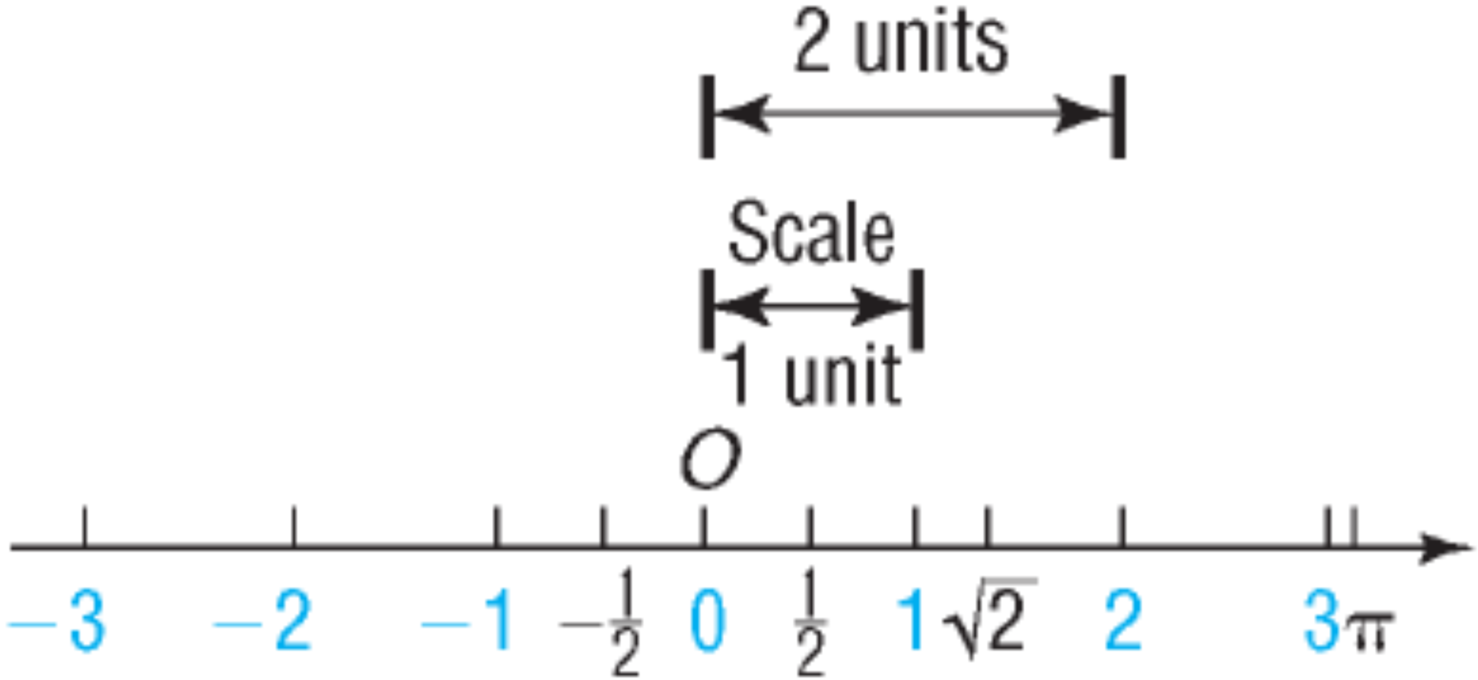
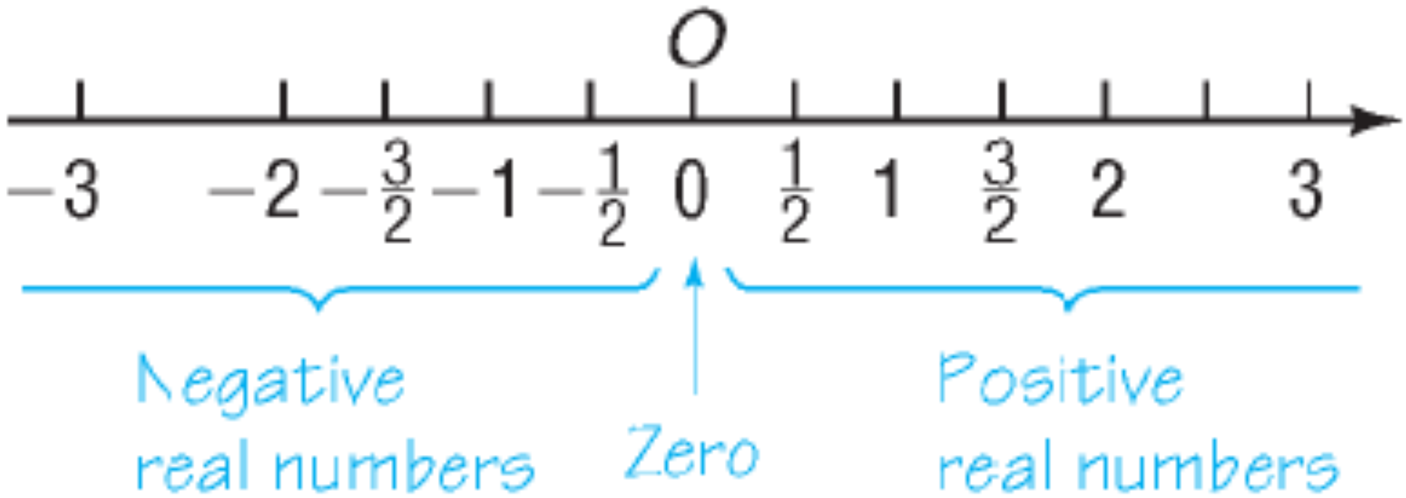
Section R.2

