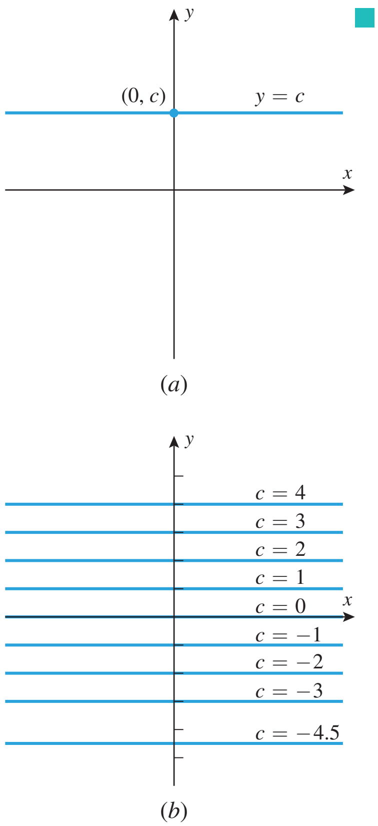
D

FAMILIES OF FUNCTIONS

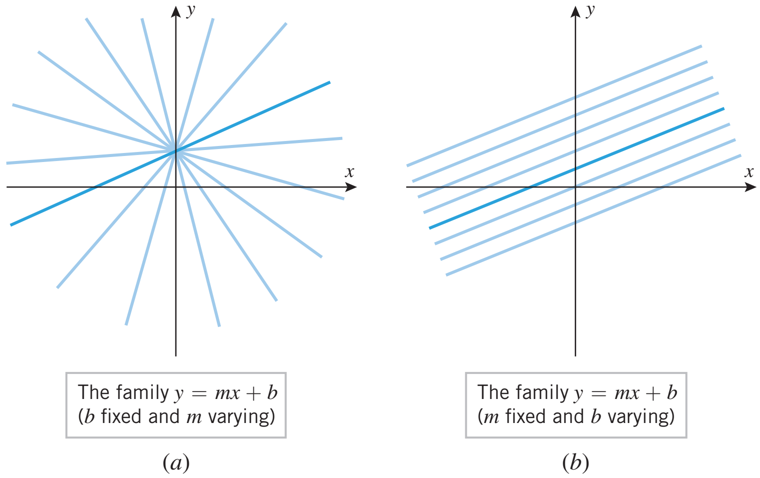
FAMILIES OF CURVES

The graph of a constant function $f(x) = c$ is the graph of the equation $y = c$, which is the horizontal line shown in Figure D.1*a*. If we vary c, then we obtain a set or ***family*** of horizontal lines such as those in Figure D.1*b*.

Constants that are varied to produce families of curves are called ***parameters***. For example, recall that an equation of the form $y = mx + b$ represents a line of slope m and y-intercept b. If we keep b fixed and treat m as a parameter, then we obtain a family of lines whose members all have y-intercept b (Figure D.2*a*), and if we keep m fixed and treat b as a parameter, we obtain a family of parallel lines whose members all have slope m (Figure D.2*b*).



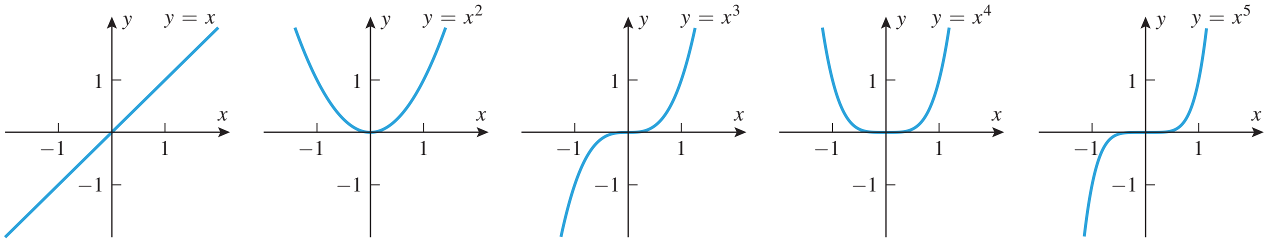
▲ **Figure D.1**



► **Figure D.2**

POWER FUNCTIONS; THE FAMILY $y = x^n$

A function of the form $f(x) = x^p$, where p is constant, is called a ***power function***. For the moment, let us consider the case where p is a positive integer, say $p = n$. The graphs of the curves $y = x^n$ for $n = 1, 2, 3, 4$, and 5 are shown in Figure D.3. The first graph is the



▲ **Figure D.3**

line with slope 1 that passes through the origin, and the second is a parabola that opens up and has its vertex at the origin (see Web Appendix I).

For $n \geq 2$ the shape of the curve $y = x^n$ depends on whether n is even or odd (Figure D.4):

- For even values of n, the functions $f(x) = x^n$ are even, so their graphs are symmetric about the y-axis. The graphs all have the general shape of the graph of $y = x^2$, and each graph passes through the points $(-1, 1)$, $(0, 0)$, and $(1, 1)$. As n increases, the graphs become flatter over the interval $-1 < x < 1$ and steeper over the intervals $x > 1$ and $x < -1$.
- For odd values of n, the functions $f(x) = x^n$ are odd, so their graphs are symmetric about the origin. The graphs all have the general shape of the curve $y = x^3$, and each graph passes through the points $(-1, -1)$, $(0, 0)$, and $(1, 1)$. As n increases, the graphs become flatter over the interval $-1 < x < 1$ and steeper over the intervals $x > 1$ and $x < -1$.

