

Matrix Assignment

A L U R U A J A Y

September 2022

Problem Statement - Find the equation of the tangent line to the curve $y = x^2 - 2x + 7$

1. parallel to the line $2x-y+9=0$.
2. perpendicular to the line $5y-15x=13$.

1 Solution

According to the question equation of a parabola is

$$y = x^2 - 2x + 7$$

The standard equation of the parabola is given as :

$$\mathbf{x}^T \mathbf{V} \mathbf{x} + 2\mathbf{u}^T \mathbf{x} + f = 0$$

$$\mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad \mathbf{u} = \begin{pmatrix} -1 \\ -0.5 \end{pmatrix}, \quad f = 7$$

1.1 parallel to the line $2x-y+9=0$

Normal vector of a given line $\mathbf{n}_1 = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$

\mathbf{V} is not invertible, Then given the normal vector \mathbf{n} the point of contact to is given by the matrix equation

$$\begin{pmatrix} (\mathbf{u} + k_1 \mathbf{n}_1)^T \\ \mathbf{V} \end{pmatrix} \mathbf{q}_1 = \begin{pmatrix} -f \\ k_1 \mathbf{n}_1 - \mathbf{u} \end{pmatrix}$$

$$\begin{pmatrix} ((-1) + \frac{1}{2} \begin{pmatrix} 2 \\ -1 \end{pmatrix})^T \\ \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \end{pmatrix} \mathbf{q}_1 = \begin{pmatrix} -7 \\ \frac{1}{2} \begin{pmatrix} 2 \\ -1 \end{pmatrix} - \begin{pmatrix} -1 \\ -0.5 \end{pmatrix} \end{pmatrix}$$

By solving the above equation, we can get the point of contact as

$$\mathbf{q}_1 = \begin{pmatrix} 2 \\ 7 \end{pmatrix}$$

Tangent Equation is

$$\mathbf{n}_1^T (\mathbf{x} - \mathbf{q}_1) = 0 \quad (3)$$

By substituting the values of \mathbf{n}_1^T & \mathbf{q}_1 in Eq(3), Then

$$2x - y + 3 = 0 \quad (4)$$

1.2 perpendicular to the line $5y-15x=13$

Normal vector of a given line $\mathbf{n}_2 = \begin{pmatrix} 1 \\ -3 \end{pmatrix}$

\mathbf{V} is not invertible, Then given the normal vector \mathbf{n} the point of contact to is given by the matrix equation

$$\begin{pmatrix} (\mathbf{u} + k_2 \mathbf{n}_2)^T \\ \mathbf{V} \end{pmatrix} \mathbf{q}_2 = \begin{pmatrix} -f \\ k_2 \mathbf{n}_2 - \mathbf{u} \end{pmatrix} \quad (5)$$

$$\begin{pmatrix} ((-1) + \frac{3}{2} \begin{pmatrix} 2 \\ -1 \end{pmatrix})^T \\ \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \end{pmatrix} \mathbf{q}_2 = \begin{pmatrix} -7 \\ \frac{3}{2} \begin{pmatrix} 2 \\ -1 \end{pmatrix} - \begin{pmatrix} -1 \\ -0.5 \end{pmatrix} \end{pmatrix}$$

(1) By solving the above equation, we can get the point of contact as

$$\mathbf{q}_2 = \begin{pmatrix} \frac{-5}{6} \\ \frac{217}{36} \end{pmatrix}$$

Tangent Equation is

$$\mathbf{n}_2^T (\mathbf{x} - \mathbf{q}_2) = 0 \quad (6)$$

By substituting the values of \mathbf{n}_2^T & \mathbf{q}_2 in Eq(6), Then

$$12x + 36y - 227 = 0 \quad (7)$$

(2) 2 Diagram

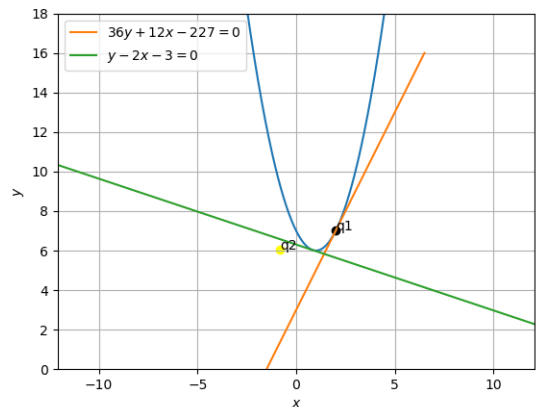


Figure 1: Parabola