

Section 4.5

Inequalities Involving Quadratic Functions

1 **Solve Inequalities Involving a Quadratic Function**

EXAMPLE

Solving an Inequality

Solve the inequality $x^2 + 5x + 4 > 0$ and graph the solution set.

Graph the function $f(x) = x^2 + 5x + 4$. **Where is this function greater than 0?**

y-intercept: $f(0) = 0^2 + 5(0) + 4 = 4$

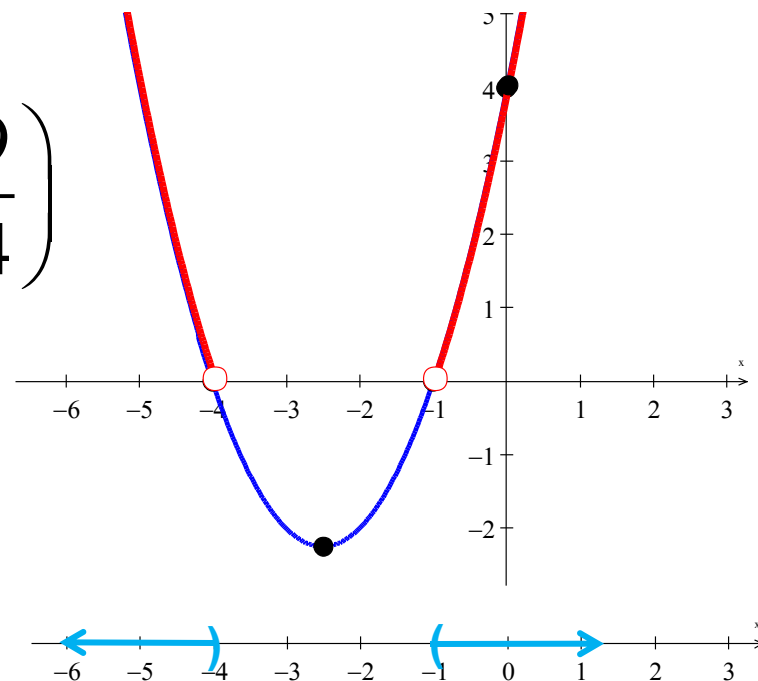
x-intercepts: $0 = x^2 + 5x + 4 = (x + 4)(x + 1)$

$x = -4$ or -1

Vertex: $x = -\frac{b}{2a} = -\frac{5}{2} \quad \left(-\frac{5}{2}, -\frac{9}{4}\right)$

$$y = \left(-\frac{5}{2}\right)^2 + 5\left(-\frac{5}{2}\right) + 4 = -\frac{9}{4}$$

$(-\infty, -4)$ or $(-1, \infty)$



EXAMPLE

Solving an Inequality

Solve the inequality $x^2 \leq x + 6$ and graph the solution set.

Method 1 Rearrange the inequality so that 0 is on the right side.

$$x^2 - x - 6 \leq 0$$

Graph the function $f(x) = x^2 - x - 6$.