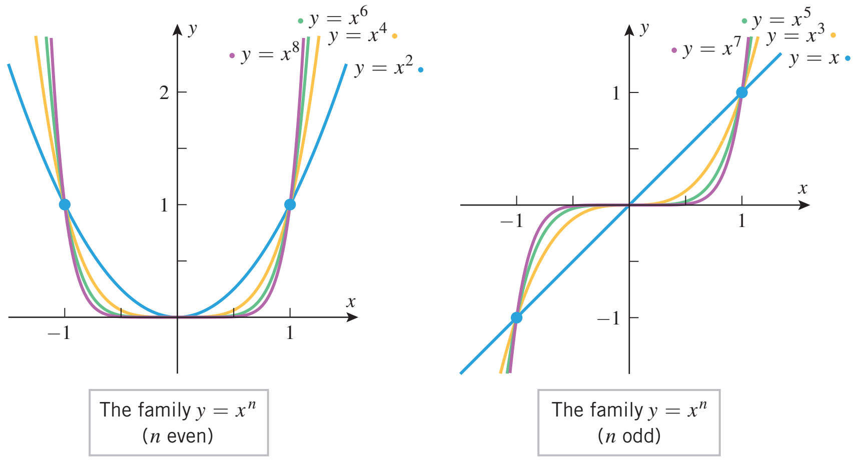


Figure D.4

REMARK The flattening and steepening effects can be understood by considering what happens when a number x is raised to higher and higher powers: If $-1 < x < 1$, then the absolute value of x^n *decreases* as n increases, thereby causing the graphs to become flatter on this interval as n increases (try raising $\frac{1}{2}$ or $-\frac{1}{2}$ to higher and higher powers). On the other hand, if $x > 1$ or $x < -1$, then the absolute value of x^n *increases* as n increases, thereby causing the graphs to become steeper on these intervals as n increases (try raising 2 or -2 to higher and higher powers).

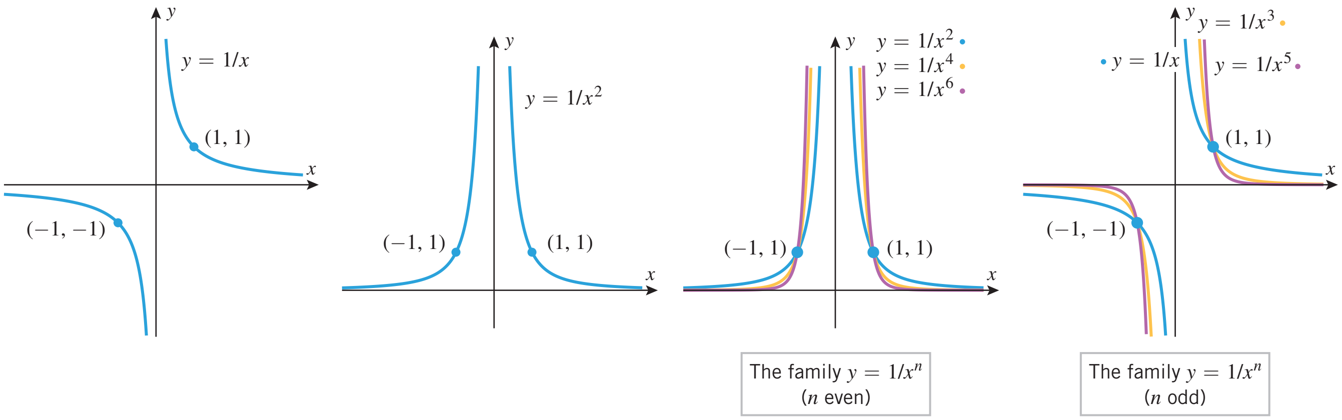
THE FAMILY $y = x^{-n}$

If p is a negative integer, say $p = -n$, then the power functions $f(x) = x^p$ have the form $f(x) = x^{-n} = 1/x^n$. Figure D.5 shows the graphs of $y = 1/x$ and $y = 1/x^2$. The graph of $y = 1/x$ is called an ***equilateral hyperbola*** (for reasons to be discussed later).

As illustrated in Figure D.5, the shape of the curve $y = 1/x^n$ depends on whether n is even or odd:

- For even values of n, the functions $f(x) = 1/x^n$ are even, so their graphs are symmetric about the y-axis. The graphs all have the general shape of the curve $y = 1/x^2$, and each graph passes through the points $(-1, 1)$ and $(1, 1)$. As n increases, the graphs become steeper over the intervals $-1 < x < 0$ and $0 < x < 1$ and become flatter over the intervals $x > 1$ and $x < -1$.
- For odd values of n, the functions $f(x) = 1/x^n$ are odd, so their graphs are symmetric about the origin. The graphs all have the general shape of the curve $y = 1/x$, and

By considering the value of $1/x^n$ for a fixed x as n increases, explain why the graphs become flatter or steeper as described here for increasing values of n.



▲ **Figure D.5**

each graph passes through the points $(1, 1)$ and $(-1, -1)$. As n increases, the graphs become steeper over the intervals $-1 < x < 0$ and $0 < x < 1$ and become flatter over the intervals $x > 1$ and $x < -1$.

- For both even and odd values of n the graph $y = 1/x^n$ has a break at the origin (called a ***discontinuity***), which occurs because division by zero is undefined.

INVERSE PROPORTIONS

Recall that a variable y is said to be ***inversely proportional to a variable x*** if there is a positive constant k, called the ***constant of proportionality***, such that

$$y = \frac{k}{x} \tag{1}$$

Since k is assumed to be positive, the graph of (1) has the same shape as $y = 1/x$ but is compressed or stretched in the y-direction. Also, it should be evident from (1) that doubling x multiplies y by $\frac{1}{2}$, tripling x multiplies y by $\frac{1}{3}$, and so forth.

Equation (1) can be expressed as $xy = k$, which tells us that the product of inversely proportional variables is a positive constant. This is a useful form for identifying inverse proportionality in experimental data.

Table D.1

x	0.8	1	2.5	4	6.25	10
y	6.25	5	2	1.25	0.8	0.5

▶ **Example 1** Table D.1 shows some experimental data.

(a) Explain why the data suggest that y is inversely proportional to x.

(b) Express y as a function of x.

