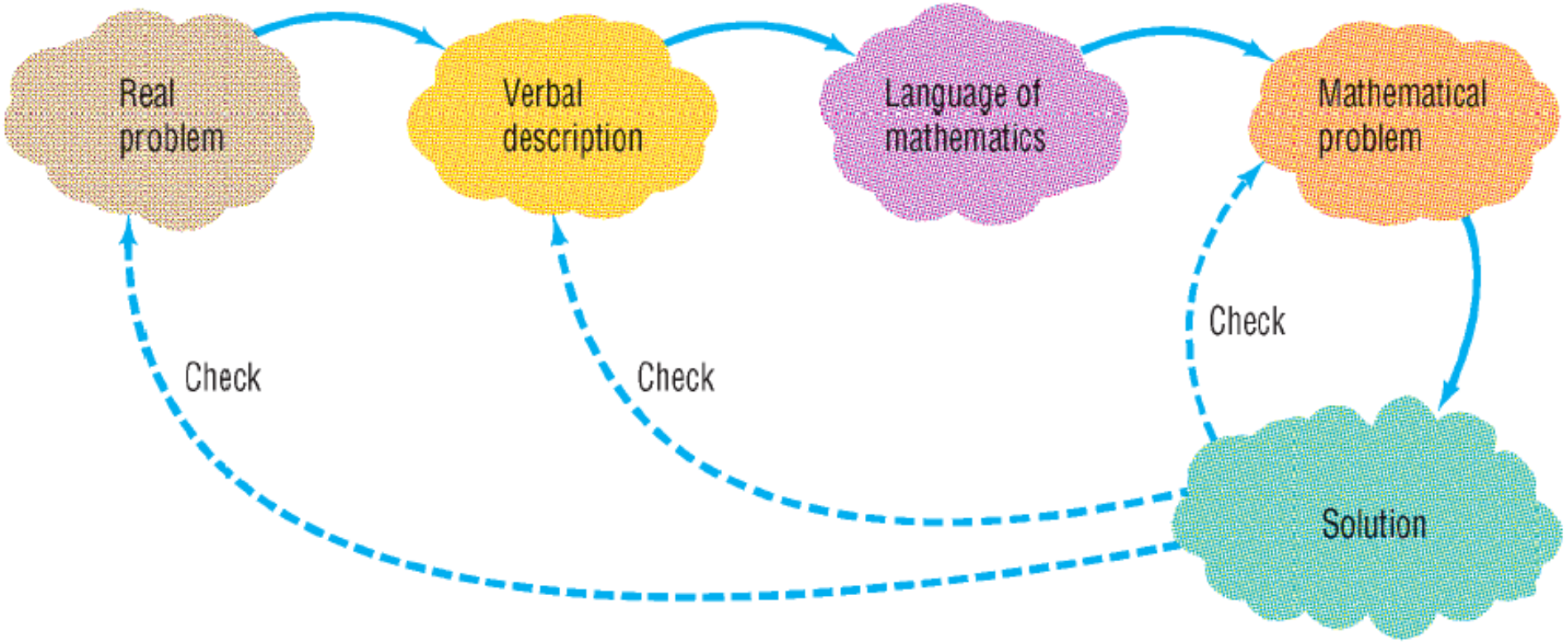
Section 1.7

Problem Solving;

Interest, Mixture, Uniform Motion, Constant Rate Job Applications



1 Translate Verbal Descriptions into Mathematical Expressions

EXAMPLE

Translating Verbal Descriptions into Mathematical Expressions

For uniform motion, the time required for an object to travel equals the distance traveled divided by the speed of the object.

If t is time, r is speed and d is distance, then $t = \frac{d}{r}$

The length of a rectangle is four times its width. $l = 4w$

Five times a number, decreased by 3 $5x - 3$

The product of 3 and a number increased by 5 $3x + 5$

Steps for Solving Applied Problems

Step 1: Read the problem carefully, perhaps two or three times. Pay particular attention to the question being asked in order to identify what you are looking for. If you can, determine realistic possibilities for the answer.

Step 2: Assign a letter (variable) to represent what you are looking for, and, if necessary, express any remaining unknown quantities in terms of this variable.

