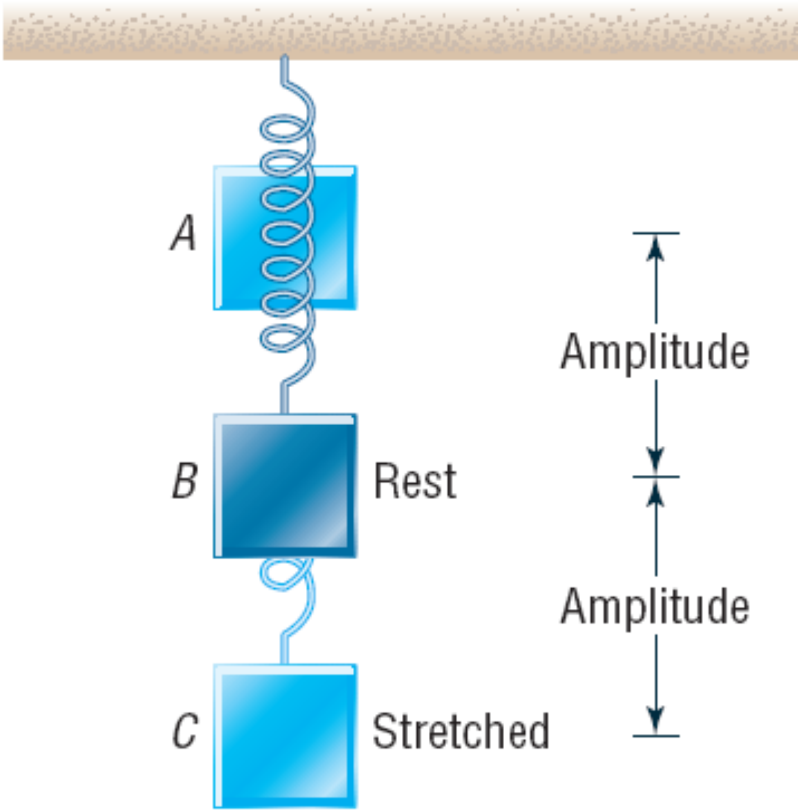
Section 9.5

Simple Harmonic Motion; Damped Motion; Combining Waves

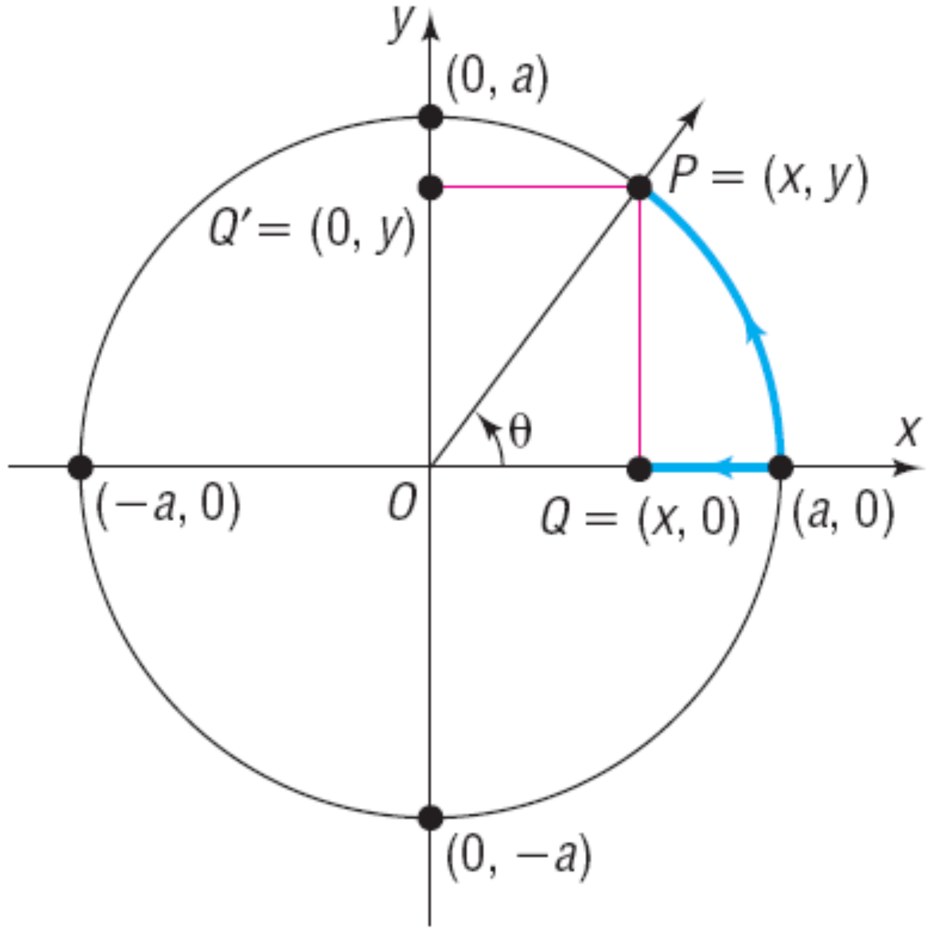
1 Build a Model for an Object in Simple Harmonic Motion

