Section 4.4

Quadratic Models; Building Quadratic Functions From Data

1 Build Quadratic Models from Verbal Descriptions

EXAMPLE **Maximizing Revenue**

The marketing department at Widgets Inc. found that, when certain widgets are sold at a price of p dollars per unit, the number x of widgets sold is given by the demand equation

$$x = 1{,}500 - 30p$$

(a) Find a model that expresses the revenue R as a function of the price p.

(b) What is the domain of R?

(c) What unit price should be used to maximize revenue?

(d) If this price is charged, what is the maximum revenue?

(a) Revenue $R = xp = \left(1{,}500 - 30p\right)p = -30p^2 + 1{,}500p$

(b) $x \geq 0$ so $1{,}500 - 30p \geq 0 \qquad -30p \geq -1{,}500 \qquad p \leq 50$

The domain of R is $\left\{p \middle| 0 < p \leq 50\right\}$.

(c) $p = -\dfrac{b}{2a} = -\dfrac{1{,}500}{2(-30)} = \$25$

(d) $R(25) = -30\left(25\right)^2 + 1500\left(25\right) = \$18{,}750$

EXAMPLE Maximizing Revenue

The marketing department at Widgets Inc. found that, when certain widgets are sold at a price of p dollars per unit, the number x of widgets sold is given by the demand equation $x = 1{,}500 - 30p$

$$R(x) = -30p^2 + 1{,}500p$$

(e) How many units are sold at this price?

(f) Graph R.

(g) What price should Widgets Inc. charge to collect at least \$12,000 in revenue?

