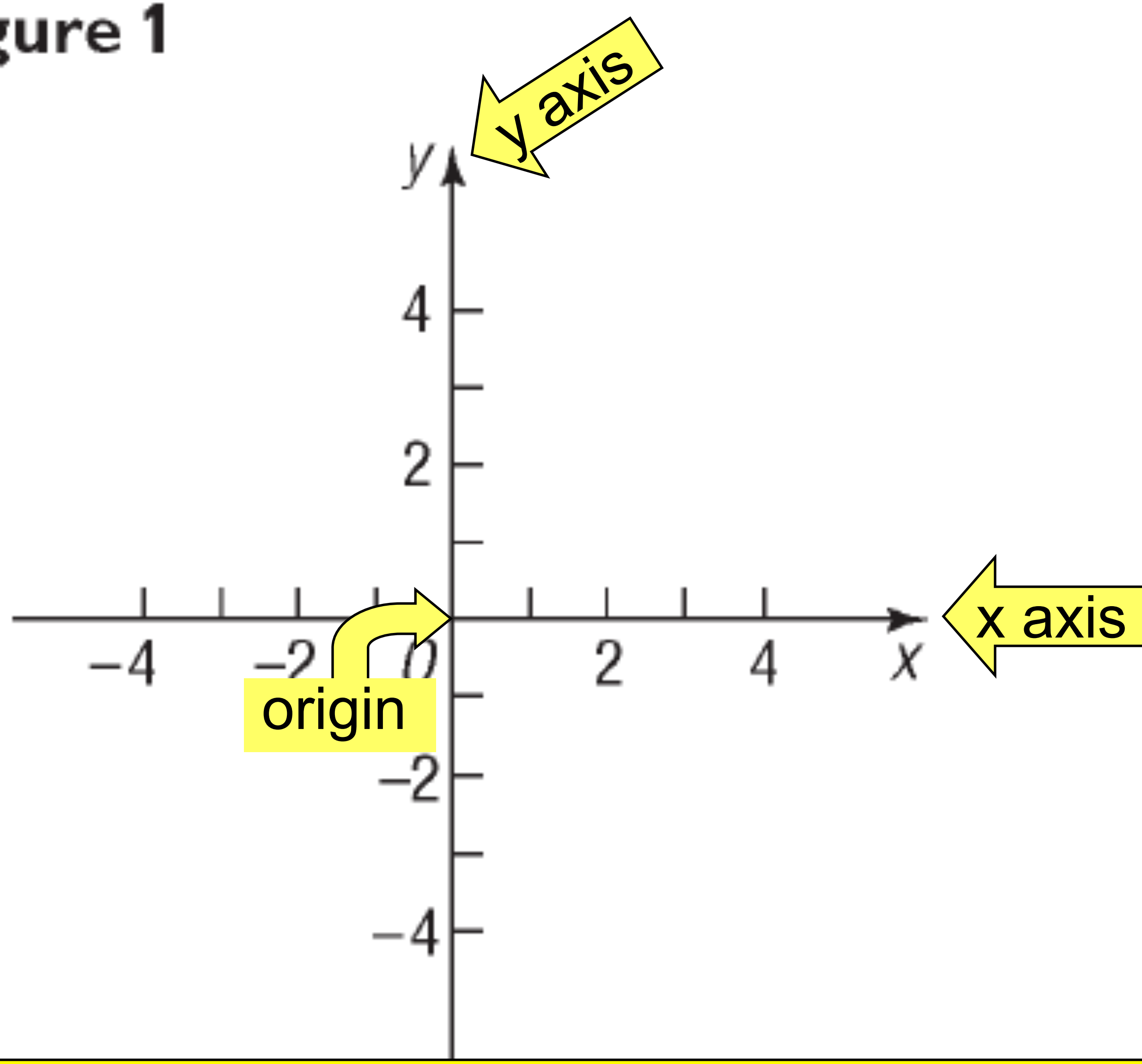
Section 2.1

The Distance and Midpoint Formulas

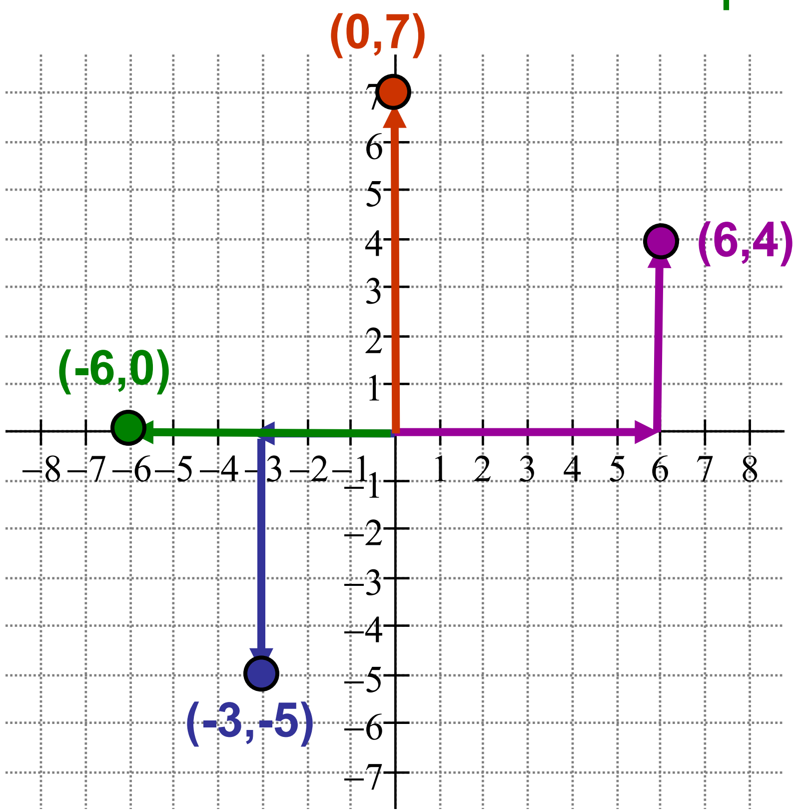