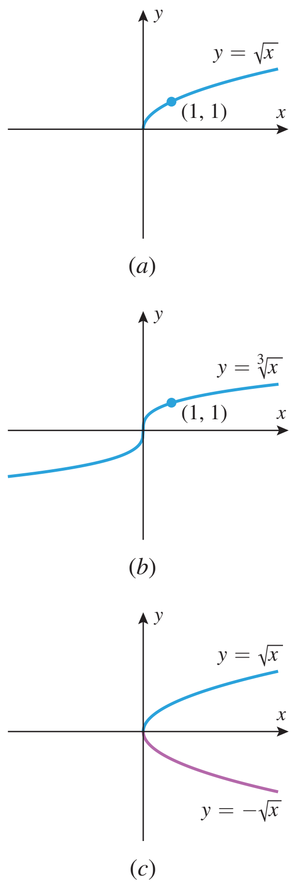
(c) Graph your function and the data together for $x > 0$.

Solution. For every data point we have $xy = 5$, so y is inversely proportional to x and $y = 5/x$. The graph of this equation with the data points is shown in Figure D.6. ◀

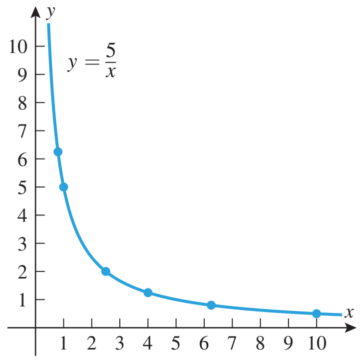
Inverse proportions arise in various laws of physics. For example, ***Boyle's law*** in physics states that *if a fixed amount of an ideal gas is held at a constant temperature, then the product of the pressure P exerted by the gas and the volume V that it occupies is constant*; that is,

$$PV = k$$

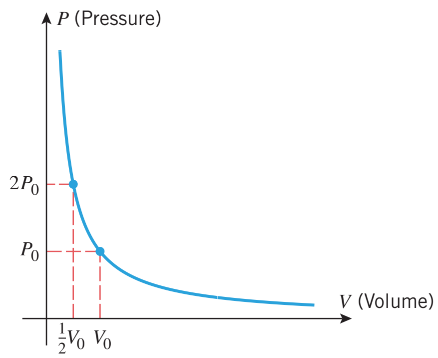
This implies that the variables P and V are inversely proportional to one another. Figure D.7 shows a typical graph of volume versus pressure under the conditions of Boyle's law. Note how doubling the pressure corresponds to halving the volume, as expected.



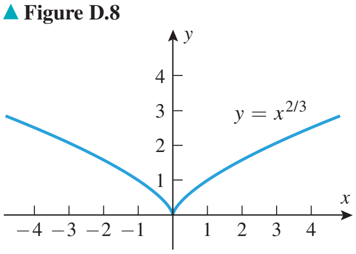
▲ **Figure D.8**



▲ **Figure D.6**



▲ **Figure D.7** Doubling pressure corresponds to halving volume



▲ **Figure D.9**

POWER FUNCTIONS WITH NONINTEGER EXPONENTS

If $p = 1/n$, where n is a positive integer, then the power functions $f(x) = x^p$ have the form

$$f(x) = x^{1/n} = \sqrt[n]{x}$$

In particular, if $n = 2$, then $f(x) = \sqrt{x}$, and if $n = 3$, then $f(x) = \sqrt[3]{x}$. The graphs of these functions are shown in parts (a) and (b) of Figure D.8.

Since every real number has a real cube root, the domain of the function $f(x) = \sqrt[3]{x}$ is $(-\infty, +\infty)$, and hence the graph of $y = \sqrt[3]{x}$ extends over the entire x-axis. In contrast, the graph of $y = \sqrt{x}$ extends only over the interval $[0, +\infty)$ because $\sqrt{x}$ is imaginary for negative x. As illustrated in Figure D.8c, the graphs of $y = \sqrt{x}$ and $y = -\sqrt{x}$ form the upper and lower halves of the parabola $x = y^2$. In general, the graph of $y = \sqrt[n]{x}$ extends over the entire x-axis if n is odd, but extends only over the interval $[0, +\infty)$ if n is even.

Power functions can have other fractional exponents. Some examples are

$$f(x) = x^{2/3}, \quad f(x) = \sqrt[5]{x^3}, \quad f(x) = x^{-7/8} \tag{2}$$

The graph of $f(x) = x^{2/3}$ is shown in Figure D.9. We will discuss expressions involving irrational exponents later.

TECHNOLOGY MASTERY

Graphing utililties sometimes omit portions of the graph of a function involving fractional exponents (or radicals). If $f(x) = x^{p/q}$, where p/q is a positive fraction in *lowest terms*, then you can circumvent this problem as follows:

- If p is even and q is odd, then graph $g(x) = |x|^{p/q}$ instead of $f(x)$.
- If p is odd and q is odd, then graph $g(x) = (|x|/x)|x|^{p/q}$ instead of $f(x)$.

Use a graphing utility to generate graphs of $f(x) = \sqrt[5]{x^3}$ and $f(x) = x^{-7/8}$ that show all of their significant features.

POLYNOMIALS

A ***polynomial in x*** is a function that is expressible as a sum of finitely many terms of the form cx^n, where c is a constant and n is a nonnegative integer. Some examples of polynomials are

$$2x + 1, \quad 3x^2 + 5x - \sqrt{2}, \quad x^3, \quad 4\,(=4x^0), \quad 5x^7 - x^4 + 3$$

The function $(x^2 - 4)^3$ is also a polynomial because it can be expanded by the binomial formula (see the inside front cover) and expressed as a sum of terms of the form cx^n:

$$(x^2 - 4)^3 = (x^2)^3 - 3(x^2)^2(4) + 3(x^2)(4^2) - (4^3) = x^6 - 12x^4 + 48x^2 - 64 \tag{3}$$

A more detailed review of polynomials appears in Web Appendix J.

