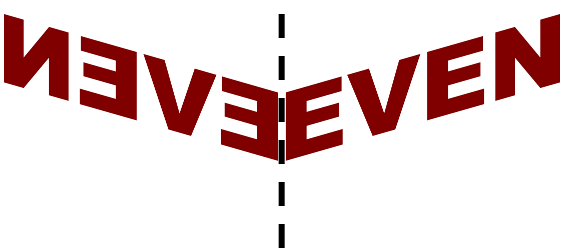
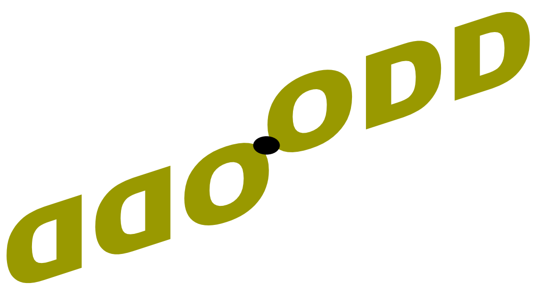
Section 3.3

Properties of Functions

1 Determine Even and Odd Functions from a Graph





DEFINITION

A function f is **even** if, for every number x in its domain, the number $-x$ is also in the domain and

$$\boxed{f(-x) = f(x)}$$

For an **even** function, for every point (x, y) on the graph, the point (-x, y) is also on the graph.

DEFINITION

A function f is **odd** if, for every number x in its domain, the number $-x$ is also in the domain and

$$\boxed{f(-x) = -f(x)}$$

So for an **odd** function, for every point (x, y) on the graph, the point ($-x$, $-y$) is also on the graph.

Theorem

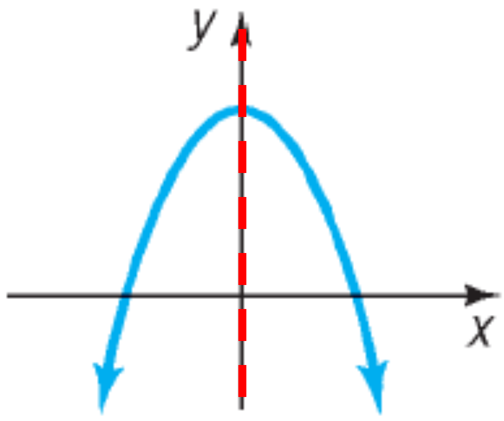
A function is even if and only if its graph is symmetric with respect to the y-axis.

A function is odd if and only if its graph is symmetric with respect to the origin.

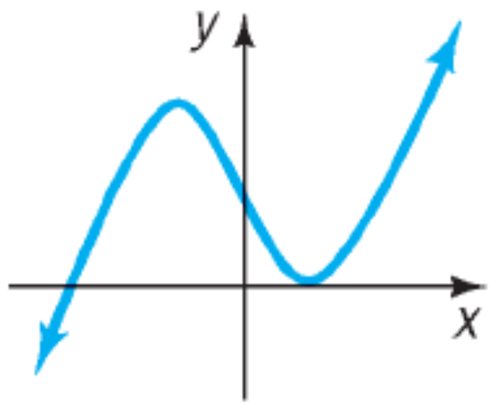
EXAMPLE

Determining Even and Odd Functions from the Graph

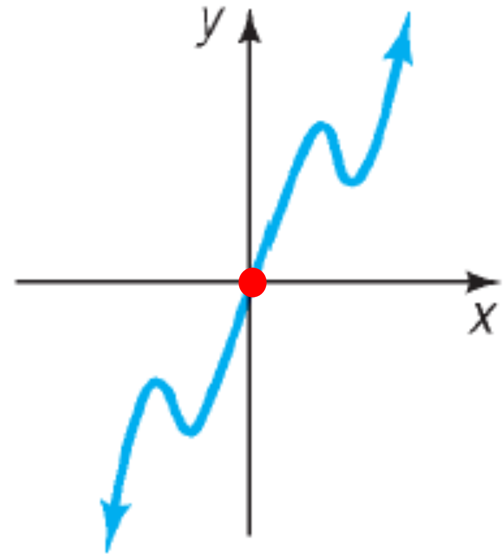
Determine whether each graph given is an even function, an odd function, or a function that is neither even nor odd.



Even function because it is symmetric with respect to the *y*-axis



Neither even nor odd because no symmetry with respect to the *y*-axis or the origin



Odd function because it is symmetric with respect to the origin

2 Identify Even and Odd Functions from the Equation

EXAMPLE

Identifying Even and Odd Functions Algebraically

Determine whether each of the following functions is even, odd, or neither. Then determine whether the graph is symmetric with respect to the y-axis or with respect to the origin.

$a)\quad f(x)=x^3+5x \qquad f(-x)=(-x)^3+5(-x)=-x^3-5x$

Odd function symmetric with respect to the origin

$$=-\left(x^3+5x\right)=-f(x)$$

$b)\quad g(x)=2x^2-3 \qquad g(-x)=2(-x)^2-3=2x^2-3=g(x)$

Even function symmetric with respect to the y-axis

$c)\quad h(x)=-4x^3+1 \qquad h(-x)=-4(-x)^3+1=4x^3+1$

Since the resulting function does not equal $h(x)$ nor $-h(x)$ this function is neither even nor odd and is not symmetric with respect to the y-axis or the origin.

