Section 1.4

Radical Equations

Equations Quadratic in Form

Absolute Value Equations

Factorable Equations

1 Solve Radical Equations

EXAMPLE Solving a Radical Equation

Find the real solutions of the equation $\sqrt[3]{3x-1}-2=0$.

$$\sqrt[3]{3x-1}=2$$

$$\left(\sqrt[3]{3x-1}\right)^3=2^3$$

$$3x-1=8$$

$$3x=9$$

$$x=3$$

The solution set is $\{3\}$.

EXAMPLE **Solving a Radical Equation**

Find the real solutions of the equation $\sqrt{x+1} = -x+1$.

$$\left(\sqrt{x+1}\right)^2 = (-x+1)^2 \qquad x+1 = x^2 - 2x + 1$$

$$x^2 - 3x = 0 \qquad x(x-3) = 0 \qquad x = 0 \text{ or } x = 3$$

Check: $\sqrt{0+1} = -0+1$

$1 = 1$

$\sqrt{3+1} = -3+1$

$2 \neq -2$

extraneous solution
(does not work in the original equation)

The solution set is $\{0\}$.

EXAMPLE Solving a Radical Equation

Find the real solutions of the equation $\sqrt{5x-1}-\sqrt{x}=1$.

$$\sqrt{5x-1}=\sqrt{x}+1$$

$$\left(\sqrt{5x-1}\right)^2=\left(\sqrt{x}+1\right)^2$$

$$5x-1=x+2\sqrt{x}+1$$

$$4x-2=2\sqrt{x}$$

$$2x-1=\sqrt{x}$$

$$\left(2x-1\right)^2=\left(\sqrt{x}\right)^2$$

$$4x^2-4x+1=x$$

$$4x^2-5x+1=0$$

$$(4x-1)(x-1)=0$$

$$x=\frac{1}{4} \text{ or } x=1$$

Check:

$$\sqrt{5\left(\frac{1}{4}\right)-1}-\sqrt{\frac{1}{4}}=1$$

$$\sqrt{\frac{1}{4}}-\sqrt{\frac{1}{4}}\neq 1$$

$$\sqrt{5(1)-1}-\sqrt{1}=1$$

$$2-1=1$$

The solution set is $\{1\}$.

2 Solve Equations Quadratic in Form

EXAMPLE Solving an Equation Quadratic in Form

Find the real solutions of the equation

$$(x-2)^2-3(x-2)+2=0.$$

Let $u = x-2$. Then $u^2=(x-2)^2$

$$u^2-3u+2=0$$

$$(u-2)(u-1)=0$$

$$u=2 \text{ or } u=1$$

$$x-2=2 \text{ or } x-2=1$$

$$x=4 \text{ or } x=3$$

The solution set is $\{3,4\}$.

EXAMPLE Solving an Equation Quadratic in Form

Find the real solutions of the equation $x - 5\sqrt{x} + 6 = 0$.

Let $u = \sqrt{x}$. Then $u^2 = x$

$u^2 - 5u + 6 = 0$

$(u-3)(u-2) = 0$

$u = 3$ or $u = 2$

$\sqrt{x} = 3$ or $\sqrt{x} = 2$

$x = 9$ or $x = 4$

The solution set is $\{4, 9\}$.

3 Solve Equations by Factoring

EXAMPLE Solving an Equation by Factoring

Solve the equation: $x^3 - 2x = x^2$

$$x^3 - x^2 - 2x = 0$$

$$x(x^2 - x - 2) = 0$$

$$x(x-2)(x+1) = 0$$

$$x = 0 \text{ or } x = 2 \text{ or } x = -1$$

The solution set is $\{-1, 0, 2\}$.

EXAMPLE Solving an Equation by Factoring

Solve the equation: $x^3 - 3x^2 - 4x + 12 = 0$

$$x^2(x-3) - 4(x-3) = 0$$

$$\left(x^2 - 4\right)(x-3) = 0$$

$$(x+2)(x-2)(x-3) = 0$$

$$x = -2 \text{ or } x = 2 \text{ or } x = 3$$

The solution set is $\{-2, 2, 3\}$.