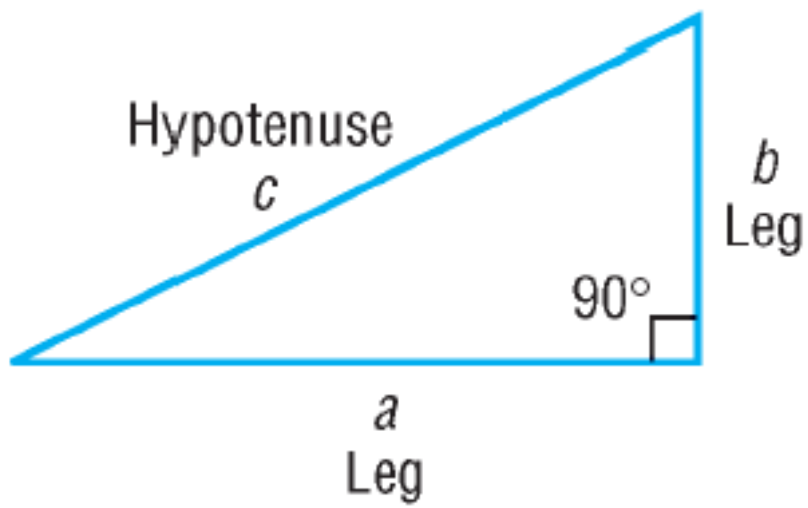
Section R.3

Geometry Essentials

1 Use the Pythagorean Theorem and Its Converse

In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs. That is, in the right triangle shown

$$c^2 = a^2 + b^2 \tag{1}$$



EXAMPLE Finding the Hypotenuse of a Right Triangle

In a right triangle, one leg is of length 4 and the other is of length 3. What is the length of the hypotenuse?

$$c^2 = 4^2 + 3^2 = 16 + 9 = 25$$

$$c = \sqrt{25} = 5$$

$$\boxed{c^2 = a^2 + b^2}$$

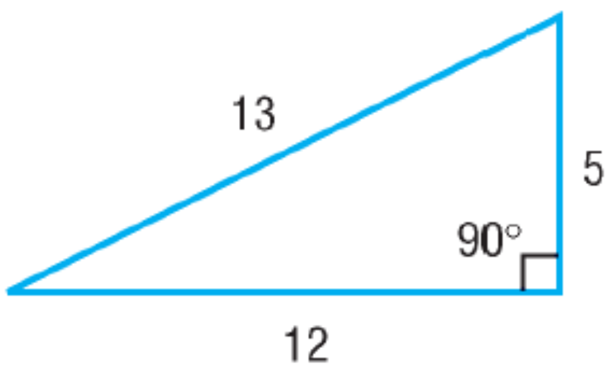
Converse of the Pythagorean Theorem

In a triangle, if the square of the length of one side equals the sum of the squares of the lengths of the other two sides, then the triangle is a right triangle. The 90° angle is opposite the longest side.

EXAMPLE **Verifying That a Triangle Is a Right Triangle**

Show that a triangle whose sides are of lengths 5, 12, and 13 is a right triangle. Identify the hypotenuse.

$$5^2 = 25, \qquad 12^2 = 144, \qquad 13^2 = 169$$



Notice that the sum of the first two squares (25 and 144) equals the third square (169). Hence, the triangle is a right triangle. The longest side, 13, is the hypotenuse.

EXAMPLE Applying the Pythagorean Theorem

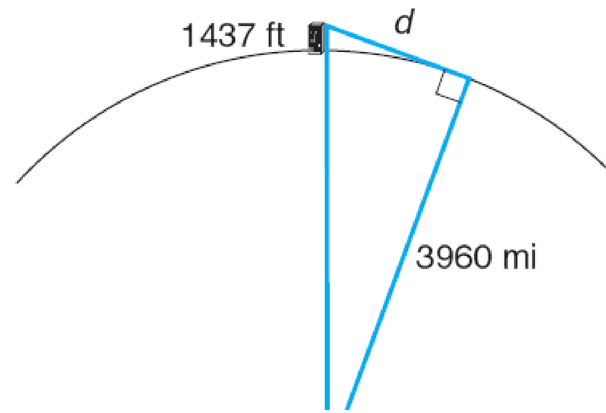
Excluding antenna, the tallest inhabited building in the world is Taipei 101 in Taipei, Taiwan. If the indoor observation deck is 1437 feet above ground level, how far can a person standing on the observation deck see (with the aid of a telescope)? Use 3960 miles for the radius of Earth.