y-intercept: $f(0) = 0^2 - (0) - 6 = -6$

x-intercepts: $0 = x^2 - x - 6 = (x - 3)(x + 2)$

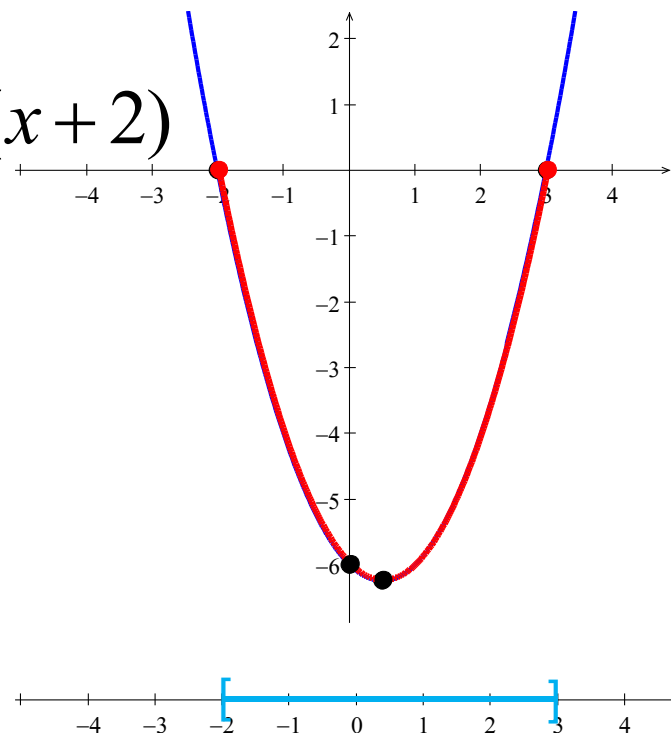
$$x = 3 \text{ or } -2$$

$$\text{Vertex: } x = -\frac{b}{2a} = \frac{1}{2} \quad \left(\frac{1}{2}, -\frac{25}{4}\right)$$

$$y = \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right) - 6 = -\frac{25}{4}$$

$$[-2, 3]$$

Where is this function less than or equal to 0?



EXAMPLE

Solving an Inequality

Solve the inequality $x^2 \leq x + 6$ and graph the solution set.

Method 2

If $f(x) = x^2$ and $g(x) = x + 6$ then we want to solve $f(x) \leq g(x)$.

Graph $f(x) = x^2$ and $g(x) = x + 6$.

Points of intersection: $x^2 = x + 6$

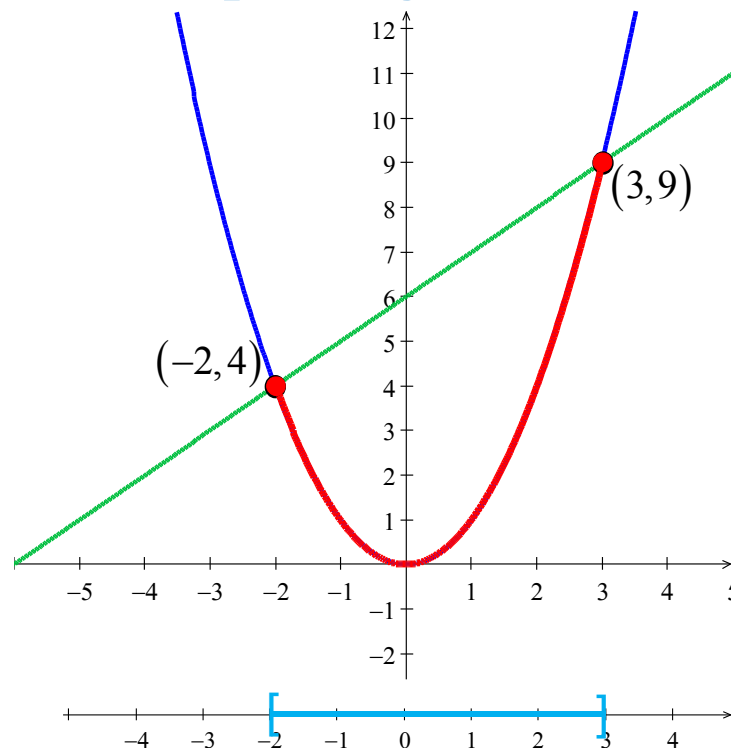
$$x^2 - x - 6 = 0 \quad (x - 3)(x + 2) = 0$$

$$x = 3 \text{ or } -2$$

$$f(3) = 9 \quad f(-2) = 4$$

$$[-2, 3]$$

Where is $f(x)$ less than or equal to $g(x)$?



EXAMPLE**Solving an Inequality**

Solve the inequality $-2x^2 + 8x - 9 < 0$ and graph the solution set.

Graph the function $f(x) = -2x^2 + 8x - 9$.

$$\text{Vertex: } x = -\frac{b}{2a} = -\frac{8}{2(-2)} = 2$$

$$y = -2(2)^2 + 8(2) - 9 = -1 \quad (2, -1)$$

$$y\text{-intercept: } f(0) = -2(0)^2 + 8(0) - 9 = -9$$

By symmetry, the point $(4, -9)$ is also on the graph.

This function is always less than 0
so the solution set is all real numbers.

