Algebra Formulas

Arithmetic Operation	$a(b+c)=ab+ac$ $\frac{a}{b}\pm\frac{c}{d}=\frac{ad\pm bc}{bd}$ $\frac{a\pm c}{b}=\frac{a}{b}\pm\frac{c}{b}$ $\frac{a/b}{c/d}=\frac{a}{b}\cdot\frac{d}{c}=\frac{ad}{bc}$ $\frac{a-b}{c-d}=\frac{b-a}{d-c}$
Exponents	$x^0=1\ (x\neq 0)$ $x^{-m}=\frac{1}{x^m}$ $\frac{1}{x^{-m}}=x^m$ $x^m\,x^n=x^{m+n}$ $(x^m)^n=x^{mn}$ $(xy)^n=x^ny^n$ $\frac{x^m}{x^n}=x^{m-n}=\frac{1}{x^{n-m}}$ $\left(\frac{x}{y}\right)^n=\frac{x^n}{y^n}$ $\left(\frac{x}{y}\right)^{-n}=\left(\frac{y}{x}\right)^n=\frac{y^n}{x^n}$
Radicals	$x^{1/n}=\sqrt[n]{x}$ $\left(\begin{array}{l}\boldsymbol{If\ n\ is\ even,\quad then\ x\geq 0}\ and\\ if\ n\ is\ odd,\quad then\ x\ is\ a\ real\ number.\end{array}\right)$ $\boxed{x^2=a \quad\Rightarrow\ x=\pm\sqrt{a}}$ $x^{m/n}=\sqrt[n]{x^m}=(x^m)^{1/m}=\left(\sqrt[n]{x}\right)^m$ $\sqrt[n]{xy}=\sqrt[n]{x}\sqrt[n]{y}$ $\sqrt[n]{\frac{x}{y}}=\frac{\sqrt[n]{x}}{\sqrt[n]{y}}$
Factoring Special Polynomials	$x^2-y^2=(x+y)(x-y)$ $x^2+y^2=Prime$ $x^4-y^4=(x^2+y^2)(x+y)(x-y)$ $x^3+y^3=(x+y)(x^2-xy+y^2)$ $x^3-y^3=(x-y)(x^2+xy+y^2)$
Quadratic Formula	$ax^2+bx+c=0,\ \ where\ a\neq 0$ $\boldsymbol{x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}}$ $If\ b^2-4ac>0,\quad there\ are\ 2\ real\ solutions$ $If\ b^2-4ac=0,\quad there\ are\ 1\ repeated\ real\ solutions$ $If\ b^2-4ac<0,\quad there\ are\ 2\ complex\ solutions\ s.t.\ \ a\pm bi$
Binomial Theorem	$(x+y)^2=x^2+2xy+y^2$ $(x+y)^3=x^3+3x^2y+3xy^2+y^3$ $(x-y)^2=x^2-2xy+y^2$ $(x-y)^3=x^3-3x^2y+3xy^2-y^3$ $(x+y)^n=\binom{n}{0}x^n+\binom{n}{1}x^{n-1}y+\binom{n}{2}x^{n-2}y^2+\cdots+\binom{n}{k}x^{n-k}y^k+\cdots+\binom{n}{n-1}xy^{n-1}+\binom{n}{n}y^n$ $=x^n+nx^{n-1}y+\frac{n(n-1)}{2}x^{n-2}y^2+\cdots+\binom{n}{k}x^{n-k}y^k+\cdots+nxy^{n-1}+y^n$ $*\ Binomial\ Coefficient=\binom{n}{k}=\frac{n(n-1)\cdots(n-k+1)}{1\cdot 2\cdot 3\cdot\ \cdots\ \cdot k}$ $*\ (x+a)^n\ has\ \ terms\ of\ x^k=\binom{n}{n-k}a^{n-k}x^k$
Distance Formula	$If\ 2\ points\ \ P_1=(x_1,y_1)\ and\ P_2=(x_2,y_2),\quad the\ \ distance\ from\ P_1\ to\ P_2\ is$ $\boldsymbol{d(P_1,P_2)=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}}$
Midpoint Formula	$M=(x,y)=\left(\frac{x_1+x_2}{2},\ \frac{y_1+y_2}{2}\right)$ $from\ 2\ points\ \ P_1=(x_1,\ \ y_1)\ \ and\ P_2=(x_2,\ \ y_2)$

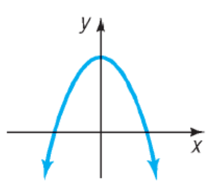
Complex Numbers	$i = \sqrt{-1}$ $\quad \boldsymbol{i^2 = -1}$ $\quad \sqrt{-a} = i\sqrt{a}\ (a \geq 0)$ $\quad$ ***Basic Form***: $\boldsymbol{a \pm bi}$ $\quad (a, b \text{ are real numbers})$ $(a+bi) \pm (c+di) = (a \pm c) + (b \pm d)i$ $\qquad (a+bi)(a-bi) = a^2 + b^2$ $(a+bi)(c+di) = (ac-bd) + (ad+bc)i$ $\qquad (a+bi)^2 = \|a+bi\|^2$ $\dfrac{a+bi}{c+di} = \dfrac{a+bi}{c+di} \cdot \dfrac{c-di}{c-di} = \dfrac{(ac+bd)-(ad-bc)i}{c^2+d^2}$ $*$ *Complex Modules*: $\sqrt{a^2+b^2} = \|a+bi\| = \|z\| = \sqrt{z\,\bar{z}}$ $\quad if\ z = a + bi.$ $*$ *Complex Conjugate*: $\overline{(a+bi)} = a - bi$ $(a+bi)\,\overline{(a+bi)} = (a+bi)(a-bi) = a^2 + b^2$
Inequalities and Absolute Value	$If\ a < b\ and\ b < c, \quad then\ a < c$ $\qquad If\ a < b, \quad then\ a \pm c < b \pm c$ $If\ a < b\ and\ c > 0, \quad then\ ac < bc\ and\ \dfrac{a}{c} < \dfrac{b}{c}$ $If\ a < b\ and\ c < 0, \quad then\ ca > cb\ and\ \dfrac{a}{c} > \dfrac{b}{c}$ $If\ a > 0, \quad then\ \|x\| = a \rightarrow \quad x = a\ or\ x = -a$ $\qquad \|a\| = \begin{cases} a & if\ a \geq 0 \\ -a & if\ a < 0 \end{cases}$ $\|x\| < a \rightarrow \quad -a < x < a$ $\|x\| > a \rightarrow \quad x > a\ or\ x < -a$ $\qquad \|a\| \geq 0 \qquad \|-a\| = \|a\|$ $* \|ab\| = \|a\|\,\|b\|$ $\qquad * \left\|\dfrac{a}{b}\right\| = \dfrac{\|a\|}{\|b\|}$ $\qquad *$ *Triangle Inequality* $\|a+b\| \leq \|a\| + \|b\|$

