Section 1.2

Quadratic Equations

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Quadratic Equations

A quadratic equation is an equation equivalent to one of the form

$$\boxed{ax^2 + bx + c = 0}$$

where a, b, and c are real numbers and $a \neq 0$.

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1 Solve a Quadratic Equation by Factoring

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EXAMPLE

Solving a Quadratic Equation by Factoring

Solve the equation: $x^2 - 4x = 0$

$$x(x-4) = 0$$

$$x = 0 \quad \text{or} \quad x - 4 = 0$$

$$x = 0 \quad \text{or} \quad x = 4$$

The solution set is $\{0, 4\}$.

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EXAMPLE

Solving a Quadratic Equation by Factoring

Solve the equation: $x^2 = 6 - x$

$$x^2 + x - 6 = 0$$

$$(x+3)(x-2) = 0$$

$$x = -3 \text{ or } x = 2$$

The solution set is $\{-3, 2\}$.

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The Square Root Method

If $x^2 = p$ and $p \geq 0$, then $x = \sqrt{p}$ or $x = -\sqrt{p}$.

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EXAMPLE

Solving a Quadratic Equation Using the Square Root Method

Solve each equation.

(a) $x^2 = 7$

$$x = \pm\sqrt{7}$$

$$x = \sqrt{7} \text{ or } x = -\sqrt{7}$$

The solution set is $\left\{-\sqrt{7}, \sqrt{7}\right\}$.

(b) $(x+3)^2 = 9$

$$x + 3 = \pm\sqrt{9}$$

$$x + 3 = \pm 3$$

$$x + 3 = 3 \text{ or } x + 3 = -3$$

$$x = 0 \text{ or } x = -6$$

The solution set is $\{0, -6\}$.

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2 Solve a Quadratic Equation by Completing the Square

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EXAMPLE

Solving a Quadratic Equation by Completing the Square

Solve by completing the square: $2x^2+6x-5=0$

$$2x^2+6x=5$$

$$\frac{2x^2}{2}+\frac{6x}{2}=\frac{5}{2}$$

$$x^2+3x+\frac{9}{4}=\frac{5}{2}+\frac{9}{4}$$

$$\left(x+\frac{3}{2}\right)^2=\frac{19}{4}$$

$$x+\frac{3}{2}=\pm\sqrt{\frac{19}{4}}$$

$$x=\pm\frac{\sqrt{19}}{2}-\frac{3}{2}$$

$$x=\frac{\sqrt{19}-3}{2} \text{ or } \frac{-\sqrt{19}-3}{2}$$

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3 Solve a Quadratic Equation Using the Quadratic Formula

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Quadratic Formula

Consider the quadratic equation

$$\boxed{ax^2 + bx + c = 0 \qquad a \neq 0}$$

If $b^2 - 4ac < 0$, this equation has no real solution.

If $b^2 - 4ac \geq 0$, the real solution(s) of this equation is (are) given by the **quadratic formula**.

$$\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

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Discriminant of a Quadratic Equation

For a quadratic equation $ax^2 + bx + c = 0$:

1. If $b^2 - 4ac > 0$, there are two unequal real solutions.
2. If $b^2 - 4ac = 0$, there is a repeated real solution, a root of multiplicity 2.
3. If $b^2 - 4ac < 0$, there is no real solution.

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EXAMPLE

Solving a Quadratic Equation Using the Quadratic Formula

Use the quadratic formula to find the real solutions, if any, of the equation

$$2x^2 - 4x + 1 = 0$$

$$ax^2 + bx + c = 0$$

$$b^2 - 4ac = (-4)^2 - 4(2)(1) = 16 - 8 = 8$$

$b^2 - 4ac > 0$ so there are two real solutions which can be found using the quadratic formula.

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EXAMPLE

Solving a Quadratic Equation Using the Quadratic Formula

Use the quadratic formula to find the real solutions, if any, of the equation

$$2x^2 - 4x + 1 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-4) \pm \sqrt{8}}{2(2)} = \frac{4 \pm \sqrt{8}}{4} = \frac{2 \pm \sqrt{2}}{2}$$

The solution set is $\left\{ \frac{2+\sqrt{2}}{2}, \frac{2-\sqrt{2}}{2} \right\}$.

