Section 1.1

Linear Equations

Procedures That Result in Equivalent Equations

1. Interchange the two sides of the equation:

$$\text{Replace} \quad 3 = x \quad \text{by} \quad x = 3$$

2. Simplify the sides of the equation by combining like terms, eliminating parentheses, and so on:

$$\text{Replace} \quad (x + 2) + 6 = 2x + (x + 1)$$

$$\text{by} \quad x + 8 = 3x + 1$$

3. Add or subtract the same expression on both sides of the equation:

$$\text{Replace} \quad 3x - 5 = 4$$

$$\text{by} \quad (3x - 5) + 5 = 4 + 5$$

4. Multiply or divide both sides of the equation by the same nonzero expression:

$$\text{Replace} \quad \frac{3x}{x - 1} = \frac{6}{x - 1} \quad x \neq 1$$

$$\text{by} \quad \frac{3x}{x - 1} \cdot (x - 1) = \frac{6}{x - 1} \cdot (x - 1)$$

5. If one side of the equation is 0 and the other side can be factored, then we may use the Zero-Product Property and set each factor equal to 0:

$$\text{Replace} \quad x(x - 3) = 0$$

$$\text{by} \quad x = 0 \quad \text{or} \quad x - 3 = 0$$

Solve the equation: $4x - 7 = 13$

$$4x - 7 + 7 = 13 + 7$$

$$4x = 20$$

$$\frac{4x}{4} = \frac{20}{4}$$

$$x = 5$$

Steps for Solving Equations

STEP 1: List any restrictions on the domain of the variable.

STEP 2: Simplify the equation by replacing the original equation by a succession of equivalent equations following the procedures listed earlier.

STEP 3: If the result of Step 2 is a product of factors equal to 0, use the Zero-Product Property and set each factor equal to 0 (procedure 5).

STEP 4: Check your solution(s).

✓1 **Solve a Linear Equation**

DEFINITION

A **linear equation in one variable** is equivalent to an equation of the form

$$ax + b = 0$$

where a and b are real numbers and $a \neq 0$.

Solving a Linear Equation

Solve the equation: $\dfrac{1}{3}(x-1)-3=\dfrac{1}{4}(3x+2)$

$$12\left[\dfrac{1}{3}(x-1)-3\right]=12\left[\dfrac{1}{4}(3x+2)\right]$$

$$4(x-1)-3\cdot12=3(3x+2)$$

$$4x-4-36=9x+6$$

$$4x-40=9x+6 \qquad 4x-40+40-9x=9x+6+40-9x$$

$$-5x=46 \qquad \dfrac{-5x}{-5}=\dfrac{46}{-5} \qquad x=-\dfrac{46}{5}$$

Solving a Linear Equation Using a Calculator

Solve the equation: $3.81x + \dfrac{4}{12.231} = 23.03$

Round the answer to two decimal places.

$$3.81x = 23.03 - \dfrac{4}{12.231}$$

$$x = \dfrac{23.03 - \dfrac{4}{12.231}}{3.81}$$

$$x \approx 5.96$$

2 Solve Equations That Lead to Linear Equations

Solving an Equation That Leads to a Linear Equation

Solve the equation: $(3x-1)(x+2)=(x+3)(3x-2)$

$$3x^2 + 5x - 2 = 3x^2 + 7x - 6$$

$$5x - 2 = 7x - 6$$

$$5x = 7x - 4$$

$$-2x = -4$$

$$x = 2$$

Solving an Equation That Leads to a Linear Equation

Solve the equation: $\dfrac{2}{x-1} = \dfrac{3}{x+3} + \dfrac{1}{(x+3)(x-1)}$

$$(x+3)(x-1)\ \dfrac{2}{x-1} = (x+3)(x-1)\left[\dfrac{3}{x+3} + \dfrac{1}{(x+3)(x-1)}\right]$$

$$2x+6 = 3(x-1)+1 \qquad\qquad 2x+6 = 3x-3+1$$

$$-x = -8 \qquad\qquad x = 8$$

Solve the equation: $\dfrac{3x}{x-2} + 1 = \dfrac{6}{x-2}$

$$\left(\dfrac{3x}{x-2} + 1\right)(x-2) = \left(\dfrac{6}{x-2}\right)(x-2), \qquad x \neq 2$$

$$3x + x - 2 = 6$$

$$4x - 2 = 6 \qquad 4x = 8 \qquad \cancel{x = 2}$$

no solution

In the United States we measure temperature in both degrees Fahrenheit (°F) and degrees Celsius (°C). They are related by the formula $C = \dfrac{5}{9}(F - 32)$.

