

Section 8.3

Trigonometric Equations

1 Solve Equations Involving a Single Trigonometric Function

EXAMPLE**Checking Whether a Given Number Is a Solution of a Trigonometric Equation**

Determine whether $\theta = \frac{\pi}{4}$ is a solution of the equation $2 \sin \theta + \sqrt{2} = 0$.

Is $\theta = \frac{5\pi}{4}$ a solution?

$$2 \sin \frac{\pi}{4} + \sqrt{2} = 2 \left(\frac{\sqrt{2}}{2} \right) + \sqrt{2} = \sqrt{2} + \sqrt{2} = 2\sqrt{2} \neq 0$$

$\frac{\pi}{4}$ is NOT a solution.

$$2 \sin \frac{5\pi}{4} + \sqrt{2} = 2 \left(-\frac{\sqrt{2}}{2} \right) + \sqrt{2} = -\sqrt{2} + \sqrt{2} = 0$$

$\frac{5\pi}{4}$ is a solution.

EXAMPLE**Finding All the Solutions of a Trigonometric Equation**

Solve the equation: $2 \cos \theta - \sqrt{3} = 0$

Give a general formula for all the solutions. List eight of the solutions.

$$\cos \theta = \frac{\sqrt{3}}{2}$$

Where is $\cos \theta = \frac{\sqrt{3}}{2}$? On the interval

$[0, 2\pi)$ there are two answers, one in quadrant I and one in quadrant IV.

On $[0, 2\pi)$, $\theta = \frac{\pi}{6}$ or $\theta = \frac{11\pi}{6}$

A general formula would include all coterminal angles

$$\theta = \frac{\pi}{6} + 2k\pi \text{ or } \theta = \frac{11\pi}{6} + 2k\pi$$

Eight solutions are: $-\frac{11\pi}{6}, -\frac{\pi}{6}, \frac{\pi}{6}, \frac{11\pi}{6}, \frac{13\pi}{6}, \frac{23\pi}{6}, \frac{25\pi}{6}, \frac{35\pi}{6}$

$k=-1 \qquad k=0 \qquad k=1 \qquad k=2$

EXAMPLE**Solving a Linear Trigonometric Equation**

Solve the equation: $2\cos(2\theta) - 1 = 0$, $0 \leq \theta < 2\pi$

$$\cos(2\theta) = \frac{1}{2} \quad \text{On the interval } [0, 2\pi) \text{ the cosine of an angle is } \frac{1}{2}$$

when the angle is $\frac{\pi}{3}$ or $\frac{5\pi}{3}$.

$2\theta = \frac{\pi}{3}$ or $2\theta = \frac{5\pi}{3}$ but since we divide by 2, adding 2π to the angle will still give us a solution for θ that falls in the interval $[0, 2\pi)$.

$$2\theta = \frac{\pi}{3}, 2\theta = \frac{\pi}{3} + 2\pi, 2\theta = \frac{5\pi}{3}, 2\theta = \frac{5\pi}{3} + 2\pi$$
$$\left\{ \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6} \right\}$$

EXAMPLE**Solving a Trigonometric Equation**

Solve the equation: $\sqrt{3} \tan(3\theta) + 1 = 0$, $0 \leq \theta < 2\pi$

$$\tan(3\theta) = -\frac{1}{\sqrt{3}}$$

On the interval $[0, 2\pi)$ the tangent of an angle is $-\frac{\sqrt{3}}{3}$ when the angle is $\frac{5\pi}{6}$ or $\frac{11\pi}{6}$.

$3\theta = \frac{5\pi}{6}$ or $3\theta = \frac{11\pi}{6}$ but since we divide by 3, adding 2π and 4π to

the angle will still give us a solution for θ that falls in the interval $[0, 2\pi)$.

$$3\theta = \frac{5\pi}{6}, 3\theta = \frac{5\pi}{6} + 2\pi, 3\theta = \frac{5\pi}{6} + 4\pi, 3\theta = \frac{11\pi}{6}, 3\theta = \frac{11\pi}{6} + 2\pi, 3\theta = \frac{11\pi}{6} + 4\pi$$

$$\left\{ \frac{5\pi}{18}, \frac{11\pi}{18}, \frac{17\pi}{18}, \frac{23\pi}{18}, \frac{29\pi}{18}, \frac{35\pi}{18} \right\}$$

EXAMPLE**Solving a Trigonometric Equation**

Solve the equation: $\cos\left(\theta - \frac{\pi}{4}\right) = 1, \quad 0 \leq \theta < 2\pi$

The cosine equals 1 when the angle is 0.

