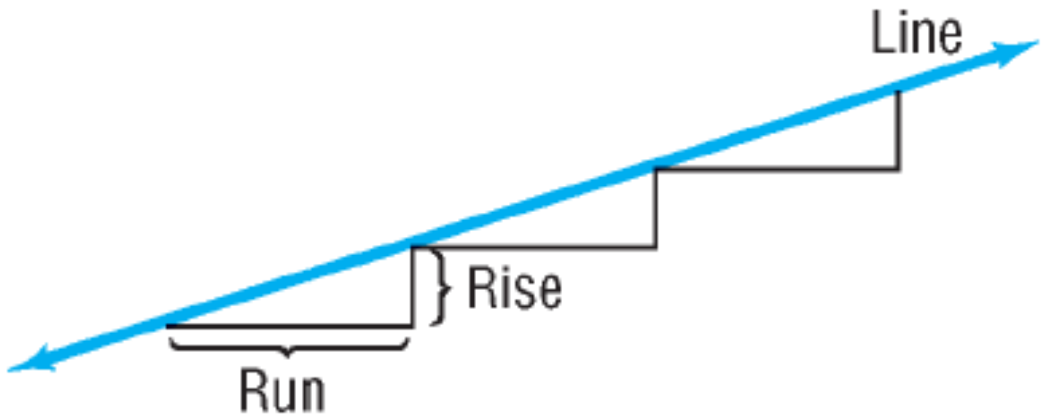
Section 2.3

Lines

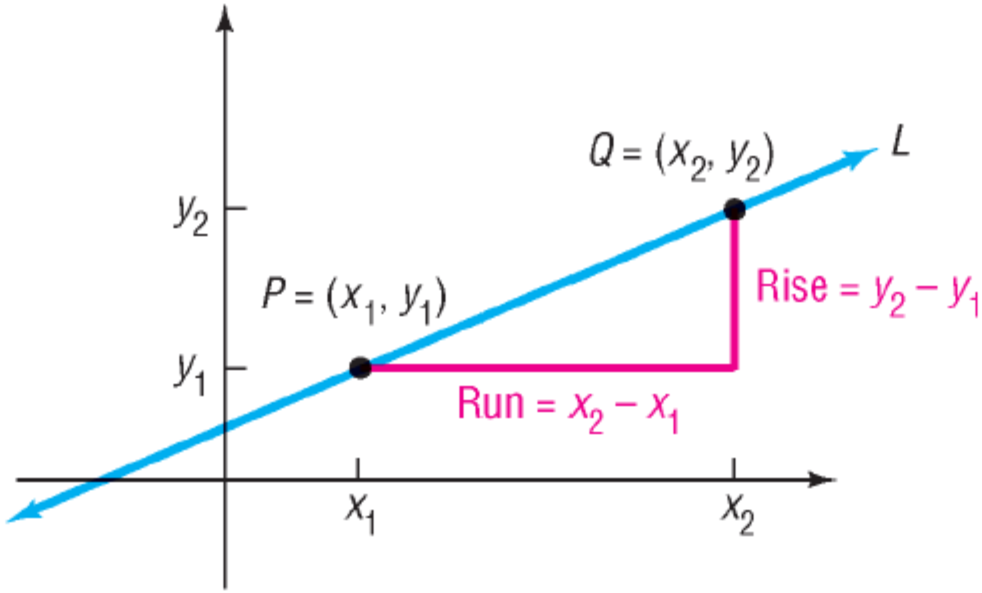
1 Calculate and Interpret the Slope of a Line



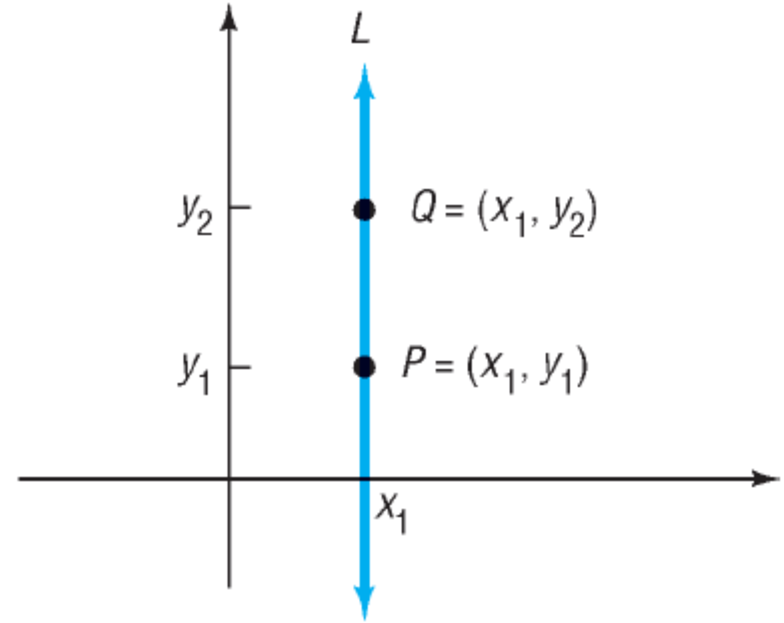
Let $P = (x_1, y_1)$ and $Q = (x_2, y_2)$ be two distinct points. If $x_1 \neq x_2$, the **slope** $\boldsymbol{m}$ of the nonvertical line L containing P and Q is defined by the formula

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad x_1 \neq x_2 \tag{1}$$

If $x_1 = x_2$, L is a **vertical line** and the slope m of L is **undefined** (since this results in division by 0).