Figure 1



Rectangular or Cartesian Coordinate System

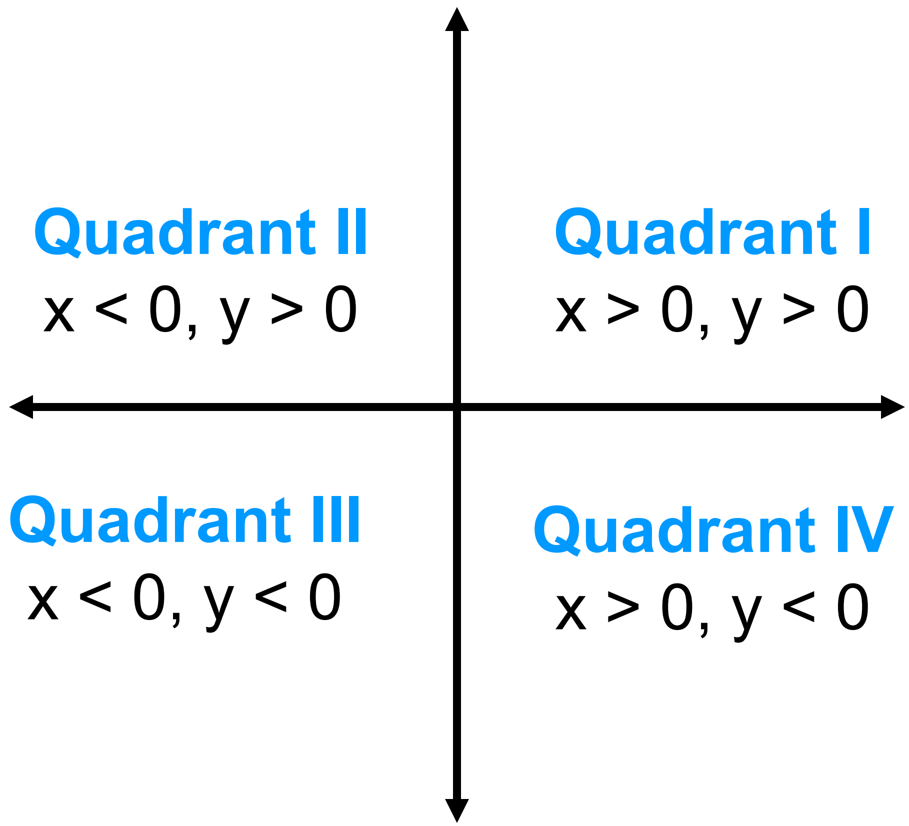
Let's plot the point (6,4).

