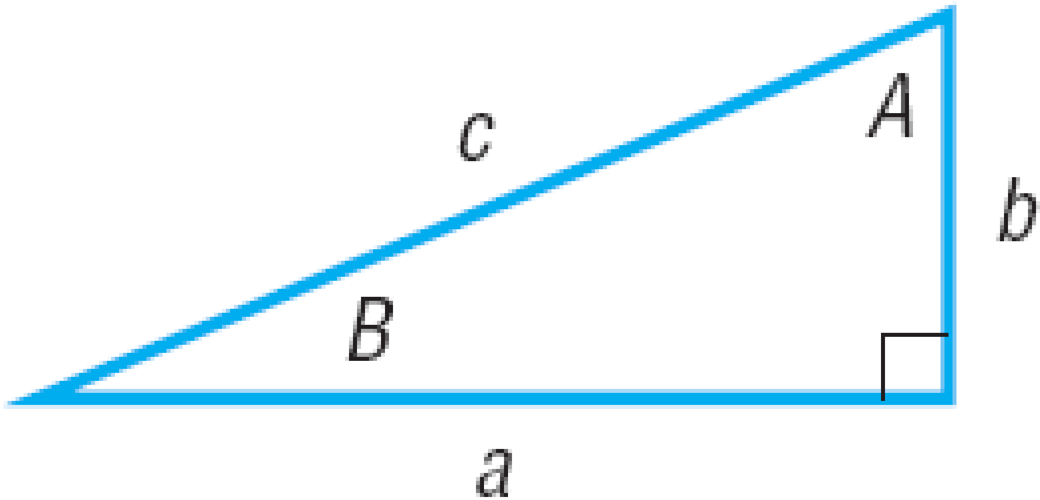
Section 9.1

Applications Involving Right Triangles

1 Solve Right Triangles

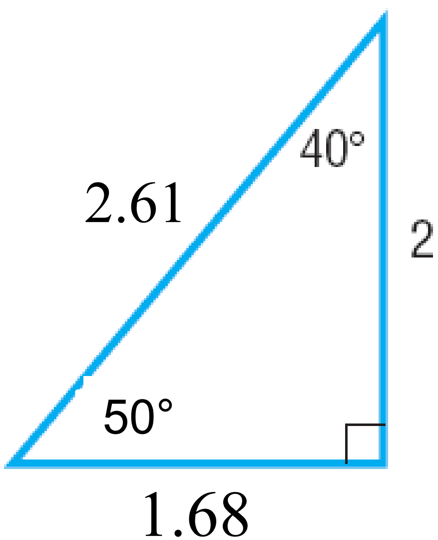


$$c^2 = a^2 + b^2 \qquad A + B = 90°$$

EXAMPLE **Solving a Right Triangle**

Use Figure 2. If $b = 2$ and $A = 40°$, find a, c, and B.

$$40 + B = 90° \text{ so } B = 50°$$



$$\tan 40^0 = \frac{a}{2} \text{ and } \cos 40^0 = \frac{2}{c}$$

$$a = 2\tan 40^0 \approx 1.68$$

$$c = \frac{2}{\cos 40^0} \approx 2.61$$

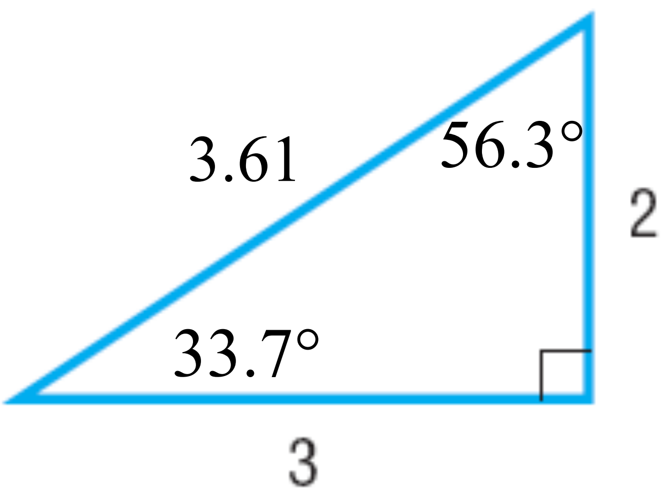
$$c^2 = a^2 + b^2 \qquad A + B = 90°$$

EXAMPLE **Solving a Right Triangle**

Use Figure 3. If $a = 3$ and $b = 2$, find c, A, and B.

