Section 12.7

Systems of Inequalities

EXAMPLE

Examples of Inequalities in Two Variables

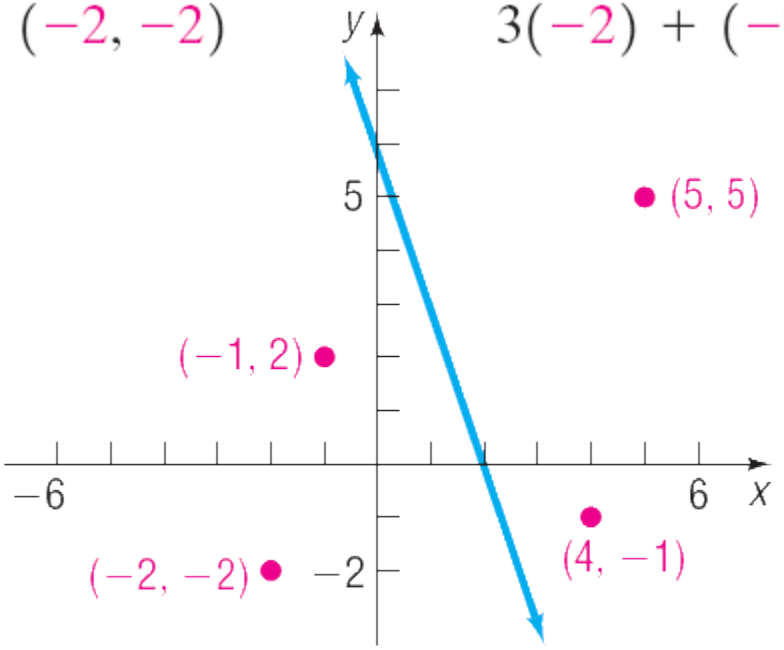
(a) $3x + y \leq 6$ (b) $x^2 + y^2 < 4$ (c) $y^2 > x$

1 Graph an Inequality

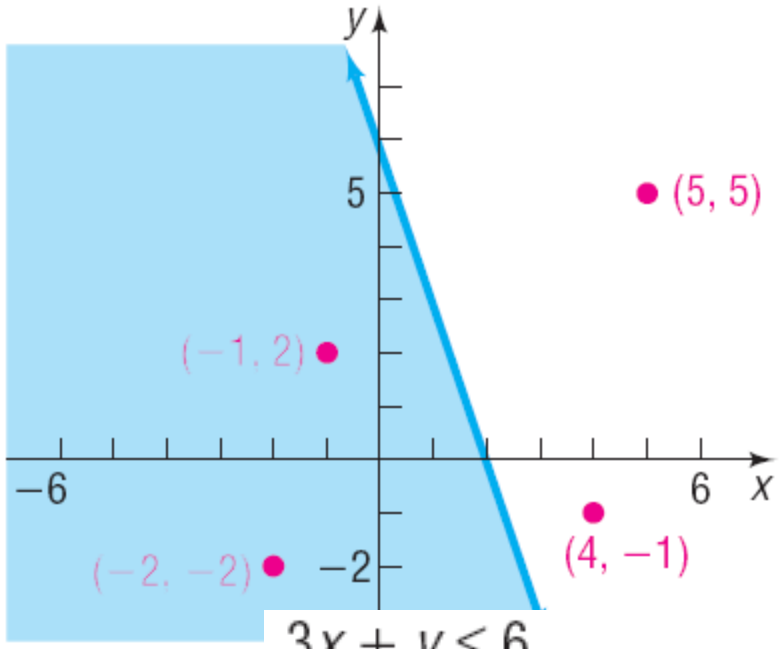
EXAMPLE **Graphing an Inequality**

Graph the linear inequality: $3x + y \leq 6$

	$3x + y \leq 6$	Conclusion
$(4, -1)$	$3(4) + (-1) = 11 > 6$	Does not belong to graph
$(5, 5)$	$3(5) + 5 = 20 > 6$	Does not belong to graph
$(-1, 2)$	$3(-1) + 2 = -1 \leq 6$	Belongs to graph
$(-2, -2)$	$3(-2) + (-2) = -8 \leq 6$	Belongs to graph



$3x + y = 6$



$3x + y \leq 6$

Steps for Graphing an Inequality

STEP 1: Replace the inequality symbol by an equal sign and graph the resulting equation. If the inequality is strict, use dashes; if it is nonstrict, use a solid mark. This graph separates the xy-plane into two or more regions.

