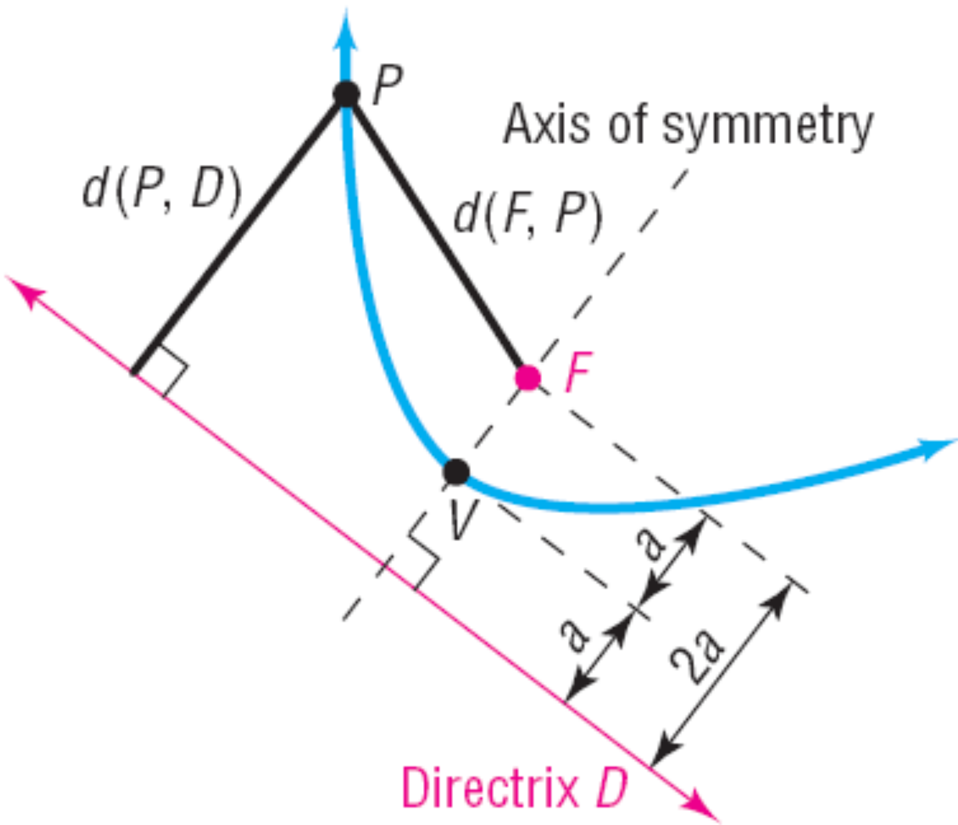
Section 11.2

The Parabola

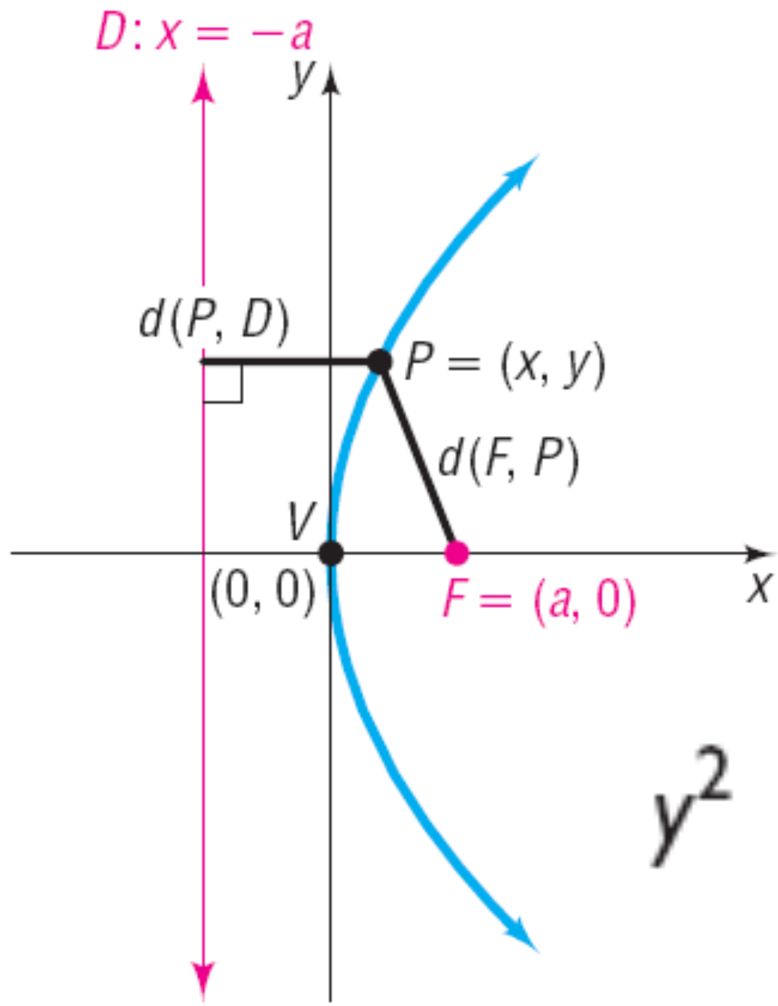
A **parabola** is the collection of all points P in the plane that are the same distance from a fixed point F as they are from a fixed line D. The point F is called the **focus** of the parabola, and the line D is its **directrix**. As a result, a parabola is the set of points P for which

$$d(F, P) = d(P, D) \qquad \textbf{(1)}$$



1 Analyze Parabolas with Vertex at the Origin

$$d(F, P) = d(P, D)$$



$$y^2 = 4ax$$

THEOREM

Equation of a Parabola

Vertex at (0, 0), Focus at (a, 0), $a > 0$

The equation of a parabola with vertex at $(0, 0)$, focus at $(a, 0)$, and directrix $x = -a$, $a > 0$, is

$$\boxed{y^2 = 4ax}$$

EXAMPLE **Finding the Equation of a Parabola and Graphing It**

Find an equation of the parabola with vertex at (0, 0) and focus at (3, 0). Graph the equation.