Algebra Essentials

Real number line



The real number associated with a point P is called the **coordinate** of P, and the line whose points have been assigned coordinates is called the **real number line**.



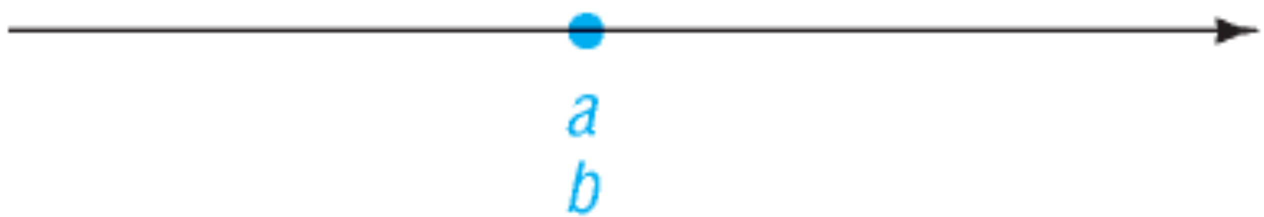
1. The **negative real numbers** are the coordinates of points to the left of the origin O.
2. The real number **zero** is the coordinate of the origin O.
3. The **positive real numbers** are the coordinates of points to the right of the origin O.

Multiplication Properties of Positive and Negative Numbers

1. The product of two positive numbers is a positive number.
2. The product of two negative numbers is a positive number.
3. The product of a positive number and a negative number is a negative number.