STEP 2: In each region, select a test point P.

(a) If the coordinates of P satisfy the inequality, so do all the points in that region. Indicate this by shading the region.

(b) If the coordinates of P do not satisfy the inequality, none of the points in that region do.

EXAMPLE Graphing an Inequality

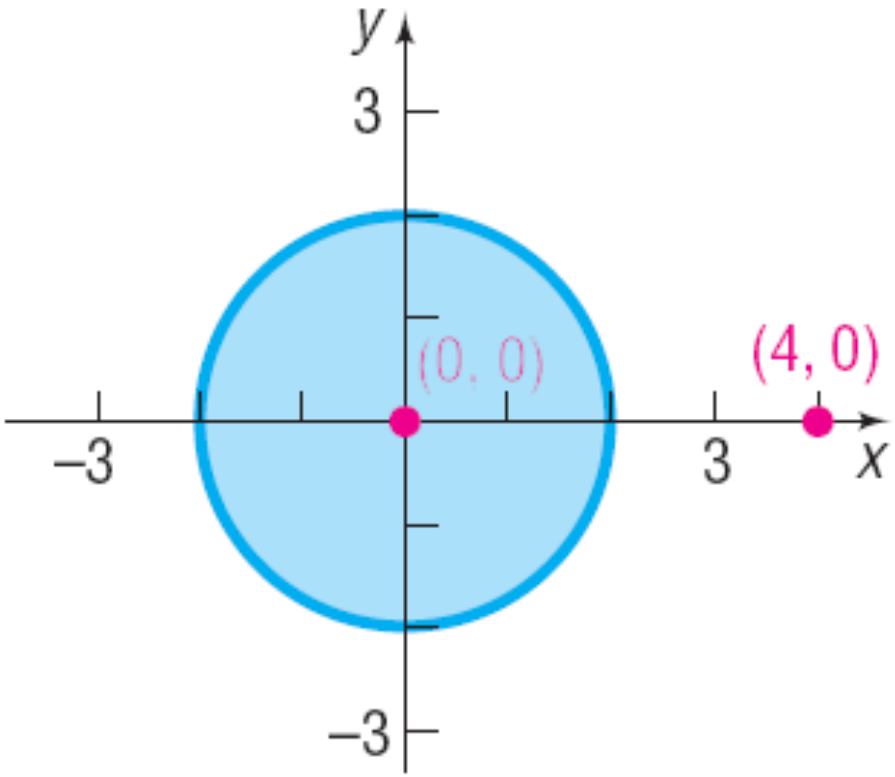
Graph: $x^2 + y^2 \leq 4$

STEP 1: Graph the equation $x^2 + y^2 = 4$, a circle of radius 2, center at the origin. A solid circle will be used because the inequality is not strict.

STEP 2: Use two test points, one inside the circle, the other outside.

