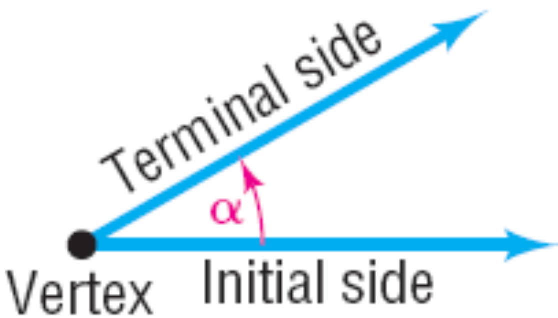
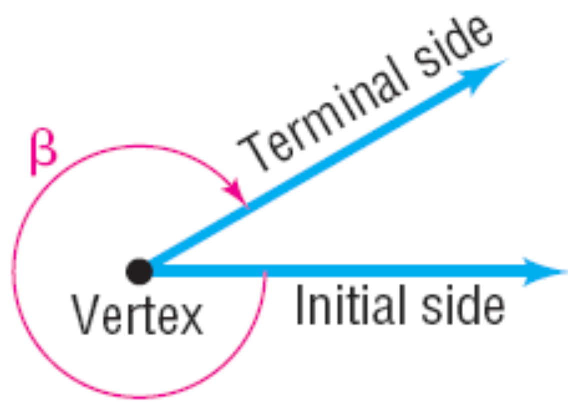
Section 7.1