A general polynomial can be written in either of the following forms, depending on whether one wants the powers of x in ascending or descending order:

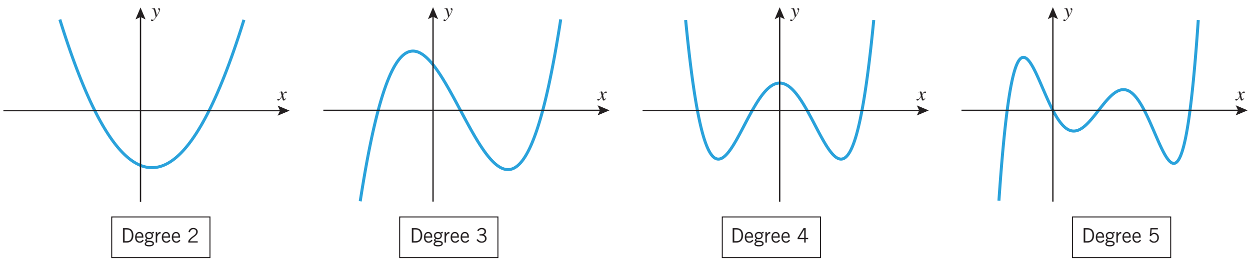
$$c_0 + c_1x + c_2x^2 + \cdots + c_nx^n$$
$$c_nx^n + c_{n-1}x^{n-1} + \cdots + c_1x + c_0$$

The constants $c_0, c_1, \ldots, c_n$ are called the ***coefficients*** of the polynomial. When a polynomial is expressed in one of these forms, the highest power of x that occurs with a nonzero coefficient is called the ***degree*** of the polynomial. Nonzero constant polynomials are considered to have degree 0, since we can write $c = cx^0$. Polynomials of degree 1, 2, 3, 4, and 5 are described as ***linear***, ***quadratic***, ***cubic***, ***quartic***, and ***quintic***, respectively. For example,

The constant 0 is a polynomial called the ***zero polynomial***. In this text we will take the degree of the zero polynomial to be undefined. Other texts may use different conventions for the degree of the zero polynomial.

$3 + 5x$	$x^2 - 3x + 1$	$2x^3 - 7$
Has degree 1 (linear)	Has degree 2 (quadratic)	Has degree 3 (cubic)
$8x^4 - 9x^3 + 5x - 3$	$\sqrt{3} + x^3 + x^5$	$(x^2 - 4)^3$
Has degree 4 (quartic)	Has degree 5 (quintic)	Has degree 6 [see (3)]

The natural domain of a polynomial in x is $(-\infty, +\infty)$, since the only operations involved are multiplication and addition; the range depends on the particular polynomial. We already know that the graphs of polynomials of degree 0 and 1 are lines and that the graphs of polynomials of degree 2 are parabolas. Figure D.10 shows the graphs of some typical polynomials of higher degree. Later, we will discuss polynomial graphs in detail, but for now it suffices to observe that graphs of polynomials are very well behaved in the sense that they have no discontinuities or sharp corners. As illustrated in Figure D.10, the graphs of polynomials wander up and down for awhile in a roller-coaster fashion, but eventually that behavior stops and the graphs steadily rise or fall indefinitely as one travels along the curve in either the positive or negative direction. We will see later that the number of peaks and valleys is less than the degree of the polynomial.



▲ **Figure D.10**

RATIONAL FUNCTIONS

A function that can be expressed as a ratio of two polynomials is called a ***rational function***. If $P(x)$ and $Q(x)$ are polynomials, then the domain of the rational function

$$f(x) = \frac{P(x)}{Q(x)}$$

consists of all values of x such that $Q(x) \neq 0$. For example, the domain of the rational function

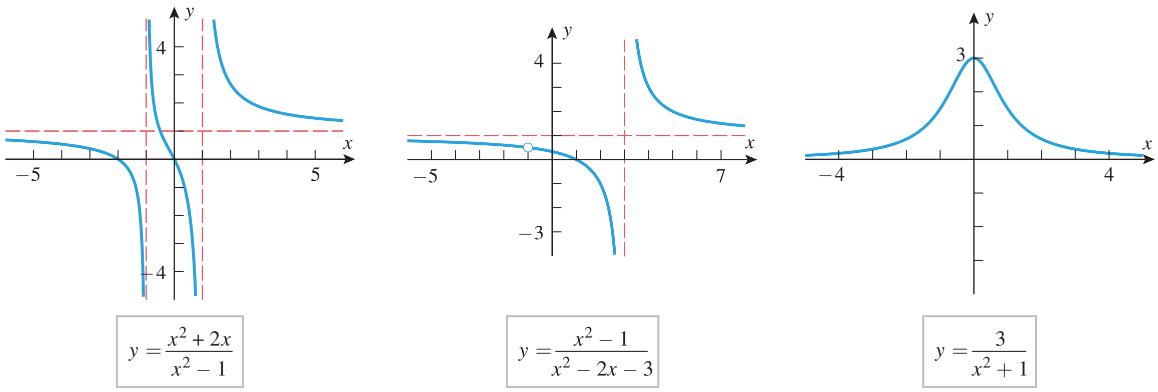
$$f(x) = \frac{x^2 + 2x}{x^2 - 1}$$

consists of all values of x, except $x = 1$ and $x = -1$. Its graph is shown in Figure D.11 along with the graphs of two other typical rational functions.

The graphs of rational functions with nonconstant denominators differ from the graphs of polynomials in some essential ways:

- Unlike polynomials whose graphs are continuous (unbroken) curves, the graphs of rational functions have discontinuities at the points where the denominator is zero.

- Unlike polynomials, rational functions may have numbers at which they are not defined. Near such points, many rational functions have graphs that closely approximate a vertical line, called a ***vertical asymptote***. These are represented by the dashed vertical lines in Figure D.11.
- Unlike the graphs of nonconstant polynomials, which eventually rise or fall indefinitely, the graphs of many rational functions eventually get closer and closer to some horizontal line, called a ***horizontal asymptote***, as one traverses the curve in either the positive or negative direction. The horizontal asymptotes are represented by the dashed horizontal lines in the first two parts of Figure D.11. In the third part of the figure the x-axis is a horizontal asymptote.



