Linear Filters

CS 554 – Computer Vision
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Image Scalin

This image is too big to fit on the screen. How can we reduce it?

How to generate a half-sized version?

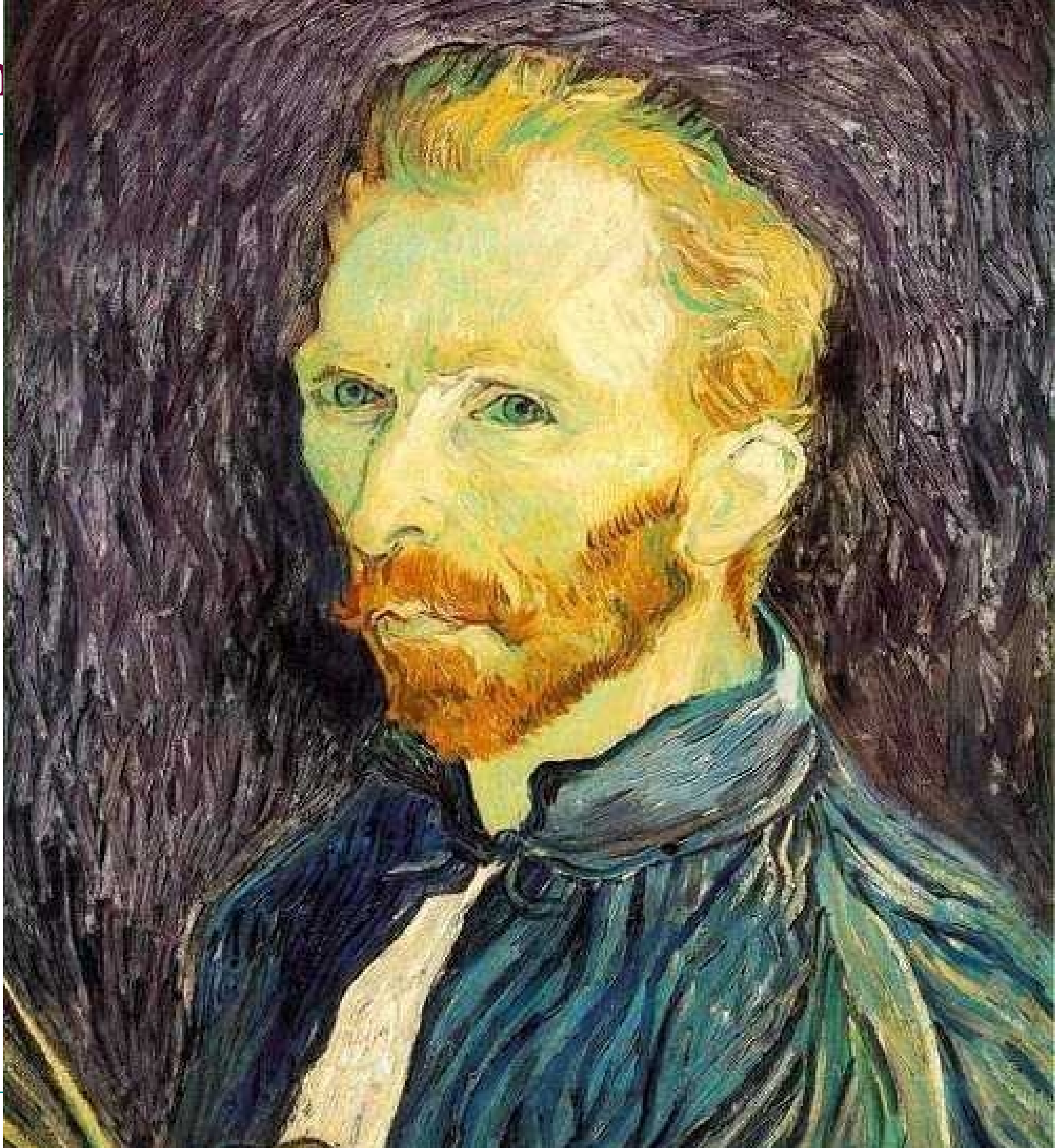
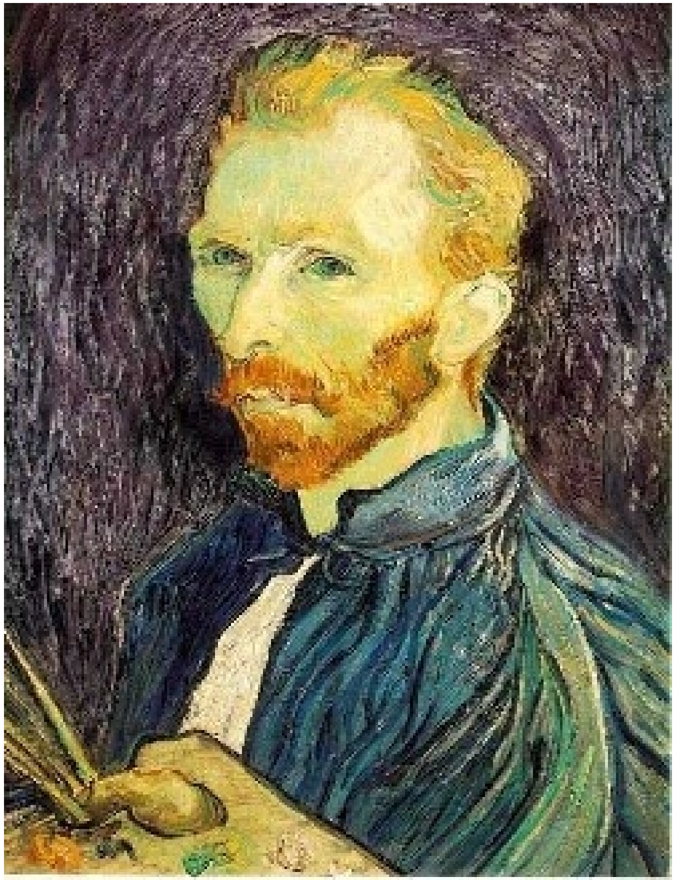
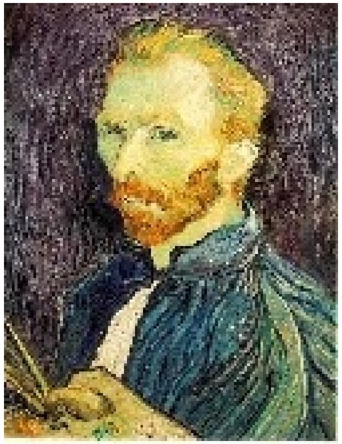


