetic field is strongest where the lines are closest together. We shall use the symbol **B** to denote magnetic field. **B** is a vector quantity.

## 4.3 The force on a moving charge

Experiment shows that a charge $q$ near a current experiences an unusual force $\mathbf{F}$.

- If $q$ is at rest there is no force, i.e. $\mathbf{F}$ depends on $q$'s velocity $\mathbf{v}$ (this cannot be an electrical force $\mathbf{F} = q\mathbf{E}$ which is independent of $\mathbf{v}$).
- The battery and wire carrying the current were originally neutral and remain so even when current flows. Thus again this force is not electrical.

Hence this is a new kind of force, needing new definitions. Apart from depending on $\mathbf{v}$, the force is also found to vary with the direction of $\mathbf{v}$. For two opposite directions of $\mathbf{v}$, there is no force on $q$, while for any other direction $\mathbf{F}$ varies in magnitude with the sine of the angle between the direction of $\mathbf{v}$ and the direction of $\mathbf{v}$ when $\mathbf{F} = 0$.




A region of space in which a particle, on account of its charge, experiences a velocity-dependent force, is termed a **magnetic field**. The magnitude of the magnetic field **B** at a point is defined by

$$F = |q| v B \sin\theta \tag{63}$$

for the force $F$ on a charge $q$ with speed $v$ at the point.

The units of **B** are

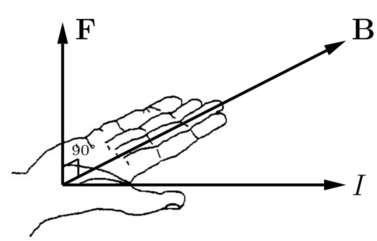
$$\frac{[F]}{[q][v]} = \frac{\mathrm{N}}{\mathrm{C\,m\,s^{-1}}} = \mathrm{kg\,C^{-1}\,s^{-1}}.$$

($1\,\mathrm{kg\,C^{-1}\,s^{-1}} = 1$ tesla or $1\,\mathrm{T}$ in SI units.)

The direction of $\mathbf{F}$ is perpendicular to that of $\mathbf{v}$ and $\mathbf{B}$ in a right-handed sense for a positive charge and the opposite for a negative charge. The right-hand rule gives the direction of $\mathbf{F}$ on a positive charge $q$.

### right-hand rule

Since force is a vector, it has a direction. The right-hand rule gives this direction. Hold your right hand flat, with thumb extended. Point your fingers in the direction of **B** and your thumb in the direction of $I$. The direction in which you would push with this flat right hand, is the direction of $\mathbf{F}$.

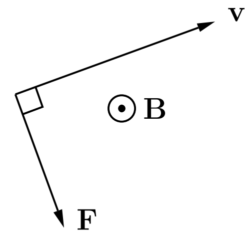



The magnetic force on a charged particle is always perpendicular to the direction of motion, hence the work done by this force on the particle is zero. Thus a static magnetic field cannot change the kinetic energy of a moving charge — it can only deflect it sideways.

If a charged particle moves through a region in which both an electric and a magnetic field are present, the resultant force is found by vector addition.

### 4.3.1 Applications of Equation (63)

**Example 4.1:** Charge moving perpendicular to a magnetic field




Suppose that a charge $q$ enters a uniform magnetic field $\mathbf{B}$, travelling such that $\mathbf{v}$ is perpendicular to $\mathbf{B}$ as shown. Describe the subsequent motion.

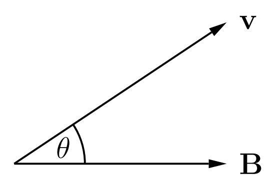
**Solution:**

The force on the charge is $F = qvB \sin \pi/2 = qvB$ which acts perpendicular to both $\mathbf{v}$ and $\mathbf{B}$. Since $\mathbf{F}$ acts perpendicular to $\mathbf{v}$ it is a centripetal force causing circular motion of radius $r$. By Newton's second law $\mathbf{F} = m\mathbf{a}$, we obtain

$$qvB = m\frac{v^2}{r} \quad \text{and} \quad r = \frac{mv}{qB}.$$

Hence the charge moves in a circular orbit with radius $r$.

**Example 4.2:** Charge moving at an angle to magnetic field




Imagine that the charge in Example 4.1 now has velocity $\mathbf{v}$ which makes an angle of $\theta$ to $\mathbf{B}$ (i.e. $\theta \neq \pi/2$).

**Solution:**

Now $\mathbf{v}$ has components $v_{\parallel} = v \cos\theta$ and $v_{\perp} = v \sin\theta$, where $v_{\parallel}$ and $v_{\perp}$ are the velocity components parallel and perpendicular to $\mathbf{B}$ respectively. The perpendicular component $v_{\perp}$ results in a force at right angles to $v_{\perp}$ and produces circular motion with radius

$$r = \frac{mv_{\perp}}{qB} = \frac{mv\sin\theta}{qB}$$

as in Example 4.1. However, $v_{\parallel}$ causes no force on $q$ as $\theta = 0°$, and hence the velocity parallel to the field remains constant.

The resultant of the circular motion in a plane perpendicular to $\mathbf{B}$ and of constant velocity parallel to $\mathbf{B}$ is *helical motion* with $\mathbf{B}$ as axis.

**Example 4.3:** Force on a charge in a combined electric and magnetic field

Suppose $\mathbf{E}$ and $\mathbf{B}$ are electric and magnetic fields respectively that are mutually perpendicular. Let the direction of $\mathbf{E}$ define the $y$ axis and the direction of $\mathbf{B}$ the $z$ axis of a Cartesian system. A particle (charge $-2\,\mu$C) enters the field region travelling with velocity $3 \times 10^4\,\mathrm{m\,s^{-1}}$ along the $x$ axis. Given $E = 5000\,\mathrm{V\,m^{-1}}$ and $B = 0.5\,\mathrm{T}$, find the resultant force on the particle.

**Solution:**

Since $q$ is negative, the electric force is along the $-y$ axis and has magnitude

$$|Eq| = |5000 \times (-2 \times 10^{-6})| = 0.01\,\mathrm{N}.$$

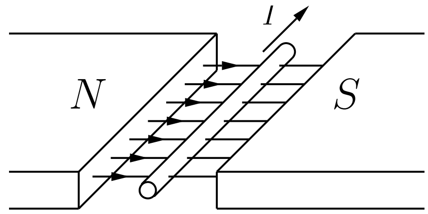
The magnetic force has magnitude

$$|qvB\sin\theta| = |-2 \times 10^{-6} \times 3 \times 10^4 \times 0.5 \times \sin 90°| = 0.03\,\mathrm{N}.$$

The right-hand rule determines the direction of this force: it is along the $+y$ axis. Hence the net force $\mathbf{F}_{\mathrm{net}} = \mathbf{F}_E + \mathbf{F}_B$ has magnitude $F = 0.03 - 0.01 = 0.02\,\mathrm{N}$ and acts along the $+y$ axis since $F_B > F_E$.

## 4.4 The force on a current-carrying wire

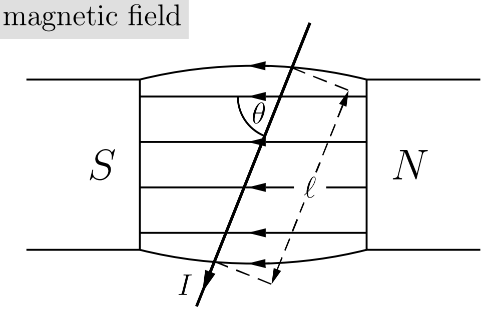
Experiment shows that a current $I$ in a straight wire of length $\ell$ perpendicular to a magnetic field experiences a force $F$ that is directly proportional to both $B$ and $\ell$. So $F \propto I\ell$ or $F = BI\ell$. The constant of proportionality is written as $B$, the magnitude of the magnetic field. The units of $B$ are $\mathrm{N\,A^{-1}\,m^{-1}}$. We use shorthand: $1\,\mathrm{N\,A^{-1}\,m^{-1}} = 1\,\mathrm{T}$ (tesla). If the wire makes an angle $\theta$ with $B$ then




$$F = BI\ell \sin\theta. \tag{64}$$

**Example 4.4:** Wire carrying current in a uniform magnetic field

A wire carrying 30 A has a length $\ell = 12\,\mathrm{cm}$ between the faces of a magnet at an angle $\theta = 60°$ as shown. The field is approximately uniform at 0.90 T. Calculate the force on the length $\ell$ of the wire.