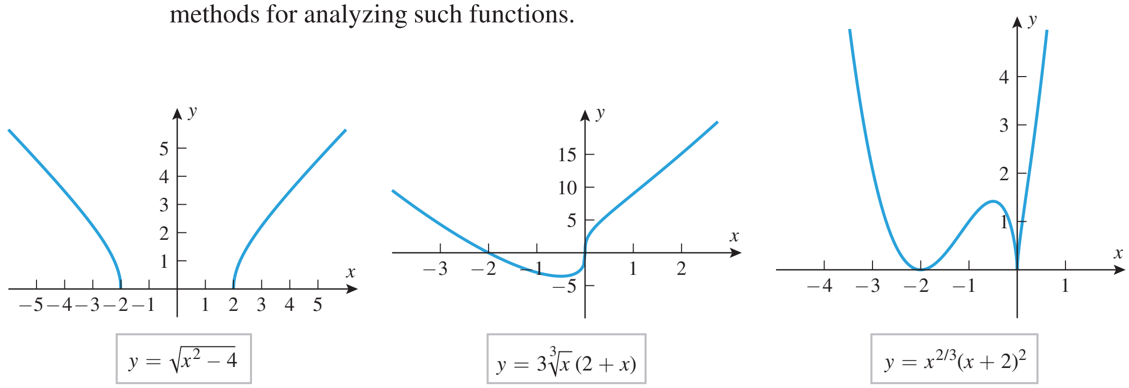
▲ **Figure D.11**

ALGEBRAIC FUNCTIONS

Functions that can be constructed from polynomials by applying finitely many algebraic operations (addition, subtraction, multiplication, division, and root extraction) are called ***algebraic functions***. Some examples are

$$f(x) = \sqrt{x^2 - 4}, \quad f(x) = 3\sqrt[3]{x}(2 + x), \quad f(x) = x^{2/3}(x + 2)^2$$

As illustrated in Figure D.12, the graphs of algebraic functions vary widely, so it is difficult to make general statements about them. Later in this text we will develop general calculus methods for analyzing such functions.



▲ **Figure D.12**

THE FAMILIES *y* = *A* sin *Bx* AND *y* = *A* cos *Bx*

Many important applications lead to trigonometric functions of the form

$$f(x) = A \sin(Bx - C) \quad \text{and} \quad g(x) = A \cos(Bx - C) \tag{4}$$

where A, B, and C are nonzero constants. The graphs of such functions can be obtained by stretching, compressing, translating, and reflecting the graphs of $y = \sin x$ and $y = \cos x$

> In this text we will assume that the independent variable of a trigonometric function is in radians unless otherwise stated. A review of trigonometric functions can be found in Appendix A.

appropriately. To see why this is so, let us start with the case where $C = 0$ and consider how the graphs of the equations

$$y = A \sin Bx \quad \text{and} \quad y = A \cos Bx$$

relate to the graphs of $y = \sin x$ and $y = \cos x$. If A and B are positive, then the effect of the constant A is to stretch or compress the graphs of $y = \sin x$ and $y = \cos x$ vertically and the effect of the constant B is to compress or stretch the graphs of $\sin x$ and $\cos x$ horizontally. For example, the graph of $y = 2 \sin 4x$ can be obtained by stretching the graph of $y = \sin x$ vertically by a factor of 2 and compressing it horizontally by a factor of 4. (Recall from Appendix C that the multiplier of x *stretches* when it is less than 1 and *compresses* when it is greater than 1.) Thus, as shown in Figure D.13, the graph of $y = 2 \sin 4x$ varies between -2 and 2, and repeats every $2\pi/4 = \pi/2$ units.

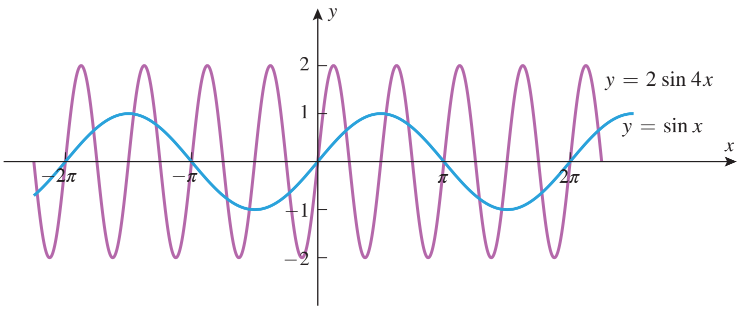


Figure D.13

In general, if A and B are positive numbers, then the graphs of

$$y = A \sin Bx \quad \text{and} \quad y = A \cos Bx$$

oscillate between $-A$ and A and repeat every $2\pi/B$ units, so we say that these functions have ***amplitude*** A and ***period*** $2\pi/B$. In addition, we define the ***frequency*** of these functions to be the reciprocal of the period, that is, the frequency is $B/2\pi$. If A or B is negative, then these constants cause reflections of the graphs about the axes as well as compressing or stretching them; and in this case the amplitude, period, and frequency are given by

$$\text{amplitude} = |A|, \quad \text{period} = \frac{2\pi}{|B|}, \quad \text{frequency} = \frac{|B|}{2\pi}$$

Example 2 Make sketches of the following graphs that show the period and amplitude.

(a) $y = 3 \sin 2\pi x$ (b) $y = -3 \cos 0.5x$ (c) $y = 1 + \sin x$

Solution (a). The equation is of the form $y = A \sin Bx$ with $A = 3$ and $B = 2\pi$, so the graph has the shape of a sine function, but it has an amplitude of $A = 3$ and a period of $2\pi/B = 2\pi/2\pi = 1$ (Figure D.14*a*).