Inside $(0, 0)$: $x^2 + y^2 = 0^2 + 0^2 = 0 \leq 4$ Belongs to the graph

Outside $(4, 0)$: $x^2 + y^2 = 4^2 + 0^2 = 16 > 4$ Does not belong to the graph

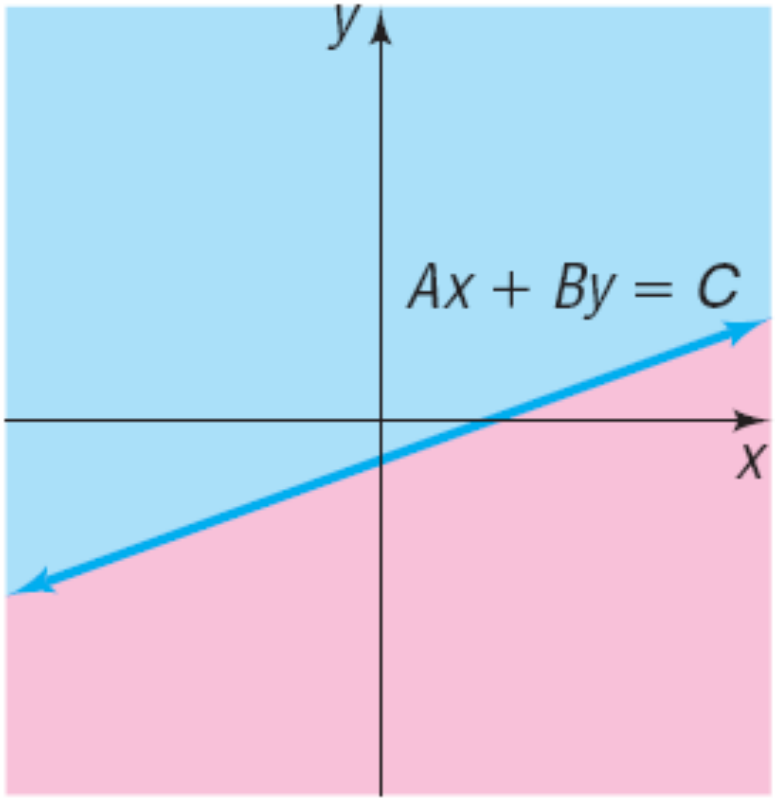


Linear Inequalities

Linear inequalities are inequalities in one of the forms

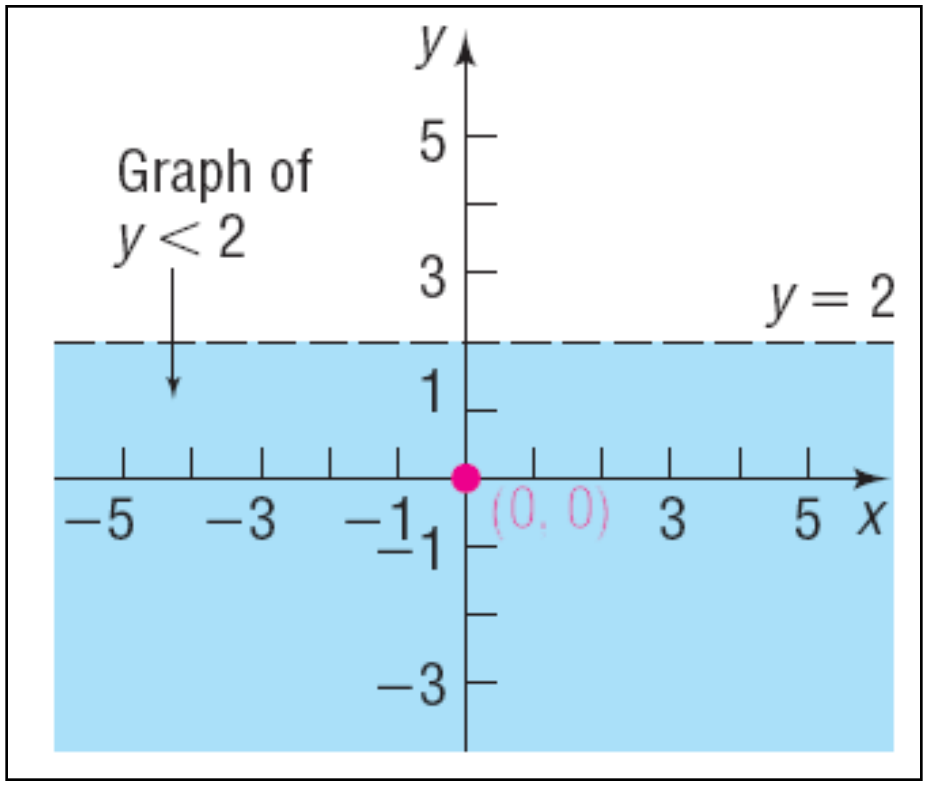
$$Ax + By < C \qquad Ax + By > C \qquad Ax + By \leq C \qquad Ax + By \geq C$$