Linear Equation

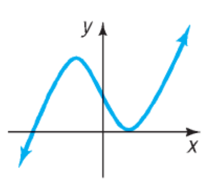
Slope Form of a linear equation	$m = a\ slope\ of\ the\ line\ and\ 2\ points\ P_1 = (x_1, y_1)\ and\ P_2 = (x_2, y_2),$ $if\ x_1 \neq x_2, \quad then\ \boldsymbol{m = \dfrac{y_2 - y_1}{x_2 - x_1}}$ $\qquad if\ x_1 = x_2, \quad then\ m = undefined$
Standard Form of a linear equation	$\boldsymbol{Ax^2 + Bx + c = 0}$ $\qquad$ **A***is must be a positive integer* $\qquad m = -\dfrac{B}{A}$
Point-Slope Equation of a Line	$a\ point\ P_1 = (x_1, y_1) \quad m = a\ slope$ $\qquad \boldsymbol{y - y_1 = m(x - x_1)}$
Slope-Intercept Equation of a Line	$m = slope\ and \quad b = y - intercept$ $\qquad \boldsymbol{y = mx + b}$

Even function If and only if the graph is *symmetric* with respect to the $\boldsymbol{y - axis}$ $\qquad \boldsymbol{f(-x) = f(x)}$

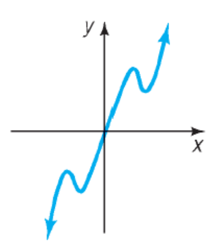
Odd function If and only if the graph is *symmetric* with respect to the ***origin*** $\qquad \boldsymbol{f(-x) = -f(x)}$



Even function

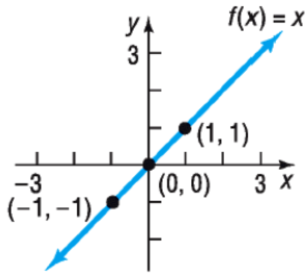
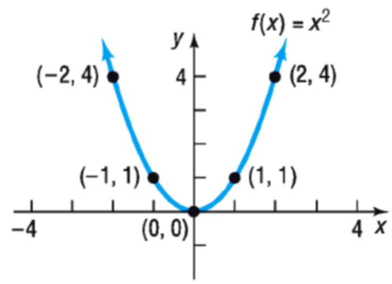
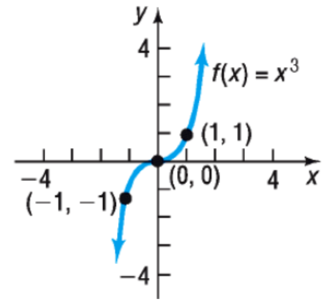


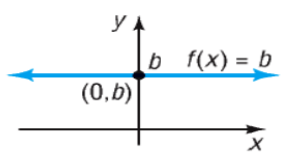
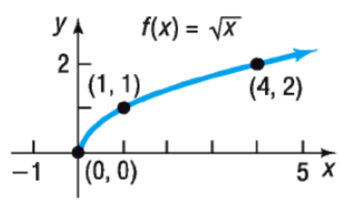
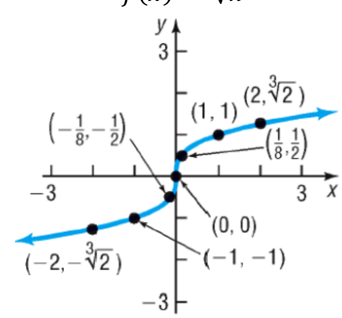
Neither

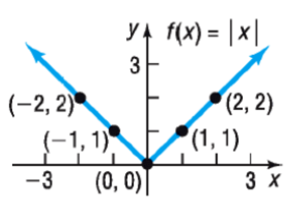
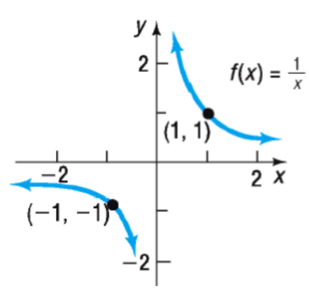
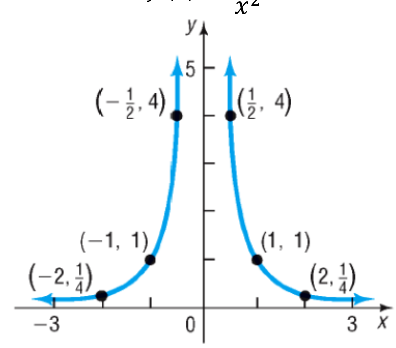


