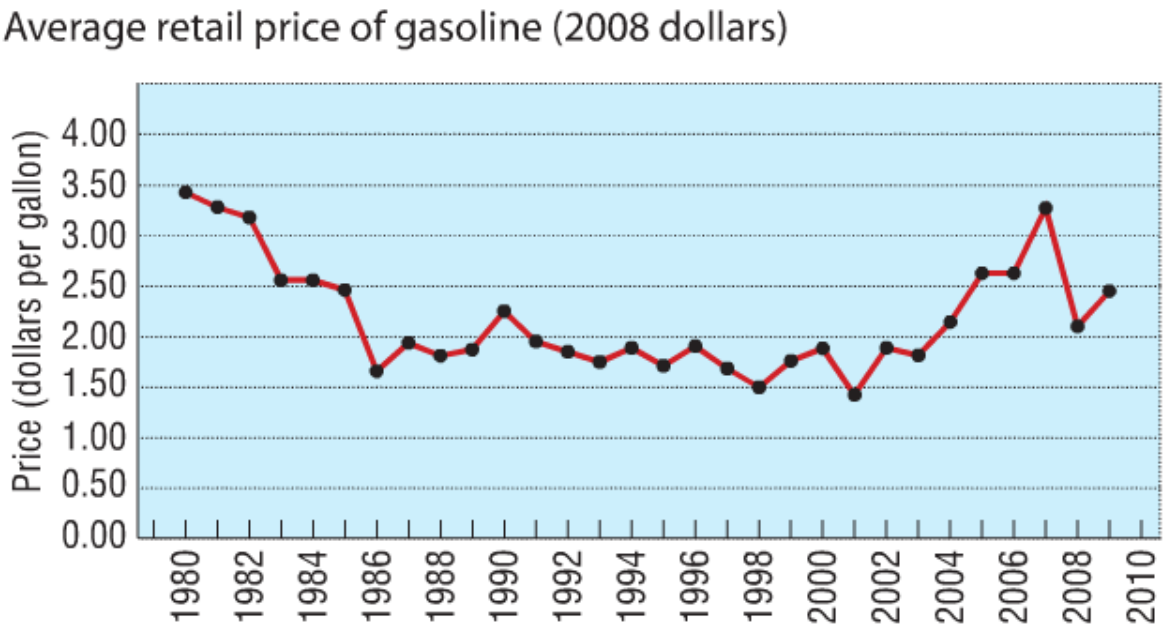
Section 3.2

The Graph of a Function

Average Price of Gasoline at a particular station in Texas

Year	Price	Year	Price	Year	Price
1980	3.41	1990	2.25	2000	1.85
1981	3.26	1991	1.90	2001	1.40
1982	3.15	1992	1.82	2002	1.86
1983	2.51	1993	1.70	2003	1.79
1984	2.51	1994	1.85	2004	2.13
1985	2.46	1995	1.68	2005	2.60
1986	1.63	1996	1.87	2006	2.62
1987	1.90	1997	1.65	2007	3.29
1988	1.77	1998	1.50	2008	2.10
1989	1.83	1999	1.73	2009	2.45



1 Identify the Graph of a Function

Theorem

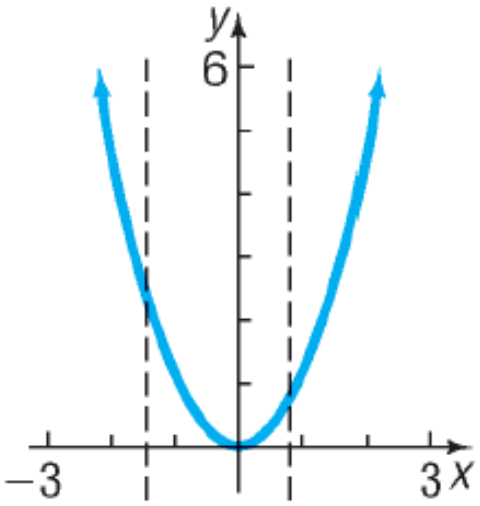
Vertical-line Test

A set of points in the xy-plane is the graph of a function if and only if every vertical line intersects the graph in at most one point.

