# Section 9.3

# The Law of Cosines

**Case 3:** Two sides and the included angle are known (SAS).

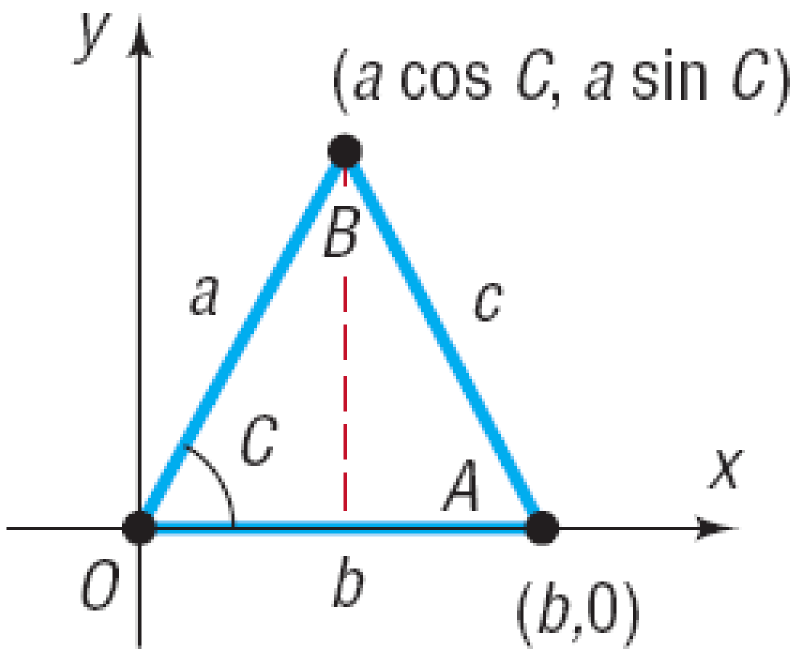
**Case 4:** Three sides are known (SSS).

## THEOREM

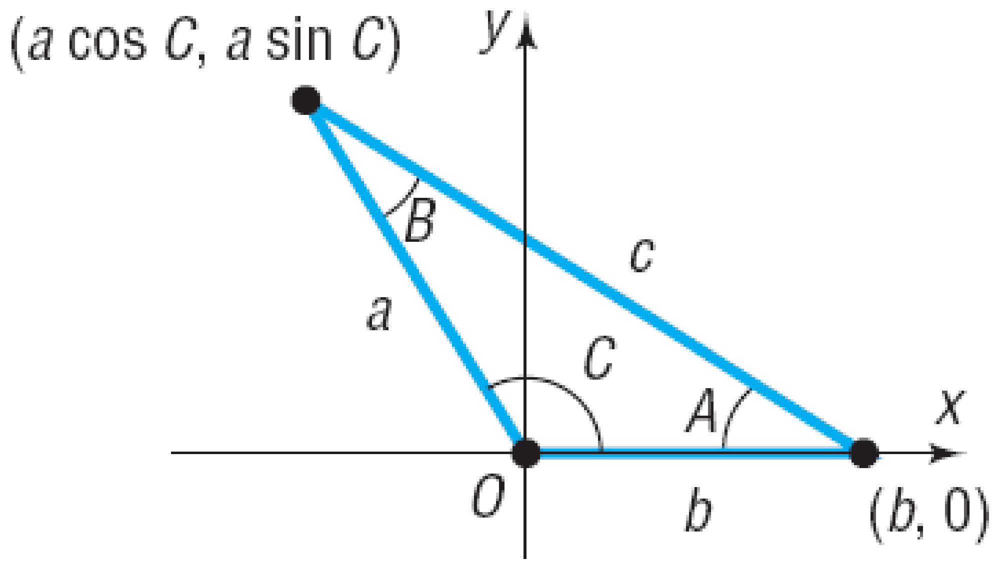
### Law of Cosines

For a triangle with sides $a$, $b$, $c$ and opposite angles $A$, $B$, $C$, respectively,

$$c^2 = a^2 + b^2 - 2ab\cos C$$
$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$a^2 = b^2 + c^2 - 2bc\cos A$$




**(a)** Angle $C$ is acute




**(b)** Angle $C$ is obtuse

## THEOREM

**Law of Cosines**

The square of one side of a triangle equals the sum of the squares of the other two sides minus twice their product times the cosine of their included angle.

# 1 Solve SAS Triangles

EXAMPLE

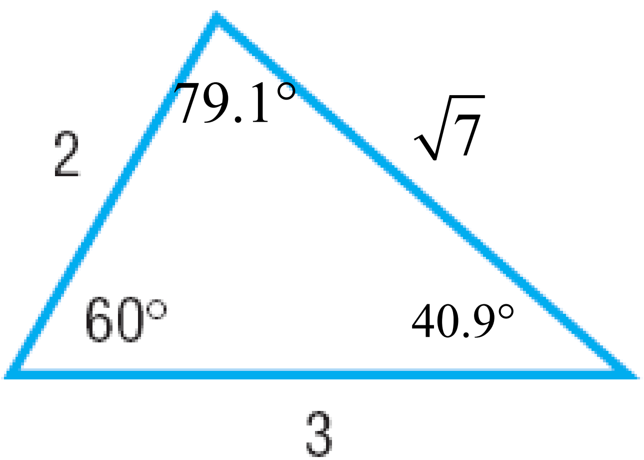
## Using the Law of Cosines to Solve an SAS Triangle