What are the Fahrenheit temperatures corresponding to Celsius temperatures of $-10°, 0°,$ and $40°$?

$$9C = 5(F - 32) \qquad 9C = 5F - 160 \qquad 9C + 160 = 5F$$
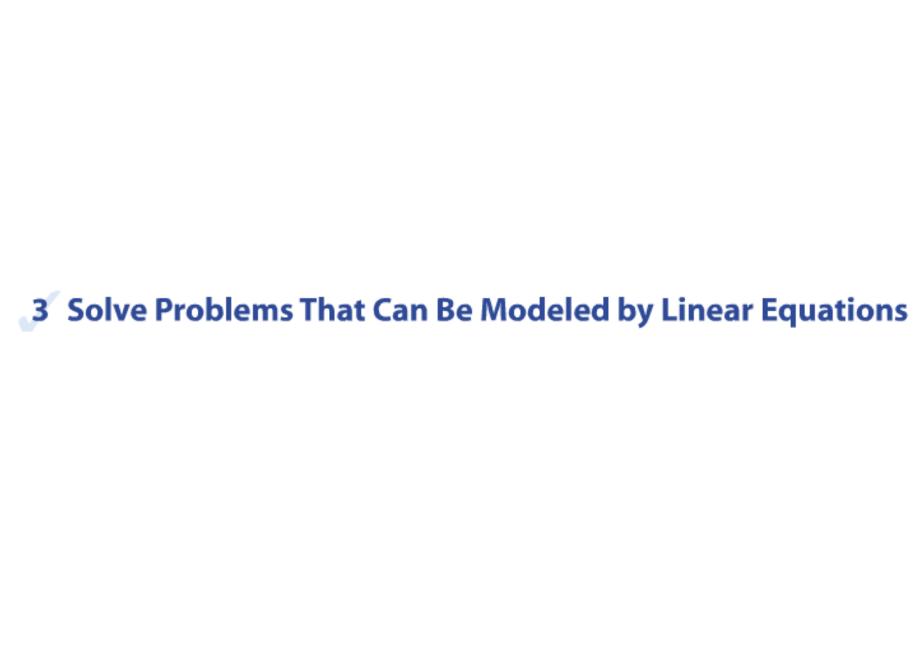
$$F = \frac{9}{5}C + 32$$

In the United States we measure temperature in both degrees Fahrenheit (°F) and degrees Celsius (°C). They are related by the formula $C = \dfrac{5}{9}(F - 32)$.

What are the Fahrenheit temperatures corresponding to Celsius temperatures of $-10°, 0°,$ and $40°$?

$$F = \dfrac{9}{5}C + 32 \qquad \text{For } -10°: \ F = \dfrac{9}{5}(-10) + 32 = 14°F$$

$$\text{For } 0°: \ F = \dfrac{9}{5}(0) + 32 = 32°F \qquad \text{For } 40°: \ F = \dfrac{9}{5}(40) + 32 = 104°F$$

3 Solve Problems That Can Be Modeled by Linear Equations

Steps for Solving Applied Problems

STEP 1: Read the problem carefully, perhaps two or three times. Pay particular attention to the question being asked in order to identify what you are looking for. If you can, determine realistic possibilities for the answer.

STEP 2: Assign a letter (variable) to represent what you are looking for, and, if necessary, express any remaining unknown quantities in terms of this variable.

STEP 3: Make a list of all the known facts, and translate them into mathematical expressions. These may take the form of an equation (or, later, an inequality) involving the variable. The equation (or inequality) is called the **model.** If possible, draw an appropriately labeled diagram to assist you. Sometimes a table or chart helps.

STEP 4: Solve the equation for the variable, and then answer the question, usually using a complete sentence.

STEP 5: Check the answer with the facts in the problem. If it agrees, congratulations! If it does not agree, try again.

Investments

A total of $16,000 is invested, some in stocks and some in bonds. If the amount invested in bonds is one fourth that invested in stocks, how much is invested in each category?

We are being asked to find the amount of two investments. These amounts total $16,000.

STEP 1: Read the problem carefully, perhaps two or three times. Pay particular attention to the question being asked in order to identify what you are looking for. If you can, determine realistic possibilities for the answer.

Investments

A total of $16,000 is invested, some in stocks and some in bonds. If the amount invested in bonds is one fourth that invested in stocks, how much is invested in each category?

If x equals the amount invested in stocks, then the rest of the money, $16,000 - x$, is the amount invested in bonds.

STEP 2: Assign a letter (variable) to represent what you are looking for, and, if necessary, express any remaining unknown quantities in terms of this variable.