Angles and Their Measure



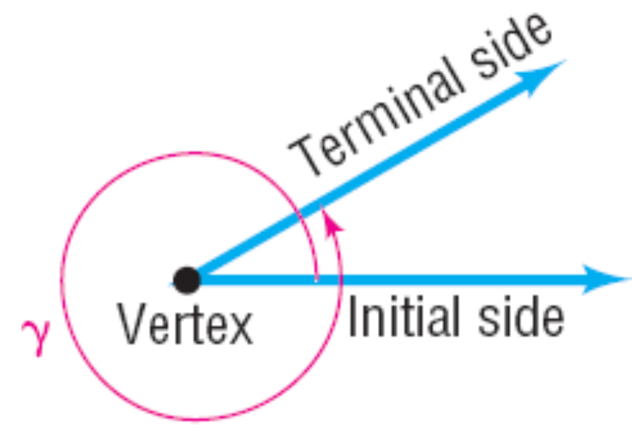
Counterclockwise rotation

Positive angle



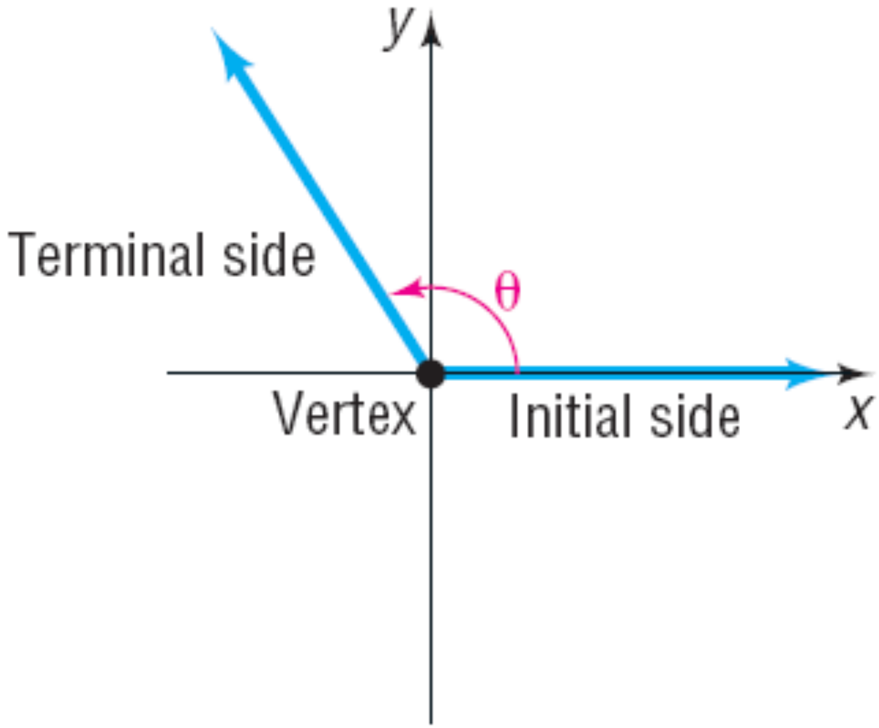
Clockwise rotation

Negative angle

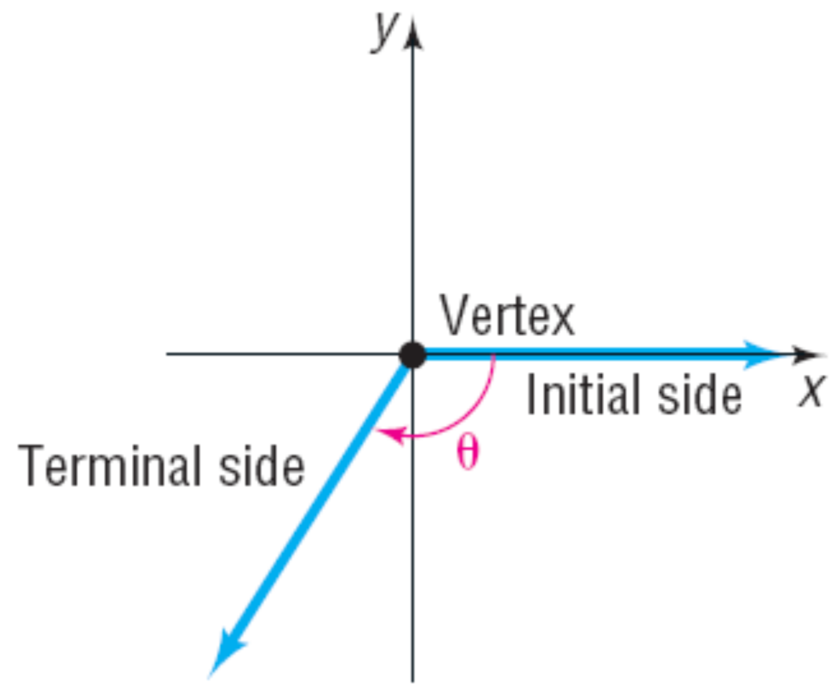


Counterclockwise rotation

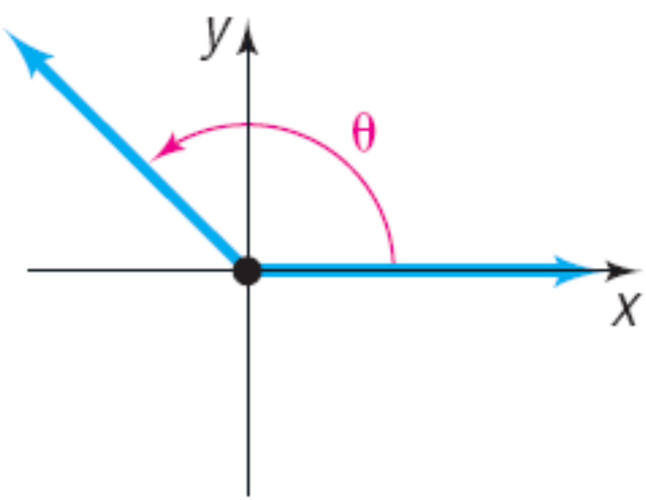
Positive angle



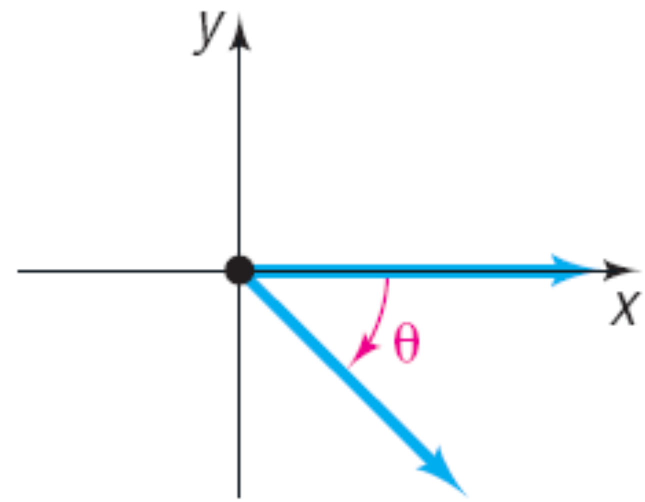
(a) θ is in standard position;
θ is positive



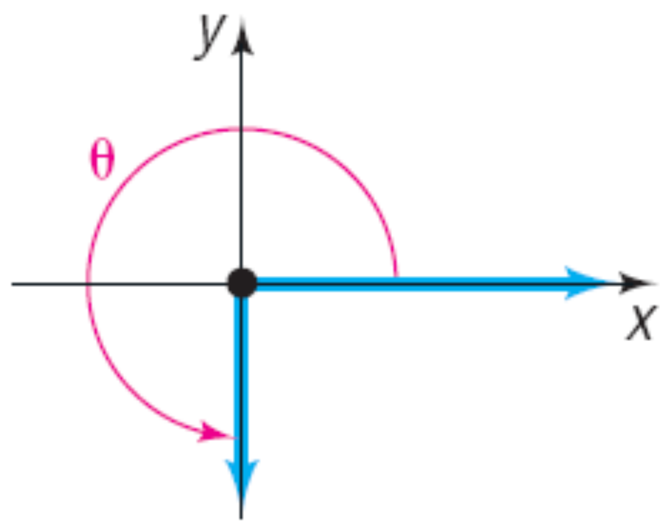
(b) θ is in standard position;
θ is negative