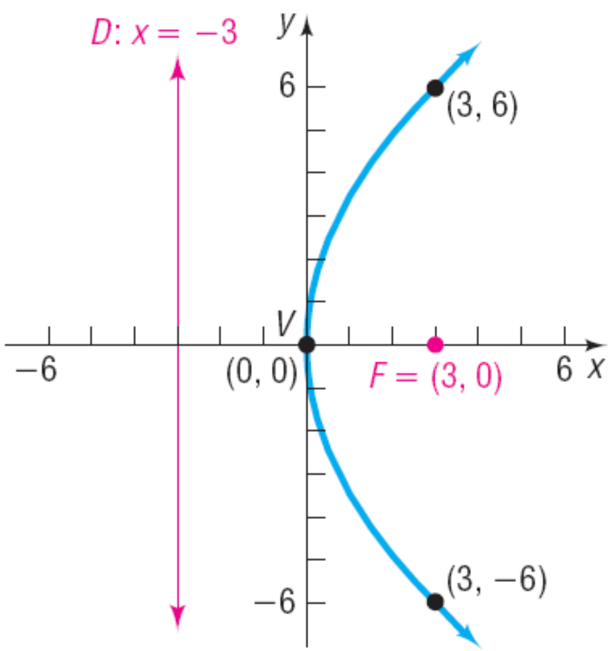
$$\boxed{y^2 = 4ax}$$

$$y^2 = 12x \qquad a = 3$$

To graph this parabola, we find the two points that determine the latus rectum by letting $x = 3$. Then

$$y^2 = 12x = 12(3) = 36$$

$$y = \pm 6$$

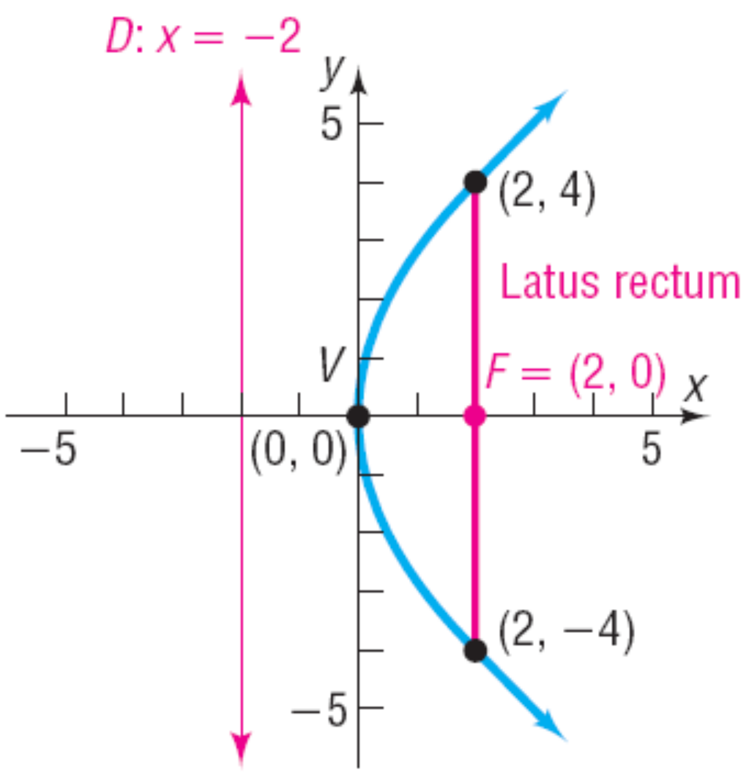


The points $(3, 6)$ and $(3, -6)$ determine the latus rectum. These points help in graphing the parabola because they determine the "opening." See Figure 5.

