Section 3.6

Mathematical Models: Building Functions

1 Build and Analyze Functions

EXAMPLE

Finding the Distance from the Origin to a Point on a Graph

Let $P = (x, y)$ be a point on the graph of $y = x^2 - 1$.

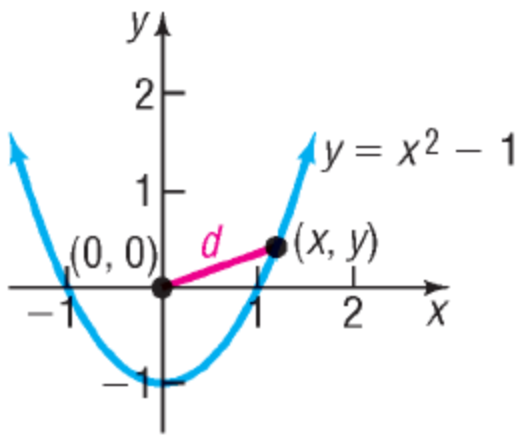
(a) Express the distance d from P to the origin O as a function of x.

(b) What is d if $x = 0$? $\quad d(0) = \sqrt{1} = 1$

(c) What is d if $x = 1$? $\quad d(1) = \sqrt{1 - 1 + 1} = 1$

(d) What is d if $x = \dfrac{\sqrt{2}}{2}$? $\quad d\left(\dfrac{\sqrt{2}}{2}\right) = \sqrt{\left(\dfrac{\sqrt{2}}{2}\right)^4 - \left(\dfrac{\sqrt{2}}{2}\right)^2 + 1} = \dfrac{\sqrt{3}}{2}$

(e) Use a graphing utility to graph the function $d = d(x)$, $x \geq 0$. Rounded to two decimal places, find the value(s) of x at which d has a local minimum. [This gives the point(s) on the graph of $y = x^2 - 1$ closest to the origin.]



$$d = \sqrt{(x - 0)^2 + (y - 0)^2} = \sqrt{x^2 + y^2}$$

$$d(x) = \sqrt{x^2 + (x^2 - 1)^2} = \sqrt{x^4 - x^2 + 1}$$

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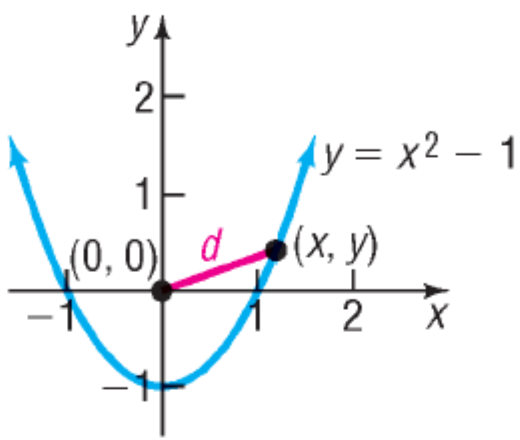
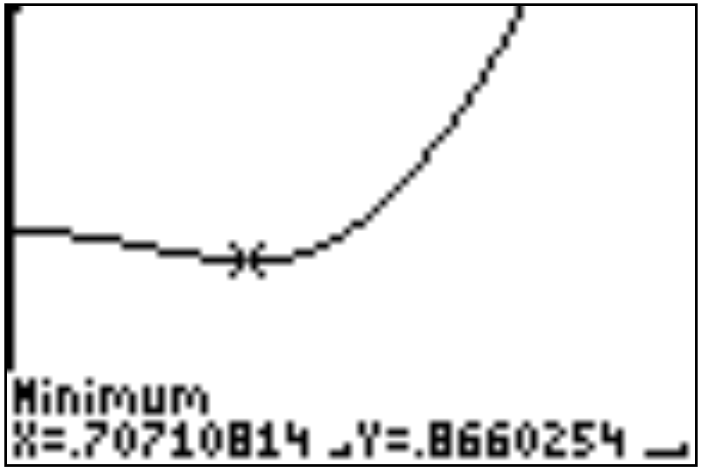