Since 1 mile $=$ 5280 feet, then 1450 feet $= \dfrac{1450}{5280}$ mile. So we have

$$d^2 + (3960)^2 = \left(3960 + \frac{1450}{5280}\right)^2$$

$$d^2 = \left(3960 + \frac{1450}{5280}\right)^2 - (3960)^2 \approx 2175.08$$

$$d \approx 46.64$$

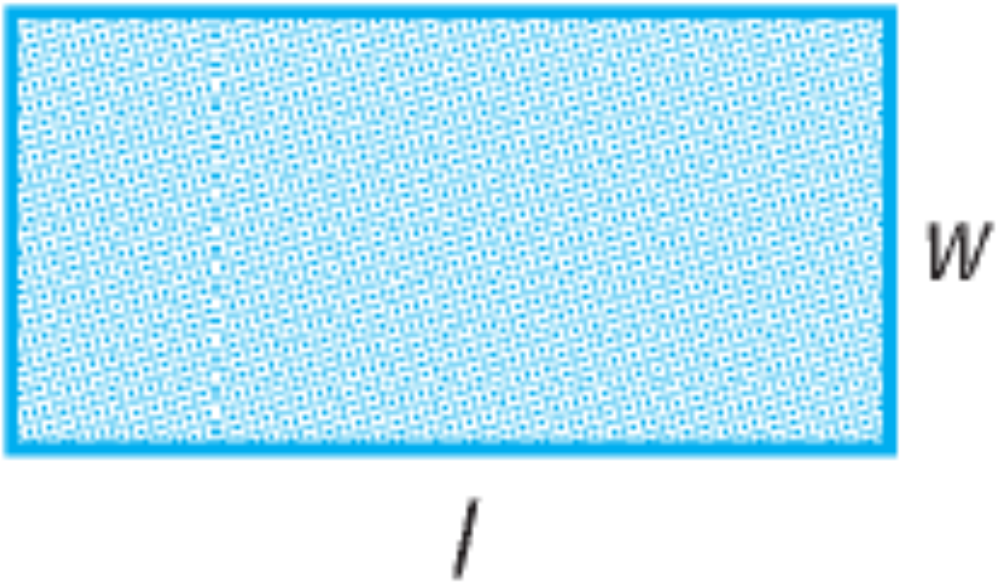


A person can see almost 47 miles from the observation tower.

2 Know Geometry Formulas

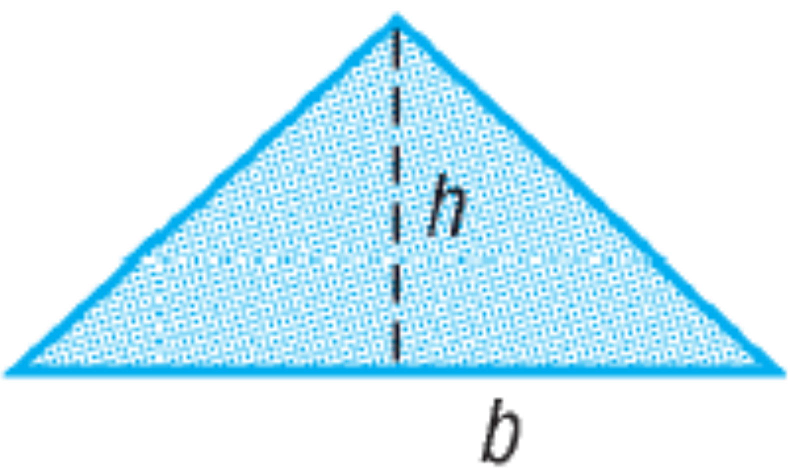
For a rectangle of length l and width w,

$$\boxed{\text{Area} = lw \qquad \text{Perimeter} = 2l + 2w}$$



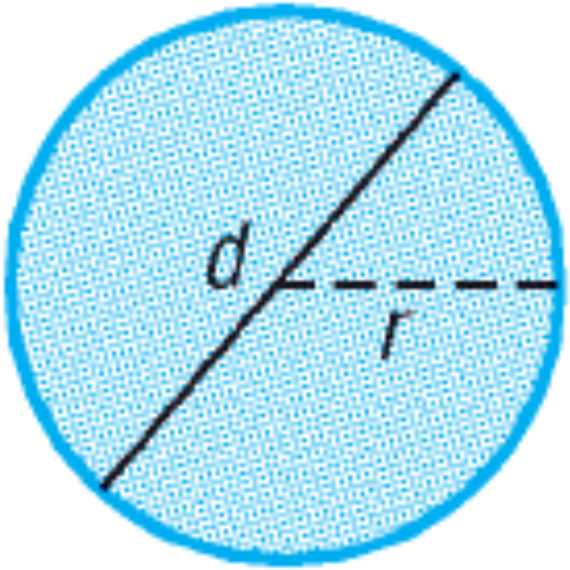
For a triangle with base b and altitude h,

