Section 12.6

Systems of Nonlinear Equations

1 Solve a System of Nonlinear Equations Using Substitution

1. If the system contains two variables and if the equations in the system are easy to graph, then graph them. By graphing each equation in the system, you can get an idea of how many solutions a system has and approximately where they are located.

2. Extraneous solutions can creep in when solving nonlinear systems, so it is imperative that all apparent solutions be checked.

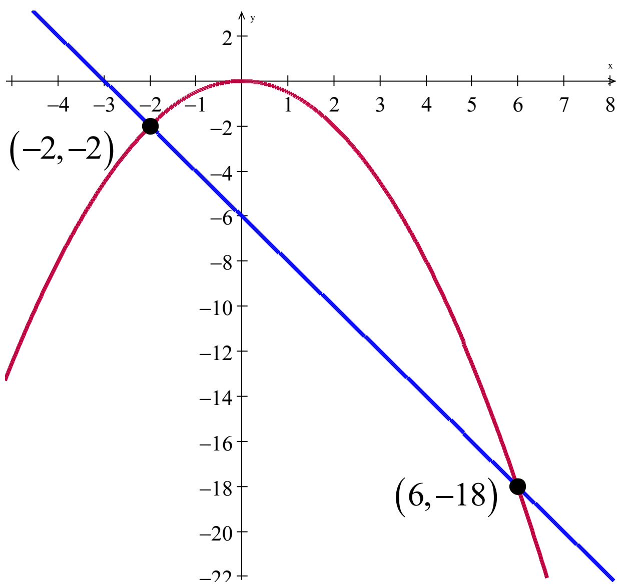
EXAMPLE Solving a System of Nonlinear Equations Using Substitution

Solve: $\begin{cases} 2x + y = -6 \\ x^2 + 2y = 0 \end{cases}$

First let's look at the graph. The first equation is a line with x intercept –6 and y intercept – 4. The second a parabola opening down.

The solutions are:

$(-2,-2)$ and $(6,-18)$



$y = -2x - 6$ Solve first equation for y

$$x^2 + 2(-2x - 6) = 0$$

$$x^2 - 4x - 12 = 0$$

$$(x+2)(x-6) = 0$$

$$x = -2 \text{ or } x = 6$$

$$y = -2(-2) - 6 = -2$$

$$y = -2(6) - 6 = -18$$

2 Solve a System of Nonlinear Equations Using Elimination

EXAMPLE Solving a System of Nonlinear Equations Using Elimination

Solve: $\begin{cases} x^2 + y^2 = 8 \\ x^2 + 2y = 0 \end{cases}$

Subtract equations: $\begin{cases} x^2 + y^2 = 8 \\ x^2 + 2y = 0 \end{cases}$

$$y^2 - 2y = 8$$

$$y^2 - 2y - 8 = 0$$

$$(y-4)(y+2) = 0$$

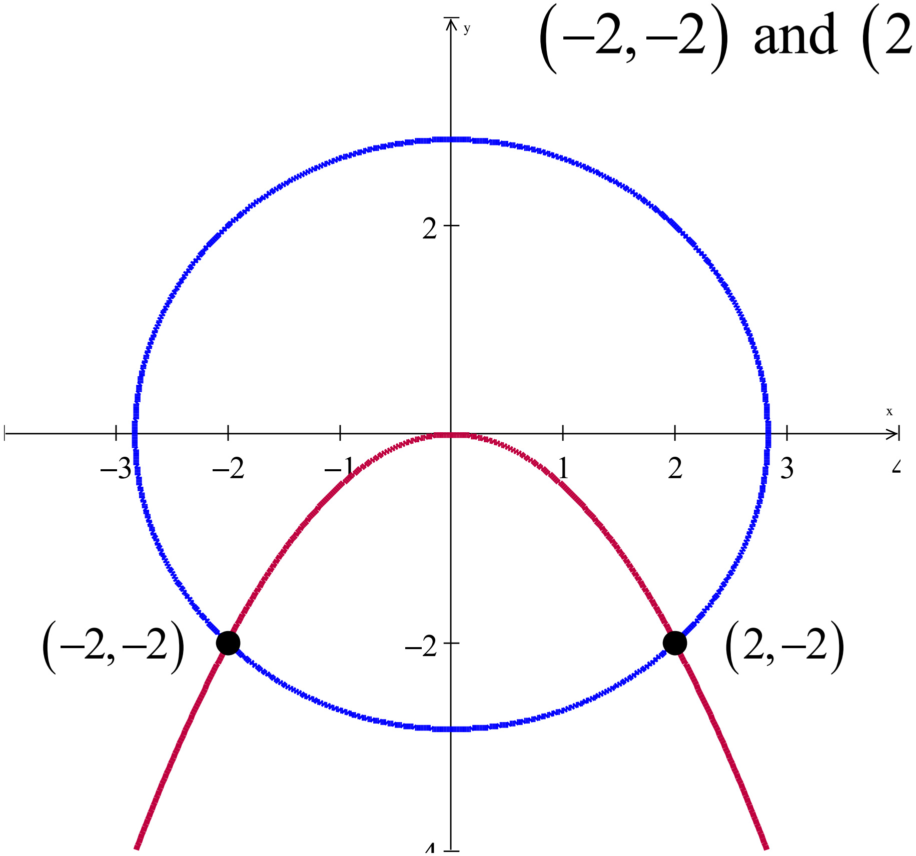
$$\cancel{y=4},\ y = -2$$

$$x^2 + 2({\color{red}-2}) = 0$$

$$x^2 = 4 \quad x = \pm 2$$

The solutions are:

$(-2,-2)$ and $(2,-2)$

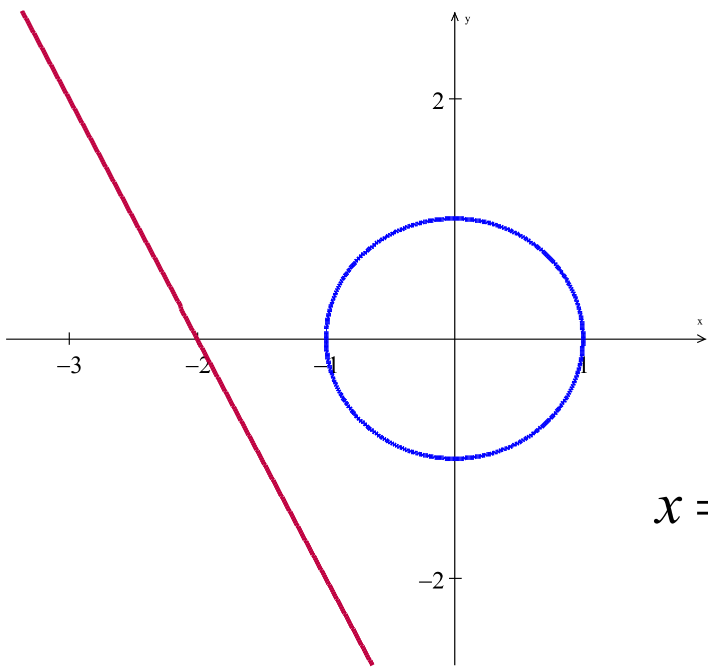


EXAMPLE Solving a System of Nonlinear Equations

Solve: $\begin{cases} 2x + y = -6 \\ x^2 + y^2 = 1 \end{cases}$

Looking at the graph, it appears there are no solutions but we'll confirm this with the algebra.