A total of $16,000 is invested, some in stocks and some in bonds. If the amount invested in bonds is one fourth that invested in stocks, how much is invested in each category?

Amount in Stocks	Amount in Bonds	Reason
x	$16{,}000 - x$	Total invested is $16,000

We also know that:

Total amount invest in bonds   is   one fourth that in stocks

$$16,000 - x \quad = \quad \frac{1}{4}x$$

STEP 3: Make a list of all the known facts, and translate them into mathematical expressions. These may take the form of an equation (or, later, an inequality) involving the variable. The equation (or inequality) is called the **model.** If possible, draw an appropriately labeled diagram to assist you. Sometimes a table or chart helps.

EXAMPLE Investments

A total of $16,000 is invested, some in stocks and some in bonds. If the amount invested in bonds is one fourth that invested in stocks, how much is invested in each category?

$$16,000 - x = \frac{1}{4}x \qquad 16,000 = x + \frac{1}{4}x \qquad 16,000 = \frac{5}{4}x$$

$$\left(\frac{4}{5}\right)16,000 = \left(\frac{4}{5}\right)\left(\frac{5}{4}x\right) \qquad 12,800 = x$$

So $12,800 is invested in stocks and $16,000 − $12,800 = $3,200 is invested in bonds.

STEP 4: Solve the equation for the variable, and then answer the question, usually using a complete sentence.

A total of $16,000 is invested, some in stocks and some in bonds. If the amount invested in bonds is one fourth that invested in stocks, how much is invested in each category?

The total invested is $3,200 + $12,800 $= $16,000 and the amount of bonds, $3,200 is one fourth of that of stocks, $12,800.

STEP 5: Check the answer with the facts in the problem. If it agrees, congratulations! If it does not agree, try again.

Determining an Hourly Wage

Andy grossed $440 one week by working 50 hours. His employer pays time-and-a-half for all hours worked in excess of 40 hours. What is Andy's hourly wage?

We are looking for an hourly wage. Our answer will be expressed in dollars per hour.

STEP 1: Read the problem carefully, perhaps two or three times. Pay particular attention to the question being asked in order to identify what you are looking for. If you can, determine realistic possibilities for the answer.

Andy grossed $440 one week by working 50 hours. His employer pays time-and-a-half for all hours worked in excess of 40 hours. What is Andy's hourly wage?

Let x represent the regularly wage; x is measured in dollars per hour.

STEP 2: Assign a letter (variable) to represent what you are looking for, and, if necessary, express any remaining unknown quantities in terms of this variable.

Determining an Hourly Wage

Andy grossed $440 one week by working 50 hours. His employer pays time-and-a-half for all hours worked in excess of 40 hours. What is Andy's hourly wage?

	Hours Worked	Hourly Wage	Salary
Regular	40	x	$40x$
Overtime	10	$1.5x$	$10(1.5x) = 15x$

The sum of regular salary plus overtime salary will equal $440. From the table, $40x + 15x = 440$.

STEP 3: Make a list of all the known facts, and translate them into mathematical expressions. These may take the form of an equation (or, later, an inequality) involving the variable. The equation (or inequality) is called the **model.** If possible, draw an appropriately labeled diagram to assist you. Sometimes a table or chart helps.

Determining an Hourly Wage

Andy grossed $440 one week by working 50 hours. His employer pays time-and-a-half for all hours worked in excess of 40 hours. What is Andy's hourly wage?

$$40x + 15x = 440$$

$$55x = 440$$

$$x = 8$$

Andy's regularly wage is $8.00 per hour.

STEP 4: Solve the equation for the variable, and then answer the question, usually using a complete sentence.

Determining an Hourly Wage

Andy grossed $440 one week by working 50 hours. His employer pays time-and-a-half for all hours worked in excess of 40 hours. What is Andy's hourly wage?

Forty hours yields a salary of $40(8.00) = \$320.00$ and 10 hours overtime yields a salary of $10(1.5)(8.00) = \$120.00$ for a total of $440.00.

STEP 5: Check the answer with the facts in the problem. If it agrees, congrat-ulations! If it does not agree, try again.

SUMMARY Steps for Solving a Linear Equation

To solve a linear equation, follow these steps:

STEP 1: List any restrictions on the variable.

STEP 2: If necessary, clear the equation of fractions by multiplying both sides by the least common multiple (LCM) of the denominators of all the fractions.

STEP 3: Remove all parentheses and simplify.

STEP 4: Collect all terms containing the variable on one side and all remaining terms on the other side.

STEP 5: Simplify and solve.

STEP 6: Check your solution(s).