$$c = \sqrt{2^2 + 3^2} = \sqrt{13} \approx 3.61$$

$$\tan A = \frac{3}{2} \text{ so } A = \tan^{-1}\frac{3}{2} \approx 56.3^\circ$$



$$56.3^\circ + B = 90^\circ \text{ so } B = 33.7^\circ$$

$$c^2 = a^2 + b^2 \qquad A + B = 90^\circ$$

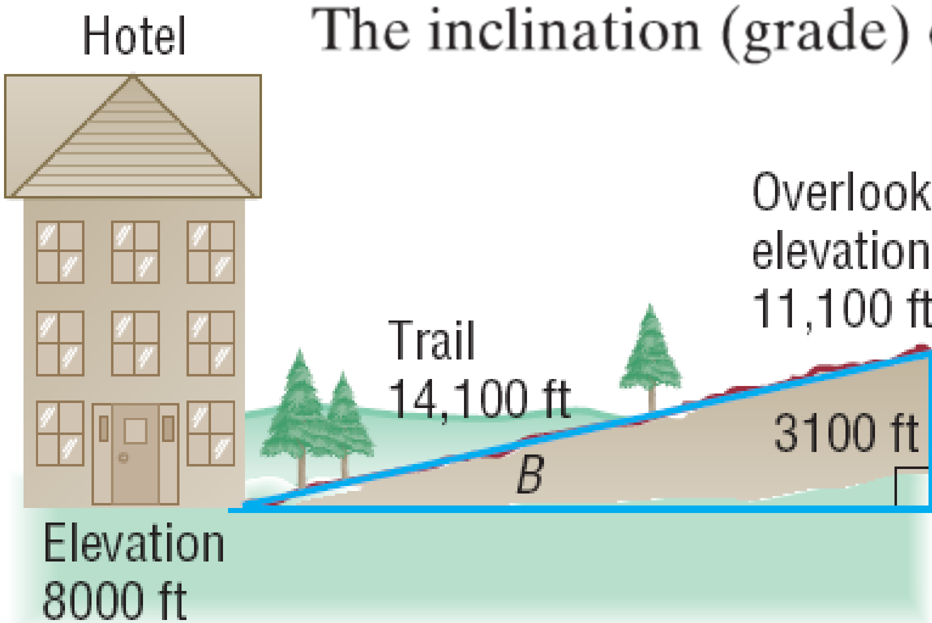
2 Solve Applied Problems

EXAMPLE **Finding the Inclination of a Mountain Trail**

A straight trail leads from the Alpine Hotel, elevation 8000 feet, to a scenic overlook, elevation 11,100 feet. The length of the trail is 14,100 feet. What is the inclination (grade) of the trail? That is, what is the angle B in Figure 4?

$$\sin B = \frac{3100}{14100} \qquad B = \sin^{-1}\frac{3100}{14100} \approx 12.7°$$

The inclination (grade) of the trail is approximately 12.7°.



EXAMPLE **The Gibb's Hill Lighthouse, Southampton, Bermuda**

In operation since 1846, the Gibb's Hill Lighthouse stands 117 feet high on a hill 245 feet high, so its beam of light is 362 feet above sea level. A brochure states that the light can be seen on the horizon about 26 miles distant. Verify the accuracy of this statement.

$$\cos\theta = \frac{3960}{3960 + \frac{362}{5280}}$$

$$\theta = \cos^{-1} 0.999982687$$
$$\approx 0.33715° \approx 20.23'$$

In either case it looks like the brochure overstated the distance.

