Section 3.4

Library of Functions;

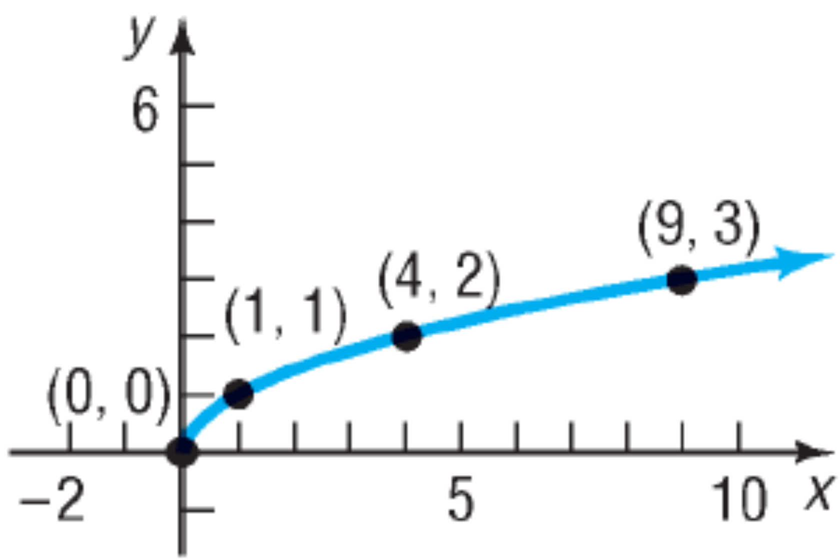
Piecewise-defined Functions

1 Graph the Functions Listed in the Library of Functions

The Square Root Function

Properties of $f(x) = \sqrt{x}$

1. The domain and the range are the set of nonnegative real numbers.
2. The x-intercept of the graph of $f(x) = \sqrt{x}$ is 0. The y-intercept of the graph of $f(x) = \sqrt{x}$ is also 0.
3. The function is neither even nor odd.
4. The function is increasing on the interval $(0, \infty)$.
5. The function has an absolute minimum of 0 at $x = 0$.



EXAMPLE

Graphing the Cube Root Function

(a) Determine whether $f(x) = \sqrt[3]{x}$ is even, odd, or neither. State whether the graph of f is symmetric with respect to the y-axis or symmetric with respect to the origin.

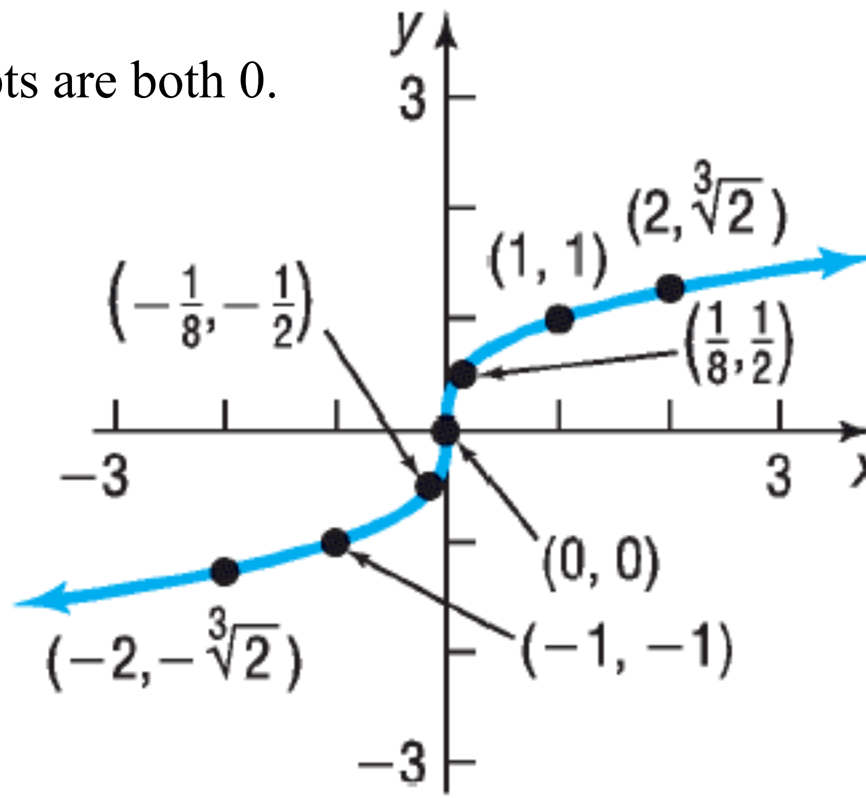
(b) Determine the intercepts, if any, of the graph of $f(x) = \sqrt[3]{x}$.