**Solution:**

The force on the wire may be calculated directly from Equation (64).

$$F = BI\ell \sin\theta = 0.9 \times 30 \times 0.12 \times \sin 60 = 2.8\,\mathrm{N}.$$

Applying the right-hand push rule, the force is found to be downwards (into the page).

## 4.5 The Magnetic field of a long, straight wire

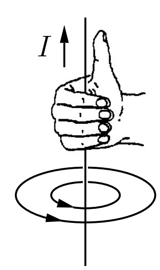
Consider a long straight wire carrying a current $I$. Experiments show that $B$ at a distance $r$ from the wire is directly proportional to $I$ and inversely proportional to $r$. Thus

$$B = \frac{\mu_0}{2\pi}\frac{I}{r}, \tag{65}$$

where $\mu_0$ is a fundamental constant known as the permeability of vacuum. Its value is fixed by the definition of the ampère and is $\mu_0 = 4\pi \times 10^{-7}\,\mathrm{T\,m\,A^{-1}}$.

### right-hand rule No.2

The figure opposite shows the magnetic field lines around a straight wire. We use the right-hand rule for finding the direction of **B**. When the thumb points in the direction of $I$, your fingers wrapped around the wire curl in the direction of the magnetic field.



## 4.6 The force between two long, current-carrying wires

We now consider two long, parallel wires a distance $L$ apart. Suppose they carry currents $I_1$ and $I_2$ respectively. Each current produces a magnetic field that is "felt" by the other. The force exerted on wire 2 is given by

$$F = B_1 I_2 \ell_2,$$

where $B_1$ is the field at wire 2 due to wire 1 and $\ell_2$ is the length of wire 2. But from Equation (65), $B_1 = k\dfrac{I_1}{L}$ so

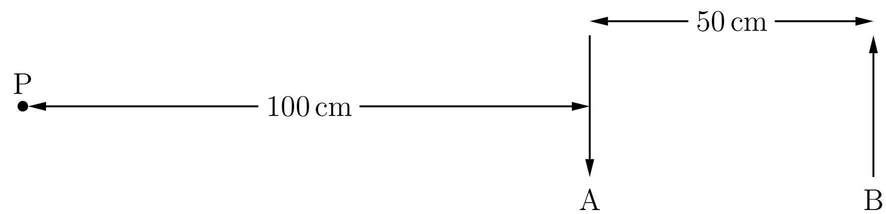
$$F = \frac{kI_1I_2\ell_2}{L} \quad \text{or} \quad \frac{F}{\ell} = \frac{kI_1I_2}{L}, \tag{66}$$

where $k = \mu_0/2\pi = 2 \times 10^{-7}\,\mathrm{T\,m\,A^{-1}}$. Equation (66) is the force per unit length experienced by either wire. By Newton's third law, the force which wire 1 exerts on wire 2 is equal in magnitude and opposite in direction to the force which wire 2 exerts on wire 1 (i.e. $\mathbf{F}_{12} = -\mathbf{F}_{21}$).

**Example 4.5:** Two parallel current carrying wires

Two long, fixed vertical wires A and B are a distance $r = 50\,\mathrm{cm}$ apart in vacuum and carry currents of 100 A and 60 A respectively in opposite directions. Determine (a) the force per unit length exerted by wire A on wire B, (b) the magnetic field at P 100 cm from A and 150 cm from B and (c) the position of any neutral point in a horizontal plane.

**Solution:**




(a) $\dfrac{F}{\ell} = \dfrac{\mu_0}{2\pi}\dfrac{I_1I_2}{r} = \dfrac{4\pi \times 10^{-7}}{2\pi} \times \dfrac{100 \times 60}{0.5} = 2.4 \times 10^{-3}\,\mathrm{N\,m^{-1}}$ away from A.

(b) By the right-hand rule, $\mathbf{B}$ at P due to A is into the page and $\mathbf{B}$ at P due to B is out of the page. The resultant field at P is therefore

$$B = B_\mathrm{A} - B_\mathrm{B} = \frac{\mu_0}{2\pi}\left(\frac{I_\mathrm{A}}{r_\mathrm{A}} - \frac{I_\mathrm{B}}{r_\mathrm{B}}\right) \tag{67}$$

$$= \frac{4\pi \times 10^{-7}}{2\pi}\left(\frac{100}{1} - \frac{60}{1.5}\right)$$

$$= 1.2 \times 10^{-5}\,\mathrm{T} \quad \text{into the page.}$$

(c) At a neutral point, $B_\mathrm{A}$ and $B_\mathrm{B}$ must be equal and opposite. Since $I_\mathrm{A} > I_\mathrm{B}$ the neutral point Q must be closer to B and further from A, such that $B_\mathrm{A}$ and $B_\mathrm{B}$ have opposite directions. Thus Q is on the right of B. Let its distance be $d$ from B. Then

$$B_\mathrm{B} = \frac{\mu_0}{2\pi}\frac{I_\mathrm{B}}{d} \quad \text{and} \quad B_\mathrm{A} = \frac{\mu_0}{2\pi}\frac{I_\mathrm{A}}{d + 0.5},$$

or

$$\frac{60}{d} = \frac{100}{d + 0.5}.$$

Solving the above equation for $d$ gives $d = 0.75\,\mathrm{m}$.

## 4.7 The definition of the Ampère

Because force can be measured accurately, as can distance, the force per unit length between parallel currents, as given by Equation (66) is used to define the SI unit of current, ampère.

**Ampère**
1 ampère (amp) is that current which, when flowing in each of two thin parallel conductors of infinite length and 1 metre apart in vacuum, causes each conductor to experience a force per unit length of $2 \times 10^{-7}$ N.

- The definition of the ampère fixes the value of the constant $\mu_0$ as follows: Substitute $I_1 = I_2 = 1\,\text{A}$ $r = 1\,\text{m}$ and $F/\ell = 2 \times 10^{-7} =$ newton/m into Equation (66)

$$\begin{aligned} 2 \times 10^{-7} &= \frac{\mu_0}{2\pi}\frac{1 \times 1}{1} \\ \therefore \quad \mu_0 &= 4\pi \times 10^{-7}\,\text{T m A}^{-1}. \end{aligned}$$

- The definition of the ampère in turns allows us to define the coulomb.

**Coulomb**
One coulomb is the quantity of electric charge that passes a given point in a conductor in one second when the current is one ampère.

Note: We can use the right-hand rule to prove the important result that **like currents attract and unlike currents repel.**

## 4.8 Ampère's law

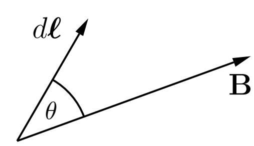
The magnetic field at a distance $r$ from a long, straight current is given by Equation (65)

$$B = \frac{\mu_0}{2\pi}\frac{I}{r},$$

or $B(2\pi r) = \mu_0 I$. This illustrates Ampère's law

$$\oint (B)(d\ell \cos\theta) = \mu_0 I_{\text{net}}, \tag{68}$$

where $d\ell \cos\theta$ is the magnitude of an infinitesimal displacement in the direction of $\mathbf{B}$ and $\theta$ is the angle between $\mathbf{B}$ and $d\ell$ as shown.




- $\oint (B\cos\theta)(d\ell)$ is the line integral around a *closed* path.
- $I_{\text{net}}$ is the net current enclosed by the path.
- The law holds for *any* closed path surrounding $I_{\text{net}}$.
- Ampère's law is one of the fundamental equations of electromagnetism. It is often useful for calculating the magnetic field of a current distribution.
- Ampère's law is best expressed in terms of a dot product

$$\oint \mathbf{B} \cdot d\boldsymbol{\ell} = \mu_0 I_{\text{net}},$$

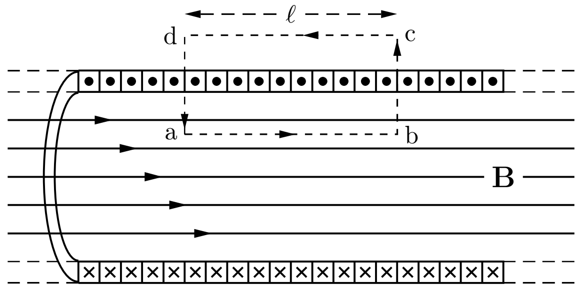
where $\mathbf{B} \cdot d\boldsymbol{\ell}$ means $Bd\ell \cos\theta$.