Image sub-sampling



Throw away every other row and column to create a 1/2 size image

- called *image sub-sampling*



1/4

1/8

Slide by Steve Seitz

Image sub-sampling

1/2 1/4 (2x zoom) 1/8 (4x zoom)

Why does this look so crufty?

Slide by Steve Seitz

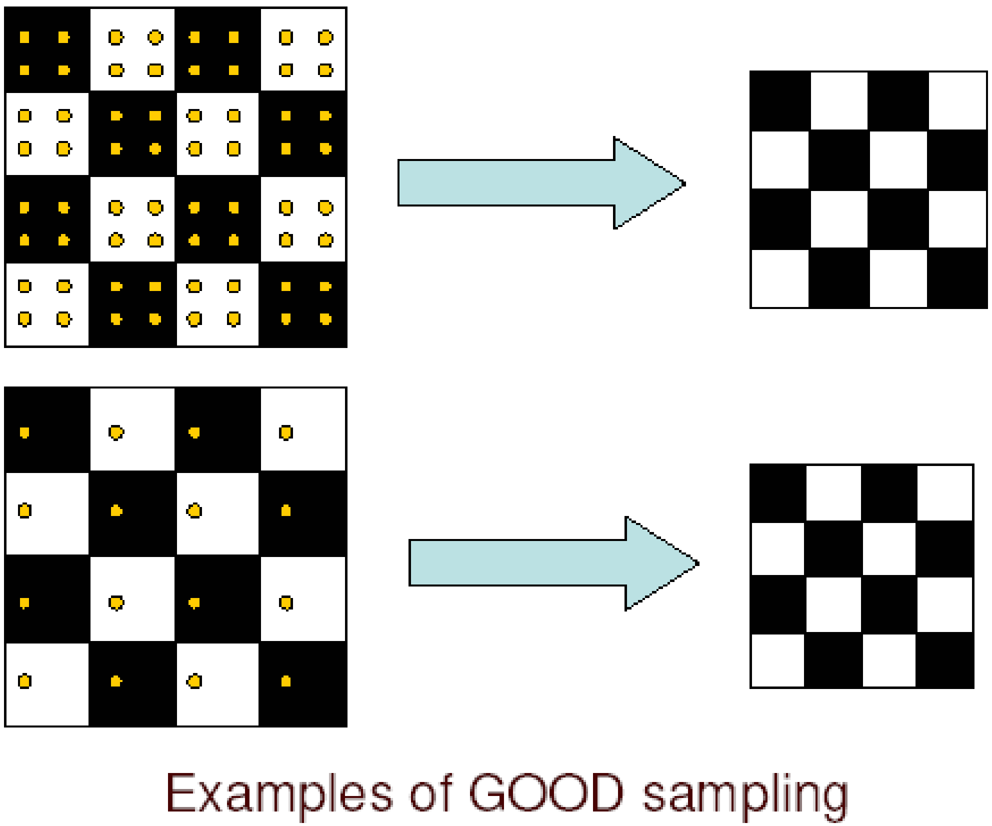
Even worse for synthetic images

Slide by Steve Seitz

Spatial frequency and Fourier transforms

- Problems
  - A discrete image cannot represent the full information
  - We cannot shrink an image by simply taking every kth pixel