(c) Graph $f(x) = \sqrt[3]{x}$.

(a) $f(-x) = \sqrt[3]{-x} = -\sqrt[3]{x} = -f(x)$ This means the function is odd and symmetric with respect to the origin.

(b) $f(0) = \sqrt[3]{0} = 0$ x and y intercepts are both 0.

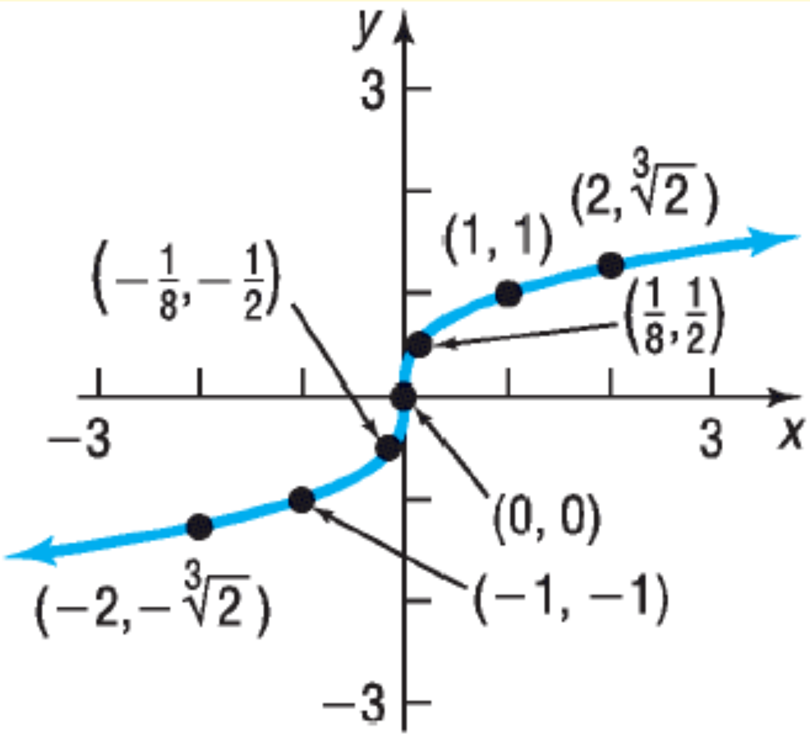
x	$y = f(x) = \sqrt[3]{x}$	(x, y)
0	0	(0, 0)
$\frac{1}{8}$	$\frac{1}{2}$	$\left(\frac{1}{8}, \frac{1}{2}\right)$
1	1	(1, 1)
2	$\sqrt[3]{2} \approx 1.26$	$(2, \sqrt[3]{2})$
8	2	(8, 2)



The Cube Root Function

Properties of $f(x) = \sqrt[3]{x}$

1. The domain and the range are the set of all real numbers.
2. The x-intercept of the graph of $f(x) = \sqrt[3]{x}$ is 0. The y-intercept of the graph of $f(x) = \sqrt[3]{x}$ is also 0.
3. The graph is symmetric with respect to the origin. The function is odd.
4. The function is increasing on the interval $(-\infty, \infty)$.
5. The function does not have any local minima or any local maxima.



EXAMPLE **Graphing the Absolute Value Function**

(a) Determine whether $f(x) = |x|$ is even, odd, or neither. State whether the graph of f is symmetric with respect to the y-axis or symmetric with respect to the origin.

(b) Determine the intercepts, if any, of the graph of $f(x) = |x|$.

(c) Graph $f(x) = |x|$.

