SUMMARY OF GEOMETRY

MEASUREMENT SYSTEMS:

METRIC SYSTEM

BASIC units: linear: **m** is a **meter**; weight or mass: **g** is a **gram** capacity: **l** is a **liter**

PREFIXES: **m**- is a **milli**- (key number is 1000);
c- is a **centi**- (key number is 100);
k- is a **kilo**- (key number is 1000);

CONVERSION:

- Using concept that if you are converting units to smaller units, you will need more of the smaller units for the same measurement and if you are converting to larger units you will need fewer of the larger units:
  a) If converting from x large units to smaller units: multiply x by the key number of the prefix.
  b) If converting from x small units to larger units: divide x by the key number of the prefix
- Alternate method: Since you are multiplying or dividing by a power of 10, the conversion can be accomplished by moving the decimal point to the right (if multiplying) or to the left (if dividing) the appropriate number of places.

To help arrive at the correct directions and appropriate number of places, you can use the memory aid of:
K H D M D C M (King Henry died miserably doing college math). Place the number under the letter representing its unit and note the direction and places to get to the letter of the unit you want to convert to.

For example: Convert 5400 cm to kilometers:

K	H	D	M	D	**C**	M
.054	←	←	←	←	5400.	

move decimal 5 places to the left.

ENGLISH SYSTEM

- You must use "conversion fractions" to get from one measurement to another.

Ex: Convert 72 cups to gallons.

$$\frac{72\ c}{1}\cdot\frac{1\ pt}{2\ c}\cdot\frac{1\ qt}{2\ pt}\cdot\frac{1\ gal}{4\ qt}=\frac{9}{2}gal=4.5gal$$

- When rounding feet and inches to nearest foot, you must have **6** or more inches to round up.
- When rounding pounds and ounces to the nearest pound, you must have **8** ounces or more to round up.
- When rounding a ruler measurement to the nearest fraction of an inch, the final answer may be in any larger unit. For example, given something that measures between 1½ in. and 1¾ in. and asked to find the measure to the nearest quarter inch, the answer could be 1½ inches, since 1½ inches is the same as $1\frac{2}{4}$ inches.

ANGLES are measured in degrees using a curved ruler called a protractor:

acute angle < 90°
right angle = 90°
obtuse angle > 90°
straight angle = 180°

PAIRS OF ANGLES:

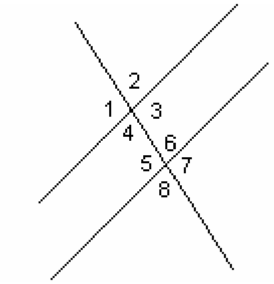
Adjacent angles: angles that share a common vertex and a common side, but have no other intersection.
NOTE: in a polygon, adjacent angles share only a common side; they are also referred to as consecutive angles.

Complementary angles: Two angles whose measures have a sum of 90°. If they are adjacent they form a right angle.

Supplementary angles: Two angles whose measures have a sum of 180°. If they are adjacent they form a straight angle.

Vertical angles: Two non-adjacent angles formed by intersecting lines. Vertical angles are equal in measure.

PARALLEL LINES CROSSED BY ONE TRANSVERSAL



Two groups of four angles are formed:

Pairs of **corresponding angles** are ∠1 and ∠5, ∠2 and ∠6, ∠3 and ∠7, ∠4 and ∠8.

Pairs of **alternate interior angles** are ∠4 and ∠6; ∠3 and ∠5

NOTE: If corresponding angles or alternate interior angles are NOT equal, then the lines are NOT parallel

If the lines are parallel, then

- **all acute angles are equal**
- **all obtuse angles are equal**
- **any acute angle and any obtuse angle are supplementary**

TRIANGLES

The sum of the interior angles of a triangle is 180°

Since all triangles have at least 2 acute angles, a triangle may be classified by the nature of its third angle, that is:

- an **acute triangle** if the third angle is also acute.
- an **obtuse triangle** if the third angle is obtuse.
- a **right triangle** if the third angle is a right angle.

Triangles may be classified by sides:

- **Scalene** -- no sides are equal (and no angles are equal).
- **Isosceles** -- two sides are equal (and the angles opposite the equal sides are equal).
- **Equilateral** -- all three sides are equal (each angle measures 60^0; thus, the triangle is also *equiangular*).