3 Use a Graph to Determine Where a Function Is Increasing, Decreasing, or Constant

INCREASING

DECREASING

CONSTANT

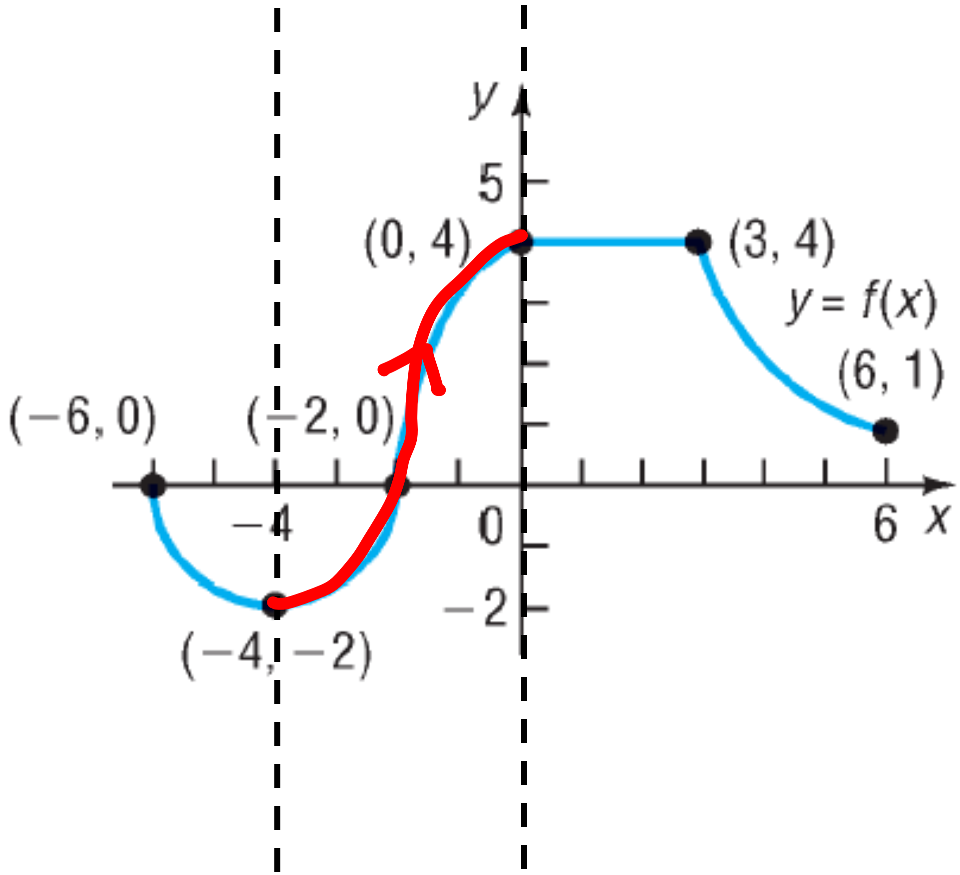
EXAMPLE

Determining Where a Function Is Increasing, Decreasing, or Constant from Its Graph

Where is the function increasing?

$$-4 < x < 0$$

$$(-4, 0)$$



EXAMPLE

Determining Where a Function Is Increasing, Decreasing, or Constant from Its Graph

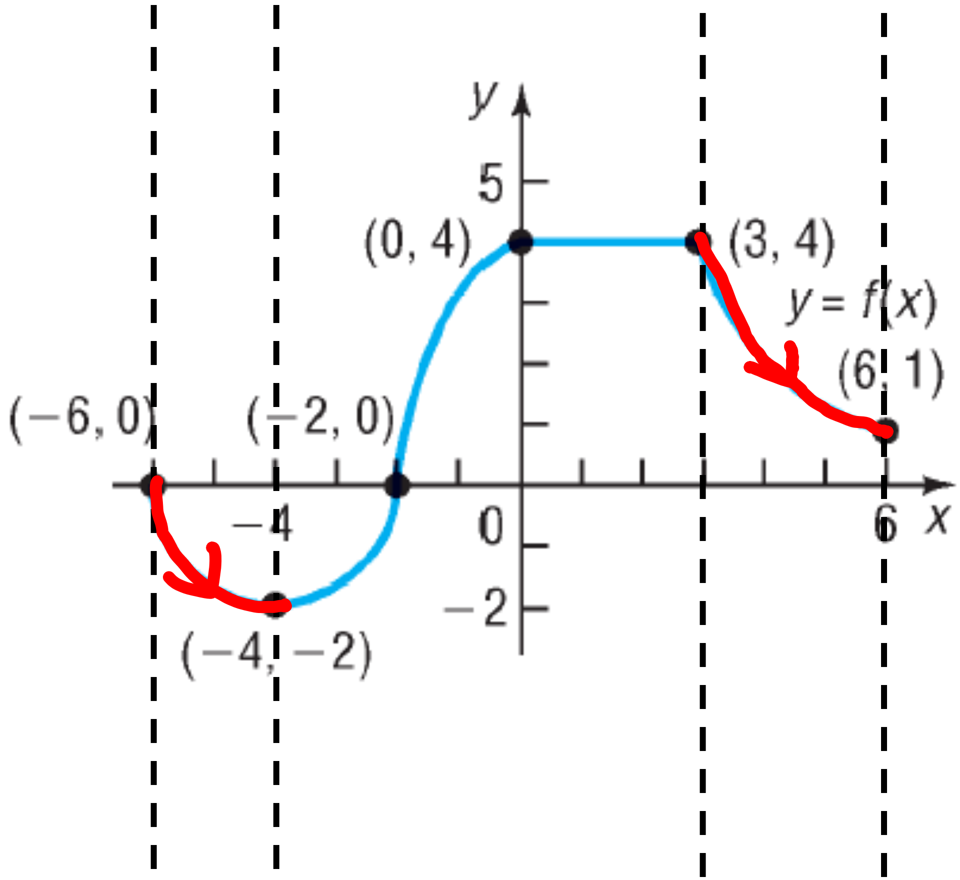
Where is the function decreasing?

$$-6 < x < -4$$

$$(-6, -4)$$

$$3 < x < 6$$

$$(3, 6)$$



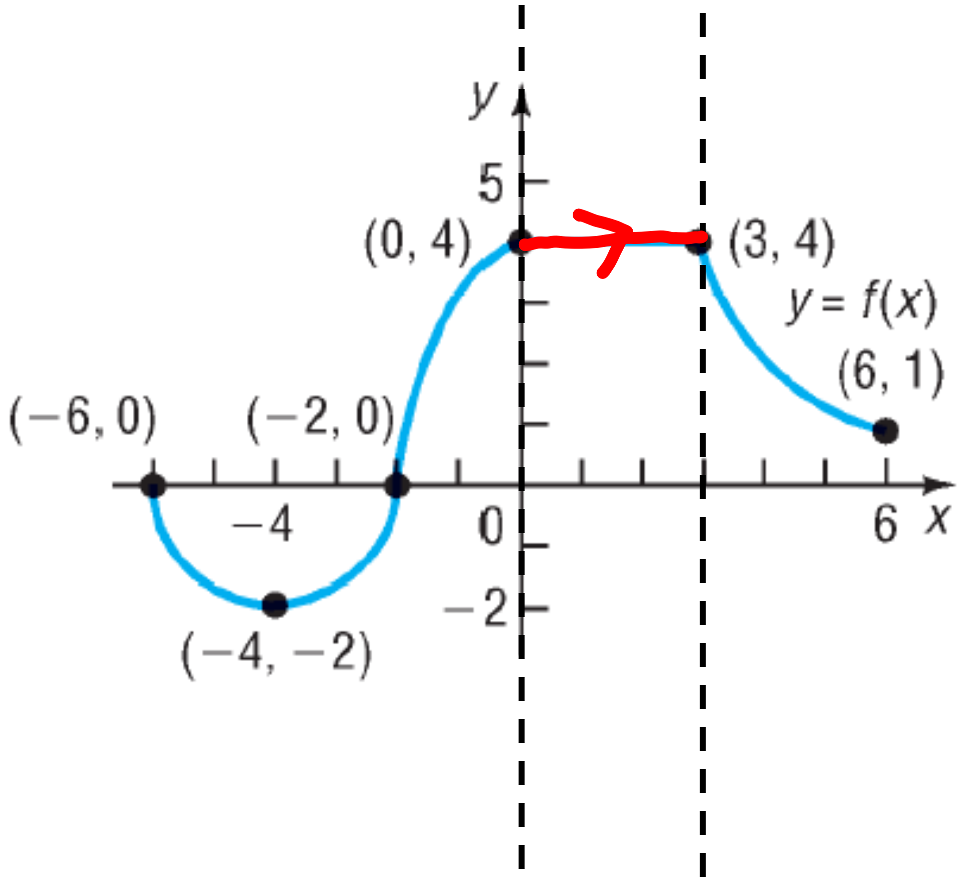
EXAMPLE

Determining Where a Function Is Increasing, Decreasing, or Constant from Its Graph