Let's plot the point (-6,0).



Let's plot the point (-3,-5).

Let's plot the point (0,7).

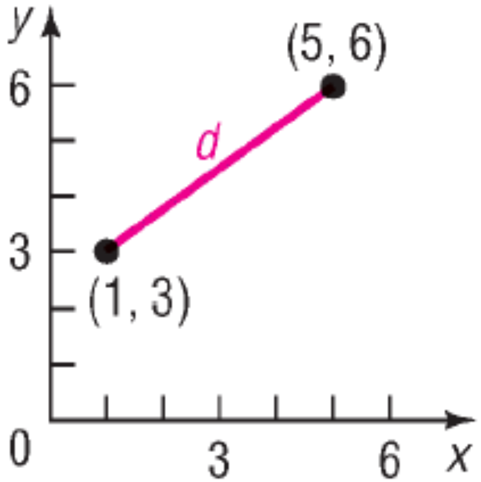

Quadrant II
x < 0, y > 0
Quadrant I
x > 0, y > 0
Quadrant III
x < 0, y < 0
Quadrant IV
x > 0, y < 0

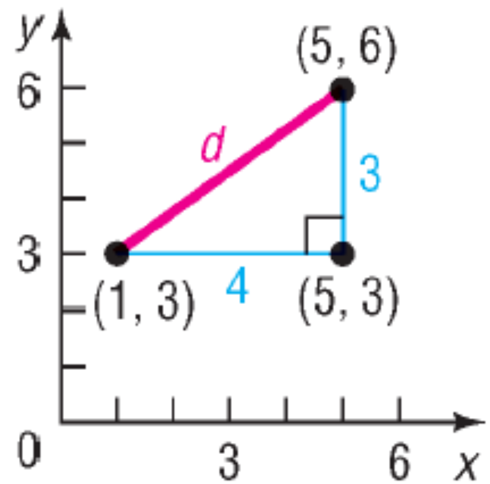
1 Use the Distance Formula

EXAMPLE

Finding the Distance between Two Points

Find the distance d between the points $(1, 3)$ and $(5, 6)$.





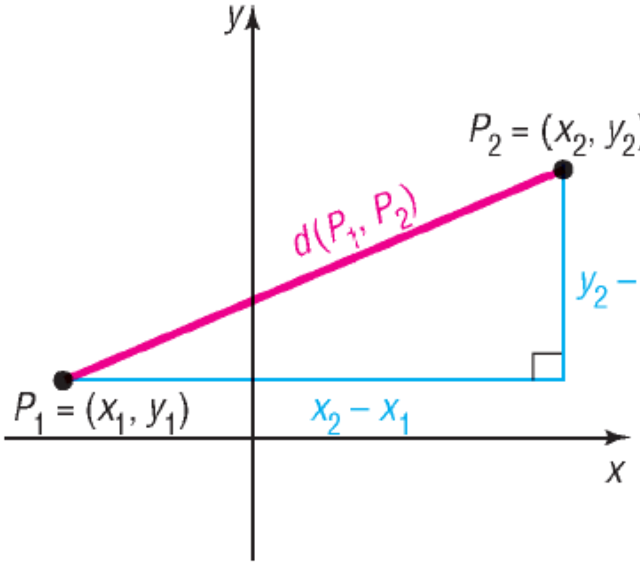
$$d^2 = 4^2 + 3^2 = 16 + 9 = 25 \qquad d = \sqrt{25} = 5$$