The shortest side of a triangle is opposite the smallest angle; the longest side is opposite the longest side.

PYTHAGOREAN THEOREM (applies to right triangles):

The square of the hypotenuse equals the sum of the squares of the other two sides.

$$H^2 = S_1^2 + S_2^2 \quad \text{or} \quad a^2 + b^2 = c^2 \text{ (where c is the hypotenuse)}$$

SIMILAR TRIANGLES:

When two triangles are similar, the measures of corresponding angles are equal, and the lengths of corresponding sides are in proportion. That is, if $\Delta ABC \sim \Delta PQR$, then:

1) $\angle A = \angle P$, $\angle B = \angle Q$ and $\angle C = \angle R$, and
2) $\dfrac{AB}{PQ} = \dfrac{BC}{QR} = \dfrac{AC}{PR}$

Two triangles will be similar if:

- two angles of one triangle are equal to two angles of the other triangle, or
- three pairs of corresponding sides are proportional, or
- two pairs of sides are proportional and the included angles are equal.

If the triangles are similar and corresponding sides are equal, the triangles are congruent.

POLYGONS:

NUMBER OF SIDES	NAME
3	Triangle
4	Quadrilateral
5	Pentagon
6	Hexagon
8	Octagon

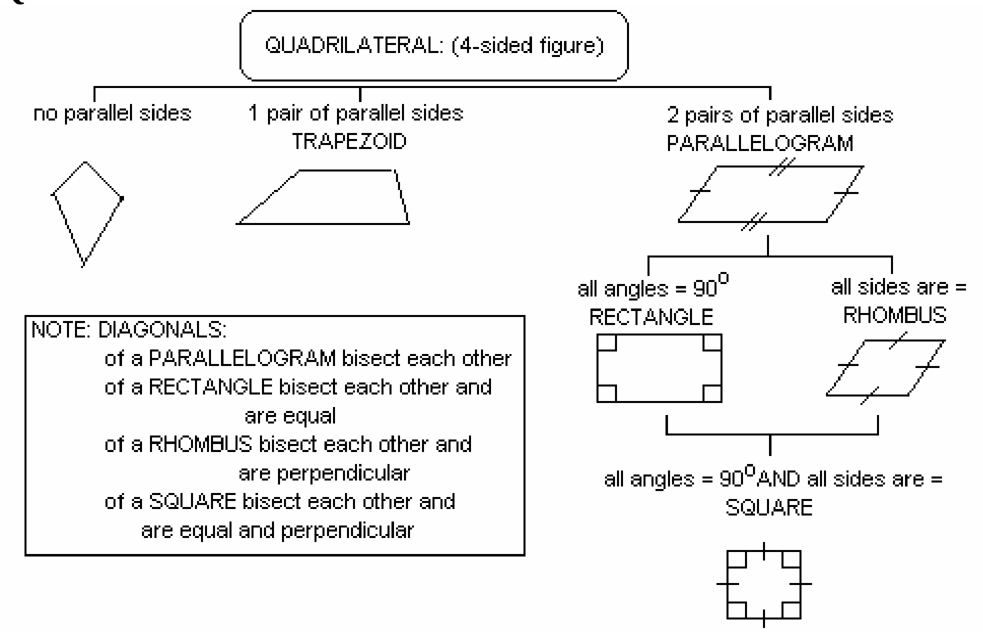
AN EXTERIOR ANGLE OF A POLYGON is formed by one side and the extension of the other side at that vertex.

- The exterior angle and the interior angle at any vertex of any polygon are supplementary.
- The exterior angle of a triangle is equal to the sum of the two remote interior angles.

Summary of Angle Measures for Polygons (in degrees):

	Sum of all (any polygon)	**Measure of one (regular polygon)**
Interior angle	(n - 2) 180	$180 - \frac{360}{n}$
Exterior angle	360	$\frac{360}{n}$

QUADRILATERALS



DISTANCE AROUND AN OBJECT:

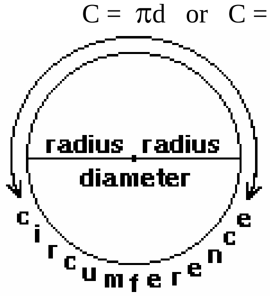
PERIMETER is the linear measure of the distance around a figure.

To find the <u>perimeter</u> of a polygon, add the lengths of all of its sides.

