Section 6.6

Logarithmic and Exponential Equations

1 Solve Logarithmic Equations

Recall:

$$y = \log_a x \quad \text{is equivalent to} \quad x = a^y \qquad a > 0, a \neq 1$$

If $\log_a M = \log_a N$, then $M = N$

M, N, and a are positive and $a \neq 1$

EXAMPLE Solving a Logarithmic Equation

Solve: $\log_3 4 = 2\log_3 x$

$$\log_3 4 = \log_3 x^2$$

$\log_a M^r = r \log_a M$

$$4 = x^2$$

If $\log_a M = \log_a N$, then $M = N$.

$x = 2$ or ~~$x = -2$~~

Reminder: The domain of logarithmic functions is positive numbers only so check for extraneous solutions.

EXAMPLE Solving a Logarithmic Equation

Solve: $\log_2(x+2)+\log_2(1-x)=1$

$$\log_2(x+2)(1-x)=1$$

$\log_a(MN) = \log_a M + \log_a N$

$$2=(x+2)(1-x)$$

$y = \log_a x$ if and only if $x = a^y$

$$2=-x^2-x+2$$

$$x^2+x=0$$

$$x(x+1)=0$$

$$x=0 \text{ or } x=-1$$

Both solutions are in the domain of the log functions.

2 Solve Exponential Equations

Recall:

If $a^u = a^v$, then $u = v$ $\quad a > 0, a \neq 1$

EXAMPLE **Solving Exponential Equations**

Solve: $3^x = 7$

$$\ln 3^x = \ln 7$$

If $M = N$, then $\log_a M = \log_a N$.

$$x \ln 3 = \ln 7$$

$\log_a M^r = r \log_a M$

$$x = \frac{\ln 7}{\ln 3} \approx 1.771$$

EXAMPLE **Solving Exponential Equations**

Solve: $5 \cdot 2^x = 3$

$$2^x = \frac{3}{5}$$

$$\ln 2^x = \ln\left(\frac{3}{5}\right)$$

If $M = N$, then $\log_a M = \log_a N$.

$$x \ln 2 = \ln\left(\frac{3}{5}\right)$$

$\log_a M^r = r \log_a M$

$$x = \frac{\ln\left(\frac{3}{5}\right)}{\ln 2} \approx -0.737$$

EXAMPLE **Solving Exponential Equations**

Solve: $2^{x-1} = 5^{2x+3}$

$$\ln 2^{x-1} = \ln 5^{2x+3}$$

If $M = N$, then $\log_a M = \log_a N$.

$$(x-1)\ln 2 = (2x+3)\ln 5$$

$\log_a M^r = r \log_a M$

$$x\ln 2 - \ln 2 = 2x\ln 5 + 3\ln 5$$

$$x\ln 2 - 2x\ln 5 = 3\ln 5 + \ln 2$$

$$x(\ln 2 - 2\ln 5) = 3\ln 5 + \ln 2$$

$$x = \frac{3\ln 5 + \ln 2}{\ln 2 - 2\ln 5} \approx -2.186$$

EXAMPLE

Solving an Exponential Equation That Is Quadratic in Form

Solve: $9^x - 3^x - 6 = 0$

Note that $9^x = \left(3^x\right)^2$ so the equation is quadratic in form.

$$u^2 - u - 6 = 0 \qquad u = 3^x \text{ and } u^2 = 9^x$$

$$(u-3)(u+2) = 0$$

$$u = 3 \text{ or } u = -2$$

$$3^x = 3 \text{ or } 3^x = -2 \qquad 3^x > 0 \text{ for all } x$$

$$x = 1$$

3 Solve Logarithmic and Exponential Equations Using a Graphing Utility

EXAMPLE Solving Equations Using a Graphing Utility

Solve: $x + e^x = 2$

Express the solution(s) rounded to two decimal places.

The solution is found by graphing $Y_1 = x + e^x$ and $Y_2 = 2$. Since Y_1 is an increasing function (do you know why?), there is only one point of intersection for Y_1 and Y_2. Figure 40 shows the graphs of Y_1 and Y_2. Using the INTERSECT command, the solution is 0.44 rounded to two decimal places.

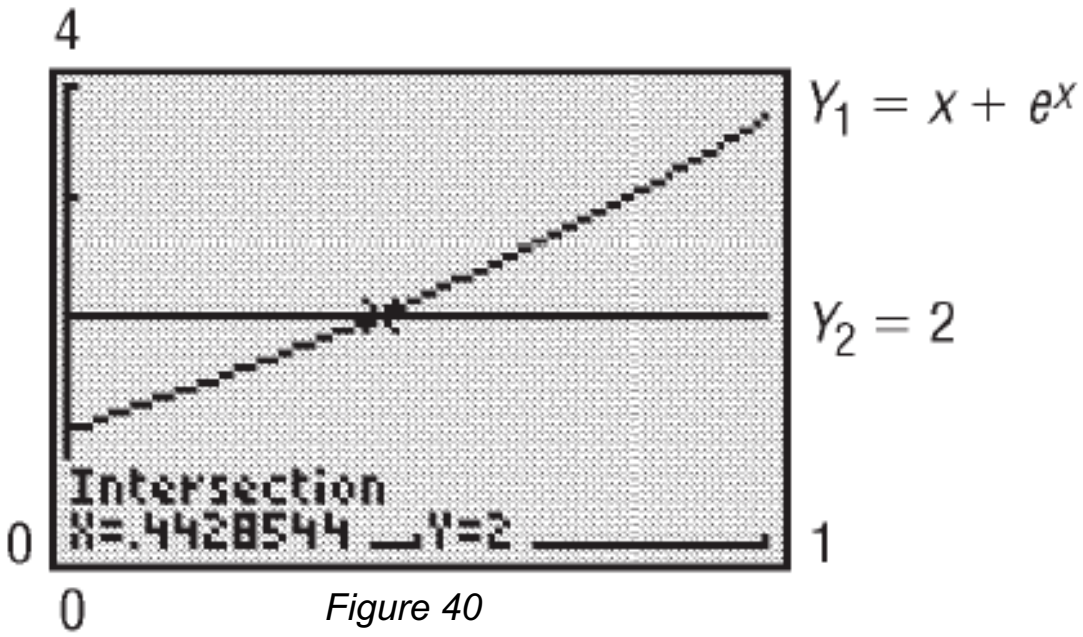

Figure 40