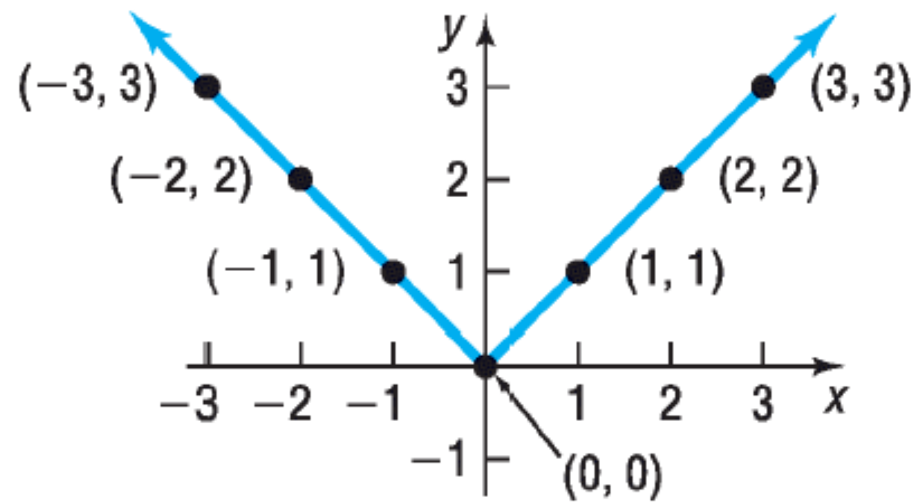
(a) $f(-x) = |-x| = |x| = f(x)$

This means the function is even and symmetric with respect to the y-axis.

(b) $f(0) = |0| = 0$

x and y intercepts are both 0.

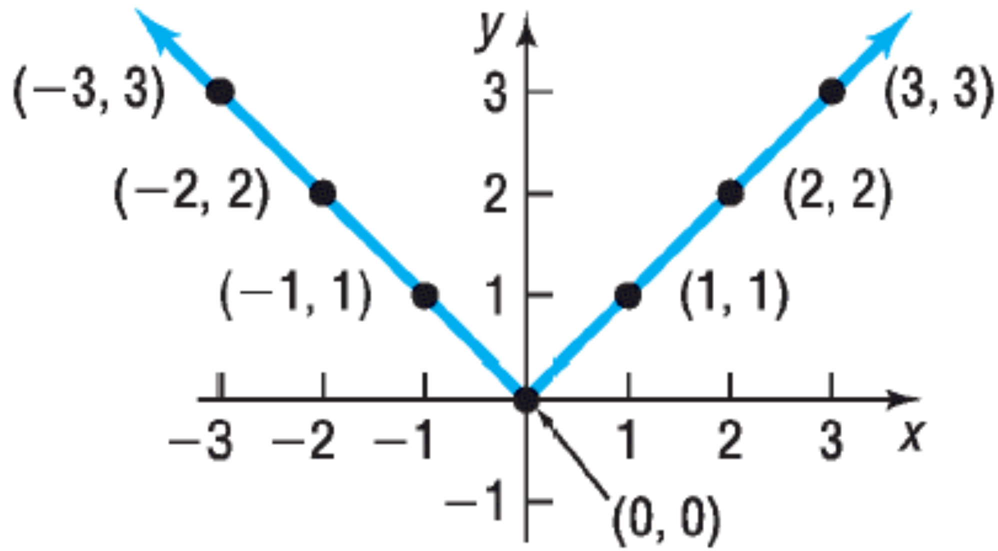
x	$y = f(x) = \|x\|$	(x, y)
0	0	(0, 0)
1	1	(1, 1)
2	2	(2, 2)
3	3	(3, 3)



The Absolute Value Function

Properties of $f(x) = |x|$

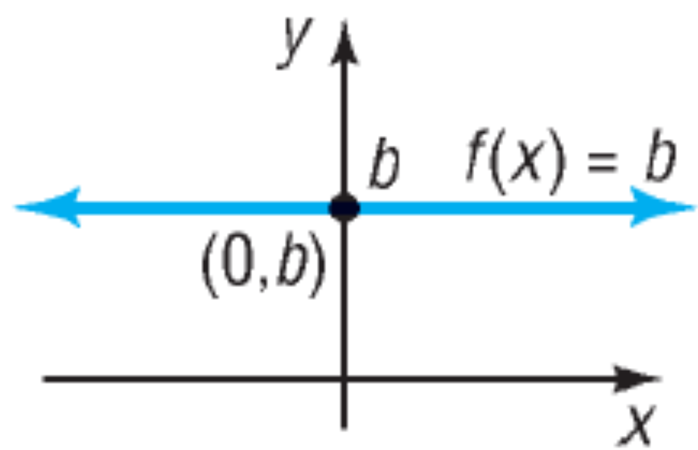
1. The domain is the set of all real numbers. The range of f is $\{y|y \geq 0\}$.
2. The x-intercept of the graph of $f(x) = |x|$ is 0. The y-intercept of the graph of $f(x) = |x|$ is also 0.
3. The graph is symmetric with respect to the y-axis. The function is even.
4. The function is decreasing on the interval $(-\infty, 0)$. It is increasing on the interval $(0, \infty)$.
5. The function has an absolute minimum of 0 at $x = 0$.



