Section R.6

Synthetic Division

1 Divide Polynomials Using Synthetic Division

$$
\begin{array}{rrrrr}
 & 2x^2 & +\ 5x & +\ 15 & \\
\cline{2-5}
x-3\,\big) & 2x^3 & -\ x^2 & & +\ 3 \\
 & 2x^3 & -\ 6x^2 & & \\
\cline{2-3}
 & & 5x^2 & & \\
 & & 5x^2 & -\ 15x & \\
\cline{3-4}
 & & & 15x & +\ 3 \\
 & & & 15x & -\ 45 \\
\cline{4-5}
 & & & & 48
\end{array}
\qquad\qquad
\begin{array}{rrrrr}
 & 2x^2 & +\ 5x & +\ 15 & \\
\cline{2-5}
x-3\,\big) & 2 & -\ 1 & 0 & 3 \\
 & & -\ 6 & & \\
\cline{2-3}
 & & 5 & & \\
 & & & -\ 15 & \\
\cline{3-5}
 & & & 15 & \\
 & & & & -\ 45 \\
\cline{4-5}
 & & & & 48
\end{array}
$$

$$
\begin{array}{r|rrrr}
 & 2x^2 & +\,5x & +\,15 & \\
\hline
x-3 & 2 & -\,1 & 0 & 3 \\
 & & -\,6 & & \\
\hline
 & & 5 & & \\
 & & & -\,15 & \\
\hline
 & & & 15 & \\
 & & & & -\,45 \\
\hline
 & & & & 48
\end{array}
$$

$$
\begin{array}{r|rrrrl}
 & 2x^2 & +\,5x & +\,15 & & \text{Row 1} \\
\hline
x-3 & 2 & -\,1 & 0 & 3 & \text{Row 2} \\
 & & -\,6 & -\,15 & -\,45 & \text{Row 3} \\
\hline
 & 0 & 5 & 15 & 48 & \text{Row 4}
\end{array}
$$

$$
\begin{array}{r|rrrrl}
x-3 & 2 & -\,1 & 0 & 3 & \text{Row 1} \\
 & & -\,6 & -\,15 & -\,45 & \text{Row 2 (subtract)} \\
\hline
 & 2 & 5 & 15 & 48 & \text{Row 3}
\end{array}
$$

$$
\begin{array}{r|rrrrl}
x-3 & 2 & -\,1 & 0 & 3 & \text{Row 1} \\
 & & 6 & 15 & 45 & \text{Row 2 (add)} \\
\hline
 & 2 & 5 & 15 & 48 & \text{Row 3}
\end{array}
$$

$$
\begin{array}{r|rrrrl}
3 & 2 & -1 & 0 & 3 & \text{Row 1} \\
 & & 6 & 15 & 45 & \text{Row 2 (add)} \\
\hline
 & 2 & 5 & 15 & 48 & \text{Row 3}
\end{array}
$$

$\underbrace{2 \quad 5 \quad 15}_{\text{Quotient}}$ $\quad$ 48 ← Remainder

$\overbrace{2x^2 + 5x + 15}$ $\quad$ 48 ← Remainder

EXAMPLE

Using Synthetic Division to Find the Quotient and Remainder

Use synthetic division to find the quotient and remainder when

$$x^3 - 4x^2 - 5 \quad \text{is divided by} \quad x - 3$$

Step 1: Write the dividend in descending powers of x. Then copy the coefficients, remembering to insert a 0 for any missing powers of x.

$$1 \quad -4 \quad 0 \quad -5 \qquad \text{Row 1}$$

Step 2: Insert the usual division symbol. In synthetic division, the divisor is of the form $x - c$, and c is the number placed to the left of the division symbol. Here, since the divisor is $x - 3$, we insert 3 to the left of the division symbol.

$$3\overline{)1 \quad -4 \quad 0 \quad -5} \qquad \text{Row 1}$$

Step 3: Bring the 1 down two rows, and enter it in row 3.

$$\begin{array}{r|rrrr l} 3 & 1 & -4 & 0 & -5 & \text{Row 1} \\ & \downarrow & & & & \text{Row 2} \\ \hline & 1 & & & & \text{Row 3} \end{array}$$

Step 4: Multiply the latest entry in row 3 by 3, and place the result in row 2, one column over to the right.

$$
\begin{array}{r|rrrrl}
3 & 1 & -4 & 0 & -5 & \text{Row 1} \\
 & & 3 & & & \text{Row 2} \\
\cline{2-5}
 & 1 & & & & \text{Row 3}
\end{array}
$$

Step 5: Add the entry in row 2 to the entry above it in row 1, and enter the sum in row 3.

$$
\begin{array}{r|rrrrl}
3 & 1 & -4 & 0 & -5 & \text{Row 1} \\
 & & 3 & & & \text{Row 2} \\
\cline{2-5}
 & 1 & -1 & & & \text{Row 3}
\end{array}
$$

Step 6: Repeat Steps 4 and 5 until no more entries are available in row 1.

$$
\begin{array}{r|rrrrl}
3 & 1 & -4 & 0 & -5 & \text{Row 1} \\
 & & 3 & -3 & -9 & \text{Row 2} \\
\cline{2-5}
 & 1 & -1 & -3 & -14 & \text{Row 3}
\end{array}
$$

Step 7: The final entry in row 3, the -14, is the remainder; the other entries in row 3, the 1, -1, and -3, are the coefficients (in descending order) of a polynomial whose degree is 1 less than that of the dividend. This is the quotient. Thus,

$$\text{Quotient} = x^2 - x - 3 \qquad \text{Remainder} = -14$$

Use synthetic division to find the quotient and remainder when

$$x^3 - 4x^2 - 5 \quad \text{is divided by} \quad x - 3$$

$$\text{Quotient} = x^2 - x - 3 \qquad \text{Remainder} = -14$$

✓Check: (Divisor)(Quotient) + Remainder

$$= (x - 3)(x^2 - x - 3) + (-14)$$
$$= (x^3 - x^2 - 3x - 3x^2 + 3x + 9) + (-14)$$
$$= x^3 - 4x^2 - 5 = \text{Dividend}$$

EXAMPLE Using Synthetic Division to Verify a Factor

Use synthetic division to show that $x + 3$ is a factor of

$$2x^5 + 5x^4 - 2x^3 + 2x^2 - 2x + 3$$

The divisor is $x + 3 = x - (-3)$, so we place -3 to the left of the division symbol. Then the row 3 entries will be multiplied by -3, entered in row 2, and added to row 1.

$$\begin{array}{r|rrrrrrl} -3 & 2 & 5 & -2 & 2 & -2 & 3 & \text{Row 1} \\ & & -6 & 3 & -3 & 3 & -3 & \text{Row 2} \\ \hline & 2 & -1 & 1 & -1 & 1 & 0 & \text{Row 3} \end{array}$$

Because the remainder is 0, we have

$$\begin{aligned} &(\text{Divisor})(\text{Quotient}) + \text{Remainder} \\ &= (x + 3)(2x^4 - x^3 + x^2 - x + 1) = 2x^5 + 5x^4 - 2x^3 + 2x^2 - 2x + 3 \end{aligned}$$

As we see, $x + 3$ is a factor of $2x^5 + 5x^4 - 2x^3 + 2x^2 - 2x + 3$.