Solve the triangle: $a = 2, \quad b = 3, \quad C = 60°$

$$c^2 = a^2 + b^2 - 2ab\cos C$$
$$= 2^2 + 3^2 - 2\cdot 2\cdot 3\cdot \cos 60° = 13 - \left(12\cdot\frac{1}{2}\right) = 7 \qquad c = \sqrt{7}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$2bc\cos A = b^2 + c^2 - a^2$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{9 + 7 - 4}{2\cdot 3\sqrt{7}} = \frac{12}{6\sqrt{7}} = \frac{2\sqrt{7}}{7}$$




$$A = \cos^{-1}\frac{2\sqrt{7}}{7} \approx 40.9°$$

$$B = 180° - 60° - 40.9° = 79.1°$$

## 2 Solve SSS Triangles

EXAMPLE

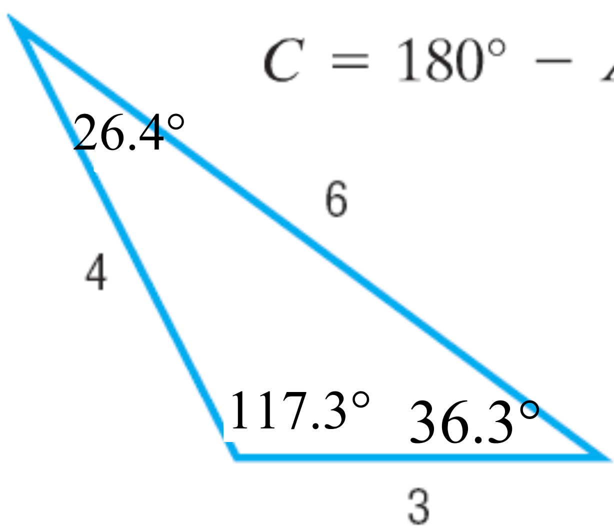
## Using the Law of Cosines to Solve an SSS Triangle

Solve the triangle: $a = 4, b = 3, c = 6$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{9 + 36 - 16}{2 \cdot 3 \cdot 6} = \frac{29}{36} \qquad A = \cos^{-1}\frac{29}{36} \approx 36.3°$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac} = \frac{16 + 36 - 9}{2 \cdot 4 \cdot 6} = \frac{43}{48} \qquad B = \cos^{-1}\frac{43}{48} \approx 26.4°$$

$$C = 180° - A - B \approx 180° - 36.3° - 26.4° = 117.3°$$



## 3 Solve Applied Problems

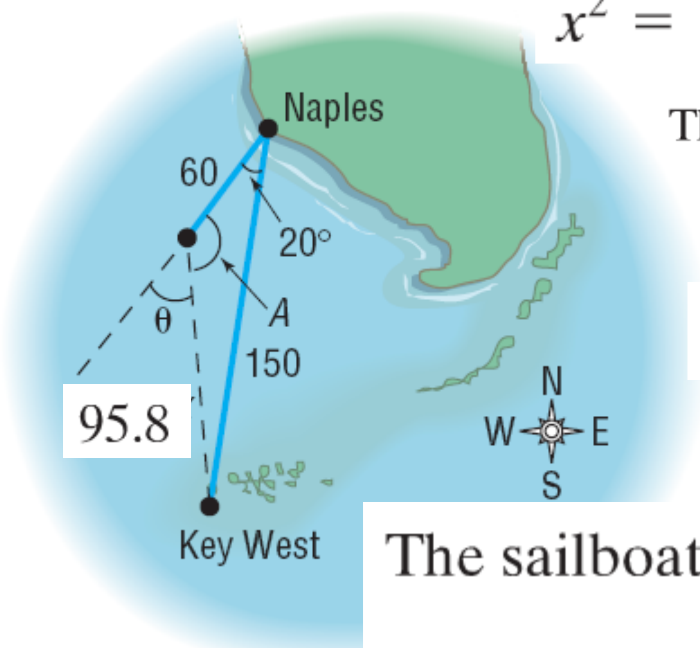
EXAMPLE **Correcting a Navigational Error**

(c) The total length of the trip is now $60 + 96 = 156$ miles. The extra 6 miles will only require about 0.4 hour or 24 minutes more if the speed of 15 miles per hour is maintained.

(a) How far is the sailboat from Key West at this time?

(b) Through what angle should the sailboat turn to correct its course?

(c) How much time has been added to the trip because of this? (Assume that the speed remains at 15 miles per hour.)




$$x^2 = 150^2 + 60^2 - 2(150)(60)\cos 20° \approx 9185.53$$

The sailboat is about 96 miles from Key West. $x \approx 95.8$

$$150^2 = 96^2 + 60^2 - 2(96)(60)\cos A$$

$$9684 = -11{,}520\cos A \qquad \cos A \approx -0.8406$$

$$A \approx 147.2°$$

The sailboat should turn through an angle of

$$\theta = 180° - A \approx 180° - 147.2° = 32.8°$$