#### 4.8.1 Application of Ampère's law to the magnetic field of an infinitely long solenoid

We now consider the important example of an ideal solenoid, assumed to possess the following properties

- infinite length
- when carrying current $I$, the magnetic field is (i) everywhere uniform inside and parallel to the axis, and (ii) zero outside.

This suggests that, as path of integration, we choose the rectangular path abcd in the figure below.




We write $\oint_C B d\ell \cos\theta$ as the sum of four integrals, one for each path segment.

$$\oint_C B d\ell \cos\theta = \int_a^b B d\ell \cos\theta + \int_b^c B d\ell \cos\theta + \int_c^d B d\ell \cos\theta + \int_d^a B d\ell \cos\theta.$$

For the path segment $ab$, $\theta = 0$ and the first integral on the right is $B\ell$ where $\ell$ is the arbitrary length of path from $a$ to $b$. The second and fourth integrals are zero because for every element of these paths $\mathbf{B}$ is perpendicular to $d\boldsymbol{\ell}$. The third integral, which includes part of the rectangle lying outside the solenoid is zero because $\mathbf{B}$ is zero for all external points of an ideal solenoid.

Thus the line integral for the entire path is

$$\oint_C B d\ell \cos\theta = B\ell + 0 + 0 + 0 = B\ell.$$

The net current that passes through the area bounded by the path of integration is

$$\text{(the number of turns enclosed)} \times \text{(current in the solenoid)} = n\ell \times I.$$

Ampère's law then gives:

$$B\ell = \mu_0 \times n\ell I$$

or

$$B = \mu_0 n I, \tag{69}$$

where $n$ is the number of turns per unit length.

We use a version of the right-hand rule to determine the direction of $\mathbf{B}$. Grasp the solenoid with your right hand such that your fingers curl in the direction of $I$. Your thumb then points in the direction of $\mathbf{B}$ along the solenoidal axis.

Although Equation (69) was derived for an infinitely long solenoid, it holds very well for actual solenoids for interior points near the centre if the length of the solenoid is much larger than its diameter.

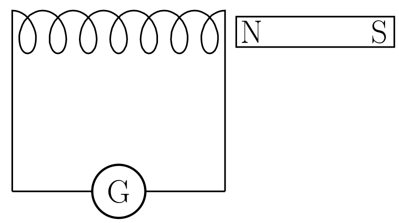
## 4.9 Magnetic flux and Faraday's law

The flux $\phi$ of magnetic field $\mathbf{B}$ is defined exactly the same way as electric flux (see Section 1.9). Equation (20) becomes

$$\phi = (B\cos\theta)(\Delta A)$$

The unit of magnetic flux is the weber, $1\,\mathrm{Wb} = 1\,\mathrm{T\,m^2}$. If $\mathbf{B}$ is constant over the area then the above equation becomes

$$\phi = \begin{pmatrix} \text{Component of } \mathbf{B} \\ \text{perpendicular to the area} \end{pmatrix} \times (\text{the area}). \tag{70}$$




In attempting to produce an electric current from a magnetic field, Faraday used apparatus similar to that shown opposite; a coil connected in series with a galvanometer G. With no source of emf in the circuit current does not flow. When a bar magnet is moved towards the coil, the galvanometer deflects, showing that an emf is induced which drives a current in the circuit.

When the magnet is moved away from the coil, the galvanometer deflects in the opposite direction. If the magnet is stationary there is no deflection, even when the magnet is close to the coil. If now the coil is moved towards the stationary magnet, there is again a deflection, which is in the same direction as when the magnet is moved towards the coil. We thus conclude that an emf is induced in the coil by relative motion of the coil and magnet. It makes no difference whether the magnet is moved towards the coil or the coil towards the magnet.

Finally, if the magnet is moved slowly towards the coil, the galvanometer deflection is less than when the same movement of the magnet is accomplished rapidly. This suggests that the induced emf depends in the rate of change of the magnet's field at the coil.

Experiments like these led Faraday to state the **law of magnetic induction**:

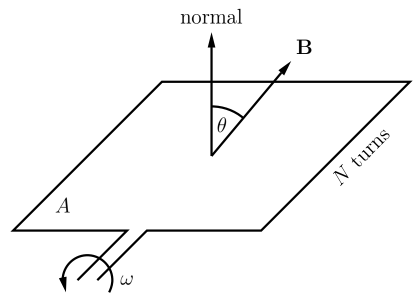
The emf that is induced around a closed path equals the negative of the rate of change of magnetic flux through that path.

$$\mathcal{E} = -\frac{d\phi}{dt}. \tag{71}$$

- The negative sign in Equation (71) indicates that the polarity of the induced emf is such that $\mathcal{E}$ opposes the flux change. For instance, if the magnetic field producing the flux in a circuit increases, the induced emf drives current in that direction which reduces the magnetic field. This is form of le Chatelier's principle, which in electromagnetism is known as Lenz's law.

- Faraday's law in the form of Equation (71) is another of the fundamental laws of electromagnetism.

### 4.9.1 The ac generator




Consider a plane coil of area $A$ and with $N$ turns, which is rotating with constant angular velocity $\omega$ about an axis perpendicular to a uniform magnetic field $\mathbf{B}$. Let the normal to the coil make an angle $\alpha$ to the field at time $t = 0$. At time $t$ the coil and its normal will have rotated trough an angle $\omega t$. Thus the angle $\theta$ between the normal and $\mathbf{B}$ at time $t$ is

$$\theta = \alpha + \omega t.$$

The magnetic flux linking each turn of the coil is

$$\phi_1 = AB\cos\theta = AB\cos(\omega t + \alpha).$$

Hence the total flux through the $N$ turns of the coil is

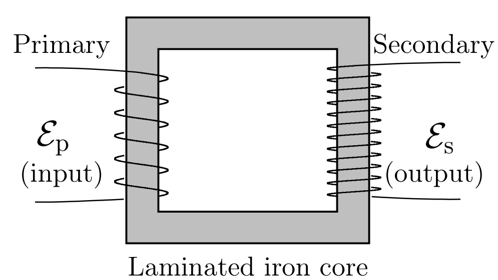
$$\mathcal{E} = -\frac{d}{dt}NAB\cos(\omega t + \alpha) = NAB\omega\sin(\omega t + \alpha) = \mathcal{E}_0\sin(\omega t + \alpha),$$

where the emf amplitude is

$$\mathcal{E}_0 = NAB\omega.$$

Thus an alternating emf is induced in a plane coil which rotates with constant angular velocity in a uniform magnetic field.

## 4.10 The transformer




A transformer is a device for increasing or decreasing an ac voltage. A transformer consists of two coils of wire wound on the same soft-iron former, which ensures good flux linkage between the coils.

An ac emf $\mathcal{E}_\mathrm{p}$ is applied to one of the coils called the primary, with $N_\mathrm{p}$ turns. Then $\mathcal{E}_\mathrm{p}$ drives an ac current $i_\mathrm{p}$ in the primary which creates an ac magnetic field. This field is 'trapped' inside the soft-iron former, giving rise to an ac flux through the $N_\mathrm{s}$ turns of the other coil, called the secondary. By Faraday's law an ac emf

$$\mathcal{E}_\mathrm{s} = -\frac{d\phi}{dt}$$

is induced in the secondary, which then drives an ac current $i_\mathrm{s}$ in an external circuit connected to the secondary coil. The ac frequency in the primary and secondary circuits is the same.

The theory of an ideal transformer depends on two assumptions:

- There are no power losses in either coil or in the former, and
- The flux per turn is the same for both the primary and secondary coils.

With these assumptions, one can show, using Faraday's law that

$$\frac{\mathcal{E}_\mathrm{s}}{\mathcal{E}_\mathrm{p}} = \frac{N_\mathrm{s}}{N_\mathrm{p}} = \frac{I_\mathrm{p}}{I_\mathrm{s}}, \tag{72}$$

where $\mathcal{E}_\mathrm{s}$ and $\mathcal{E}_\mathrm{p}$ are the maximum values of emf on secondary and primary windings respectively. The ratio $N_\mathrm{s}/N_\mathrm{p}$ is called the **turns ratio*** $n$. If $n > 1$, we have a step-up transformer (output $\mathcal{E} >$ input $\mathcal{E}$). If $n < 1$, we have a step-down transformer (output $\mathcal{E} <$ input $\mathcal{E}$). Note that transformers will not work with a dc emf in the primary.

