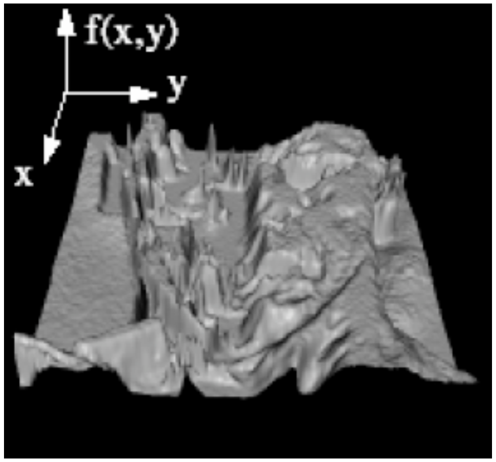
What is an image?

- Ideally, we think of an **image** as a 2-dimensional light intensity function, $f(x,y)$, where x and y are spatial coordinates, and f at (x,y) is related to the brightness or color of the image at that point.
- In practice, most images are defined over a rectangle.
- Continous in amplitude („continous-tone“)
- Continous in space: no pixels!



Digital Images and Pixels

- A **digital image** is the representation of a continuous image $f(x,y)$ by a 2-d array of discrete samples. The amplitude of each sample is quantized to be represented by a finite number of bits.
- Each element of the 2-d array of samples is called a **pixel** or **pel** (from „picture element“)
- Think of pixels as point samples, without extent.

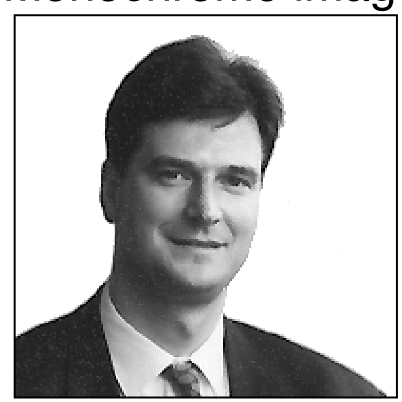
Image Resolution

200x200 100x100 50x50 25x25

- These images were produced by simply picking every n-th sample horizontally and vertically and replicating that value nxn times.
- We can do better
  - *prefiltering before subsampling to avoid aliasing*
  - *Smooth interpolation*

Color Components

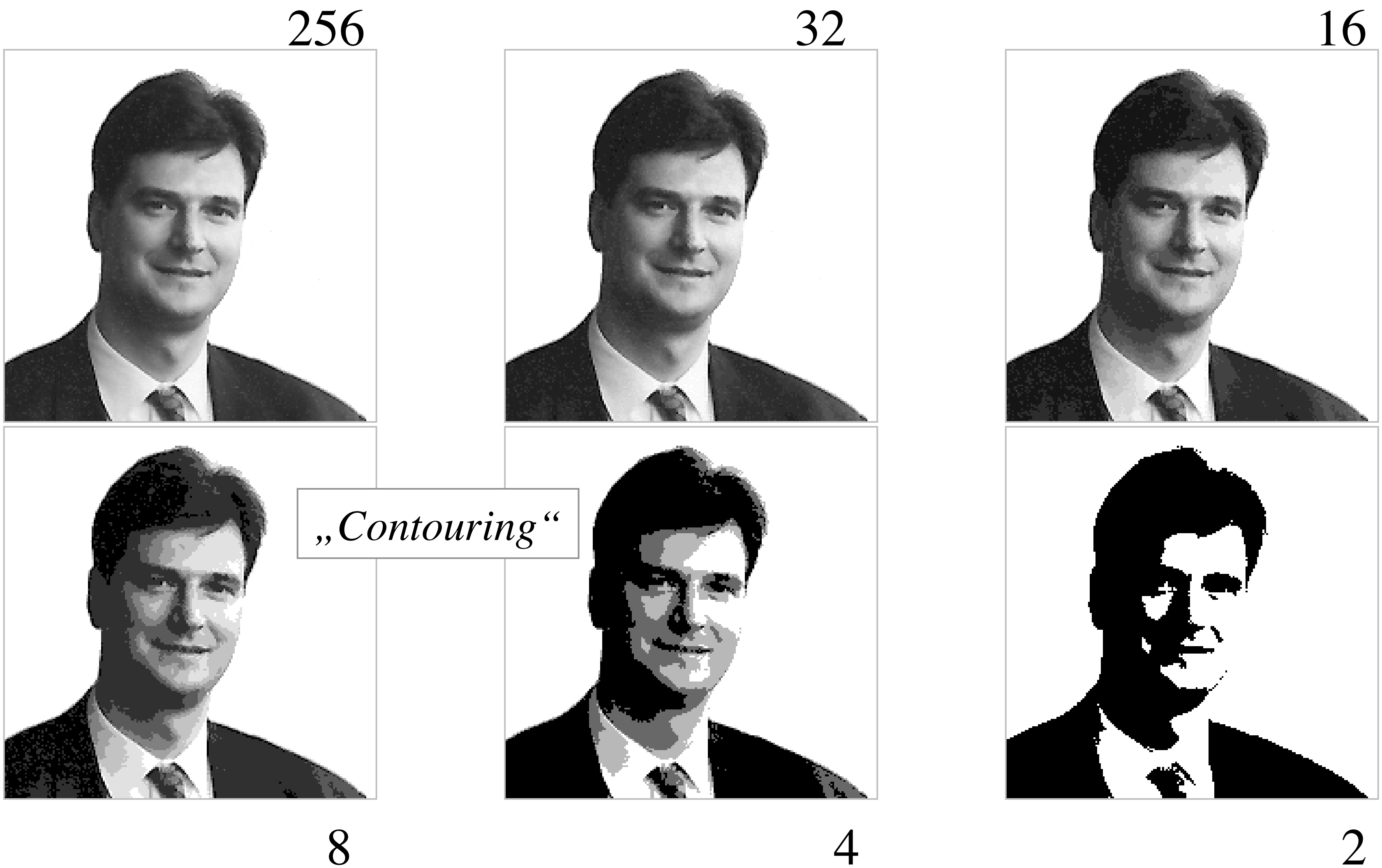
Monochrome image



$R(x,y) = G(x,y) = B(x,y)$

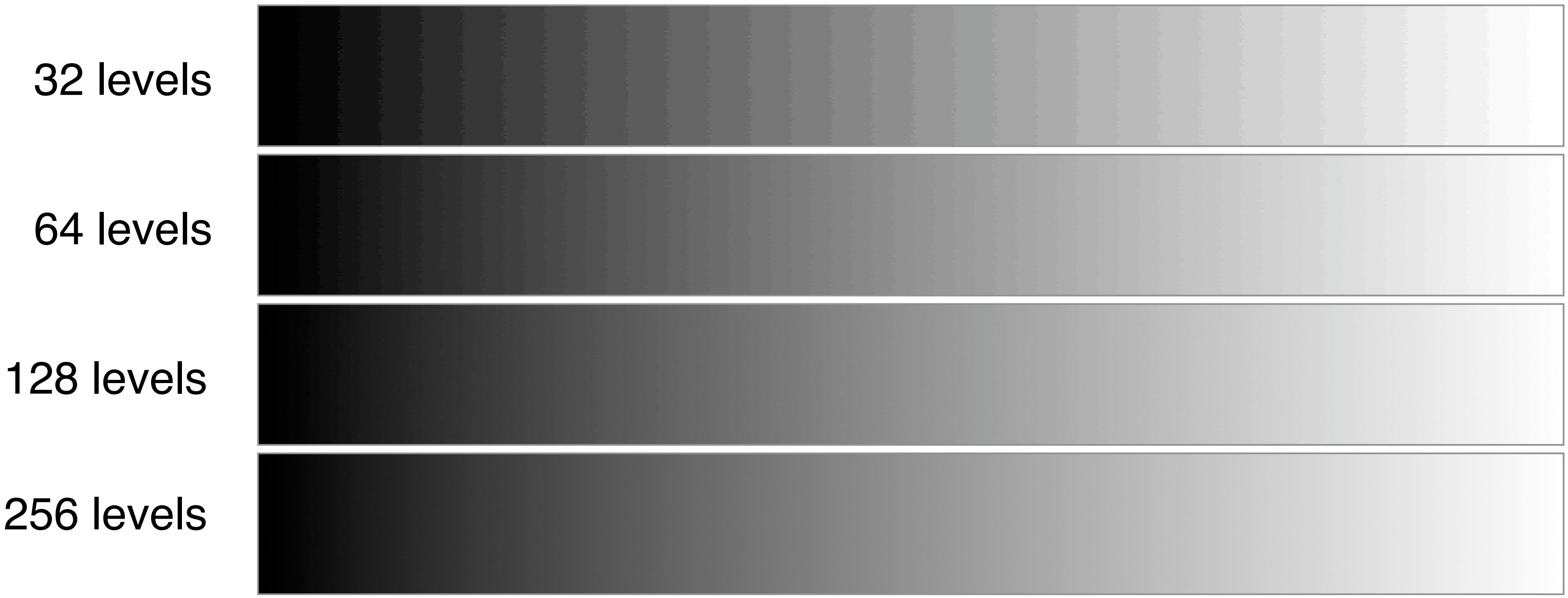
Red $R(x,y)$ Green $G(x,y)$ Blue $B(x,y)$

Different numbers of gray levels



How many gray levels are required?