Simple harmonic motion is a special kind of vibrational motion in which the acceleration a of the object is directly proportional to the negative of its displacement d from its rest position. That is, $a = -kd$, $k > 0$.

Vibrating tuning fork



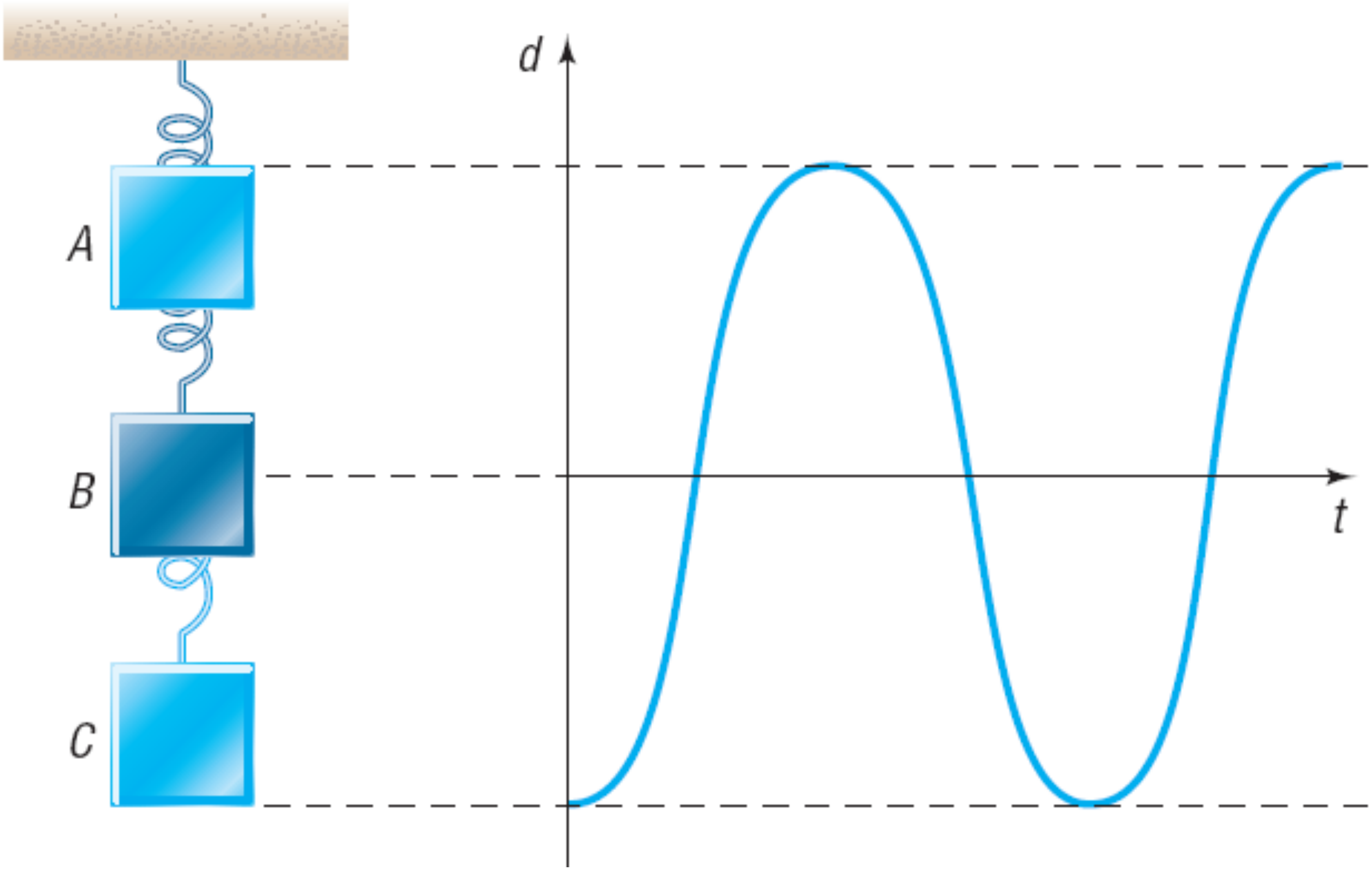
Coiled spring



$$\theta = \omega t$$

$$x = a\cos\theta = a\cos(\omega t)$$

$$y = a\sin\theta = a\sin(\omega t)$$


d
t
A
B
C

THEOREM

Simple Harmonic Motion

An object that moves on a coordinate axis so that the distance d from its rest position at time t is given by either

$$\boxed{d = a\cos(\omega t) \quad \text{or} \quad d = a\sin(\omega t)}$$

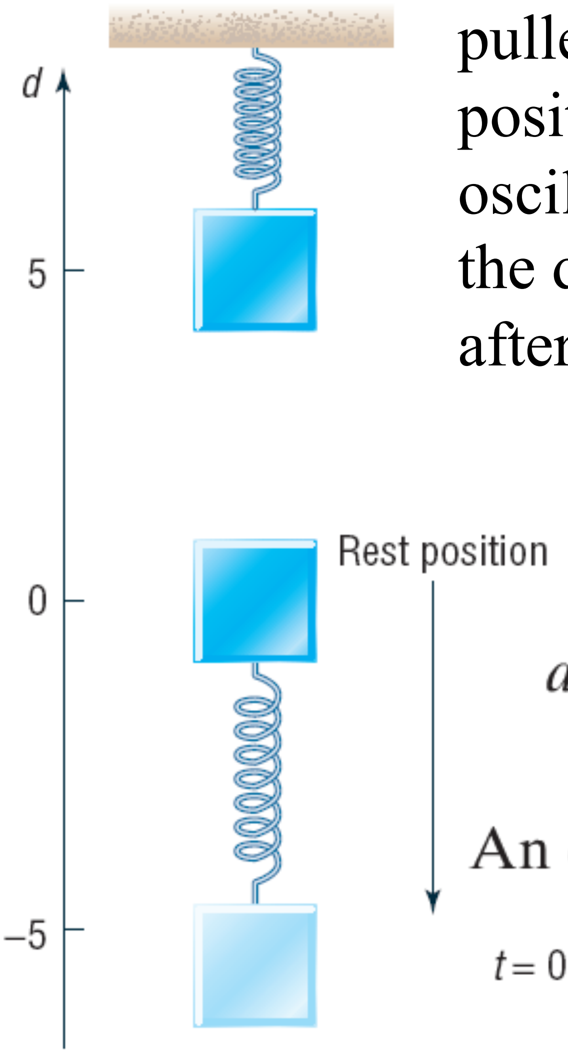
where a and $\omega > 0$ are constants, moves with simple harmonic motion. The motion has amplitude $|a|$ and period $\frac{2\pi}{\omega}$.

Frequency

$$\boxed{f = \frac{\omega}{2\pi} \qquad \omega > 0}$$

EXAMPLE

Build a Model for an Object in Harmonic Motion



Suppose that an object attached to a coiled spring is pulled down a distance of 5 inches from its rest position and then released. If the time for one oscillation is 3 seconds, write an equation that relates the displacement *d* of the object from its rest position after time *t* (in seconds). Assume no friction.

$$d = a\cos(\omega t)$$

$$a = -5 \quad \text{and} \quad \frac{2\pi}{\omega} = \text{period} = 3, \quad \text{so} \ \ \omega = \frac{2\pi}{3}$$

An equation that models the motion of the object is

$$d = -5\cos\left[\frac{2\pi}{3}t\right]$$

2 Analyze Simple Harmonic Motion

EXAMPLE Analyzing the Motion of an Object

Suppose that the displacement d (in meters) of an object at time t (in seconds) satisfies the equation

$$d = 25\cos(4t)$$

(a) Describe the motion of the object.

(b) What is the maximum displacement from its resting position?

(c) What is the time required for one oscillation?

(d) What is the frequency?

(a) The motion is simple harmonic.