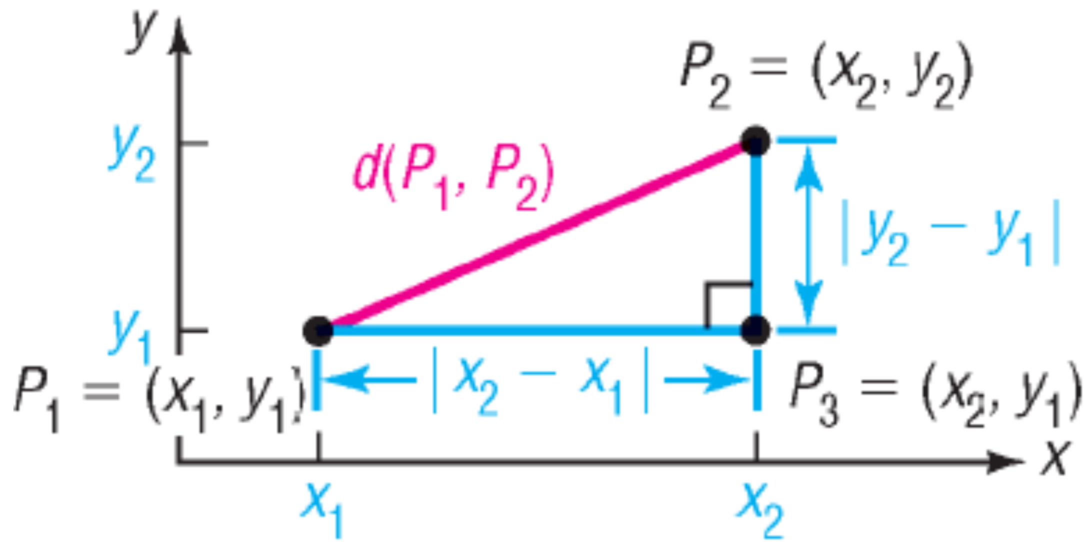
Distance Formula

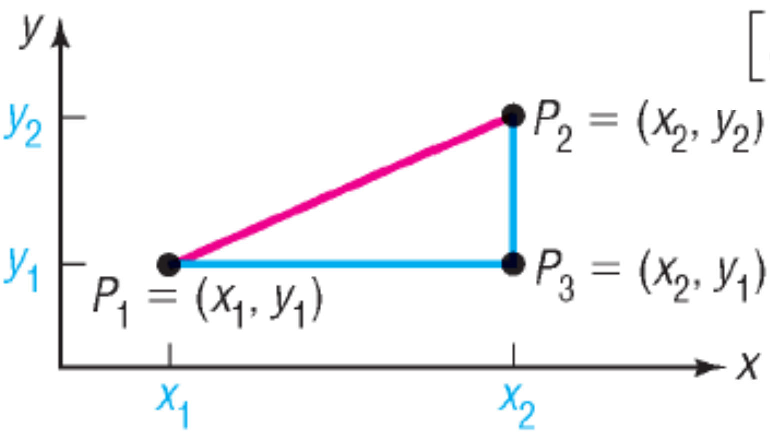
The distance between two points $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$, denoted by $d(P_1, P_2)$, is