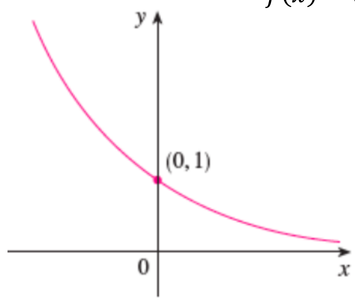
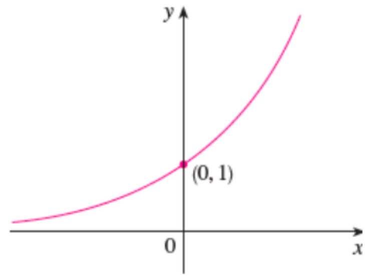
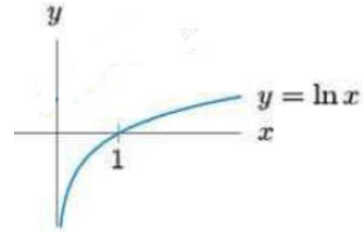
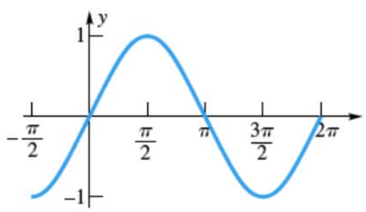
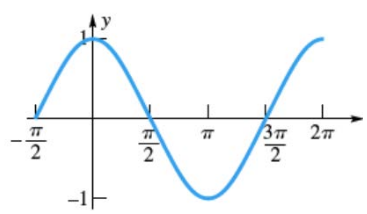
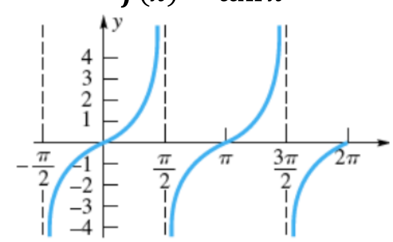
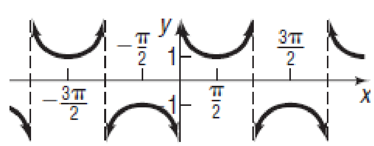
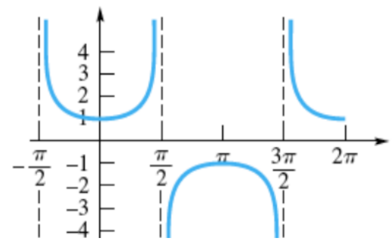
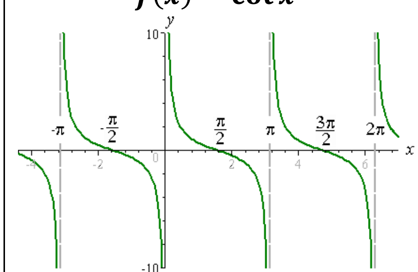
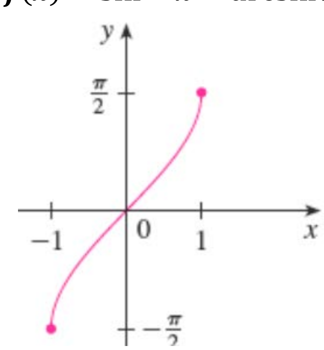
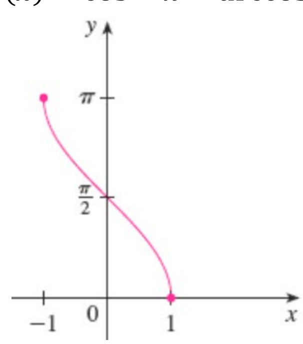
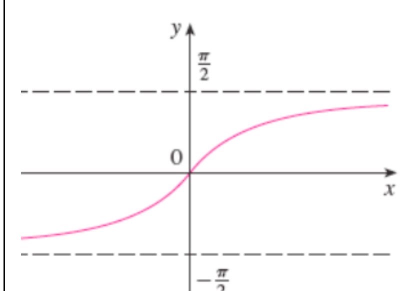
Odd function

Library of Functions

	Identity Function	**Square Function**	**Cube Function**
	$f(x) = x$	$f(x) = x^2$	$f(x) = x^3$
			
Domain	$(-\infty, \infty)$	$(-\infty, \infty)$	$(-\infty, \infty)$
Range	$(-\infty, \infty)$	$[0, \infty)$	$(-\infty, \infty)$

	Constant Function	**Square Root Function**	**Cube Root Function**
	$f(x) = b \quad , b \text{ is a real } \#$	$f(x) = \sqrt{x}$	$f(x) = \sqrt[3]{x}$
			
Domain	$(-\infty, \infty)$	$[0, \infty)$	$(-\infty, \infty)$
Range	$\{b\}$	$[0, \infty)$	$(-\infty, \infty)$

	Absolute Function	**Reciprocal Function**	**Squared Reciprocal Function**
	$f(x) = \lvert x \rvert$ $\begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$	$f(x) = \dfrac{1}{x}$	$f(x) = \dfrac{1}{x^2}$
			
Domain	$(-\infty, \infty)$	$(-\infty, 0) \cup (0, \infty)$	$(-\infty, 0) \cup (0, \infty)$
Range	$[0, \infty)$	$(-\infty, 0) \cup (0, \infty)$	$(0, \infty)$

	Exponential Function $f(x) = a^x$ or $f(x) = e^x$		**Natural Logarithmic Function** $f(x) = \ln x$
	 $0 < a < 1$ or $k < 0$	 $a > 1$ or $k > 0$	
Domain	$(-\infty, \infty)$	$(-\infty, \infty)$	$(0, \infty)$
Range	$(0, \infty)$	$(0, \infty)$	$(-\infty, \infty)$
	$\boldsymbol{f(x) = \sin x}$	$\boldsymbol{f(x) = \cos x}$	$\boldsymbol{f(x) = \tan x}$
			
Domain	$(-\infty, \infty)$	$(-\infty, \infty)$	$\left\{x \middle\vert x \neq (2n+1)\frac{\pi}{2}\right\}$ *where n is an integer.*
Range	$[-1, 1]$	$[-1, 1]$	$(-\infty, \infty)$
Period	2π	2π	π
	$\boldsymbol{f(x) = \csc x}$	$\boldsymbol{f(x) = \sec x}$	$\boldsymbol{f(x) = \cot x}$
			