(a) θ lies in quadrant II

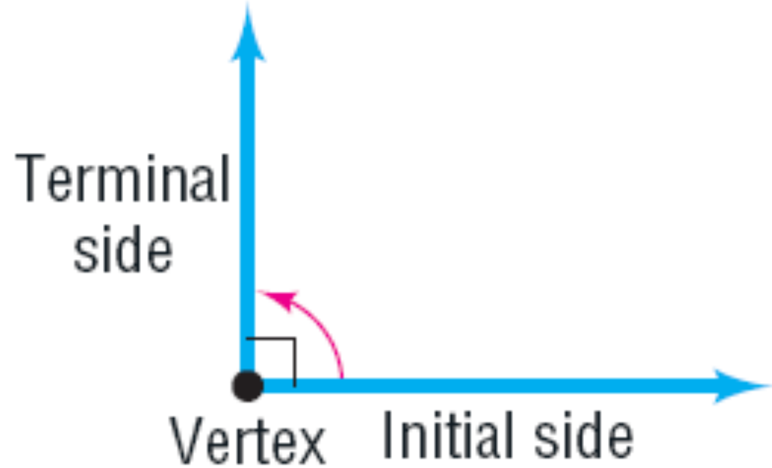


(b) θ lies in quadrant IV



(c) θ is a quadrantal angle

(a) 1 revolution counterclockwise, 360°



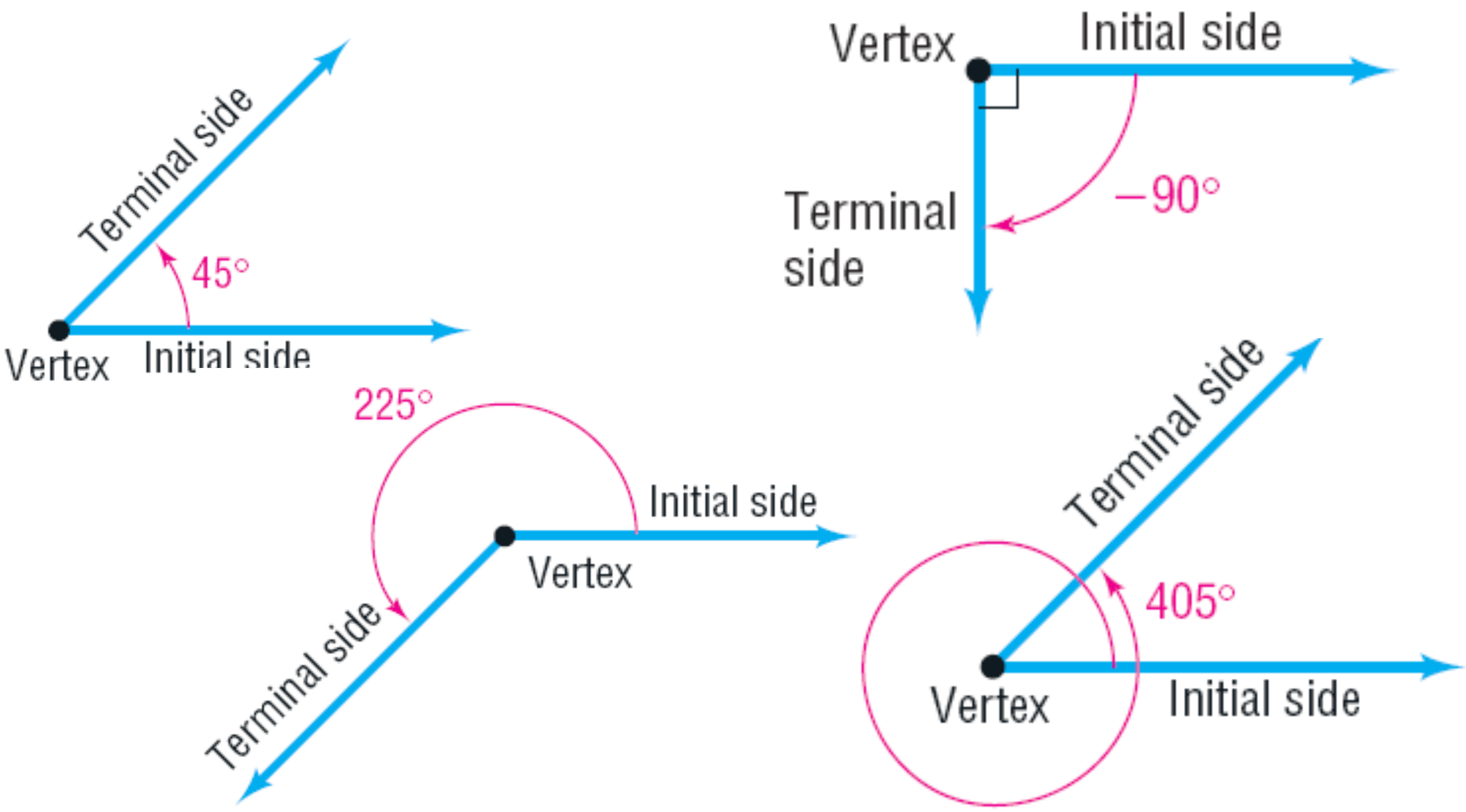
(b) right angle, $\frac{1}{4}$ revolution counter-clockwise, 90°