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EXAMPLE

Solving a Quadratic Equation Using the Quadratic Formula

Use the quadratic formula to find the real solutions, if any, of the equation

$$\frac{1}{2}x^2 - 6x + 18 = 0$$

$$x^2 - 12x + 36 = 0$$

$$ax^2 + bx + c = 0$$

$$b^2 - 4ac = (-12)^2 - 4(1)(36) = 144 - 144 = 0$$

$b^2 - 4ac = 0$ so there is a repeated solution which can be found using the quadratic formula.

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EXAMPLE

Solving a Quadratic Equation Using the Quadratic Formula

Use the quadratic formula to find the real solutions, if any, of the equation

$$\frac{1}{2}x^2 - 6x + 18 = 0$$

$$x^2 - 12x + 36 = 0$$

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-12) \pm \sqrt{0}}{2(1)} = \frac{12}{2} = 6$$

The solution set is $\{6\}$.

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EXAMPLE

Solving a Quadratic Equation Using the Quadratic Formula

Use the quadratic formula to find the real solutions, if any, of the equation

$$2x^2 + 3 = 2x$$

$$2x^2 - 2x + 3 = 0$$

$$ax^2 + bx + c = 0$$

$$b^2 - 4ac = (-2)^2 - 4(2)(3)$$

$$= 4 - 24 = -20$$

Since $b^2 - 4ac < 0$,

there is no real solution.

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EXAMPLE

Solving a Quadratic Equation Using the Quadratic Formula

Use the quadratic formula to find the real solutions if any, of the equation

$$6+\frac{1}{x}-\frac{2}{x^2}=0$$

$$6x^2+x-2=0$$

$$b^2-4ac=(1)^2-4(6)(-2)$$

$$=1+48=49$$

Since $b^2-4ac>0$,

there are two real solutions.

$$x=\frac{-1\pm\sqrt{49}}{2(6)}=\frac{-1\pm 7}{12}$$

$$=\frac{-1+7}{12}=\frac{1}{2}$$

$$=\frac{-1-7}{12}=-\frac{2}{3}$$

The solution set is $\left\{\frac{1}{2},-\frac{2}{3}\right\}$.

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SUMMARY Procedure for Solving a Quadratic Equation

To solve a quadratic equation, first put it in standard form:

$$ax^2 + bx + c = 0$$

Then:

Step 1: Identify a, b, and c.

Step 2: Evaluate the discriminant, $b^2 - 4ac$.

Step 3: (a) If the discriminant is negative, the equation has no real solution.
(b) If the discriminant is zero, the equation has one real solution, a repeated root.
(c) If the discriminant is positive, the equation has two distinct real solutions.

If you can easily spot factors, use the factoring method to solve the equation. Otherwise, use the quadratic formula or the method of completing the square.

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4 Solve Problems That Can Be Modeled by Quadratic Equations

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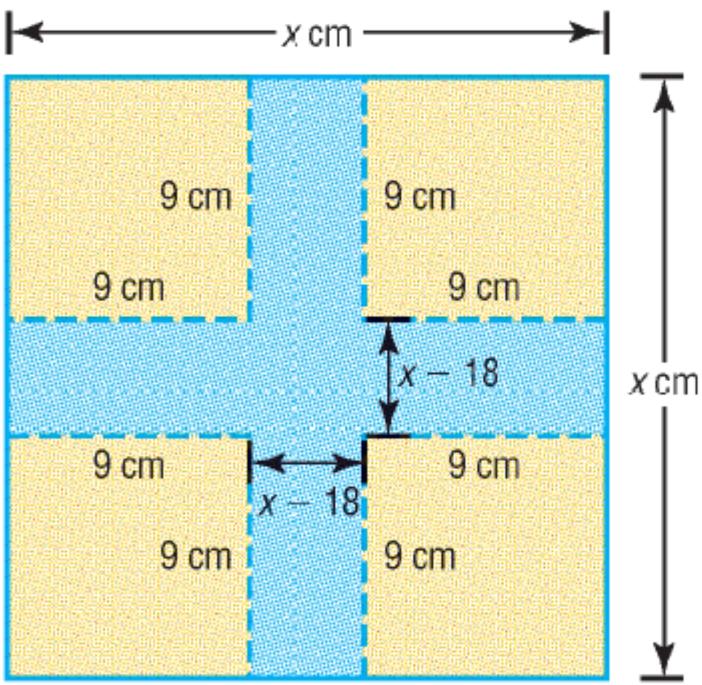
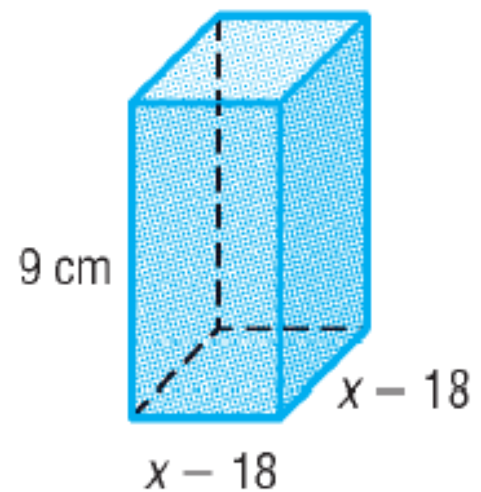
EXAMPLE Constructing a Box