The distance s in statute miles is given by the formula $s = r\theta$, where θ is measured in radians. Then, since

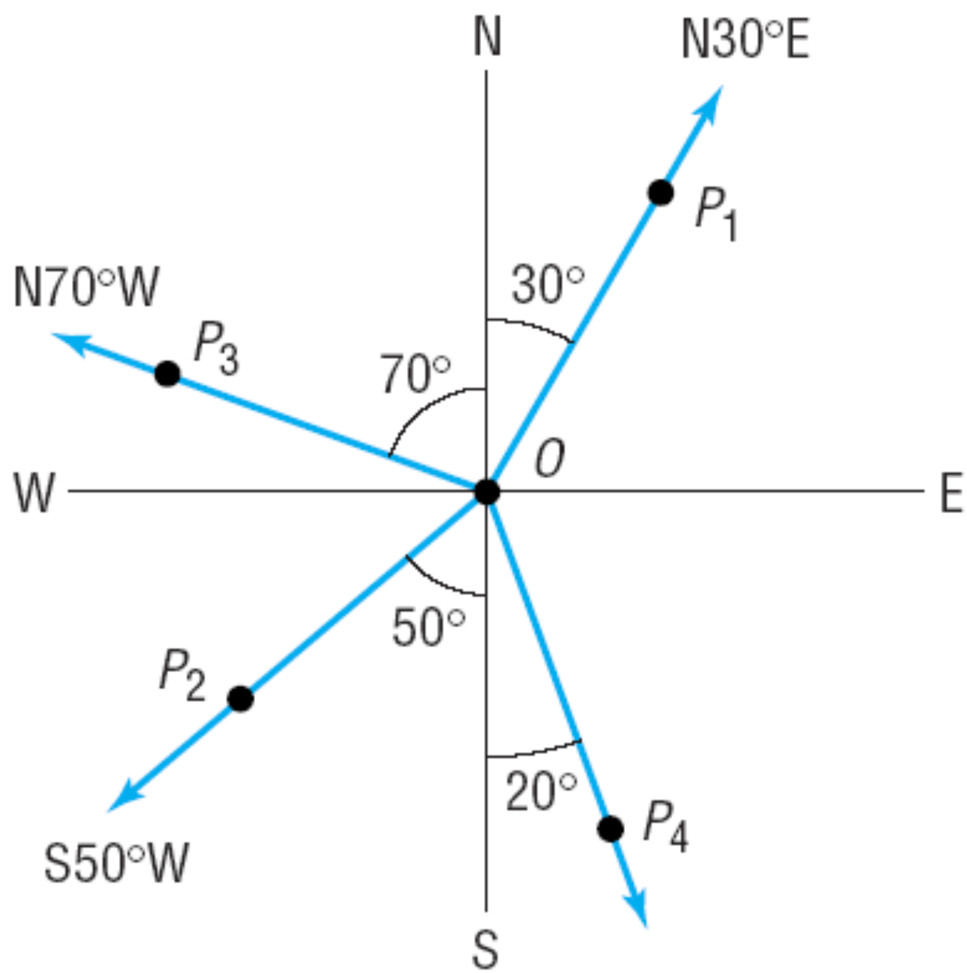
$$\theta \approx 20.23' \approx 0.33715° \approx 0.00588 \text{ radian}$$

$$s = r\theta \approx (3960)(0.00588) \approx 23.3 \text{ miles}$$

The brochure does not indicate whether the distance is measured in nautical miles or statute miles. Let's calculate both distances.

The distance s in nautical miles (refer to Problem 114, p. 516) is the measure of the angle θ in minutes, so $s \approx 20.23$ nautical miles.

In navigation and surveying, the **direction** or **bearing** from a point O to a point P equals the acute angle θ between the ray OP and the vertical line through O, the north–south line.