Domain	$\{x \vert x \neq n\,\pi\}$	$\left\{x \middle\vert x \neq (2n+1)\frac{\pi}{2}\right\}$	$\{x \vert x \neq n\,\pi\}$
Range	$(-\infty, -1] \cup [1, \infty)$	$(-\infty, -1] \cup [1, \infty)$	$(-\infty, \infty)$
Period	2π	2π	π
	$\boldsymbol{f(x) = \sin^{-1} x = \arcsin x}$	$\boldsymbol{f(x) = \cos^{-1} x = \arccos x}$	$\boldsymbol{f(x) = \tan^{-1} x = \arctan x}$
			
Domain	$[-1, 1]$	$[-1, 1]$	$(-\infty, \infty)$
Range	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	$[0, \pi]$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Summary of Graphing Techniques

To Graph	Draw the Graph of f and:	Functional change to $f(x)$
Vertical Shifts $y = f(x) + \boldsymbol{k}, \quad k > 0$ $y = f(x) - \boldsymbol{k}, \quad k > 0$	⇧ **Raise** the graph of f by $\boldsymbol{k}$ units ⇩ **Lower** the graph of f by $\boldsymbol{k}$ units	Add $\boldsymbol{k}$ to $f(x)$ Subtract $\boldsymbol{k}$ from $f(x)$
Horizontal Shifts $y = f(x + \boldsymbol{h}),\ h > 0$ $y = f(x - \boldsymbol{h}),\ h > 0$	⇦ Shift the graph of f to the **left** $\boldsymbol{h}$ units ⇨ Shift the graph of f to the **right** $\boldsymbol{h}$ units	Replace x by $\boldsymbol{x + h}$ Replace x by $\boldsymbol{x - h}$
	Compressing or Stretching	
Vertically $y = \boldsymbol{a}f(x),\ if\ \ 0 < a < 1$ $if\ \ a > 1$	Multiply each y - coordinate of $y = f(x)$ by a. **Compress** the graph of f **vertically** **Stretch** the graph of f **vertically**	Multiply $f(x)$ by a
Horizontally $y = f(\boldsymbol{a}x),\ if\ \ a > 1$ $if\ \ 0 < a < 1$	Multiply each x - coordinate of $y = f(x)$ by $1/a$ **Compress** the graph of f **horizontally** **Stretch** the graph of f **horizontally**	Replace x by ax
Reflection about the axis $y = -f(x)$ $y = f(-\boldsymbol{x})$	Reflection about the $\boldsymbol{x - axis}$ Reflection about the $\boldsymbol{y - axis}$	Multiply $f(x)$ by -1 Replace x by $-x$

$$\boldsymbol{y = a\sin k(x - b) + c} \qquad \boldsymbol{y = a\cos k(x - b) + c} \qquad \boldsymbol{y = a\tan k(x - b) + c}$$

$\boldsymbol{a}$ *is* $\boldsymbol{Amplitude}$. *If* a *is a negative* $\rightarrow$ *flip* $\qquad \boldsymbol{b} \rightarrow \boldsymbol{shift}\ left/right$, & $\qquad \boldsymbol{c} \rightarrow shift\ \ up/down$

$\boldsymbol{k}$ *is* $\boldsymbol{period}$*icity*. $\sin x\ \&\cos x\ \ cases \rightarrow \dfrac{2\pi}{k} \qquad \tan x \rightarrow \dfrac{\pi}{k} \qquad (\boldsymbol{b}, \boldsymbol{c}) = \boldsymbol{new\ origin}.$

Analyzing the Graph of a Polynomial Function $F(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0 \quad a_n \neq 0$

Step 1: *Determine* ***the end behavior***: *The highest degree* $\rightarrow$ ***even*** # ↖ ↗ *or* ↙ ↘ ***odd*** # ↙ ↗ *or* ↖ ↘

Step 2: *Find the* ***x and y intercepts***.

Step 3: *At a zero of* ***even*** *multiplicity*: *The grapg of* ***touches*** *the* $x - axis$

At a zero of ***odd*** *multiplicity*: *The grapg of* ***crosses*** *the* $x - axis$

Step 4: *Determine* $\boldsymbol{maximum\ of\ turning\ ponts = n - 1}$

Step 5: *Determine the graph* ***behavior between zeros***, *the graph increases or decreases*

Analyzing the Graph of a Rational Function R $\quad R(x) = \frac{p(x)}{q(x)} \quad (q(x) \neq 0)$

Step 1: *Factor* $p(x)$ *and* $q(x) \rightarrow$ *Find the domain.*

Step 2: *Write* $R(x)$ *in lowest terms*

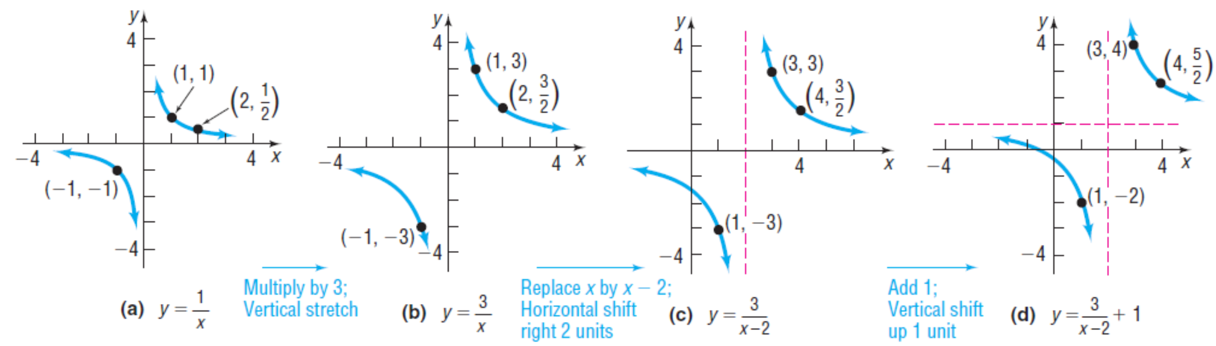
Step 3: *Find the* x & y *intercepts* $(x - int.\ \ from\ p(x) = 0)$
Determine the behavior of the graph near each $x - intercepts$.