1 Graph Inequalities

(a) $a < b$



(b) $a = b$

(c) $a > b$

EXAMPLE

Using Inequality Symbols

(a) $3 < 7$

(b) $-8 > -16$

(c) $-6 < 0$

(d) $-8 < -4$

(e) $4 > -1$

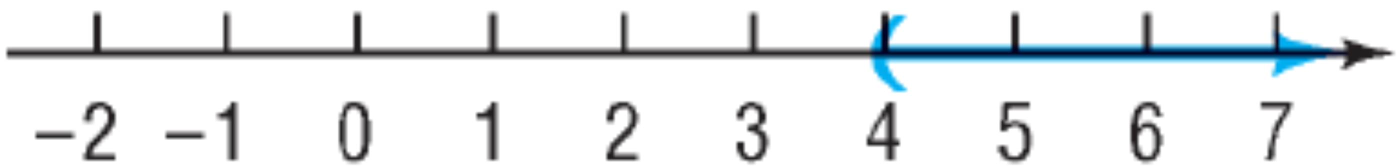
(f) $8 > 0$

$a > 0$ is equivalent to a is positive

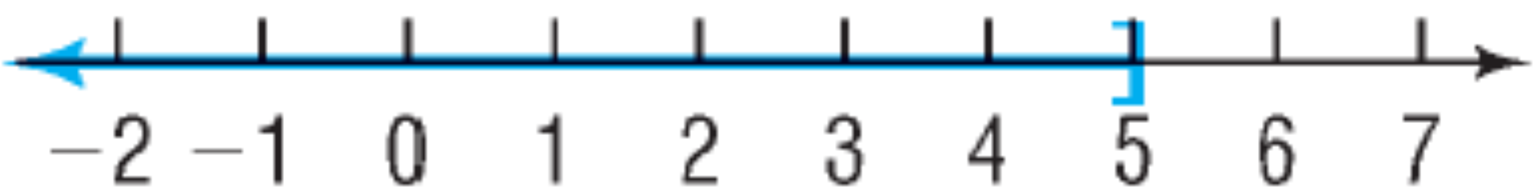
$a < 0$ is equivalent to a is negative

EXAMPLE **Graphing Inequalities**

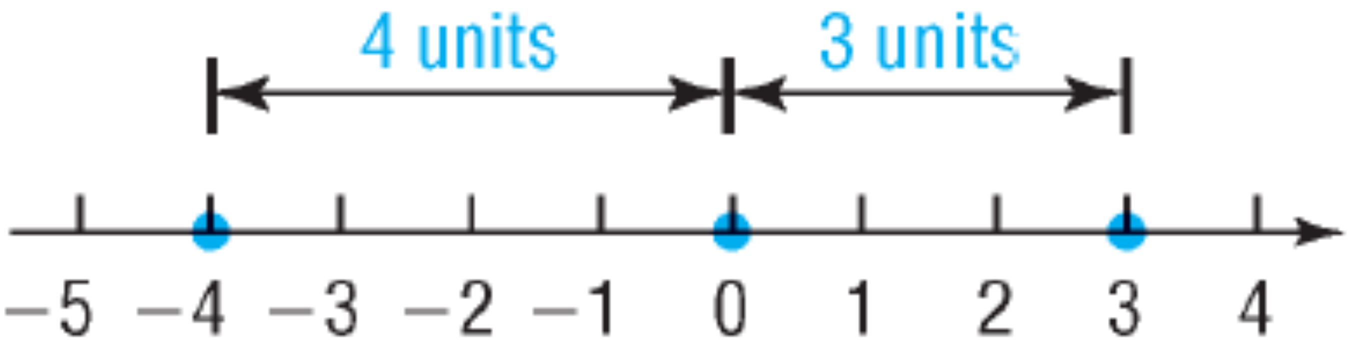
(a) On the real number line, graph all numbers x for which $x > 4$.



(b) On the real number line, graph all numbers x for which $x \leq 5$.



2 Find Distance on the Real Number Line



The **absolute value** of a real number a, denoted by the symbol $|a|$, is defined by the rules

$$\boxed{|a| = a \quad \text{if } a \geq 0 \qquad \text{and} \qquad |a| = -a \quad \text{if } a < 0}$$

EXAMPLE Computing Absolute Value

(a) $|8| = 8$

(b) $|0| = 0$

(c) $|-15| = -(-15) = 15$

If P and Q are two points on a real number line with coordinates a and b, respectively, the **distance between P and Q,** denoted by $d(P, Q)$, is

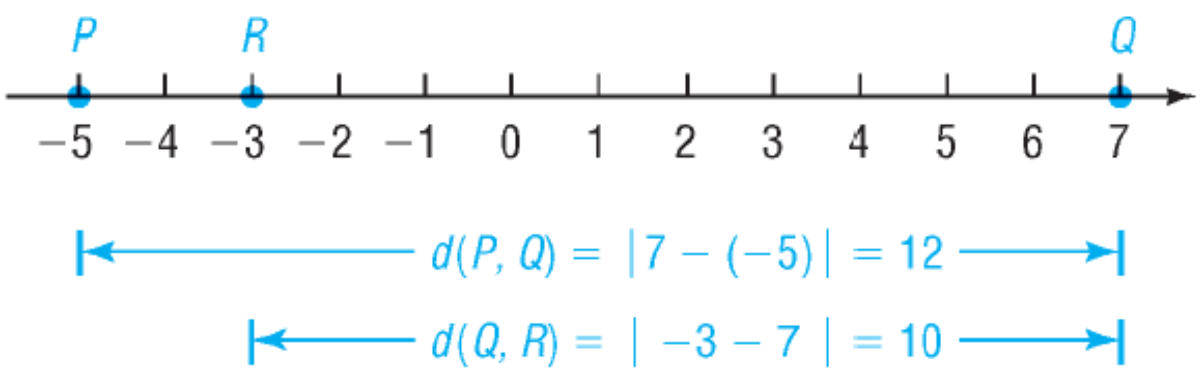
$$\boxed{d(P, Q) = |b - a|}$$

Since $|b - a| = |a - b|$, it follows that $d(P, Q) = d(Q, P)$.

EXAMPLE Finding Distance on a Number Line

Let P, Q, and R be points on a real number line with coordinates -5, 7, and -3, respectively. Find the distance

(a) between P and Q (b) between Q and R



(a) $d(P, Q) = |7 - (-5)| = |12| = 12$

(b) $d(Q, R) = |-3 - 7| = |-10| = 10$

3 Evaluate Algebraic Expressions

EXAMPLE

Evaluating an Algebraic Expression

Evaluate each expression if $x = 3$ and $y = -1$.