(a) Slope of L is $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

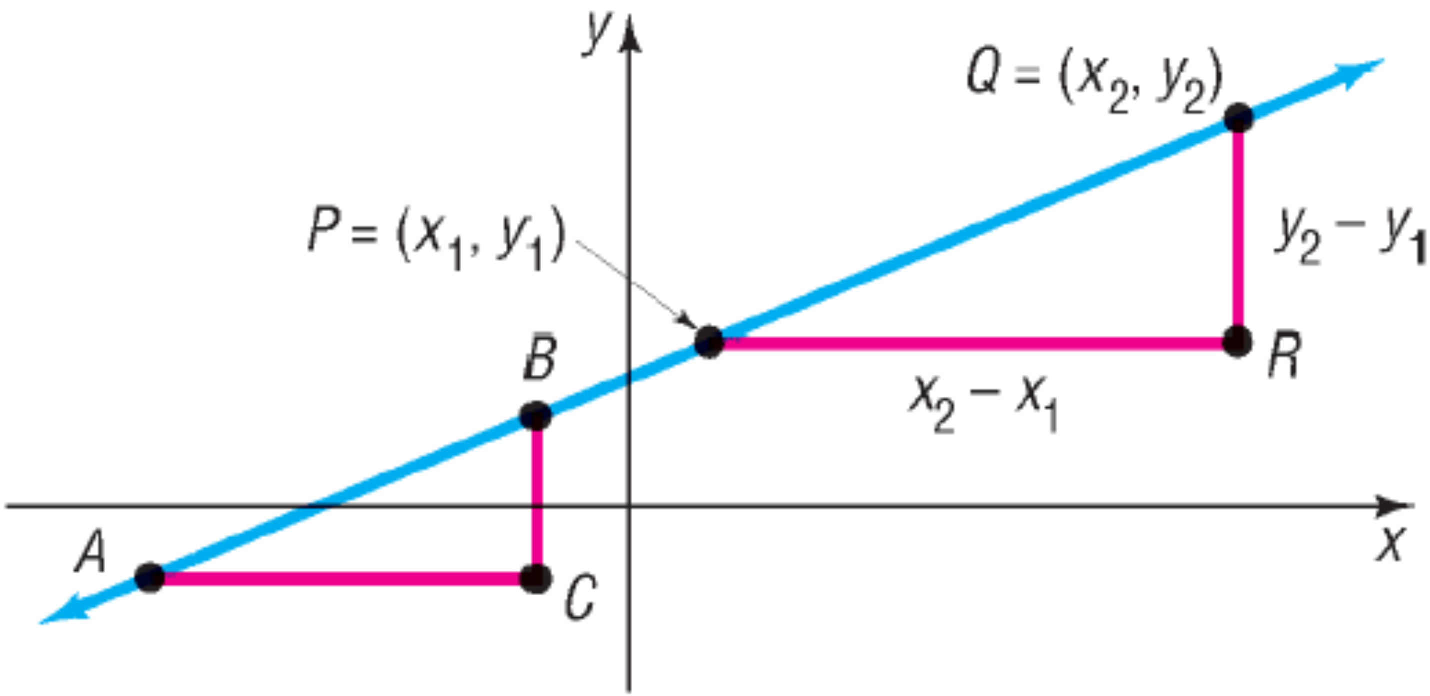


(b) Slope is undefined; L is vertical

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{Rise}}{\text{Run}}$$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{Change in } y}{\text{Change in } x} = \frac{\Delta y}{\Delta x}$$

Any two distinct points on the line can be used to compute the slope of the line.



Since any two distinct points can be used to compute the slope of a line, the average rate of change of a line is always the same number.

The slope of a line may be computed from

$P = (x_1, y_1)$ to $Q = (x_2, y_2)$ or from Q to P because

$$\boxed{\frac{y_2 - y_1}{x_2 - x_1} = \frac{y_1 - y_2}{x_1 - x_2}}$$

EXAMPLE

Finding and Interpreting the Slope of a Line Given Two Points

Find the slope of the line containing the points (-1, 4) and (2, -3).

$$\boxed{m = \frac{y_2 - y_1}{x_2 - x_1}}$$

$$\boxed{m = \frac{y_1 - y_2}{x_1 - x_2}}$$

$$m = \frac{-3-4}{2+1} = -\frac{7}{3}$$

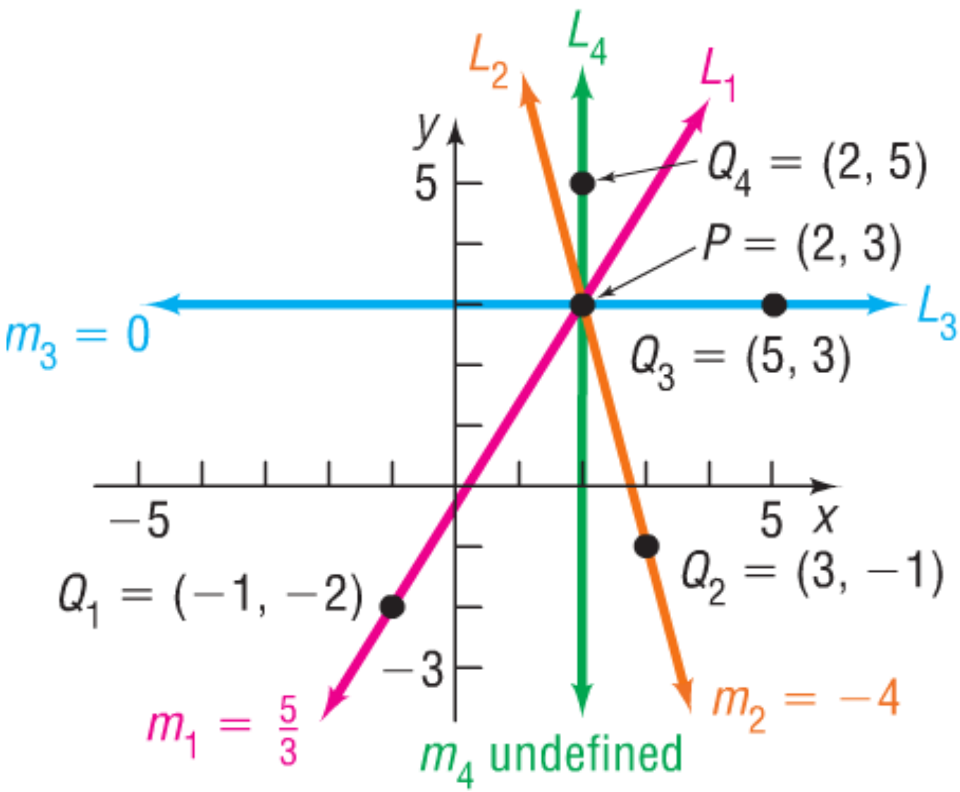
$$m = \frac{4+3}{-1-2} = -\frac{7}{3}$$

The average rate of change of y with respect to x is $-\frac{7}{3}$.

EXAMPLE

Finding the Slopes of Various Lines Containing the Same Point (2, 3)

Compute the slopes of the lines L_1, L_2, L_3, and L_4 containing the following pairs of points. Graph all four lines on the same set of coordinate axes.

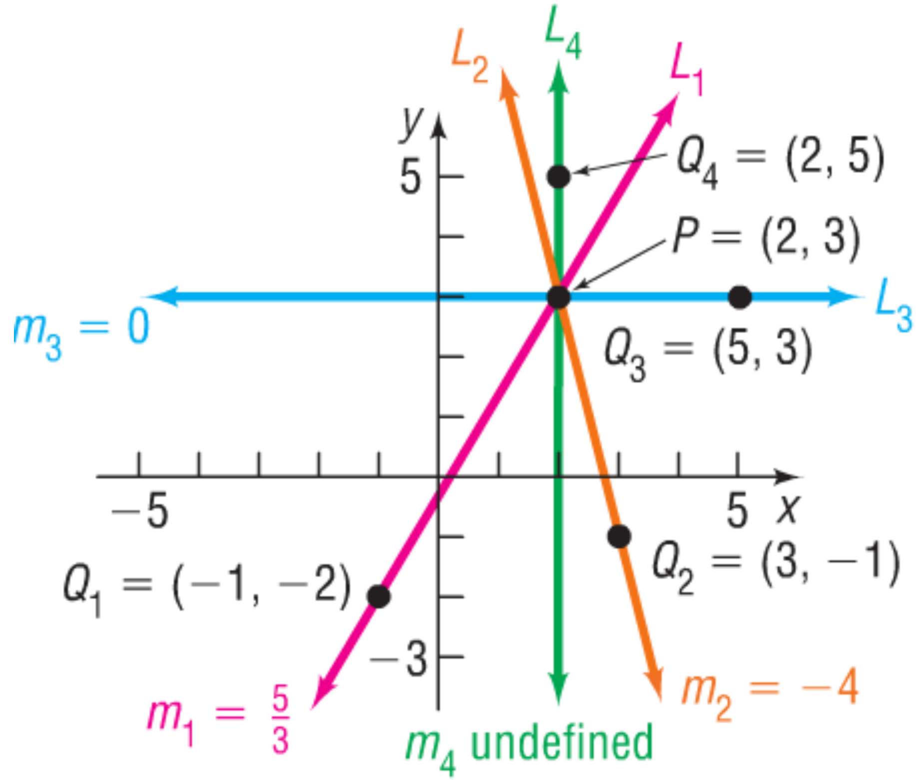


$$m_1 = \frac{-2 - 3}{-1 - 2} = \frac{-5}{-3} = \frac{5}{3}$$

$$m_2 = \frac{-1 - 3}{3 - 2} = \frac{-4}{1} = -4$$

$$m_3 = \frac{3 - 3}{5 - 2} = \frac{0}{3} = 0$$

m_4 is undefined



1. When the slope of a line is positive, the line slants upward from left to right (L_1).
2. When the slope of a line is negative, the line slants downward from left to right (L_2).
3. When the slope is 0, the line is horizontal (L_3).
4. When the slope is undefined, the line is vertical (L_4).