Step 4: *Determine Vertical, Horizontal, or Oblique Asymtotes*

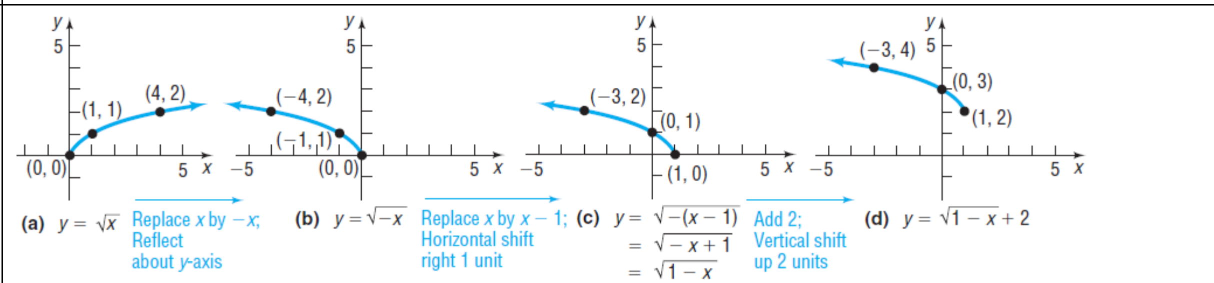
Step 5: *Determine points, if any, at Horizontal, or Oblique Asymtotes*

Step 6: *Determine the graph behavior between zeros and all asymtotes.*

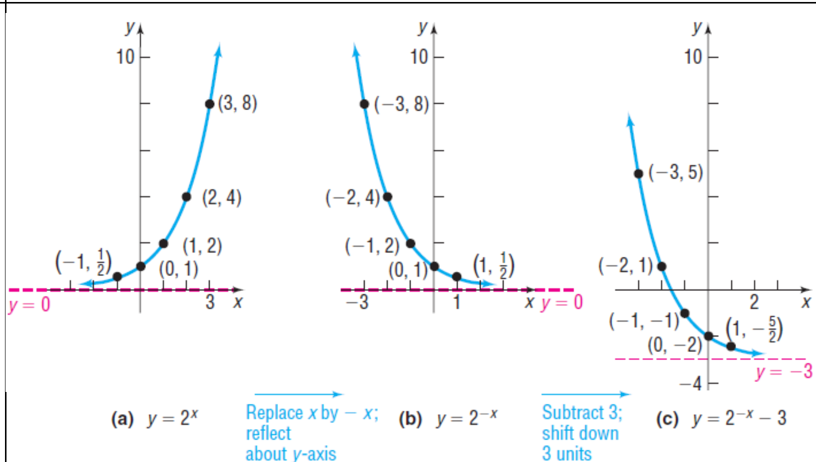
Ex) $f(x) = \dfrac{3}{x-2} + 1$



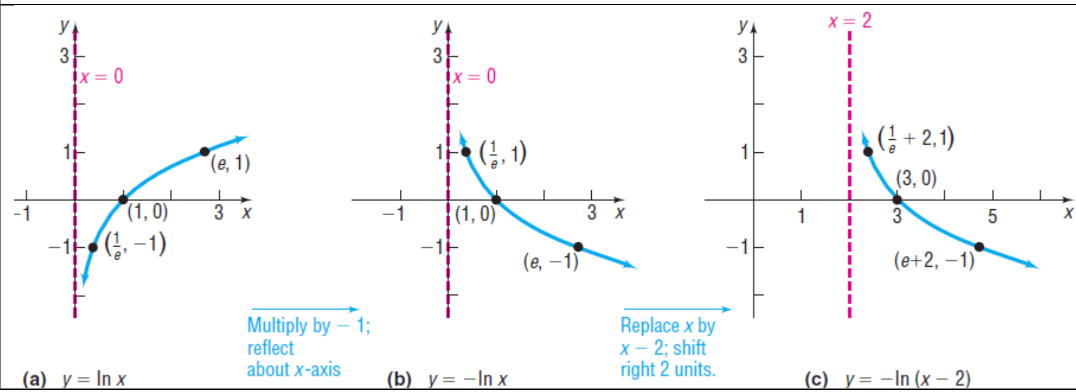
Ex) $f(x) = \sqrt{1-x} + 2$



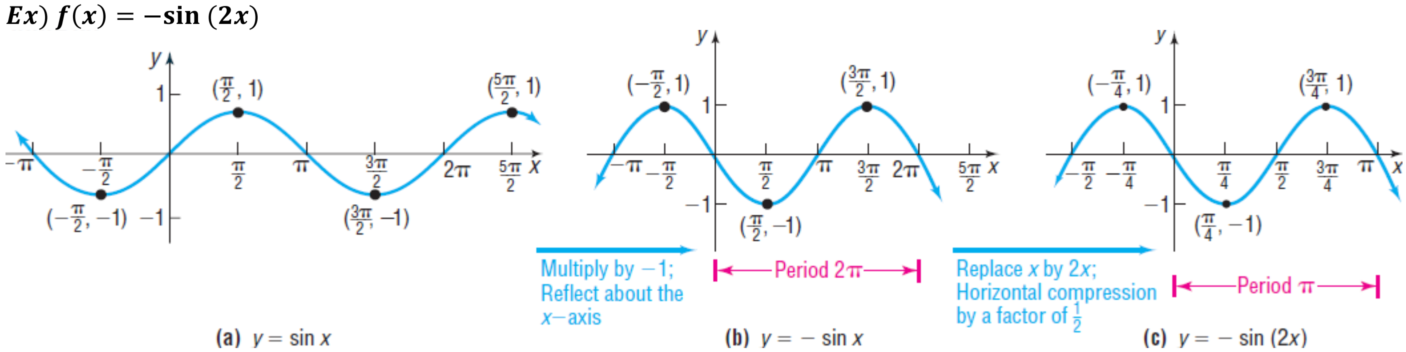
Ex) $f(x) = 2^{-x} - 3$



Ex) $f(x) = -\ln(x-2)$



Ex) $f(x) = -\sin(2x)$



Rational Function $\boldsymbol{R(x)=\frac{p(x)}{q(x)}}$ $(q(x)\neq 0)$

Vertical Asymptote (V.A.) $\boldsymbol{x=r}$

$R(x)=\frac{p(x)}{q(x)}$ *in lowest terms* *r is a real zero of the denominator q.*

