

Optimization Advanced Assignment

A L U R U A J A Y

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Problem Statement - Show that $\frac{x^2-7x+6}{x-10}$ has a maximum value when $x=4$ and a minimum when $x=16$

1 Solution

Given function is,

$$f(x) = \frac{x^2 - 7x + 6}{x - 10} \quad (1)$$

1.1 Calculation of Maxima and Minima using normal differentiation

Differentiating above Eq(1), we get,

$$\nabla f(x) = \frac{x^2 - 20x + 64}{(x-10)^2}$$

$$\Rightarrow 0 = \frac{x^2 - 20x + 64}{(x-10)^2}$$

$$\Rightarrow 0 = x^2 - 20x + 64$$

On simplifying,

$a = 1$, $b = -20$, $c = 64$

$$x_{max} = \frac{-b - \sqrt{b^2 - 4ac}}{2a}, \quad x_{min} = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad (2)$$

$$x = 4$$

$$x = 16$$

1.2 Calculation of Maxima using gradient ascent algorithm

Maxima of the above equation (1), can be calculated from the following expression,

$$x_{n+1} = x_n + \alpha \nabla f(x_n) \quad (3)$$

Taking $x_0 = 0.5$, $\alpha = 0.001$ and precision = 0.00000001, values obtained using python are:

$$\boxed{\text{Maxima} = 1}$$

$$\boxed{\text{Maxima Point} = 4}$$

1.3 Calculation of Minima using gradient descent algorithm

Maxima of the above equation (1), can be calculated from the following expression,

$$x_{n+1} = x_n - \alpha \nabla f(x_n) \quad (4)$$

Taking $x_0 = 0.5$, $\alpha = 0.001$ and precision = 0.00000001, values obtained using python are:

$$\boxed{\text{Maxima} = 25}$$

$$\boxed{\text{Maxima Point} = 16}$$

2 Plot to find maxima and minima of the function

Plot of the function $\frac{(x^2-7x+6)}{(x-10)}$ is shown in the figure 1.

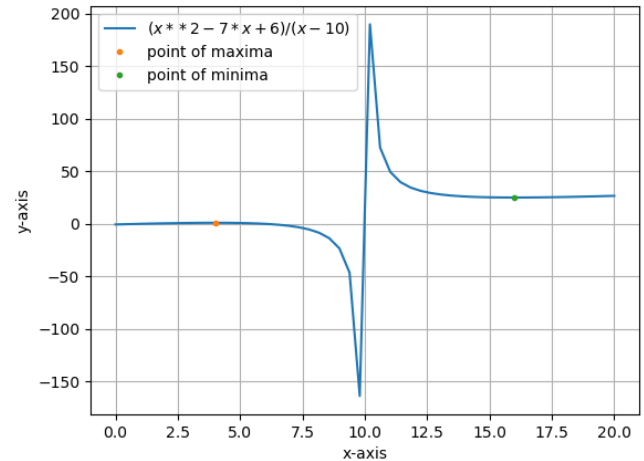


Figure 1: Plot of $f(x)$ to find Maxima and Minima

3 Conclusion

1. At first, the given function has been differentiated and it is solved by setting $f'(x)$ equal to zero. By using x values, $f(x)$ values are calculated.
2. Later, the given function $f(x)$ is solved by gradient ascent algorithm to find maxima and the point at which $f(x)$ is maximum.

3. Then, the given function $f(x)$ is solved by gradient descent algorithm to find minima and the point at which $f(x)$ is minimum.

4. Maxima and Minima and related points are,

Maxima point, $\text{Max}=(4, 1)$ and

Minima point, $\text{Min}=(16, 25)$