Section 2.2

Graphs of Equations In Two Variables; Intercepts; Symmetry

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1 Graph Equations by Plotting Points

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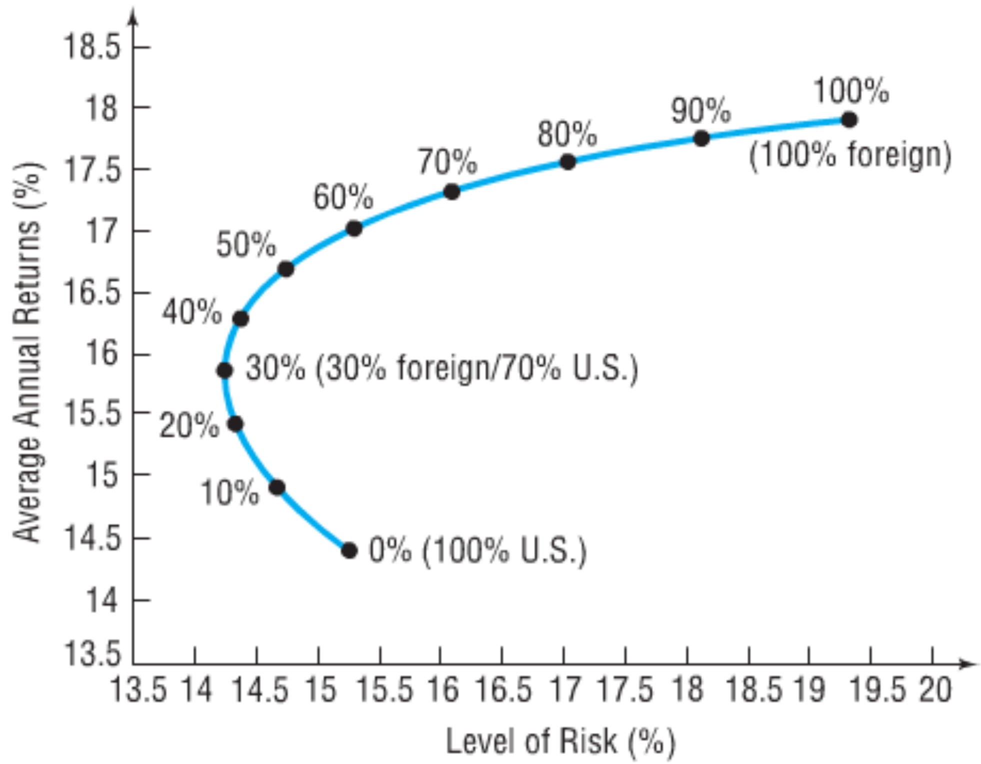
equation in two variables

$$x^2 + y^2 = 5 \qquad 2x - y = 6 \qquad y = 2x + 5 \qquad x^2 = y$$



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EXAMPLE

Determining Whether a Point Is on the Graph of an Equation

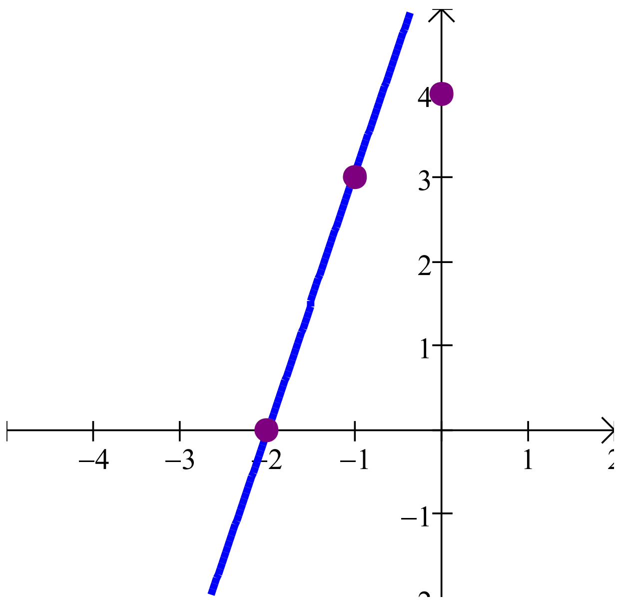
Determine if the following points are on the graph of the equation $-3x+y=6$

(a) $(0, 4)$ (b) $(-2, 0)$ (c) $(-1, 3)$

$$-3(0)+4=4\neq 6$$

$$-3(-2)+0=6$$

$$-3(-1)+3=3+3=6$$



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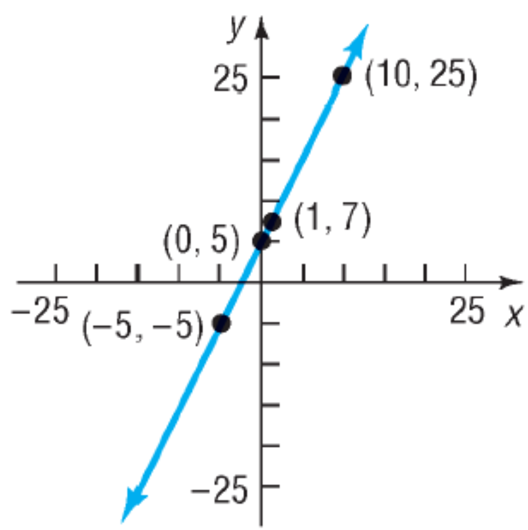
EXAMPLE