(c) straight angle, $\frac{1}{2}$ revolution counter-clockwise, 180°

EXAMPLE Drawing an Angle

Draw each angle.

(a) 45° (b) -90° (c) 225° (d) 405°



1 Convert between Decimals and Degrees, Minutes, Seconds Measures for Angles

$$1 \text{ counterclockwise revolution} = 360°$$

$$1° = 60' \qquad 1' = 60''$$

EXAMPLE **Converting between Degrees, Minutes, Seconds, and Decimal Forms**

(a) Convert 40° 12'5" to a decimal in degrees. Round the answer to four decimal places.

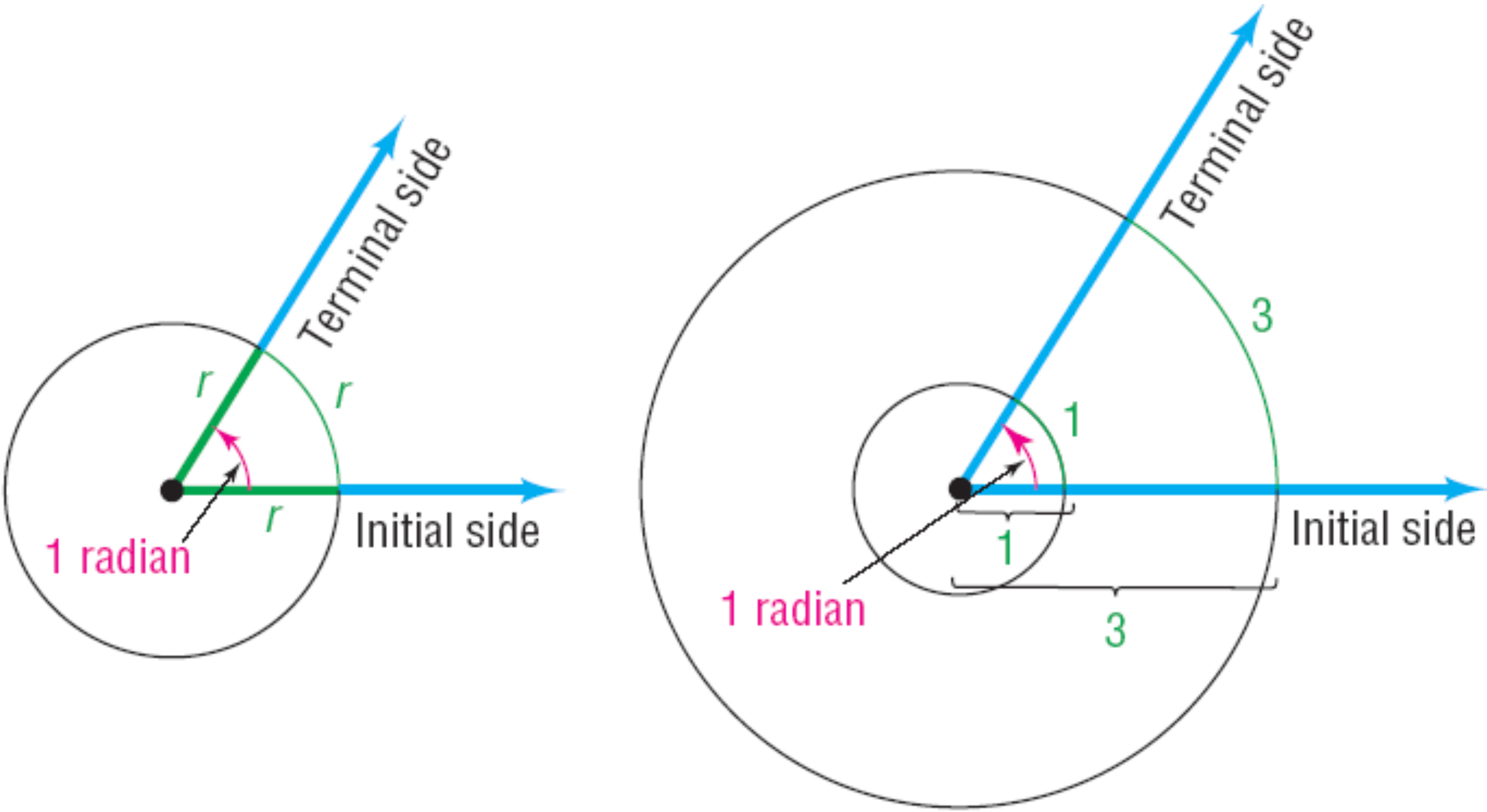
(b) Convert 78.562° to the D° M'S" form. Round the answer to the nearest second.