EXAMPLE

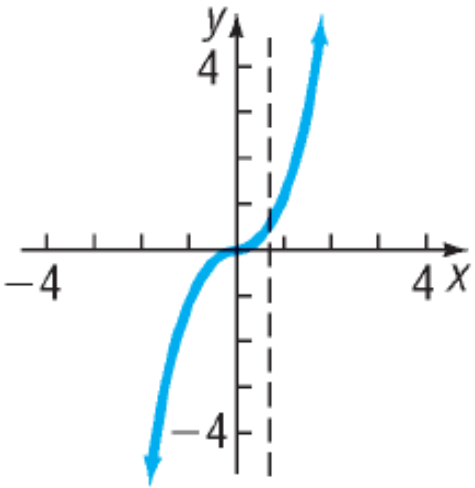
Identifying the Graph of a Function

Which of the following are graphs of functions?



(a) $y = x^2$

A Function



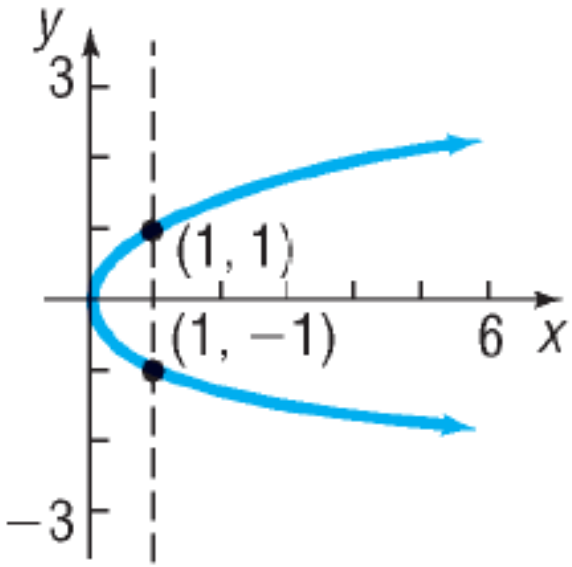
(b) $y = x^3$

A Function

EXAMPLE

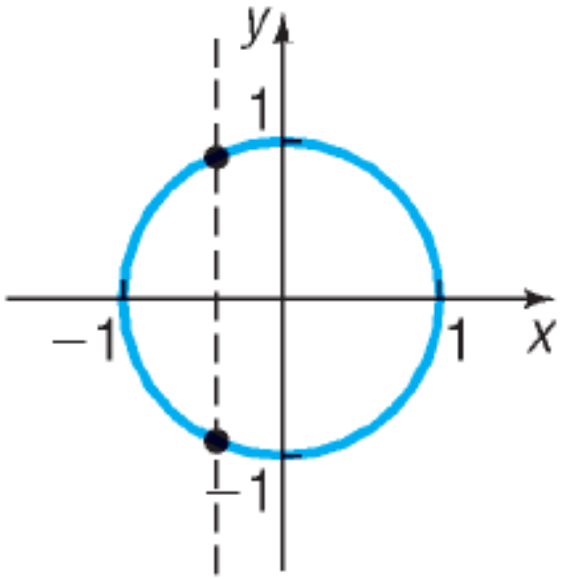
Identifying the Graph of a Function

Which of the following are graphs of functions?



(c) $x = y^2$

Not A Function



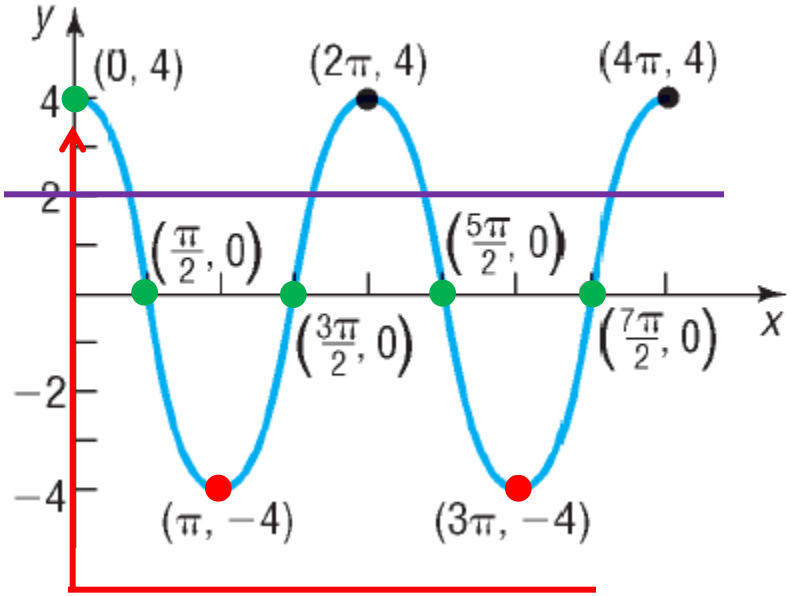
(d) $x^2 + y^2 = 1$

Not A Function

2 Obtain Information from or about the Graph of a Function

EXAMPLE

Obtaining Information from the Graph of a Function



(a) What are $f(0)$, $f\left(\frac{3\pi}{2}\right)$, and $f(3\pi)$?