$$\boxed{\text{Area} = \frac{1}{2}bh}$$



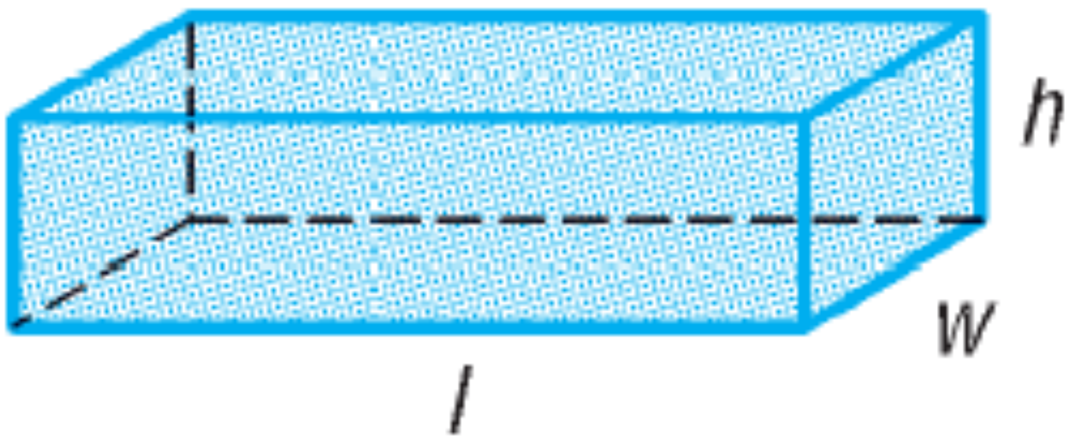
For a circle of radius r (diameter $d = 2r$),

$$\boxed{\text{Area} = \pi r^2 \qquad \text{Circumference} = 2\pi r = \pi d}$$



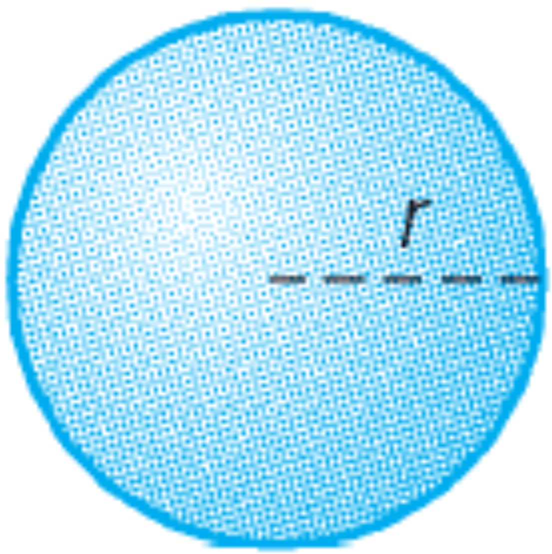
For a closed rectangular box of length l, width w, and height h,