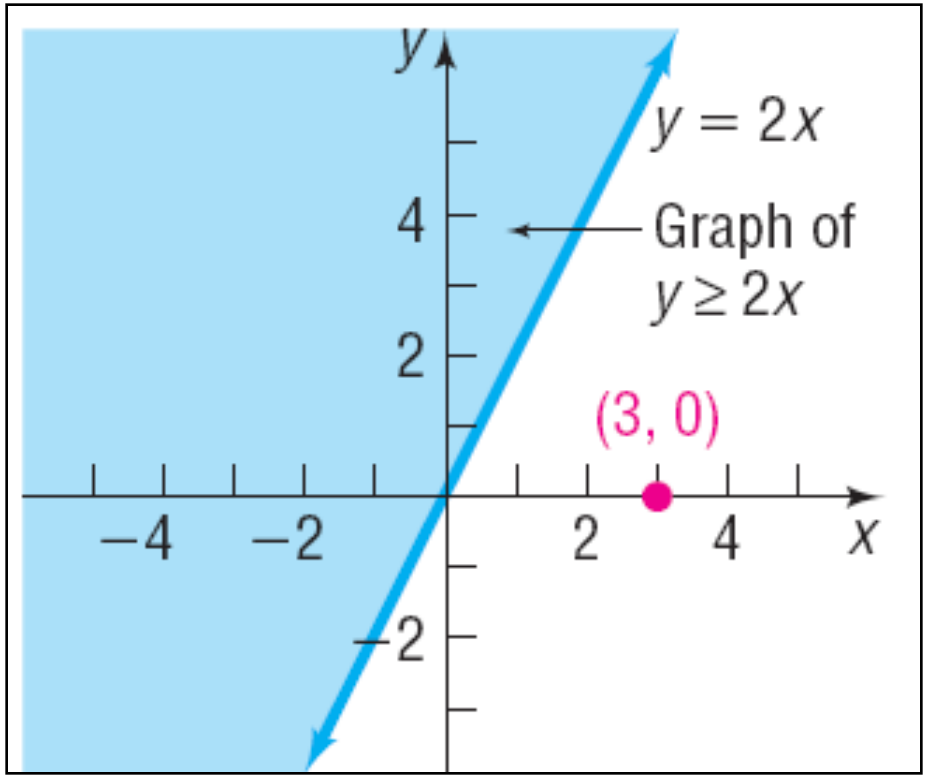
Half-planes



EXAMPLE **Graphing Linear Inequalities**

Graph: (a) $y < 2$ (b) $y \geq 2x$

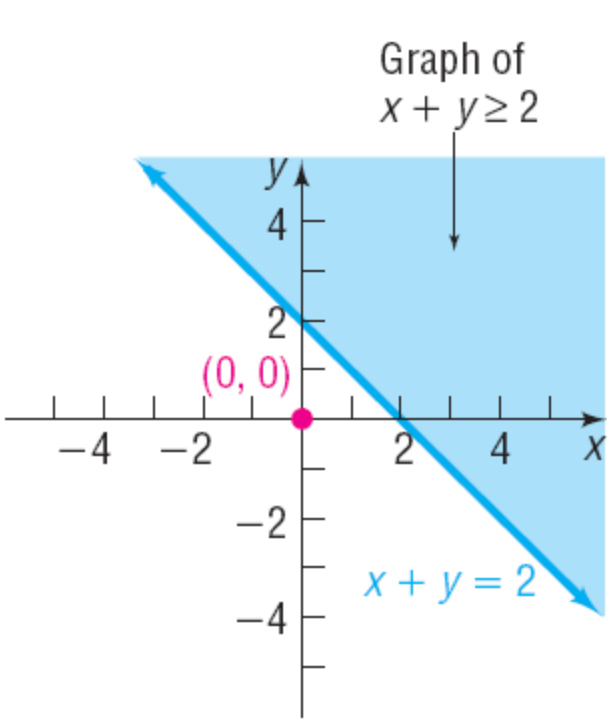