EXAMPLE **Analyzing the Equation of a Parabola**

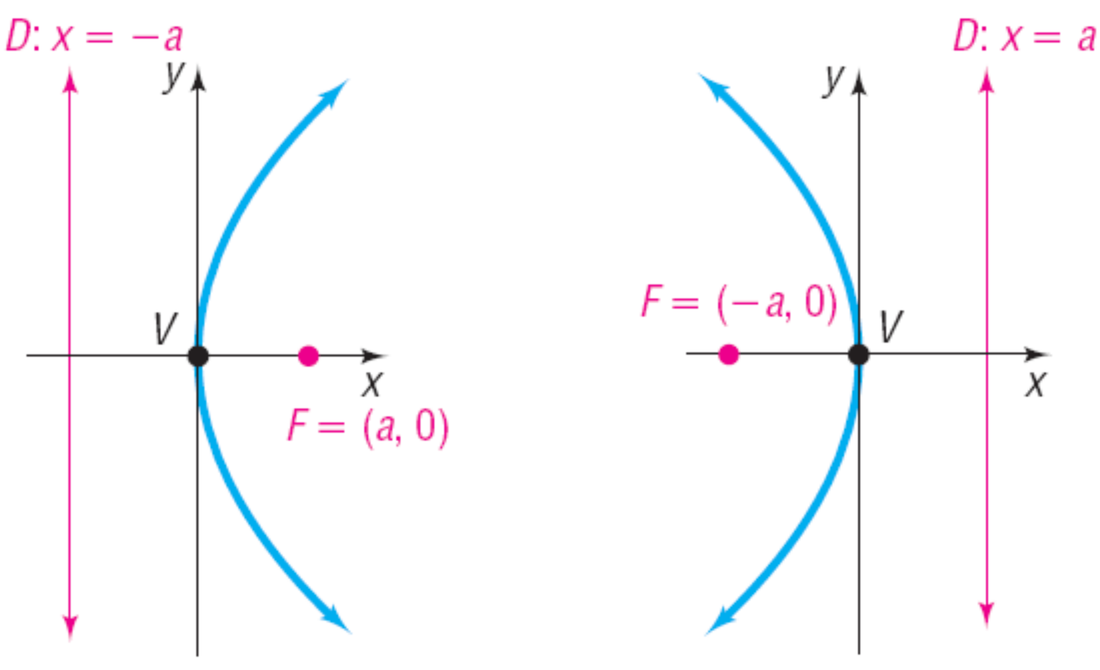
Analyze the equation: $y^2 = 8x$

Solution The equation $y^2 = 8x$ is of the form $y^2 = 4ax$, where $4a = 8$, so $a = 2$. Consequently, the graph of the equation is a parabola with vertex at $(0, 0)$ and focus on the positive x-axis at $(a, 0) = (2, 0)$. The directrix is the vertical line $x = -2$. The two points that determine the latus rectum are obtained by letting $x = 2$. Then $y^2 = 16$, so $y = \pm 4$. The points $(2, -4)$ and $(2, 4)$ determine the latus rectum. See Figure 6 for the graph.



EQUATIONS OF A PARABOLA
VERTEX AT (0, 0); FOCUS ON AN AXIS; $a > 0$

Vertex	Focus	Directrix	Equation	Description
$(0, 0)$	$(a, 0)$	$x = -a$	$y^2 = 4ax$	Parabola, axis of symmetry is the x-axis, opens right
$(0, 0)$	$(-a, 0)$	$x = a$	$y^2 = -4ax$	Parabola, axis of symmetry is the x-axis, opens left



(a) $y^2 = 4ax$ **(b)** $y^2 = -4ax$

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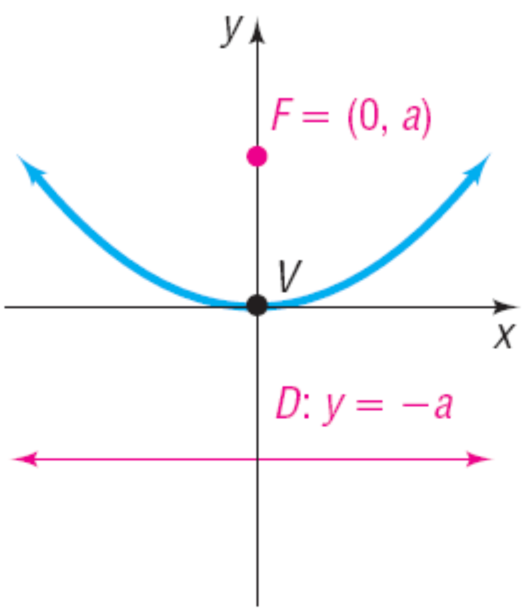
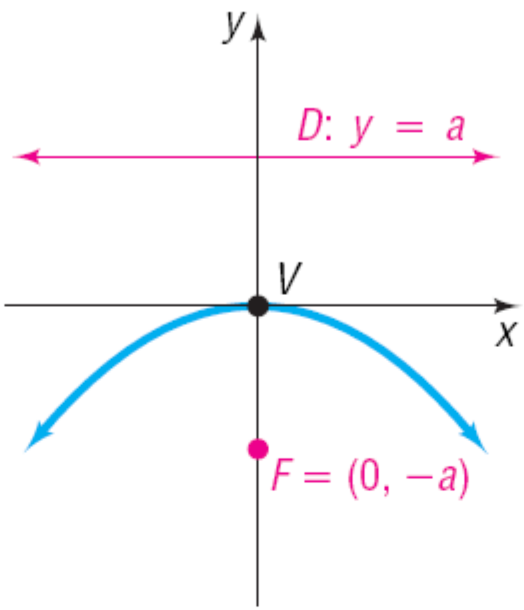
EQUATIONS OF A PARABOLA
VERTEX AT (0, 0); FOCUS ON AN AXIS; $a > 0$

Vertex	Focus	Directrix	Equation	Description
$(0, 0)$	$(0, a)$	$y = -a$	$x^2 = 4ay$	Parabola, axis of symmetry is the y-axis, opens up
$(0, 0)$	$(0, -a)$	$y = a$	$x^2 = -4ay$	Parabola, axis of symmetry is the y-axis, opens down