$ex)\ \frac{2x}{x^2-9}=\frac{2x}{(x+3)(x-3)}$ $\Rightarrow V.A.:x=\pm 3$ $Dmain\ \{x|x\neq 3, x\neq -3\}$

$ex)\ \frac{x^2-1}{x+1}=\frac{(x+1)(x-1)}{x+1}=x-1$ $\Rightarrow V.A.\ none$ $Dmain\ \{x|x\neq -1\}$

$ex)\ \frac{x^2-x-6}{x^2-9}=\frac{(x-3)(x+2)}{(x+3)(x-3)}=\frac{(x+2)}{(x+3)}$ $\Rightarrow V.A.:x=-3$ $Dmain\ \{x|x\neq 3, x\neq -3\}$

Horizontal (H.A.) or Oblique Asymptote (O.A.)

$R(x)=\frac{p(x)}{q(x)}=\frac{a_nx^n+a_{n-1}x^{n-1}+\cdots+a_1x+a_0}{b_mx^m+b_{m-1}x^{m-1}+\cdots+b_1x+a_0}\sim f(x)=\boldsymbol{\frac{a_nx^n}{b_mx^m}}$ $\begin{cases}\boldsymbol{n} & \text{is the highest degree of numerator}\\ \boldsymbol{m} & \text{is the highest degree of numerator}\end{cases}$

$n<m$	$H.A.:\ y=0$	$\frac{x-5}{3x^2+x-1}\sim\frac{x}{3x^2}=\frac{1}{3x}$	$H.A.:\ y=0$
$n=m$	$H.A.:\ y=\frac{a_n}{b_m}$ $(=a\ number)$	$\frac{5-2x^3}{3x^3-4x+1}\sim\frac{-2x^3}{3x^3}=-\frac{2}{3}$	$H.A.:\ y=-\frac{2}{3}$
$n=m+1$	$O.A.:\ y=mx+b$ $(a\ line)$	$\frac{(1-x)^3}{x^2}=(3-x)+\frac{1-3x}{x^2}$ $\left(\sim-\frac{x^3}{x^2}\right)$	$O.A.:\ y=-x+3$
$n>m+1$	*Neither H.A. nor O.A.*	$\frac{x^5+7x^2-3}{6x^2-x-1}\sim\frac{x^5}{6x^2}$	

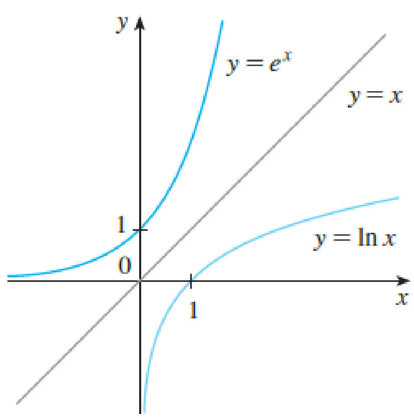
Permutations & Combinations	$n!=n\cdot(n-1)\cdot(n-2)\cdot\cdots\cdot 3\cdot 2\cdot 1$ $\quad 0!=1!=1$ ${}_nP_r=P(n,r)=\frac{n!}{(n-r)!}$ $\quad 0\le r\le n$ $\quad {}_nP_n=n!$ $\quad {}_nP_1=n$ $\quad {}_nP_0=1$ $\binom{n}{r}={}_nC_r=C(n,r)=\frac{n!}{(n-r)!\,r!}$ $\quad 0\le r\le n$ $\quad {}_nC_n={}_nC_0=1$ ${}_nC_1=\binom{n}{1}=\binom{n}{n-1}=n$

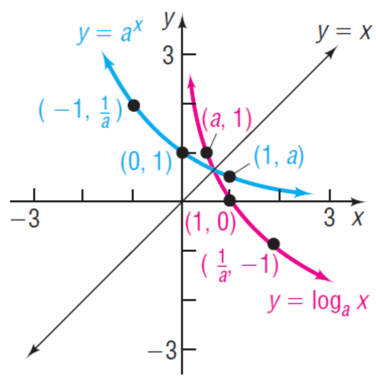
Arithmetic Sequence	$a_1+(a_1+d)+(a_1+2d)+\cdots+[a_1+(n-1)d]=\frac{n}{2}[2a_1+(n-1)d]=\frac{n}{2}[a_1+a_n]$
Geometric Sequence	$a_1+a_1r+a_1r^2+\cdots+a_1r^{n-2}+a_1r^{n-1}=a_1\cdot\frac{1-r^n}{1-r}$ $\quad r\neq 0,1$
Sums of Sequences	$\sum_{k=1}^{n}k=\frac{n(n+1)}{2}$ $\quad \sum_{k=1}^{n}k^2=\frac{n(n+1)(2n+1)}{6}$ $\quad \sum_{k=1}^{n}k^3=\frac{n(n+1)(2n+1)}{6}$
Geometric Series	$If\ \lvert r\rvert<1,\ a_1+a_1r+a_1r^2+\cdots+a_1r^{n-1}+a_1r^n+\cdots=\sum_{k=1}^{\infty}a_1r^{k-1}=\frac{a_1}{1-r}$ $\rightarrow converges$