Graphing an Equation by Plotting Points

Graph the equation: $y = 2x + 5$

If	Then	Point on Graph
$x = 0$	$y = 2(0) + 5 = 5$	$(0, 5)$
$x = 1$	$y = 2(1) + 5 = 7$	$(1, 7)$
$x = -5$	$y = 2(-5) + 5 = -5$	$(-5, -5)$
$x = 10$	$y = 2(10) + 5 = 25$	$(10, 25)$



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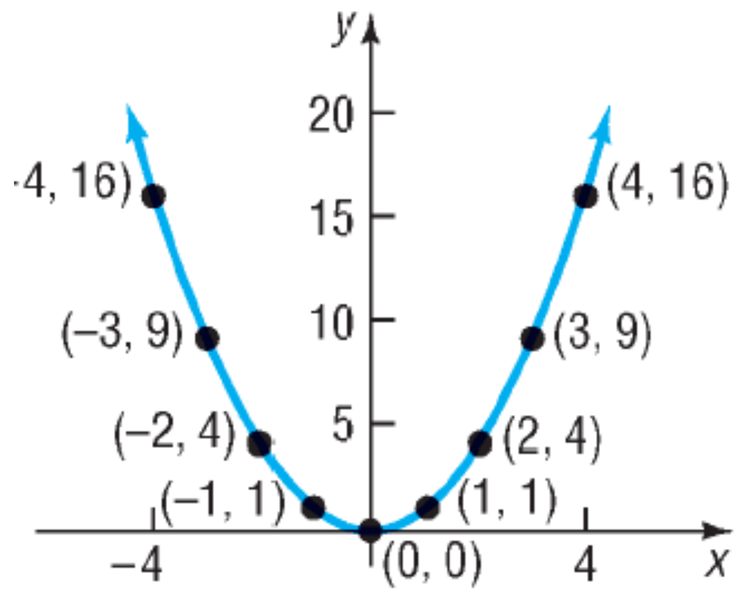
EXAMPLE

Graphing an Equation by Plotting Points

Graph the equation: $y = x^2$

x	$y = x^2$	(x, y)
-4	16	$(-4, 16)$
-3	9	$(-3, 9)$
-2	4	$(-2, 4)$
-1	1	$(-1, 1)$
0	0	$(0, 0)$
1	1	$(1, 1)$
2	4	$(2, 4)$
3	9	$(3, 9)$
4	16	$(4, 16)$



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2 Find Intercepts from a Graph

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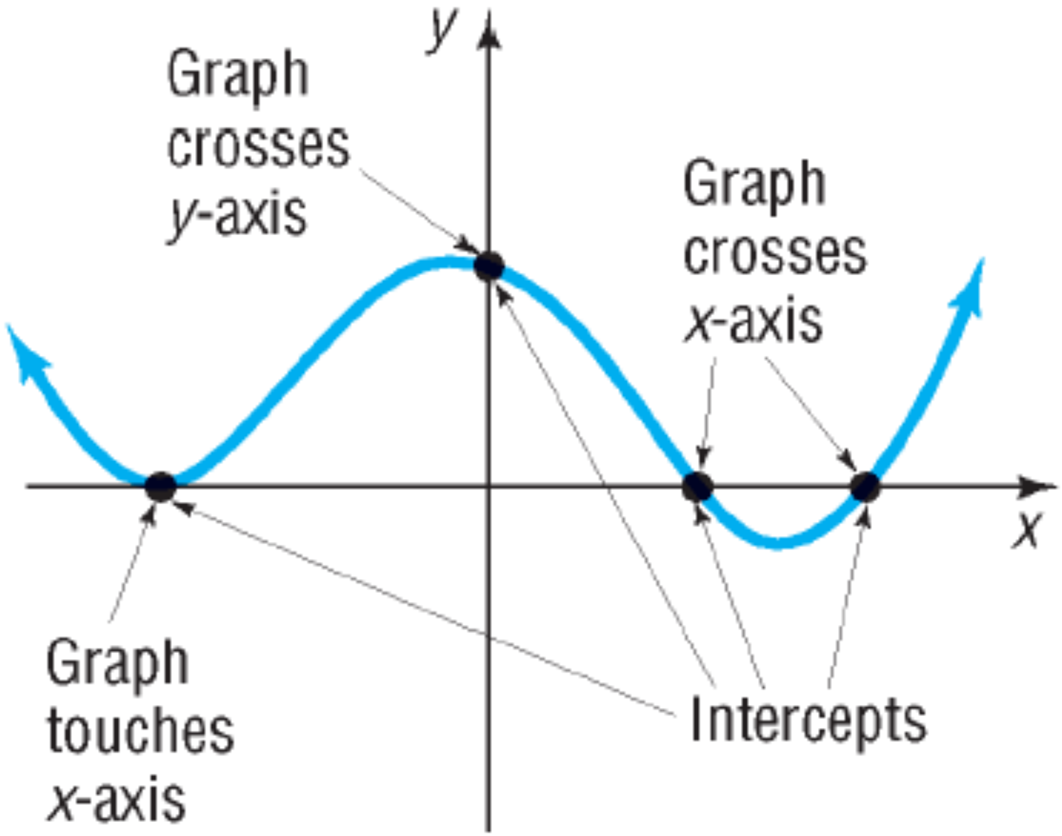

