Section 1.5

Solving Inequalities

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1 Use Interval Notation

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DEFINITION

Let a and b represent two real numbers with $a < b$:

A **closed interval**, denoted by $[\boldsymbol{a}, \boldsymbol{b}]$, consists of all real numbers x for which $a \leq x \leq b$.

An **open interval**, denoted by $(\boldsymbol{a}, \boldsymbol{b})$, consists of all real numbers x for which $a < x < b$.

The **half-open**, or **half-closed, intervals** are $(\boldsymbol{a}, \boldsymbol{b}]$, consisting of all real numbers x for which $a < x \leq b$, and $[\boldsymbol{a}, \boldsymbol{b})$, consisting of all real numbers x for which $a \leq x < b$.

$[\boldsymbol{a}, \infty)$	consists of all real numbers x for which $x \geq a$
$(\boldsymbol{a}, \infty)$	consists of all real numbers x for which $x > a$
$(-\infty, \boldsymbol{a}]$	consists of all real numbers x for which $x \leq a$
$(-\infty, \boldsymbol{a})$	consists of all real numbers x for which $x < a$
$(-\infty, \infty)$	consists of all real numbers x

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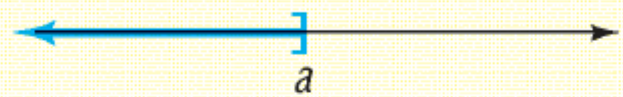
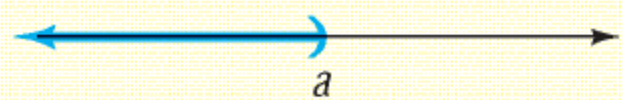
Interval	Inequality	Graph
The open interval (a, b)	$a < x < b$	
The closed interval $[a, b]$	$a \leq x \leq b$	
The half-open interval $[a, b)$	$a \leq x < b$	
The half-open interval $(a, b]$	$a < x \leq b$	
The interval $[a, \infty)$	$x \geq a$	
The interval (a, ∞)	$x > a$	
The interval $(-\infty, a]$	$x \leq a$	
The interval $(-\infty, a)$	$x < a$	
The interval $(-\infty, \infty)$	All real numbers	

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EXAMPLE

Writing Inequalities Using Interval Notation

Write each inequality using interval notation:

$a)\quad -2 \le x \le 4$ $\qquad [-2, 4]$

$b)\quad 2 < x < 7$ $\qquad (-2, 7)$

$c)\quad x \ge 6$ $\qquad [6, \infty)$

$d)\quad x < -3$ $\qquad (-\infty, -3)$

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EXAMPLE

Writing Intervals Using Inequality Notation

Write each interval as an inequality involving *x*.

$a)\quad (-1, 2] \qquad -1 < x \le 2$

$b)\quad [-2, 0] \qquad -2 \le x \le 0$

$c)\quad (5, \infty) \qquad x > 5$

$d)\quad (-\infty, 1) \qquad x < 1$

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2 Use Properties of Inequalities

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Nonnegative Property

For any real number a,

$$\boxed{a^2 \geq 0}$$

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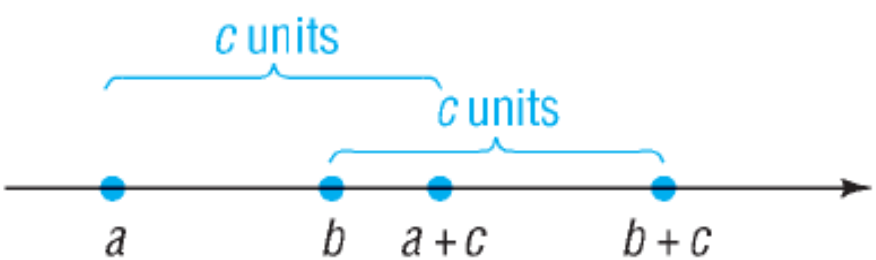
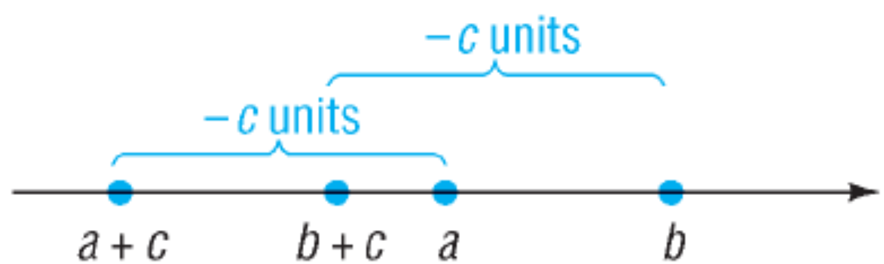
Addition Property of Inequalities

For real numbers a, b, and c,

$$\boxed{\begin{array}{l} \text{if } a < b, \text{ then } a + c < b + c \\ \text{if } a > b, \text{ then } a + c > b + c \end{array}}$$



(a) If $a < b$ and $c > 0$, then $a + c < b + c$.