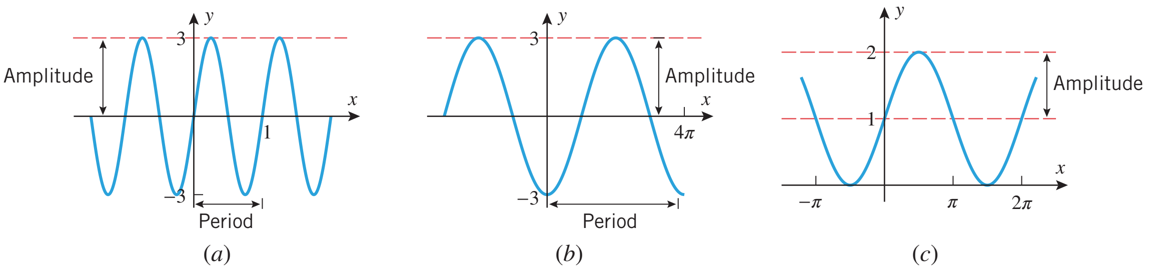


Figure D.14

Solution (b). The equation is of the form $y = A\cos Bx$ with $A = -3$ and $B = 0.5$, so the graph has the shape of a cosine curve that has been reflected about the x-axis (because $A = -3$ is negative), but with amplitude $|A| = 3$ and period $2\pi/B = 2\pi/0.5 = 4\pi$ (Figure D.14*b*).

Solution (c). The graph has the shape of a sine curve that has been translated up 1 unit (Figure D.14*c*). ◀

THE FAMILIES $y = A\sin(Bx - C)$ AND $y = A\cos(Bx - C)$

To investigate the graphs of the more general families

$$y = A\sin(Bx - C) \quad \text{and} \quad y = A\cos(Bx - C)$$

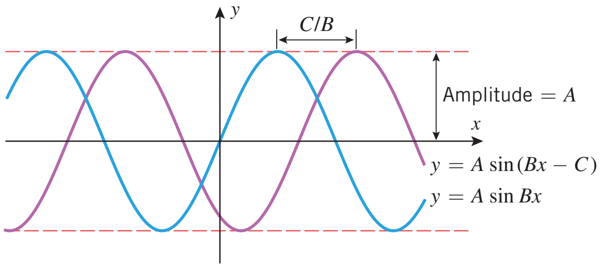
it will be helpful to rewrite these equations as

$$y = A\sin\left[B\left(x - \frac{C}{B}\right)\right] \quad \text{and} \quad y = A\cos\left[B\left(x - \frac{C}{B}\right)\right]$$

In this form we see that the graphs of these equations can be obtained by translating the graphs of $y = A\sin Bx$ and $y = A\cos Bx$ to the left or right, depending on the sign of C/B. For example, if $C/B > 0$, then the graph of

$$y = A\sin[B(x - C/B)] = A\sin(Bx - C)$$

can be obtained by translating the graph of $y = A\sin Bx$ to the right by C/B units (Figure D.15). If $C/B < 0$, the graph of $y = A\sin(Bx - C)$ is obtained by translating the graph of $y = A\sin Bx$ to the left by $|C/B|$ units.



▶ **Figure D.15**

▶ Example 3 Find the amplitude and period of

$$y = 3\cos\left(2x + \frac{\pi}{2}\right)$$

and determine how the graph of $y = 3\cos 2x$ should be translated to produce the graph of this equation. Confirm your results by graphing the equation on a calculator or computer.

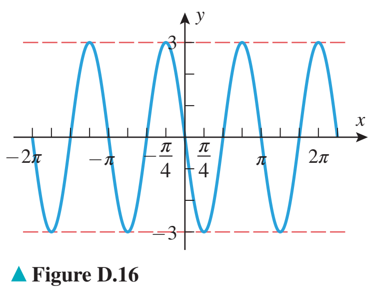
Solution. The equation can be rewritten as

$$y = 3\cos\left[2x - \left(-\frac{\pi}{2}\right)\right] = 3\cos\left[2\left(x - \left(-\frac{\pi}{4}\right)\right)\right]$$

which is of the form

$$y = A\cos\left[B\left(x - \frac{C}{B}\right)\right]$$

with $A = 3$, $B = 2$, and $C/B = -\pi/4$. It follows that the amplitude is $A = 3$, the period is $2\pi/B = \pi$, and the graph is obtained by translating the graph of $y = 3\cos 2x$ left by $|C/B| = \pi/4$ units (Figure D.16). ◀



▲ **Figure D.16**

EXERCISE SET D

Graphing Utility

1. (a) Find an equation for the family of lines whose members have slope $m = 3$.
(b) Find an equation for the member of the family that passes through $(-1, 3)$.
(c) Sketch some members of the family, and label them with their equations. Include the line in part (b).

2. Find an equation for the family of lines whose members are perpendicular to those in Exercise 1.

3. (a) Find an equation for the family of lines with y-intercept $b = 2$.
(b) Find an equation for the member of the family whose angle of inclination is $135°$.
(c) Sketch some members of the family, and label them with their equations. Include the line in part (b).

4. Find an equation for
(a) the family of lines that pass through the origin
(b) the family of lines with x-intercept $a = 1$
(c) the family of lines that pass through the point $(1, -2)$
(d) the family of lines parallel to $2x + 4y = 1$.

5. Find an equation for the family of lines tangent to the circle with center at the origin and radius 3.

6. Find an equation for the family of lines that pass through the intersection of $5x - 3y + 11 = 0$ and $2x - 9y + 7 = 0$.

7. The U.S. Internal Revenue Service uses a 10-year linear depreciation schedule to determine the value of various business items. This means that an item is assumed to have a value of zero at the end of the tenth year and that at intermediate times the value is a linear function of the elapsed time. Sketch some typical depreciation lines, and explain the practical significance of the y-intercepts.

8. Find all lines through $(6, -1)$ for which the product of the x- and y-intercepts is 3.

FOCUS ON CONCEPTS

9–10 State a geometric property common to all lines in the family, and sketch five of the lines.