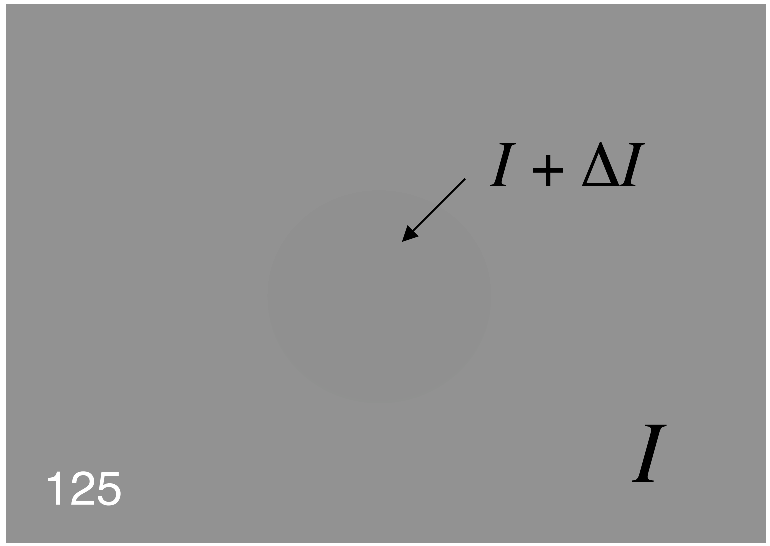
- Contouring is most visible for a ramp



- Digital images typically are quantized to 256 gray levels.

Brightness discrimination experiment

- Can you see the circle?



Note: I is luminance, measured in cd/m^2

- Visibility threshold

$$\boxed{\Delta I / I \approx K_{Weber} \approx 1 \ldots 2\%}$$

„Weber fraction“
„Weber‘s Law“

Contrast with 8 Bits According to Weber‘s Law

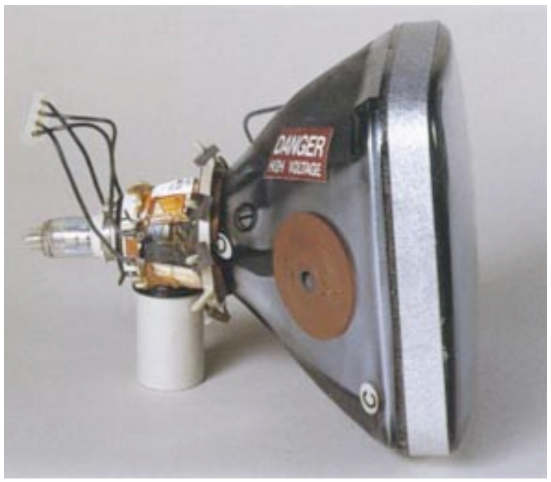
- Assume that the luminance difference between two successive representative levels is just at visibility threshold

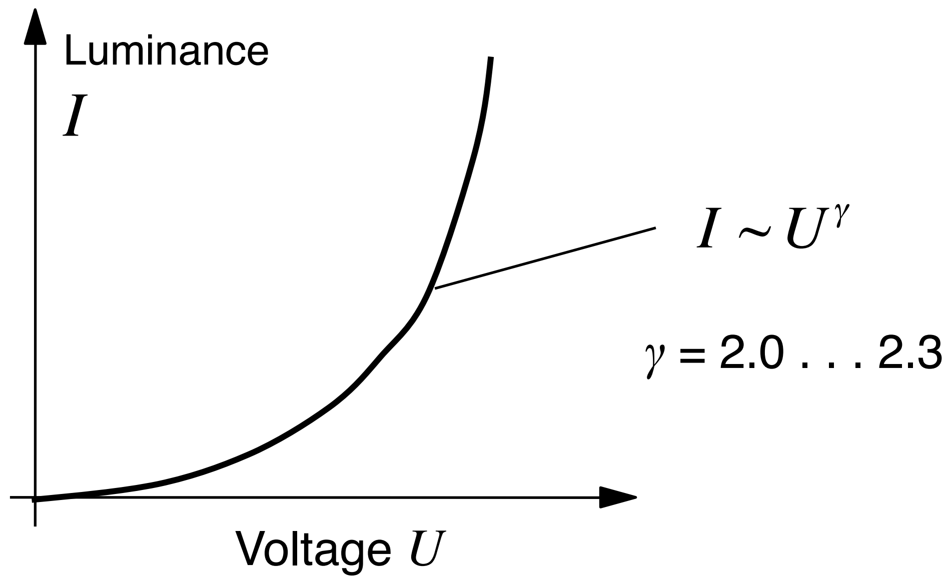
$$\boxed{\frac{I_{\max}}{I_{\min}} = \left(1 + K_{Weber}\right)^{255}}$$

- For $K_{Weber} = 0.01 \cdots 0.02$ $\qquad \dfrac{I_{\max}}{I_{\min}} = 13 \cdots 156$
- Typical display contrast
  - Cathode ray tube 100:1
  - Print on paper 10:1
- Suggests uniform perception in the $\log(I)$ domain („Fechner‘s Law“)