$$\text{Volume} = lwh \qquad \text{Surface area} = 2lh + 2wh + 2lw$$



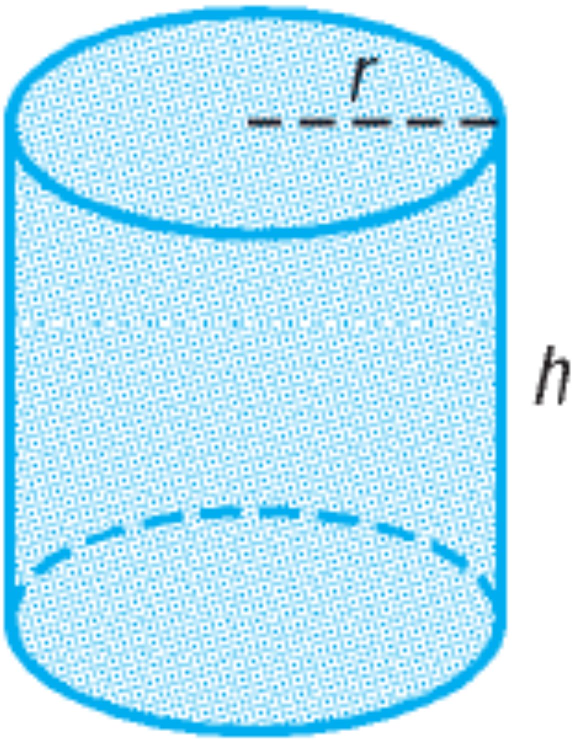
For a sphere of radius r,

$$\text{Volume} = \frac{4}{3}\pi r^3 \qquad \text{Surface area} = 4\pi r^2$$



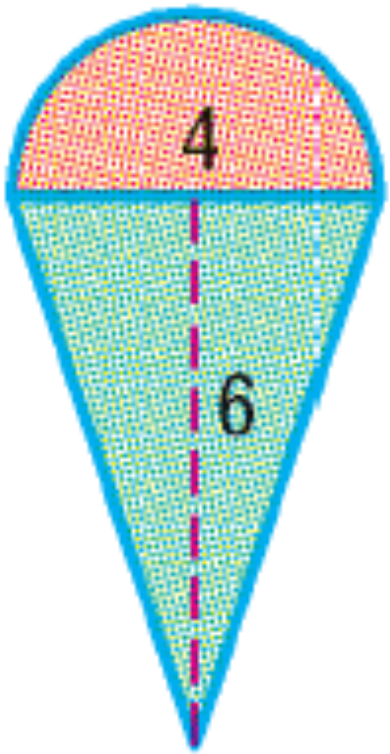
For a right circular cylinder of height h and radius r,

$$\text{Volume} = \pi r^2 h \qquad \text{Surface area} = 2\pi r^2 + 2\pi r h$$



EXAMPLE Using Geometry Formulas

A Christmas tree ornament is in the shape of a semicircle on top of a triangle. How many square centimeters (cm) of copper is required to make the ornament if the height of the triangle is 6 cm and the base is 4 cm?



$$\text{Area} = \text{Area of triangle} + \text{Area of semicircle}$$

$$= \frac{1}{2}bh + \frac{1}{2}\pi r^2 = \frac{1}{2}(4)(6) + \frac{1}{2}\pi \cdot 2^2$$

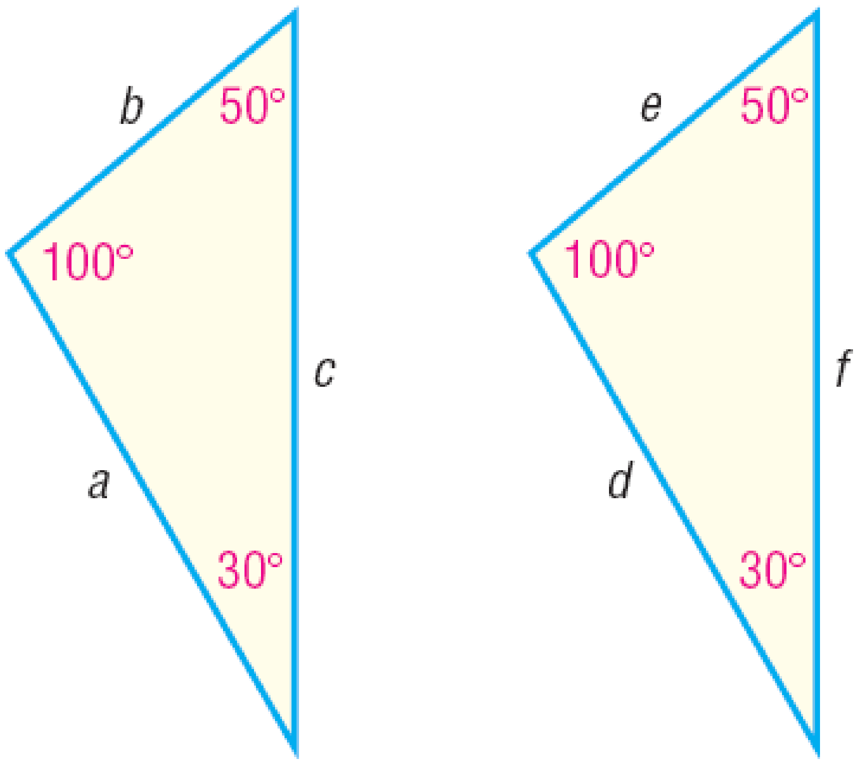
$$= 12 + 2\pi \approx 18.28 \text{ cm}^2$$

About 18.28 cm^2 of copper is required.