Step 3: Make a list of all the known facts, and translate them into mathematical expressions. These may take the form of an equation or an inequality involving the variable. If possible, draw an appropriately labeled diagram to assist you. Sometimes a table or chart helps.

Step 4: Solve the equation for the variable, and then answer the question.

Step 5: Check the answer with the facts in the problem. If it agrees, congratulations! If it does not agree, try again.

2 Solve Interest Problems

Simple Interest Formula

If a principal of P dollars is borrowed for a period of t years at a per annum interest rate r, expressed as a decimal, the interest I charged is

$$I = Prt$$

EXAMPLE Finance: Computing Interest on a Loan

Suppose you borrow $1000 for 6 months at the simple interest rate of 6% per annum. What is the interest you will be charged on this loan? If you pay the loan back at the end of 6 months, what is the amount you must pay?

$$I = Prt$$

$$I = (1000)(0.06)\left(\frac{1}{2}\right) = \$30.00$$

After 6 months you would owe the amount you borrowed plus the interest: $1000 + $30.00 = $1030

EXAMPLE Financial Planning

Candy has $70,000 to invest and requires an overall rate of return of 9%. She can invest in a safe, government-insured certificate of deposit, but it only pays 8%. To obtain 9%, she agrees to invest some of her money in noninsured corporate bonds paying 12%. How much should be placed in each investment to achieve her goal?

STEP 1: The question is asking for two dollar amounts: the principal to invest in the corporate bonds and the principal to invest in the certificate of deposit.

STEP 2: We let x represent the amount (in dollars) to be invested in the bonds. Then $70{,}000 - x$ is the amount that will be invested in the certificate. (Do you see why?)

Candy has $70,000 to invest and requires an overall rate of return of 9%. She can invest in a safe, government-insured certificate of deposit, but it only pays 8%. To obtain 9%, she agrees to invest some of her money in noninsured corporate bonds paying 12%. How much should be placed in each investment to achieve her goal?

STEP 3: We set up a table:

	Principal ($)	Rate	Time (yr)	Interest ($)
Bonds	x	12% = 0.12	1	$0.12x$
Certificate	$70{,}000 - x$	8% = 0.08	1	$0.08(70{,}000 - x)$
Total	70,000	9% = 0.09	1	$0.09(70{,}000) = 6300$

Since the total interest from the investments is equal to $0.09(70{,}000) = 6300$, we must have the equation

$$0.12x + 0.08(70{,}000 - x) = 6300$$

(Note that the units are consistent: the unit is dollars on each side.)

STEP 4:
$$\begin{aligned} 0.12x + 5600 - 0.08x &= 6300 \\ 0.04x &= 700 \\ x &= 17{,}500 \end{aligned}$$

Candy should place $17,500 in the bonds and $70,000 − $17,500 = $52,500 in the certificate.

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STEP 4:
$$0.12x + 5600 - 0.08x = 6300$$
$$0.04x = 700$$
$$x = 17{,}500$$

Candy should place $17,500 in the bonds and $70,000 − $17,500 = $52,500 in the certificate.

STEP 5: The interest on the bonds after 1 year is 0.12($17,500) = $2100; the interest on the certificate after 1 year is 0.08($52,500) = $4200. The total annual interest is $6300, the required amount. ◀

