Section 1.6

Equations and Inequalities Involving Absolute Valves

1 Solve Equations Involving Absolute Value

THEOREM

If a is a positive real number and if u is any algebraic expression, then

$$\boxed{|u| = a \quad \text{is equivalent to} \quad u = a \quad \text{or} \quad u = -a}$$

EXAMPLE

Solving an Equation Involving Absolute Value

Solve the equations:

a) $|x-3|=10$

$x-3=10 \text{ or } x-3=-10$

$x=13 \text{ or } x=-7$

The solution set is $\{-7,13\}$.

b) $|2x+1|-3=7$

$|2x+1|=10$

$2x+1=10 \text{ or } 2x+1=-10$

$2x=9 \text{ or } 2x=-11$

$x=\frac{9}{2} \text{ or } x=-\frac{11}{2}$

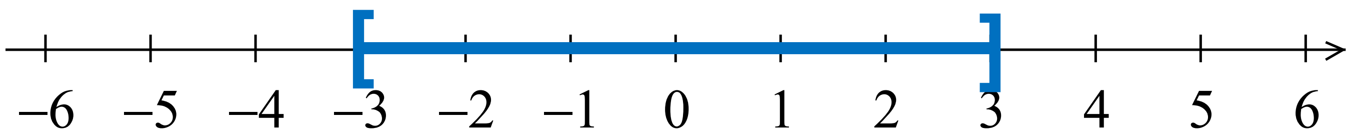
The solution set is $\left\{-\frac{11}{2},\frac{9}{2}\right\}$.

2 Solve Inequalities Involving Absolute Value

EXAMPLE

Solving an Inequality Involving Absolute Value

Solve the inequality: $|x| \le 3$



$$-3 \le x \le 3 \text{ or } [-3,3]$$

THEOREM

If a is a positive number and if u is an algebraic expression, then

$$|u| < a \quad \text{is equivalent to} \quad -a < u < a \qquad \textbf{(2)}$$

$$|u| \le a \quad \text{is equivalent to} \quad -a \le u \le a \qquad \textbf{(3)}$$

In other words, $|u| < a$ is equivalent to $-a < u$ and $u < a$.

EXAMPLE

Solving an Inequality Involving Absolute Value

Solve the inequality: $|3x-1| \le 5$. Graph the solution set.

$$-5 \le 3x-1 \le 5$$

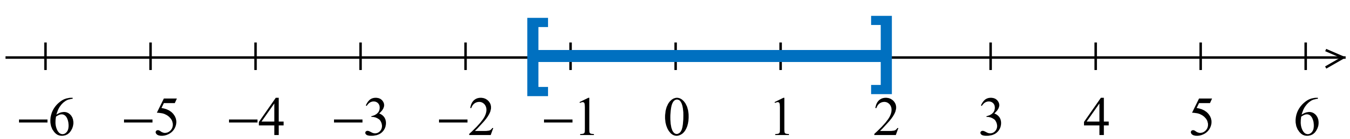
$$-5+1 \le 3x-1+1 \le 5+1$$

$$-4 \le 3x \le 6$$

$$\frac{-4}{3} \le \frac{3x}{3} \le \frac{6}{3}$$

$$-\frac{4}{3} \le x \le 2$$

The solution set is $\left\{x \middle| -\frac{4}{3} \le x \le 2\right\}$ or $\left[-\frac{4}{3}, 2\right]$.



EXAMPLE

Solving an Inequality Involving Absolute Value

Solve the inequality: $|3-2x|<4$. Graph the solution set.

$$-4<3-2x<4$$

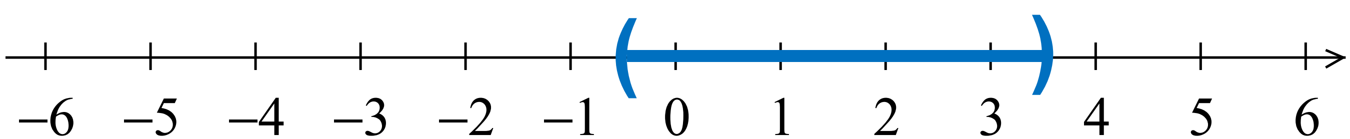
$$-4-3<3-2x-3<4-3$$

$$-7<-2x<1$$

$$\frac{-7}{-2}>\frac{-2x}{-2}>\frac{1}{-2}$$

$$\frac{7}{2}>x>-\frac{1}{2}$$

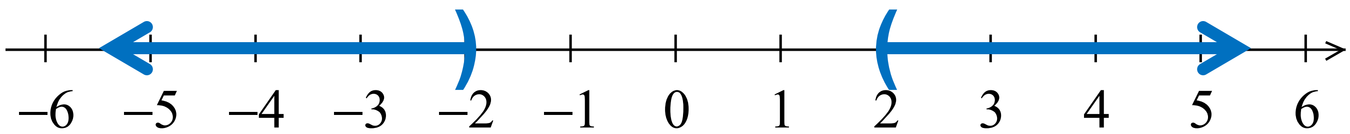
The solution set is $\left\{x \middle| -\frac{1}{2}<x<\frac{7}{2}\right\}$ or $\left(-\frac{1}{2},\frac{7}{2}\right)$.



EXAMPLE

Solving an Inequality Involving Absolute Value

Solve the inequality: $|x| > 2$



$x < -2$ or $x > 2$

$(-\infty, -2) \cup (2, \infty)$

THEOREM

If a is a positive number and u is an algebraic expression, then

$$|u| > a \quad \text{is equivalent to} \quad u < -a \quad \text{or} \quad u > a \tag{4}$$

$$|u| \geq a \quad \text{is equivalent to} \quad u \leq -a \quad \text{or} \quad u \geq a \tag{5}$$

EXAMPLE

Solving an Inequality Involving Absolute Value

Solve the inequality: $|3x-2|>7$. Graph the solution set.

$$3x-2<-7 \quad \text{or} \quad 3x-2>7$$

$$3x<-5 \quad \text{or} \quad 3x>9$$

$$x<-\frac{5}{3} \quad \text{or} \quad x>3$$

The solution set is $\left\{x \middle| x<-\frac{5}{3} \text{ or } x>3\right\}$. $\left(-\infty,-\frac{5}{2}\right)\cup(3,\infty)$