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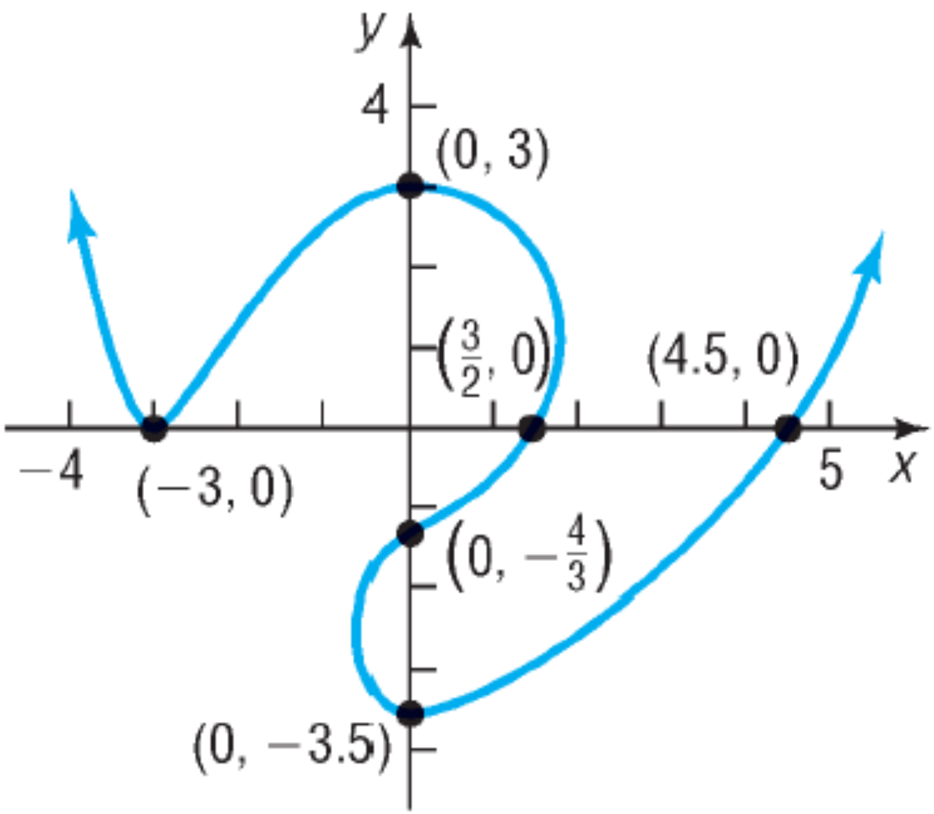
EXAMPLE

Finding Intercepts from a Graph

Find the intercepts of the graph.



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3 Find Intercepts from an Equation

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Procedure for Finding Intercepts

1. To find the x-intercept(s), if any, of the graph of an equation, let $y = 0$ in the equation and solve for x, where x is a real number.
2. To find the y-intercept(s), if any, of the graph of an equation, let $x = 0$ in the equation and solve for y, where y is a real number.

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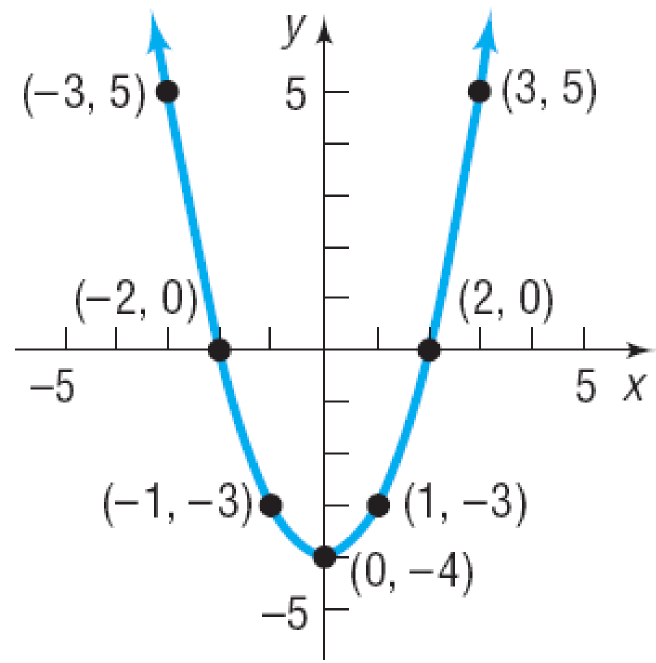
EXAMPLE

Finding Intercepts from an Equation

Find the x-intercept(s) and the y-intercept(s) of the graph of $y = x^2 - 4$ then graph by plotting points.

x	$y = x^2 - 4$	(x, y)
−3	5	(−3, 5)
−1	−3	(−1, −3)
1	−3	(1, −3)
3	5	(3, 5)



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4 Test an Equation for Symmetry with Respect to the x-Axis, the y-Axis, and the Origin