Gamma characteristic

- Cathode ray tubes (CRT) are nonlinear

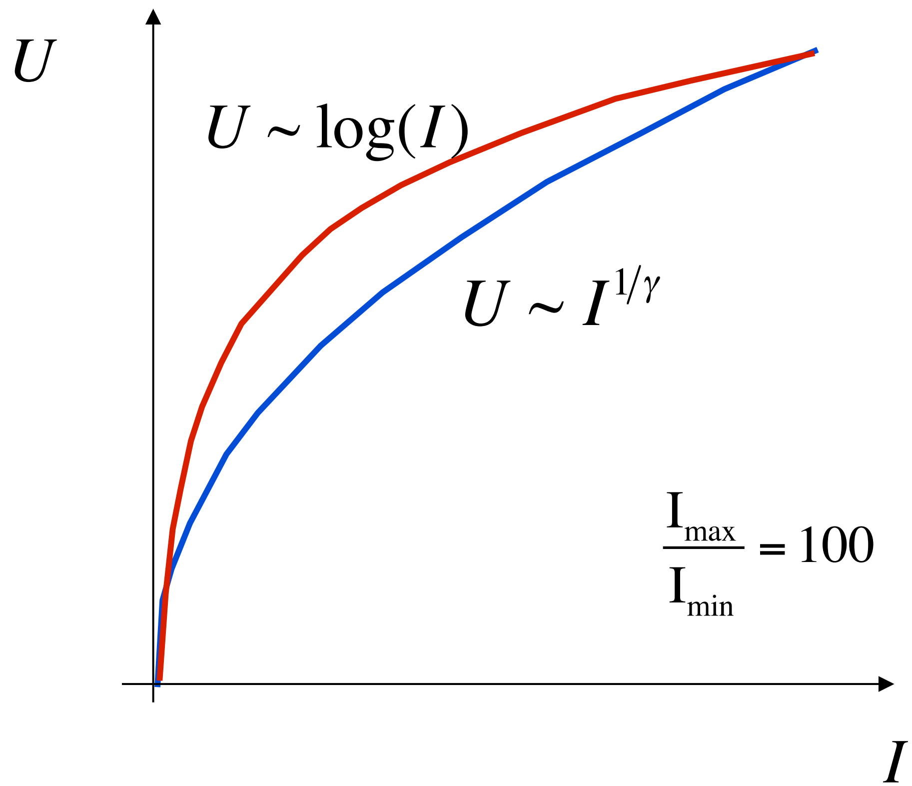




- Cameras contain γ-predistortion circuit

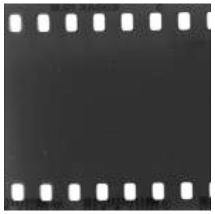
$$U \sim I^{1/\gamma}$$

log vs. γ-predistortion

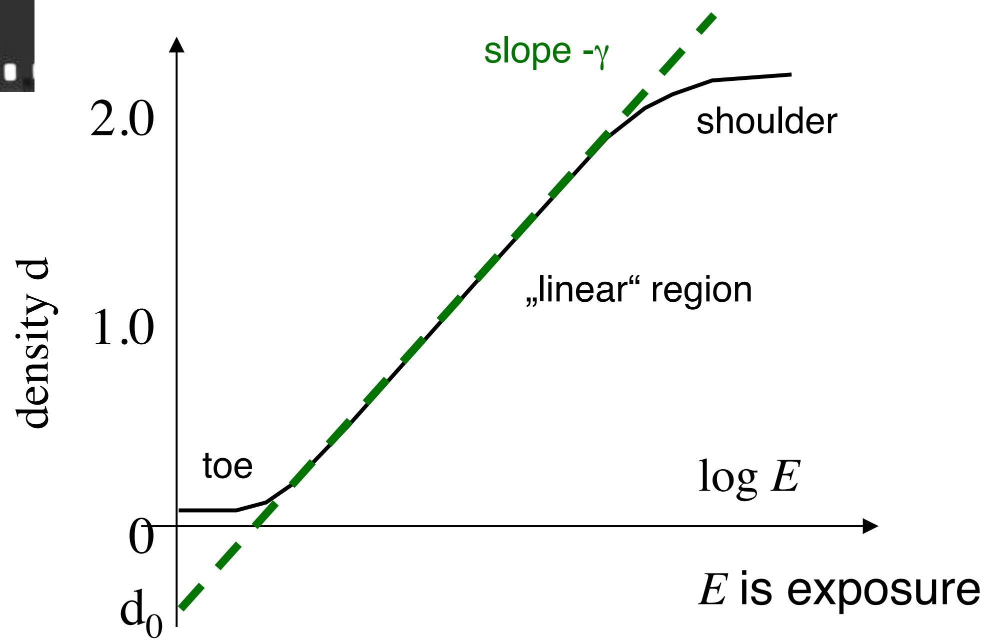


- Similar enough for most practical applications

Photographic film



Hurter & Driffield curve (H&D curve)
for photographic negative



Luminance

$$I = I_0 \cdot 10^{-d}$$
$$= I_0 \cdot 10^{-\left(-\gamma \log E + d_0\right)}$$
$$= I_0 \cdot 10^{-d_0} \cdot E^{\gamma}$$

- γ measures film contrast
  - General purpose films: γ = -0.7 . . . -1.0
  - High-contrast films: γ = -1.5 . . . -10
- Lower speed films tend to have higher absolute γ

Intensity Scaling

Original image

$f(x,y)$

Scaled image

$a \cdot f(x,y)$

Scaling in the γ-domain is equivalent to scaling in the linear luminance domain

$$I \sim \left(a \cdot f(x,y)\right)^{\gamma} = a^{\gamma} \cdot \left(f(x,y)\right)^{\gamma}$$

. . . same effect as adjusting camera exposure time.

Adjusting γ

Original image

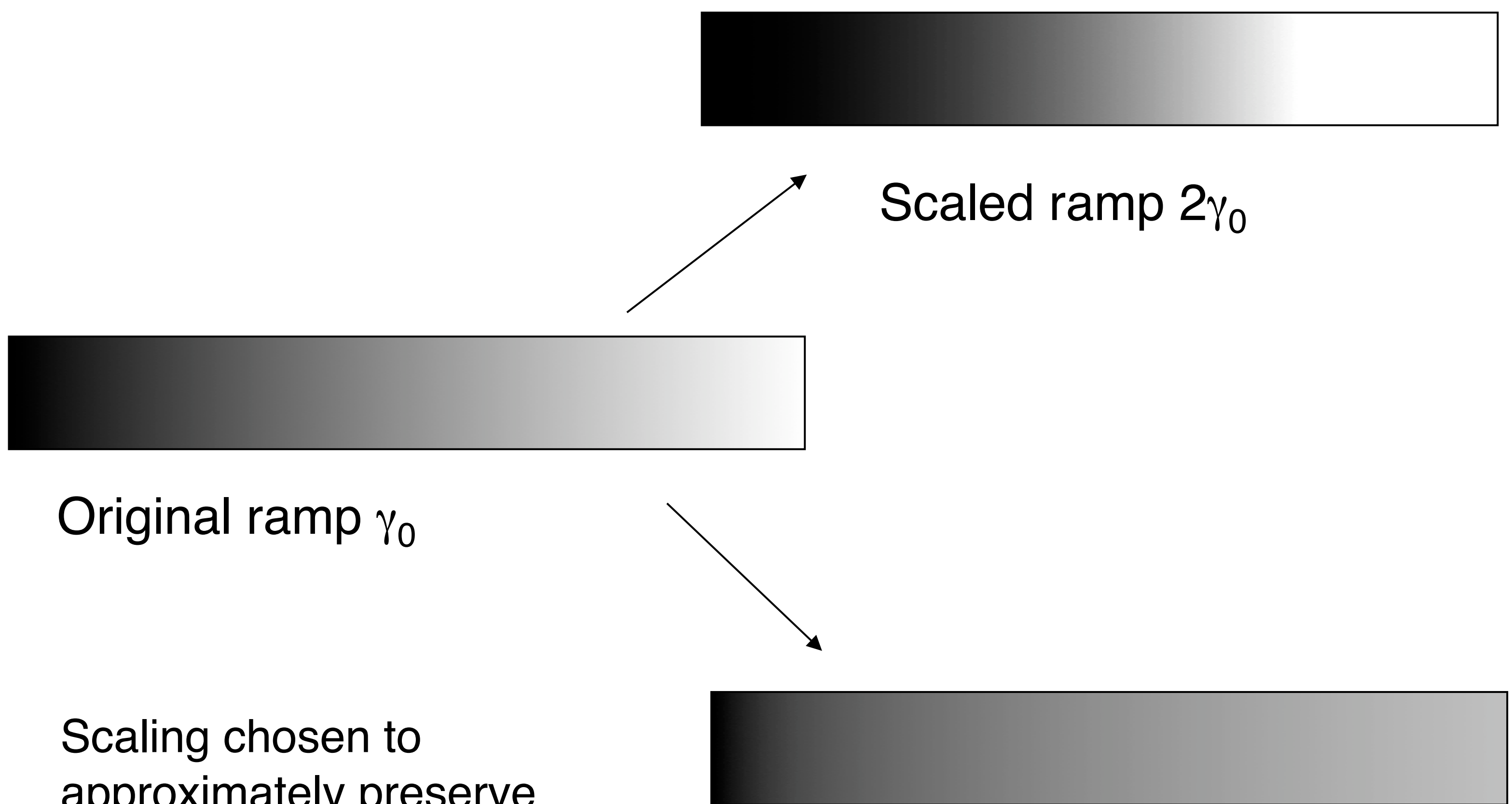
$f(x,y)$

γ increased by 50%

$a \cdot \left(f(x,y) \right)^{\gamma}$ with $\gamma = 1.5$

. . . same effect as using a different photographic film . . .