Constant Function

$$f(x) = b, \qquad b \text{ is a real number}$$

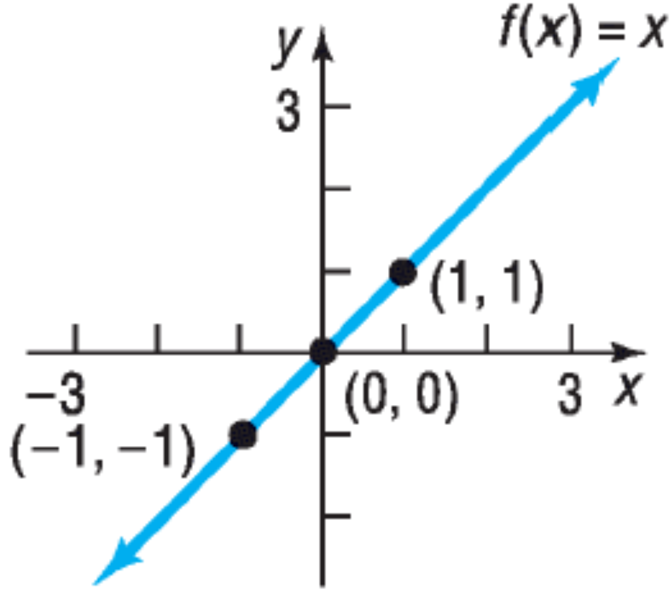
Constant Function



Identity Function

$$\boxed{f(x) = x}$$

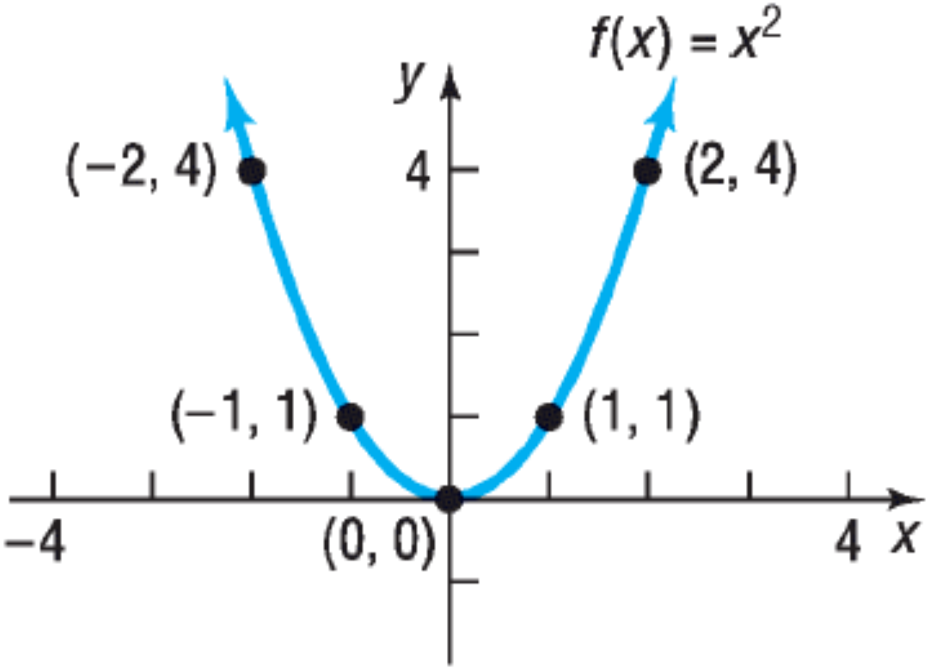
Identity Function



Square Function

$$\boxed{f(x) = x^2}$$

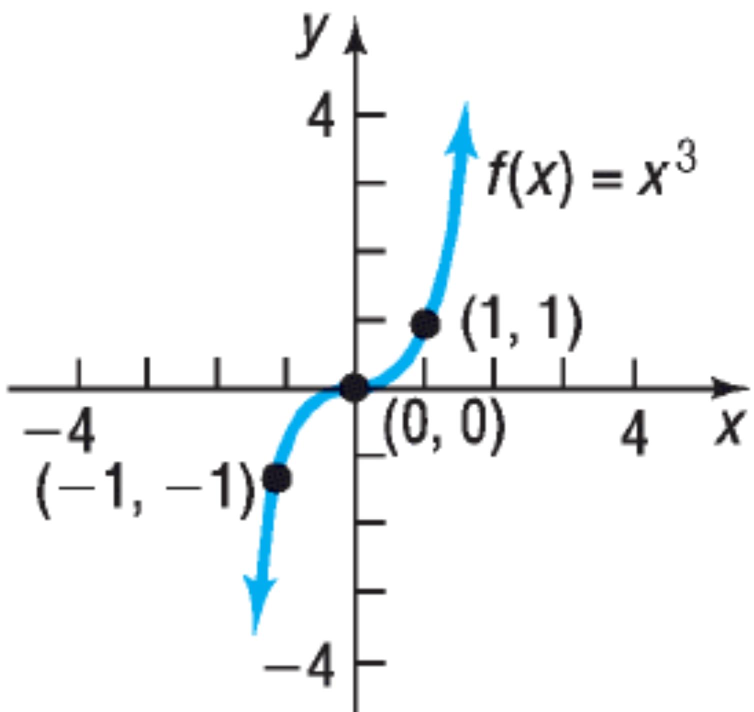
Square Function



Cube Function

$$\boxed{f(x) = x^3}$$

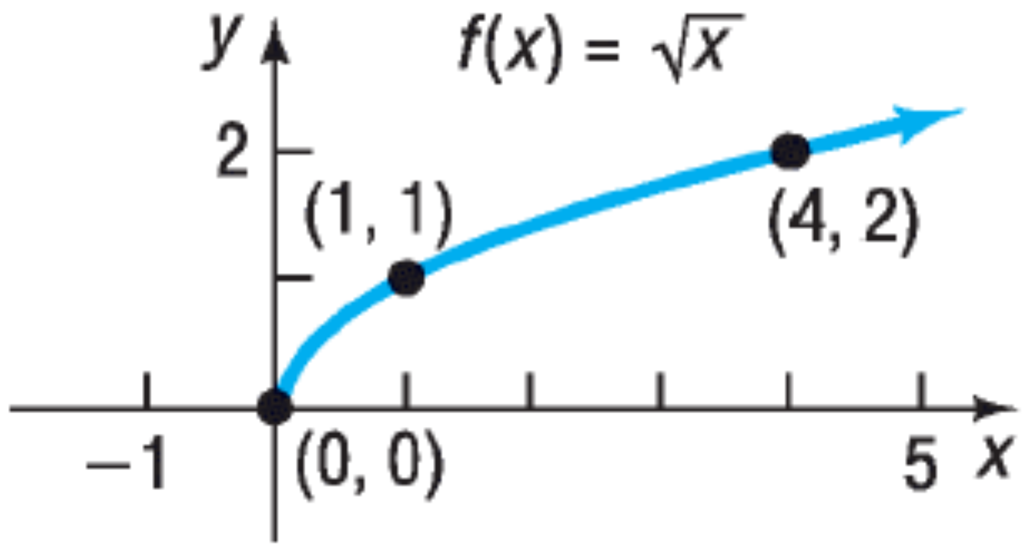
Cube Function