The system is inconsistent and has no real solution.



$$y = -2x - 6$$

$$x^2 + (-2x - 6)^2 = 1$$

$$x^2 + 4x^2 - 24x + 36 = 1$$

$$5x^2 - 24x + 35 = 0$$

$$x = \frac{-(-24) \pm \sqrt{(-24)^2 - 4(5)(35)}}{2(5)} = \frac{24 \pm \sqrt{-124}}{10}$$

EXAMPLE **Solving a System of Nonlinear Equations Using Elimination**

Solve: $\begin{cases} x^2 + x + y^2 - 3y + 2 = 0 \\ x + 1 + \dfrac{y^2 - y}{x} = 0 \end{cases}$

Since it is not straightforward how to graph the equations in the system, we proceed directly to use the method of elimination.

$$\begin{cases} x^2 + x + y^2 - 3y + 2 = 0 \\ x^2 + x + y^2 - y = 0 \end{cases}$$

$$-2y + 2 = 0$$

$$y = 1$$

$$x^2 + x + (1)^2 - 3(1) + 2 = 0$$

$$x^2 + x = 0 \qquad x(x+1) = 0$$

$$\cancel{x = 0} \text{ or } x = -1$$

The solution is (–1, 1)

EXAMPLE Solving a System of Nonlinear Equations

Solve: $\begin{cases} 3xy - 2y^2 = -2 \\ 9x^2 + 4y^2 = 10 \end{cases}$

$$\begin{cases} 6xy - 4y^2 = -4 \\ 9x^2 + 4y^2 = 10 \end{cases}$$

$$9x^2 + 6xy = 6$$

$$3x^2 + 2xy = 2$$

$$y = \frac{2 - 3x^2}{2x}$$

$$9x^2 + 4\left(\frac{2 - 3x^2}{2x}\right)^2 = 10$$

$$9x^2 + \frac{4 - 12x^2 + 9x^4}{x^2} = 10$$

$$18x^4 - 22x^2 + 4 = 0$$

$$9x^4 - 11x^2 + 2 = 0$$

$$(9x^2 - 2)(x^2 - 1) = 0$$

$$9x^2 - 2 = 0 \quad \text{or} \quad x^2 - 1 = 0$$

$$x^2 = \frac{2}{9} \quad \text{or} \quad x^2 = 1$$

$$x = \pm\sqrt{\frac{2}{9}} = \pm\frac{\sqrt{2}}{3} \quad \text{or} \quad x = \pm 1$$

There are four solutions:

$$\left(\frac{\sqrt{2}}{3}, \sqrt{2}\right), \left(-\frac{\sqrt{2}}{3}, -\sqrt{2}\right),$$

$$\left(1, -\frac{1}{2}\right), \left(-1, \frac{1}{2}\right)$$

EXAMPLE

Running a Long Distance Race

In a 50-mile race, the winner crosses the finish line 1 mile ahead of the second-place runner and 4 miles ahead of the third-place runner. Assuming that each runner maintains a constant speed throughout the race, by how many miles does the second-place runner beat the third-place runner?

We seek the distance d of the third-place runner from the finish at time t_2. At time t_2, the third-place runner has gone a distance of $v_3 t_2$ miles, so the distance d remaining is $50 - v_3 t_2$. Now

$$\begin{aligned} d &= 50 - v_3 t_2 \\ &= 50 - v_3\left(t_1 \cdot \frac{t_2}{t_1}\right) \\ &= 50 - (v_3 t_1) \cdot \frac{t_2}{t_1} \\ &= 50 - 46 \cdot \frac{\frac{50}{v_2}}{\frac{50}{v_1}} \\ &= 50 - 46 \cdot \frac{50}{49} \approx 3.06 \text{ miles} \end{aligned}$$

$$\left\{ \begin{array}{l} \text{From (3), } v_3 t_1 = 46 \\ \text{From (4), } t_2 = \dfrac{50}{v_2} \\ \text{From (1), } t_1 = \dfrac{50}{v_1} \end{array} \right.$$

$$\left\{ \begin{array}{llll} 50 = v_1 t_1 & (1) & \text{First-place runner goes 50 miles in } t_1. \\ 49 = v_2 t_1 & (2) & \text{Second-place runner goes 49 miles in } t_1. \\ 46 = v_3 t_1 & (3) & \text{Third-place runner goes 46 miles in } t_1. \\ 50 = v_2 t_2 & (4) & \text{Second-place runner goes 50 miles in } t_2. \end{array} \right.$$