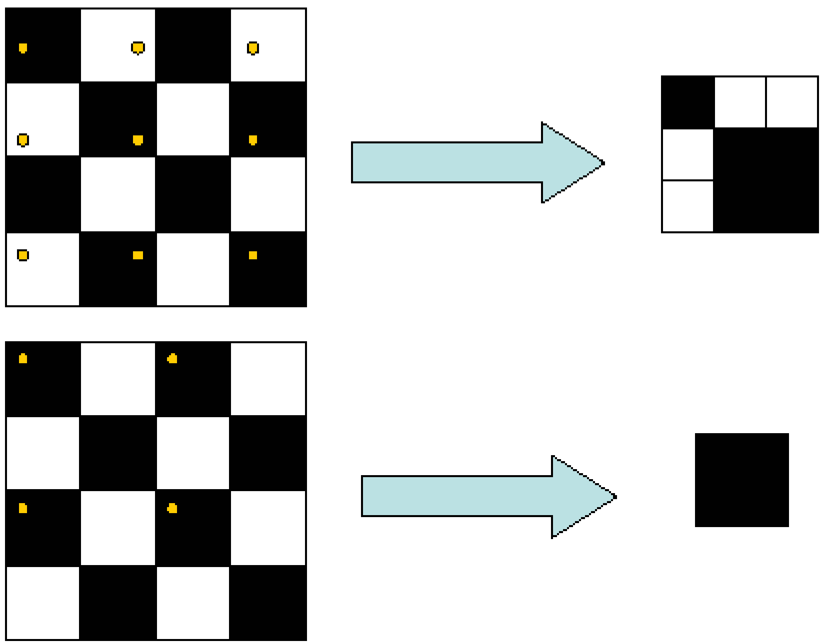
Sampling and Aliasing



Resample the checkerboard by taking one sample at each circle. In the case of the top, new representation is reasonable.
Bottom also yields a reasonable representation.

Sampling and Aliasing

Undersampling



Examples of BAD sampling -> Aliasing

Bottom is all black (dubious) and top has checks that are too big.

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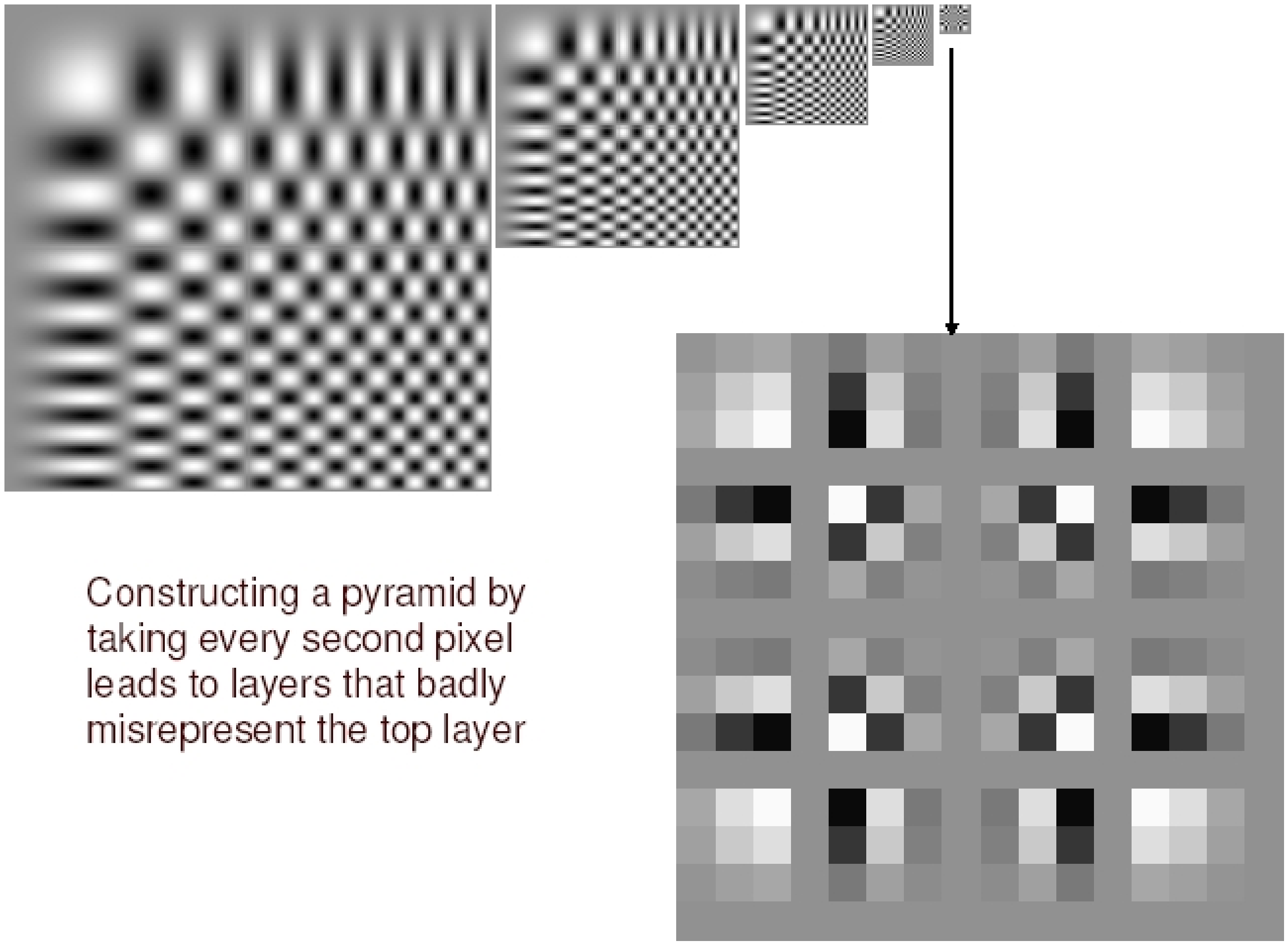
Sampling