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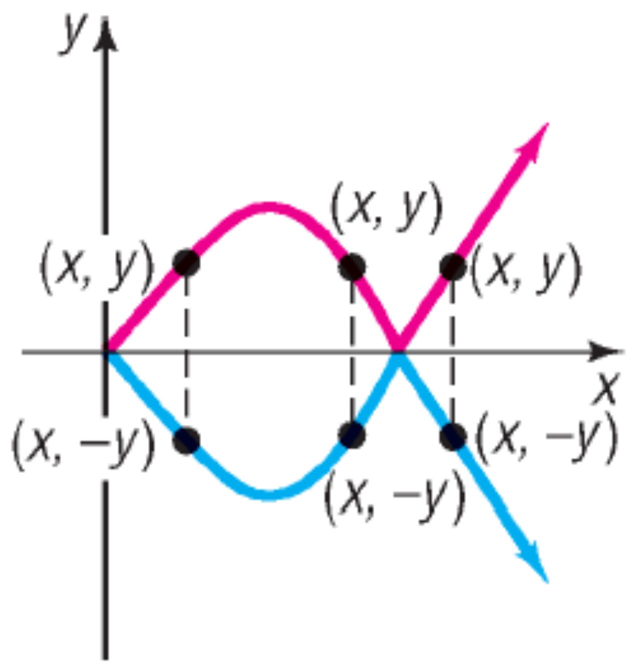
DEFINITION

A graph is said to be **symmetric with respect to the *x*-axis** if, for every point (x, y) on the graph, the point $(x, -y)$ is also on the graph.



Symmetry with respect to the *x*-axis

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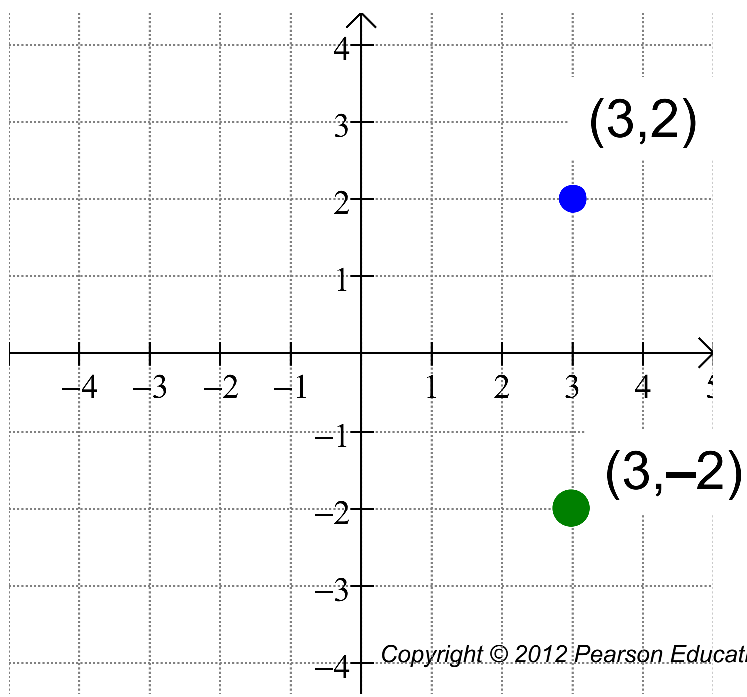
EXAMPLE

Points Symmetric with Respect to the *x*-Axis

If a graph is symmetric with respect to the x-axis and the point (3,2) is on the graph, what other point is also on the graph?



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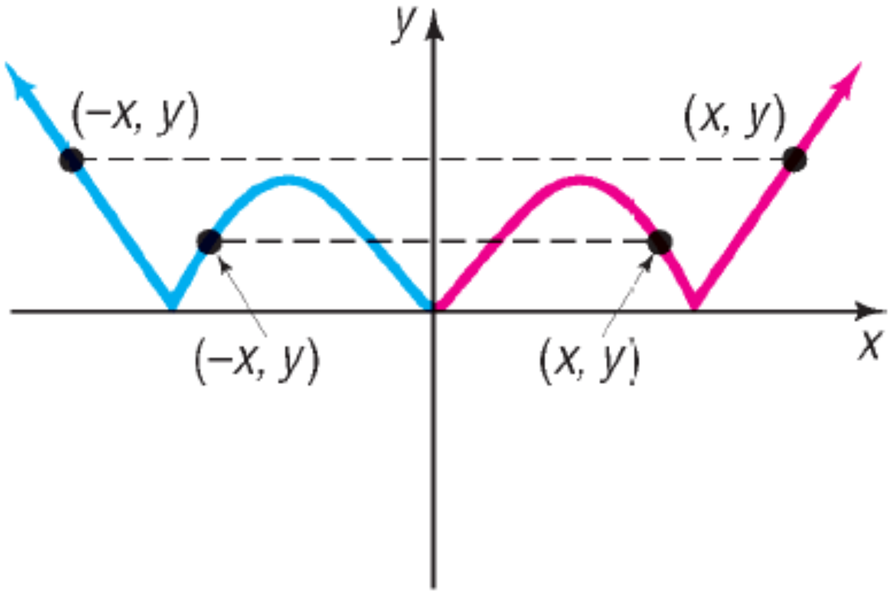
DEFINITION

A graph is said to be **symmetric with respect to the *y*-axis** if, for every point (x, y) on the graph, the point $(-x, y)$ is also on the graph.



