Section 14.1

Counting

1 Find All the Subsets of a Set

EXAMPLE

Finding All the Subsets of a Set

Write down all the subsets of the set $\{a, b, c\}$.

0 Elements	**1 Element**	**2 Elements**	**3 Elements**
$\emptyset$	$\{a\}, \{b\}, \{c\}$	$\{a, b\}, \{b, c\}, \{a, c\}$	$\{a, b, c\}$

2 Count the Number of Elements in a Set

If A is a set with n elements, A has 2^n subsets.

EXAMPLE Analyzing Survey Data

In a survey of 100 college students, 35 were registered in College Algebra, 52 were registered in Computer Science I, and 18 were registered in both courses.

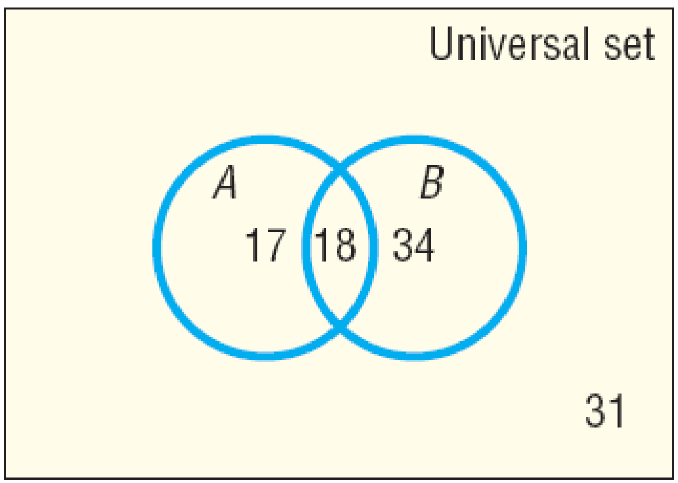
(a) How many students were registered in College Algebra or Computer Science I?

(b) How many were registered in neither course?

(a) First, let A = set of students in College Algebra

B = set of students in Computer Science I

$$n(A) = 35 \qquad n(B) = 52 \qquad n(A \cap B) = 18$$



Those in Set A that are not in Set B = 35 – 18 = 17

Those in Set B that are not in Set A = 52 – 18 = 34

So total is 18 + 17 + 34 = 69 students.

(b) Since 100 students were surveyed, it follows that $100 - 69 = 31$ were registered in neither course.

THEOREM

Counting Formula

If A and B are finite sets, then

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

THEOREM

Addition Principle of Counting

If two sets A and B have no elements in common, that is, if $A \cap B = \emptyset$, then $n(A \cup B) = n(A) + n(B)$

THEOREM

General Addition Principle of Counting

If, for n sets $A_1, A_2, \ldots, A_n$, no two have elements in common, then

$$n(A_1 \cup A_2 \cup \cdots \cup A_n) = n(A_1) + n(A_2) + \cdots + n(A_n)$$

EXAMPLE Counting

Table 1 lists the level of education for all United States residents 25 years of age or older in 2007.

Level of Education	Number of U.S. Residents at Least 25 Years Old
Not a high school graduate	27,787,000
High school graduate	61,404,000
Some college, but no degree	32,451,000
Associate's degree	16,711,000
Bachelor's degree	36,726,000
Advanced degree	19,237,000

(a) How many U.S. residents 25 years of age or older had an associate's degree or bachelor's degree?

(b) How many U.S. residents 25 years of age or older had an associate's degree, a bachelor's degree, or an advanced degree?

Let A represent the set of associate's degree holders, B represent the set of bachelor's degree holders, and C represent the set of advanced degree holders. No two of the sets A, B, or C have elements in common

EXAMPLE Counting

Level of Education	Number of U.S. Residents at Least 25 Years Old
Not a high school graduate	27,787,000
High school graduate	61,404,000
Some college, but no degree	32,451,000
Associate's degree	16,711,000
Bachelor's degree	36,726,000
Advanced degree	19,237,000

(a) How many U.S. residents 25 years of age or older had an associate's degree or bachelor's degree?