Seeing the Concept

On the same square screen, graph the following equations:

$$Y_1 = 0$$

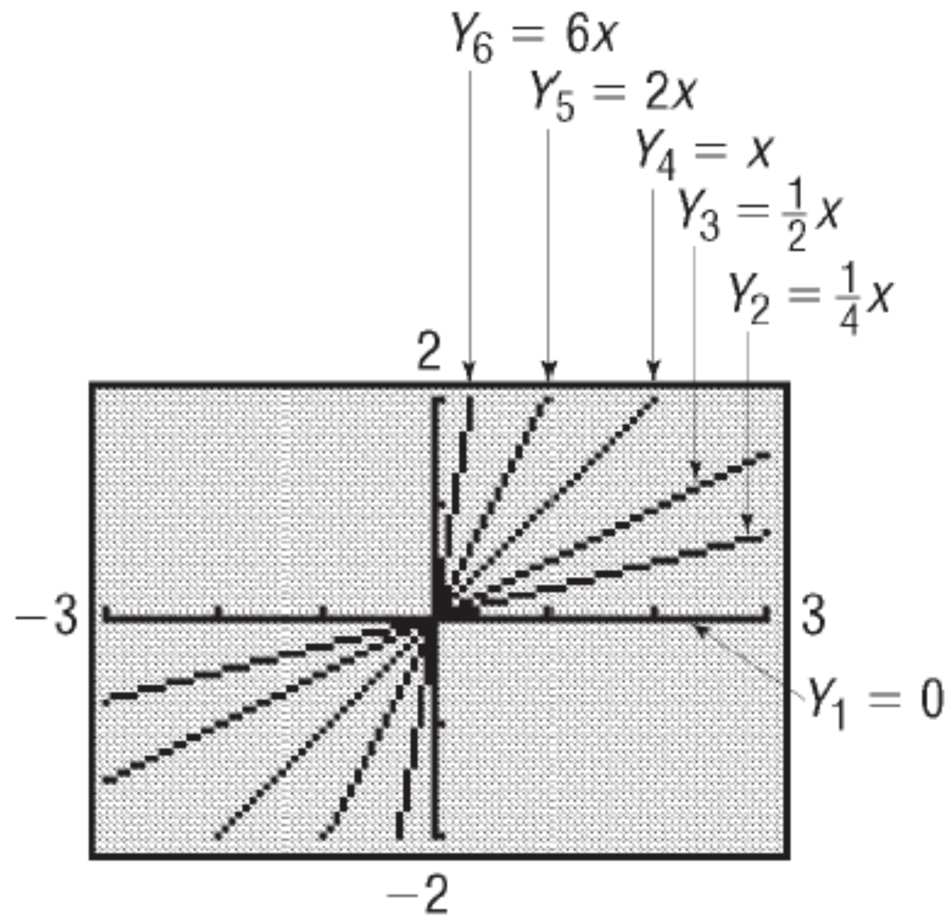
$$Y_2 = -\frac{1}{4}x$$

$$Y_3 = -\frac{1}{2}x$$

$$Y_4 = -x$$

$$Y_5 = -2x$$

$$Y_6 = -6x$$



Seeing the Concept

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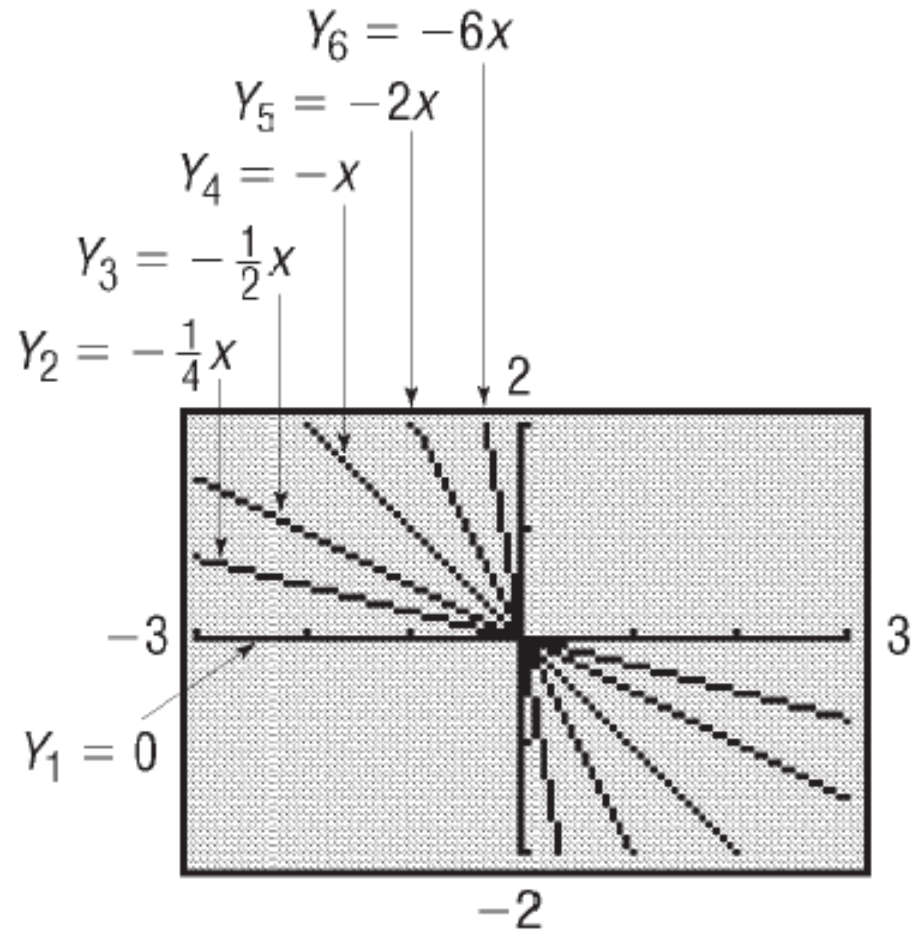


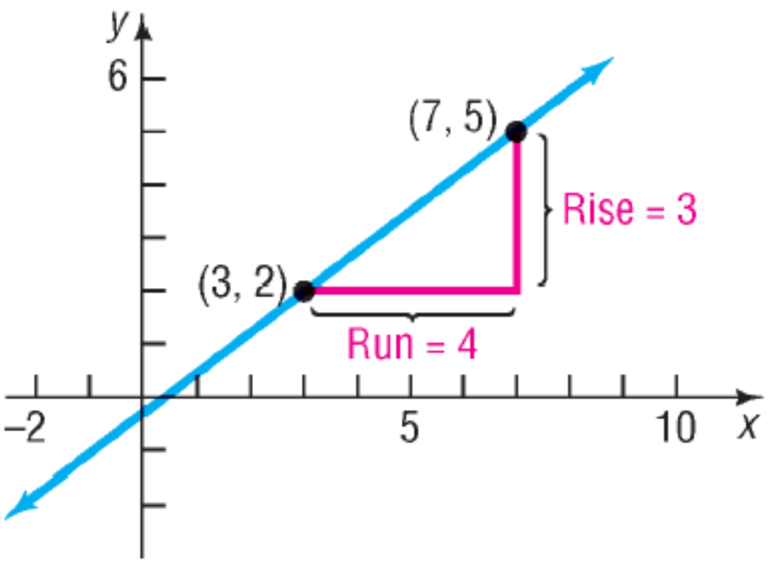
2 Graph Lines Given a Point and the Slope