From each corner of a square piece of sheet metal, remove a square of side 9 centimeters. Turn up the edges to form an open box. If the box is to hold 144 cubic centimeters (cm^3), what should be the dimensions of the piece of sheet metal?





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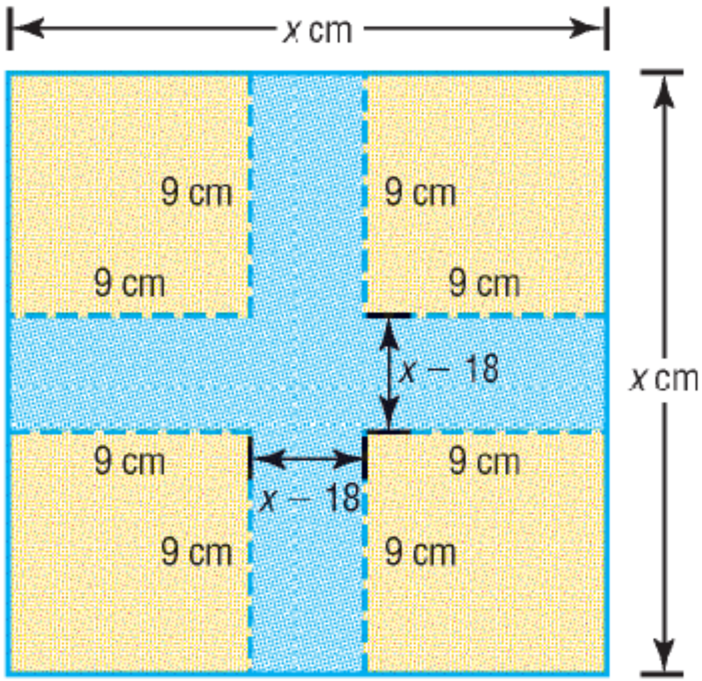
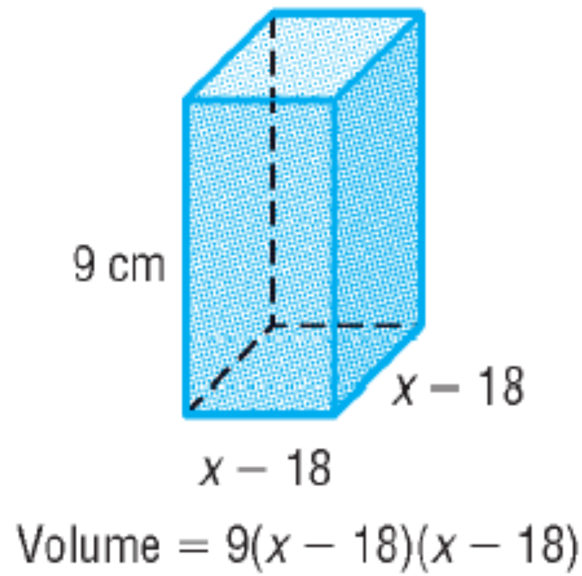
EXAMPLE **Constructing a Box**

From each corner of a square piece of sheet metal, remove a square of side 9 centimeters. Turn up the edges to form an open box. If the box is to hold 144 cubic centimeters (cm^3), what should be the dimensions of the piece of sheet metal?





$$9(x-18)^2 = 144 \qquad (x-18)^2 = 16 \qquad x - 18 = \pm 4$$

$$x = 18 \pm 4 = 22 \text{ or } \cancel{14}$$

22 centimeters by 22 centimeters

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