Exponential & Logarithmic functions

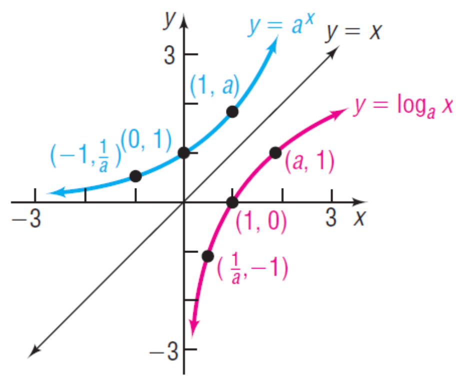
Exponential & Logarithmic functions	*Law of Logarithms*	*Cancellation Equations*
$\log_a 1 = 0 \qquad \log_a a = 1$ $\log_a x = y \quad \Leftrightarrow \quad a^y = x\,, \qquad x > 0$ $ex)\ 2^3 = 8 \quad \Leftrightarrow \quad \log_2 8 = 3$	$\log_a xy = \log_a x + \log_a y$ $\log_a \left(\frac{x}{y}\right) = \log_a x - \log_a y$ $\log_a(x^r) = r \log_a x$	$\log_a(a^x) = x \qquad a^{\log_a x} = x$ $ln(e^x) = x \qquad e^{ln\,x} = x$ $a^x = e^{\ln a^x} = e^{x \ln a}$
$\log x = \log_{10} x$ $\ln x = \log_e x \quad where \ln e = 1$ $\ln x = y \quad \Leftrightarrow \quad e^y = x$	***Change-of- base*** $\qquad a, b\ \neq 1$ $\log_a x = \frac{\log_b x}{\log_b a} \approx \frac{\log x}{\log a} \approx \frac{\ln x}{\ln a}$	$M = N \quad \Leftrightarrow \quad a^M = a^N$ $M = N \quad \Leftrightarrow \quad \log_a M = \log_a N$ $e = 2.71828\cdots$

Graphs



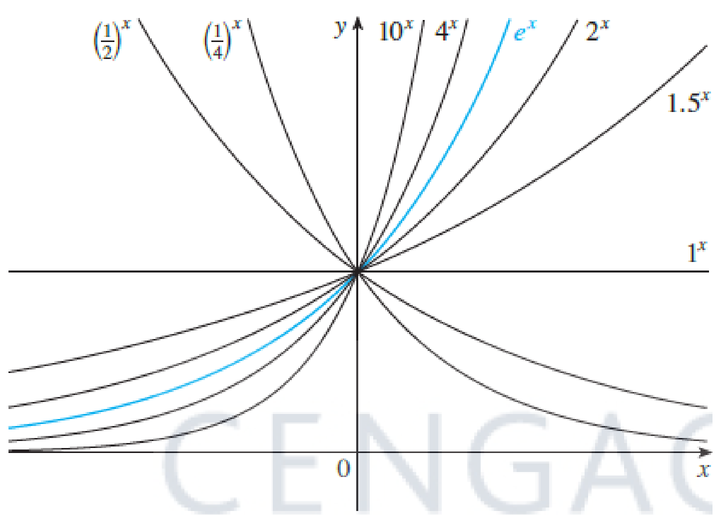


(a) $0 < a < 1$

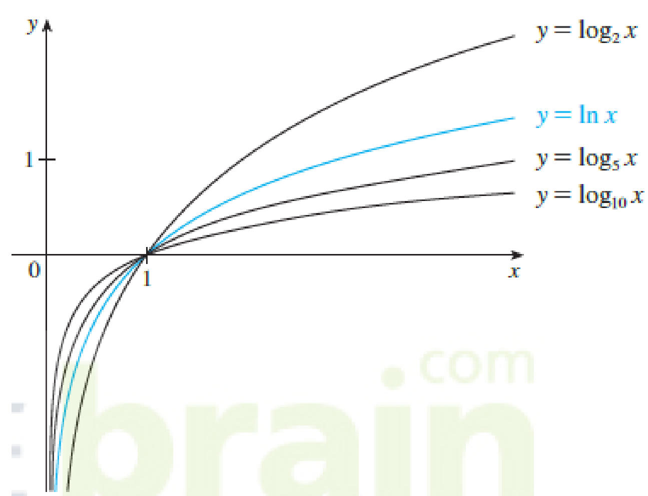


(b) $a > 1$

Limit $\qquad \lim_{x\to -\infty} e^x = 0 \qquad \lim_{x\to \infty} e^x = \infty \qquad \lim_{x\to 0^+} (ln\ x) = 0 \qquad \lim_{x\to \infty} (ln\ x) = \infty$



Exponential functions



Logarithmic functions

Analytic Geometry

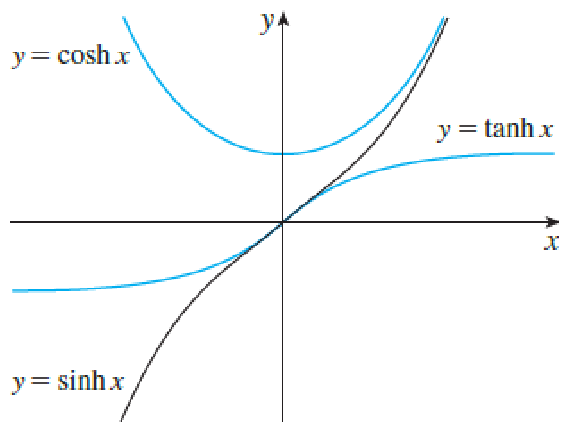
Parabola / Quadratic function $a > 0$ *upward* $a < 0$ *downward*	*Completing Square*: $f(x) = y = x^2 + bx + c \to \left(x + \frac{b}{2}\right)^2$ & $c = \left(\frac{b}{2}\right)^2$ $If\ \ f(x) = \boldsymbol{ax^2 + bx + c = a(x - h)^2 + k} \quad (a \neq 0)$ $\qquad * f(x) = y$ $\boldsymbol{Vertex}$ *is at* $(h,k) = \left(-\frac{\boldsymbol{b}}{\boldsymbol{2a}},\ f\left(-\frac{b}{2a}\right)\right) \qquad h = -\frac{b}{2a}$ & $k = \frac{4ac - b^2}{4a}$
Standard Equation of a Circle	$\boldsymbol{(x - h)^2 + (y - k)^2 = r^2} \qquad r = Radius, \qquad (h,k) = center$
Ellipse	$\frac{\boldsymbol{(x - h)^2}}{\boldsymbol{a^2}} + \frac{\boldsymbol{(y - k)^2}}{\boldsymbol{b^2}} = \boldsymbol{1} \qquad (h,k) = center$ *vertices* "$\boldsymbol{a}$" *units* left/right *from the center* *vertices* "$\boldsymbol{b}$" *units* up/down *from the center*
Hyperbola	$\frac{\boldsymbol{(x - h)^2}}{\boldsymbol{c^2}} - \frac{\boldsymbol{(y - k)^2}}{\boldsymbol{d^2}} = \boldsymbol{1} \qquad$ *opens left & right* *vertices* "$\boldsymbol{c}$" *units* left/right *from the center* $(h,k) = center \qquad$ *asymptotes that pass through center with slope* $\pm\, {}^{d}/_{c}$ $\frac{\boldsymbol{(x - h)^2}}{\boldsymbol{d^2}} - \frac{\boldsymbol{(y - k)^2}}{\boldsymbol{c}} = \boldsymbol{1} \qquad$ *opens up & down* *vertices* "$\boldsymbol{d}$" *units* up/down *from the center* $(h,k) = center \qquad$ *asymptotes that pass through center with slope* $\pm\, {}^{d}/_{c}$