**Example 4.6:** Rotating coil

A circular coil of 8000 turns and radius 5 cm rotates in 0.2 s about a vertical axis from a position where its plane is parallel to a horizontal magnetic field $B$ to a position where it is perpendicular to $B$. An emf of 6 mV is induced in the coil. Calculate $B$.

**Solution:**

Use $\phi = BAN\cos\theta$ with $A = \pi r^2$.
Initially: $\theta = 90°$ and $\phi_i = 0$.
Finally: $\theta = 0°$ and $\phi_f = BAN = B\pi\left(5 \times 10^{-2}\right)^2 \times 8000 = 62.93B$.
Hence $\Delta\phi = \phi_f - \phi_i = 62.83B - 0 = 62.83B$. Then Faraday's law

$$\mathcal{E} = -\frac{\Delta\phi}{\Delta t}$$

gives

$$6 \times 10^{-3} = \frac{62.83B}{0.2},$$

hence

$$B = 1.91 \times 10^{-5}\,\mathrm{T}.$$

**Example 4.7:** Emf induced in a coil

A long solenoid has 7500 turns/m and carries a current of 5 A. A close-packed coil, 3 cm in diameter and carrying 200 turns, is placed near the centre of the solenoid such that **B** is parallel to the coil's axis. The current in the solenoid is reduced to zero and then raised to 5 A in the other direction at a steady rate over a period of 50 ms. Calculate the average emf induced in the coil whilst the current is being changed.

**Solution:**

The field in the solenoid is $B = \mu_0 n I = 4\pi \times 10^{-7} \times 7500 \times 5 = 4.7 \times 10^{-2}\,\mathrm{T}$.
The initial flux through the coil $\phi_i = BAN\cos 0° = BAN$ and the final flux through the coil $\phi_f = BAN\cos 180° = -BAN$. Hence

$$\begin{aligned}\Delta\phi = \phi_f - \phi_i &= -2BAN \\ &= -2 \times 4.7 \times 10^{-2} \times \pi \times \left(1.5 \times 10^{-2}\right)^2 \times 200 \\ &= -1.33 \times 10^{-2}\,\mathrm{Wb}.\end{aligned}$$

*There does not appear to be consensus whether the turns ration is $N_\mathrm{s}/N_\mathrm{p}$ or $N_\mathrm{p}/N_\mathrm{s}$. Most elementary physics textbooks give the definition provided here, whereas most electrical engineering texts prefer $N_\mathrm{p}/N_\mathrm{s}$. To avoid confusion it is advisable to state which ratio is referred to, or to state whether the transformer is a step-up or step-down transformer.

Then

$$\mathcal{E} = -\frac{\Delta\phi}{\Delta t}$$

gives

$$\mathcal{E} = \frac{1.33d-2}{50d-3} = 0.266\,\mathrm{V}.$$

**Example 4.8:** Change of flux

A coil is connected to a current integrator ( a device for measuring charge). The total resistance in the circuit is $120\,\Omega$. A flux change $\Delta\phi$ occurs in the coil and the integrator measures a charge of $4\,\mu\mathrm{C}$. Calculate $\Delta\phi$.

**Solution:**

Faraday's law with $\mathcal{E} = RI$ gives

$$RI = \frac{\Delta\phi}{\Delta t}.$$

But

$$I = \frac{\Delta q}{\Delta t}$$

so

$$R\frac{\Delta q}{\Delta t} = \frac{\Delta\phi}{\Delta t}.$$

Hence $\Delta\phi = R\Delta q = 120 \times 4 \times 10^{-6} = 480 \times 10^{-6}\,\mathrm{C}$.

**Example 4.9:** A step-down transformer

A transformer reduces 240 V ac to 9 V ac to operate a cd player. The secondary coil contains 30 turns and the cd player draws 400 mA. Calculate (a) the number of turns in the primary coil, (b) the power transformed and (c) the current in the primary coil.

**Solution:**

Since the voltage is stepped down, this is a step-down transformer.

(a) From Equation (72):

$$N_\mathrm{p} = N_\mathrm{s}\frac{V_\mathrm{p}}{V_\mathrm{s}} = 30 \times \left(\frac{240}{9}\right) = 800\,\text{turns}.$$

(b) The power transformed may be calculated from Equation (59). Thus

$$\text{Power} = V_\mathrm{s}I_\mathrm{s} = 9 \times 0.4 = 3.6\,\mathrm{W}.$$

(c) Assuming power loss is negligible, then

$$V_\mathrm{p}I_\mathrm{p} = V_\mathrm{s}I_\mathrm{s} = 3.6\,\mathrm{W}$$

and

$$I_\mathrm{p} = 3.6/240 = 0.015\,\mathrm{A}.$$

Transformers are used in power transmission from one place to another (e.g. from a power station to a city). By stepping up the voltage, the current is decreased. Since power depends on $I^2R$ (see Equation (38)), the joule losses are reduced.

# TUTORIAL QUESTIONS

## Coulomb's law

A1 A small sphere A carries a charge of $+120\,\mu C$. Calculate the magnitude and direction of the force which it exerts on a second small sphere B carrying a charge of $-300\,\mu C$ if the distance between them is 30 cm.

A2 A small metal sphere is given a charge of $20\,\mu C$ and a second identical sphere located 25 cm away is given a charge of $-5\,\mu C$.

(a) What is the force of attraction between the charges?

(b) Calculate the number of excess electrons on the negative sphere.

A3 The two spheres of Question 2 are allowed to touch and are again placed 25 cm apart.

(a) What charge is transferred between the spheres?

(b) What force exists between them now?

A4 Two non-conducting spheres carry a total charge of $190\,\mu C$. When placed 1.0 m apart, the force each exerts on the other is 54.0 N and is repulsive.

(a) What is the charge on each?

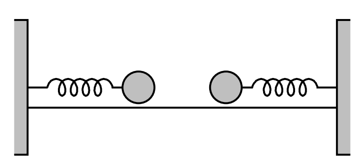
(b) What if the force were attractive?

A5 A $4.5\,\mu C$ and a $-0.5\,\mu C$ charge are placed 18 cm apart. Where can a third charge be placed so that it experiences no net force?

A6 Two equally charged balls, each of mass 0.1 g, are suspended from the same point by threads 13 cm long. The balls come to rest 10 cm apart due to repulsion. Determine the charge $q$ on each ball.

A7 Three point charges of $+2, +3$ and $+4\,\mu C$ are at the vertices of the equilateral triangle ABC having sides of 10 cm. What is the magnitude of the resultant force $R$ acting on the $+4\,\mu C$ charge?

A8 Two small spheres are mounted on identical horizontal springs and rest on a frictionless table as shown. When the spheres are uncharged, the spacing between them is 5 cm and the springs are unstressed. When each sphere carries a charge of $1.6\,\mu C$, the spacing doubles. Determine a value for the spring constant.



A9 Two small identical spheres are placed 20 cm apart. Each carries a different charge and they attract each other with a force of 1.2 N. The spheres are brought into contact so that the net charge is shared equally between them. They are now returned to their original positions and are found to repel each other with a force equal in magnitude to the initial attractive force. Calculate the initial charge on each object. Note that there are two answers.

# Electric field and electric current

B1 Consider a point 2.0 m away from a $-3.0\,\mu$C point charge. Calculate

(a) the electric field $\mathbf{E}$ and

(b) the electric potential $V$ at this point.

B2 An electron in a uniform electric field experiences a force of $8.0 \times 10^{-16}$ N. What is the magnitude and direction of $\mathbf{E}$ at this point?

B3 A point charge of $-36\,\mu$C is located at the origin. Find the electric field (a) on the $x$ axis at $x = 2$ m, (b) on the $y$ axis at $y = -3$ m and (c) at the point with coordinates $x = 1$ m, $y = 1$ m.

B4 A charge of $16 \times 10^{-9}$ C is fixed at the origin of coordinates, a second charge of unknown magnitude is at $x = 3$ m, $y = 0$, and a third charge of $12 \times 10^{-9}$ C is at $x = 6$ m, $y = 0$. What is the magnitude of the unknown charge if the resultant field at $x = 8$ m, $y = 0$ is $20.25\,\hat{\mathbf{x}}\,\mathrm{N\,C^{-1}}$?

B5 A small object carrying a charge $-5 \times 10^{-9}$ C experiences a downward force of $20 \times 10^{-9}$ N when placed at a certain point in an electric field.

(a) What is the electric field at the point?