$$\boxed{d(P_1, P_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}$$

Proof of the Distance Formula





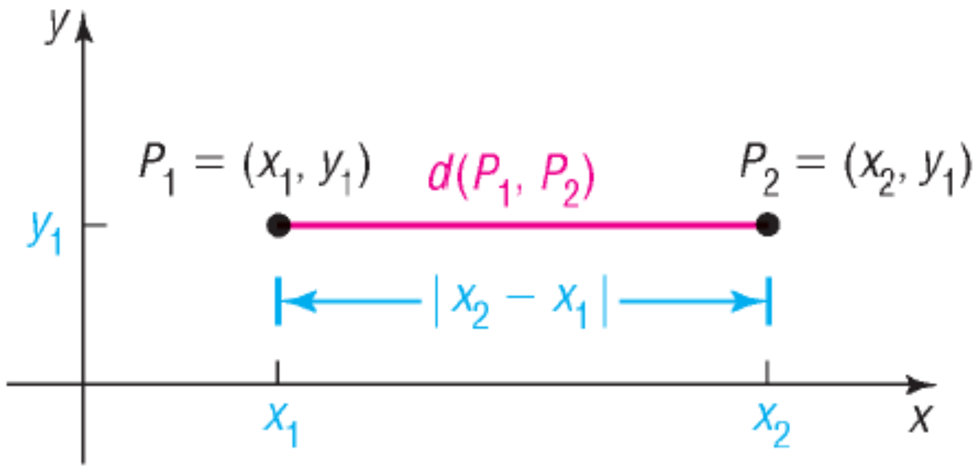


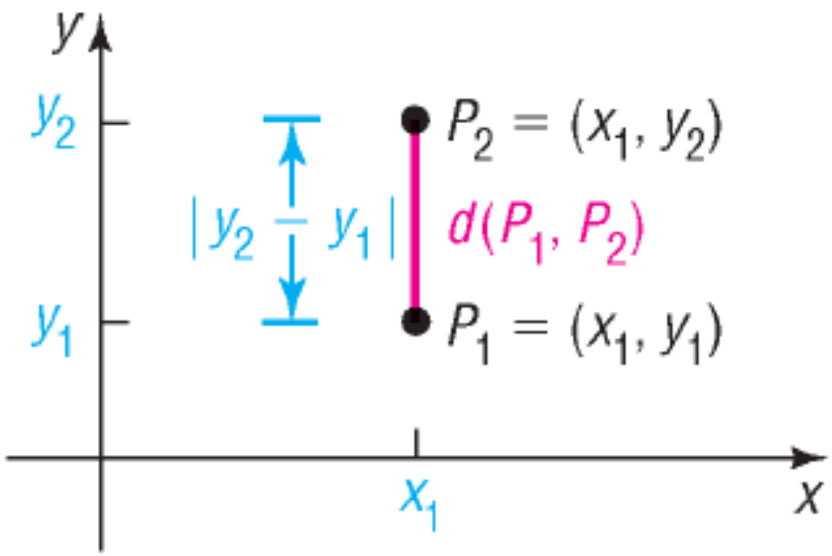
$$[d(P_1, P_2)]^2 = |x_2 - x_1|^2 + |y_2 - y_1|^2$$

$$= (x_2 - x_1)^2 + (y_2 - y_1)^2$$

$$d(P_1, P_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Horizontal or Vertical Segments





EXAMPLE

Using the Distance Formula

Find the distance d between the points (2, – 4) and (–1, 3).

$$d = \sqrt{(-1-2)^2 + \left(3-(-4)\right)^2}$$

$$d = \sqrt{(-3)^2 + (7)^2} = \sqrt{9+49} = \sqrt{58} \approx 7.62$$

$$d(P_1, P_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

EXAMPLE Using Algebra to Solve Geometry Problems

Consider the three points $A = (-2, 1)$, $B = (2, 3)$, and $C = (3, 1)$.

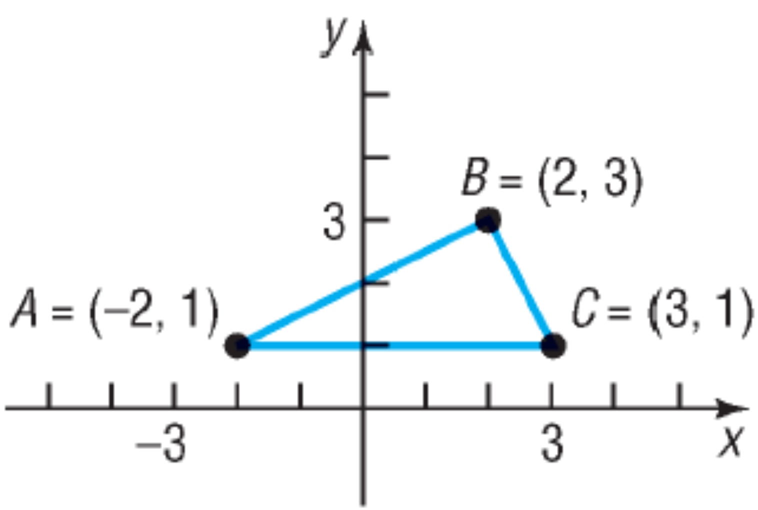
(a) Plot each point and form the triangle ABC.