(c) $x^2 = 4ay$



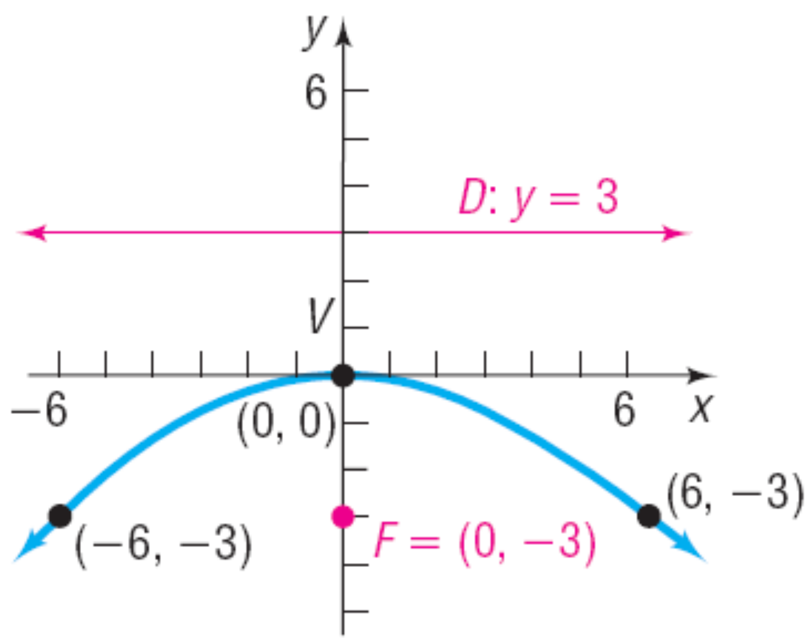
(d) $x^2 = -4ay$

all.

EXAMPLE **Analyzing the Equation of a Parabola**

Analyze the equation: $x^2 = -12y$

The equation $x^2 = -12y$ is of the form $x^2 = -4ay$, with $a = 3$. Consequently, the graph of the equation is a parabola with vertex at $(0, 0)$, focus at $(0, -3)$, and directrix the line $y = 3$. The parabola opens down, and its axis of symmetry is the y-axis. To obtain the points defining the latus rectum, let $y = -3$. Then $x^2 = 36$, so $x = \pm 6$. The points $(-6, -3)$ and $(6, -3)$ determine the latus rectum. See Figure 8 for the graph.



EXAMPLE **Finding the Equation of a Parabola**

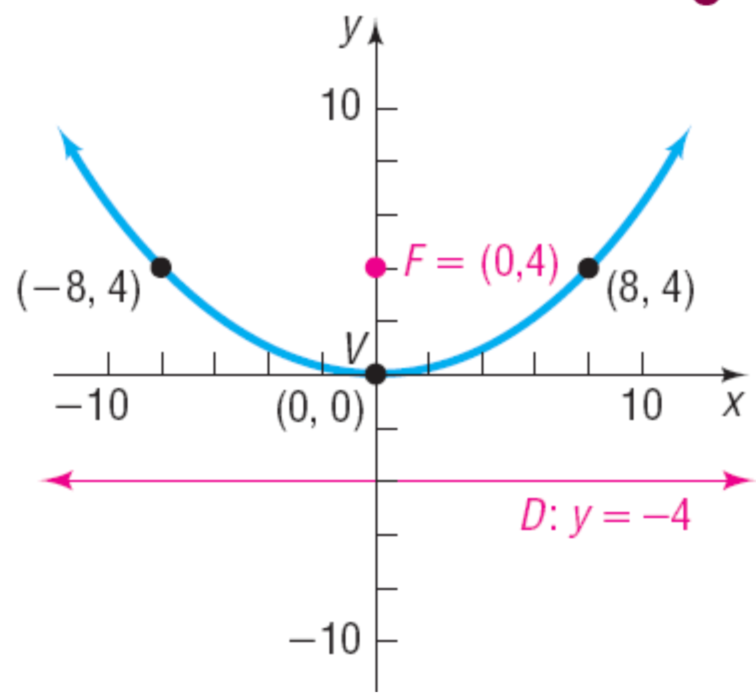
Find the equation of the parabola with focus at (0, 4) and directrix the line $y = -4$. Graph the equation.

The points (8, 4) and (–8, 4) determine the latus rectum. Figure 9 shows the graph of $x^2 = 16y$.