Constructing a pyramid by taking every second pixel leads to layers that badly misrepresent the top layer

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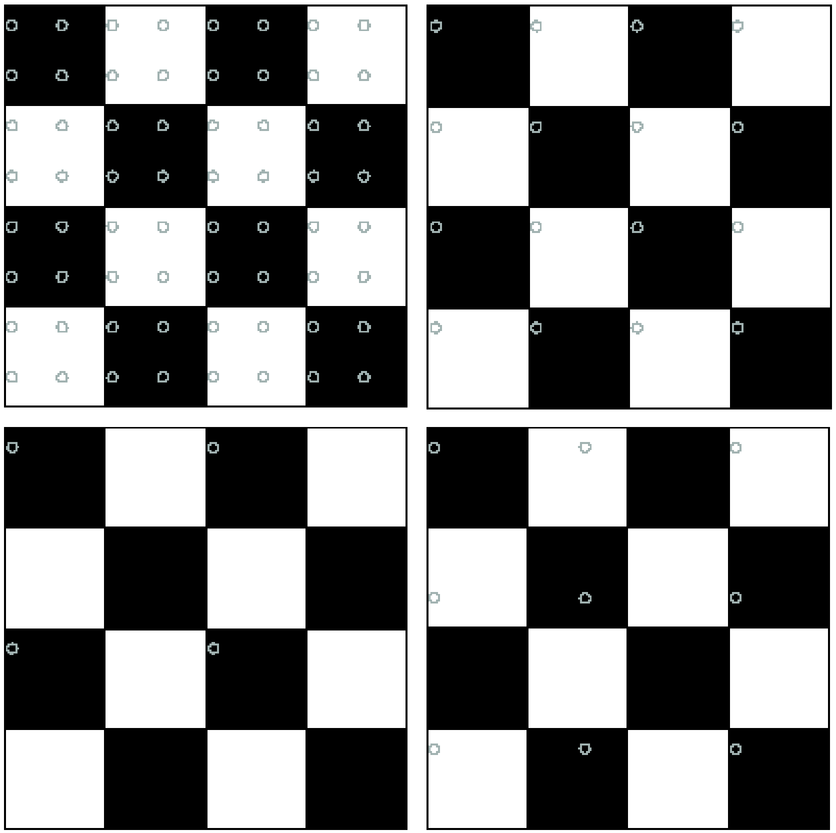
Aliasing

- Can't shrink an image by taking every second pixel
- If we do, characteristic errors appear
  - Common phenomenon
    - High spatial frequency components of the image appear as low spatial frequency components
  - Examples
    - Wagon wheels rolling the wrong way in movies
    - Checkerboards misrepresented in ray tracing
    - Striped shirts look funny on color television

adapted from David Forsyth, UC Berkeley

Problems



Difference between discrete and continuous image

Aliasing is related to passing from one to the other

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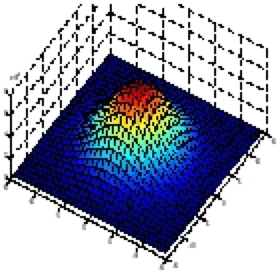
Problems

What content is lost from the original image?