3 Understand Congruent Triangles and Similar Triangles

DEFINITION

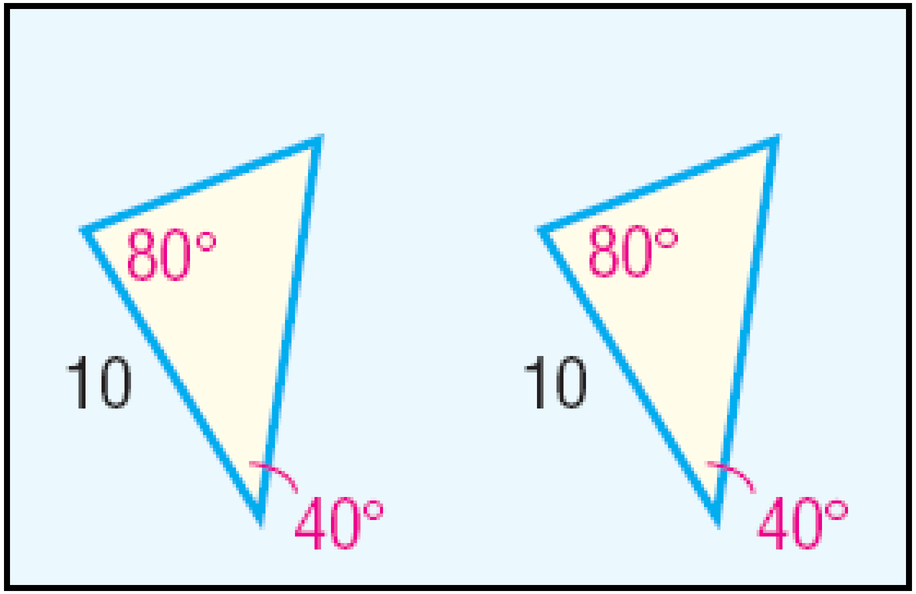
Two triangles are **congruent** if each of the corresponding angles is the same measure and each of the corresponding sides is the same length.