$$x^2 = 4ay = 4(4)y = 16y$$

$$\uparrow$$

$$a = 4$$



$$x^2 = 16y$$

$$\boxed{x^2 = 4ay}$$

EXAMPLE

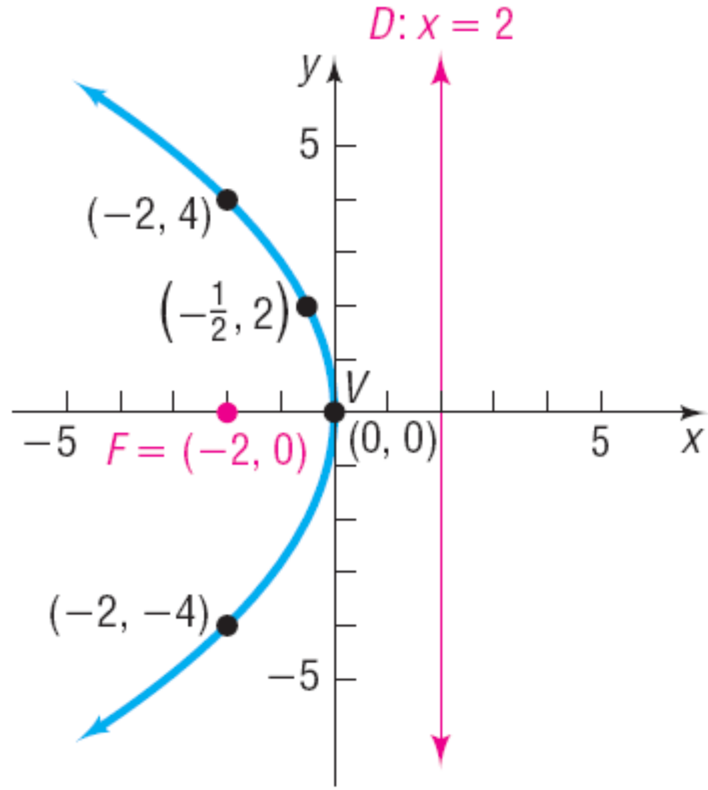
Finding the Equation of a Parabola

Find the equation of a parabola with vertex at $(0, 0)$ if its axis of symmetry is the x-axis and its graph contains the point $\left(-\frac{1}{2}, 2\right)$. Find its focus and directrix, and graph the equation.

$$4 = -4a\left(-\frac{1}{2}\right) \quad y^2 = -4ax;\ x = -\frac{1}{2},\ y = 2$$

$$a = 2$$

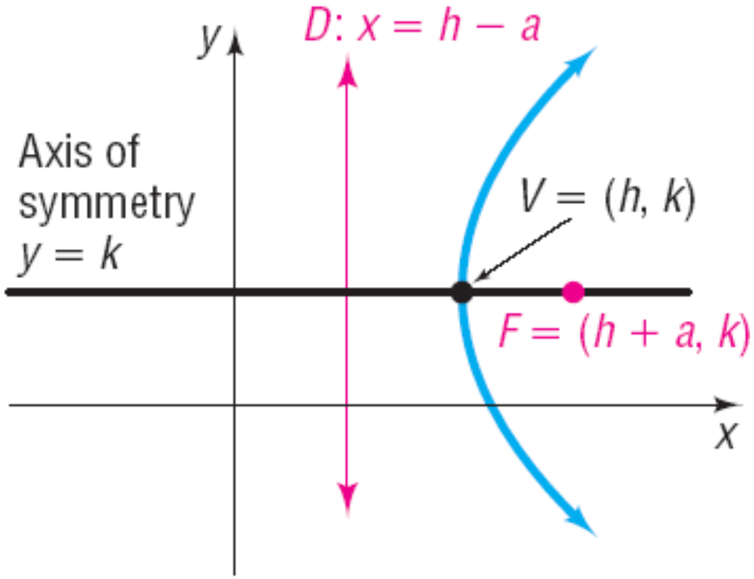
$$y^2 = -4(2)x = -8x$$



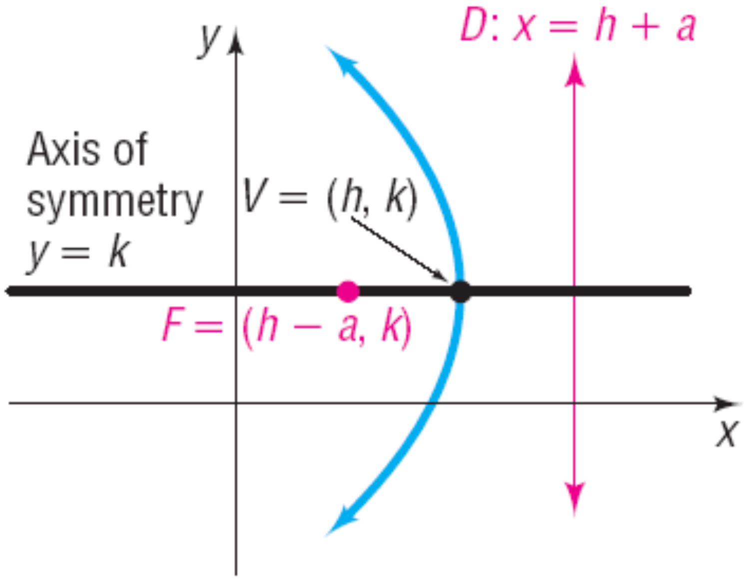
The focus is at $(-2, 0)$ and the directrix is the line $x = 2$. Let $x = -2$. Then $y^2 = 16$, so $y = \pm 4$. The points $(-2, 4)$ and $(-2, -4)$ determine the latus rectum. See Figure 10.

2 Analyze Parabolas with Vertex at (h, k)

Vertex	Focus	Directrix	Equation	Description
(h, k)	$(h + a, k)$	$x = h - a$	$(y - k)^2 = 4a(x - h)$	Parabola, axis of symmetry parallel to x-axis, opens right
(h, k)	$(h - a, k)$	$x = h + a$	$(y - k)^2 = -4a(x - h)$	Parabola, axis of symmetry parallel to x-axis, opens left