(e) $x = 1{,}500 - 30(25) = 750$

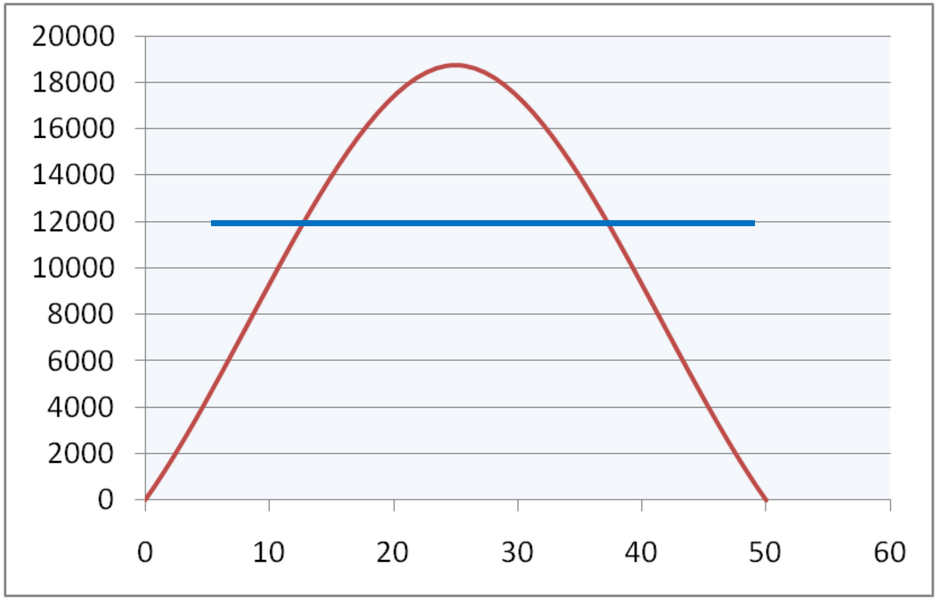
(g) $12{,}000 = -30p^2 + 1{,}500p$

$$30p^2 - 15{,}00p + 12{,}000 = 0$$

$$30\left(p^2 - 50p + 40\right) = 0$$

$$30(p-10)(p-40) = 0$$

$$p = 10 \text{ or } p = 40$$

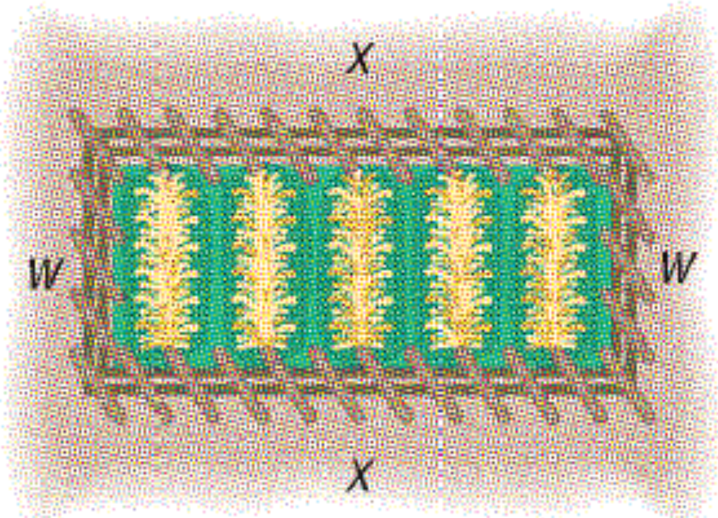


So the company should charge between \$10 and \$40 to earn at least \$12,000 in revenue.

EXAMPLE

Maximizing the Area Enclosed by a Fence

A farmer has 1600 yards of fence to enclose a rectangular field. What are the dimensions of the rectangle that encloses the most area?



$$A = xw$$

$$2x + 2w = 1600$$

$$w = 800 - x$$

$$A = x(800 - x) = -x^2 + 800x$$

$$x = -\frac{b}{2a} = -\frac{800}{2(-1)} = 400 \qquad w = 800 - 400 = 400$$

The farmer should make the rectangle 400 yards by 400 yards to enclose the most area.

EXAMPLE **Analyzing the Motion of a Projectile**

A projectile is fired from a cliff 500 feet above the water at an inclination of 45° to the horizontal, with a muzzle velocity of 400 feet per second. In physics, it is established that the height h of the projectile above the water is given by

$$h(x) = \frac{-32x^2}{(400)^2} + x + 500$$

where x is the horizontal distance of the projectile from the base of the cliff.

(a) Find the maximum height of the projectile.

$$x = -\frac{b}{2a} = -\frac{1}{2\left(\frac{-32}{400^2}\right)} = -\frac{1}{2\left(\frac{-1}{5000}\right)} = \frac{5000}{2} = 2500$$

$$f(2500) = \frac{-1}{5000}(2500)^2 + 2500 + 500 = -1250 + 3000 = 1750 \text{ ft}$$

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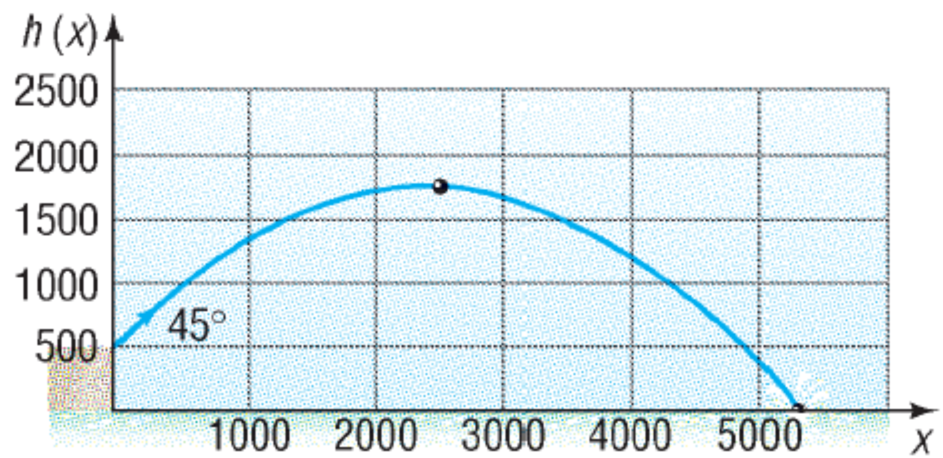
where x is the horizontal distance of the projectile from the base of the cliff.

(b) How far from the base of the cliff will the projectile strike the water?

$$h(x) = \frac{-1}{5000}x^2 + x + 500 = 0$$

$$x = \frac{-1 \pm \sqrt{1^2 - 4\left(\frac{-1}{5000}\right)(500)}}{2\left(-\frac{1}{5000}\right)}$$



$$x \approx -458 \text{ or } 5458$$

Solution cannot be negative so the projectile will hit the water about 5458 feet from the base of the cliff.

Seeing the Concept

Graph

$$h(x) = \frac{-1}{5000}x^2 + x + 500$$

$$0 \le x \le 5500$$

Use MAXIMUM to find the maximum height of the projectile, and use ROOT or ZERO to find the distance from the base of the cliff to where it strikes the water.