EXAMPLE **Area of a Rectangle**

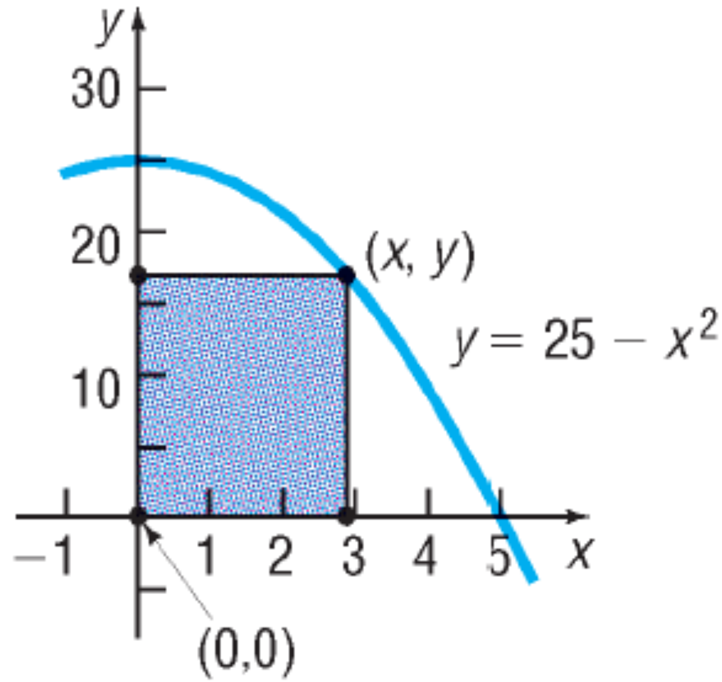
A rectangle has one corner on the graph of $y = 25 - x^2$, another at the origin, a third on the positive y-axis, and the fourth on the positive x-axis.

(a) Express the area A of the rectangle as a function of x.

(b) What is the domain of A?

(c) Graph $A = A(x)$.

(d) For what value of x is the area largest?



(a) $A = xy = x\left(25 - x^2\right) = 25x - x^3$

$A(x) = 25x - x^3$

(b) $\{x \mid 0 < x < 5\}$

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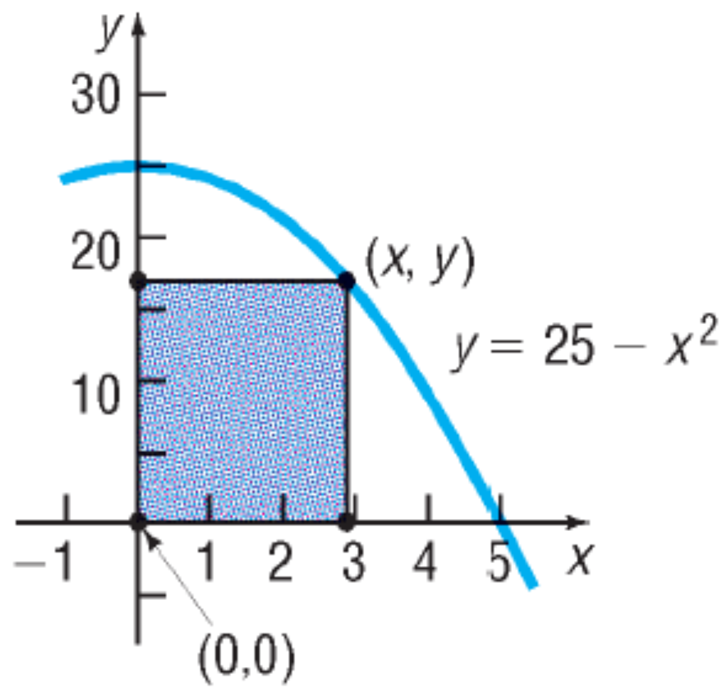

