

Two Popular Discrete Distributions

- The Binomial
- □ The Poisson



Bernoulli Process

A situation where the random variable has only two mutually exclusive outcomes.

```
Exam grade \longrightarrow pass / fail

Coin toss \longrightarrow heads / tails

Lottery \longrightarrow win / do-not-win
```



Multiple Trials of the Bernoulli Process

```
Game of Dice Win (if you roll a 6)
Lose (otherwise)
```



Multiple Trials of the Bernoulli Process

Game of Dice
$$\begin{cases} Win & \text{(if you roll a 6)} \\ Lose & \text{(otherwise)} \end{cases}$$

Probability of winning = 1/6 = 0.1667



Multiple Trials of the Bernoulli Process

Probability of winning at least 4 times in 10 rolls of the dice?



Multiple Trials of the Bernoulli Process

Probability of winning at least 4 times in 10 rolls of the dice?

Probability of winning at least 1 time in 10 rolls of the dice?



Multiple Trials of the Bernoulli Process

Probability of winning at least 4 times in 10 rolls of the dice?

Probability of winning at least 1 time in 10 rolls of the dice?

Probability of winning exactly 3 times in 10 rolls of the dice?



Multiple Ten Trials of the Bernoulli Process

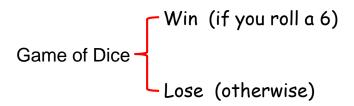
```
Game of Dice \begin{cases} Win & \text{(if you roll a 6)} \\ Lose & \text{(otherwise)} \end{cases}
```

Random Variable

Number of times you win when you roll the dice 10 times



Multiple Ten Trials of the Bernoulli Process



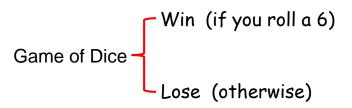
Random Variable

Number of times you win when you roll the dice 10 times

A Binomial Distribution



Multiple Ten Trials of the Bernoulli Process



Random Variable

Number of times you win when you roll the dice 10 times

→ A Binomial Distribution

 \longrightarrow 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10



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Define random variable **X** to denote number of successes in n trials.

Then this random variable is said to have a Binomial distribution.

In our example...

$$n = 10$$
, $p = 1/6 = 0.1667$, success = getting a 6 in one roll

X = Number of times you win when the dice is rolled ten times



Probability Mass Function

$$P(X = x) = \frac{n!}{x! (n-x)!} p^{x} (1-p)^{n-x}$$



Probability Mass Function

$$P(X = x) = \frac{n!}{x!(n-x)!}p^{x}(1-p)^{n-x}$$

the BINOM.DIST function



=BINOM.DIST(x, n, p, FALSE/TRUE)



```
=BINOM.DIST(x, n, p, FALSE/TRUE)

P(successes = x) P(successes ≤ x)
```



probability that you win three times in ten rolls of a dice?

P(X=3) = BINOM.DIST(3, 10, 0.1667, FALSE) = 0.1551



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probability that you win at most five times in ten rolls of a dice?

 $P(X \le 5) = BINOM.DIST(5, 10, 0.1667, TRUE) = 0.9976$

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P(X=3) = BINOM.DIST(3, 10, 0.1667, FALSE) = 0.1551

probability that you win at the most five times in ten rolls of a dice?

 $P(X \le 5) = BINOM.DIST(5, 10, 0.1667, TRUE) = 0.9976$

probability that you win less than five times in ten rolls of a dice?

 $P(X<5) = P(X\le4) = BINOM.DIST(4, 10, 0.1667, TRUE) = 0.9845$

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probability that you win at least three times in ten rolls of a dice?

$$P(X \ge 3) = 1 - P(X \le 2) = 1 - BINOM.DIST(2, 10, 0.1667, TRUE) = 0.2249$$