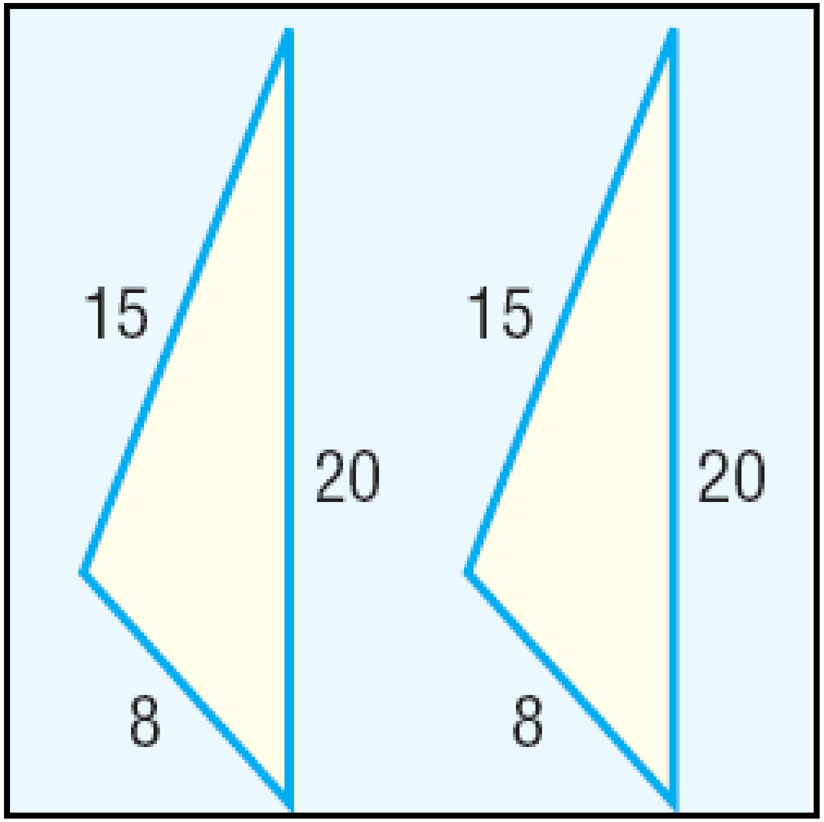
Congruent triangles

Determining Congruent Triangles

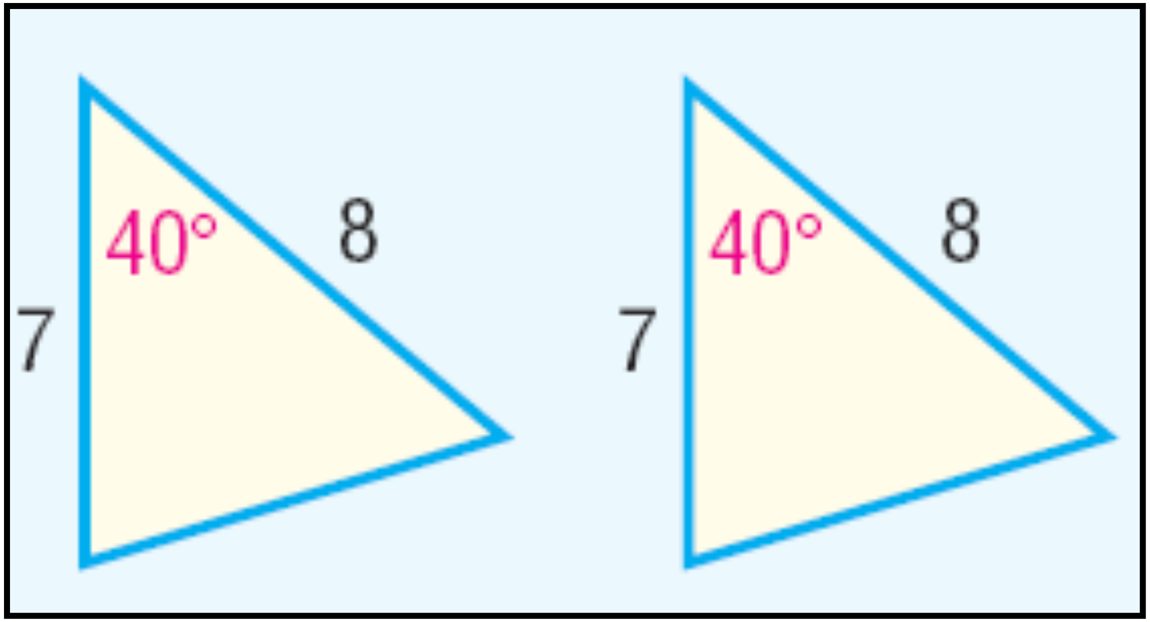
1. **Angle–Side–Angle Case** Two triangles are congruent if two of the angles are equal and the lengths of the corresponding sides between the two angles are equal.



2. Side–Side–Side Case Two triangles are congruent if the lengths of the corresponding sides of the triangles are equal.



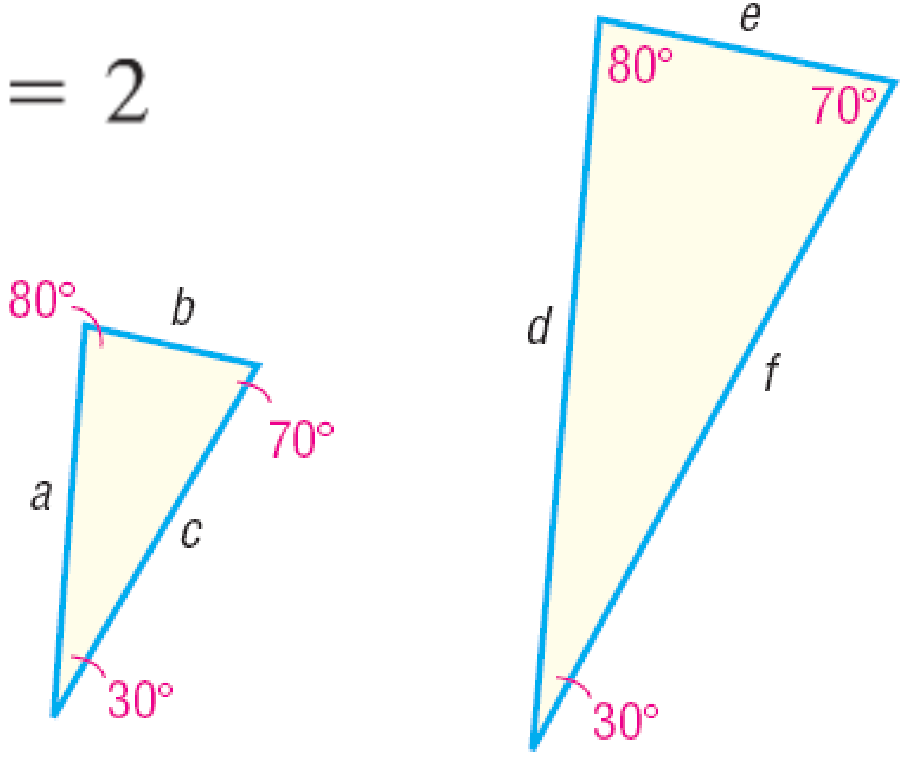
3. Side–Angle–Side Case Two triangles are congruent if the lengths of two corresponding sides are equal and the angles between the two sides are the same.