Changing gradation by γ-adjustment



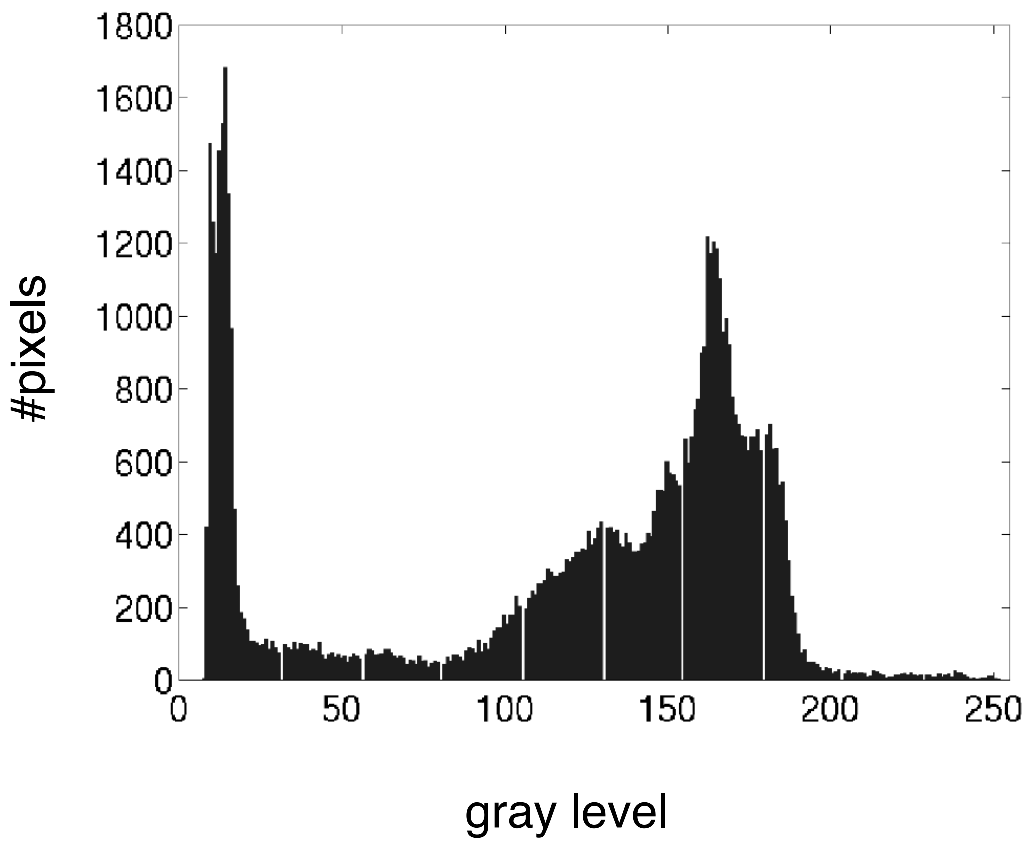
Scaling chosen to approximately preserve brightness of mid-gray

Scaled ramp $0.5\gamma_0$

Histograms

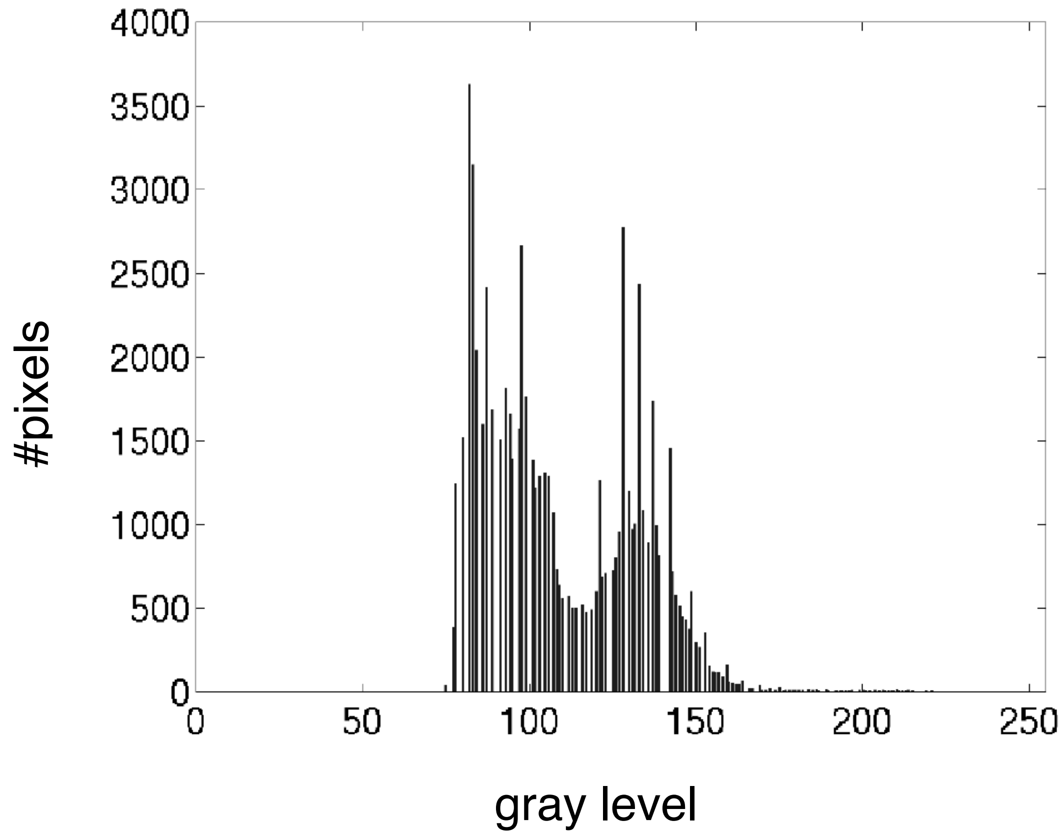
- Distribution of gray-levels can be judged by measuring a histogram:
  - For B-bit image, initialize 2^B counters with 0
  - Loop over all pixels x,y
  - When encountering gray level $f(x,y)=i$, increment counter $\#\iota$
- Histogram can be interpreted as an estimate of the probability density function (pdf) of an underlying random process.
- You can also use fewer, larger bins to trade off amplitude resolution against sample size.

Example histogram



Cameraman
image

Example histogram



Pout
image

Histogram equalization

- Idea: find a non-linear transformation

$$g = T\left(f\right)$$

  to be applied to each pixel of the input image $f(x,y)$, such that a uniform distribution of gray levels in the entire range results for the output image $g(x,y)$.
- Analyse ideal, continuous case first, assuming
  - $0 \le f \le 1 \qquad 0 \le g \le 1$
  - $T(f)$ is strictly monotonically increasing, hence, there exists

$$f = T^{-1}\left(g\right) \qquad 0 \le g \le 1$$

  - Goal: pdf $p_g(g) = const.$ over the range

Histogram equalization for continuous case

- From basic probability theory

$$p_g(g) = \left[p_f(f) \frac{df}{dg} \right]_{f=T^{-1}(g)}$$

- Consider the transformation function

$$g = T(f) = \int_0^f p_f(\alpha) d\alpha \qquad 0 \le f \le 1$$

- Then . . .

$$\frac{dg}{df} = p_f(f)$$

$$p_g(g) = \left[p_f(f) \frac{df}{dg} \right]_{f=T^{-1}(g)} = \left[p_f(f) \frac{1}{p_f(f)} \right]_{f=T^{-1}(g)} = 1 \qquad 0 \le g \le 1$$

Histogram equalization for discrete case

- Now, f only assumes discrete amplitude values $f_0, f_1, \cdots, f_{L-1}$ with „probabilities“

$$P_0 = \frac{n_0}{n} \qquad P_1 = \frac{n_1}{n} \qquad \cdots \qquad P_{L-1} = \frac{n_{L-1}}{n}$$

- Discrete approximation of $g = T\left(f\right) = \int_0^f p_f\left(\alpha\right)\, d\alpha$

$$g_k = T\left(f_k\right) = \sum_{i=0}^{k} P_i$$

- The resulting values g_k are in the range $[0,1]$ and need to be scaled and rounded appropriately.