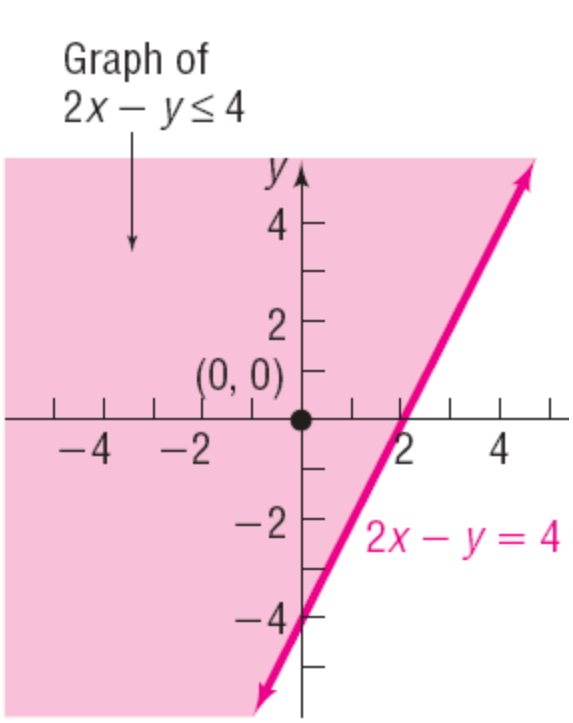


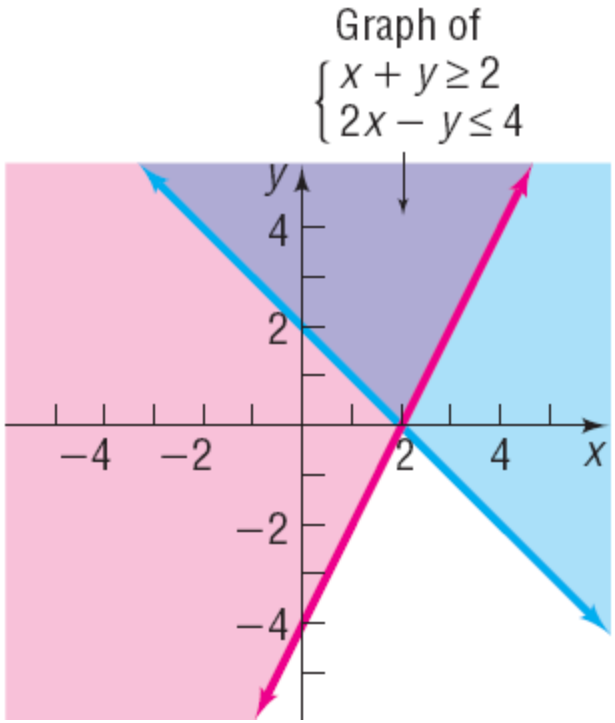
2 Graph a System of Inequalities

EXAMPLE **Graphing a System of Linear Inequalities**