DEFINITION

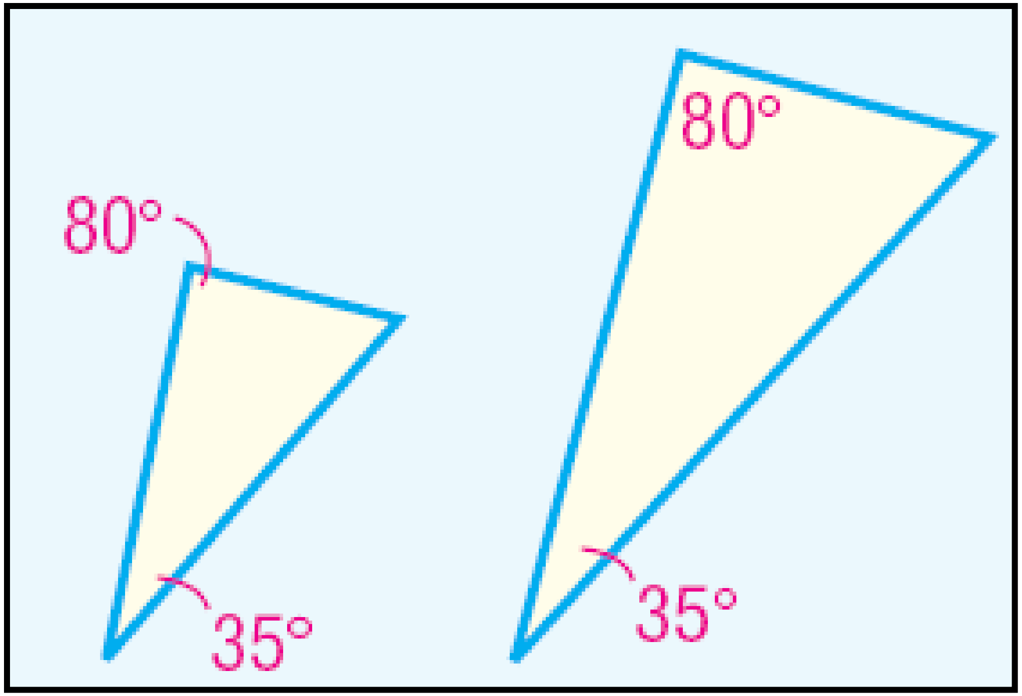
Two triangles are **similar** if the corresponding angles are equal and the lengths of the corresponding sides are proportional.

$$\frac{d}{a} = \frac{e}{b} = \frac{f}{c} = 2$$

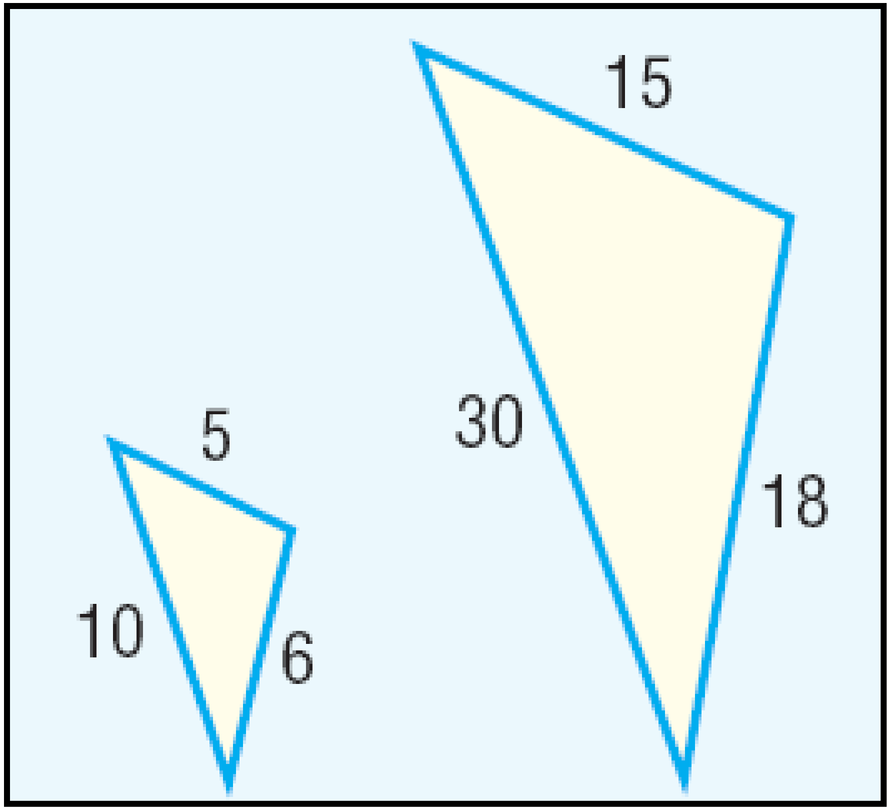


Determining Similar Triangles

1. **Angle–Angle Case** Two triangles are similar if two of the corresponding angles are equal.