Where is the function constant?

$$0 < x < 3$$

$$(0, 3)$$

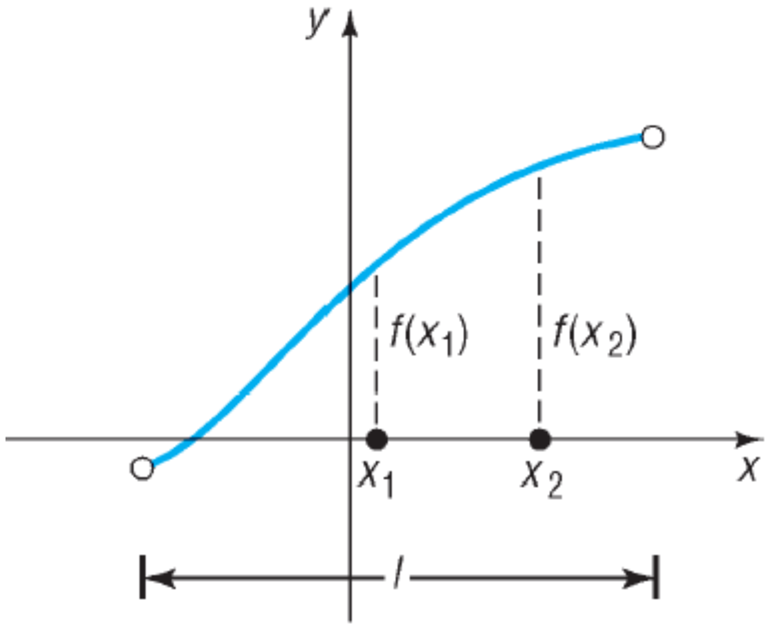


DEFINITIONS

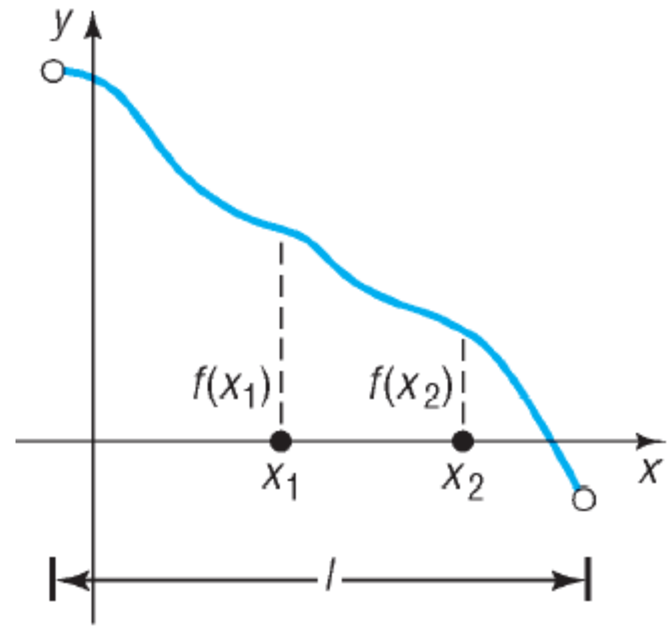
A function f is **increasing** on an open interval I if, for any choice of x_1 and x_2 in I, with $x_1 < x_2$, we have $f(x_1) < f(x_2)$.

A function f is **decreasing** on an open interval I if, for any choice of x_1 and x_2 in I, with $x_1 < x_2$, we have $f(x_1) > f(x_2)$.

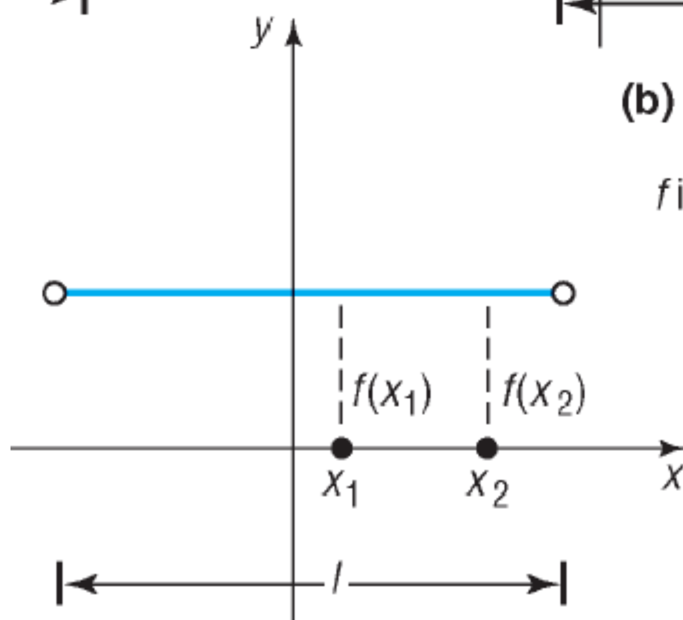
A function f is **constant** on an interval I if, for all choices of x in I, the values $f(x)$ are equal.



(a) For $x_1 < x_2$ in I,
$f(x_1) < f(x_2)$;
f is increasing on I



(b) For $x_1 < x_2$ in I,
$f(x_1) > f(x_2)$;
f is decreasing on I



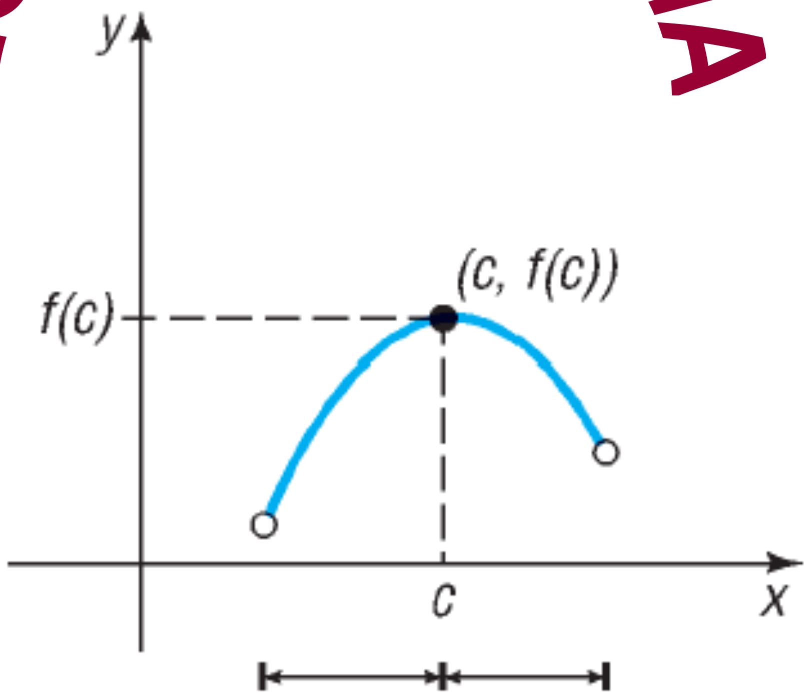
(c) For all x in I, the values of
f are equal; f is constant on I