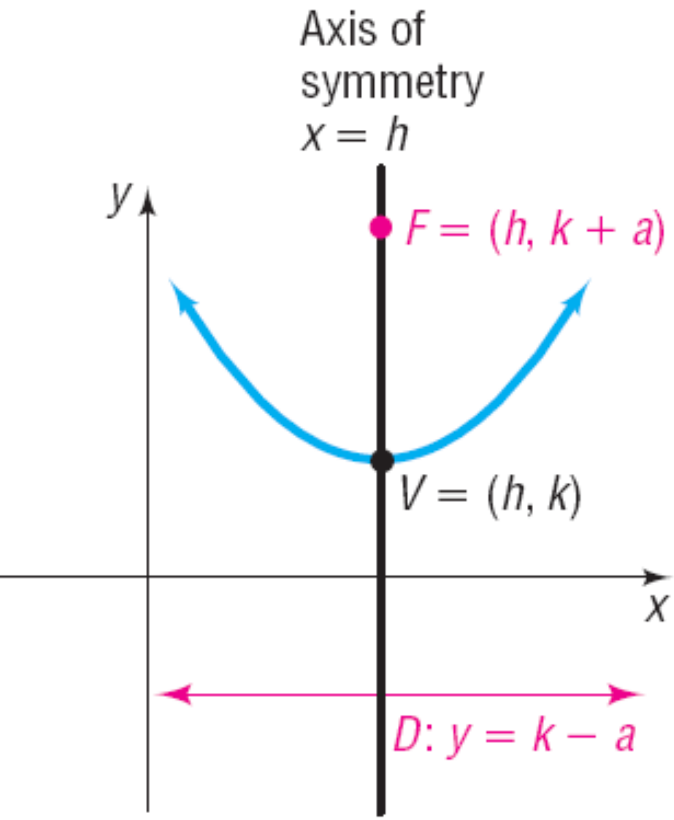


(a) $(y - k)^2 = 4a(x - h)$

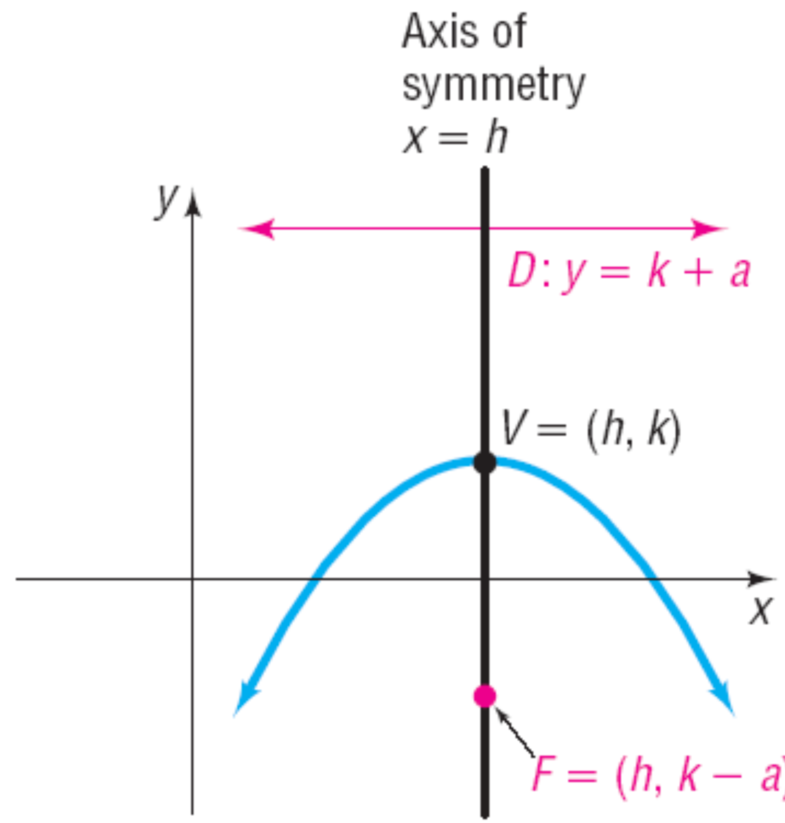


(b) $(y - k)^2 = -4a(x - h)$

Vertex	Focus	Directrix	Equation	Description
(h, k)	$(h, k + a)$	$y = k - a$	$(x - h)^2 = 4a(y - k)$	Parabola, axis of symmetry parallel to y-axis, opens up
(h, k)	$(h, k - a)$	$y = k + a$	$(x - h)^2 = -4a(y - k)$	Parabola, axis of symmetry parallel to y-axis, opens down



(c) $(x - h)^2 = 4a(y - k)$

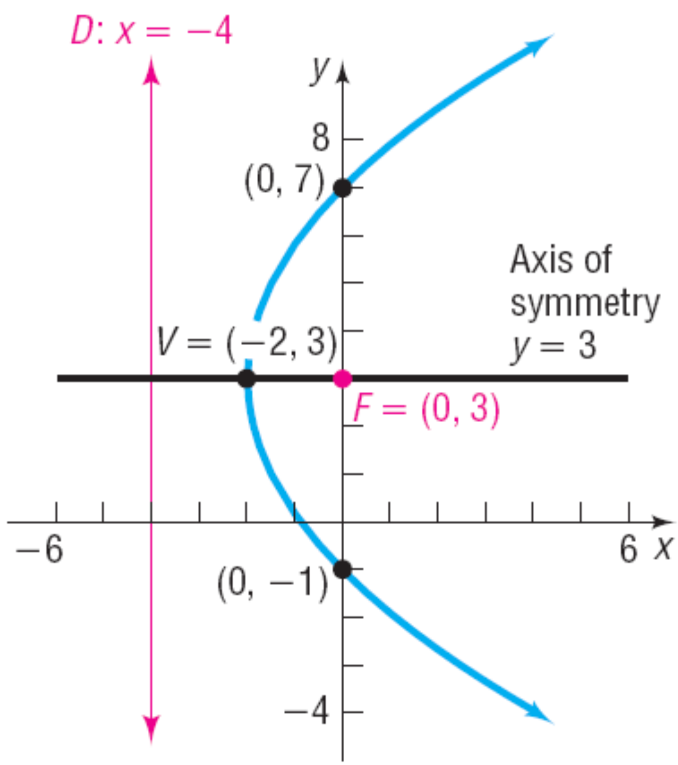


(d) $(x - h)^2 = -4a(y - k)$

EXAMPLE **Finding the Equation of a Parabola, Vertex Not at the Origin**

Find an equation of the parabola with vertex at (–2, 3) and focus at (0, 3). Graph the equation.