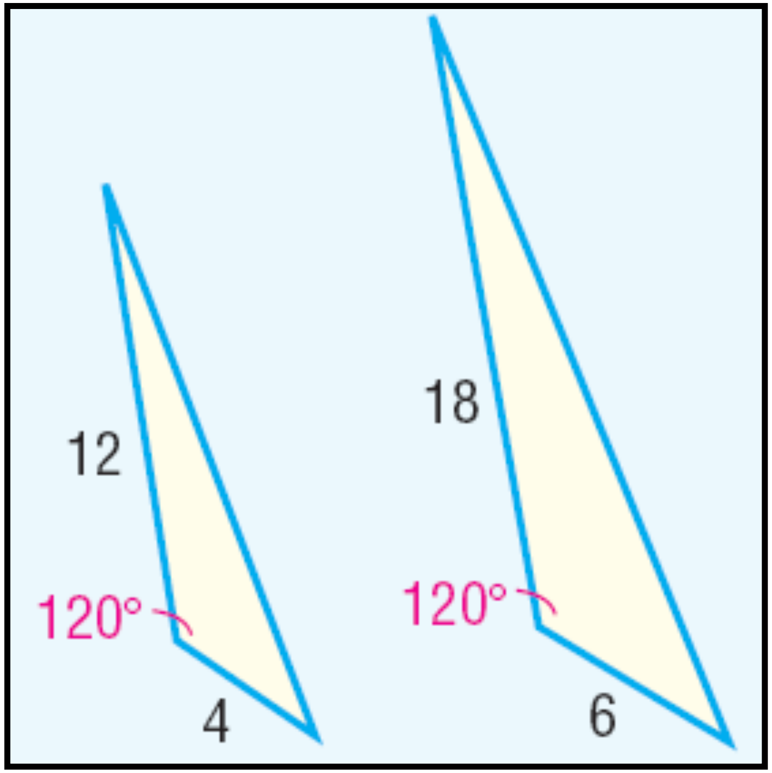


2. Side–Side–Side Case Two triangles are similar if the lengths of all three sides of each triangle are proportional.



$$\frac{10}{30} = \frac{5}{15} = \frac{6}{18} = \frac{1}{3}$$

3. Side–Angle–Side Case Two triangles are similar if two corresponding sides are proportional and the angles between the two sides are equal.



$$\frac{4}{6} = \frac{12}{18} = \frac{2}{3}$$

EXAMPLE

Using Similar Triangles

Given that the triangles in Figure 24 are similar, find the missing length x and the angles A, B, and C.

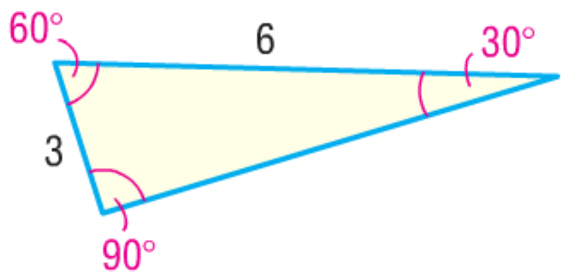
Because the triangles are similar, corresponding angles are equal. So $A = 90°$, $B = 60°$, and $C = 30°$. Also, the corresponding sides are proportional. That is, $\frac{3}{5} = \frac{6}{x}$. We solve this equation for x.

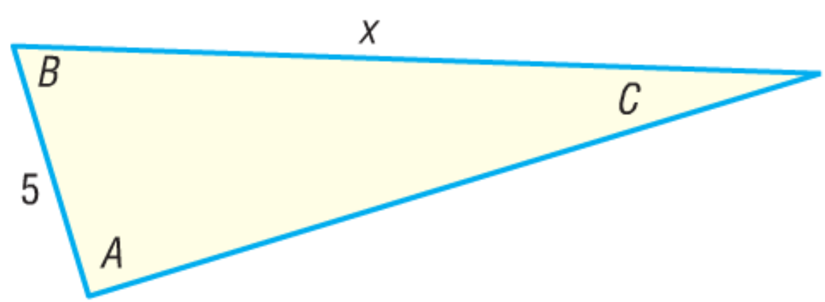
$$\frac{3}{5} = \frac{6}{x} \qquad 5x \cdot \frac{3}{5} = 5x \cdot \frac{6}{x}$$

$$3x = 30 \qquad x = 10$$

The missing length is 10 units.

Figure 24





Proof of the Pythagorean Theorem