4 Use a Graph to Locate Local Maxima and Local Minima

LOCAL MAXIMA

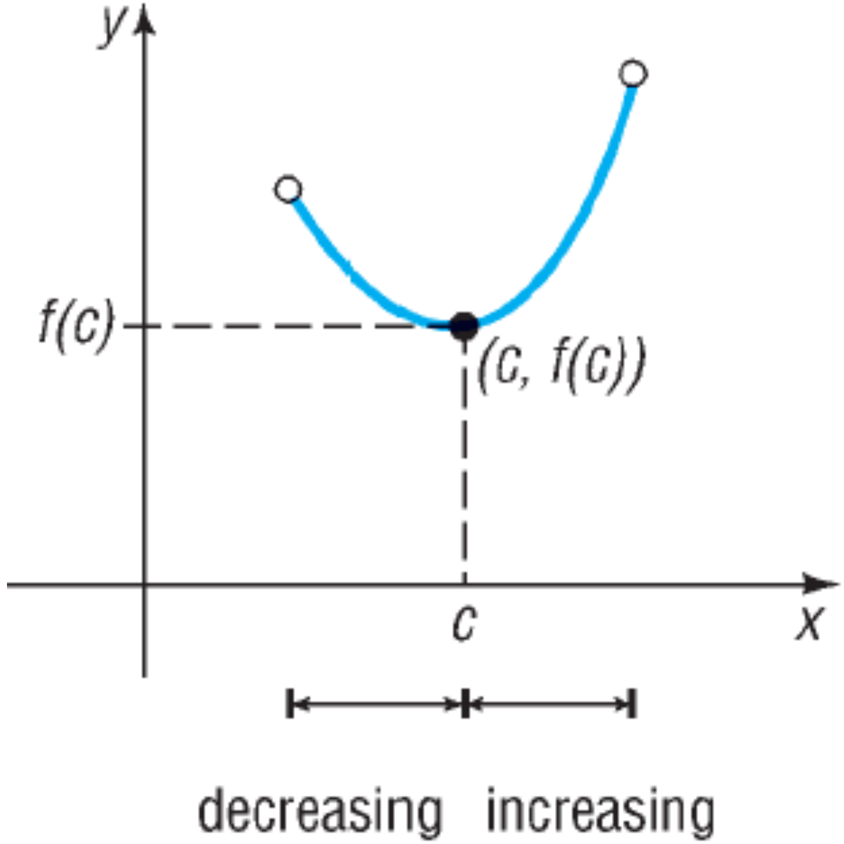
LOCAL MINIMA

LOCAL MAXIMA



increasing decreasing

The local maximum is $f(c)$ and occurs at $x = c$.



The local minimum is $f(c)$ and occurs at $x = c$.

LOCAL MINIMA

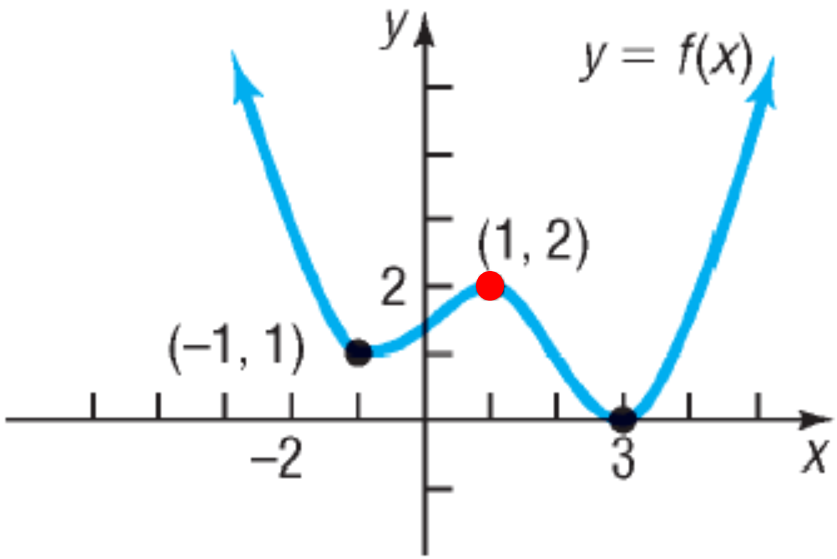
DEFINITIONS

A function f has a **local maximum** at c if there is an open interval I containing c so that, for all $x \neq c$ in I, $f(x) \leq f(c)$. We call $f(c)$ a **local maximum of f**.

A function f has a **local minimum** at c if there is an open interval I containing c so that, for all $x \neq c$ in I, $f(x) \geq f(c)$. We call $f(c)$ a **local minimum of f**.

EXAMPLE

Finding Local Maxima and Local Minima from the Graph of a Function and Determining Where the Function Is Increasing, Decreasing, or Constant



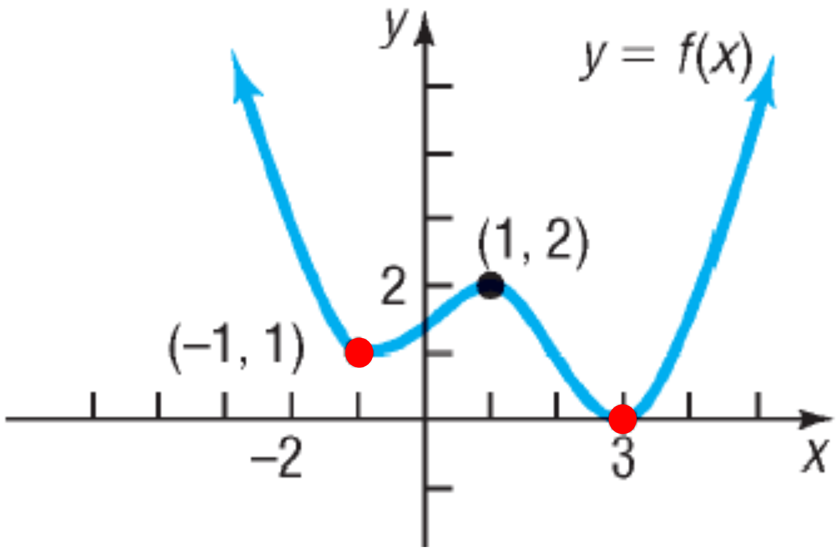
(a) At what number(s), if any, does f have a local maximum?

There is a local maximum when $x = 1$.

(b) What are the local maxima? The local maximum value is 2.