$(h, k) = (-2, 3)$ and $a = 2$. Therefore, the equation is



$$(y - 3)^2 = 4 \cdot 2[x - (-2)]$$
$$(y - 3)^2 = 8(x + 2)$$

To find the points that define the latus rectum, let $x = 0$, so that $(y - 3)^2 = 16$. Then $y - 3 = \pm 4$, so $y = -1$ or $y = 7$. The points $(0, -1)$ and $(0, 7)$ determine the latus rectum; the line $x = -4$ is the directrix. See Figure 12.

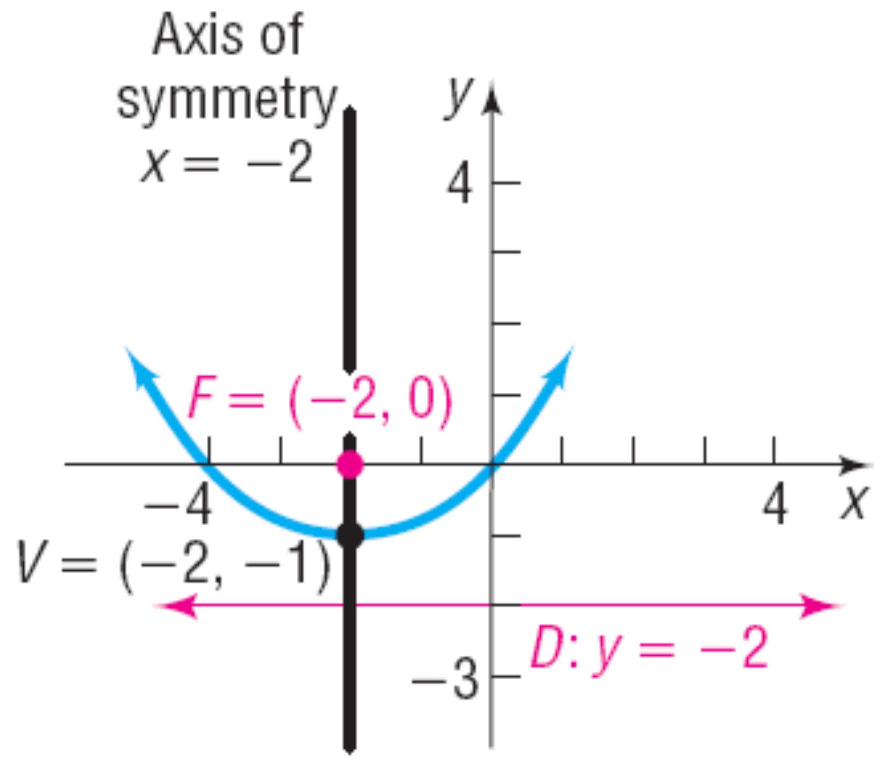
EXAMPLE **Analyzing the Equation of a Parabola**