Hyperbolic Functions

$$\sinh x = \frac{e^x - e^{-x}}{2} \qquad \operatorname{csch} x = \frac{1}{\sinh x}$$

$$\cosh x = \frac{e^x + e^{-x}}{2} \qquad \operatorname{sech} x = \frac{1}{\cosh x}$$

$$\tanh x = \frac{\sinh x}{\cosh x} \qquad \coth x = \frac{\cosh x}{\sinh x}$$



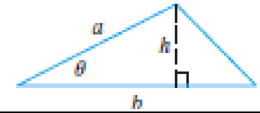
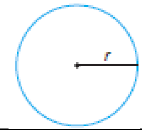
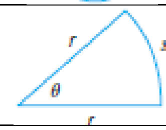
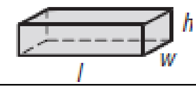
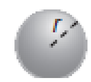
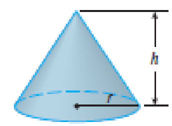
Inverse Hyperbolic Functions

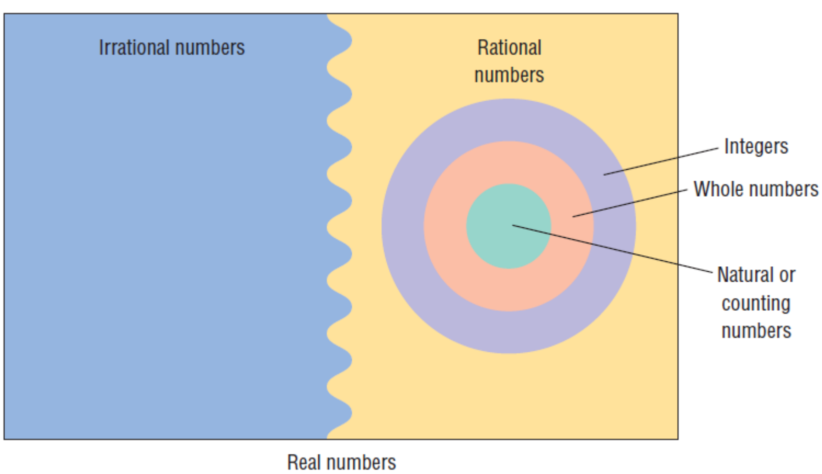
$$y = \sinh^{-1} x \iff \sinh y = x \qquad \sinh^{-1} x = \ln\left(x + \sqrt{x^2 + 1}\right)$$

$$y = \cosh^{-1} x \iff \cosh y = x \text{ and } y \geqslant 0 \qquad \cosh^{-1} x = \ln\left(x + \sqrt{x^2 - 1}\right)$$

$$y = \tanh^{-1} x \iff \tanh y = x \qquad \tanh^{-1} x = \tfrac{1}{2} \ln\left(\frac{1 + x}{1 - x}\right)$$

Geometric formulas $\boldsymbol{h} = Hight(Altitude)$, $\boldsymbol{A} = Area$, $,\boldsymbol{P} = Perimeter$, $\boldsymbol{S} = Surface\ area$, $\boldsymbol{V} = volume$

Triangle	$A = \frac{1}{2}bh = \frac{1}{2}ab\sin\theta$ $(b = Base)$	
Rectangle	$A = lw$ $P = 2l + 2w = 2(l + w)$ $(l = Length \quad w = Width)$	
Circle	$A = \pi r^2$ $C = 2\pi r$ $(C = Circumference \quad r = radius)$	
Sector of Circle	$A = \frac{1}{2}r^2\theta$ $s = r\theta$ $(r = in\ radians)$	
Rectangular Box	$V = lwh$ $S = 2(lw + lh + wh) = 2lw + 2lh + 2wh$	
Sphere	$V = \frac{4}{3}\pi r^3$ $S = 4\pi r^2$	
Right Circular Cylinder	$V = \pi r^2 h$ $S = 2\pi r^2 + 2\pi rh$	
Cone	$V = \frac{1}{3}\pi r^2 h$ $A = \pi r\sqrt{r^2 + h^2}$	



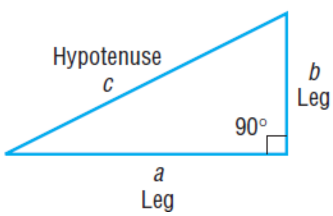
Order of Operations : PEMDAS

Parentheses or Brackets

Exponents

Multiply **or D**ivide

Add **or S**ubtract



Pythagorean Theorem

$$a^2 + b^2 = c^2$$