EXAMPLE

Finding Local Maxima and Local Minima from the Graph of a Function and Determining Where the Function Is Increasing, Decreasing, or Constant



(c) At what number(s), if any, does f have a local minimum?

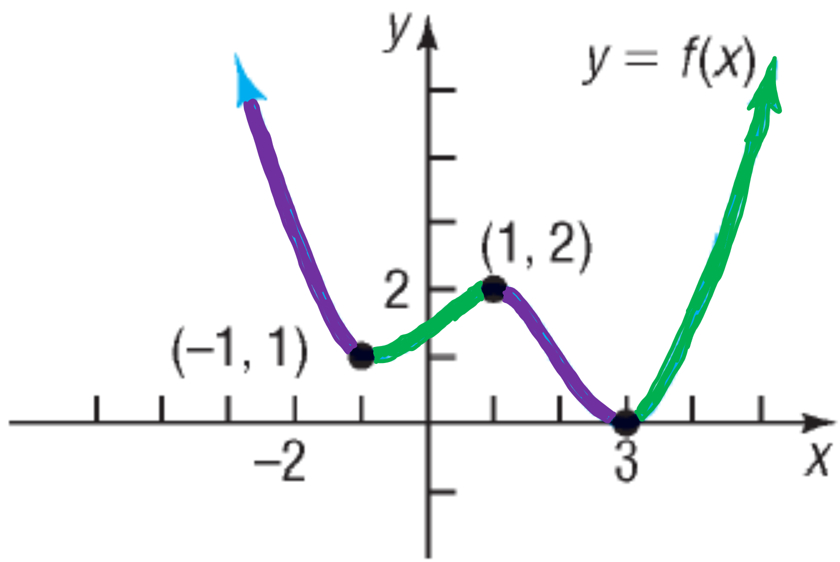
There is a local minimum when $x = -1$ and $x = 3$.

(d) What are the local minima?

The local minima values are 1 and 0.

EXAMPLE

Finding Local Maxima and Local Minima from the Graph of a Function and Determining Where the Function Is Increasing, Decreasing, or Constant



(e) List the intervals on which f is **increasing**. $(-1,1)$ and $(3,\infty)$

(f) List the intervals on which f is **decreasing**. $(-\infty,-1)$ and $(1,3)$

5 Use a Graph to Locate the Absolute Maximum and the Absolute Minimum

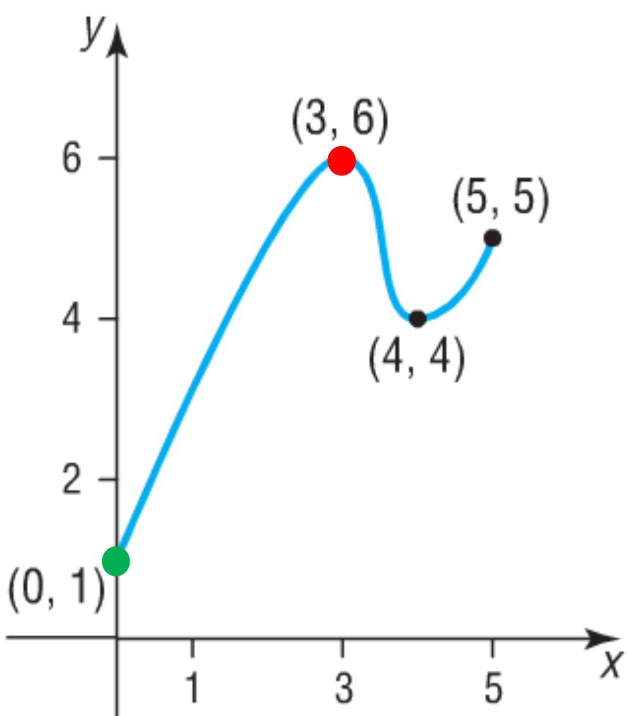
DEFINITION Let f denote a function defined on some interval I. If there is a number u in I for which $f(x) \leq f(u)$ for all x in I, then $f(u)$ is the **absolute maximum of f** on I and we say **the absolute maximum of f occurs at u**.

If there is a number v in I for which $f(x) \geq f(v)$ for all x in I, then $f(v)$ is the **absolute minimum of f** on I and we say **the absolute minimum of f occurs at v.**

EXAMPLE

Finding the Absolute Maximum and the Absolute Minimum from the Graph of a Function

Find the absolute maximum and the absolute minimum, if they exist.



(a)

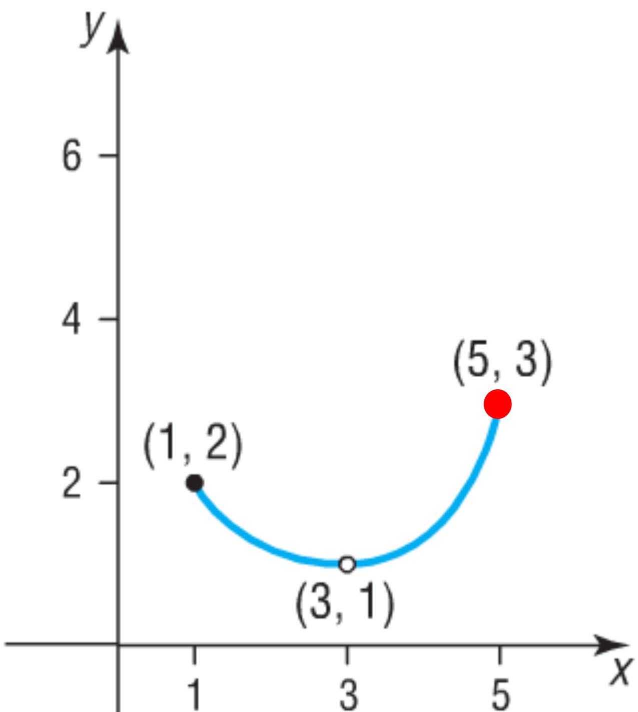
The **absolute maximum** of 6 occurs when $x = 3$.

The **absolute minimum** of 1 occurs when $x = 0$.

EXAMPLE

Finding the Absolute Maximum and the Absolute Minimum from the Graph of a Function

Find the absolute maximum and the absolute minimum, if they exist.