Convolution with Gaussian kernel



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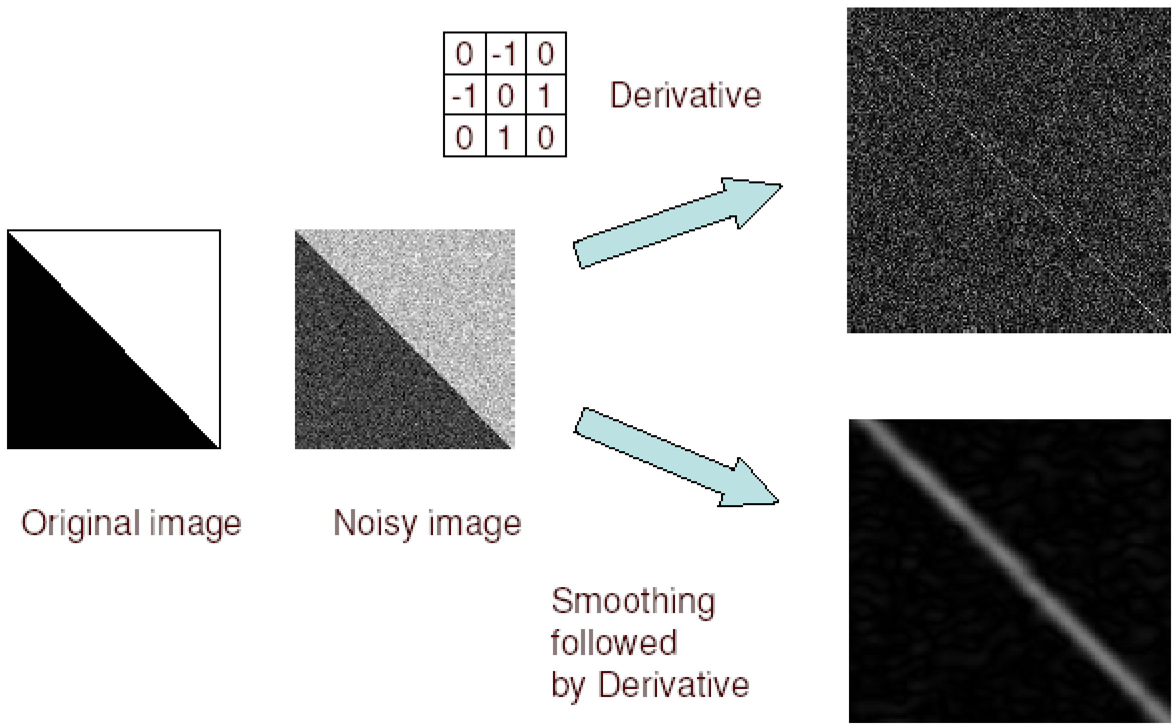
Problems

Differentiation emphasizes noise
Because derivatives are large at fast changes, and noise pixels tend to be different from their neighbors
Smoothing helps



adapted from Martial Hebert, CMU

Problems

- What causes the differentiation to emphasize noise, and Gaussian convolution to remove fine features
- How do we get discrete images differ from continuous images?
- How do we avoid aliasing while sampling?

adapted from David Forsyth,UC Berkeley

Solution

A continuous signal can be represented as weighted sum of basis functions

The simplest basis is a box function

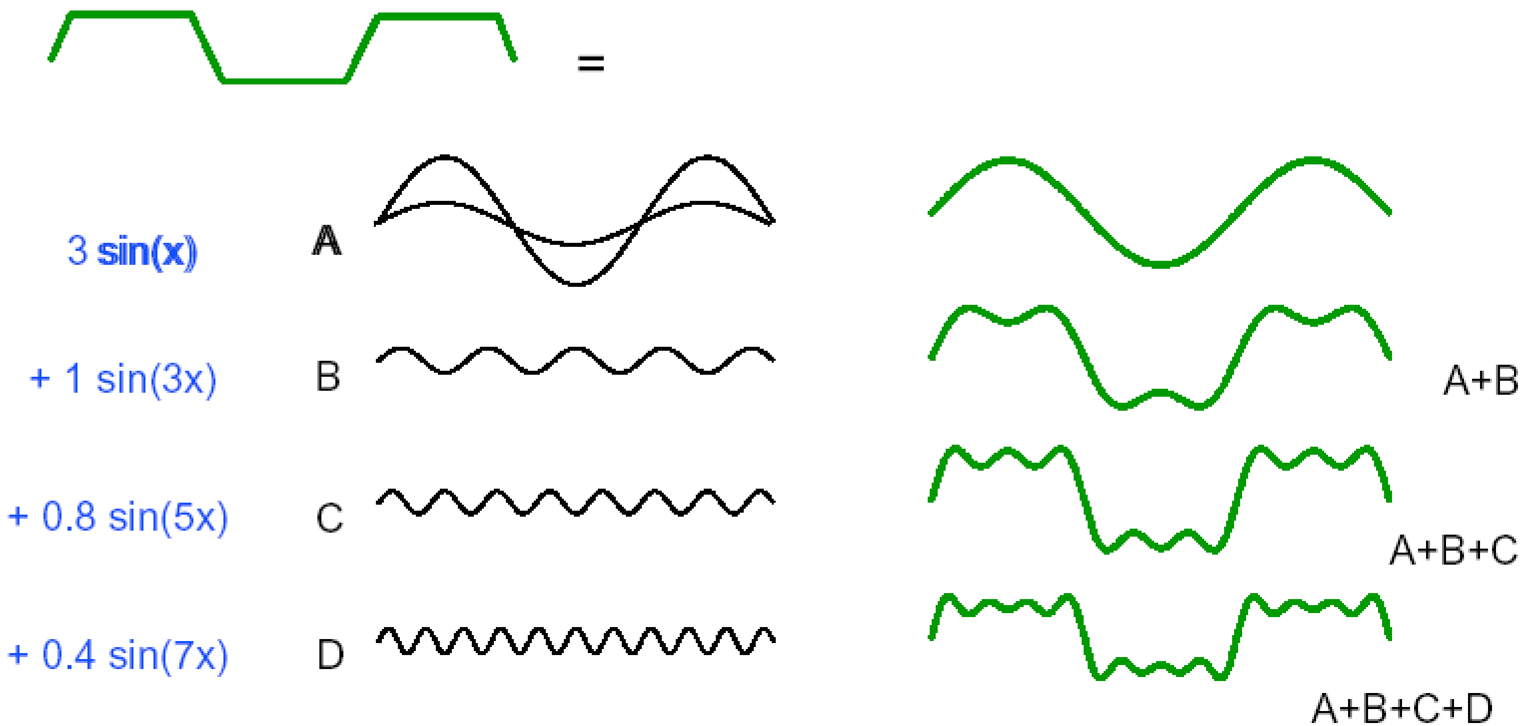
For a better representation, change basis

e.g. set of sinusoids

represent the signal as an infinite weighted sum of infinite number of sinusoids

Jean Baptiste Fourier (1768-1830)