9. (a) The family $y = -x + b$
(b) The family $y = mx - 1$
(c) The family $y = m(x + 4) + 2$
(d) The family $x - ky = 1$

10. (a) The family $y = b$
(b) The family $Ax + 2y + 1 = 0$
(c) The family $2x + By + 1 = 0$
(d) The family $y - 1 = m(x + 1)$

11. In each part, match the equation with one of the accompanying graphs.
(a) $y = \sqrt[5]{x}$ (b) $y = 2x^5$
(c) $y = -1/x^8$ (d) $y = \sqrt{x^2 - 1}$
(e) $y = \sqrt[4]{x - 2}$ (f) $y = -\sqrt[5]{x^2}$

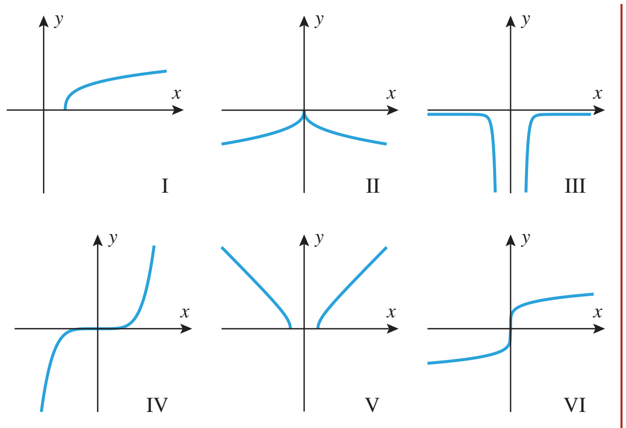


Figure Ex-11

12. The accompanying table gives approximate values of three functions: one of the form kx^2, one of the form kx^{-3}, and one of the form $kx^{3/2}$. Identify which is which, and estimate k in each case.

x	0.25	0.37	2.1	4.0	5.8	6.2	7.9	9.3
$f(x)$	640	197	1.08	0.156	0.0513	0.0420	0.0203	0.0124
$g(x)$	0.0312	0.0684	2.20	8.00	16.8	19.2	31.2	43.2
$h(x)$	0.250	0.450	6.09	16.0	27.9	30.9	44.4	56.7

Table Ex-12

13–14 Sketch the graph of the equation for $n = 1$, 3, and 5 in one coordinate system and for $n = 2$, 4, and 6 in another coordinate system. If you have a graphing utility, use it to check your work.

13. (a) $y = -x^n$ (b) $y = 2x^{-n}$ (c) $y = (x - 1)^{1/n}$

14. (a) $y = 2x^n$ (b) $y = -x^{-n}$
(c) $y = -3(x + 2)^{1/n}$

15. (a) Sketch the graph of $y = ax^2$ for $a = \pm 1, \pm 2$, and ± 3 in a single coordinate system.
(b) Sketch the graph of $y = x^2 + b$ for $b = \pm 1, \pm 2$, and ± 3 in a single coordinate system.
(c) Sketch some typical members of the family of curves $y = ax^2 + b$.

16. (a) Sketch the graph of $y = a\sqrt{x}$ for $a = \pm 1, \pm 2$, and ± 3 in a single coordinate system.
(b) Sketch the graph of $y = \sqrt{x} + b$ for $b = \pm 1, \pm 2$, and ± 3 in a single coordinate system.
(c) Sketch some typical members of the family of curves $y = a\sqrt{x} + b$.

17–18 Sketch the graph of the equation by making appropriate transformations to the graph of a basic power function. If you have a graphing utility, use it to check your work.

17. (a) $y = 2(x+1)^2$ (b) $y = -3(x-2)^3$
(c) $y = \dfrac{-3}{(x+1)^2}$ (d) $y = \dfrac{1}{(x-3)^5}$

18. (a) $y = 1 - \sqrt{x+2}$ (b) $y = 1 - \sqrt[3]{x+2}$
(c) $y = \dfrac{5}{(1-x)^3}$ (d) $y = \dfrac{2}{(4+x)^4}$

19. Sketch the graph of $y = x^2 + 2x$ by completing the square and making appropriate transformations to the graph of $y = x^2$.

20. (a) Use the graph of $y = \sqrt{x}$ to help sketch the graph of $y = \sqrt{|x|}$.
(b) Use the graph of $y = \sqrt[3]{x}$ to help sketch the graph of $y = \sqrt[3]{|x|}$.

21. As discussed in this section, Boyle's law states that at a constant temperature the pressure P exerted by a gas is related to the volume V by the equation $PV = k$.
(a) Find the appropriate units for the constant k if pressure (which is force per unit area) is in newtons per square meter (N/m^2) and volume is in cubic meters (m^3).
(b) Find k if the gas exerts a pressure of 20,000 N/m^2 when the volume is 1 liter (0.001 m^3).
(c) Make a table that shows the pressures for volumes of 0.25, 0.5, 1.0, 1.5, and 2.0 liters.
(d) Make a graph of P versus V.

22. A manufacturer of cardboard drink containers wants to construct a closed rectangular container that has a square base and will hold $\frac{1}{10}$ liter (100 cm^3). Estimate the dimensions of the container that will require the least amount of material for its manufacture.

23–24 A variable y is said to be ***inversely proportional to the square of a variable*** x if y is related to x by an equation of the form $y = k/x^2$, where k is a nonzero constant, called the ***constant of proportionality***. This terminology is used in these exercises.

23. According to ***Coulomb's law***, the force F of attraction between positive and negative point charges is inversely proportional to the square of the distance x between them.
(a) Assuming that the force of attraction between two point charges is 0.0005 newton when the distance between them is 0.3 meter, find the constant of proportionality (with proper units).
(b) Find the force of attraction between the point charges when they are 3 meters apart.
(c) Make a graph of force versus distance for the two charges.
(d) What happens to the force as the particles get closer and closer together? What happens as they get farther and farther apart?