(b)

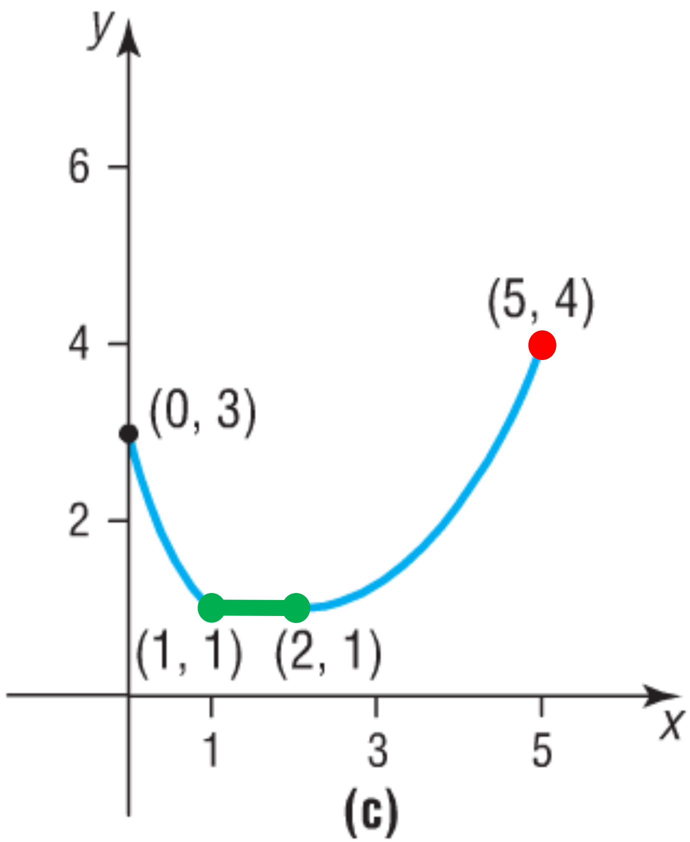
The **absolute maximum** of 3 occurs when $x = 5$.

There is no **absolute minimum** because of the “hole” at $x = 3$.

EXAMPLE

Finding the Absolute Maximum and the Absolute Minimum from the Graph of a Function

Find the absolute maximum and the absolute minimum, if they exist.



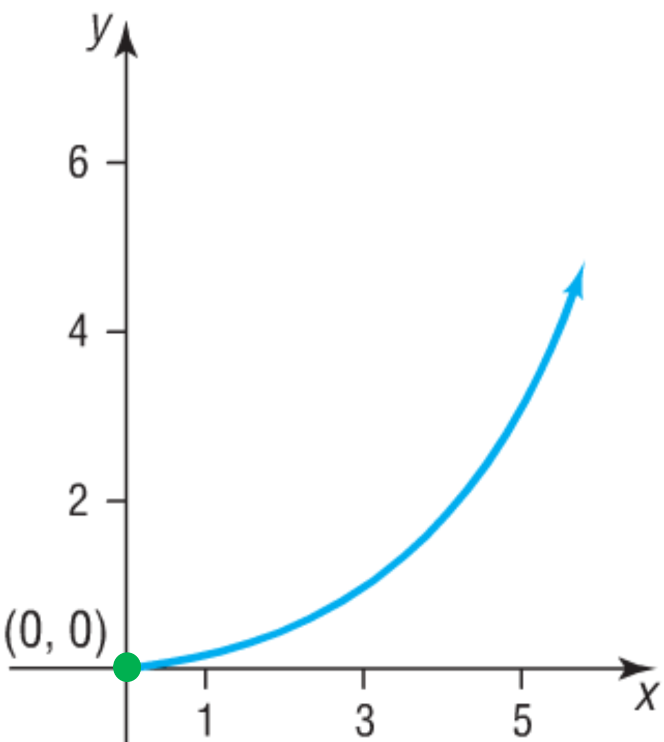
The **absolute maximum** of 4 occurs when $x = 5$.

The **absolute minimum** of 1 occurs on the interval [1,2].

EXAMPLE

Finding the Absolute Maximum and the Absolute Minimum from the Graph of a Function

Find the absolute maximum and the absolute minimum, if they exist.



(d)

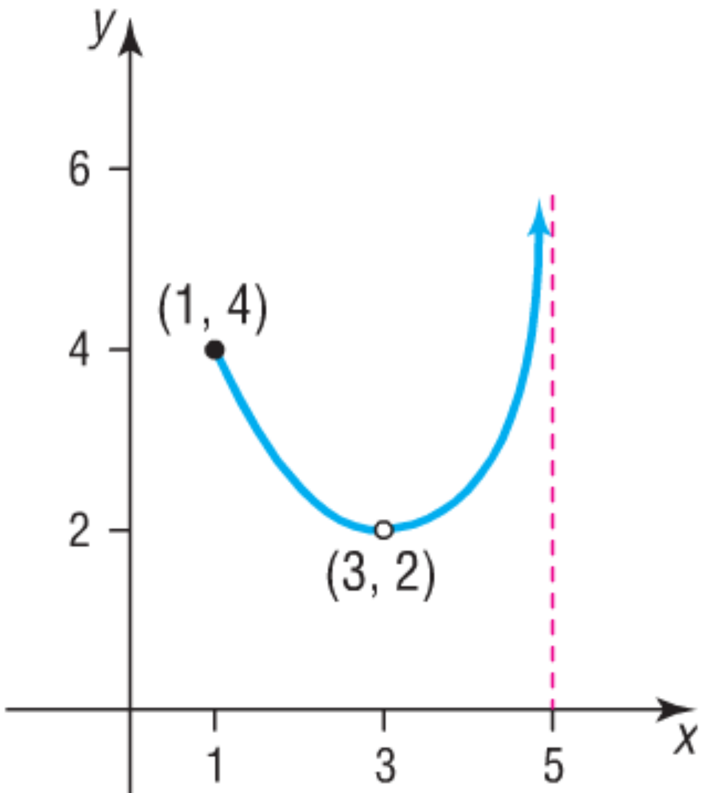
There is no **absolute maximum**.

The **absolute minimum** of 0 occurs when $x = 0$.

EXAMPLE

Finding the Absolute Maximum and the Absolute Minimum from the Graph of a Function

Find the absolute maximum and the absolute minimum, if they exist.



(e)

There is no **absolute maximum**.

There is no **absolute minimum**.

THEOREM

Extreme Value Theorem

If f is a continuous function* whose domain is a closed interval $[a, b]$, then f has an absolute maximum and an absolute minimum on $[a, b]$.

* Although it requires calculus for a precise definition, we'll agree for now that a continuous function is one whose graph has no gaps or holes and can be traced without lifting the pencil from the paper.

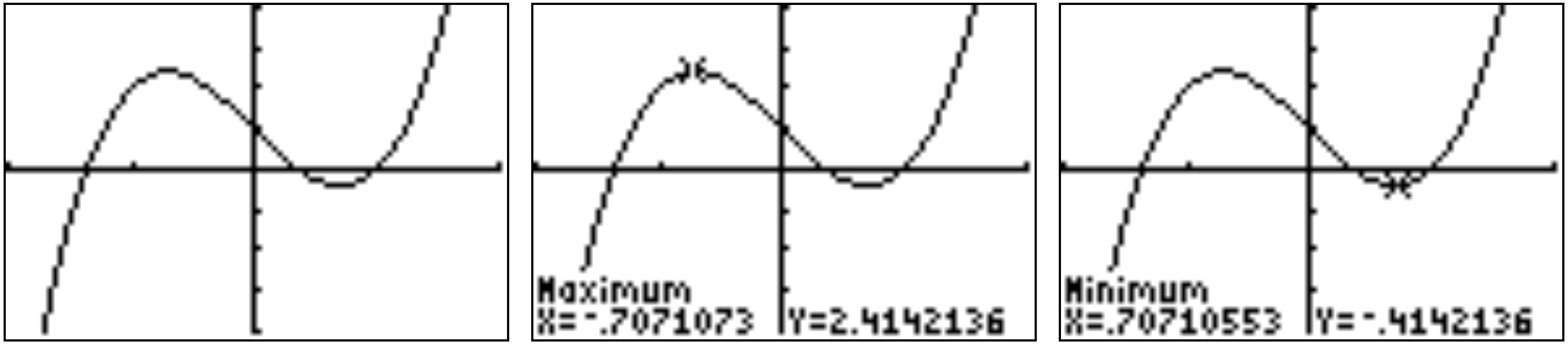
6 Use a Graphing Utility to Approximate Local Maxima and Local Minima and to Determine Where a Function Is Increasing or Decreasing