Symmetry with respect to the *y*-axis

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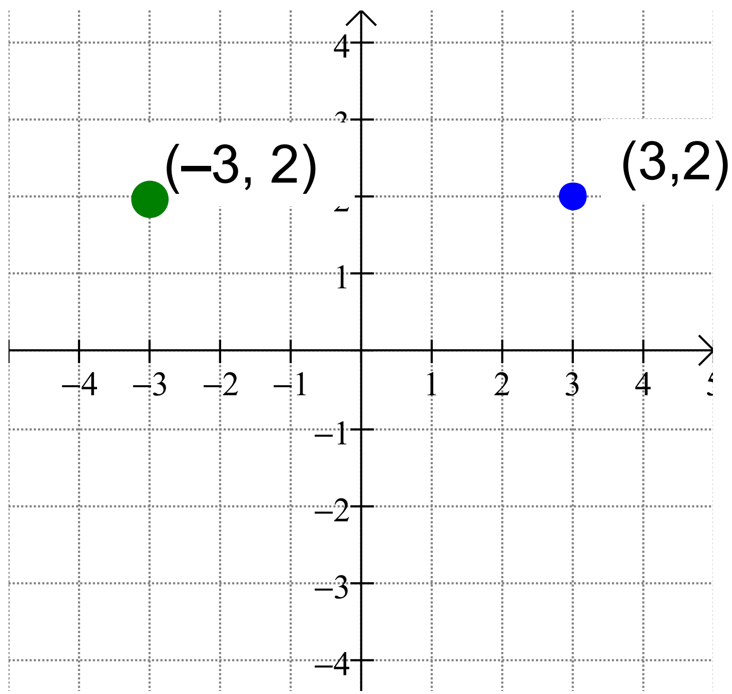
EXAMPLE

Points Symmetric with Respect to the y-Axis

If a graph is symmetric with respect to the *y*-axis and the point (3,2) is on the graph, what other point is also on the graph?



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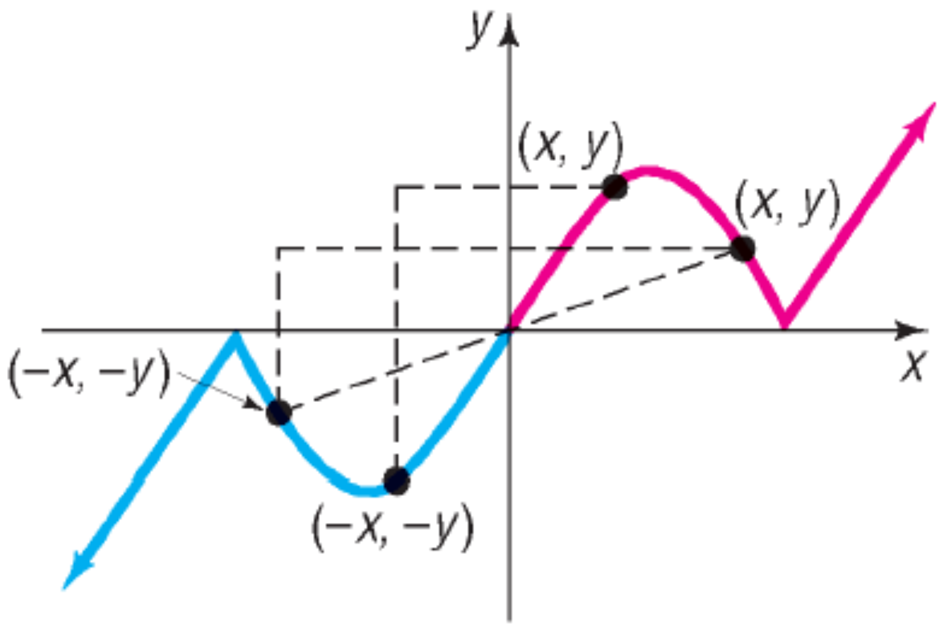
DEFINITION

A graph is said to be **symmetric with respect to the origin** if, for every point (x, y) on the graph, the point $(-x, -y)$ is also on the graph.



Symmetry with respect to the origin

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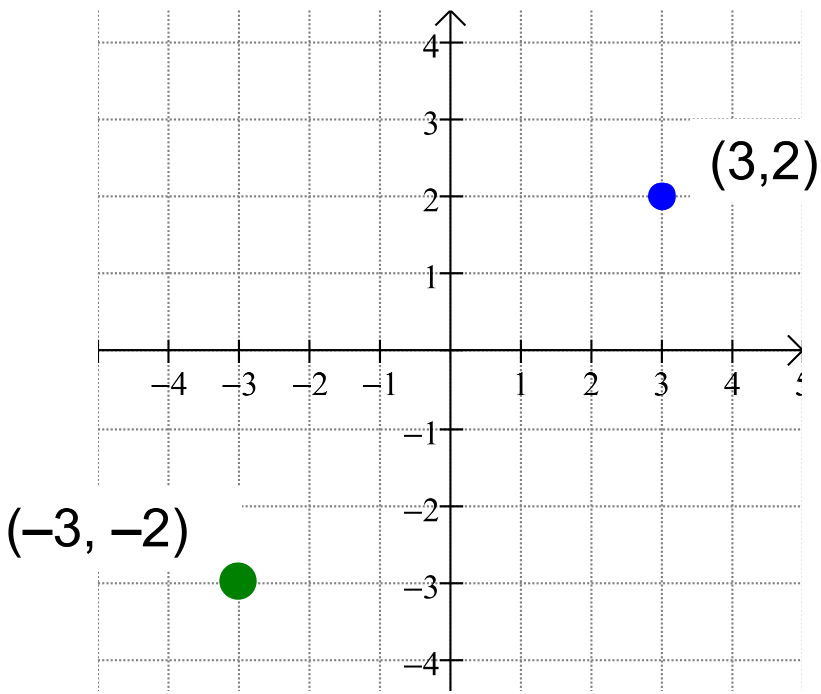
EXAMPLE