$$1' = \left(\frac{1}{60}\right)° \text{ and } 1'' = \left(\frac{1}{60} \cdot \frac{1}{60}\right)°$$

(a) $40° + 12' + 5'' = 40° + 12\left(\frac{1}{60}\right)° + 5\left(\frac{1}{60} \cdot \frac{1}{60}\right)° = 40° + 0.2° + 0.0014°$
$= 40.2014°$

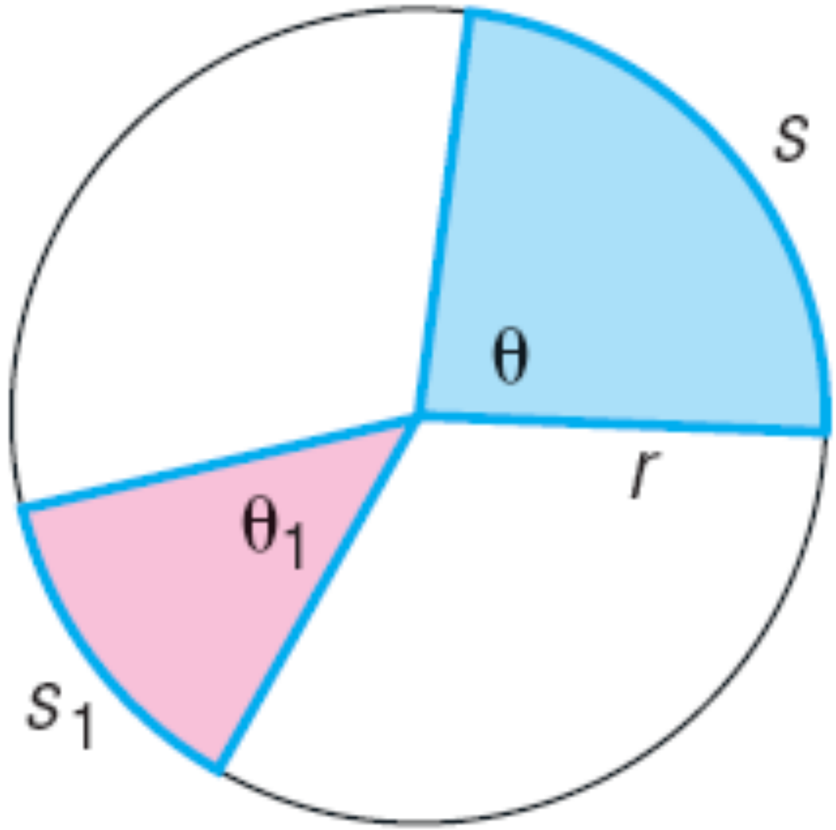
(b) $78° + 0.562° = 78° + (0.562)(60') = 78° + 33.72' = 78° + 33' + 0.72'$
$= 78° + 33' + (0.72)(60'') = 78° + 33' + 43.2'' = 78°33'43''$

Radians



Terminal side
r
r
r
1 radian
Initial side
Terminal side
3
1
1
3
1 radian
Initial side

2 Find the Length of an Arc of a Circle



$$\frac{\theta}{\theta_1} = \frac{s}{s_1}$$

THEOREM

Arc Length

For a circle of radius r, a central angle of θ radians subtends an arc whose length s is

$$\boxed{s = r\theta}$$

EXAMPLE

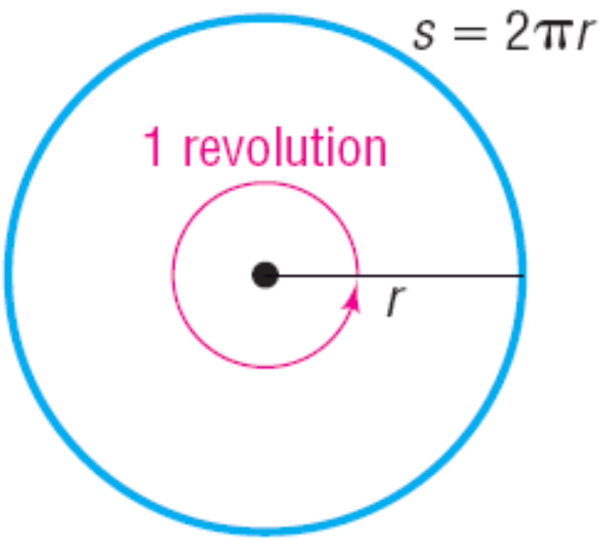
Finding the Length of an Arc of a Circle