(a) $x + 3y$ $\quad x + 3y = 3 + 3(-1) = 3 + (-3) = 0$

$x = 3, y = -1$

(b) $5xy$ $\quad 5xy = 5(3)(-1) = -15$

(c) $\dfrac{3y}{2 - 2x}$ $\quad \dfrac{3y}{2 - 2x} = \dfrac{3(-1)}{2 - 2(3)} = \dfrac{-3}{2 - 6} = \dfrac{-3}{-4} = \dfrac{3}{4}$

(d) $|-4x + y|$

$$|-4x + y| = |-4(3) + (-1)| = |-12 + (-1)| = |-13| = 13$$

4 Determine the Domain of a Variable

DEFINITION

The set of values that a variable may assume is called the **domain of the variable.**

EXAMPLE Finding the Domain of a Variable

The domain of the variable x in the expression

$$\frac{5}{x-2}$$

is $\{x \mid x \neq 2\}$, since, if $x = 2$, the denominator becomes 0, which is not defined.

EXAMPLE Circumference of a Circle

In the formula for the circumference C of a circle of radius r,

$$C = 2\pi r$$

the domain of the variable r, representing the radius of the circle, is the set of positive real numbers. The domain of the variable C, representing the circumference of the circle, is also the set of positive real numbers.

5 Use the Laws of Exponents

If a is a real number and n is a positive integer, then the symbol $\boldsymbol{a^n}$ represents the product of n factors of a. That is,

$$a^n = \underbrace{a \cdot a \cdot \ldots \cdot a}_{n \text{ factors}} \qquad \mathbf{(1)}$$

Here it is understood that $a^1 = a$.

$$a^0 = 1 \quad \text{if } a \neq 0$$

$$a^{-n} = \frac{1}{a^n} \quad \text{if } a \neq 0$$

EXAMPLE

Evaluating Expressions Containing Negative Exponents

(a) $2^{-3} = \dfrac{1}{2^3} = \dfrac{1}{8}$

(b) $x^{-4} = \dfrac{1}{x^4}$

(c) $\left(\dfrac{1}{5}\right)^{-2} = \dfrac{1}{\left(\dfrac{1}{5}\right)^2} = \dfrac{1}{\dfrac{1}{25}} = 25$

Laws of Exponents

$$a^m a^n = a^{m+n} \qquad (a^m)^n = a^{mn} \qquad (ab)^n = a^n b^n$$

$$\frac{a^m}{a^n} = a^{m-n} = \frac{1}{a^{n-m}}, \quad \text{if } a \neq 0 \qquad \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}, \quad \text{if } b \neq 0$$

EXAMPLE **Using the Laws of Exponents**

(a) $x^{-3} \cdot x^5 = x^{-3+5} = x^2 \quad x \neq 0$

(b) $(x^{-3})^2 = x^{-3 \cdot 2} = x^{-6} = \dfrac{1}{x^6} \quad x \neq 0$

(c) $(2x)^3 = 2^3 \cdot x^3 = 8x^3$

(d) $\left(\dfrac{2}{3}\right)^4 = \dfrac{2^4}{3^4} = \dfrac{16}{81}$

(e) $\dfrac{x^{-2}}{x^{-5}}$

$= x^{-2-(-5)} = x^3 \quad x \neq 0$

6 Evaluate Square Roots

If a is a nonnegative real number, the nonnegative number b, such that $b^2 = a$ is the **principal square root** of a, is denoted by $b = \sqrt{a}$.

1. Negative numbers do not have square roots (in the real number system), because the square of any real number is *nonnegative*. For example, $\sqrt{-4}$ is not a real number, because there is no real number whose square is -4.

2. The principal square root of 0 is 0, since $0^2 = 0$. That is, $\sqrt{0} = 0$.

3. The principal square root of a positive number is positive.

4. If $c \geq 0$, then $\left(\sqrt{c}\right)^2 = c$. For example, $\left(\sqrt{2}\right)^2 = 2$ and $\left(\sqrt{3}\right)^2 = 3$.