EXAMPLE The Golden Gate Bridge

The Golden Gate Bridge, a suspension bridge, spans the entrance to San Francisco Bay. Its 746-foot-tall towers are 4200 feet apart. The bridge is suspended from two huge cables more than 3 feet in diameter; the 90-foot-wide roadway is 220 feet above the water. The cables are parabolic in shape* and touch the road surface at the center of the bridge. Find the height of the cable at a distance of 1000 feet from the center.

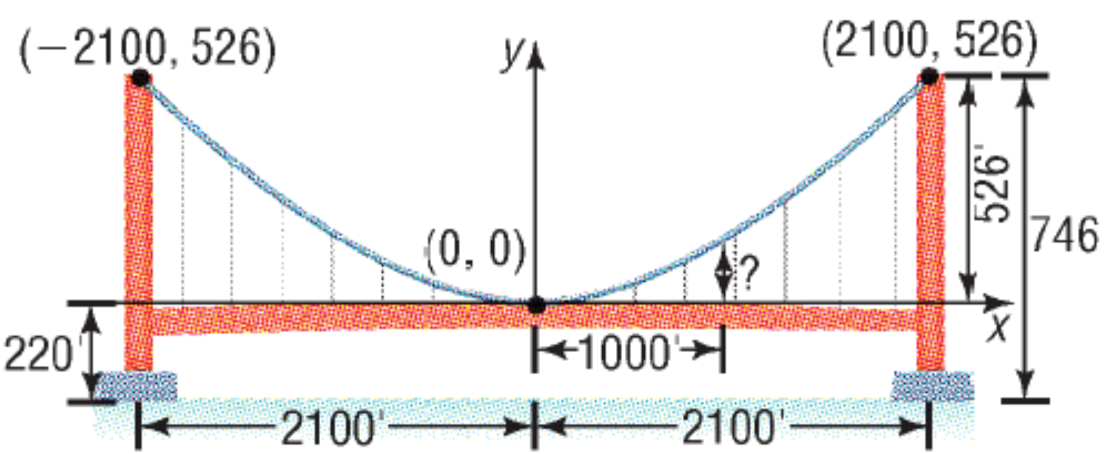
$$y = ax^2 \qquad 526 = a(2100)^2 \qquad a = \frac{526}{(2100)^2}$$

The equation of the parabola is

$$y = \frac{526}{(2100)^2}x^2$$

The height of the cable when $x = 1000$ is

$$y = \frac{526}{(2100)^2}(1000)^2 \approx 119.3 \text{ feet}$$



2 Build Quadratic Models from Data

$y = ax^2 + bx + c,\ a > 0$

$y = ax^2 + bx + c,\ a < 0$

EXAMPLE

Fitting a Quadratic Function to Data

A farmer collected the data given in Table 3, which shows crop yields Y for various amounts of fertilizer used, x.

(a) With a graphing utility, draw a scatter diagram of the data. Comment on the type of relation that may exist between the two variables.