(b) How many U.S. residents 25 years of age or older had an associate's degree, a bachelor's degree, or an advanced degree?

$n(A) = 16{,}711{,}000 \quad n(B) = 36{,}726{,}000 \quad n(C) = 19{,}237{,}000$

(a) Using formula (2),

$$n(A \cup B) = n(A) + n(B) = 16{,}711{,}000 + 36{,}726{,}000 = 53{,}437{,}000$$

There were 53,437,000 U.S. residents 25 years of age or older who had an associate's degree or bachelor's degree.

EXAMPLE Counting

Level of Education	Number of U.S. Residents at Least 25 Years Old
Not a high school graduate	27,787,000
High school graduate	61,404,000
Some college, but no degree	32,451,000
Associate's degree	16,711,000
Bachelor's degree	36,726,000
Advanced degree	19,237,000

(a) How many U.S. residents 25 years of age or older had an associate's degree or bachelor's degree?

(b) How many U.S. residents 25 years of age or older had an associate's degree, a bachelor's degree, or an advanced degree?

(b) Using formula (3),

$$\begin{aligned} n(A \cup B \cup C) &= n(A) + n(B) + n(C) \\ &= 16{,}711{,}000 + 36{,}726{,}000 + 19{,}237{,}000 \\ &= 72{,}674{,}000 \end{aligned}$$

There were 72,674,000 U.S. residents 25 years of age or older who had an associate's degree, bachelor's degree, or advanced degree.

3 Solve Counting Problems Using the Multiplication Principle

EXAMPLE Counting the Number of Possible Meals

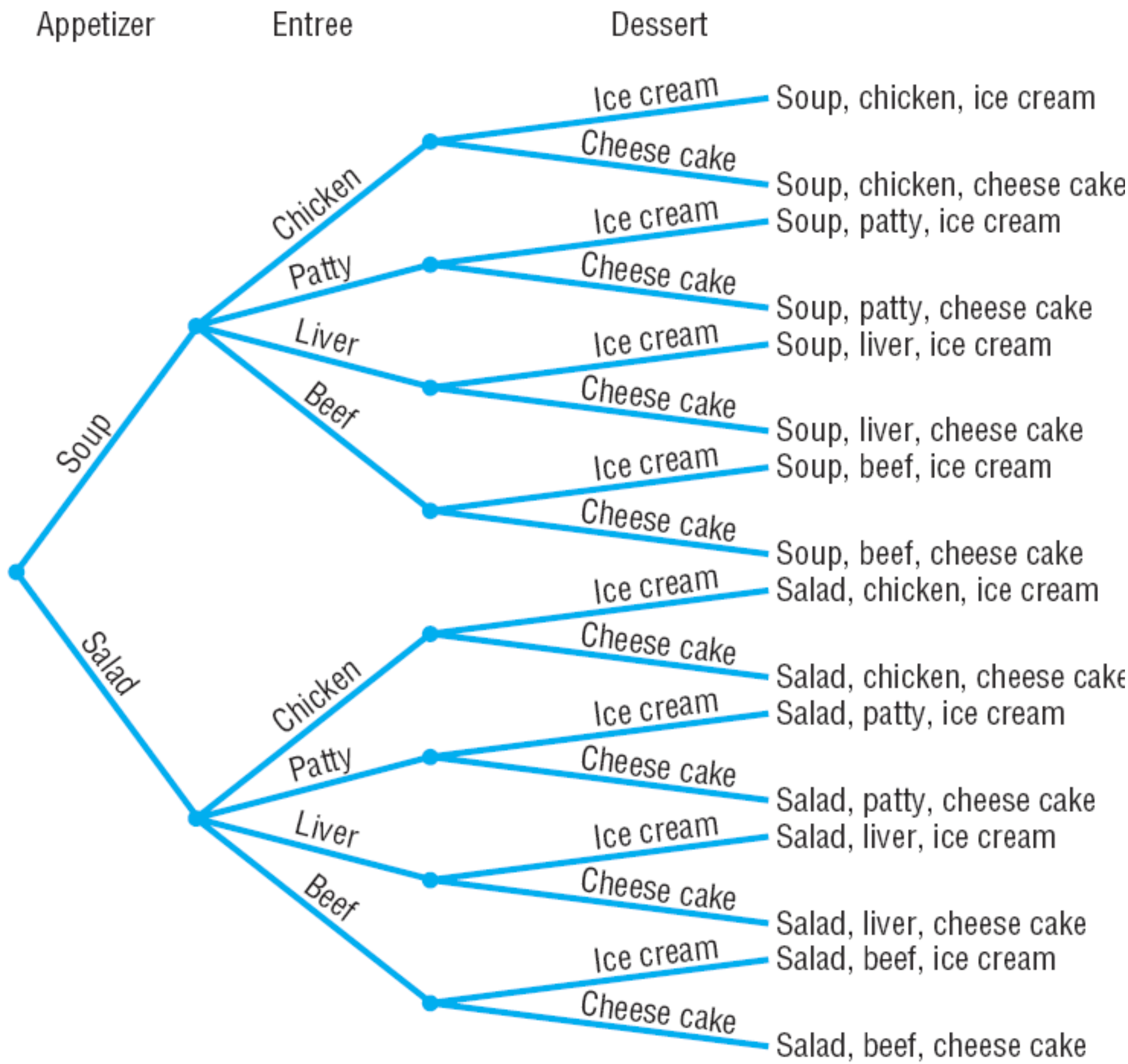
The fixed-price dinner at Mabenka Restaurant provides the following choices:

Appetizer:	soup or salad
Entree:	baked chicken, broiled beef patty, baby beef liver, or roast beef au jus
Dessert:	ice cream or cheese cake

How many different meals can be ordered?

Choose an Appetizer	**Choose an Entree**	**Choose a Dessert**
2 choices	4 choices	2 choices

$$\text{Number of Choices} = 2 \cdot 4 \cdot 6 = 16$$


Appetizer
Entree
Dessert
Soup
Salad
Chicken
Patty
Liver
Beef
Chicken
Patty
Liver
Beef
Ice cream
Cheese cake
Ice cream
Cheese cake
Ice cream
Cheese cake
Ice cream
Cheese cake
Ice cream
Cheese cake
Ice cream
Cheese cake
Ice cream
Cheese cake
Ice cream
Cheese cake
Soup, chicken, ice cream
Soup, chicken, cheese cake
Soup, patty, ice cream
Soup, patty, cheese cake
Soup, liver, ice cream
Soup, liver, cheese cake
Soup, beef, ice cream
Soup, beef, cheese cake
Salad, chicken, ice cream
Salad, chicken, cheese cake
Salad, patty, ice cream
Salad, patty, cheese cake
Salad, liver, ice cream
Salad, liver, cheese cake
Salad, beef, ice cream
Salad, beef, cheese cake

THEOREM

Multiplication Principle of Counting

If a task consists of a sequence of choices in which there are p selections for the first choice, q selections for the second choice, r selections for the third choice, and so on, then the task of making these selections can be done in

$$p \cdot q \cdot r \cdot \ldots$$

different ways.

EXAMPLE Forming Codes

How many two-symbol code words can be formed if the first symbol is an uppercase letter and the second symbol is a digit?

It sometimes helps to begin by listing some of the possibilities. The code consists of an uppercase letter followed by a digit, so some possibilities are A1, A2, B3, X0, and so on. The task consists of making two selections: the first selection requires choosing an uppercase letter (26 choices) and the second task requires choosing a digit (10 choices). By the Multiplication Principle, there are

$$26 \cdot 10 = 260$$

different code words of the type described.