(b) What would be the magnitude and direction of the force acting on an electron placed at the point?

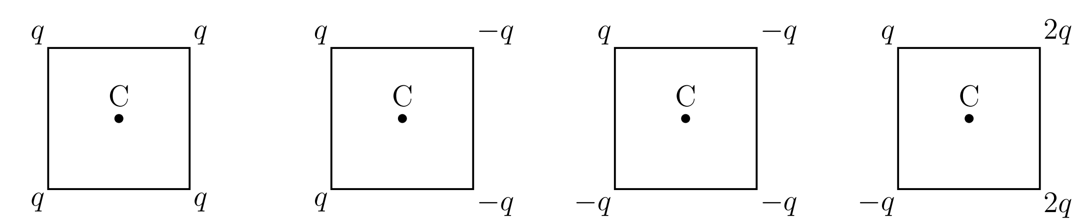
(c) What is the acceleration of the electron?

B6 A uniform electric field exists in the region between two oppositely charged, plane, parallel plates. An electron is released from rest at the surface of the negatively charged plate and strikes the surface of the opposite plate, 2 cm distant from the first, in a time interval of $1.5 \times 10^{-8}$ s.

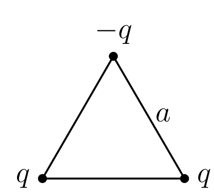
(a) Find the electric field.

(b) Find the velocity of the electron when it strikes the second plate.

B7 What is the direction of the electric field at the centre C of each of the charge distributions shown below? The charges are located at the corners of a square.




B8 Three charges are located at the corners of an equilateral triangle of side $a$ as show. (a) Determine the magnitude and direction of $\mathbf{E}$ at the centre of the triangle. (b) Find the magnitude and direction of the resultant force on charge $-q$.

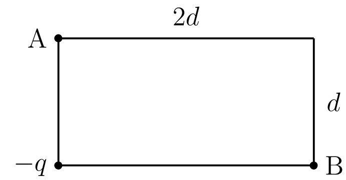



B9 Two charges $q_1$ and $q_2$ are placed 10 cm apart in vacuum. At what point on the line joining the two charges is the electric field zero if

(a) $q_1 = +1\,\mu\text{C}$ and $q_2 = +2\,\mu\text{C}$?

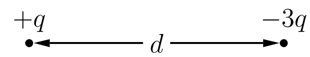
(b) $q_1 = +1\,\mu\text{C}$ and $q_2 = -2\,\mu\text{C}$?

B10 A negative charge $-q$ is fixed to one corner of a rectangle as shown. What positive charge must be fixed to corner A and what positive charge must be fixed to corner B so that the total electric field at the remaining corner is zero? Express your answers in terms of $q$.

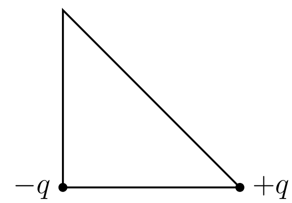



B11 Calculate the field and potential of a point charge $-4\,\text{nC}$ at a distance of $20\,\text{cm}$.

B12 In the figure opposite locate the points (a) where $V = 0$ and (b) where $\mathbf{E} = 0$. Consider only points on the inter-charge axis. Choose $q = 0.02\,\mu\text{C}$ and $d = 1\,\text{m}$. Roughly sketch several electric-field lines and draw the equipotential surface corresponding to zero volts.




B13 Two point charges each with a magnitude $q = 2.00\,\mu\text{C}$ are fixed to adjacent corners of a right-angled isosceles triangle as shown. One charge is positive and one is negative. What charge (both magnitude and sign) must be fixed to the mid-point of the hypotenuse so that the electric potential at the empty corner is zero?




B14 Charges $2\,\text{nC}$ and $-3\,\text{nC}$ lie on two corners of an equilateral triangle of side $20\,\text{cm}$. Find the potential at (a) the midpoint P of the two charges, and (b) the third corner A. (c) Find the potential difference between A and P and (d) the work done to take a charge of $4\,\text{nC}$ from P to A.

B15 The straight line connecting two points that are $15\,\text{cm}$ apart lies parallel to a uniform electric field of $360\,\text{N}\,\text{C}^{-1}$. What is the pd between the two points and which is at higher potential?

B16 Two positive point charges each of magnitude $q$, are fixed on the $y$ axis at points $y = +a$ and $y = -a$.

(a) Draw a diagram showing the positions of the charges.

(b) Show that the potential at any point on the $x$ axis is

$$V = \frac{1}{4\pi\varepsilon_0}\frac{2q}{\sqrt{a^2 + x^2}}.$$

(c) Suppose a positively charged particle of charge $q_0$ and mass $m$ is displaced slightly from the origin in the direction of the $x$ axis. What is its speed at infinity?

(d) Sketch a graph of the velocity of the particle as a function of $x$.

B17 A particle with charge $3\,\text{nC}$ is in a uniform electric field acting to the west. It is released from rest and travels a distance $5\,\text{cm}$, after which its kinetic energy is found to be $45\,\mu\text{J}$.

(a) What work was done by the electric field?

(b) What is the magnitude of the electric field?

(c) What is the potential of the starting point relative to the end point?

B18 A positive charge $+q_1$ is located to the left of a negative charge $-q_2$. On a line passing through the two charges there are two places where the electric potential is zero. The first is between the charges 4 cm to the left of $-q_2$ and the second is 7 cm to the right of $-q_2$. Find

(a) the distance between the charges and

(b) the ratio of the magnitudes of the charges $q_1/q_2$.

B19 Two point charges 20 nC and −12 nC are 50 cm apart. An electron is placed on the line between the charges at a point 10 cm from the negative charge. It is then released. What is its velocity when it is 10 cm from the positive charge? (Hint: Use the definition of pd to find the work, and work done on an object gives the kinetic energy.)

B20 A 5.0 g object carries a net charge of 3.8 µC. It acquires a speed $v$ when accelerated from rest through a potential difference $V$. A 2.0 g object acquires twice the speed under the same circumstances. What is its charge?

B21 Two electrons, initially far apart, are aimed directly at each other. Each has a speed of $1000\,\mathrm{m\,s^{-1}}$. How close do they come before reversing direction and flying apart?

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# Capacitance

C1 How much charge flows from a 12.0 V battery when it is connected to a 2.0 µF capacitor?

C2 The two plates of a capacitor carry +1500 µC and −1500 µC of charge respectively, when the potential difference is 300 V. Calculate the capacitance.

C3 Calculate the magnitude of the electric field between the plates of a 20 µF capacitor if they are 2.0 mm apart and each has a charge of 300 µC.

C4 Three capacitors having capacitances of 0.16 µF, 0.22 µF and 0.47 µF are connected in parallel and charged to a potential difference of 240 V.

(a) Determine the charge on each capacitor.

(b) What is the total capacitance of the combination?

(c) What is the total charge acquired?

C5 A 6.0 µF and a 4.0 µF capacitor are connected in series to a 60.0 V battery.

(a) Calculate the equivalent capacitance.

(b) What is the charge on each capacitor?

(c) Determine the voltage across each capacitor.

C6 Two identical capacitors are connected in series and found to have charge of magnitude 200 nC on each plate when the combination is connected across a power supply of 50 V. Find the capacitance of each capacitor.

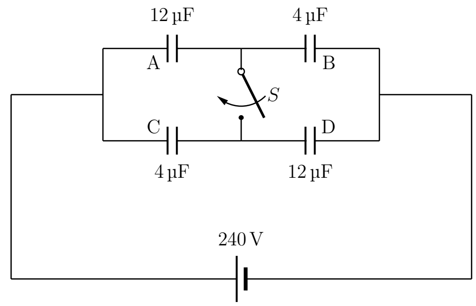
C7 A 0.1 nF capacitor is charged to 50 V. The charging battery is then disconnected and replaced by a second capacitor. If the measured potential difference drops to 35 V, what is the capacitance of this second capacitor?

C8 Calculate the charge on each capacitor shown opposite

(a) when switch $S$ is open and

(b) when switch $S$ is closed.

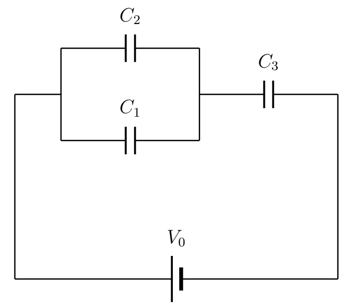
Determine also (c) the change in energy stored in the capacitor A when $S$ is closed.