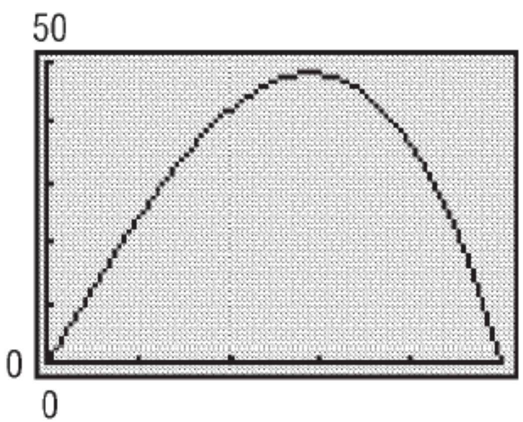

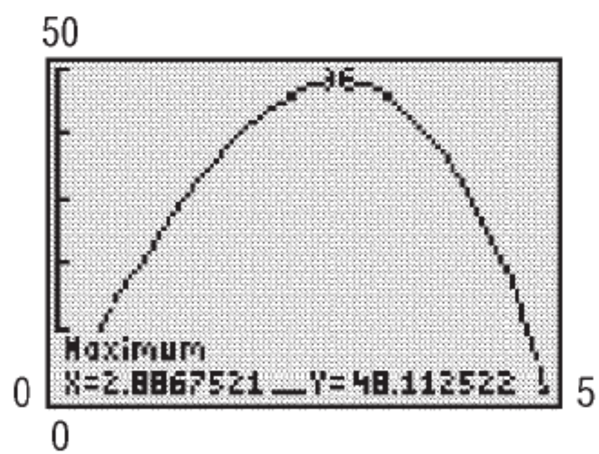

EXAMPLE Making a Playpen*

A manufacturer of children's playpens makes a square model that can be opened at one corner and attached at right angles to a wall or, perhaps, the side of a house. If each side is 3 feet in length, the open configuration doubles the available area in which the child can play from 9 square feet to 18 square feet. See Figure 65.

Now suppose that we place hinges at the outer corners to allow for a configuration like the one shown in Figure 66.

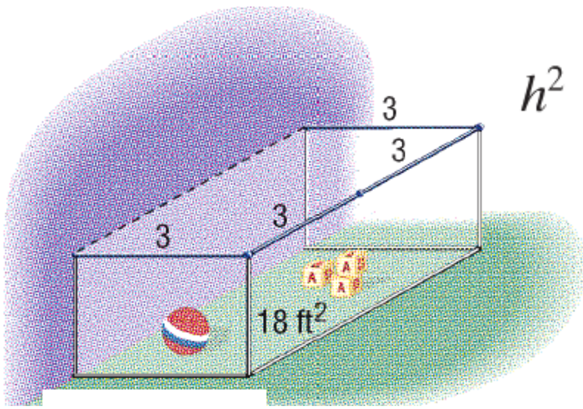

Figure 65

$$h^2 + \left(\frac{x}{2}\right)^2 = 3^2 \qquad h^2 = 3^2 - \left(\frac{x}{2}\right)^2$$

$$= 9 - \frac{x^2}{4} = \frac{36 - x^2}{4}$$

$$h = \frac{1}{2}\sqrt{36 - x^2}$$

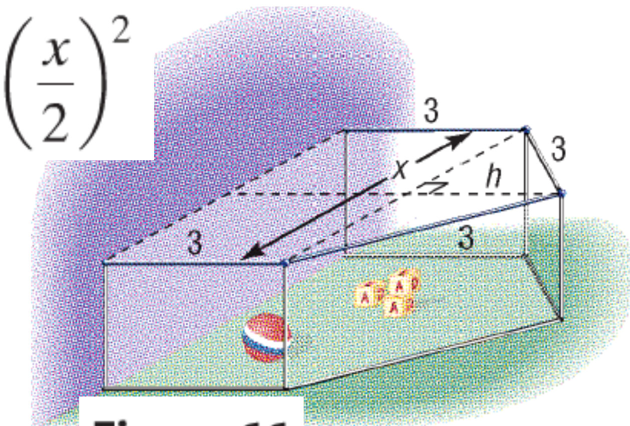

Figure 66

$$A = \text{area of rectangle} + \text{area of triangle} = 3x + \frac{1}{2}x\left(\frac{1}{2}\sqrt{36 - x^2}\right)$$

(a) Build a model that expresses the area A of the configuration shown in Figure 66 as a function of the distance x between the two parallel sides.