Points Symmetric with Respect to the Origin

If a graph is symmetric with respect to the origin and the point (3,2) is on the graph, what other point is also on the graph?



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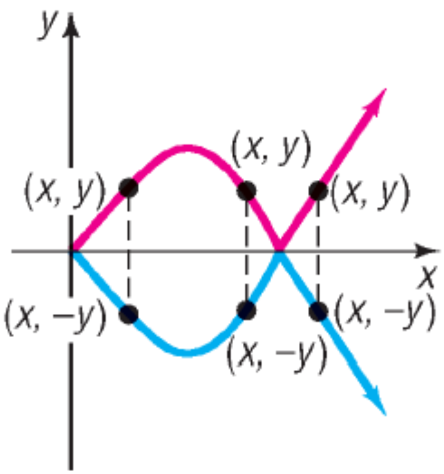
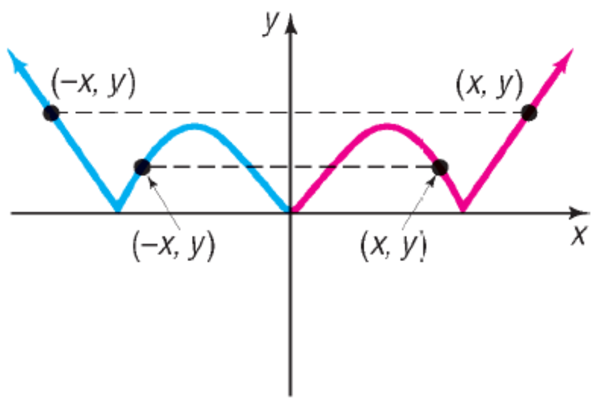
A graph is said to be **symmetric with respect to the x-axis** if, for every point (x, y) on the graph, the point $(x, -y)$ is also on the graph.

A graph is said to be **symmetric with respect to the y-axis** if, for every point (x, y) on the graph, the point $(-x, y)$ is also on the graph.

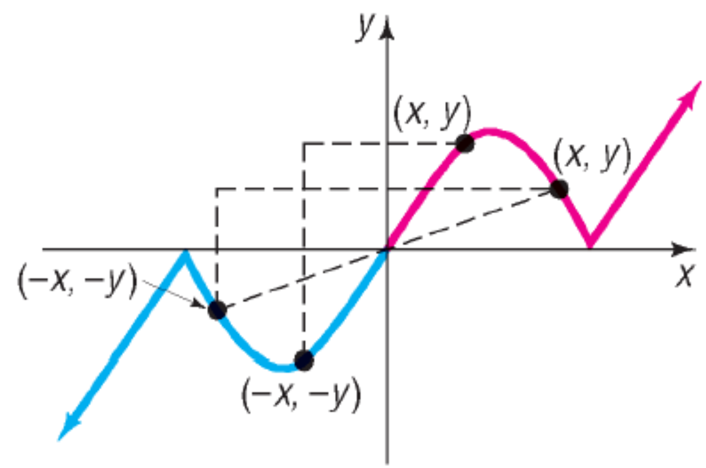
A graph is said to be **symmetric with respect to the origin** if, for every point (x, y) on the graph, the point $(-x, -y)$ is also on the graph.



Symmetry with respect to the x-axis



Symmetry with respect to the y-axis



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Symmetry with respect to the origin

Tests for Symmetry

To test the graph of an equation for symmetry with respect to the

x-Axis	Replace y by $-y$ in the equation and simplify. If an equivalent equation results, the graph of the equation is symmetric with respect to the x-axis.
y-Axis	Replace x by $-x$ in the equation and simplify. If an equivalent equation results, the graph of the equation is symmetric with respect to the y-axis.
Origin	Replace x by $-x$ and y by $-y$ in the equation and simplify. If an equivalent equation results, the graph of the equation is symmetric with respect to the origin.

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EXAMPLE Testing an Equation for Symmetry

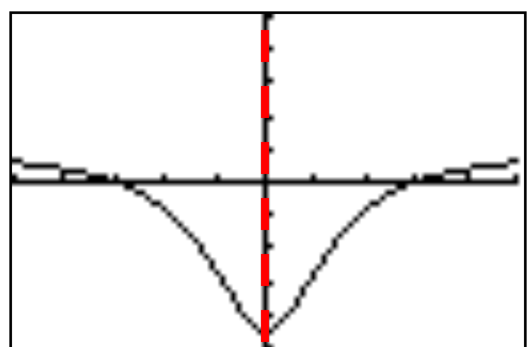
$$\text{Test } y=\frac{x^2-9}{x^2+2} \text{ for symmetry.}$$

x-Axis: $-y=\dfrac{x^2-9}{x^2+2}$ Not equivalent so not symmetric with respect to the x-axis.

y-Axis: $y=\dfrac{(-x)^2-9}{(-x)^2+2}$ IS equivalent so symmetric with respect to the y-axis.