EXAMPLE

Using a Graphing Utility to Approximate Local Maxima and Minima and to Determine Where a Function Is Increasing or Decreasing

Use a graphing utility to graph $f(x) = 2x^3 - 3x + 1$ for $-2 < x < 2$.

Approximate where f has any local maxima or local minima.

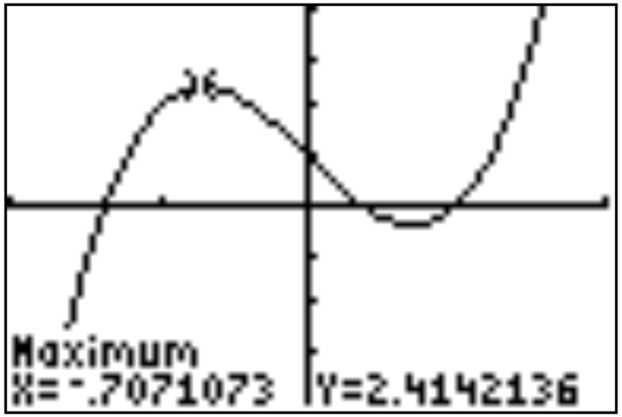


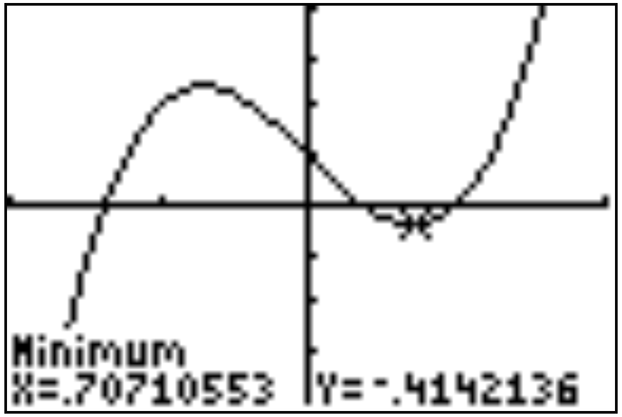
EXAMPLE

Using a Graphing Utility to Approximate Local Maxima and Minima and to Determine Where a Function Is Increasing or Decreasing

Use a graphing utility to graph $f(x) = 2x^3 - 3x + 1$ for $-2 < x < 2$.

Determine where f is increasing and where it is decreasing.





7 Find the Average Rate of Change of a Function

DEFINITION

If a and b, $a \neq b$, are in the domain of a function $y = f(x)$, the **average rate of change of f** from a to b is defined as

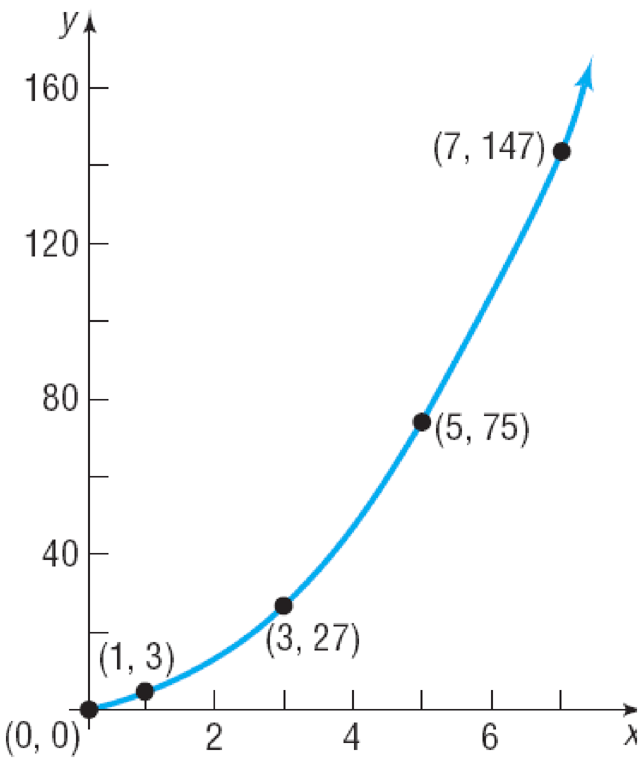
$$\boxed{\text{Average rate of change} = \frac{\Delta y}{\Delta x} = \frac{f(b) - f(a)}{b - a} \qquad a \neq b \qquad \textbf{(1)}}$$

EXAMPLE

Finding the Average Rate of Change

Find the average rate of change of $f(x) = 3x^2$:

a) From 1 to 3



$$\frac{\Delta y}{\Delta x} = \frac{f(3) - f(1)}{3 - 1} = \frac{27 - 3}{3 - 1} = \frac{24}{2} = 12$$

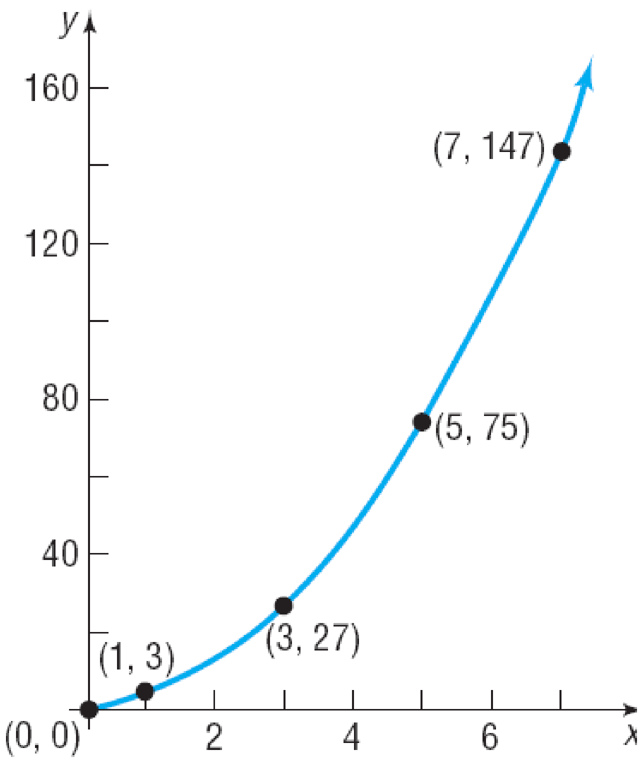
$$\boxed{\frac{\Delta y}{\Delta x} = \frac{f(x) - f(c)}{x - c}}$$