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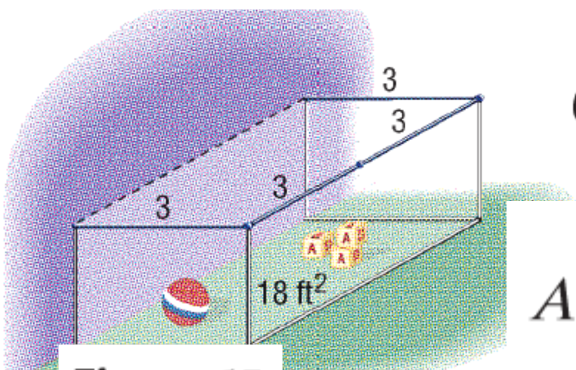

Figure 65

(b) Find the domain of A.

$$A(x) = 3x + \frac{x\sqrt{36 - x^2}}{4}$$

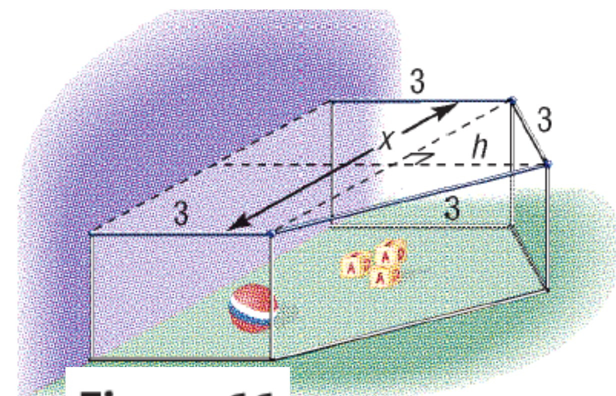

Figure 66

(b) To find the domain of A, notice that $x > 0$, since x is a length. Also, the expression under the square root must be positive, so

$$\begin{aligned} 36 - x^2 &> 0 \\ x^2 &< 36 \\ -6 < x &< 6 \end{aligned}$$

Combining these restrictions, the domain of A is $0 < x < 6$, or $(0, 6)$ using interval notation.

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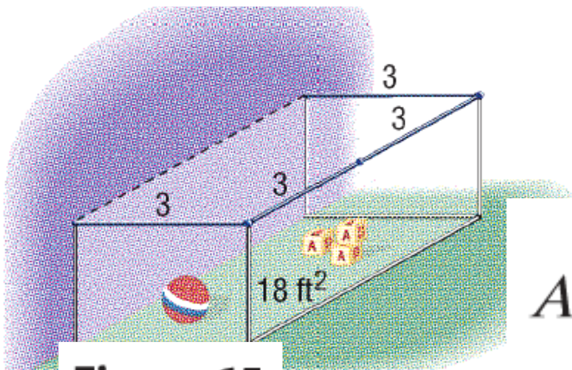

Figure 65

(c) Find A if $x = 5$.

$$A(x) = 3x + \frac{x\sqrt{36 - x^2}}{4}$$

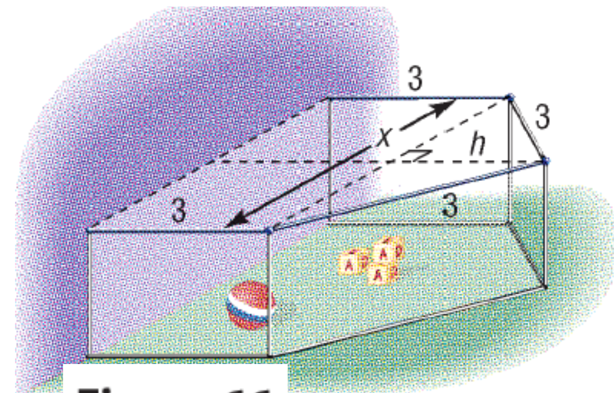

Figure 66

$$A(5) = 3(5) + \frac{5}{4}\sqrt{36 - (5)^2} \approx 19.15 \text{ square feet}$$

EXAMPLE Making a Playpen*

(d) Graph $A = A(x)$. For what value of x is the area largest? What is the maximum area?