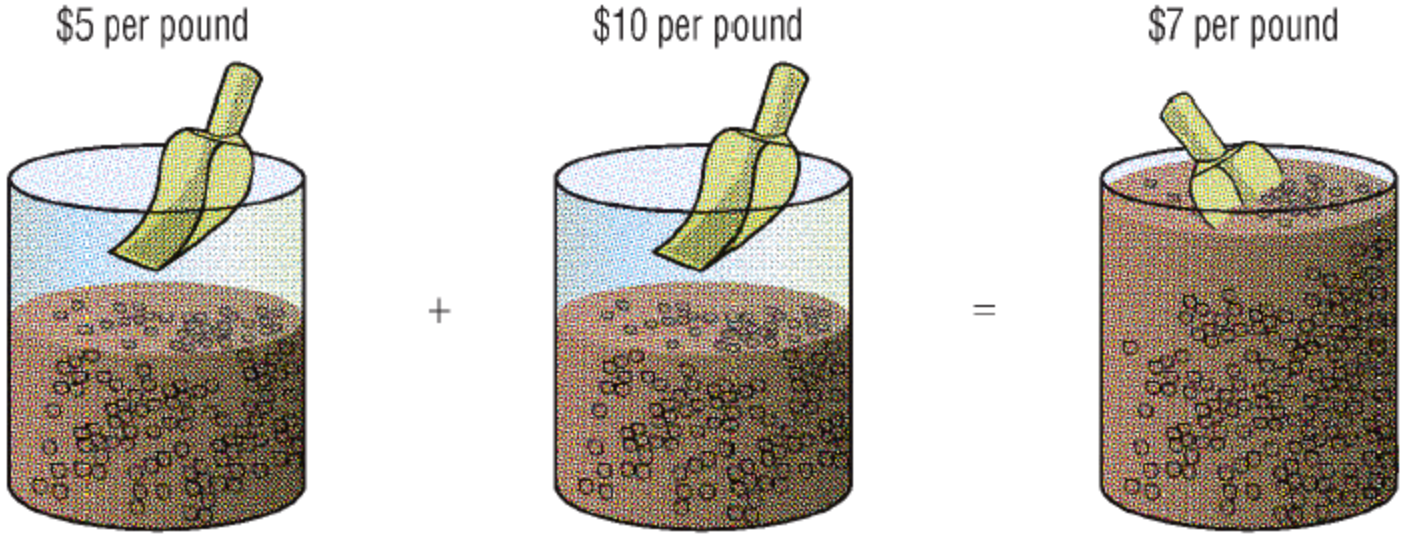
3 Solve Mixture Problems

EXAMPLE

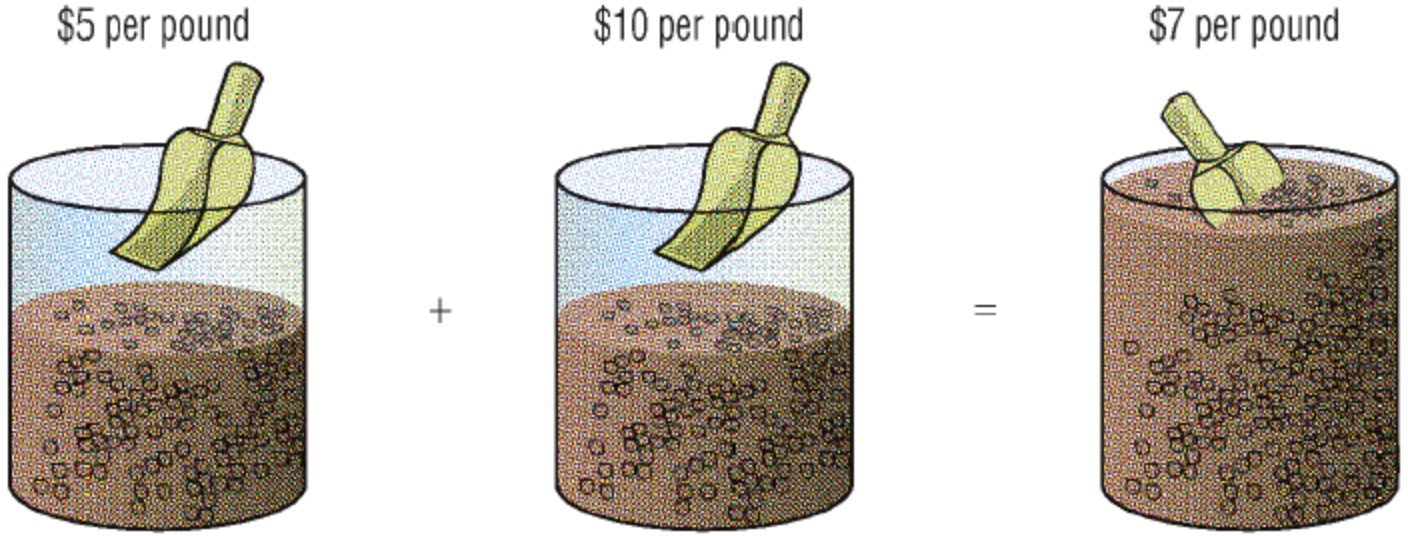
Blending Coffees

The manager of a Starbucks store decides to experiment with a new blend of coffee. She will mix some B grade Colombian coffee that sells for $5 per pound with some A grade Arabica coffee that sells for $10 per pound to get 100 pounds of the new blend. The selling price of the new blend is to be $7 per pound, and there is to be no difference in revenue from selling the new blend versus selling the other types. How many pounds of the B grade Colombian and A grade Arabica coffees are required?