Analyze the equation: $x^2 + 4x - 4y = 0$

$$x^2 + 4x = 4y$$

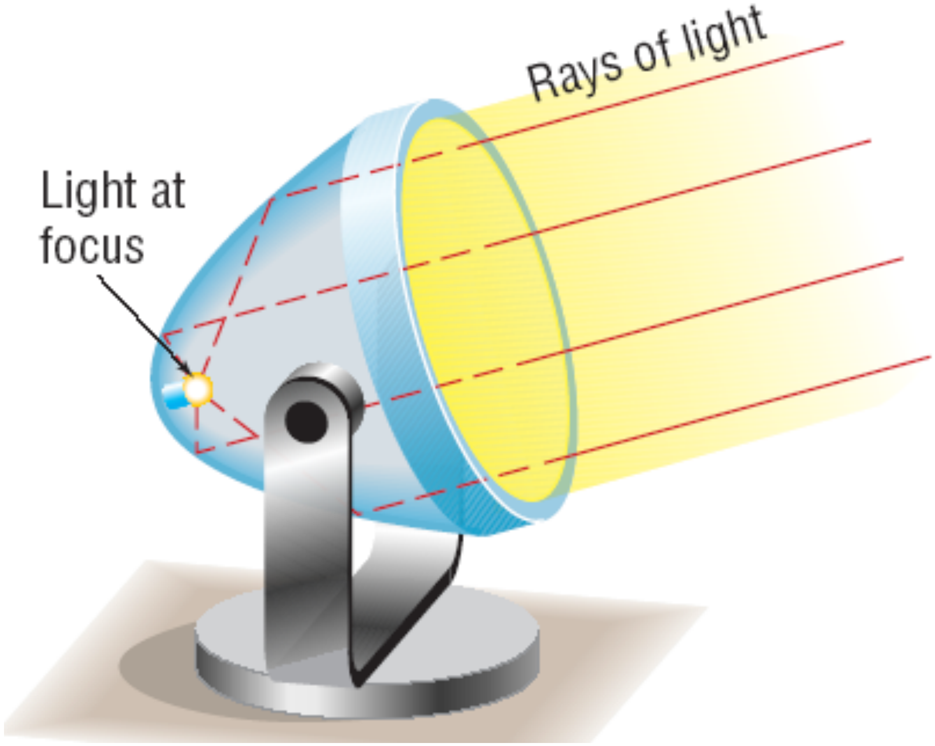
$$x^2 + 4x + 4 = 4y + 4$$

$$(x + 2)^2 = 4(y + 1)$$

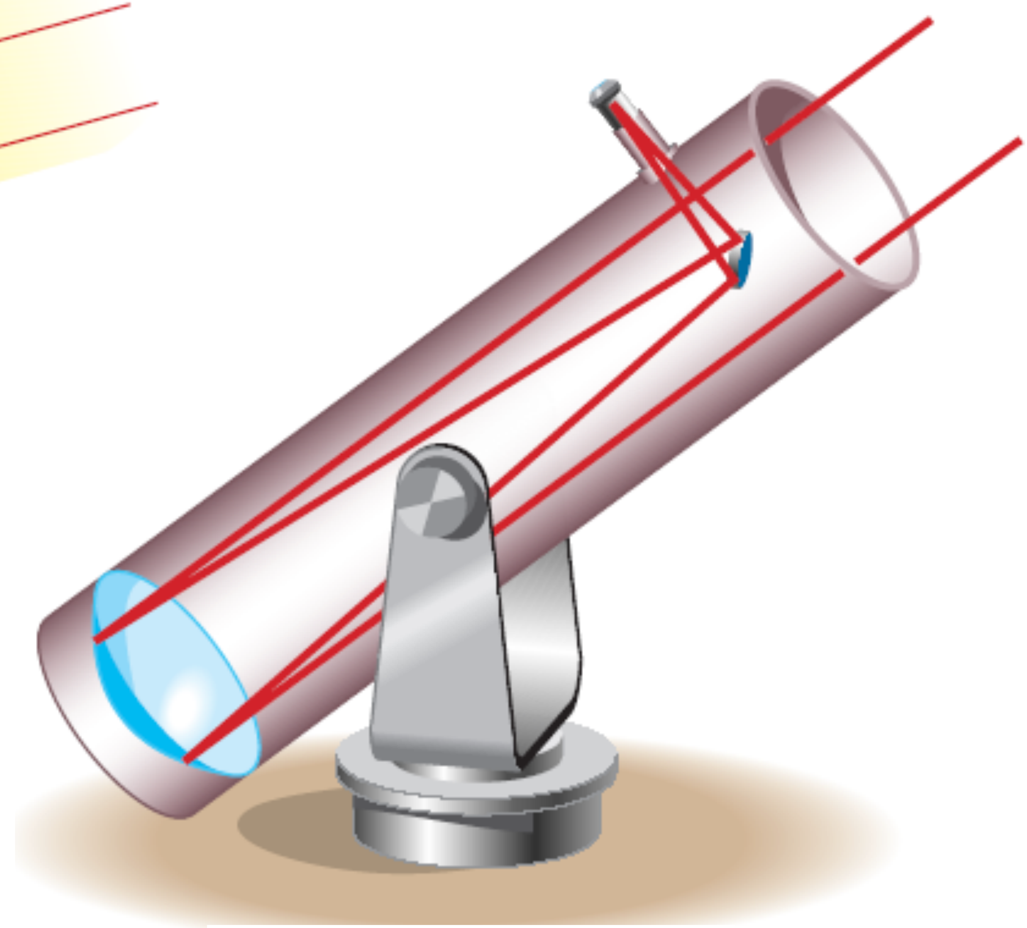


This equation is of the form $(x - h)^2 = 4a(y - k)$, with $h = -2, k = -1$, and $a = 1$. The graph is a parabola with vertex at $(h, k) = (-2, -1)$ that opens up. The focus is at $(-2, 0)$, and the directrix is the line $y = -2$. See Figure 13.

3 Solve Applied Problems Involving Parabolas



Searchlight



Telescope

EXAMPLE

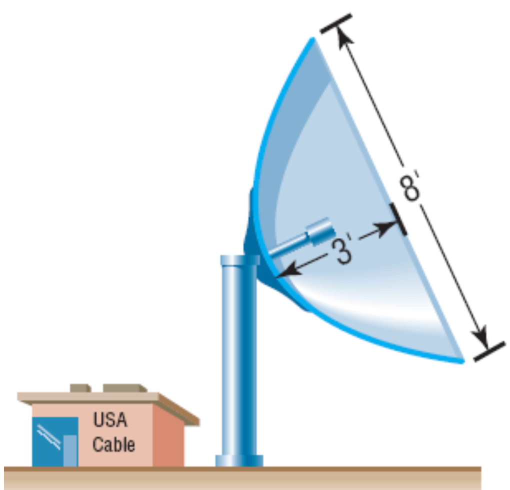
Satellite Dish

A satellite dish is shaped like a paraboloid of revolution. The signals that emanate from a satellite strike the surface of the dish and are reflected to a single point, where the receiver is located. If the dish is 8 feet across at its opening and 3 feet deep at its center, at what position should the receiver be placed?

Since (4, 3) is a point on the graph, we have

$$4^2 = 4a(3) \qquad x^2 = 4ay;\ x = 4,\ y = 3$$

$$a = \frac{4}{3} \qquad \text{Solve for } a.$$



The receiver should be located 1 foot 4 inches from the base of the dish, along its axis of symmetry.