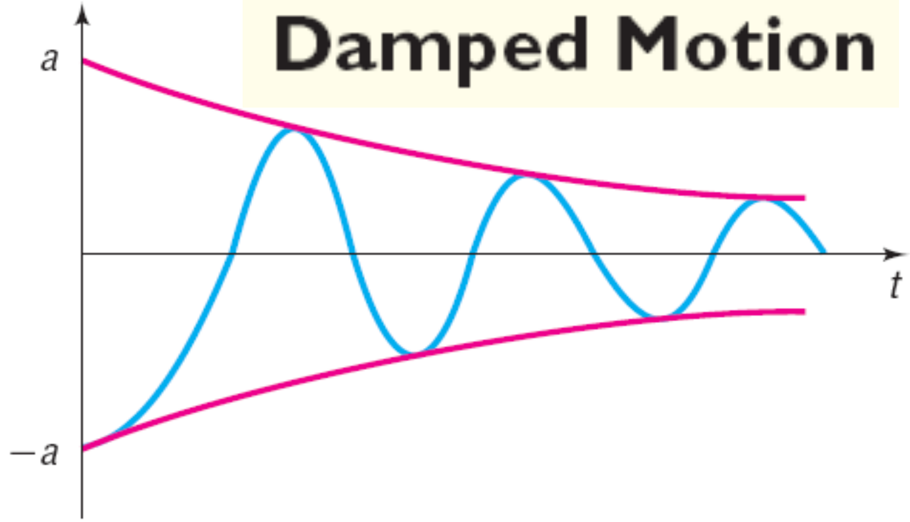
(b) Maximum displacement is $|a| = 25$ meters.

(c) Period $\dfrac{2\pi}{\omega} = \dfrac{2\pi}{4} = \dfrac{\pi}{2}$ seconds

(d) Frequency $= \dfrac{\omega}{2\pi} = \dfrac{2}{\pi}$ oscillations per second

3 Analyze an Object in Damped Motion

Damped Motion



Damped Motion

The displacement d of an oscillating object from its at-rest position at time t is given by

$$d(t) = ae^{-bt/(2m)}\cos\left(\sqrt{\omega^2 - \frac{b^2}{4m^2}}\,t\right)$$

where b is the **damping factor** or **damping coefficient** and m is the mass of the oscillating object. Here $|a|$ is the displacement at $t = 0$ and $\frac{2\pi}{\omega}$ is the period under simple harmonic motion (no damping).

EXAMPLE Analyzing a Damped Vibration Curve

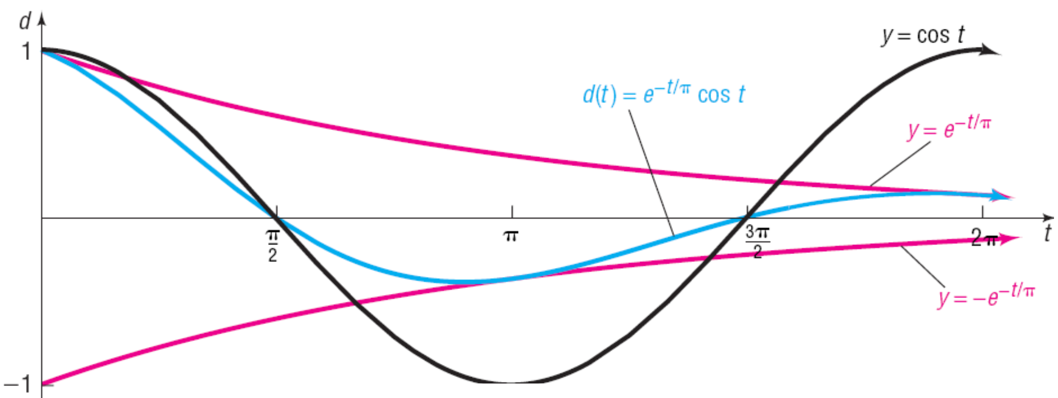
Analyze the damped vibration curve $d(t) = e^{-t/\pi} \cos t, \quad t \geq 0$

$$|d(t)| = |e^{-t/\pi} \cos t| = |e^{-t/\pi}||\cos t| \leq |e^{-t/\pi}| = e^{-t/\pi}$$

$$-e^{-t/\pi} \leq d(t) \leq e^{-t/\pi}$$

t	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$e^{-t/\pi}$	1	$e^{-1/2}$	e^{-1}	$e^{-3/2}$	e^{-2}
$\cos t$	1	0	-1	0	1
$d(t) = e^{-t/\pi} \cos t$	1	0	$-e^{-1}$	0	e^{-2}
Point on graph of d	$(0, 1)$	$\left(\frac{\pi}{2}, 0\right)$	$(\pi, -e^{-1})$	$\left(\frac{3\pi}{2}, 0\right)$	$(2\pi, e^{-2})$