EXAMPLE **Evaluating Square Roots**

(a) $\sqrt{64} = 8$

(b) $\sqrt{\dfrac{1}{16}} = \dfrac{1}{4}$

(c) $\left(\sqrt{1.4}\right)^2 = 1.4$

$$\sqrt{a^2} = |a| \qquad a \text{ any real number} \tag{2}$$

EXAMPLE Using Equation (2)

(a) $\sqrt{(2.3)^2} = |2.3| = 2.3$

(b) $\sqrt{(-2.3)^2} = |-2.3| = 2.3$

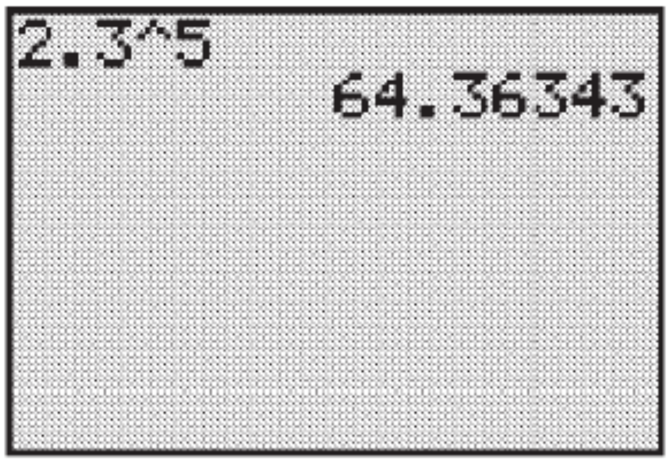
(c) $\sqrt{x^2} = |x|$

7 Use a Calculator to Evaluate Exponents

EXAMPLE

Exponents on a Graphing Calculator

Evaluate: $(2.3)^5$



8 Use Scientific Notation

When a number has been written as the product of a number x, where $1 \leq x < 10$, times a power of 10, it is said to be written in **scientific notation**.

Converting a Decimal to Scientific Notation

To change a positive number into scientific notation:

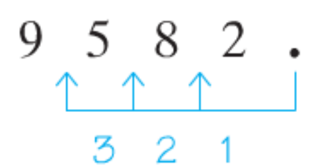
1. Count the number N of places that the decimal point must be moved to arrive at a number x, where $1 \leq x < 10$.
2. If the original number is greater than or equal to 1, the scientific notation is $x \times 10^N$. If the original number is between 0 and 1, the scientific notation is $x \times 10^{-N}$.

EXAMPLE Using Scientific Notation

Write each number in scientific notation.

(a) 9582 (b) 1.245 (c) 0.285 (d) 0.000561

(a) The decimal point in 9582 follows the 2. Count left from the decimal point



stopping after three moves, because 9.582 is a number between 1 and 10. Since 9582 is greater than 1, we write

$$9582 = 9.582 \times 10^3$$

(b) The decimal point in 1.245 is between the 1 and 2. Since the number is already between 1 and 10, the scientific notation for it is $1.245 \times 10^0 = 1.245$.

Using Scientific Notation

Write each number in scientific notation.

(a) 9582 (b) 1.245 (c) 0.285 (d) 0.000561

(c) The decimal point in 0.285 is between the 0 and the 2. We count

$$0\,.\,2\;8\;5$$

$$1$$

stopping after one move, because 2.85 is a number between 1 and 10. Since 0.285 is between 0 and 1, we write

$$0.285 = 2.85 \times 10^{-1}$$

(d) The decimal point in 0.000561 is moved as follows:

$$0\,.\,0\;0\;0\;5\;6\;1$$

$$1\quad 2\quad 3\quad 4$$

As a result,

$$0.000561 = 5.61 \times 10^{-4}$$

EXAMPLE Changing from Scientific Notation to Decimals

Write each number as a decimal.

(a) 2.1×10^{4} (b) 3.26×10^{-5} (c) 1×10^{-2}

(a) $2.1 \times 10^{4} = 2\ .\ 1\ 0\ 0\ 0 \times 10^{4} = 21{,}000$

1 2 3 4

(b) $3.26 \times 10^{-5} = 0\ 0\ 0\ 0\ 0\ 3\ .\ 2\ 6 \times 10^{-5} = 0.0000326$

5 4 3 2 1

(c) $1 \times 10^{-2} = 0\ 0\ 1\ .\ 0 \times 10^{-2} = 0.01$

2 1

EXAMPLE

Using Scientific Notation

(a) The diameter of the smallest living cell is only about 0.00001 centimeter (cm).* Express this number in scientific notation.

(b) The surface area of Earth is about 1.97×10^8 square miles.† Express the surface area as a whole number.

(a) $0.00001 \text{ cm} = 1 \times 10^{-5} \text{ cm}$ because the decimal point is moved five places and the number is less than 1.

(b) 1.97×10^8 square miles $= 197{,}000{,}000$ square miles.

COMMENT *On a calculator, a number such as 3.615×10^{12} is usually displayed as* $\boxed{3.615\text{E}12.}$