Let x represent the number of pounds of the B grade Colombian coffee. Then $100 - x$ equals the number of pounds of the A grade Arabica coffee.



all.



B Grade Colombian
x pounds

+

A Grade Arabica
$100 - x$ pounds

=

Blend
100 pounds

$$\left\{\begin{matrix}\text{Price per pound}\\ \text{of B grade}\end{matrix}\right\}\left\{\begin{matrix}\text{\# Pounds}\\ \text{B grade}\end{matrix}\right\} + \left\{\begin{matrix}\text{Price per pound}\\ \text{of A grade}\end{matrix}\right\}\left\{\begin{matrix}\text{\# Pounds}\\ \text{A grade}\end{matrix}\right\} = \left\{\begin{matrix}\text{Price per pound}\\ \text{of blend}\end{matrix}\right\}\left\{\begin{matrix}\text{\# Pounds}\\ \text{blend}\end{matrix}\right\}$$

$$\$5 \cdot x + \$10 \cdot (100 - x) = \$7 \cdot 100$$

$$\begin{aligned} 5x + 10(100 - x) &= 700 \\ 5x + 1000 - 10x &= 700 \\ -5x &= -300 \\ x &= 60 \end{aligned}$$

The manager should blend 60 pounds of B grade Colombian coffee with $100 - 60 = 40$ pounds of A grade Arabica coffee.

4 Solve Uniform Motion Problems

Uniform Motion Formula

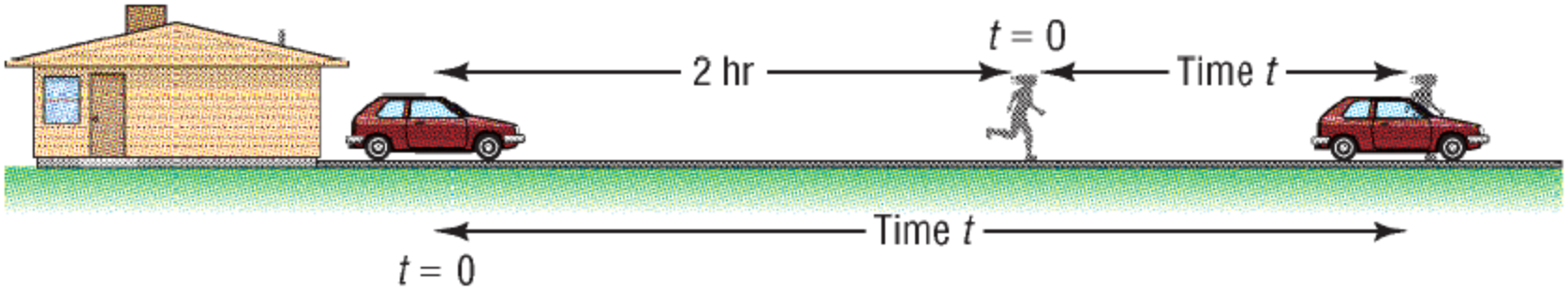
If an object moves at an average speed (rate) r, the distance d covered in time t is given by the formula

$$d = rt \qquad (2)$$

That is, Distance $=$ Rate $\cdot$ Time.

EXAMPLE Physics: Uniform Motion

Tanya, who is a long-distance runner, runs at an average velocity of 8 miles per hour (mi/hr). Two hours after Tanya leaves your house, you leave in your Honda and follow the same route. If your average velocity is 40 mi/hr, how long will it be before you catch up to Tanya? How far will each of you be from your home?