Find the length of the arc of a circle of radius 4 meters subtended by a central angle of 0.5 radian.

$$s = (4)(0.5) = 2 \text{ meters}$$

$$\boxed{s = r\theta}$$

3 Convert from Degrees to Radians and from Radians to Degrees



$$\boxed{1 \text{ revolution} = 2\pi \text{ radians}}$$

$$\boxed{180^\circ = \pi \text{ radians}}$$

$$\boxed{1 \text{ degree} = \frac{\pi}{180} \text{ radian} \qquad 1 \text{ radian} = \frac{180}{\pi} \text{ degrees}}$$

EXAMPLE Converting from Degrees to Radians

Convert each angle in degrees to radians.

(a) 30° (b) 120° (c) -60° (d) 270° (e) 104°

$$\text{(a)}\ 30^0\left(\frac{\pi \text{ radian}}{180^\circ}\right)=\frac{\pi}{6}\text{ radians}$$

$$\text{(b)}\ 120^0\left(\frac{\pi \text{ radian}}{180^\circ}\right)=\frac{2\pi}{3}\text{ radians}$$

$$\text{(c)}\ -60^0\left(\frac{\pi \text{ radian}}{180^\circ}\right)=-\frac{\pi}{3}\text{ radians}$$

$$\text{(d)}\ 270^0\left(\frac{\pi \text{ radian}}{180^\circ}\right)=\frac{3\pi}{2}\text{ radians}$$

$$\text{(e)}\ 104^0\left(\frac{\pi \text{ radian}}{180^\circ}\right)\approx 1.815\text{ radians}$$

$$1 \text{ degree} = \frac{\pi}{180}\text{ radian} \qquad 1 \text{ radian} = \frac{180}{\pi}\text{ degrees}$$

EXAMPLE Converting from Degrees to Radians

Convert each angle in radians to degrees.

(a) $\frac{\pi}{3}$ radian (b) $-\frac{\pi}{2}$ radian (c) $\frac{5\pi}{6}$ radians (d) 5 radians

(a) $\frac{\pi}{3}\left(\frac{180°}{\pi}\right)=60°$

(b) $-\frac{\pi}{2}\left(\frac{180°}{\pi}\right)=-90°$

(c) $\frac{5\pi}{6}\left(\frac{180°}{\pi}\right)=150°$

(d) $5\left(\frac{180°}{\pi}\right)=286.48°$

$$1 \text{ degree} = \frac{\pi}{180} \text{ radian} \qquad 1 \text{ radian} = \frac{180}{\pi} \text{ degrees}$$

Degrees	0°	30°	45°	60°	90°	120°	135°	150°	180°
Radians	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
Degrees		210°	225°	240°	270°	300°	315°	330°	360°
Radians		$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	2π

EXAMPLE Finding the Distance between Two Cities

See Figure 13(a). The latitude of a location L is the angle formed by a ray drawn from the center of Earth to the Equator and a ray drawn from the center of Earth to L. See Figure 13(b). Glasgow, Montana, is due north of Albuquerque, New Mexico. Find the distance between Glasgow ($48°9'$ north latitude) and Albuquerque ($35°5'$ north latitude). Assume that the radius of Earth is 3960 miles.

The measure of the central angle between the two cities is $48°9' - 35°5' = 13°4'$.

In order to use $s = r\theta$, θ must be in radians.

$$\theta = 13°4' \approx 13.0667° = 13.0667 \cdot \frac{\pi}{180} \text{ radian} \approx 0.228 \text{ radian}$$

$$4' = 4\left(\frac{1}{60}\right)^{\circ}$$

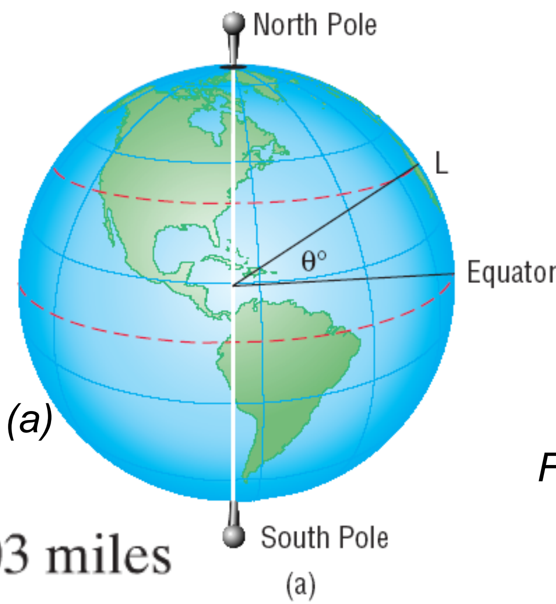


Figure 13 (a)