EXAMPLE

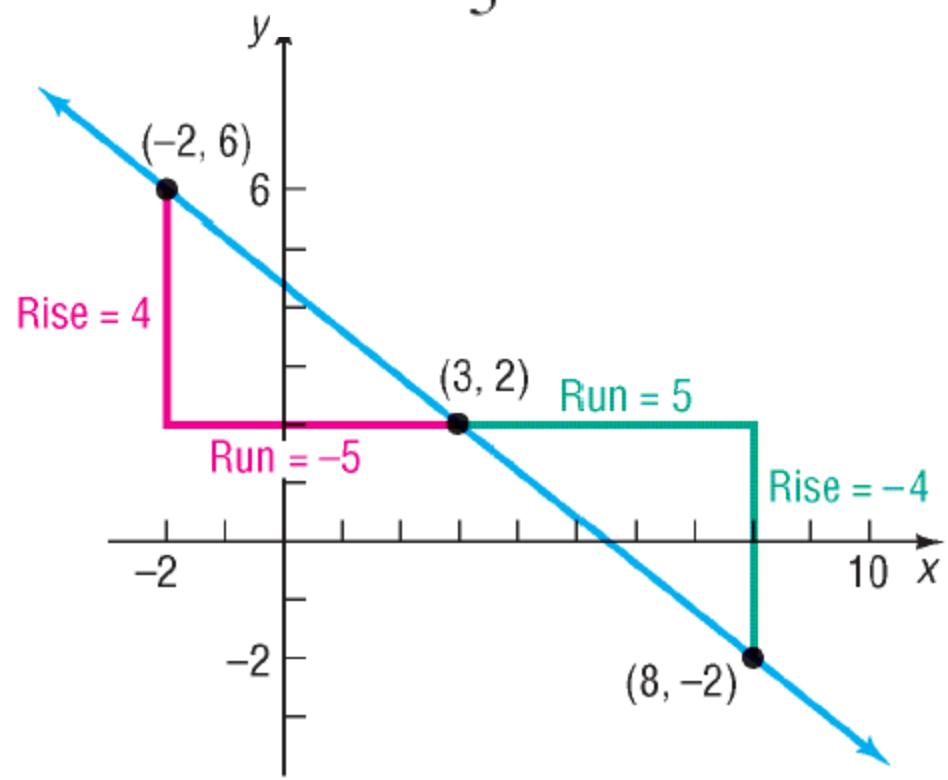
Graphing a Line Given a Point and a Slope

Draw a graph of the line that contains the point $(3, 2)$ and has a slope of:

(a) $\frac{3}{4}$



(b) $-\frac{4}{5}$

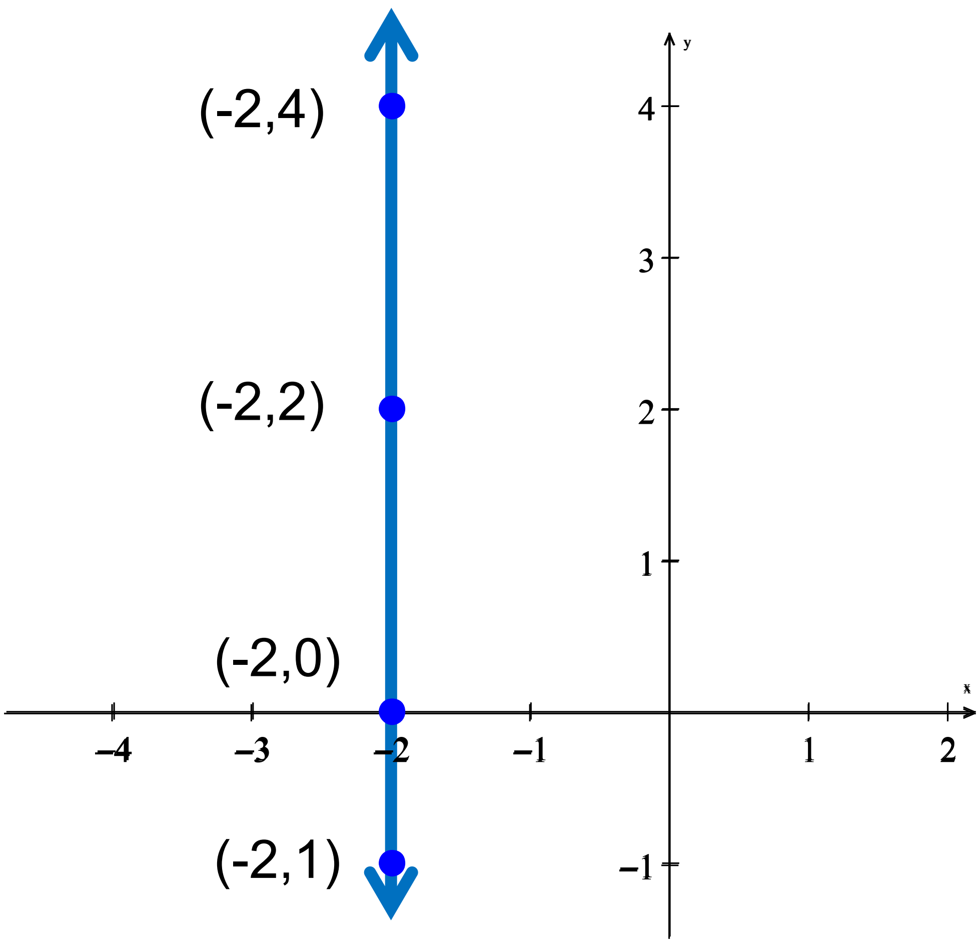


3 Find the Equation of a Vertical Line

EXAMPLE

Graphing a Line

Graph the equation: $x = -2$



Theorem

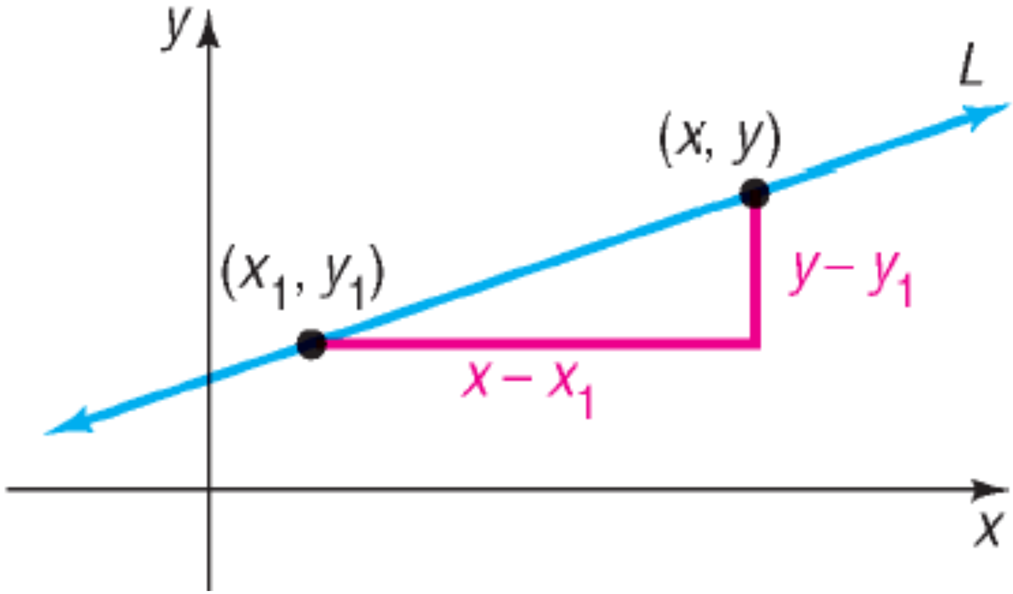
Equation of a Vertical Line

A vertical line is given by an equation of the form

$$\boxed{x = a}$$

where a is the x-intercept.