Square Root Function

$$f(x) = \sqrt{x}$$

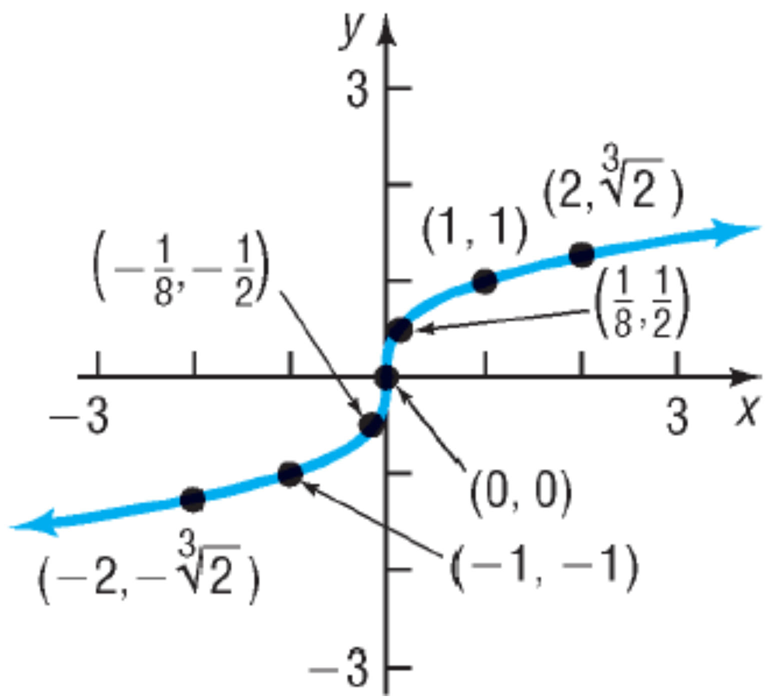
Square Root Function



Cube Root Function

$$f(x) = \sqrt[3]{x}$$

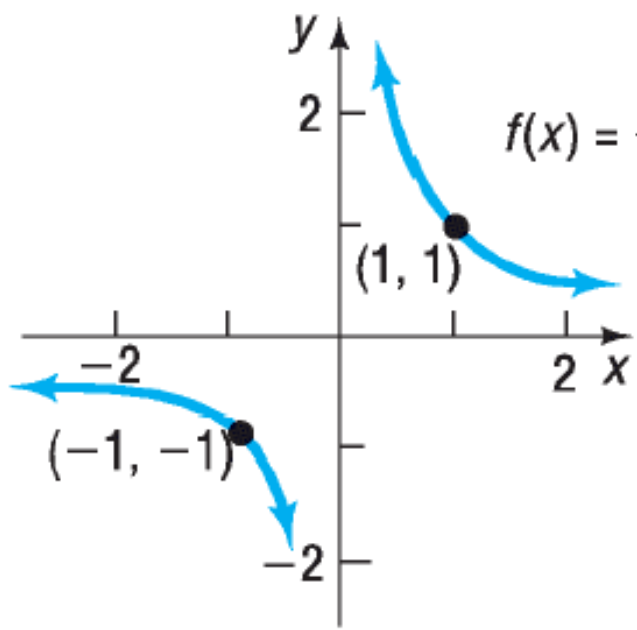
Cube Root Function



Reciprocal Function

$$f(x) = \frac{1}{x}$$

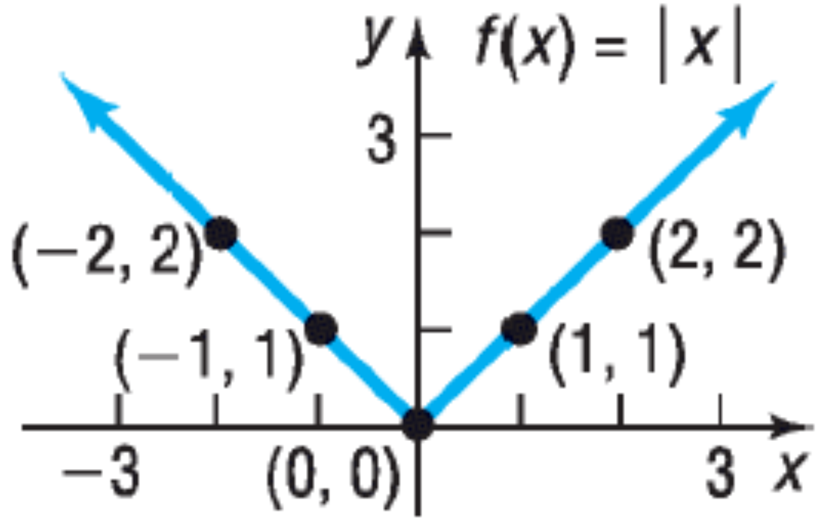
Reciprocal Function



Absolute Value Function

$$f(x) = |x|$$

Absolute Value Function

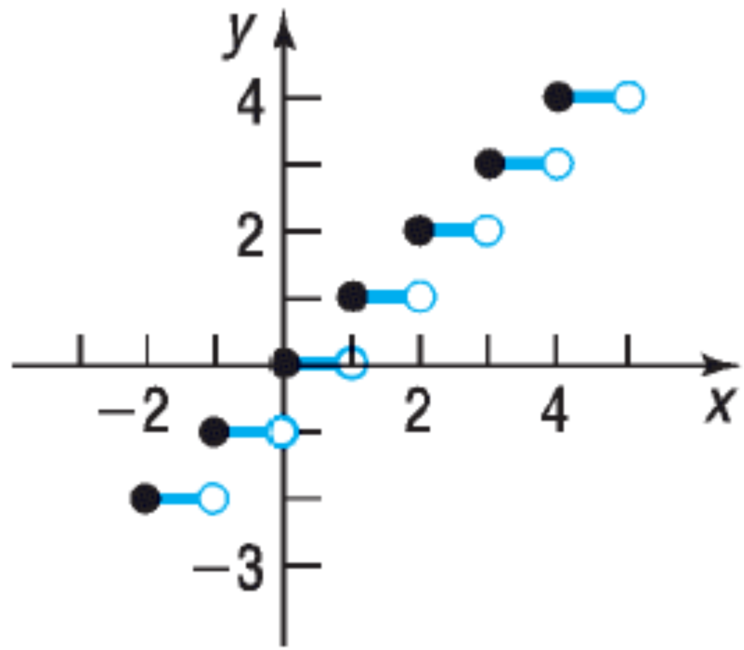


Greatest Integer Function

$$f(x) = \text{int}(x)^* = \text{greatest integer less than or equal to } x$$

x	$y = f(x) = \text{int}(x)$	(x, y)
-1	-1	$(-1, -1)$
$-\frac{1}{2}$	-1	$\left(-\frac{1}{2}, -1\right)$
$-\frac{1}{4}$	-1	$\left(-\frac{1}{4}, -1\right)$
0	0	$(0, 0)$
$\frac{1}{4}$	0	$\left(\frac{1}{4}, 0\right)$
$\frac{1}{2}$	0	$\left(\frac{1}{2}, 0\right)$
$\frac{3}{4}$	0	$\left(\frac{3}{4}, 0\right)$