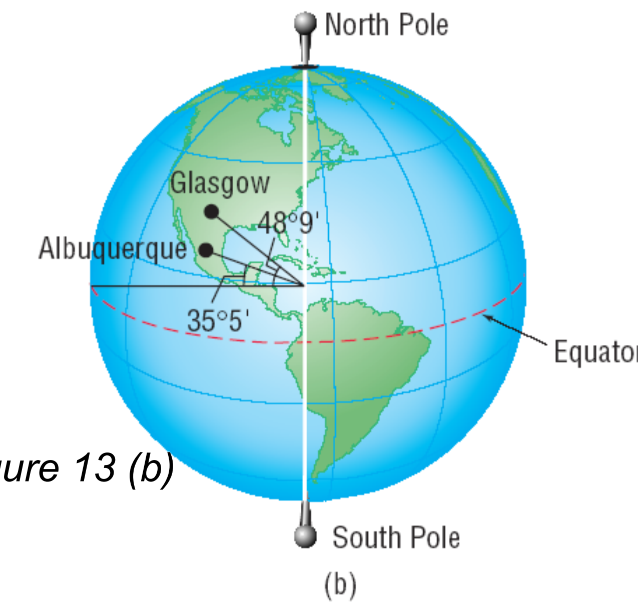
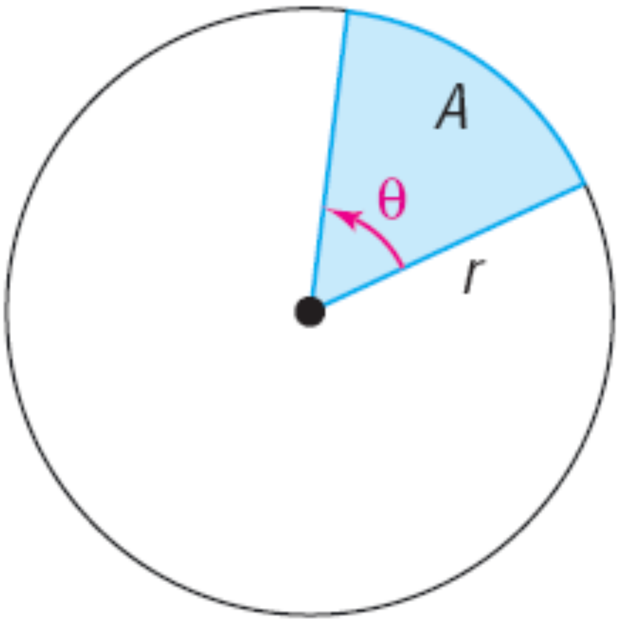


Figure 13 (b)

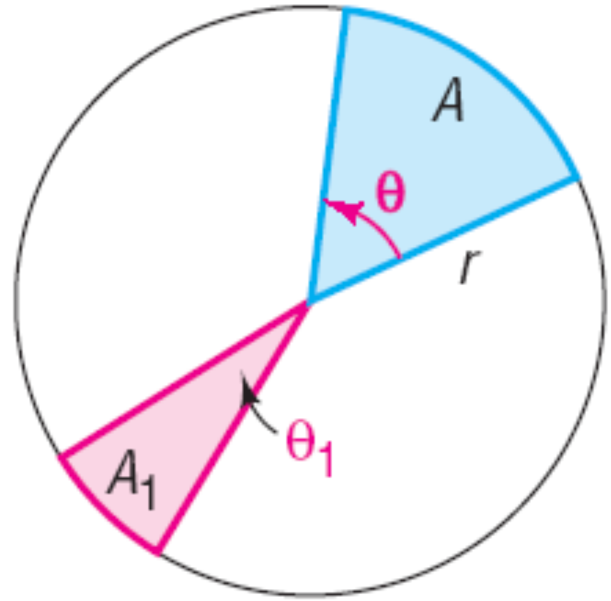
$$s = r\theta = 3960 \cdot 0.228 \approx 903 \text{ miles}$$

4 Find the Area of a Sector of a Circle

Area of a Sector



$$\frac{\theta}{\theta_1} = \frac{A}{A_1}$$



The area A of the sector of a circle of radius r formed by a central angle of θ radians is

$$\boxed{A = \frac{1}{2}r^2\theta}$$

EXAMPLE Finding the Area of a Sector of a Circle

Find the area of the sector of a circle of radius 5 feet formed by an angle of 60° . Round the answer to two decimal places.