(b) Find the length of each side of the triangle.

$$d(A, B) = \sqrt{[2-(-2)]^2 + (3-1)^2} = \sqrt{16+4} = \sqrt{20} = 2\sqrt{5}$$

$$d(B, C) = \sqrt{(3-2)^2 + (1-3)^2} = \sqrt{1+4} = \sqrt{5}$$

$$d(A, C) = \sqrt{[3-(-2)]^2 + (1-1)^2} = \sqrt{25+0} = 5$$



EXAMPLE Using Algebra to Solve Geometry Problems

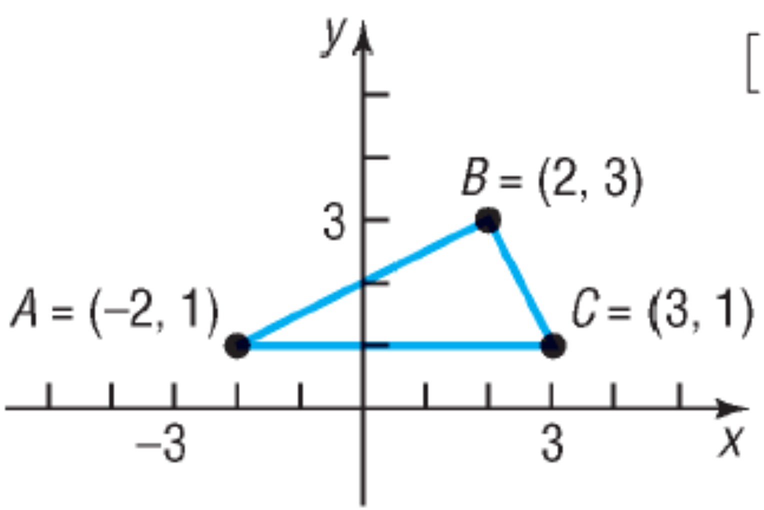
Consider the three points $A = (-2, 1)$, $B = (2, 3)$, and $C = (3, 1)$.

(c) Verify that the triangle is a right triangle.

$$d(A, B) = \sqrt{[2-(-2)]^2 + (3-1)^2} = \sqrt{16+4} = \sqrt{20} = 2\sqrt{5}$$

$$d(B, C) = \sqrt{(3-2)^2 + (1-3)^2} = \sqrt{1+4} = \sqrt{5}$$

$$d(A, C) = \sqrt{[3-(-2)]^2 + (1-1)^2} = \sqrt{25+0} = 5$$



$$[d(A, B)]^2 + [d(B, C)]^2 = [d(A, C)]^2$$

$$\left(2\sqrt{5}\right)^2 + \left(\sqrt{5}\right)^2$$

$$= 20 + 5 = 25 = [d(A, C)]^2$$

EXAMPLE

Using Algebra to Solve Geometry Problems

Consider the three points $A = (-2, 1)$, $B = (2, 3)$, and $C = (3, 1)$.

(d) Find the area of the triangle.

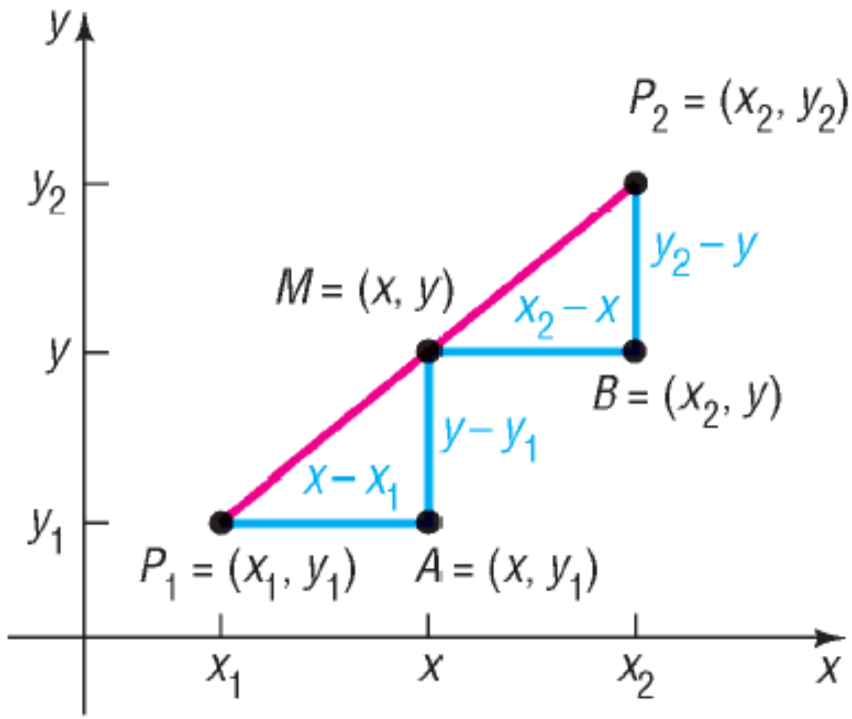
$$d(A, B) = \sqrt{[2-(-2)]^2 + (3-1)^2} = \sqrt{16+4} = \sqrt{20} = 2\sqrt{5}$$