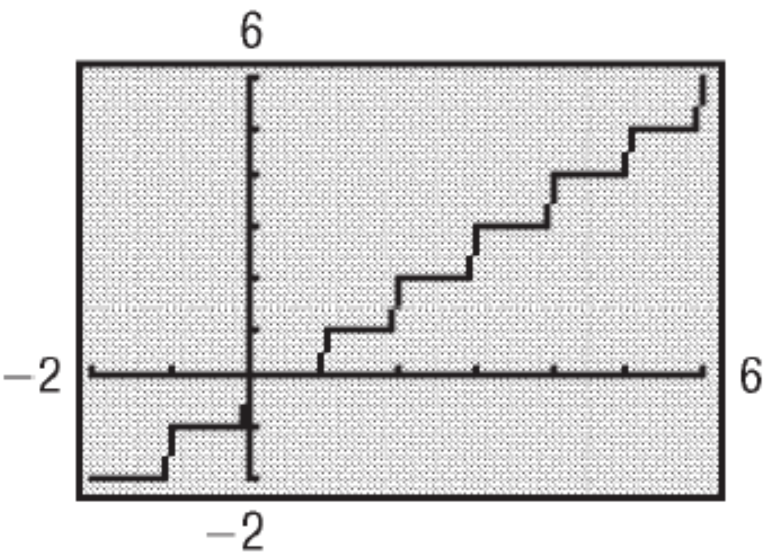
Greatest Integer Function



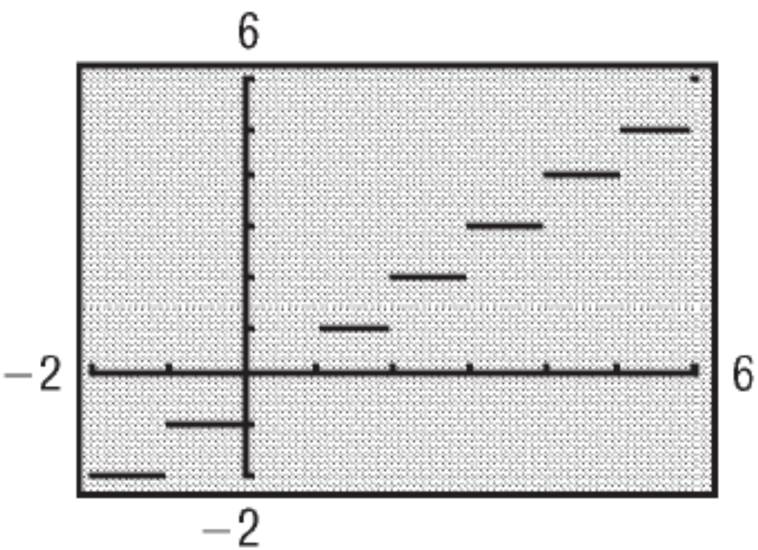
Greatest Integer Function

$$f(x) = \text{int}(x)^* = \text{greatest integer less than or equal to } x$$

$$f(x) = \text{int}(x)$$



(a) Connected mode



(b) Dot mode

2 Graph Piecewise-defined Functions

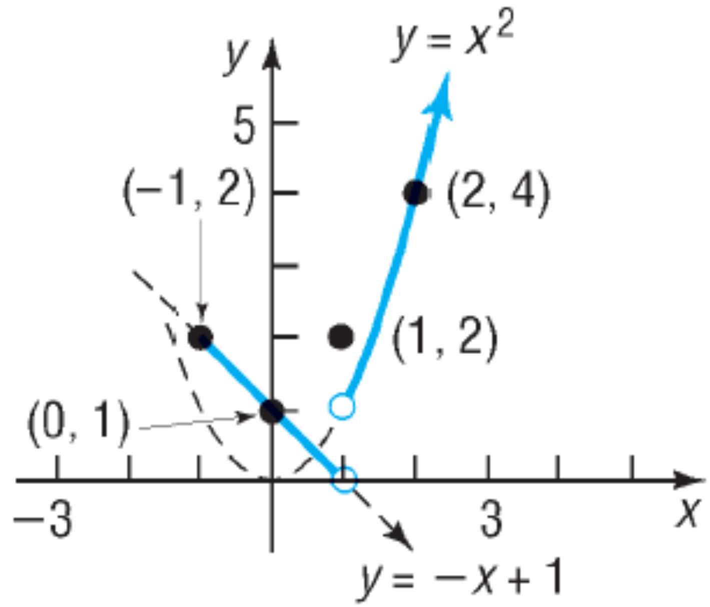
PIECE WISE FUNCTIONS

EXAMPLE

Analyzing a Piecewise-defined Function

The function f is defined as

$$f(x) = \begin{cases} -x + 1 & \text{if } -1 \le x < 1 \\ 2 & \text{if } x = 1 \\ x^2 & \text{if } x > 1 \end{cases}$$



(a) Find $f(0)$, $f(1)$, and $f(2)$.

(b) Determine the domain of f.

(c) Graph f by hand.

(d) Use the graph to find the range of f.

(a) $f(0) = -(0) + 1 = 1$ $\quad f(1) = 2$ $\quad f(2) = (2)^2 = 4$

(b) The domain of f is $[-1, \infty)$.

(d) The range of f is $(0, \infty)$.

EXAMPLE Cost of Electricity

In the summer of 2009, Duke Energy supplied electricity to residences of Ohio for a monthly customer charge of \$4.50 plus 4.2345¢ per kilowatt-hour (kWhr) for the first 1000 kWhr supplied in the month and 5.3622¢ per kWhr for all usage over 1000 kWhr in the month.

(a) What is the charge for using 300 kWhr in a month?

(b) What is the charge for using 1500 kWhr in a month?