EXAMPLE

Finding the Average Rate of Change

Find the average rate of change of $f(x) = 3x^2$:

b) From 1 to 5



$$\frac{\Delta y}{\Delta x} = \frac{f(5) - f(1)}{5 - 1} = \frac{75 - 3}{5 - 1} = \frac{72}{4} = 18$$

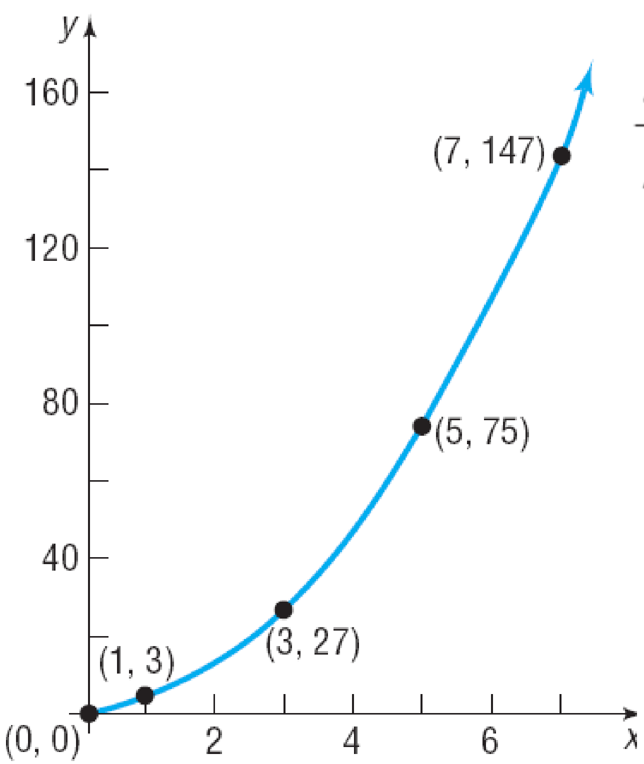
$$\boxed{\frac{\Delta y}{\Delta x} = \frac{f(x) - f(c)}{x - c}}$$

EXAMPLE

Finding the Average Rate of Change

Find the average rate of change of $f(x) = 3x^2$:

c) From 1 to 7

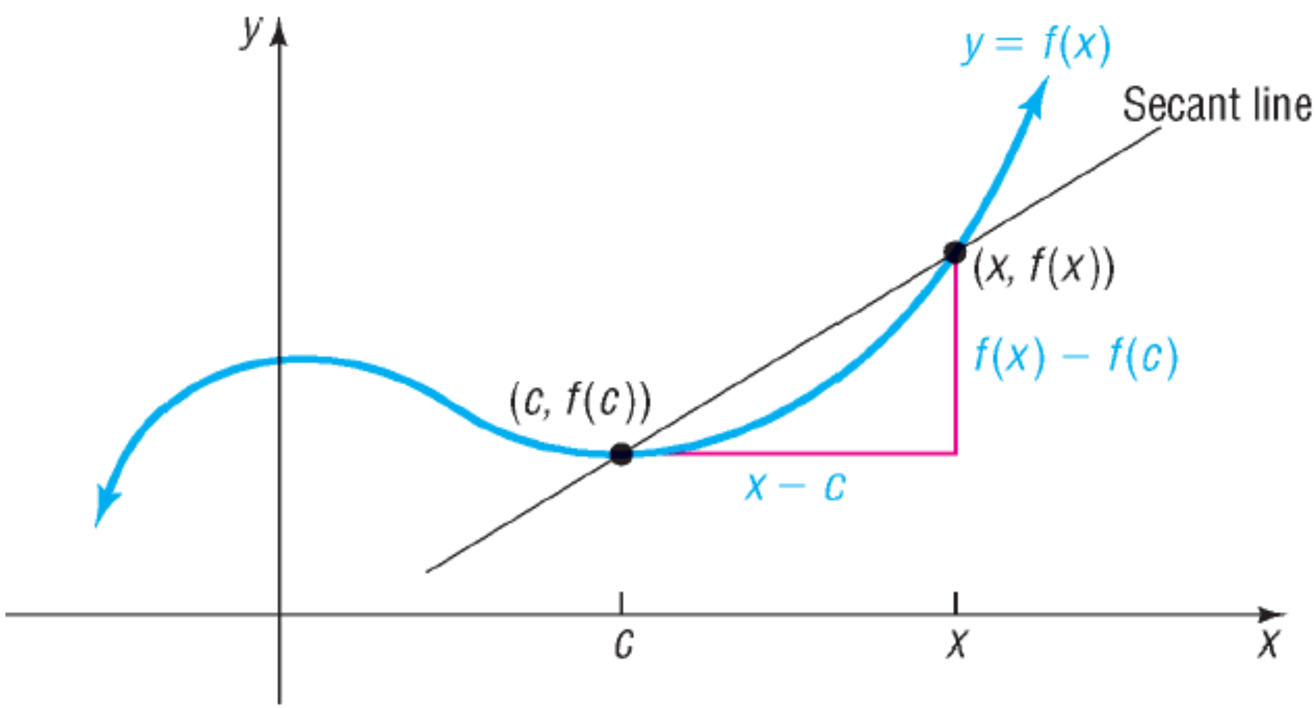


$$\frac{\Delta y}{\Delta x} = \frac{f(7) - f(1)}{7 - 1} = \frac{147 - 3}{7 - 1} = \frac{144}{6} = 24$$

$$\boxed{\frac{\Delta y}{\Delta x} = \frac{f(x) - f(c)}{x - c}}$$

The Secant Line

$$m_{\text{sec}} = \frac{f(x) - f(c)}{x - c}$$



Theorem

Slope of the Secant Line

The average rate of change of a function from a to b equals the slope of the secant line containing the two points $(a, f(a))$ and $(b, f(b))$ on its graph.

EXAMPLE **Finding the Equation of a Secant Line**

Suppose that $g(x) = -2x^2 + 4x - 3.$

(a) Find the average rate of change of g from -2 to 1.

(b) Find an equation of the secant line containing $(-2, g(-2))$ and $(1, g(1))$.

(c) Using a graphing utility, draw the graph of g and the secant line obtained in part (b) on the same screen.

(a) $$\frac{\Delta y}{\Delta x} = \frac{-2(1)^2 + 4(1) - 3 - \left(-2(-2)^2 + 4(-2) - 3\right)}{1-(-2)} = \frac{18}{3} = 6$$

$$y - y_1 = m_{\text{sec}}(x - x_1)$$

(b) $$y - (-19) = 6(x - (-2))$$

$$y + 19 = 6x + 12$$

$$y = 6x - 7$$