Graph the system: $\begin{cases} x + y \geq 2 \\ 2x - y \leq 4 \end{cases}$

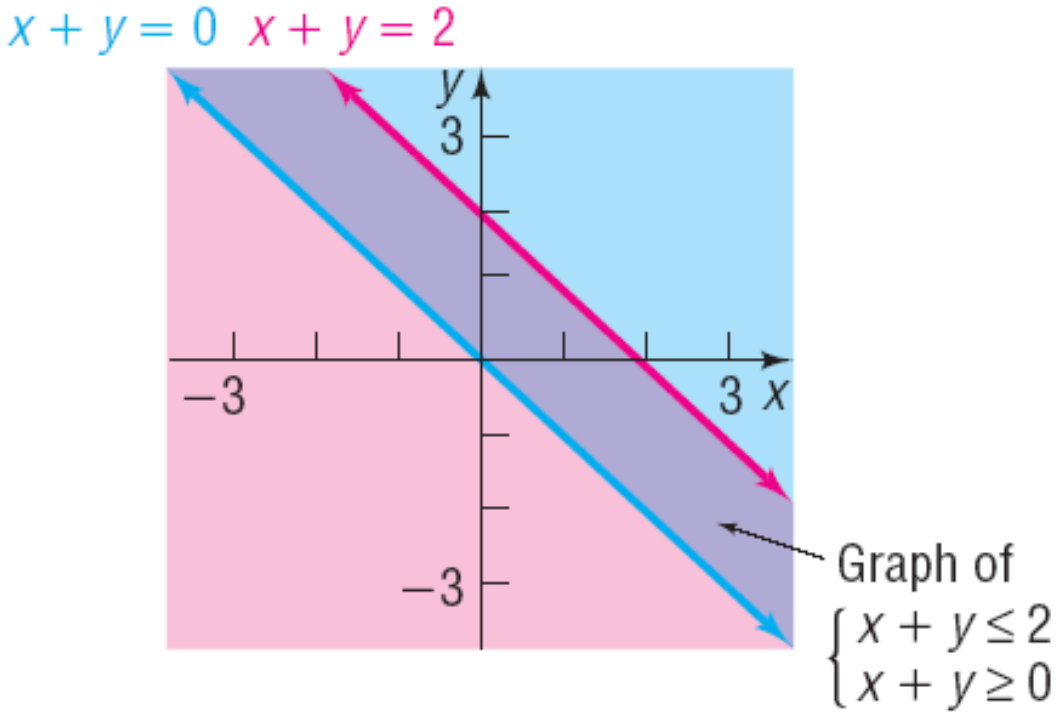






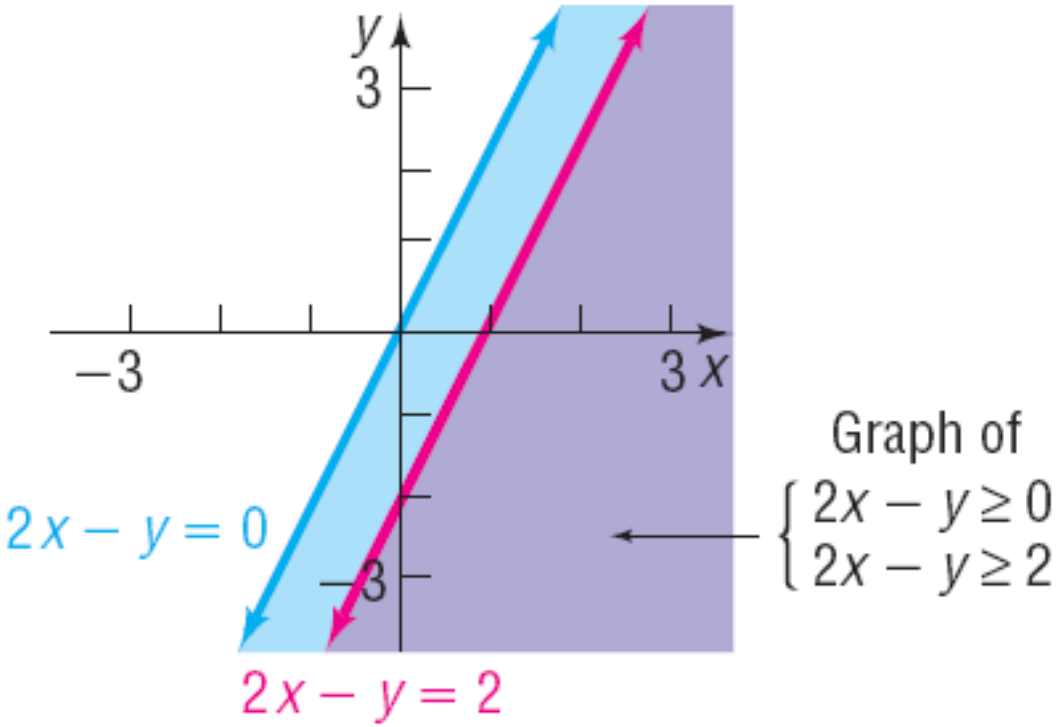
EXAMPLE Graphing a System of Linear Inequalities