4 Use the Point–Slope Form of a Line; Identify Horizontal Lines



Theorem

Point–Slope Form of an Equation of a Line

An equation of a nonvertical line of slope m that contains the point (x_1, y_1) is

$$\boxed{y - y_1 = m(x - x_1)}$$

EXAMPLE Using the Point–Slope Form of a Line

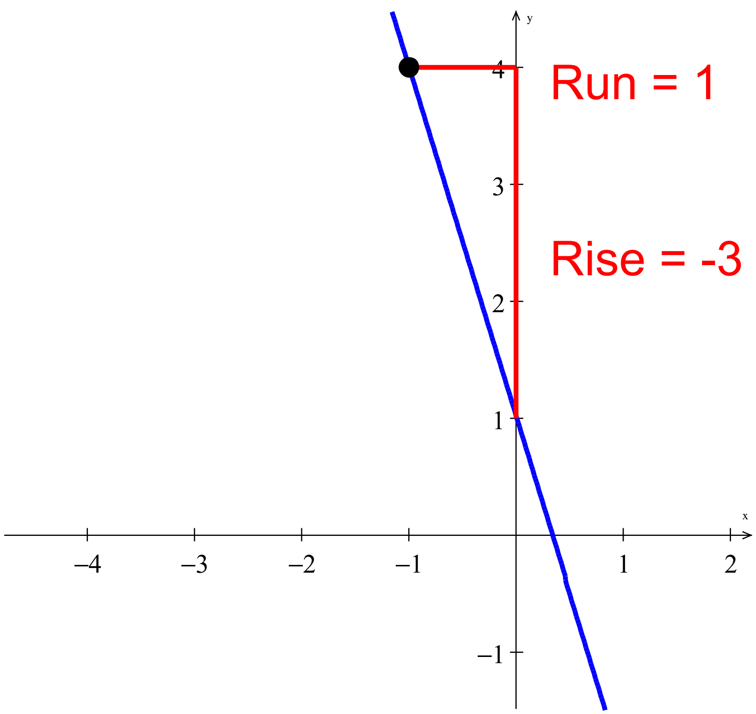
Find the equation of a line with slope –3 and containing the point (–1, 4).

$$y - y_1 = m(x - x_1)$$

$$y-4=-3\left(x-(-1)\right)$$

$$y-4=-3x-3$$

$$y=-3x+1$$



EXAMPLE

Finding the Equation of a Horizontal Line

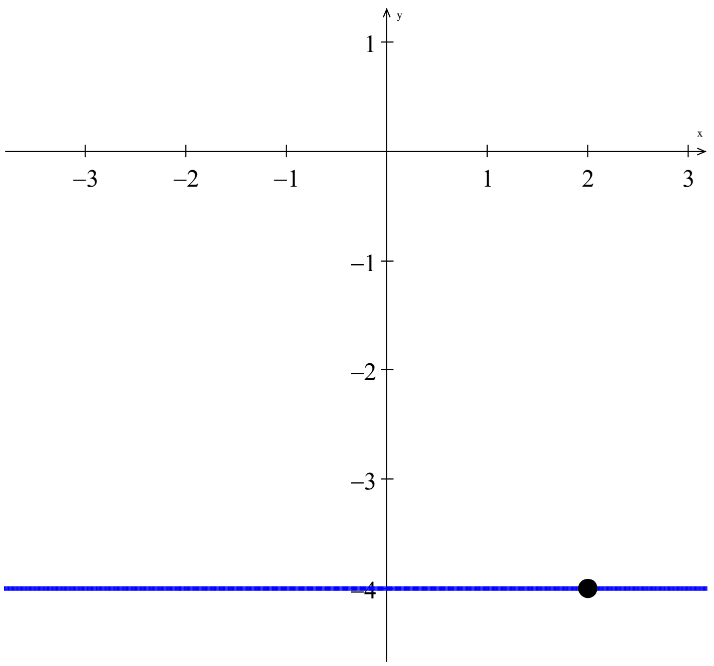
Find the equation of a horizontal line containing the point (2, – 4).

$$y - y_1 = m(x - x_1)$$

$$y - (-4) = 0 \cdot (x - 2)$$

$$y + 4 = 0$$

$$y = -4$$



Theorem

Equation of a Horizontal Line

A horizontal line is given by an equation of the form

$$\boxed{y = b}$$

where b is the y-intercept.

5 Find the Equation of a Line Given Two Points

EXAMPLE

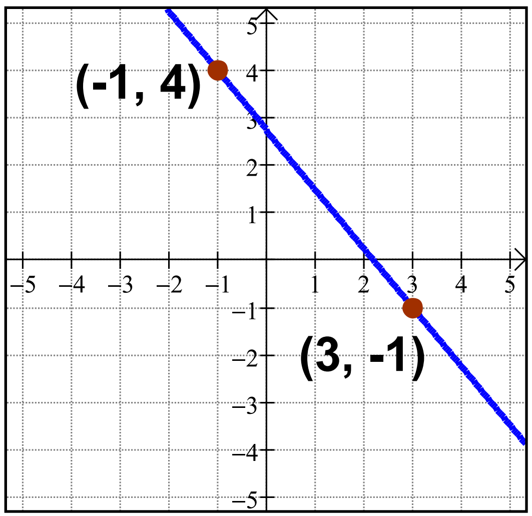
Finding an Equation of a Line Given Two Points

Find an equation of the line L containing the points (–1, 4) and (3, –1). Graph the line L.