C9 In the circuit shown, assume that $C_1 = 10\,\mu\text{F}$, $C_2 = 5\,\mu\text{F}$, $C_3 = 4\,\mu\text{F}$ and $V_0 = 100\,\text{V}$.

(a) Find the equivalent capacitance of the combination.

(b) Suppose that the capacitor $C_3$ breaks down electrically becoming equivalent to a zero-resistance path. What changes to the charge and potential difference occur for capacitor $C_1$?




C10 Calculate the energy stored in a 600 pF capacitor that is charged to 100 V.

C11 It takes 6.0 J of energy to move a 2000 µC charge from one plate of a 5.0 µF capacitor to the other. Calculate the charge on each plate.

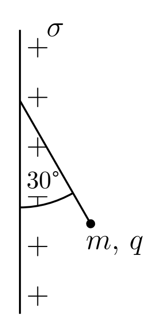
C12 A 16.0 µF and a 4.0 µF capacitor are connected in parallel and charged by a 22.0 V battery. What voltage is required to charge a series combination of the two capacitors with the same total energy?

# Gauss' law

D1 A thin-walled metal sphere has a radius of 25 cm and carries a charge of $2 \times 10^{-7}$ C. Find **E** for a point (a) inside the sphere (b) just outside the sphere (c) 3 m from the centre of the sphere.

D2 Two charged concentric spheres have radii of 10 cm and 15 cm. The charge on the inner sphere is $4 \times 10^{-8}$ C and that on the outer sphere $-2 \times 10^{-8}$ C. Find **E** at (a) $r = 12$ cm, (b) $r = 20$ cm.

D3 A small sphere whose mass $m$ is 1 mg carries a charge $q$ of 0.02 µC. It hangs from a silk thread which makes an angle of 30° with a large, charged conducting sheet as shown. Calculate the charge density $\sigma$ for the sheet.




D4 Two uniformly charged planes with surface charge densities of $3 \times 10^{-9}\,\text{C}\,\text{m}^{-2}$ and $-1 \times 10^{-9}\,\text{C}\,\text{m}^{-2}$ lie parallel to each other a distance 8 cm apart in vacuum. Draw the field lines between the plates and behind each plate and find **E** everywhere. (Hint: Use superposition.)

D5 An infinite charged sheet has a charge density $\sigma$ of $0.1\,\text{nC}\,\text{mm}^{-2}$. How far apart are the equipotential surfaces whose potentials differ by 5 kV?

# Current, resistance and resistivity

E1 Most of the wiring in a typical house can safely handle about 15 A of current. At this current level, how much charge flows through a wire in one hour?

E2 A wire carries a current of 5 A. How many electrons are flowing past any point in this wire per minute?

E3 Calculate the resistance of a 2.0 m length of copper wire 0.15 mm in diameter. Take $\rho_{\text{Cu}} = 1.7 \times 10^{-8}\,\Omega\,\text{m}$.

E4 A wire of length 0.24 m and diameter $3.0 \times 10^{-5}$ m has a resistance of 160 Ω. Calculate the resistivity of its material.

E5 Consider a cube 5 mm on a side, made of carbon. Estimate the resistance between a pair of opposite faces given $\rho_{\text{C}} = 3.5 \times 10^{-5}\,\Omega\,\text{m}$.

E6 A 0.5 Ω wire is drawn out ("stretched") to four times its original length. Assuming that the density of the wire does not change, calculate its new resistance.

E7 A 33 Ω resistor is made from a coil of copper wire whose total mass is 12 g. What is the diameter of the wire and how long is it? Take $d_{\text{Cu}} = 8.9 \times 10^{3}\,\text{kg}\,\text{m}^{-3}$ and $\rho_{\text{Cu}} = 1.7 \times 10^{-8}\,\Omega\,\text{m}$.

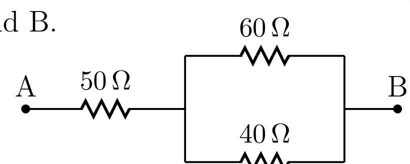
E8 A 100 W light bulb has a resistance of about 12 Ω when cold and 140 Ω when "on" (hot). Estimate the temperature of the filament when "on", assuming a mean temperature-coefficient of resistance of $6 \times 10^{-3}\,{}^{\circ}\text{C}^{-1}$.

E9 A coil of wire has a resistance $R_0$ at 0 °C and a temperature coefficient of resistance $\alpha$. If its resistance is 20 Ω at 25 °C and 25 Ω at 100 °C, calculate $\alpha$ and $R_0$.

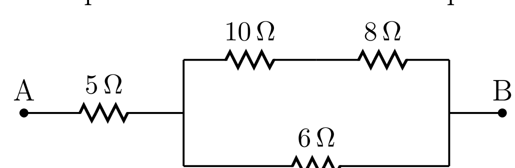
E10 An iron wire has a resistance of 5.90 Ω at 20 °C and a gold wire has a resistance of 6.70 Ω at the same temperature. At what temperature $T$°C do the wires have the same resistance? (Take the mean temperature coefficients of resistance of iron and gold over the range from 20 °C to $T$°C as $5.0 \times 10^{-3}\,{}^{\circ}\text{C}^{-1}$ and $3.4 \times 10^{-3}\,{}^{\circ}\text{C}^{-1}$, respectively.)

E11 Three 100 Ω resistors can be connected together in four different ways, making series and/or parallel combinations. What are these four ways and what is the net resistance in each case?

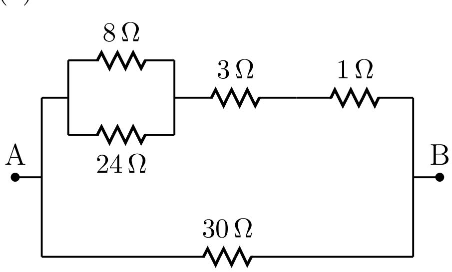
E12 In each of the combinations below, calculate the equivalent resistance between points A and B.



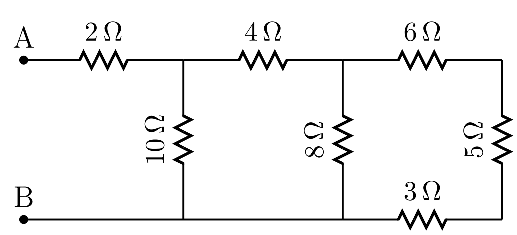
(a)



(b)



(c)



(d)

E13 A standard resistor marked 5 Ω is tested and found to have an actual resistance of 5.05 Ω. What length of nichrome wire of resistance $135\,\Omega\,\mathrm{m}^{-1}$ must be connected in parallel with the resistor to make the combined resistance of 5 Ω?

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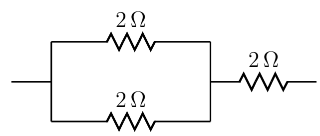
# DC circuits

F1 A 1000 Ω 2 W resistor is needed, but only 1000 Ω 1 W resistors are available. (a) How can the required resistance and power rating be obtained by a combination of the available units? (b) What power is then dissipated in each resistor?

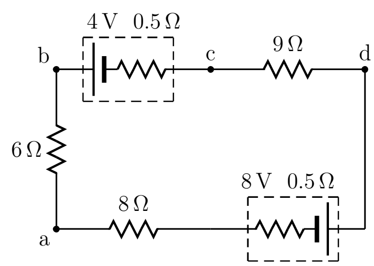
F2 Each of the three resistors in the figure has a resistance of 2 Ω and can dissipate a maximum of 18 W without becoming excessively heated. What maximum power can the circuit dissipate?




F3 (a) Find the potential difference $V_{ad}$ in the circuit

(b) Find the potential difference across the 4 V cell

(c) A battery of emf 17 V and internal resistance 1 Ω is inserted in the circuit at d with its positive terminal connected to the positive terminal of the 8 V battery. Find $V_{bc}$ between the terminals of the 4 V battery.




F4 A dry cell having an emf of 1.55 V and an internal resistance of 0.08 Ω supplies current to a 2.0 Ω resistor.

(a) Determine the current in the circuit.

(b) Calculate the terminal voltage of the cell.

F5 How many cells, each having an emf of 1.5 V and an internal resistance of 0.50 Ω must be connected in series to supply a current of $5/3$ A to operate an instrument having a resistance of 6 Ω?