24. It follows from Newton's Law of Universal Gravitation that the weight W of an object (relative to the Earth) is inversely proportional to the square of the distance x between the object and the center of the Earth, that is, $W = C/x^2$.
(a) Assuming that a weather satellite weighs 2000 pounds on the surface of the Earth and that the Earth is a sphere of radius 4000 miles, find the constant C.
(b) Find the weight of the satellite when it is 1000 miles above the surface of the Earth.
(c) Make a graph of the satellite's weight versus its distance from the center of the Earth.
(d) Is there any distance from the center of the Earth at which the weight of the satellite is zero? Explain your reasoning.

25–28 True–False Determine whether the statement is true or false. Explain your answer.

25. Each curve in the family $y = 2x + b$ is parallel to the line $y = 2x$.

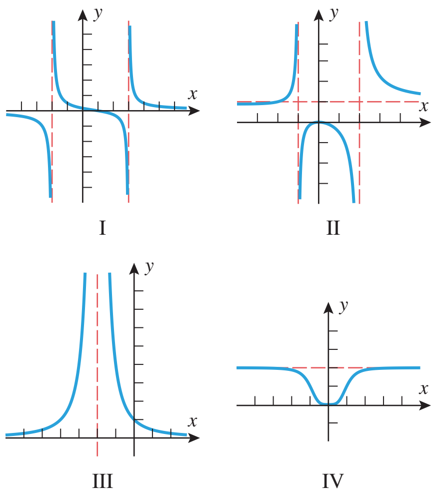
26. Each curve in the family $y = x^2 + bx + c$ is a translation of the graph of $y = x^2$.

27. If a curve passes through the point (2, 6) and y is inversely proportional to x, then the constant of proportionality is 3.

28. Curves in the family $y = -5\sin(A\pi x)$ have amplitude 5 and period $2/|A|$.

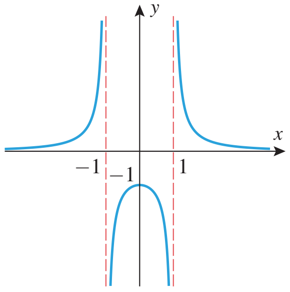
FOCUS ON CONCEPTS

29. In each part, match the equation with one of the accompanying graphs, and give the equations for the horizontal and vertical asymptotes.
(a) $y = \dfrac{x^2}{x^2 - x - 2}$ (b) $y = \dfrac{x-1}{x^2 - x - 6}$
(c) $y = \dfrac{2x^4}{x^4 + 1}$ (d) $y = \dfrac{4}{(x+2)^2}$



▲ **Figure Ex-29**

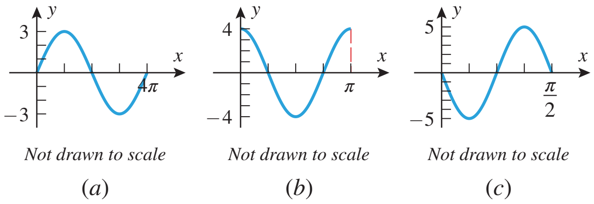
30. Find an equation of the form $y = k/(x^2 + bx + c)$ whose graph is a reasonable match to that in the accompanying figure. If you have a graphing utility, use it to check your work.



◀ **Figure Ex-30**

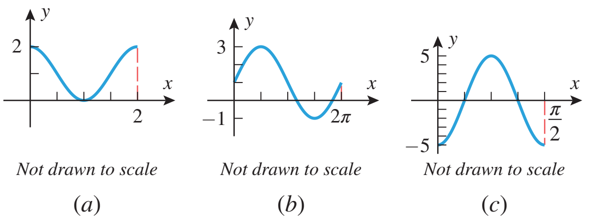
31–32 Find an equation of the form $y = D + A \sin Bx$ or $y = D + A \cos Bx$ for each graph.

31.



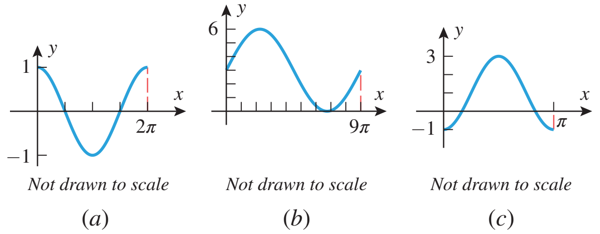
▲ **Figure Ex-31**

32.



▲ **Figure Ex-32**

33. In each part, find an equation for the graph that has the form $y = y_0 + A \sin(Bx - C)$.



▲ **Figure Ex-33**

34. In the United States, a standard electrical outlet supplies sinusoidal electrical current with a maximum voltage of $V = 120\sqrt{2}$ volts (V) at a frequency of 60 hertz (Hz). Write an equation that expresses V as a function of the time t, assuming that $V = 0$ if $t = 0$. [*Note:* 1 Hz = 1 cycle per second.]

35–36 Find the amplitude and period, and sketch at least two periods of the graph by hand. If you have a graphing utility, use it to check your work.

35. (a) $y = 3 \sin 4x$ (b) $y = -2 \cos \pi x$
(c) $y = 2 + \cos\left(\dfrac{x}{2}\right)$

36. (a) $y = -1 - 4 \sin 2x$ (b) $y = \frac{1}{2} \cos(3x - \pi)$
(c) $y = -4 \sin\left(\dfrac{x}{3} + 2\pi\right)$

37. Equations of the form

$$x = A_1 \sin \omega t + A_2 \cos \omega t$$

arise in the study of vibrations and other periodic motion. Express the equation

$$x = 5\sqrt{3} \sin 2\pi t + \tfrac{5}{2} \cos 2\pi t$$

in the form $x = A \sin(\omega t + \theta)$, and use a graphing utility to confirm that both equations have the same graph.

38. Determine the number of solutions of $x = 2 \sin x$, and use a graphing or calculating utility to estimate them.