(b) If $a < b$ and $c < 0$, then $a + c < b + c$.

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EXAMPLE

Addition Property of Inequalities

(a) If $x < -5$, then $x + 5 < -5 + 5$ or $x + 5 < 0$.

(b) If $x > 2$, then $x + (-2) > 2 + (-2)$ or $x - 2 > 0$.

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EXAMPLE

Multiplying an Inequality by a Positive Number

Express as an inequality the result of multiplying each side of the inequality $3 < 5$ by 2.

$$3(2) < 5(2)$$

$$6 < 10$$

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EXAMPLE

Multiplying an Inequality by a Negative Number

Express as an inequality the result of multiplying each side of the inequality $3 < 5$ by -2.

$$3(-2) \;?\; 5(-2)$$

$$-6 > -10$$

Note to keep this inequality true, the inequality symbol must be reversed.

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Multiplication Properties for Inequalities

For real numbers a, b, and c,

if $a < b$ and if $c > 0$, then $ac < bc$.

if $a < b$ and if $c < 0$, then $ac > bc$.

if $a > b$ and if $c > 0$, then $ac > bc$.

if $a > b$ and if $c < 0$, then $ac < bc$.

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EXAMPLE

Multiplication Property of Inequalities

(a) If $2x < 6$, then $\frac{1}{2}(2x) < \frac{1}{2}(6)$ or $x < 3$.

(b) If $\frac{x}{-3} > 12$, then $-3\left(\frac{x}{-3}\right) < -3(12)$ or $x < -36$.

(c) If $-4x > -8$, then $\frac{-4x}{-4} < \frac{-8}{-4}$ or $x < 2$.

(d) If $-x < 8$, then $(-1)(-x) > (-1)(8)$ or $x > -8$.

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Reciprocal Property for Inequalities

If $a > 0$, then $\frac{1}{a} > 0$ If $\frac{1}{a} > 0$, then $a > 0$

If $a < 0$, then $\frac{1}{a} < 0$ If $\frac{1}{a} < 0$, then $a < 0$

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3 Solve Inequalities

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Procedures That Leave the Inequality Symbol Unchanged

1. Simplify both sides of the inequality by combining like terms and eliminating parentheses:

$$\text{Replace} \quad (x + 2) + 6 > 2x + 5(x + 1)$$
$$\text{by} \quad x + 8 > 7x + 5$$

2. Add or subtract the same expression on both sides of the inequality:

$$\text{Replace} \quad 3x - 5 < 4$$
$$\text{by} \quad (3x - 5) + 5 < 4 + 5$$

3. Multiply or divide both sides of the inequality by the same positive expression:

$$\text{Replace} \quad 4x > 16 \quad \text{by} \quad \frac{4x}{4} > \frac{16}{4}$$

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Procedures That Reverse the Sense or Direction of the Inequality Symbol

1. Interchange the two sides of the inequality:

$$\text{Replace} \quad 3 < x \quad \text{by} \quad x > 3$$

2. Multiply or divide both sides of the inequality by the same *negative* expression:

$$\text{Replace} \quad -2x > 6 \quad \text{by} \quad \frac{-2x}{-2} < \frac{6}{-2}$$

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EXAMPLE

Solving an Inequality

Solve the inequality: $5-3x \geq -1$ and graph the solution set.

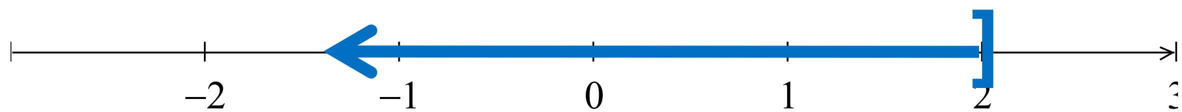
$$5-3x-5 \geq -1-5$$

$$-3x \geq -6$$

$$\frac{-3x}{-3} \leq \frac{-6}{-3}$$

$$x \leq 2$$

$$\{x \mid x \leq 2\} \text{ or } (-\infty, 2]$$



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EXAMPLE

Solving an Inequality

Solve the inequality: $4x+3<2x-1$ and graph the solution set.

$$4x+3-3<2x-1-3$$

$$4x<2x-4$$

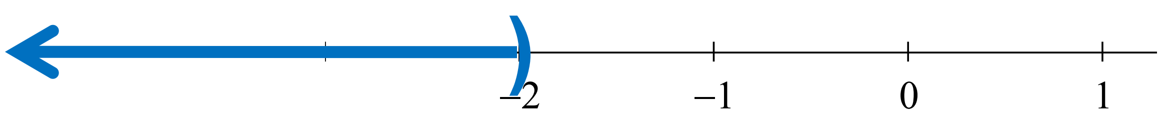
$$4x-2x<2x-4-2x$$

$$2x<-4$$

$$\frac{2x}{2}<\frac{-4}{2}$$

$$x<-2$$

$\{x \mid x<-2\}$ or $(-\infty,-2)$



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4 Solve Combined Inequalities

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EXAMPLE

Solving a Combined Inequality

Solve the inequality: $-1 < 3x + 2 < 5$ and graph the solution set.