	Rate mi/hr	Time hr	Distance mi
Tanya	8	$t + 2$	$8(t + 2)$
Honda	40	t	$40t$

	Rate mi/hr	Time hr	Distance mi
Tanya	8	$t + 2$	$8(t + 2)$
Honda	40	t	$40t$

$$8(t+2) = 40t$$

$$8t + 16 = 40t$$

$$16 = 32t$$

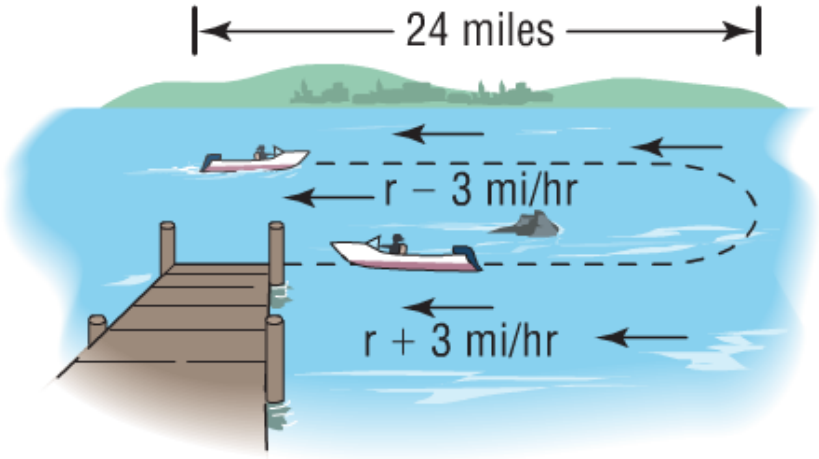
$$\frac{1}{2} = t$$

It will take the Honda $\frac{1}{2}$ hour to catch up to Tanya. Each will have gone 20 miles.

EXAMPLE

Physics: Uniform Motion

A motorboat heads upstream a distance of 24 miles on a river whose current is running at 3 miles per hour (mi/hr). The trip up and back takes 6 hours. Assuming that the motorboat maintained a constant speed relative to the water, what was its speed?



$$\text{Time} = \frac{\text{Distance}}{\text{Rate}}$$

	Rate mi/hr	Distance mi	Time hr $= \frac{\text{Distance}}{\text{Rate}}$
Upstream	$r - 3$	24	$\frac{24}{r-3}$
Downstream	$r + 3$	24	$\frac{24}{r+3}$

	Rate mi/hr	Distance mi	Time hr $= \frac{\text{Distance}}{\text{Rate}}$
Upstream	$r - 3$	24	$\frac{24}{r-3}$
Downstream	$r + 3$	24	$\frac{24}{r+3}$

$$\frac{24}{r-3} + \frac{24}{r+3} = 6$$

$$\frac{24(r+3) + 24(r-3)}{(r-3)(r+3)} = 6$$

$$\frac{48r}{r^2 - 9} = 6$$

$$48r = 6(r^2 - 9)$$

$$6r^2 - 48r - 54 = 0$$

$$r^2 - 8r - 9 = 0$$

$$(r - 9)(r + 1) = 0$$

$$r = 9 \quad \text{or} \quad r = -1$$

We discard the solution $r = -1$ mi/hr, so the speed of the motorboat relative to the water is 9 mi/hr.

5 Solve Constant Rate Job Problems

EXAMPLE Working Together to Do a Job

At 10 AM Danny is asked by his father to weed the garden. From past experience, Danny knows that this will take him 4 hours, working alone. His older brother, Mike, when it is his turn to do this job, requires 6 hours. Since Mike wants to go golfing with Danny and has a reservation for 1 PM, he agrees to help Danny. Assuming no gain or loss of efficiency, when will they finish if they work together? Can they make the golf date?

Working together, the job can be done in $\frac{12}{5}$ hours, or 2 hours, 24 minutes. They should make the golf date, since they will finish at 12:24 PM. ◀

	Hours to Do Job	Part of Job Done in 1 Hour
Danny	4	$\frac{1}{4}$
Mike	6	$\frac{1}{6}$
Together	t	$\frac{1}{t}$

$$\frac{1}{4} + \frac{1}{6} = \frac{1}{t}$$

$$\frac{3}{12} + \frac{2}{12} = \frac{1}{t}$$

$$\frac{5}{12} = \frac{1}{t}$$

$$5t = 12$$

$$t = \frac{12}{5}$$