Origin: $-y=\dfrac{(-x)^2-9}{(-x)^2+2}$ Not equivalent so not symmetric with respect to the origin.

Seeing the Concept



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5 Know How to Graph Key Equations

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EXAMPLE

Graphing the Equation $y = x^3$ by Finding Intercepts, Checking for Symmetry, and Plotting Points

Graph the equation $y = x^3$ by hand by plotting points. Find any intercepts and check for symmetry first.

First, find the intercepts. When $x = 0$, then $y = 0$; and when $y = 0$, then $x = 0$. The origin $(0, 0)$ is the only intercept. Now test for symmetry.

x-Axis: Replace y by $-y$. Since $-y = x^3$ is not equivalent to $y = x^3$, the graph is not symmetric with respect to the x-axis.

y-Axis: Replace x by $-x$. Since $y = (-x)^3 = -x^3$ is not equivalent to $y = x^3$, the graph is not symmetric with respect to the y-axis.

Origin: Replace x by $-x$ and y by $-y$. Since $-y = (-x)^3 = -x^3$ is equivalent to $y = x^3$ (multiply both sides by -1), the graph is symmetric with respect to the origin.

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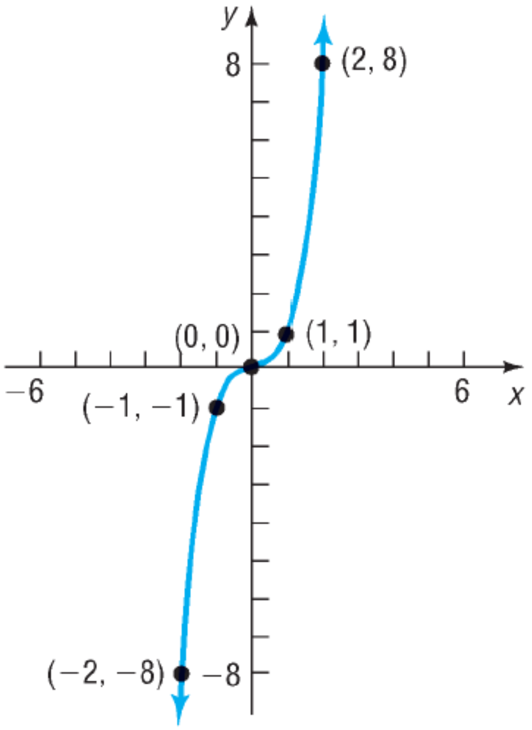
EXAMPLE

Graphing the Equation $y = x^3$ by Finding Intercepts, Checking for Symmetry, and Plotting Points

Graph the equation $y = x^3$ by hand by plotting points. Find any intercepts and check for symmetry first.

x	$y = x^3$	(x, y)
0		
1		
2		
3		



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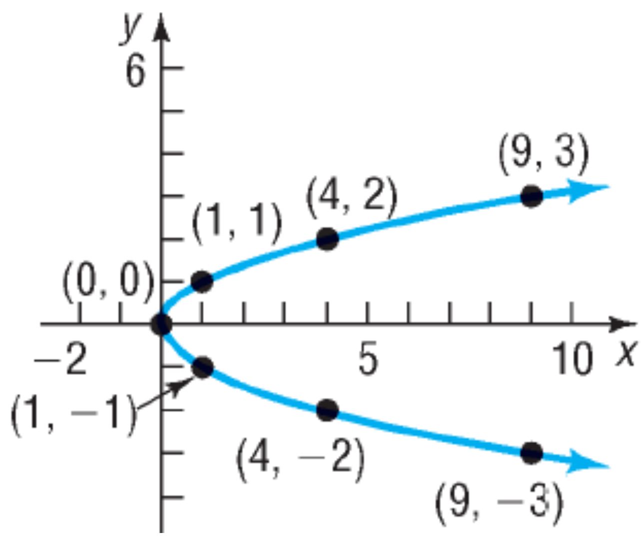
EXAMPLE Graphing the Equation $x = y^2$

Graph the equation $x = y^2$.

Find any intercepts and check for symmetry first.

The lone intercept is $(0, 0)$. The graph is symmetric with respect to the x-axis. (Do you see why? Replace y by $-y$.) Figure 22 shows the graph.