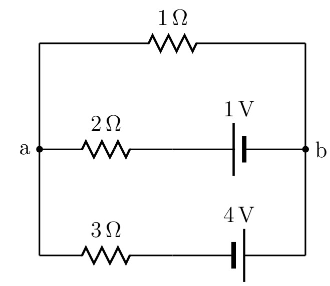
F6 A battery has an internal resistance of 0.50 Ω. A number of identical light bulbs, each with a resistance of 15 Ω, are connected in parallel across the battery terminals. The terminal voltage of the battery is observed to be half the emf of the battery. How many bulbs are connected?

F7 A galvanometer has a full-scale reading of 100 µA and a coil resistance of $R_c = 50.0\,\Omega$. Determine the resistance that must be connected

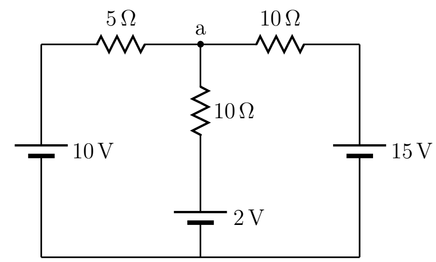
(a) in series with the coil to produce a voltmeter that will register a full-scale reading of 0.500 V.

(b) in parallel with the coil to produce an ammeter that will register a full-scale reading of 60.0 mA.

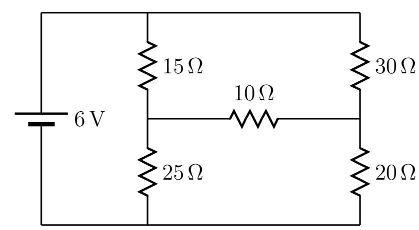
F8 Find the magnitude and direction of the current in the $2.0\,\Omega$ resistor in the diagram opposite.




F9 Determine the voltage across the $5.0\,\Omega$ resistor in the circuit opposite. Which end of the resistor is at the higher potential?




F10 Determine the current through each of the resistors in the figure opposite.



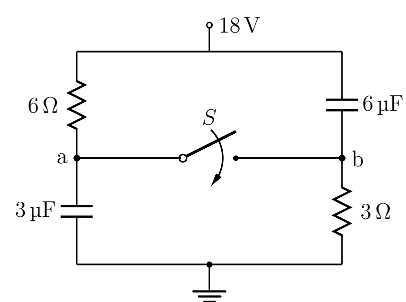

F11 In the figures opposite, we employ a convention often used in circuit diagrams. The battery, or other power supply, is not shown explicitly. It is understood that the point labelled 18 V is connected to the positive terminal of an 18 V battery having negligible internal resistance and the ground symbol is connected to its negative terminal. Answer the following questions considering each circuit in turn.

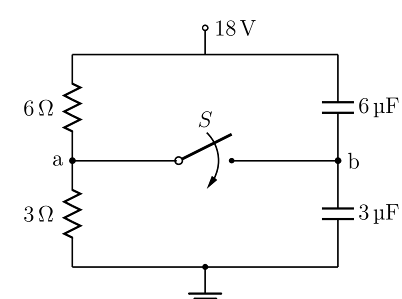
(a) What is the potential of point a with respect to point b when switch $S$ is open?

(b) Which point, a or b, is at the higher potential?

(c) What is the final potential of point b when $S$ is closed?

(d) How much does the charge on each capacitor change when $S$ is closed?



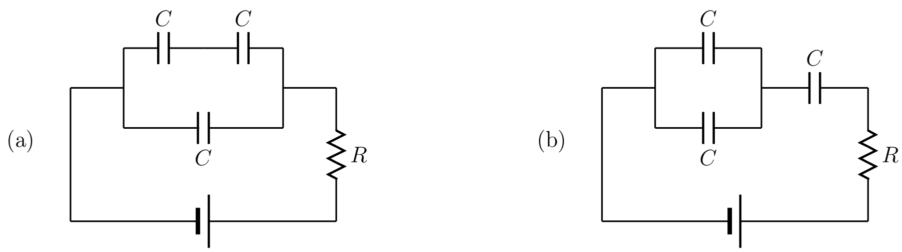

F12 Consider a series $RC$ circuit for which $C = 6\,\mu\mathrm{F}$, $R = 2 \times 10^6\,\Omega$ and $\mathcal{E} = 20\,\mathrm{V}$. Find (a) the time constant of the circuit, (b) the maximum charge on the capacitor after a switch in the circuit is closed, and (c) the current in the circuit at the instant just after the switch in the circuit is closed.

F13 A 2 nF capacitor with an initial charge of 5.1 µC is discharged through a 1300 Ω resistor. Calculate (a) the maximum current through the resistor, (b) the current through the resistor 9.0 µs after the resistor is connected across the terminals of the capacitor, and (c) the charge remaining on the capacitor after 8.0 µs.

F14 How many time constants must lapse before a capacitor in a series $RC$ circuit is charged to within 0.1 % of its equilibrium charge?

F15 An electronic flash attachment for a camera produces a flash by using the energy stored in a 750 µF capacitor. Between flashes, the capacitor recharges through a resistor whose resistance is chosen so that the capacitor will recharge with a time constant of 3 s. Determine the value of the resistance.

F16 Three identical capacitors are connected with a resistor in two different ways. When they are connected as in Figure (a), the time constant is 20 ms. What is the time constant when they are connected with the same resistor as in Figure (b)?




## Alternating currents and voltages

G1 At what frequency does a 7.50 µF capacitor have a reactance of 168 Ω?

G2 What is the inductance of an inductor that has a reactance of 1.8 kΩ at a frequency of 4.2 kHz?

G3 What voltage is needed to create a current of 29 mA in a circuit containing only a 0.565 µF capacitor, when the frequency is 2.60 kHz?

G4 Three capacitors are connected in parallel across the terminals of a 440 Hz generator. The capacitances are 2.0, 4.0, and 7.0 µF. (a) Find the equivalent capacitance of these capacitors. (b) If the generator supplies a total current of 0.62 A, what is the voltage of the generator? (c) What is the current supplied to each of the three capacitors?

G5 A coil has a resistance of 20 Ω. At a frequency of 100 Hz, the voltage across the coil leads the current in it by 30°. Determine the inductance of the coil.

G6 A series circuit has an impedance of 50 Ω and a power factor of 0.6 at 60 Hz, the voltage leading the current.

(a) Should an inductor or capacitor be placed in series with the circuit to raise its power factor?

(b) What size element will raise the power factor to unity?

G7 A series circuit has a resistance of 75 Ω and an impedance of 150 Ω. What power is consumed in the circuit when 120 V (rms) is impressed across it?

G8 Resistance of 500 Ω and capacitance of 2 μF are connected in series across an ac source with emf amplitude of 282 V and frequency 60 Hz. Find $X_C$, $Z$, $i_0$, $V_{0C}$ and $\alpha$.

G9 An inductor having a reactance of 25 Ω and a resistance $R$ gives off heat at the rate 10 W when it carries a current of 0.5 A (rms). What is the impedance of the inductor?

G10 Given that $i(t) = 1.2 \sin \omega t$ and $v(t) = 75 \sin(\omega t - 0.733)$ for a series $LRC$ circuit.

(a) Is the circuit largely inductive or largely capacitative in nature?

(b) Is the circuit in resonance?

(c) Must there be (i) a capacitor in the circuit (ii) an inductor (iii) a resistor?

G11 In a series $RCL$ circuit the instantaneous current is $I = 4 \cos 1200t$ amps ($t$ in seconds). Given that $R = 40\,\Omega$, $C = 25\,\mu\text{F}$ and $L = 40\,\text{mH}$, find the instantaneous voltage across $R$, $C$ and $L$ and also across $C$ and $L$ in series.

G12 The instantaneous potential difference across $RCL$ in series is $V = 80 \sin(400t)$ V ($t$ in seconds). Given that $R = 200\,\Omega$, $C = 5\,\mu\text{F}$ and $L = 800\,\text{mH}$, calculate the impedance of the circuit, the current amplitude, the resonant frequency and the instantaneous current.

G13 In a series $LRC$ circuit the voltage amplitude is 50 V and the angular frequency is $1000\,\text{rad}\,\text{s}^{-1}$. $R = 300\,\Omega$, $L = 0.9\,\text{H}$, $C = 2\,\mu\text{F}$. Calculate (a) the impedance of the circuit, (b) the current amplitude, (c) the voltage amplitude across the resistor and across the inductor, (d) the phase angle between $v(t)$ and $i(t)$.