$$m=\frac{-1-4}{3-(-1)}=-\frac{5}{4}$$

$$y-4=-\frac{5}{4}\left(x-(-1)\right)$$

$$y-4=-\frac{5}{4}(x+1)$$



6 Write the Equation of a Line in Slope–Intercept Form

Theorem

Slope–Intercept Form of an Equation of a Line

An equation of a line L with slope m and y-intercept b is

$$\boxed{y = mx + b}$$

Seeing the Concept

Graph the following lines on the same square screen

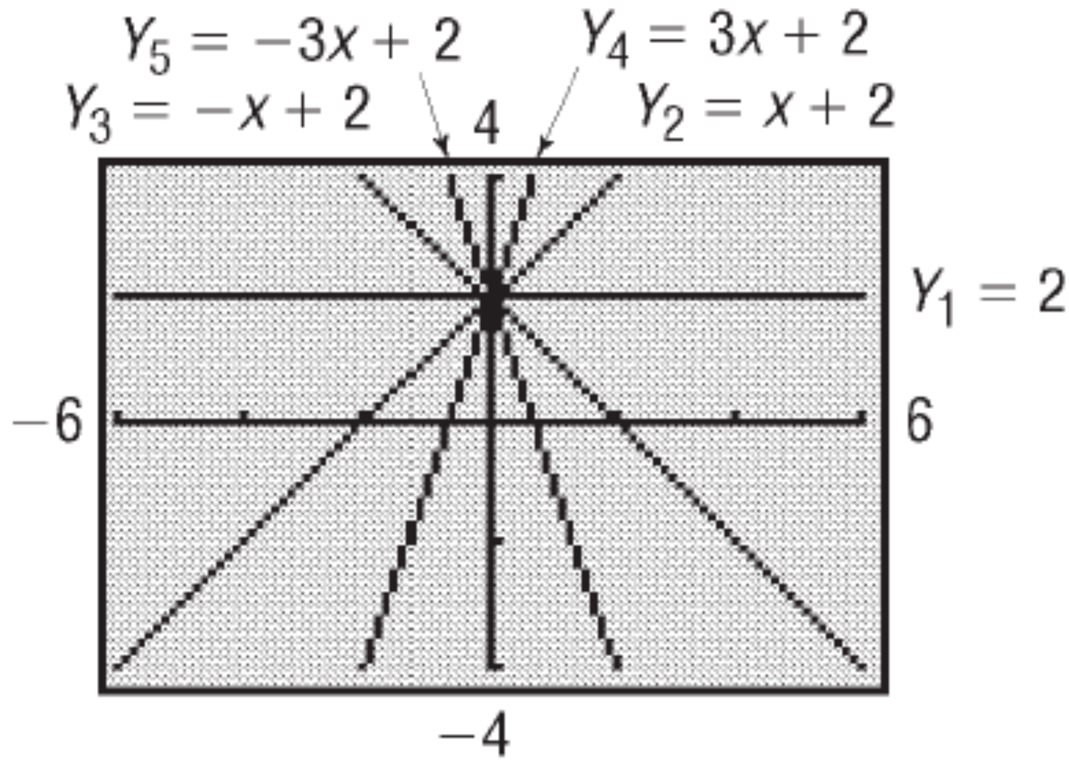
$Y_1 = 2$

$Y_2 = x + 2$

$Y_3 = -x + 2$

$Y_4 = 3x + 2$

$Y_5 = -3x + 2$



What do you conclude about the lines $y = mx + 2$?

Seeing the Concept

Graph the following lines on the same square screen

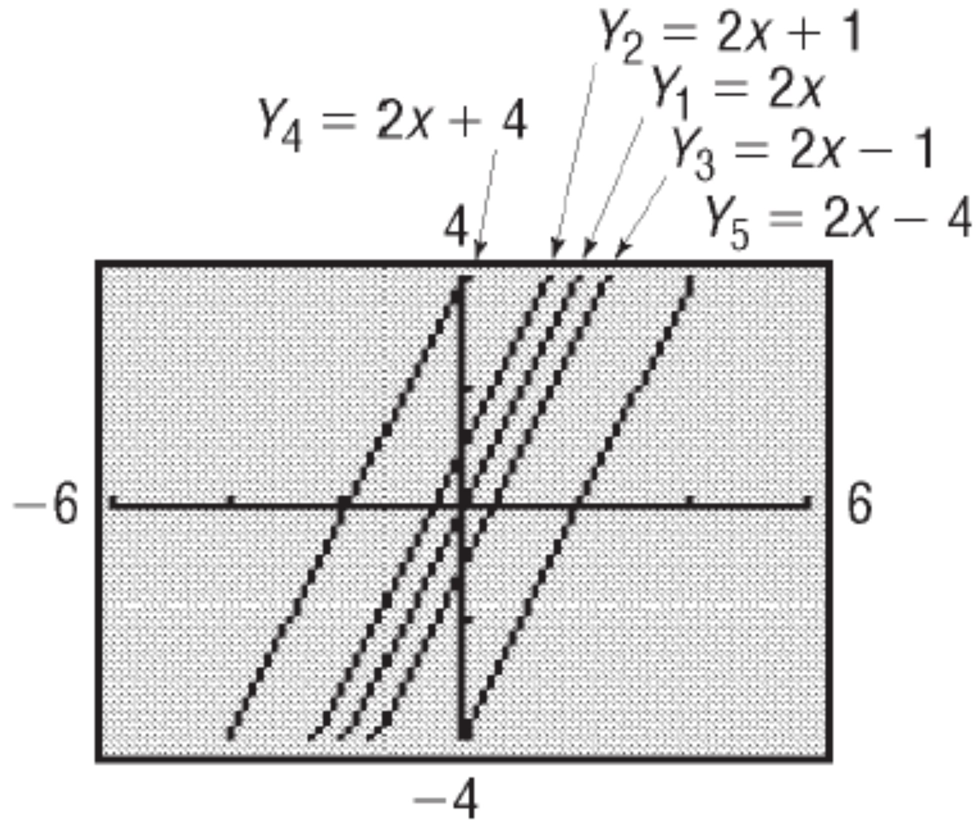
$Y_1 = 2x$

$Y_2 = 2x + 1$

$Y_3 = 2x - 1$

$Y_4 = 2x + 4$

$Y_5 = 2x - 4$



What do you conclude about the lines $y = 2x + b$?

7 Identify the Slope and *y*-Intercept of a Line from Its Equation

EXAMPLE

Finding the Slope and y-Intercept

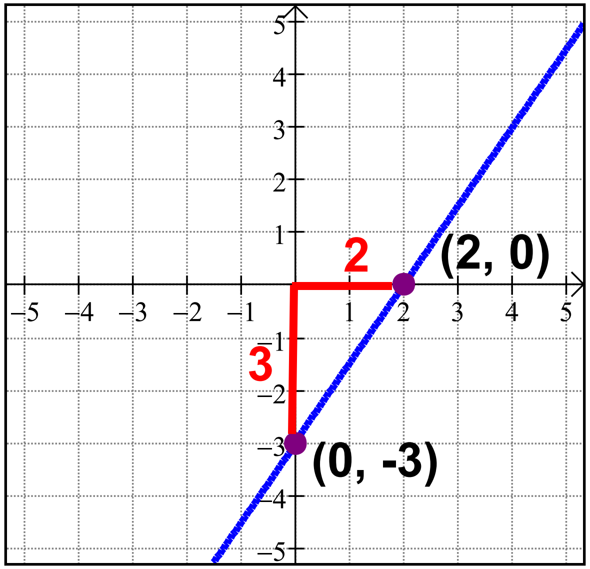
Find the slope m and y-intercept b of the equation $3x - 2y = 6$. Graph the equation.

$y = mx + b$

$$3x - 2y = 6$$

$$-2y = -3x + 6$$

$$y = \frac{3}{2}x - 3$$



$$y = \frac{3}{2}x - 3$$

8 Graph Lines Written in General Form Using Intercepts

The equation of a line L is in **general form** when it is written as

$$\boxed{Ax + By = C}$$

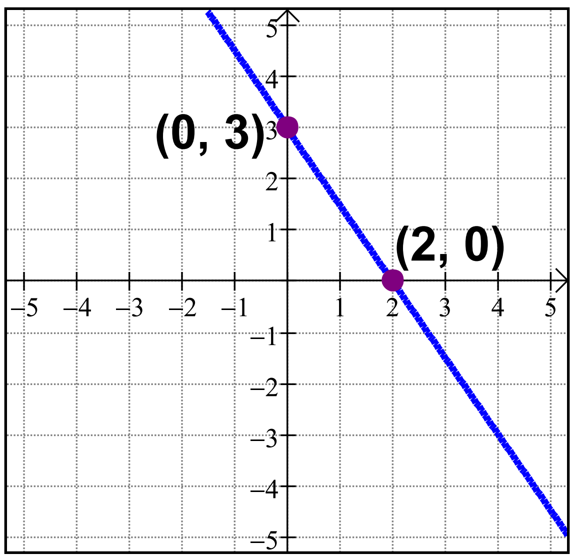
where A, B, and C are real numbers and A and B are not both 0.

EXAMPLE

Graphing an Equation in General Form Using Its Intercepts

Graph the linear equation $3x + 2y = 6$ by finding its intercepts.