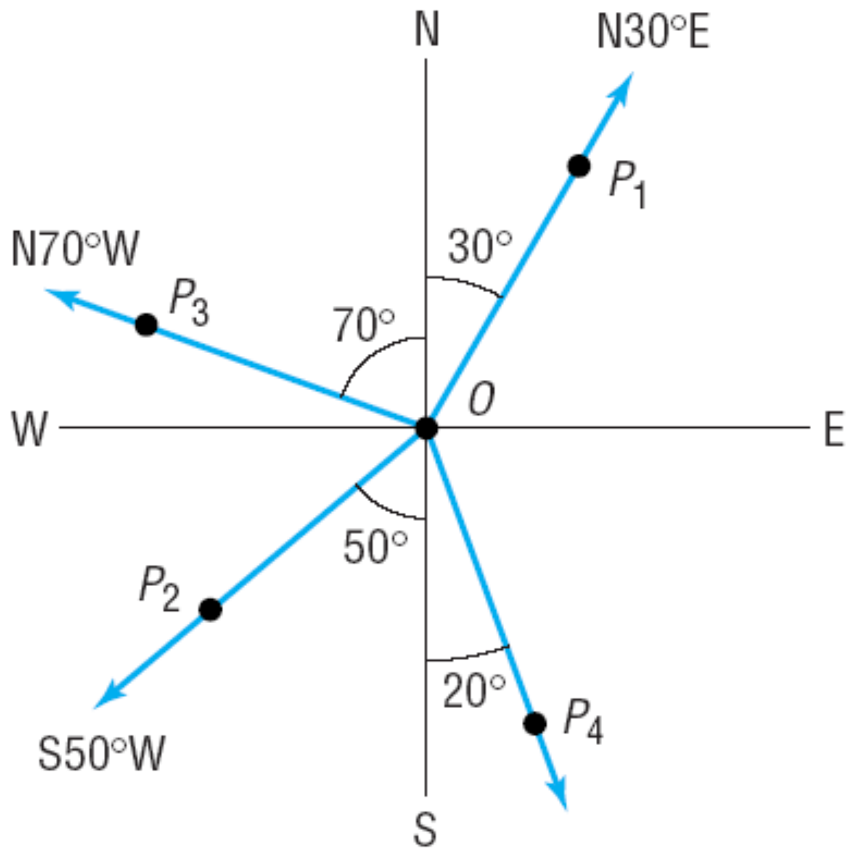


EXAMPLE

Finding the Bearing of an Object

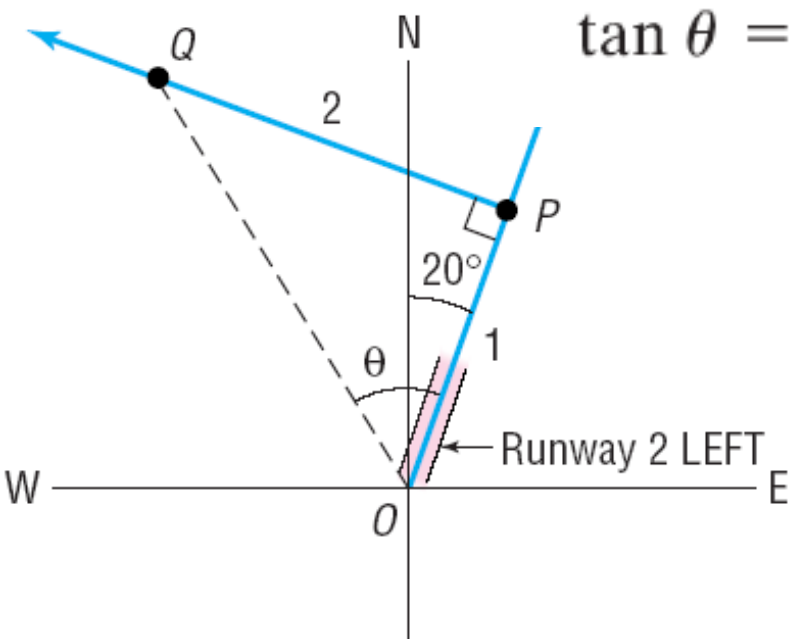
In Figure 6, what is the bearing from O to an object at P_4?

The acute angle between the ray OP_4 and the north–south line through O is 20°. The bearing from O to P_4 is S20°E.



EXAMPLE Finding the Bearing of an Airplane

A Boeing 777 aircraft takes off from O'Hare Airport on runway 2 LEFT, which has a bearing of N20°E.* After flying for 1 mile, the pilot of the aircraft requests permission to turn 90° and head toward the northwest. The request is granted. After the plane goes 2 miles in this direction, what bearing should the control tower use to locate the aircraft?



$$\tan\theta = \frac{2}{1} = 2 \quad \text{so} \quad \theta = \tan^{-1} 2 \approx 63.4^\circ$$

The acute angle between north and the ray OQ is $63.4^\circ - 20^\circ = 43.4^\circ$. The bearing of the aircraft from O to Q is N43.4°W.