# Section 1.4 Radical Equations **Equations Quadratic in Form Absolute Value Equations Factorable Equations**

# 1 Solve Radical Equations

# **Solving a Radical Equation**

Find the real solutions of the equation  $\sqrt[3]{3x-1} - 2 = 0$ .

$$\sqrt[3]{3x-1} = 2$$

$$\left(\sqrt[3]{3x-1}\right)^3 = 2^3$$

$$3x - 1 = 8$$

$$3x = 9$$

$$x = 3$$

The solution set is  $\{3\}$ .

# Solving a Radical Equation

Find the real solutions of the equation  $\sqrt{x+1} = -x+1$ .

$$\left(\sqrt{x+1}\right)^2 = \left(-x+1\right)^2$$

$$x + 1 = x^2 - 2x + 1$$

$$x^2 - 3x = 0$$

$$x(x-3) = 0$$

$$x = 0 \text{ or } x \neq 3$$



**Check:** 
$$\sqrt{0+1} = -0+1$$
  $1=1$ 

extraneous solution (does not work in the original equation)

$$\sqrt{3+1} = -3+1$$
$$2 \neq -2$$

The solution set is  $\{0\}$ .

# Solving a Radical Equation

Find the real solutions of the equation  $\sqrt{5}x - 1 - \sqrt{x} = 1$ .

$$\sqrt{5x-1} = \sqrt{x} + 1$$

$$\left(\sqrt{5x-1}\right)^2 = \left(\sqrt{x} + 1\right)^2$$

$$5x-1 = x + 2\sqrt{x} + 1$$

$$4x-2 = 2\sqrt{x}$$

$$4x - 2 = 2\sqrt{x}$$

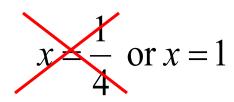
$$2x - 1 = \sqrt{x}$$

$$\left(2x-1\right)^2 = \left(\sqrt{x}\right)^2$$

$$4x^2 - 4x + 1 = x$$

$$4x^2 - 5x + 1 = 0$$

$$(4x-1)(x-1)=0$$





The solution set is  $\{1\}$ .

$$\sqrt{5\left(\frac{1}{4}\right) - 1} - \sqrt{\frac{1}{4}} = 1$$

$$\sqrt{5(1) - 1} - \sqrt{1} = 1$$

$$2 - 1 = 1$$

$$\sqrt{\frac{1}{4}} - \sqrt{\frac{1}{4}} \neq 1$$

# 2 Solve Equations Quadratic in Form

# **EXAMPLE** Solving an Equation Quadratic in Form

Find the real solutions of the equation

$$(x-2)^2-3(x-2)+2=0.$$

Let 
$$u = x - 2$$
. Then  $u^2 = (x - 2)^2$ 

$$u^2 - 3u + 2 = 0$$

$$(u-2)(u-1)=0$$

$$u = 2 \text{ or } u = 1$$

$$x-2=2 \text{ or } x-2=1$$

$$x = 4 \text{ or } x = 3$$

The solution set is  $\{3,4\}$ .

# **EXAMPLE** Solving an Equation Quadratic in Form

Find the real solutions of the equation  $x - 5\sqrt{x} + 6 = 0$ .

Let 
$$u = \sqrt{x}$$
. Then  $u^2 = x$ 

$$u^2 - 5u + 6 = 0$$

$$(u-3)(u-2) = 0$$

$$u = 3$$
 or  $u = 2$ 

$$\sqrt{x} = 3 \text{ or } \sqrt{x} = 2$$

$$x = 9 \text{ or } x = 4$$

The solution set is  $\{4,9\}$ .

# **3** Solve Equations by Factoring

# Solving an Equation by Factoring

Solve the equation:  $x^3 - 2x = x^2$ 

$$x^{3}-x^{2}-2x=0$$

$$x(x^{2}-x-2)=0$$

$$x(x-2)(x+1)=0$$

$$x = 0$$
 or  $x = 2$  or  $x = -1$ 

The solution set is  $\{-1,0,2\}$ .

# Solving an Equation by Factoring

Solve the equation: 
$$x^3 - 3x^2 - 4x + 12 = 0$$
  
 $x^2(x-3) - 4(x-3) = 0$ 

$$(x^2-4)(x-3)=0$$

$$(x+2)(x-2)(x-3) = 0$$

$$x = -2$$
 or  $x = 2$  or  $x = 3$ 

The solution set is  $\{-2,2,3\}$ .