G14 The power dissipated in a series $RCL$ circuit is 65.0 W, and the current is 0.530 A. The circuit is at resonance. Determine the peak voltage of the generator.

G15 At resonance in a series $RLC$ circuit the rms current is 0.1 A and the rms potential differences across $R$ and $C$ are 40 V and 20 V respectively. Find the rms potential difference across $L$. Calculate the impedance in the circuit at twice the resonant frequency.

G16 Suppose you have a number of capacitors. Each of these capacitors is identical to the capacitor that is already in a series $RCL$ circuit. How many of these additional capacitors must be inserted in series in the circuit so that the resonance frequency triples?

G17 The capacitor in a series $RCL$ circuit ($R = 250\,\Omega$, $C = 0.02\,\mu\text{F}$ and $L = 0.5\,\text{H}$) can withstand a peak voltage of 350 V. To what maximum value should the rms voltage of an oscillator connected to this circuit be restricted at resonance?

G18 At resonance in an $RCL$ circuit ($R = 100\,\Omega$), the rms potential difference across the resistor and inductor are 40 V and 30 V respectively. The resonance frequency is $10^5$ Hz. Determine $L$ and $C$.

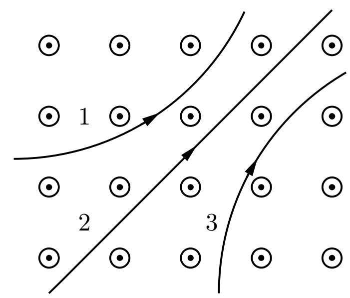
G19 In a series $RCL$ circuit the dissipated power drops by a factor of two when the frequency of the generator is changed from the resonance frequency to a non-resonance frequency. The peak voltage is held constant while this change is made. Determine the power factor of the circuit at the non-resonance frequency.

# Magnetic fields and forces

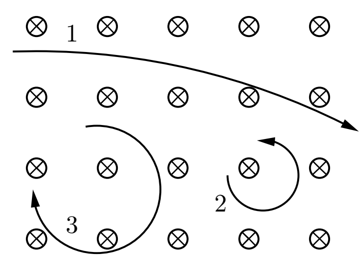
H1 A charge of $12\,\mu C$, travelling with a speed of $9.0 \times 10^{6}\,m\,s^{-1}$ in a direction perpendicular to a magnetic field, experiences a magnetic force of $8.7 \times 10^{-3}\,N$. Calculate the magnitude of the field.

H2 A particle, moving with a velocity of $8.0 \times 10^{4}\,m\,s^{-1}$ at an angle of $30°$ with respect to a magnetic field of $5.6 \times 10^{-6}\,T$, experiences a force of $2.0 \times 10^{-4}\,N$. Calculate the magnitude of the particle's charge.

H3 Particles 1, 2 and 3 follow the paths shown opposite as they pass through the magnetic field. What can one conclude about each particle?




H4 Three particles have identical charges and masses. They enter a uniform magnetic field and follow the paths shown in the diagram. Which particle is moving fastes and which is moving the slowest? Justify your answer.




H5 A wire 1.0 m long carries a current of 10 A and makes an angle of $30°$ with a uniform magnetic field with $B = 1.5\,T$. Calculate the magnitude and direction of the force on the wire.

H6 (a) Calculate the force per unit length on a wire carrying a current of 0.5 A when perpendicular to a 4.0 T magnetic field.

(b) What if the angle between the wire and the field is $45°$?

H7 The force on a wire carrying 20 A is a maximum of 3.6 N when placed between the pole faces of a magnet. If the wire is 15 cm long, what is the approximate magnitude of $\mathbf{B}$?

H8 How far from a long, straight wire carrying 10 A will the magnetic field be $1.0 \times 10^{-2}\,T$?

H9 Determine the magnetic field midway between two long, straight wires 10 cm apart if one carries 10 A and the other 8.0 A and these currents are

(a) in the same direction, and

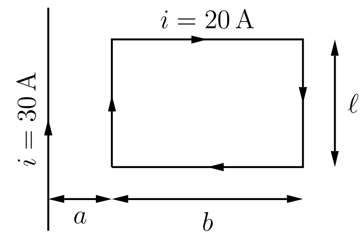
(b) in opposite directions.

H10 Two long, parallel wires 12 cm apart carry 15 A currents in the same direction. Determine the magnitude of the magnetic field $\mathbf{B}$ at a point 10 cm from one wire and 20 cm from the other. (Hint: make a drawing in a plane containing the field lines and recall the rules for vector addition.)

H11 Calculate the magnitude and direction of the force between two parallel wires 85 m long and 30 cm apart, each carrying 60 A in the same direction.

H12 A vertical, straight wire carrying 5 A exerts an attractive force per unit length $6 \times 10^{-4}\,\mathrm{N\,m^{-1}}$ on a second, parallel wire 8.0 cm away. What current (magnitude and direction) flows in the second wire?

H13 The figure shows a long wire carrying a current of 30 A. The rectangular loop carries a current of 20 A. Calculate the resultant force acting on the loop. Assume that $a = 1$ cm, $b = 8$ cm and $\ell = 30$ cm.

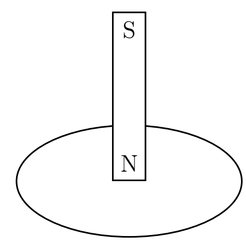



H14 A solenoid 120 cm long and 3 cm in mean diameter has five layers of windings with 840 turns on each. The current in the solenoid is 5 A. Calculate

(a) the magnetic field at the centre of the solenoid and

(b) the magnetic flux per turn for a cross section of the solenoid at its centre.

H15 The north pole of a magnet is moved away from a metallic ring as shown. Looking from above, in which direction does the current circulate?



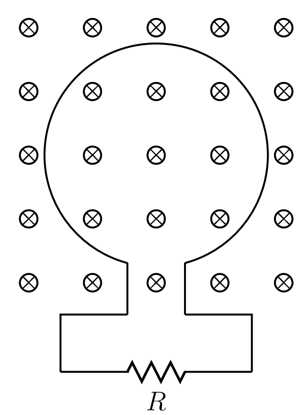

H16 In the figure alongside, magnetic flux through the loop perpendicular to the plane of the coil and directed into the paper is varying according to the relation

$$\phi = 6t^2 + 7t + 1,$$

where $\phi$ is in milliwebers and $t$ is in seconds.

(a) What is the magnitude of the emf induced in the loop when $t = 2$ s?

(b) What is the direction of the current through $R$?




H17 A closely-wound rectangular coil of 50 turns has dimensions of 12 cm × 25 cm. The plane of the coil is rotated in time $t = 0.1$ s from a position where it makes an angle of 60° with a uniform magnetic field of strength 2 T to a position perpendicular to the field. What is the average emf induced in the coil?

H18 A rectangular coil having 10 turns with dimensions of 20 cm × 30 cm rotates with an angular velocity of 600 rpm in a uniform magnetic field of strength 0.10 T. The axis of rotation is perpendicular to the field. Find the maximum emf produced.

H19 A solenoid of cross-sectional area $6\,\mathrm{cm}^2$ and length 30 cm has two layers of wire, one with 300 turns and the other with 240 turns. The current is 0.5 A in the same direction in both layers. A secondary winding of 2 turns encircles the solenoid. When the primary circuit is opened, the magnetic field of the solenoid becomes zero in 0.05 s. What is the average induced emf in the secondary?

H20 A closely-wound coil has an area of $4\,\text{cm}^2$, 160 turns, and a resistance of $50\,\Omega$. It is connected to a charge-measuring instrument whose resistance is $30\,\Omega$. When the coil is rotated quickly from a position parallel to a uniform magnetic field to one perpendicular to the field, the instrument indicates a charge of $4 \times 10^{-5}\,\text{C}$. What is the magnitude of the field?

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## Transformers

J1 A transformer changes 12 V to 18 000 V and there are 6000 turns in the secondary. How many turns are there in the primary?

J2 A transformer has 145 turns in the primary and 55 in the secondary. What kind of transformer is this, and by what factor does it change the voltage?

J3 A step-up transformer increases 30 V to 120 V. What is the current in the secondary as compared to the primary?

J4 Describe a transformer that could be used to light a 6 V bulb from a 240 V, 50 Hz source.

J5 A transformer has 1500 primary turns and 120 secondary turns. The input voltage is 240 V and the output current is 8.0 A. What is the secondary voltage and primary current?

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