$3x + 2(0) = 6$	$3(0) + 2y = 6$
$3x = 6$	$2y = 6$
$x = 2$	$y = 3$



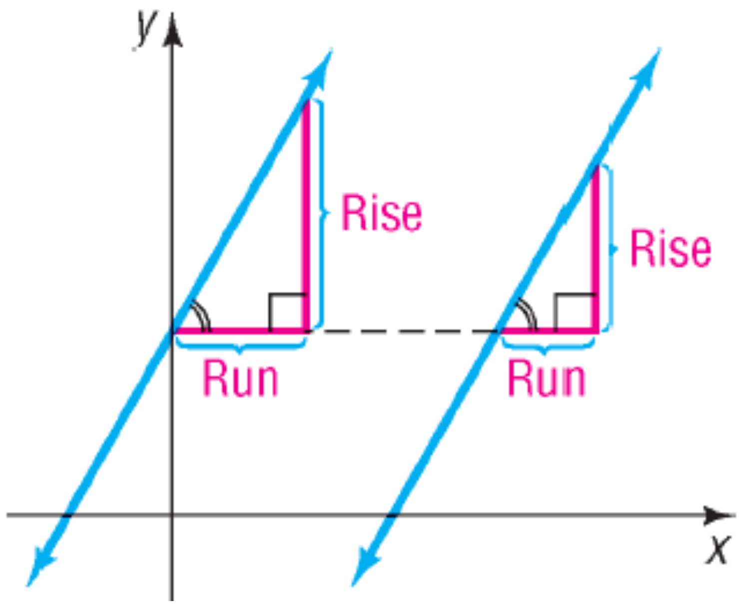
The x-intercept is at the point (2, 0).

The y-intercept is at the point (0, 3).

9 Find Equations of Parallel Lines

Criterion for Parallel Lines

Two nonvertical lines are parallel if and only if their slopes are equal and they have different *y*-intercepts.



If two nonvertical lines are parallel, then their slopes are equal and they have different *y*-intercepts.

If two nonvertical lines have equal slopes and they have different *y*-intercepts, then they are parallel.

EXAMPLE Showing That Two Lines Are Parallel

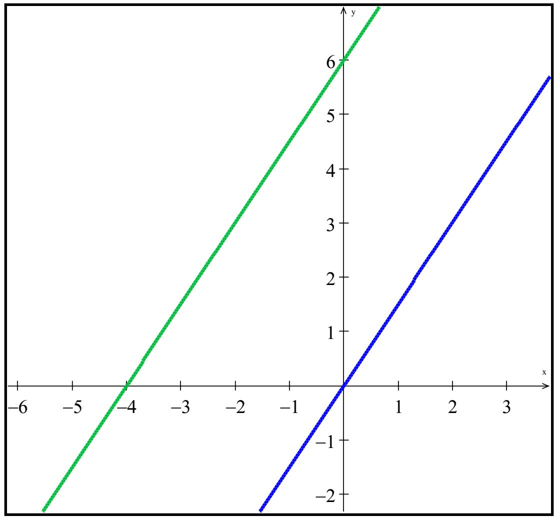
Show that the lines given by the following equations are parallel:

$$L_1 : -3x + 2y = 12$$
$$2y = 3x + 12$$
$$y = \frac{3}{2}x + 6$$

$$L_2 : 6x - 4y = 0$$
$$-4y = -6x$$
$$y = \frac{3}{2}x$$

$\text{Slope} = \frac{3}{2}$; y-intercept $= 6$

$\text{Slope} = \frac{3}{2}$; y-intercept $= 0$

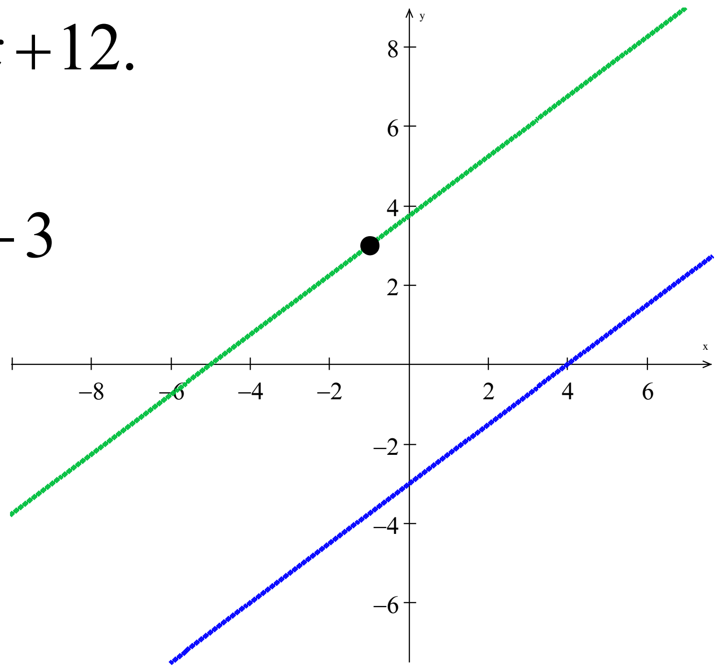


Finding a Line That Is Parallel to a Given Line

Find an equation for the line that contains the point $(-1,3)$ and is parallel to the line $3x-4y=12$.

$$-4y=-3x+12.$$

$$y=\frac{3}{4}x-3$$



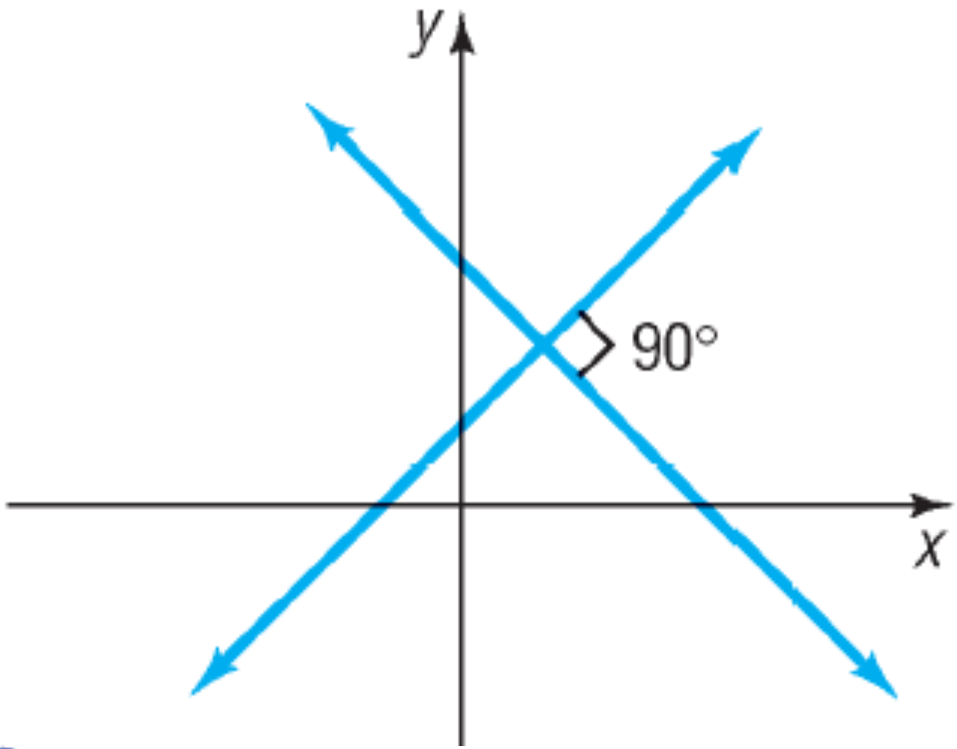
$$y-y_1=m\left(x-x_1\right)$$

$$y-3=\frac{3}{4}\left(x-(-1)\right)$$

$$y=\frac{3}{4}x+\frac{15}{4}$$

So a line parallel to this one would have a slope of $\frac{3}{4}$.