Graph the system: $\begin{cases} x + y \le 2 \\ x + y \ge 0 \end{cases}$



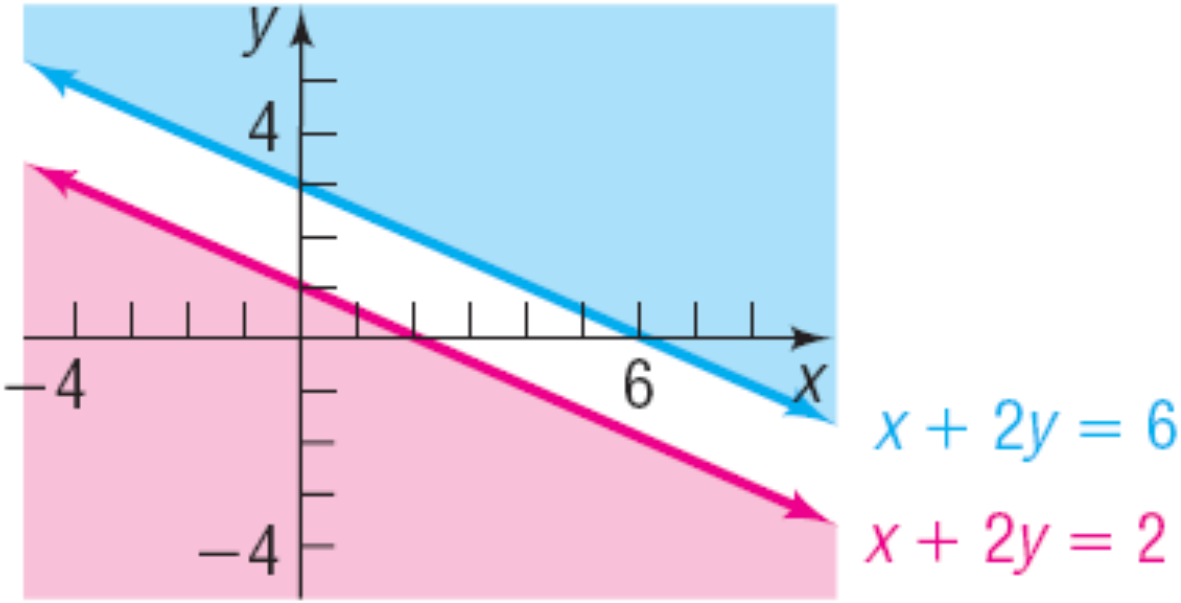
EXAMPLE Graphing a System of Linear Inequalities

Graph the system: $\begin{cases} 2x - y \geq 0 \\ 2x - y \geq 2 \end{cases}$



EXAMPLE **Graphing a System of Linear Inequalities**

Graph the system: $\begin{cases} x + 2y \leq 2 \\ x + 2y \geq 6 \end{cases}$



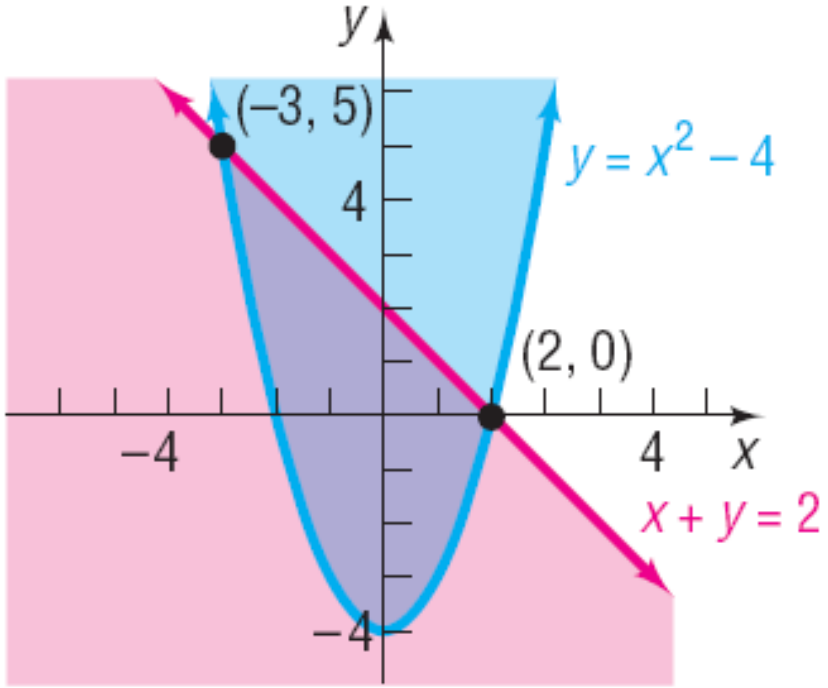
EXAMPLE

Graphing a System of Nonlinear Inequalities

Graph the region below the graph of $x + y = 2$ and above the graph of $y = x^2 - 4$ by graphing the system:

$$\begin{cases} y \geq x^2 - 4 \\ x + y \leq 2 \end{cases}$$

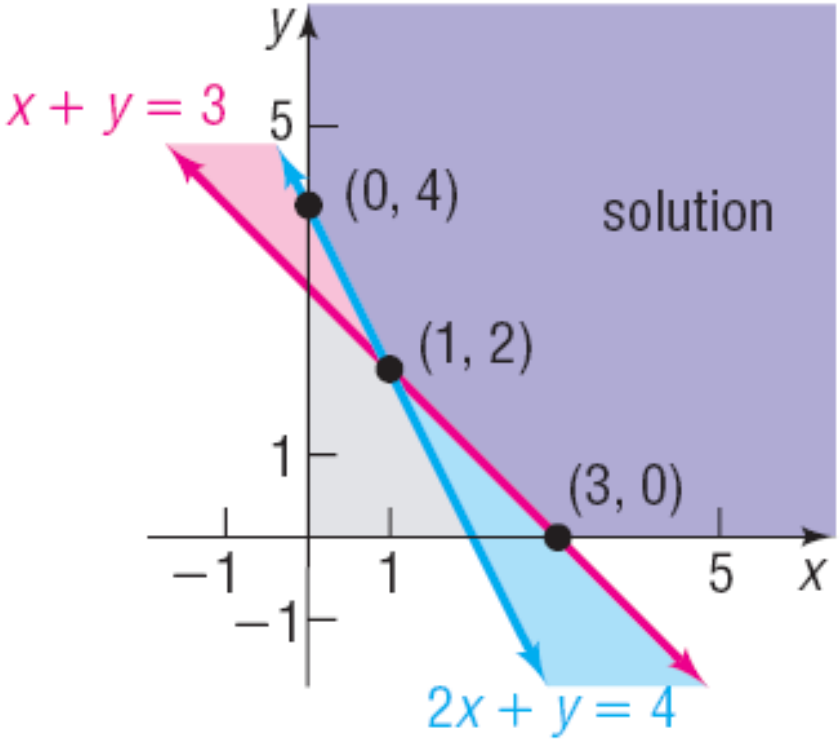
Label all points of intersection.



EXAMPLE **Graphing a System of Four Linear Inequalities**

Graph the system:

$$\begin{cases} x + y \geq 3 \\ 2x + y \geq 4 \\ x \geq 0 \\ y \geq 0 \end{cases}$$



EXAMPLE Financial Planning

A retired couple has up to \$25,000 to invest. As their financial adviser, you recommend that they place at least \$15,000 in Treasury bills yielding 6% and at most \$5000 in corporate bonds yielding 9%.

(a) Using x to denote the amount of money invested in Treasury bills and y the amount invested in corporate bonds, write a system of linear inequalities that describes the possible amounts of each investment. We shall assume that x and y are in thousands of dollars.

(b) Graph the system.

The system of linear inequalities is

$$\begin{cases} x \geq 0 \\ y \geq 0 \\ x + y \leq 25 \\ x \geq 15 \\ y \leq 5 \end{cases}$$

x and y are nonnegative variables since they represent money invested in thousands of dollars.

The total of the two investments, $x + y$, cannot exceed \$25,000.

At least \$15,000 in Treasury bills.

At most \$5000 in corporate bonds.

EXAMPLE Financial Planning

A retired couple has up to $25,000 to invest. As their financial adviser, you recommend that they place at least $15,000 in Treasury bills yielding 6% and at most $5000 in corporate bonds yielding 9%.

(a) Using x to denote the amount of money invested in Treasury bills and y the amount invested in corporate bonds, write a system of linear inequalities that describes the possible amounts of each investment. We shall assume that x and y are in thousands of dollars.

(b) Graph the system.