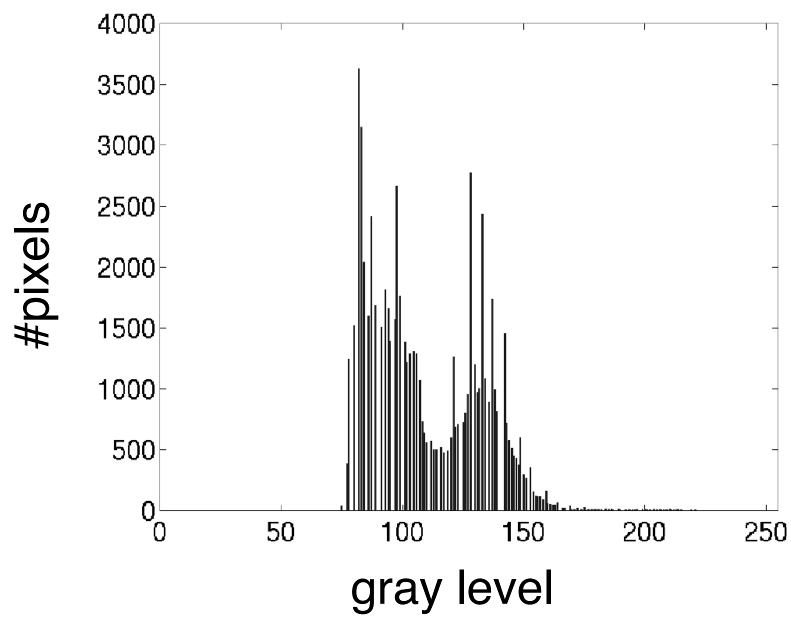
Histogram equalization example

Original image *Pout*

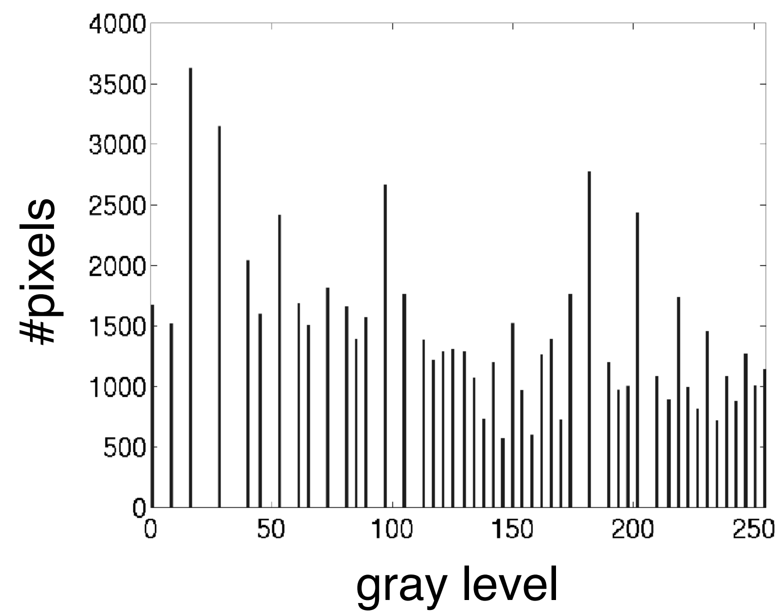
Pout
after histogram equalization

Histogram equalization example

Original image *Pout*



. . . after histogram equalization



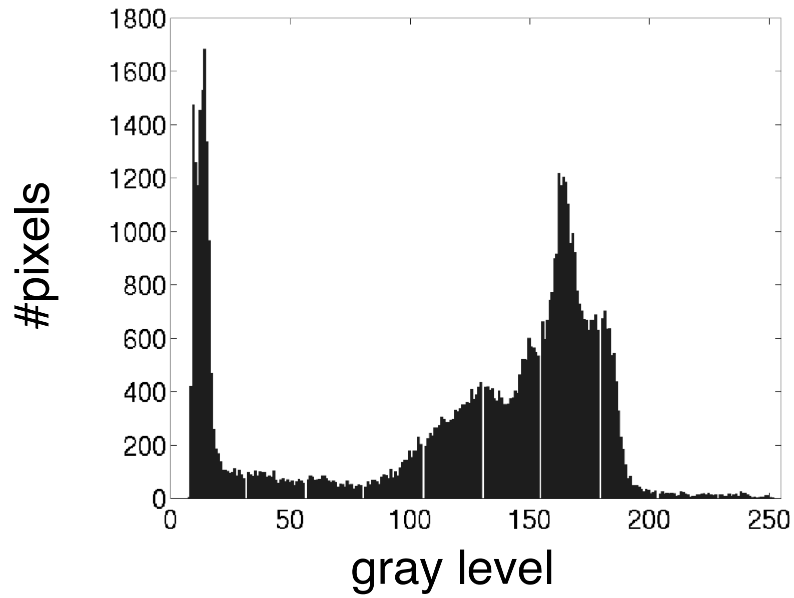
Histogram equalization example

Original image
Cameraman

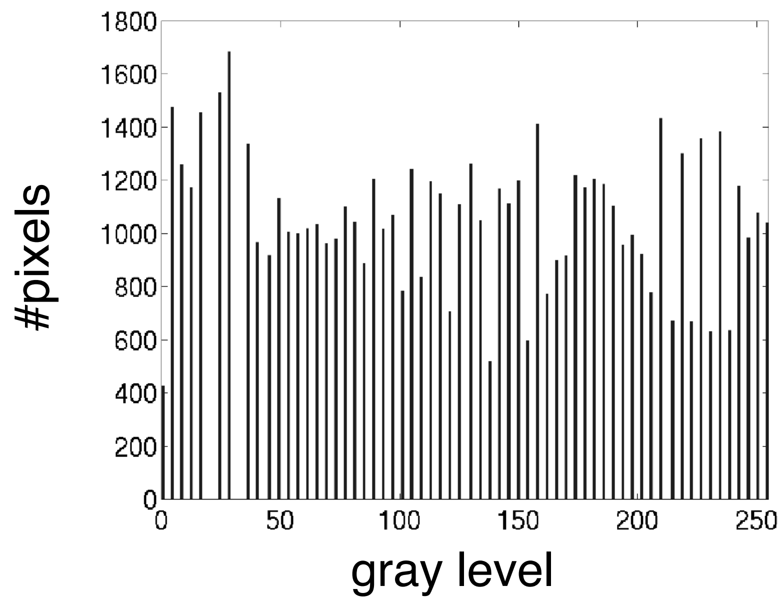
Cameraman
after histogram equalization

Histogram equalization example

Original image *Cameraman*



. . . after histogram equalization



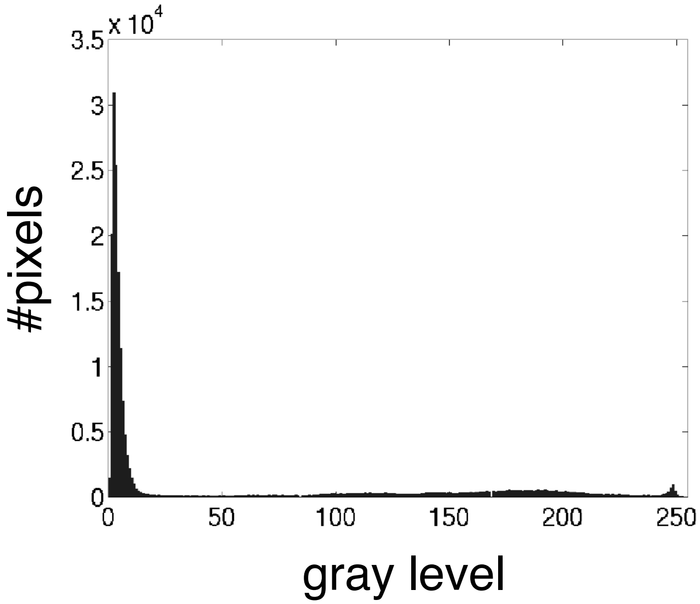
Histogram equalization example

Original image *Moon*

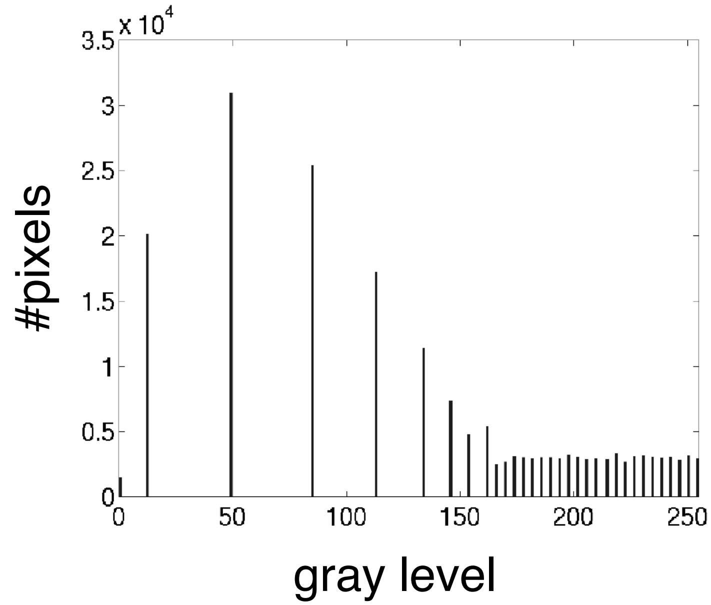
Moon
after histogram equalization

Histogram equalization example

Original image *Moon*



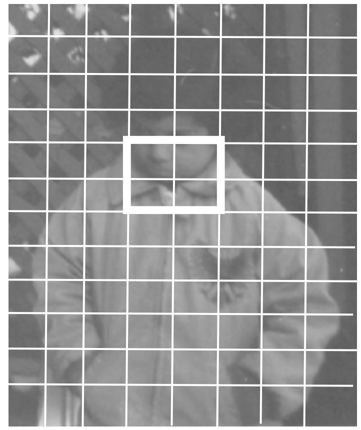
. . . after histogram equalization



Adaptive Histogram Equalization

- Apply histogram equalization based on a histogram obtained from a portion of the image