(a) $f(0) = 4$

(b) What is the domain of f?

(b) $\{x \mid 0 \le x \le 4\pi\}$ or $[0, 4\pi]$

(c) What is the range of f?

(c) $\{y \mid -4 \le y \le 4\}$ or $[-4, 4]$

(d) List the intercepts. $(0, 4), \left(\frac{\pi}{2}, 0\right), \left(\frac{3\pi}{2}, 0\right), \left(\frac{5\pi}{2}, 0\right), \left(\frac{7\pi}{2}, 0\right)$

(e) How often does the line $y = 2$ intersect the graph? Four times.

(f) For what values of x does $f(x) = -4$? $x = \pi$ and $x = 3\pi$

(g) For what values of x is $f(x) > 0$? $\left[0, \frac{\pi}{2}\right) \cup \left(\frac{3\pi}{2}, \frac{5\pi}{2}\right) \cup \left(\frac{7\pi}{2}, 4\pi\right]$

EXAMPLE

Obtaining Information from the Graph of a Function

Consider the function $f(x) = \dfrac{x}{x+1}$

(a) Is the point $\left(1, \dfrac{1}{2}\right)$ on the graph of f? Yes $\quad f(1) = \dfrac{1}{1+1} = \dfrac{1}{2}$

(b) If $x = 2$, what is $f(x)$? What point is on the graph of f?

$$f(2) = \frac{2}{2+1} = \frac{2}{3} \qquad \left(2, \frac{2}{3}\right)$$

(c) If $f(x) = 2$, what is x? What point is on the graph of f?

$$f(x) = 2 = \frac{x}{x+1} \qquad 2(x+1) = x \qquad 2x + 2 = x \qquad x = -2 \qquad (-2, 2)$$

EXAMPLE Average Cost Function

The average cost $\overline{C}$ of manufacturing x computers per day is given by the function

$$\overline{C}(x) = 0.56x^2 - 34.39x + 1212.57 + \frac{20{,}000}{x}$$

Determine the average cost of manufacturing:

(a) 30 computers in a day

(b) 40 computers in a day

(c) 50 computers in a day

(d) Graph the function $\overline{C} = \overline{C}(x), 0 < x \leq 80$.

(e) Create a TABLE with TblStart $= 1$ and ΔTbl $= 1$.

Which value of x minimizes the average cost?

EXAMPLE Average Cost Function

$$\overline{C}(x) = 0.56x^2 - 34.39x + 1212.57 + \frac{20{,}000}{x}$$

(a) 30 computers in a day = \$1351.54

(b) 40 computers in a day = \$1232.97

(c) 50 computers in a day = \$1293.07

(d) Graph the function $\overline{C} = \overline{C}(x), 0 < x \leq 80.$

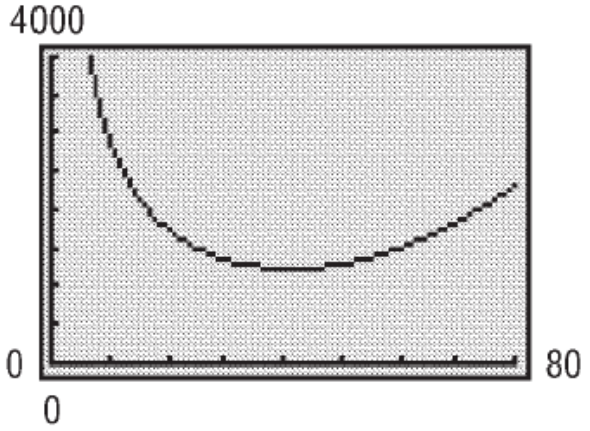
(e) Create a TABLE with TblStart = 1 and ΔTbl = 1

X	Y1
1	21179
2	11146
3	7781.1
4	6084
5	5054.6
6	4359.7
7	3856.4

Y1■.56X²−34.39X...

X	Y1
38	1240.7
39	1235.9
40	1233
41	[illegible]
42	1232.2
43	1234.4
44	1238.1

Y1=1231.74487805



Which value of x minimizes the average cost? $x = 41.$

SUMMARY

Graph of a Function	The collection of points (x, y) that satisfies the equation $y = f(x)$.
Vertical Line Test	A collection of points is the graph of a function provided that every vertical line intersects the graph in at most one point.