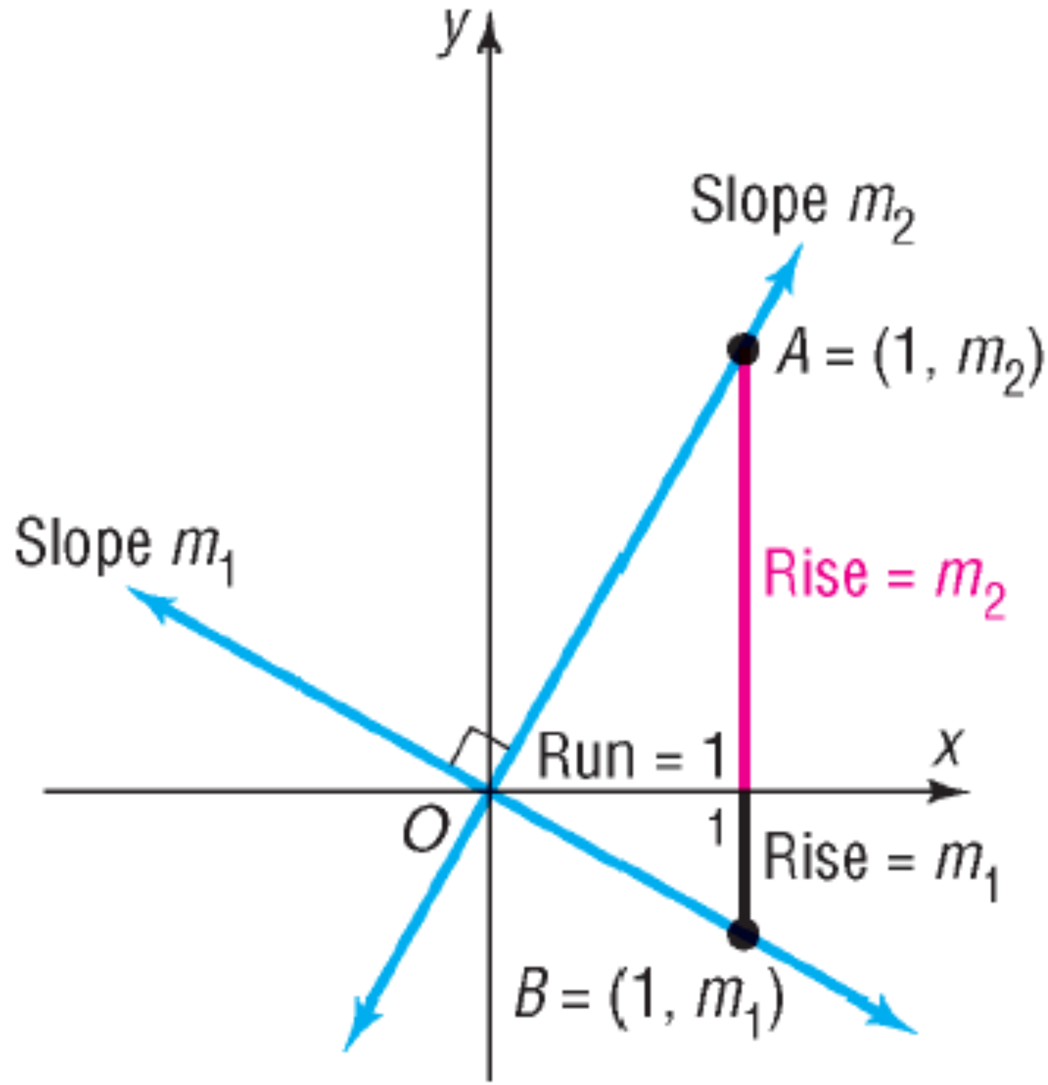
10 Find Equations of Perpendicular Lines



Theorem

Criterion for Perpendicular Lines

Two nonvertical lines are perpendicular if and only if the product of their slopes is -1.



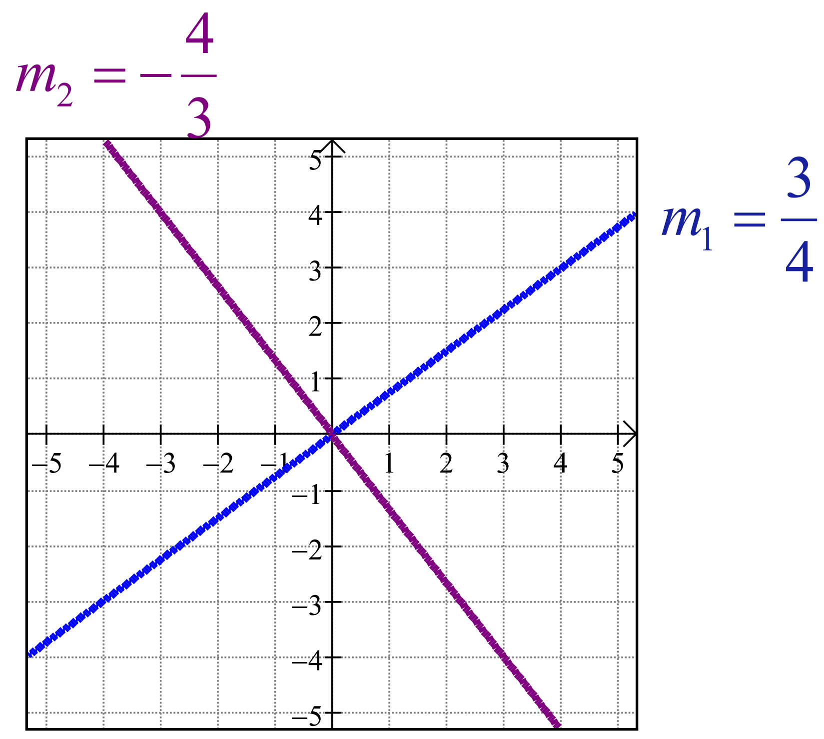
y
Slope m_2
$A = (1, m_2)$
Slope m_1
Rise = m_2
Run = 1
x
O
1
Rise = m_1
$B = (1, m_1)$

EXAMPLE

Finding the Slope of a Line Perpendicular to Another Line

Find the slope of a line perpendicular to a line with slope $\dfrac{3}{4}$.

$$m_{\text{perpendicular}} = -\frac{4}{3}$$



EXAMPLE

Finding the Equation of a Line Perpendicular to a Given Line

Find an equation for the line that contains the point $(-1, 3)$ and is perpendicular to the line $3x - 4y = 12$.

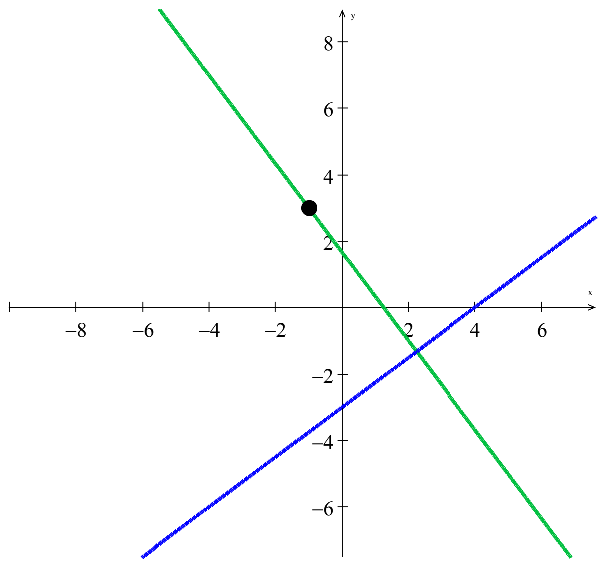
$$-4y = -3x + 12.$$

$$y = \frac{3}{4}x - 3$$

$$y - y_1 = m(x - x_1)$$

$$y - 3 = -\frac{4}{3}(x - (-1))$$

$$y = -\frac{4}{3}x + \frac{5}{3}$$



So a line perpendicular to this one would have a slope of $-\frac{4}{3}$.