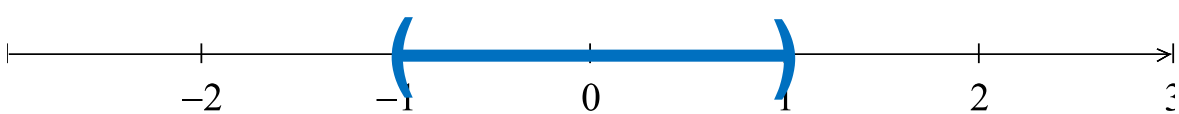
$$-1 < 3x + 2 \quad \text{and} \quad 3x + 2 < 5$$

$$-1 - 2 < 3x + 2 - 2 \qquad\qquad 3x + 2 - 2 < 5 - 2$$

$$-3 < 3x \qquad\qquad 3x < 3$$

$$\frac{-3}{3} < \frac{3x}{3} \qquad\qquad \frac{3x}{3} < \frac{3}{3}$$

$$-1 < x \quad \text{and} \quad x < 1$$



$$\{x \mid -1 < x < 1\} \quad \text{or} \quad (-1, 1)$$

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EXAMPLE

Solving a Combined Inequality

Solve the inequality: $1 \le \dfrac{5-2x}{3} \le 3$ and graph the solution set.

$$3(1) \le 3\left(\frac{5-2x}{3}\right) \le 3(3)$$

$$3 \le 5-2x \le 9$$

$$3-5 \le 5-5-2x \le 9-5$$

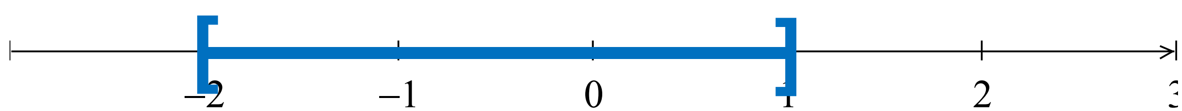
$$-2 \le -2x \le 4$$

$$\frac{-2}{-2} \ge \frac{-2x}{-2} \ge \frac{4}{-2}$$

$$1 \ge x \ge -2$$

$$-2 \le x \le 1$$

$$\{x \mid -2 \le x \le 1\} \text{ or } [-2,1]$$



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EXAMPLE

Using the Reciprocal Property to Solve an Inequality

Solve the inequality: $(3x+6)^{-1} > 0$ and graph the solution set.

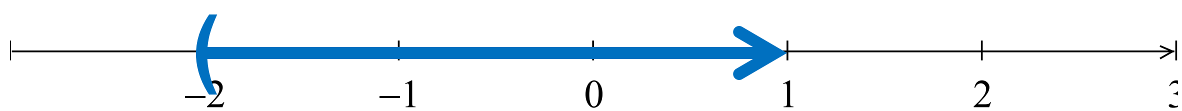
$$(3x+6)^{-1} = \frac{1}{3x+6} > 0$$

By the reciprocal property: $3x+6>0$

$$3x > -6$$

$$x > -2$$

$$\{x \mid x > -2\} \text{ or } (-2, \infty)$$



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EXAMPLE

Creating Equivalent Inequalities

If $-3 < x < 2$, find a and b so that $a < 3x + 2 < b$.

$$-3 < x < 2$$

$$3(-3) < 3x < 3(2)$$

$$-9 < 3x < 6$$

$$-9 + 2 < 3x + 2 < 6 + 2$$

$$-7 < 3x + 2 < 8$$

So $a = -7$ and $b = 8$

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Application

EXAMPLE **Physics: Ohm's Law**

In electricity, Ohm's law states that $E = IR$, where E is the voltage (in volts). I is the current (in amperes), and R is the resistance (in ohms). An air-conditioning unit is rated at a resistance of 10 ohms. If the voltage varies from 110 to 120 volts, inclusive, what corresponding range of current will the air conditioner draw?

$$110 \le E \le 120$$

$$110 \le IR \le 120$$

$$110 \le I(10) \le 120$$

$$\frac{110}{10} \le \frac{I(10)}{10} \le \frac{120}{10}$$

$$11 \le I \le 12$$

The air conditioner will draw between 11 and 12 amperes of current, inclusive.

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