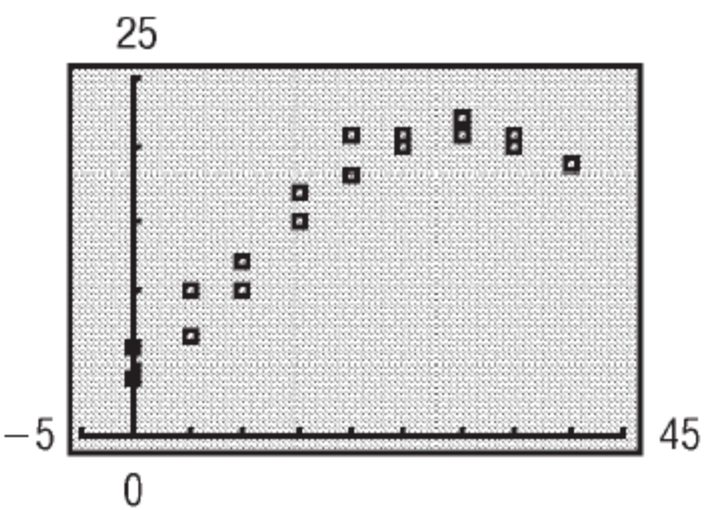


Table 3

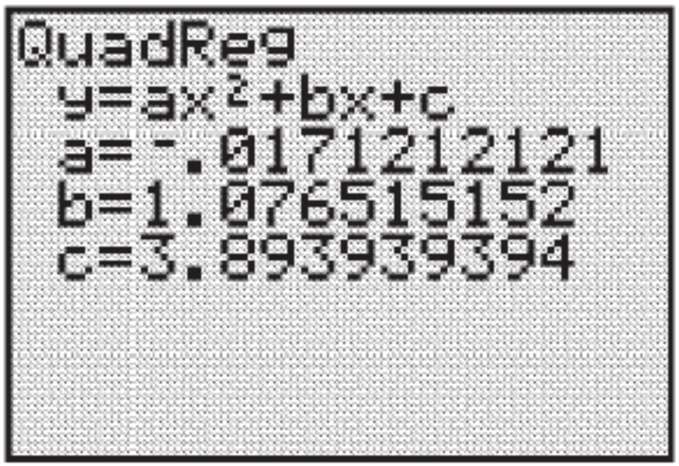
Plot	Fertilizer, x (Pounds/100 ft^2)	Yield (Bushels)
1	0	4
2	0	6
3	5	10
4	5	7
5	10	12
6	10	10
7	15	15
8	15	17
9	20	18
10	20	21
11	25	20
12	25	21
13	30	21
14	30	22
15	35	21
16	35	20
17	40	19
18	40	19

EXAMPLE

Fitting a Quadratic Function to Data

A farmer collected the data given in Table 3, which shows crop yields Y for various amounts of fertilizer used, x.

(b) Use a graphing utility to find the quadratic function of best fit to these data.



$$Y(x) = -0.0171x^2 + 1.0765x + 3.8939$$

***Table** 3*

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14	30	22
15	35	21
16	35	20
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18	40	19

EXAMPLE

Fitting a Quadratic Function to Data

(c) Use the function found in part (b) to determine the optimal amount of fertilizer to apply.

$$-\frac{b}{2a} = -\frac{1.0765}{2(-0.0171)} \approx 31 \text{ lbs}/100 \text{ ft}^2$$

(d) Use the function found in part (b) to predict crop yield when the optimal amount of fertilizer is applied.

$$-0.0171(31.5)^2 + 1.0765(31.5) + 3.8939$$

$$\approx 21 \text{ bushels}$$

$$Y(x) = -0.0171x^2 + 1.0765x + 3.8939$$

Table 3

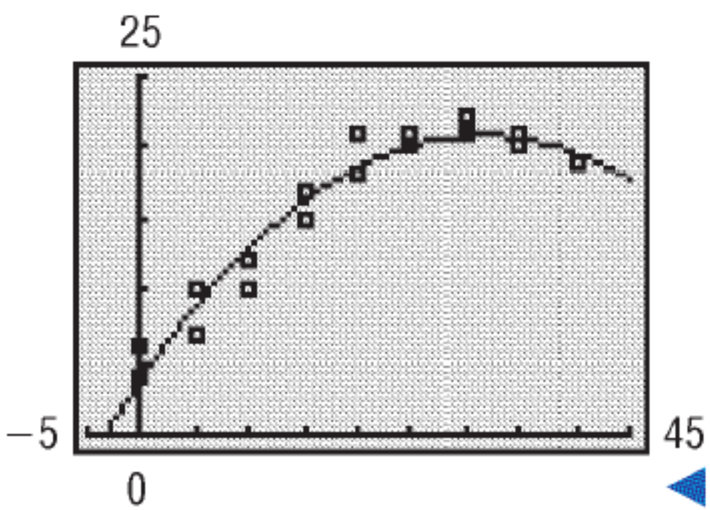
Plot	Fertilizer, *x* (Pounds/100 ft^2)	Yield (Bushels)
1	0	4
2	0	6
3	5	10
4	5	7
5	10	12
6	10	10
7	15	15
8	15	17
9	20	18
10	20	21
11	25	20
12	25	21
13	30	21
14	30	22
15	35	21
16	35	20
17	40	19
18	40	19

EXAMPLE

Fitting a Quadratic Function to Data

A farmer collected the data given in Table 3, which shows crop yields Y for various amounts of fertilizer used, x.

(e) Draw the quadratic function of best fit on the scatter diagram.



$$Y(x) = -0.0171x^2 + 1.0765x + 3.8939$$ n Ec

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