Fourier Transform



adapted from Michael Black, Brown University

Fourier Transform

A representation for image changes

- We need a change of basis to move from pixel intensities (space domain) to frequency domain.

"dot"

= A measure of image content at this frequency and orientation

adapted from Martial Hebert, CMU

Fourier Transform

- Represent function on a new basis
  - Think of functions as vectors, with many components
  - We now apply a linear transformation to transform the basis
    - dot product with each basis element
- In the expression, u and v select the basis element, so a function of x and y becomes a function of u and v
- basis elements have the form $e^{-i2\pi(ux+vy)}$

$$F(g(x,y))(u,v) = \iint_{\mathbb{R}^2} g(x,y)\, e^{-i2\pi(ux+vy)}\, dx\, dy$$

Fourier Transform

$$e^{-i2\pi(ux+vy)}$$

cos(2Pi (ux + vy)) + i sin (2Pi(ux+vy))

Fourier Transform

To get some sense of what basis elements look like, we plot a basis element --- or rather, its real part --- as a function of x,y for some fixed u, v. We get a function that is constant when (ux+vy) is constant. The magnitude of the vector (u, v) gives a frequency, and its direction gives an orientation. The function is a sinusoid with this frequency along the direction, and constant perpendicular to the direction.

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Fourier Transform

Here u and v are larger than in the previous slide.

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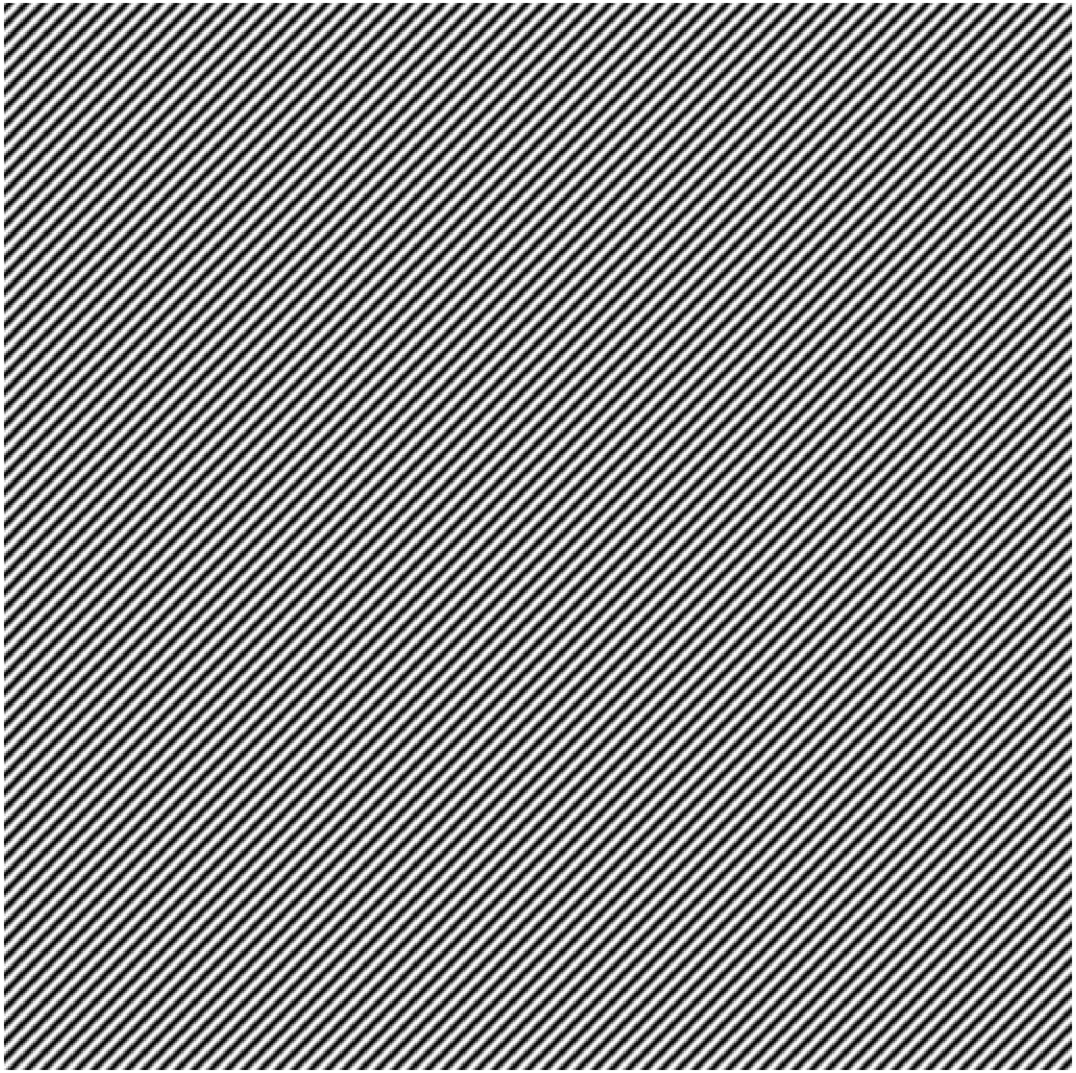
Fourier Transform