Also, the graph of d will touch these graphs when $|\cos t| = 1$

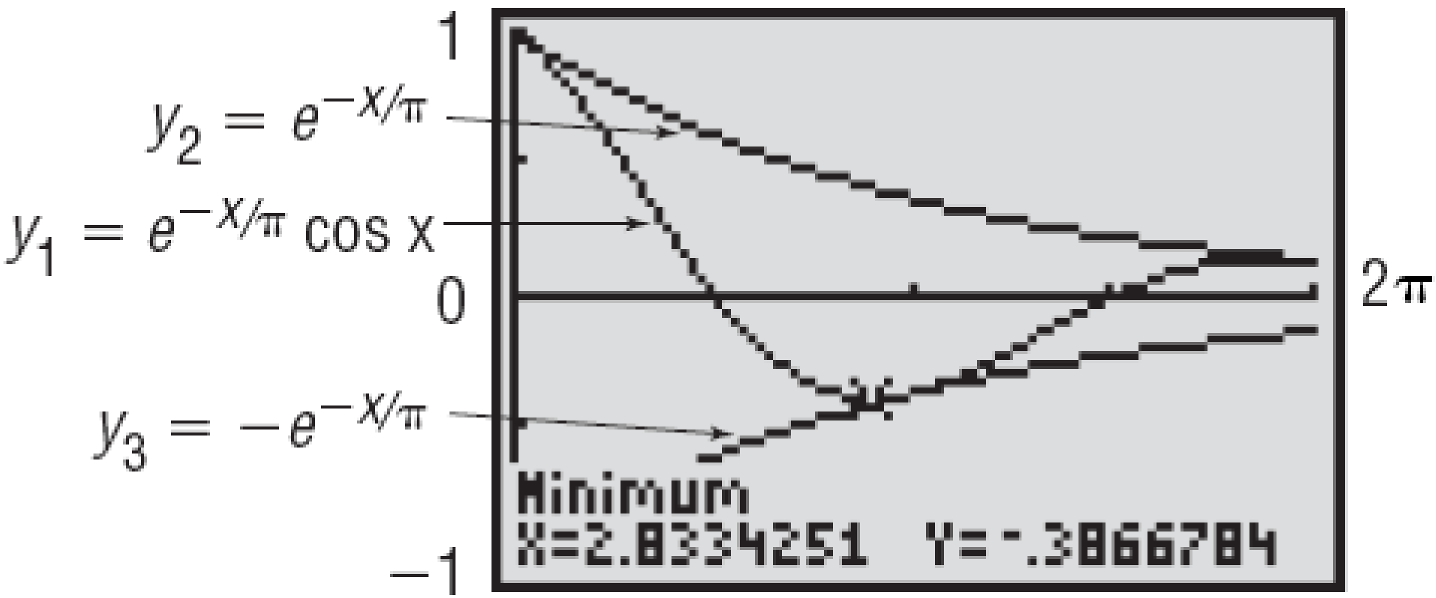


The -intercepts of the graph of d occur when $\cos t = 0$

This means that the graph of d will lie between the graphs of $y = e^{-t/\pi}$ and $y = -e^{-t/\pi}$, the **bounding curves** of d.

Exploration

Graph $Y_1 = e^{-x/\pi} \cos x$, along with $Y_2 = e^{-x/\pi}$, and $Y_3 = -e^{-x/\pi}$, for $0 \le x \le 2\pi$. Determine where Y_1 has its first turning point (local minimum). Compare this to where Y_1 intersects Y_3.