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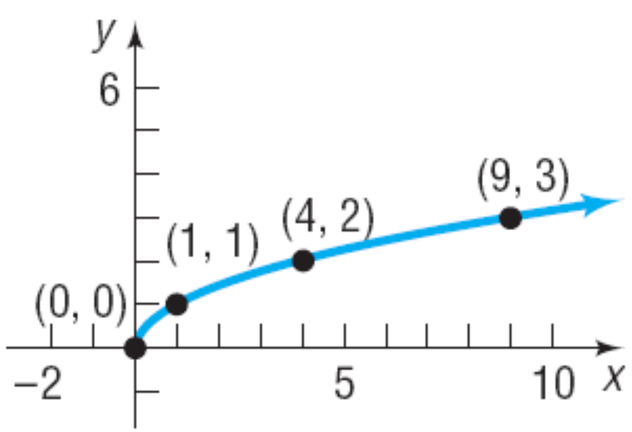
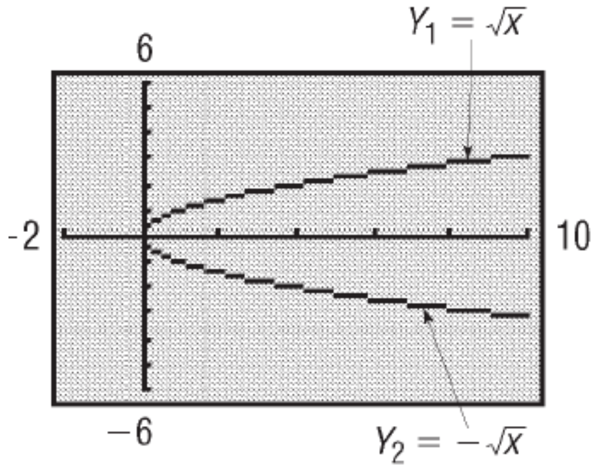
EXAMPLE Graphing the Equation $x = y^2$

Graph the equation $x = y^2$.

Graph $x = y^2$, $y \geq 0$.

If we restrict y so that $y \geq 0$, the equation $x = y^2$, $y \geq 0$, may be written equivalently as $y = \sqrt{x}$. The portion of the graph of $x = y^2$ in quadrant I is therefore the graph of $y = \sqrt{x}$.





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EXAMPLE Graphing the Equation $y = \frac{1}{x}$

Graph the equation $y = \dfrac{1}{x}$.

Find any intercepts and check for symmetry first.

Check for intercepts first. If we let $x = 0$, we obtain 0 in the denominator, which makes y undefined. We conclude that there is no y-intercept. If we let $y = 0$, we get the equation $\dfrac{1}{x} = 0$, which has no solution. We conclude that there is no x-intercept. The graph of $y = \dfrac{1}{x}$ does not cross or touch the coordinate axes.

x-Axis: Replacing y by $-y$ yields $-y = \dfrac{1}{x}$, which is not equivalent to $y = \dfrac{1}{x}$.

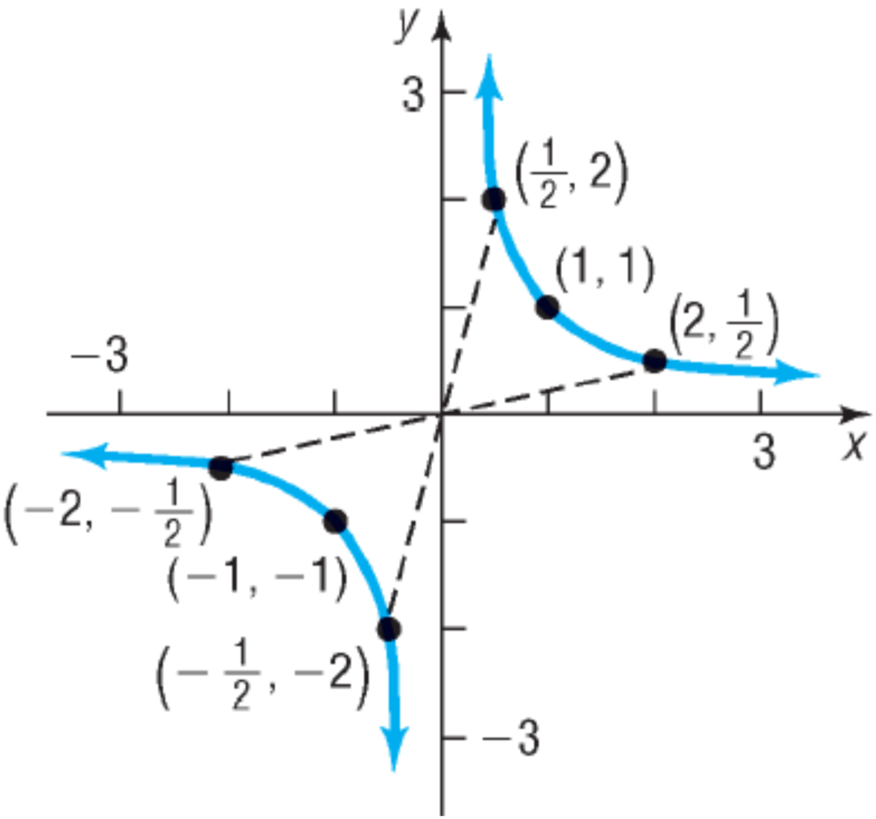
y-Axis: Replacing x by $-x$ yields $y = \dfrac{1}{-x} = -\dfrac{1}{x}$, which is not equivalent to

Origin: Replacing x by $-x$ and y by $-y$ yields $-y = -\dfrac{1}{x}$, which is equivalent to $y = \dfrac{1}{x}$. The graph is symmetric with respect to the origin.

EXAMPLE

Graphing the Equation $y = \dfrac{1}{x}$

Graph the equation $y = \dfrac{1}{x}$.



x	$y = \frac{1}{x}$	(x, y)
$\frac{1}{10}$		
$\frac{1}{3}$		
$\frac{1}{2}$		
1		
2		
3		
10		

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