And larger still...



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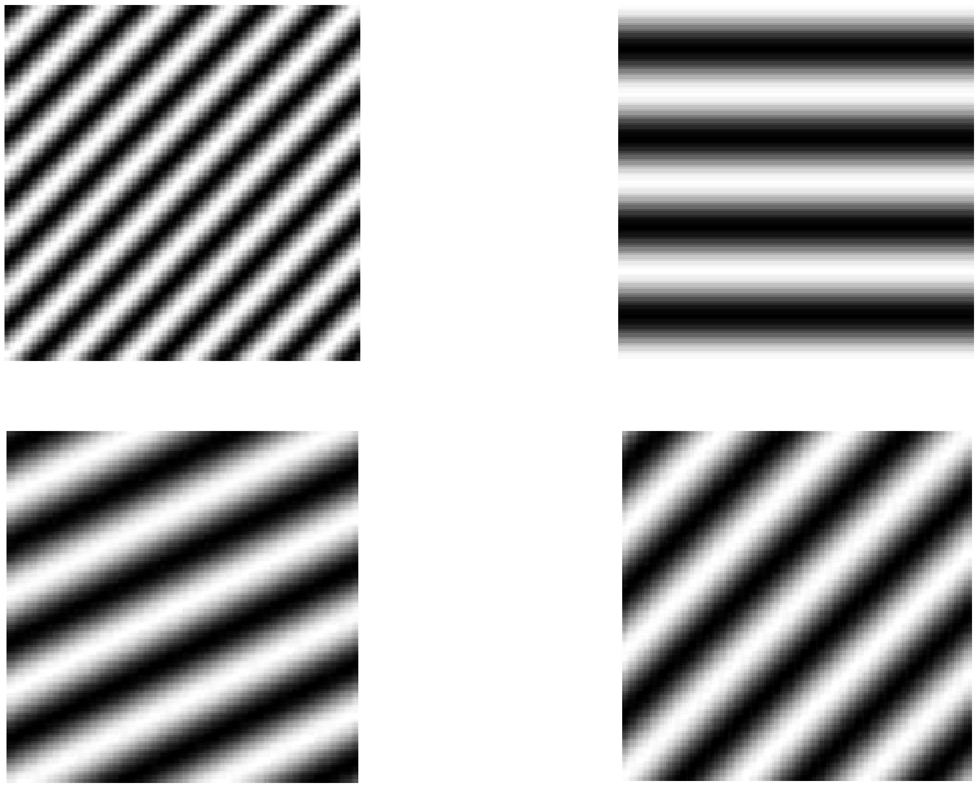
Fourier Transform



…etc.

Fourier basis

adapted from Martial Hebert, CMU

Fourier Transform

Phase and Magnitude

- The magnitude spectra of all natural images quite similar
  - Heavy on low-frequencies, falling off in high frequences
  - Will any image be like that, or is it a property of the world we live in?
- Most information in the image is carried in the phase, not the amplitude
  - Seems to be a fact of life
  - Not quite clear why
- Demonstration
  - Take two pictures, swap the phase transforms, compute the inverse - what does the result look like?

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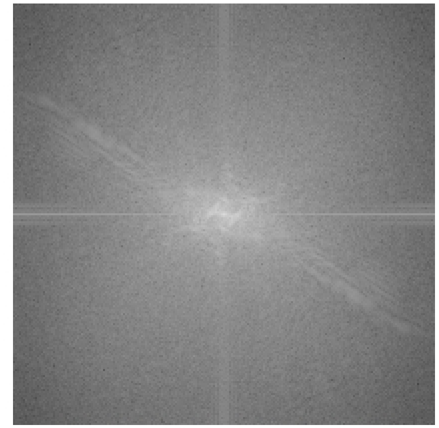
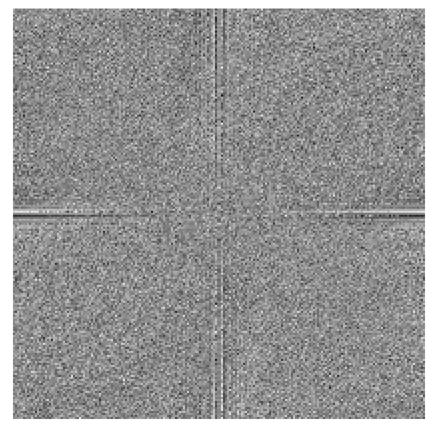
Fourier Transform