(c) If C is the monthly charge for x kWhr, develop a model relating the monthly charge and kilowatt-hours used. That is, express C as a function of x.

(a) $\text{Charge} = \$4.50 + \$0.042345(300) = \$17.20$

(b) $\text{Charge} = \$4.50 + \$0.042345(1000) + \$0.053622(500) = \$73.66$

EXAMPLE **Cost of Electricity**

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(c) If C is the monthly charge for x kWhr, develop a model relating the monthly charge and kilowatt-hours used. That is, express C as a function of x.

(c) Let x represent the number of kilowatt-hours used. If $0 \leq x \leq 1000$, the monthly charge C (in dollars) can be found by multiplying x times \$0.042345 and adding the monthly customer charge of \$4.50. So, if $0 \leq x \leq 1000$, then $C(x) = 0.042345x + 4.50$.

For $x > 1000$, the charge is $0.042345(1000) + 4.50 + 0.053622(x - 1000)$, since $x - 1000$ equals the usage in excess of 1000 kWhr, which costs \$0.053622 per kWhr. That is, if $x > 1000$, then

$$\begin{aligned} C(x) &= 0.042345(1000) + 4.50 + 0.053622(x - 1000) \\ &= 46.845 + 0.053622(x - 1000) \\ &= 0.053622x - 6.777 \end{aligned}$$

EXAMPLE **Cost of Electricity**

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$$C(x) = \begin{cases} 0.042345x + 4.50 & \text{if } 0 \le x \le 1000 \\ 0.053622x - 6.777 & \text{if } x > 1000 \end{cases}$$