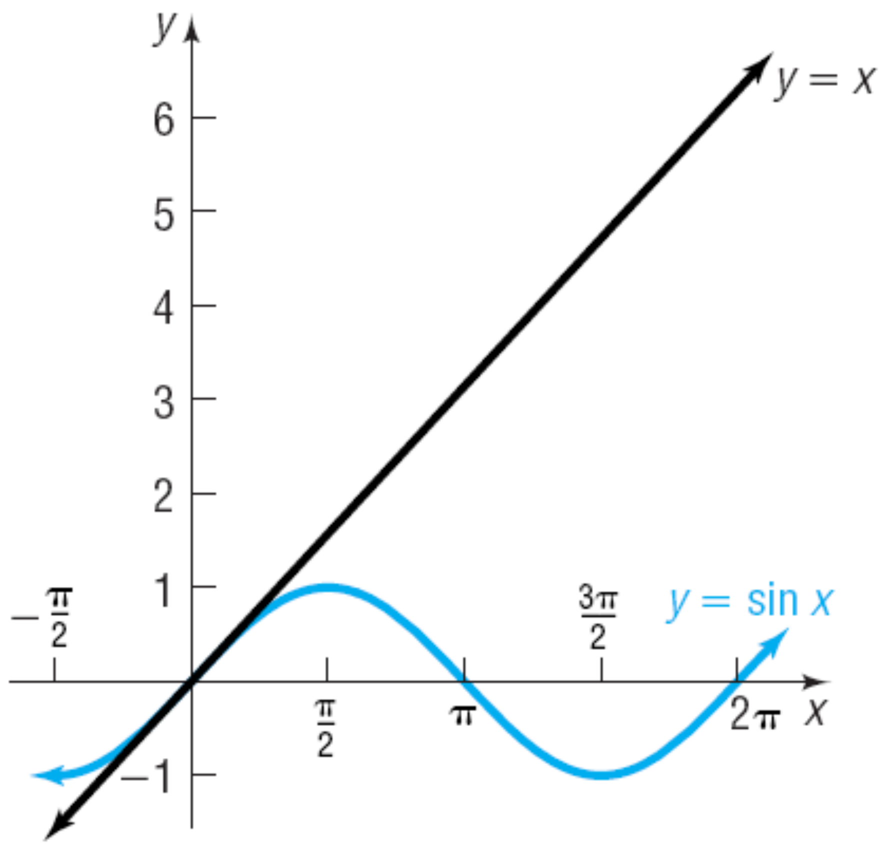
4 Graph the Sum of Two Functions

EXAMPLE

Graphing the Sum of Two Functions

Use the method of adding y-coordinates to graph $f(x) = x + \sin x$.

$$y = f_1(x) = x \qquad y = f_2(x) = \sin x$$



EXAMPLE Graphing the Sum of Two Functions

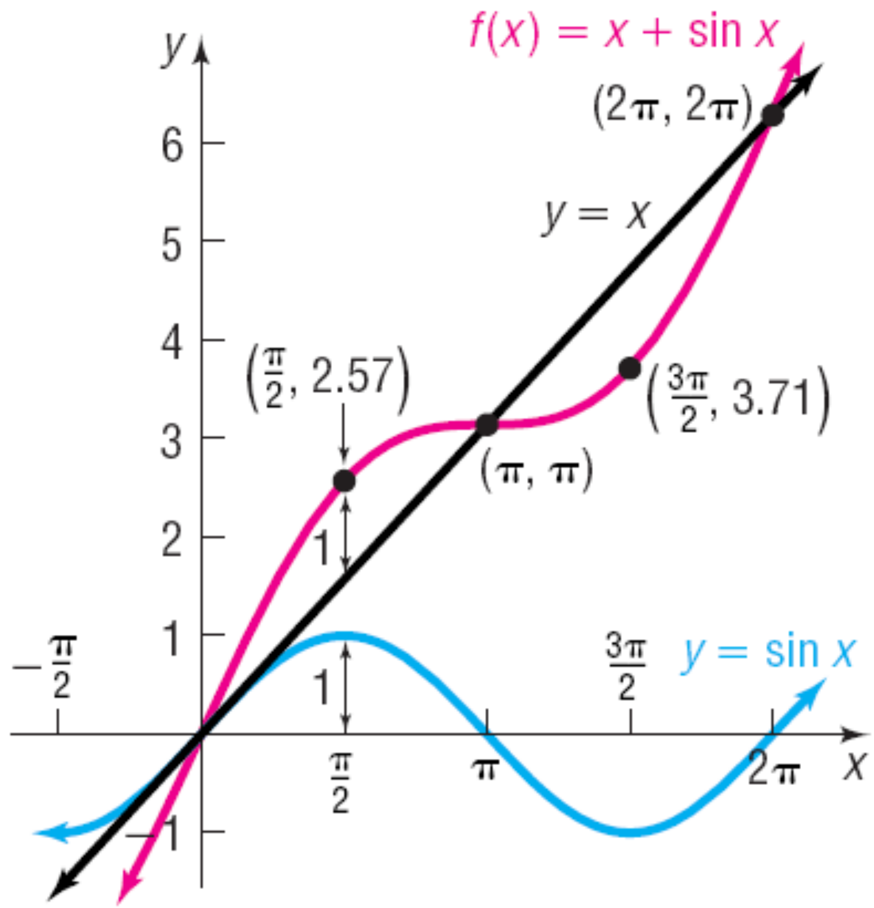
Use the method of adding y-coordinates to graph $f(x) = x + \sin x$.