Sliding window approach:
different histogram (and mapping) for every pixel



Tiling approach:
subdivide into overlapping regions, mitigate blocking effect by smooth blending between neighboring tiles

- Must limit contrast expansion in flat regions of the image, e.g. by clipping individual histogram values to a maximum

Adaptive Histogram Equalization




Original

Global histogram

Tiling
8x8 histograms

Tiling
32x32 histograms

Adaptive histogram equalization

Original image *Tire*

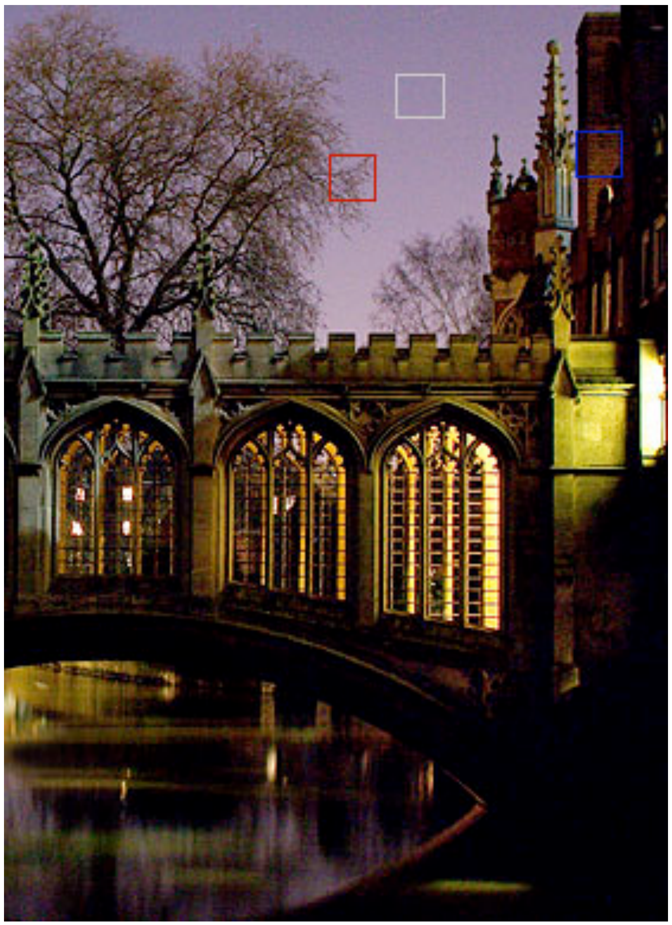
Tire after
equalization of
global histogram

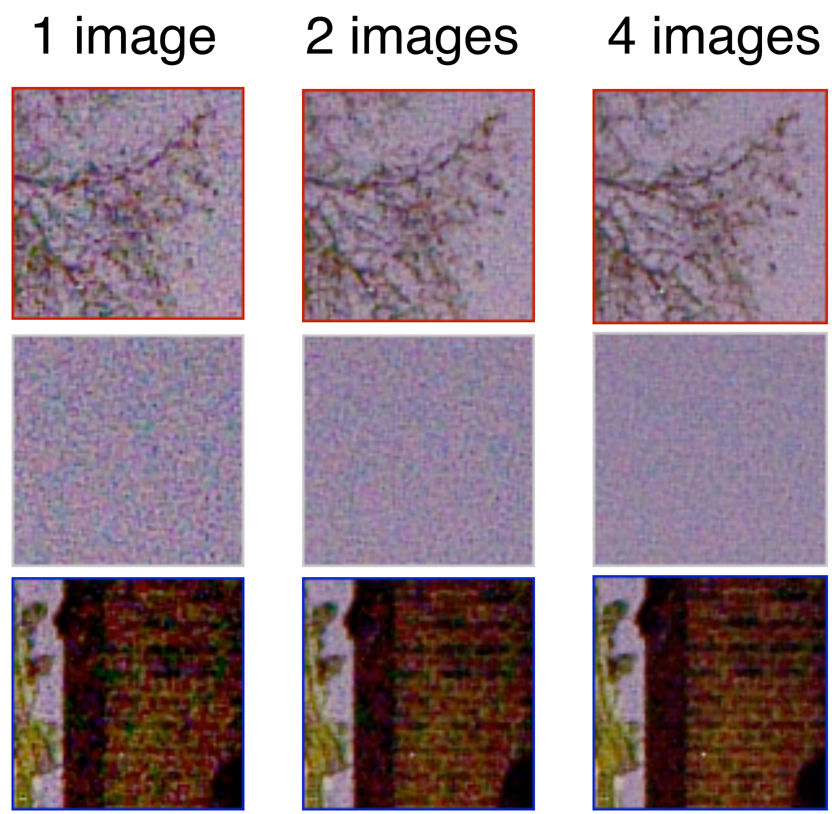
Tire after
adaptive histogram equalization
8x8 tiles

Point Operations Combining Images

- Image averaging for noise reduction
- Combination of different exposure for high-dynamic range imaging
- Image subtraction for change detection
- Accurate alignment is always a requirement

Image averaging for noise reduction





http://www.cambridgeincolour.com/tutorials/noise-reduction.htm

Image averaging for noise reduction

- Take N aligned images $f_1(x,y), f_2(x,y), \cdots, f_N(x,y)$
- Average image: $\overline{f(x,y)} = \frac{1}{N}\sum_{i=1}^{N} f_i(x,y)$
- Mean squared error vs. noise-free image g

$$E\left\{\left(\overline{f}-g\right)^2\right\} = E\left\{\left(\left(\frac{1}{N}\sum_i f_i\right)-g\right)^2\right\} = E\left\{\left(\left(\frac{1}{N}\sum_i \left(g+n_i\right)\right)-g\right)^2\right\}$$

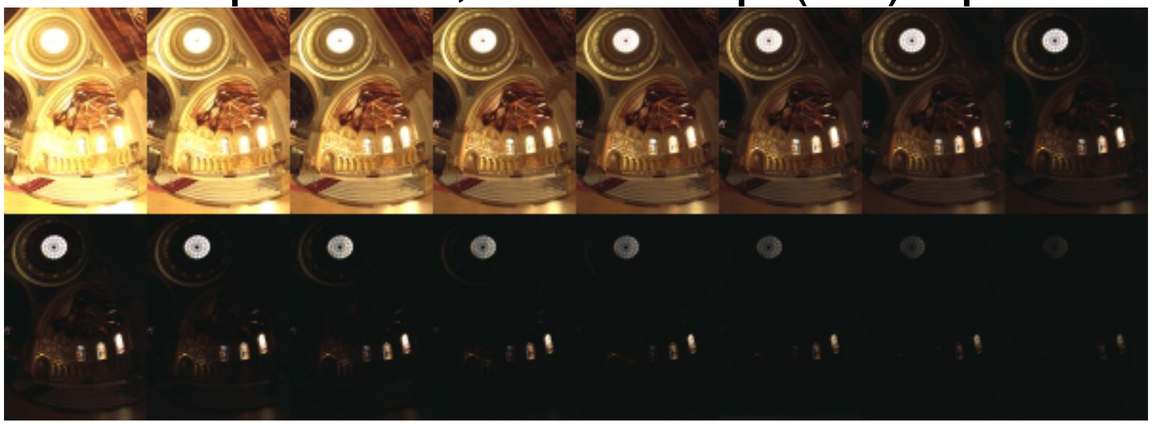
$$= E\left\{\left(\frac{1}{N}\sum_i n_i\right)^2\right\} = \frac{1}{N^2}\sum_i E\left\{n_i^2\right\} = \frac{1}{N}E\left\{n^2\right\}$$