$$d(B, C) = \sqrt{(3-2)^2 + (1-3)^2} = \sqrt{1+4} = \sqrt{5}$$

$$d(A, C) = \sqrt{[3-(-2)]^2 + (1-1)^2} = \sqrt{25+0} = 5$$

$$\text{Area} = \frac{1}{2}(\text{Base})(\text{Height}) = \frac{1}{2}\left(2\sqrt{5}\right)\left(\sqrt{5}\right) = 5 \text{ square units}$$

2 Use the Midpoint Formula



$$x - x_1 = x_2 - x$$
$$2x = x_1 + x_2$$
$$x = \frac{x_1 + x_2}{2}$$

$$y - y_1 = y_2 - y$$
$$2y = y_1 + y_2$$
$$y = \frac{y_1 + y_2}{2}$$

Midpoint Formula

The midpoint $M = (x, y)$ of the line segment from $P_1 = (x_1, y_1)$ to $P_2 = (x_2, y_2)$ is

$$\boxed{M = (x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)}$$

EXAMPLE

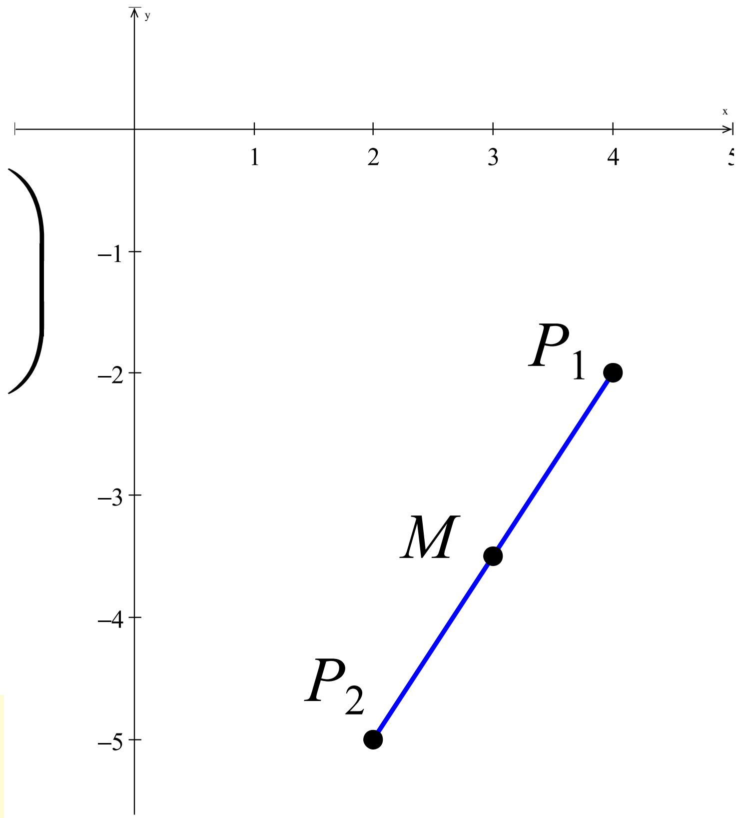
Finding the Midpoint of a Line Segment

Find the midpoint of the line segment from $P_1 = (4, -2)$ to $P_2 = (2, -5)$. Plot the points and their midpoint.

$$x = \frac{4+2}{2} = 3$$

$$M = \left(3, -\frac{7}{2}\right)$$

$$y = \frac{-2-5}{2} = -\frac{7}{2}$$



$$M = (x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$