Select several values of x and compute $f_1(x)$, $f_2(x)$ and $f(x) = f_1(x) + f_2(x)$

x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$y = f_1(x) = x$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$y = f_2(x) = \sin x$	0	1	0	-1	0
$f(x) = x + \sin x$	0	$\frac{\pi}{2} + 1 \approx 2.57$	π	$\frac{3\pi}{2} - 1 \approx 3.71$	2π
Point on graph of f	$(0, 0)$	$\left(\frac{\pi}{2}, 2.57\right)$	(π, π)	$\left(\frac{3\pi}{2}, 3.71\right)$	$(2\pi, 2\pi)$

EXAMPLE Graphing the Sum of Two Functions

Use the method of adding y-coordinates to graph $f(x) = x + \sin x$.



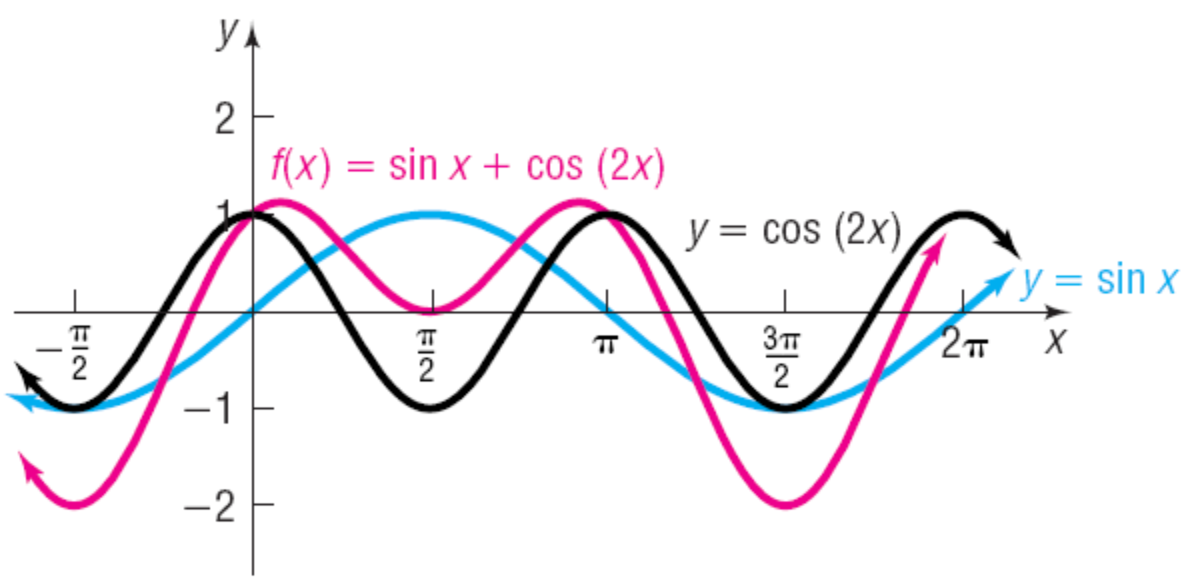
EXAMPLE

Graphing the Sum of Two Sinusoidal Functions

Use the method of adding *y*-coordinates to graph

$$f(x) = \sin x + \cos(2x)$$

x	$-\frac{\pi}{2}$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$y = f_1(x) = \sin x$	-1	0	1	0	-1	0
$y = f_2(x) = \cos(2x)$	-1	1	-1	1	-1	1
$f(x) = \sin x + \cos(2x)$	-2	1	0	1	-2	1
Point on graph of f	$\left(-\frac{\pi}{2}, -2\right)$	$(0, 1)$	$\left(\frac{\pi}{2}, 0\right)$	$(\pi, 1)$	$\left(\frac{3\pi}{2}, -2\right)$	$(2\pi, 1)$



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