$$\theta - \frac{\pi}{4} = 0$$

$$\theta = \frac{\pi}{4}$$

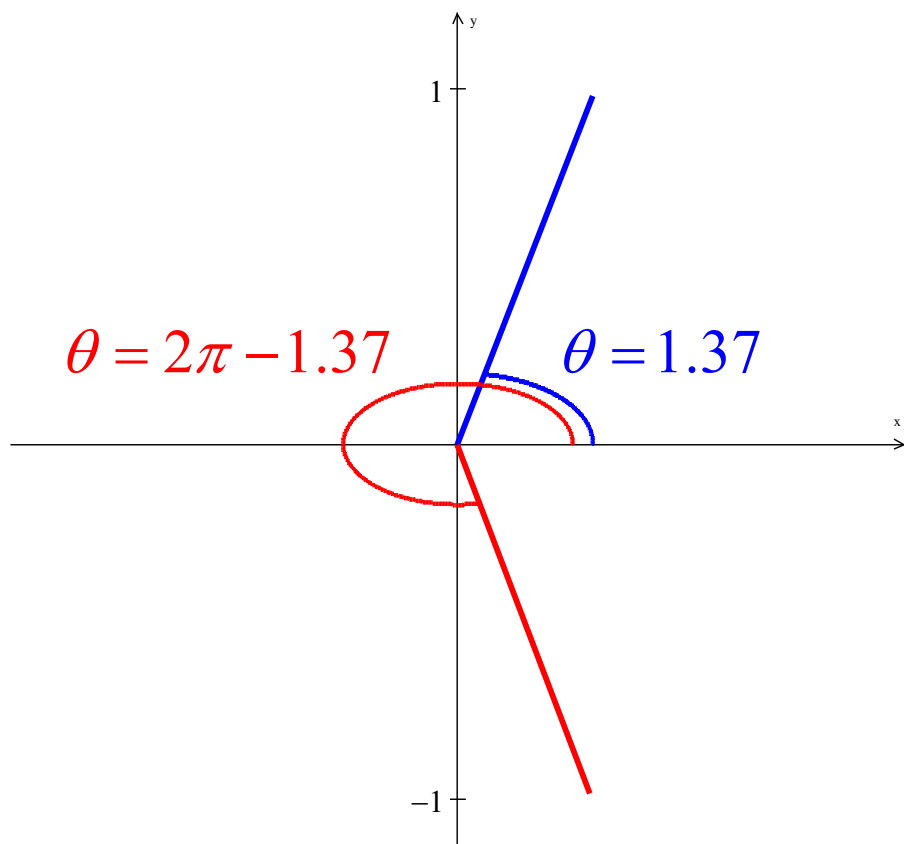
2 Solve Trigonometric Equations Using a Calculator

EXAMPLE**Solving a Trigonometric Equation with a Calculator**

Use a calculator to solve the equation: $\cos \theta = 0.2$, $0 \leq \theta < 2\pi$ Express any solutions in radians, rounded to two decimal places.

$$\theta = \cos^{-1} 0.2 \approx 1.37$$

For inverse cosine the calculator will only give an angle from 0 to π . There is another angle in the interval from 0 to 2π with that cosine value in quadrant IV.



$$\{1.37, 4.91\}$$

3 Solve Trigonometric Equations Quadratic in Form

EXAMPLE**Solving a Trigonometric Equation Quadratic in Form**

Solve the equation: $2 \cos^2 \theta - \cos \theta - 1 = 0$, $0 \leq \theta < 2\pi$

$$(2 \cos \theta + 1)(\cos \theta - 1) = 0$$

$$2 \cos \theta + 1 = 0 \text{ or } \cos \theta - 1 = 0$$

$$\cos \theta = -\frac{1}{2} \text{ or } \cos \theta = 1$$

$$\theta = \frac{2\pi}{3}, \theta = \frac{4\pi}{3}, \theta = 0$$

$$\left\{ 0, \frac{2\pi}{3}, \frac{4\pi}{3} \right\}$$

4 Solve Trigonometric Equations Using Fundamental Identities

EXAMPLE**Solving a Trigonometric Equation Using Identities**

Solve the equation: $\sin^2 \theta - \sin \theta = \cos^2 \theta$, $0 \leq \theta < 2\pi$

$$\sin^2 \theta - \sin \theta = 1 - \sin^2 \theta$$

$$2\sin^2 \theta - \sin \theta - 1 = 0$$

$$(2\sin \theta + 1)(\sin \theta - 1) = 0$$

$$2\sin \theta + 1 = 0 \text{ or } \sin \theta - 1 = 0$$

$$\sin \theta = -\frac{1}{2} \text{ or } \sin \theta = 1$$

$$\theta = \frac{7\pi}{6}, \theta = \frac{11\pi}{6}, \theta = \frac{\pi}{2} \qquad \left\{ \frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6} \right\}$$

EXAMPLE**Solving a Trigonometric Equation Using Identities**

Solve the equation: $3\sin^2 \theta = \cos^2 \theta$, $0 \leq \theta < 2\pi$

$$3\sin^2 \theta = 1 - \sin^2 \theta \quad 4\sin^2 \theta = 1 \quad \sin^2 \theta = \frac{1}{4}$$

$$\sin \theta = \pm \sqrt{\frac{1}{4}} = \pm \frac{1}{2}$$

$$\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6} \quad \left\{ \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6} \right\}$$



5 Solve Trigonometric Equations Using a Graphing Utility

EXAMPLE

Solving a Trigonometric Equation Using a Graphing Utility

Solve: $3 \cos x + x = 4$

Express the solution(s) rounded to two decimal places.

This cannot be solved by previous methods but can be solved using a graphing utility.

$$Y_1 = 3 \cos x + x \quad Y_2 = 4$$

Use INTERSECT to find solutions.

$$x = 4.53$$

