Section 2.5

Variation

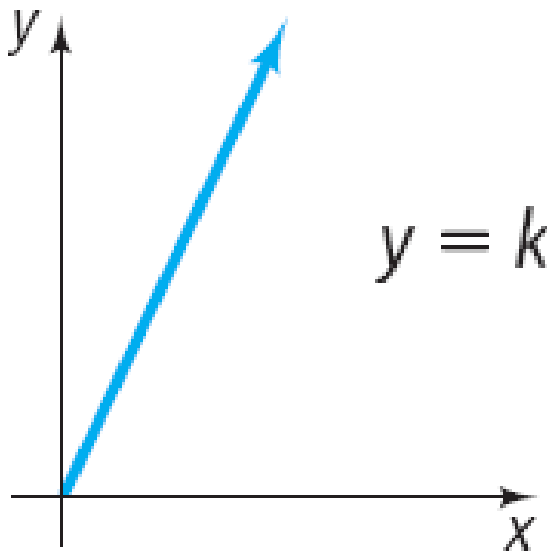
1 Construct a Model Using Direct Variation

DEFINITION

Let x and y denote two quantities. Then y **varies directly** with x, or y is **directly proportional to** x, if there is a nonzero number k such that

$$y = kx$$

The number k is called the **constant of proportionality.**



$y = kx; k > 0, x \geq 0$

EXAMPLE Mortgage Payments

The monthly payment p on a mortgage varies directly with the amount borrowed B. If the monthly payment on a 30-year mortgage is \$5.95 for every \$1000 borrowed, find a formula that relates the monthly payment p to the amount borrowed B for a mortgage with these terms. Then find the monthly payment p when the amount borrowed B is \$125,000

Because p varies directly with B, $p = kB$ for some constant k.

Because $p = 5.95$ when $B = 1000$, $5.95 = k(1000)$. $k = 0.00595$

$$p = k \quad (125{,}000)$$

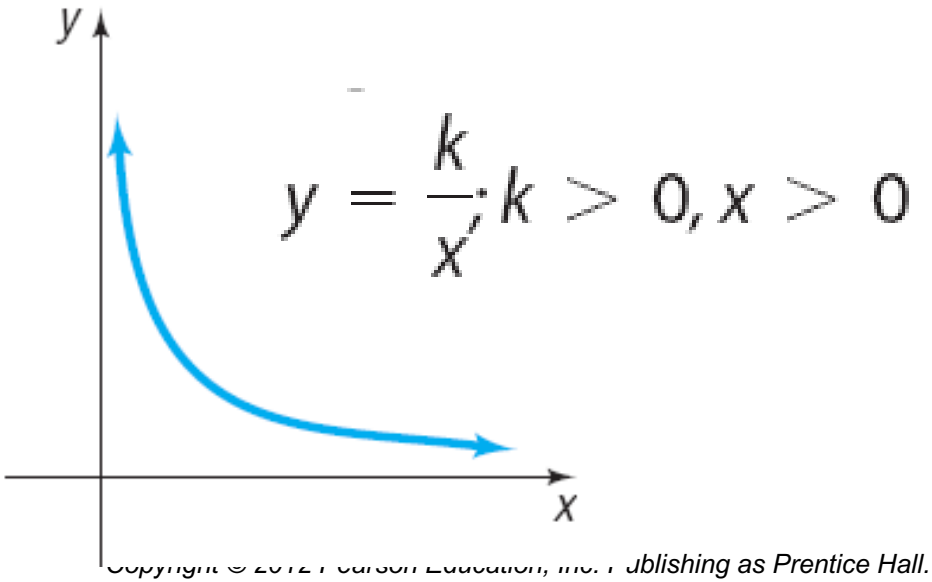
$$p = \$743.75$$

2 Construct a Model Using Inverse Variation

DEFINITION

Let x and y denote two quantities. Then y **varies inversely** with x, or y is **inversely proportional to** x, if there is a nonzero constant k such that