CIRCUMFERENCE is the linear measure of the distance around a **<u>circle</u>**.

Circumference is Pi times the diameter or Pi times twice the radius.

$C = \pi d$ or $C = 2\pi r$

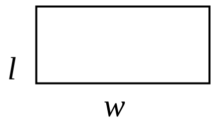


MEASUREMENT OF GEOMETRIC FIGURES:

1) **Linear:** used when measuring length, width, depth, distance around an object (in., ft., yd., m., cm, km, etc.)
2) **Square:** used when measuring area (in^2., ft^2., yd^2., m^2., cm^2, km^2, etc.)
3) **Cubic:** used when measuring volume (in^3., ft^3., yd^3., m^3., cm^3, km^3, etc.)

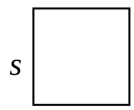
Area Formulas

1. Rectangle



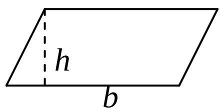
$A = l \cdot w$ (length x width)

2. Square



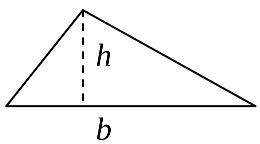
$A = s^2$ (side squared)

3. Parallelogram



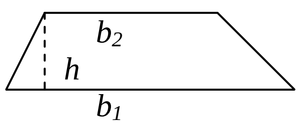
$A = b \cdot h$ (base x height)

4. Triangle



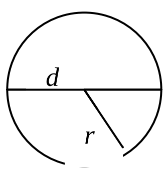
$A = ½\, b\, h$ (1/2 base x height)

5. Trapezoid



$A = ½\, (b_1 + b_2)\, h$ (1/2 sum of bases x height)

6. Circle



$A = \pi r^2$ (pi x radius squared)
(Circumference: $C = \pi d$ or $2\pi r$)

Notes:

1. Area measures the amount of surface enclosed in a region and is measured in square units:
   sq. in. = in^2 or sq. cm = cm^2 NOTE: 9 sq. ft = 1 sq. yd.
2. The area of any figure made up of a combination of the above figures can be found by adding up the individual parts.
3. The area of a shaded region can be found by first finding the figure's total area and then subtracting out the unshaded area.
4. Leave area and circumference in terms of π unless otherwise indicated.

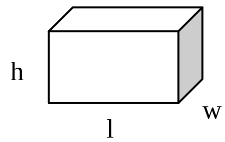
Volume Formulas

In general, the volume of a solid figure (in <u>cubic units</u>) will be:

a) the area of the base · height , if the figure goes "straight up" from the base.

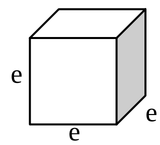
b) 1/3 the area of the base · height, if the figure rises to a point (e.g. cone or pyramid).

1. Rectangular Solid (box)



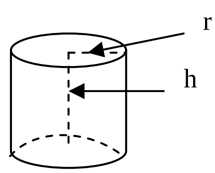
Volume = base area · height
$= lw \cdot h = lwh$
(Surface Area = area of 6 faces = $2 \cdot lw + 2 \cdot lh + 2 \cdot wh$)

2. Cube



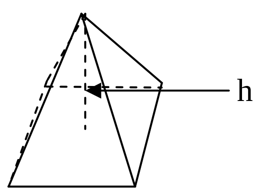
Volume = base area · height
$= e^2 \cdot e = e^3$

3. Cylinder



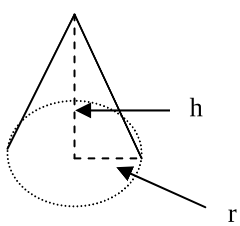
Volume = base area · height
$= \pi r^2 \cdot h = \pi r^2 h$
(Surface Area = area of ends + area of "label")
$= 2 \cdot \pi r^2 + 2\pi rh$

4. Pyramid



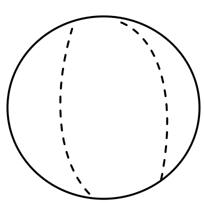
Volume = 1/3 · base area · height
$= 1/3 \cdot lw \cdot h = 1/3\ lwh$

5. Cone



Volume = 1/3 · base area · height
$= 1/3 \cdot \pi r^2 \cdot h = 1/3\ \pi r^2 h$

6. Sphere



Volume = $4/3 \cdot \pi r^3$