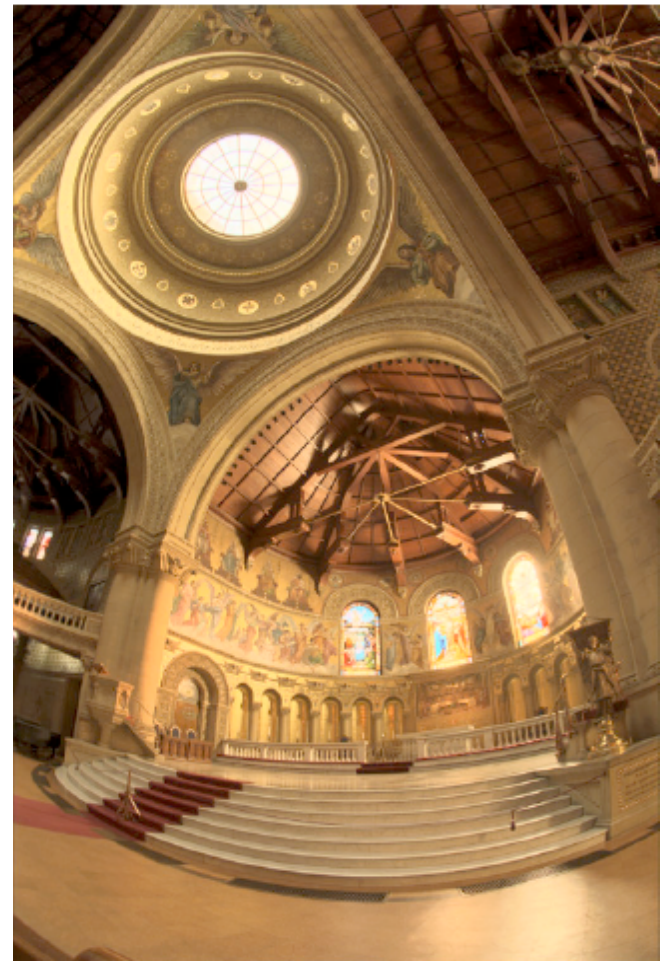
provided $E\left\{n_i n_j\right\} = 0 \; \forall i,j$

$E\left\{n_i\right\} = E\left\{n\right\} \forall i$

High-dynamic range imaging

16 exposures, one f-stop (2X) apart



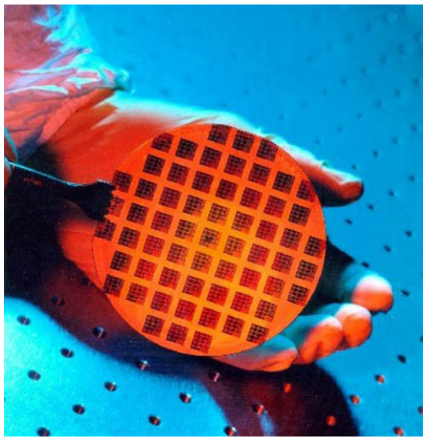


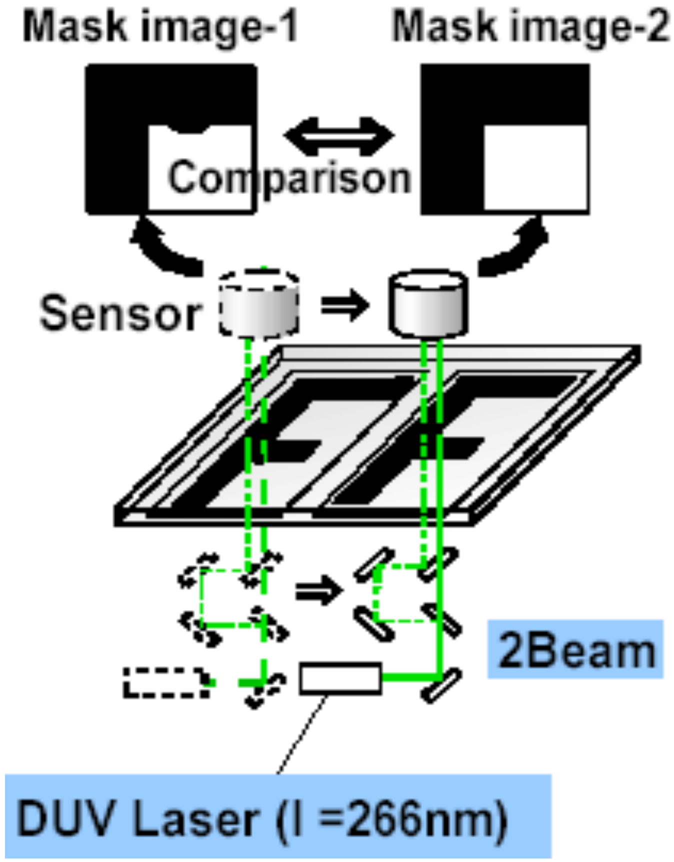
Combined image

[Debevec, Malik, 1997]

Image subtraction

- Find differences/changes between 2 mostly identical images
- Example from IC manufacturing: defect detection in photomasks by die-to-die comparison





Where is the Defect?

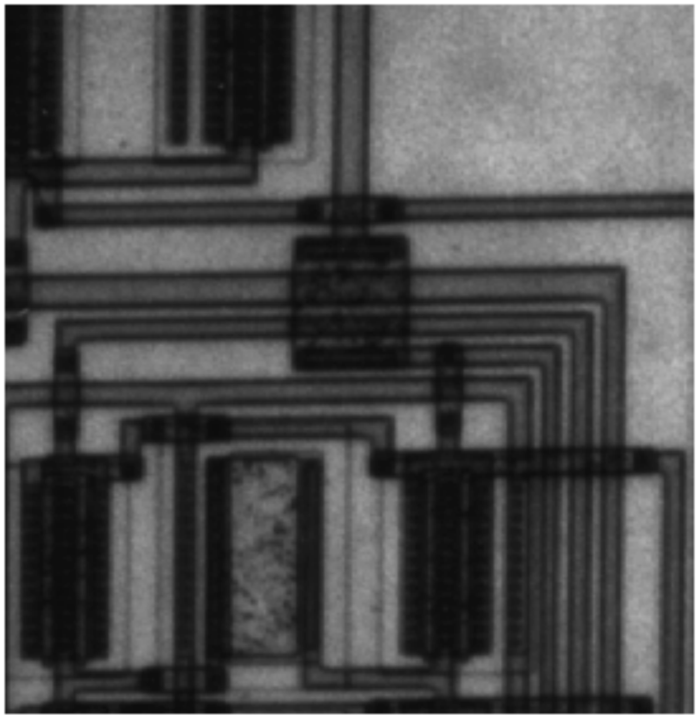


Image A (no defect)