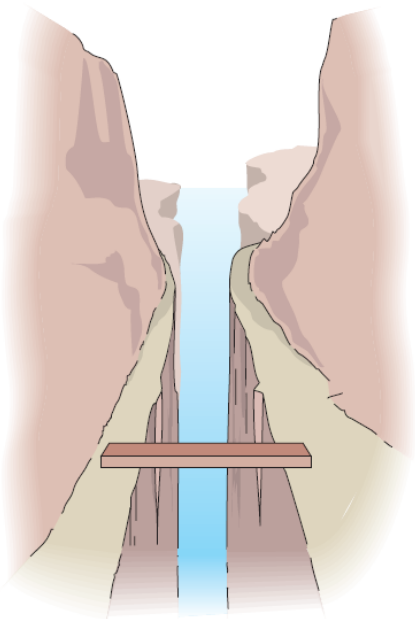
$$y = \frac{k}{x}$$



EXAMPLE

Maximum Weight That Can Be Supported by a Piece of Pine

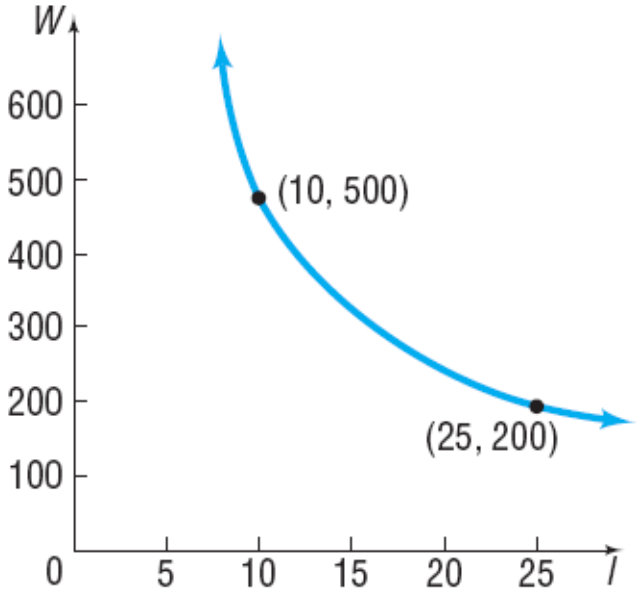
See Figure 57. The maximum weight W that can be safely supported by a 2-inch by 4-inch piece of pine varies inversely with its length l. Experiments indicate that the maximum weight that a 10-foot-long 2-by-4 piece of pine can support is 500 pounds. Write a general formula relating the maximum weight W (in pounds) to length l (in feet). Find the maximum weight W that can be safely supported by a length of 25 feet.



$$W = \frac{k}{l} \qquad 500 = \frac{k}{10}$$

$$5000 = k$$

$$W = \frac{5000}{25} = 200 \text{ lbs}$$



3 Construct a Model Using Joint Variation or Combined Variation

EXAMPLE Loss of Heat Through a Wall

The loss of heat through a wall varies jointly with the area of the wall and the difference between the inside and outside temperatures and varies inversely with the thickness of the wall. Write an equation that relates these quantities.

L = Heat loss $\quad$ T = Temperature difference

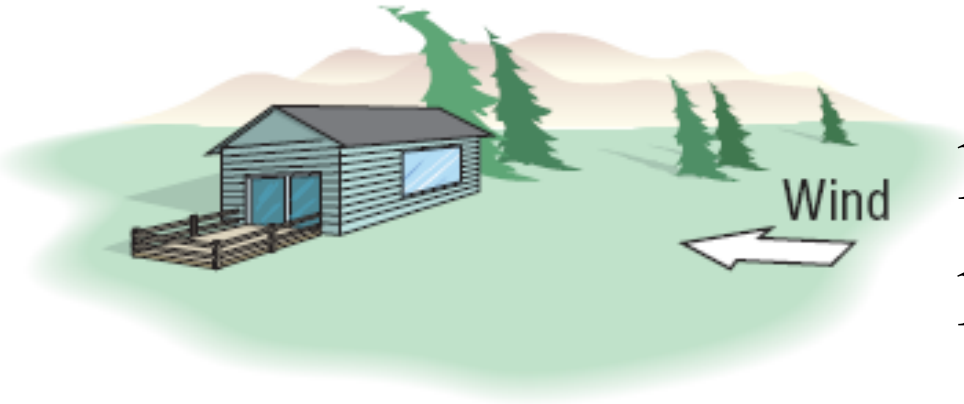
A = Area of wall $\quad$ d = Thickness of wall

$$L = k\frac{AT}{d}$$

EXAMPLE

Force of the Wind on a Window

The force F of the wind on a flat surface positioned at a right angle to the direction of the wind varies jointly with the area A of the surface and the square of the speed v of the wind. A wind of 30 miles per hour blowing on a window measuring 4 feet by 5 feet has a force of 150 pounds. See Figure 59. What is the force on a window measuring 2 feet by 3 feet caused by a wind of 70 miles per hour?



$$F = kAv^2$$

$$150 = k(20)(30)^2$$

$$150 = 18000k$$

$$\frac{1}{120} = k$$

$$F = kAv^2$$

$$F = \frac{1}{120}(6)(70)^2 = 245 \text{ lbs}$$