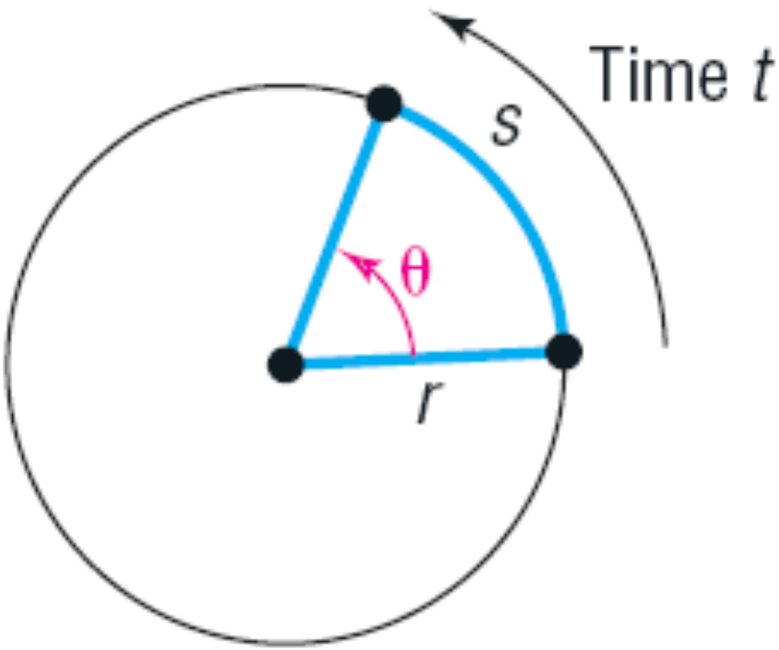
In order to use the equation for the area of a sector, θ must be in radians.

$$\theta = 60^\circ\left(\frac{\pi}{180^\circ}\right) = \frac{\pi}{3}$$

$$A = \frac{1}{2}(5)^2\left(\frac{\pi}{3}\right) = \frac{25\pi}{6} \approx 13.09 \text{ square feet}$$

$$\boxed{A = \frac{1}{2}r^2\theta}$$

5 Find the Linear Speed of an Object Traveling in Circular Motion



$$v = \frac{s}{t}$$

Linear Speed

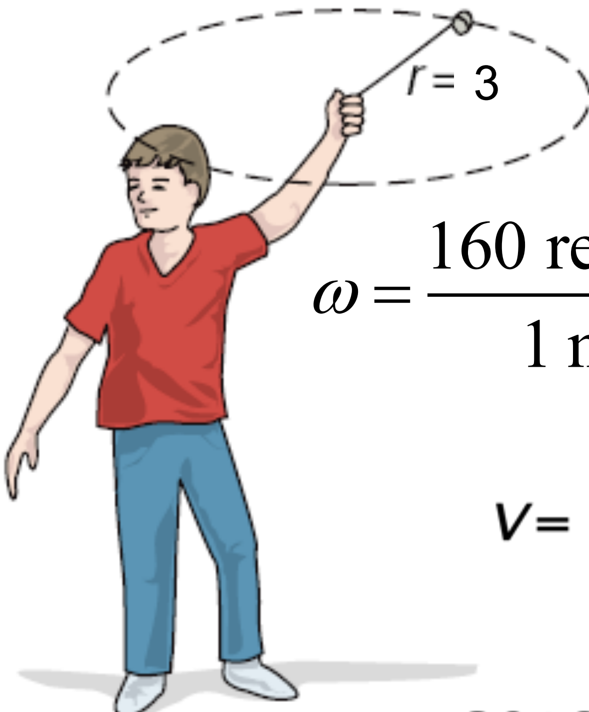
$$\omega = \frac{\theta}{t}$$

Angular Speed

$$v = r\omega$$

EXAMPLE Finding Linear Speed

A child is spinning a rock at the end of a 3-foot rope at the rate of 160 revolutions per minute (rpm). Find the linear speed of the rock when it is released.



$$\omega = \frac{160 \text{ revolutions}}{1 \text{ minute}} \cdot \frac{2\pi \text{ radians}}{1 \text{ revolution}} = 320\pi \frac{\text{radians}}{\text{minute}}$$

$$v = rw = 3 \text{ feet} \cdot 320p \frac{\text{radians}}{\text{minute}} \approx 3016 \frac{\text{feet}}{\text{minute}}$$

$$v \approx 3016 \frac{\text{feet}}{\text{minute}} \cdot \frac{1 \text{ mile}}{5280 \text{ feet}} \cdot \frac{60 \text{ minutes}}{1 \text{ hour}} \approx 34.3 \text{ mph}$$