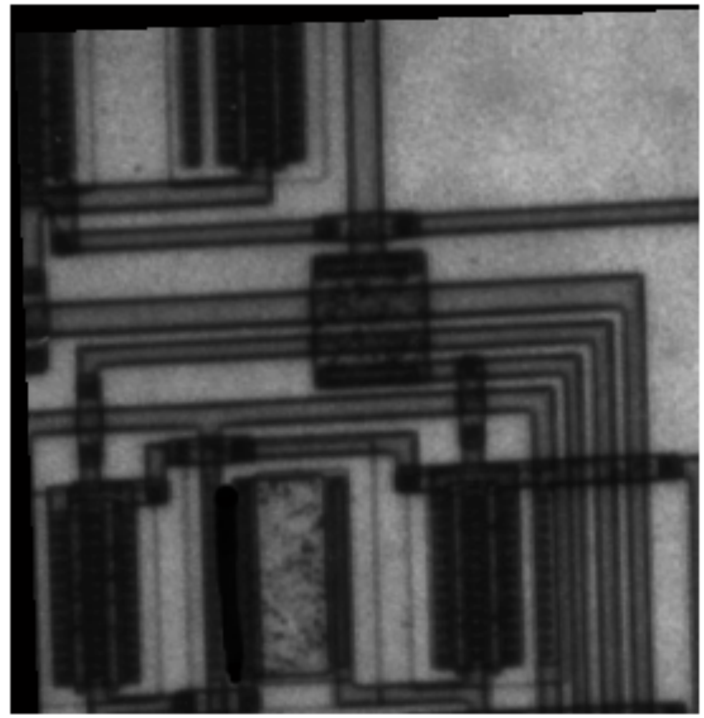
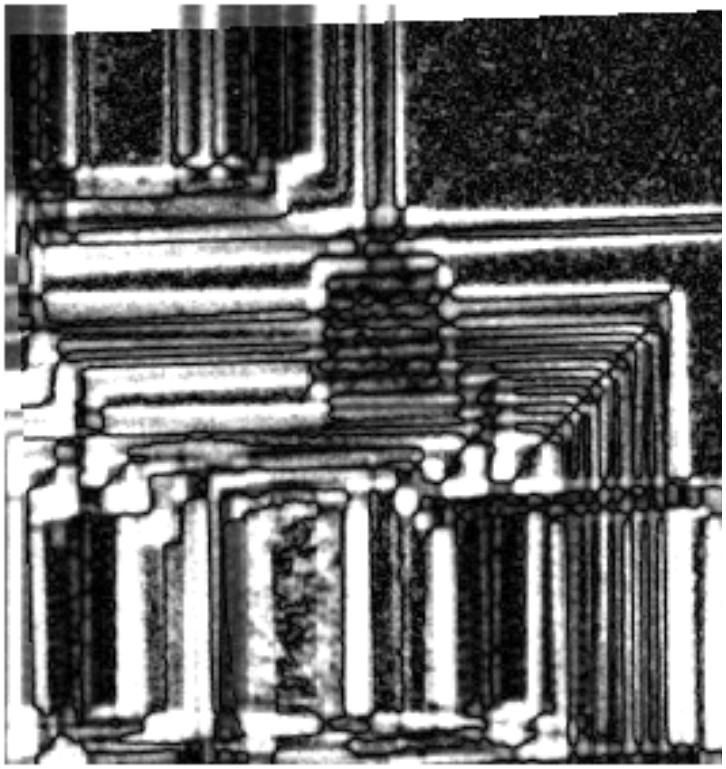
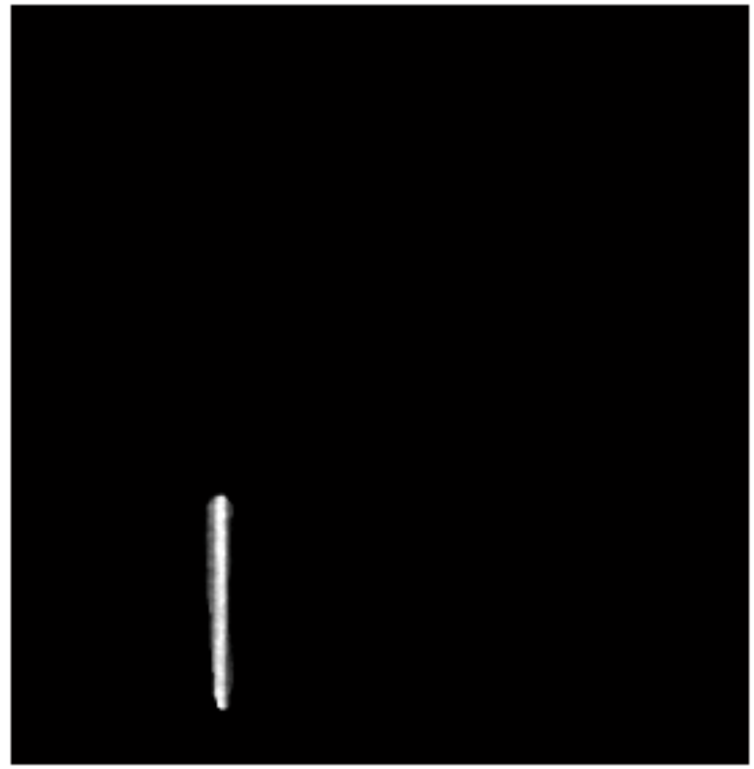


Image B (w/ defect)

Absolute Difference Between Two Images

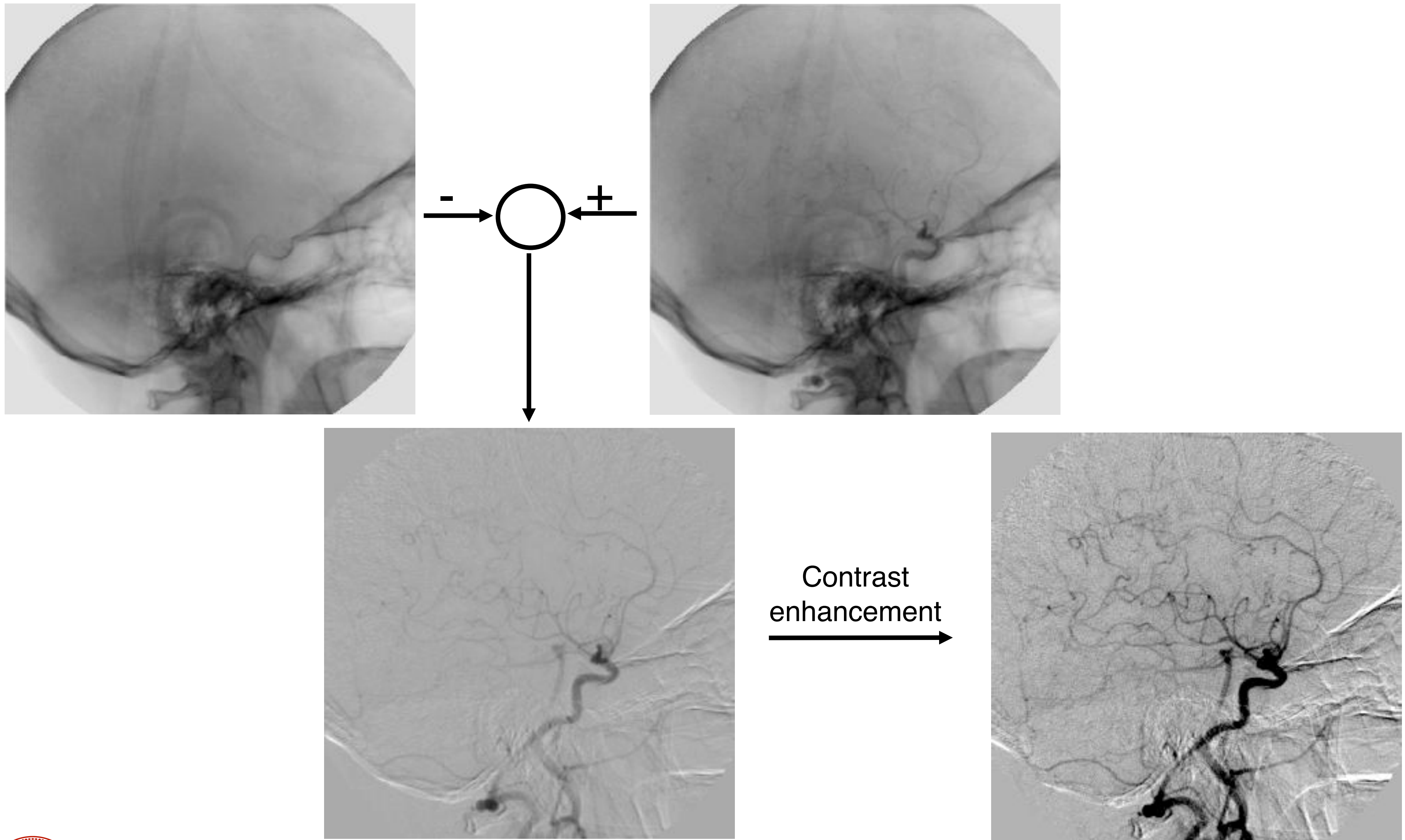


w/o alignment



w/ alignment

Digital Subtraction Angiography



http://www.isi.uu.nl/Research/Gallery/DSA/