Magnitude spectrum



Phase spectrum

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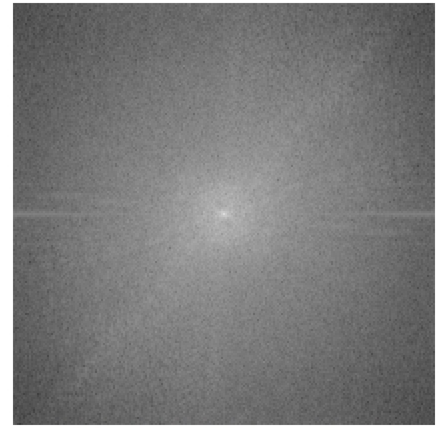
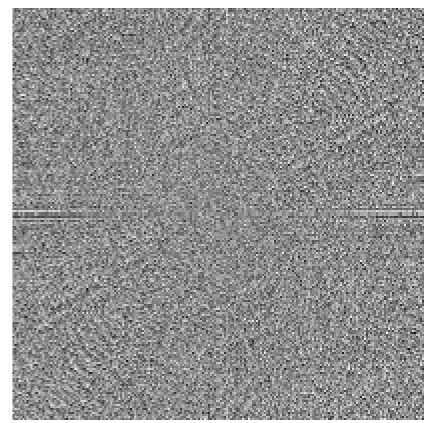
Fourier Transform



Magnitude spectrum



Phase spectrum

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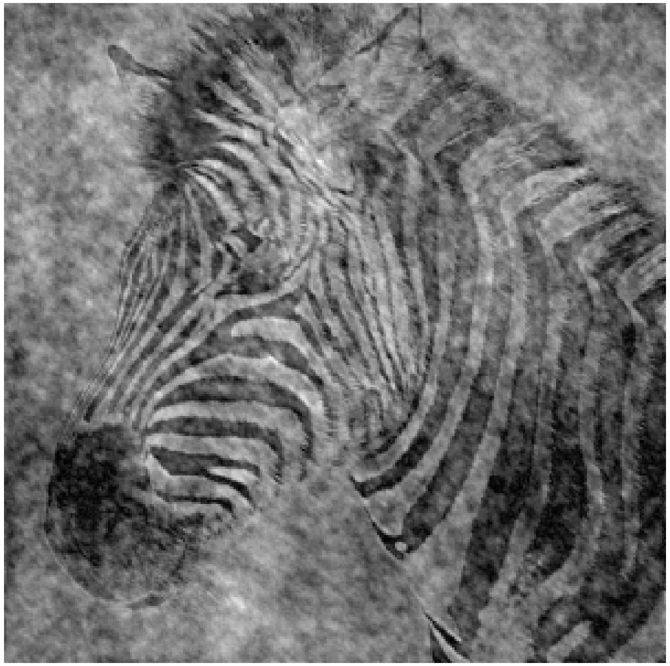
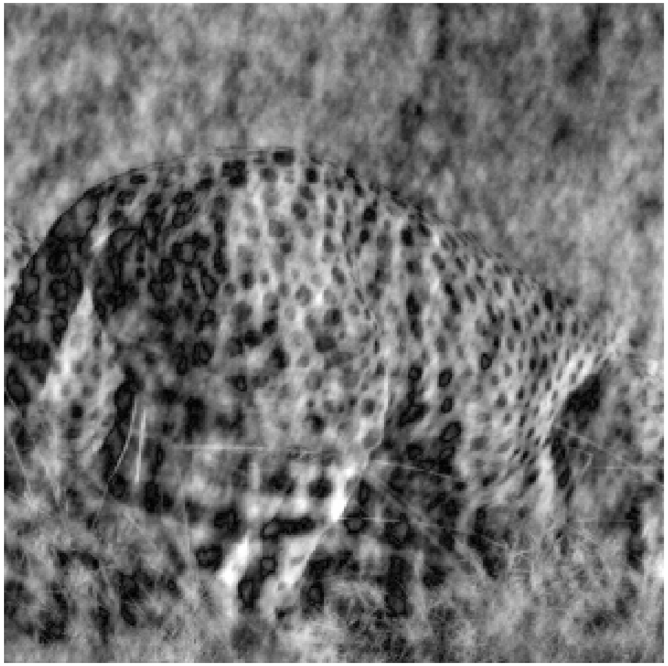
Fourier Transform



Reconstruction with zebra phase, cheetah magnitude



Reconstruction with cheetah phase, zebra magnitude

- Phase spectrum seems to be more important for perception.
- We're however going to look at magnitude spectra for analyzing filters

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Fourier Transform

Various Fourier Transform Pairs

- Important facts
  - The Fourier transform is linear
  - There is an inverse FT
  - if you scale the function's argument, then the transform's argument scales the other way. This makes sense --- if you multiply a function's argument by a number that is larger than one, you are stretching the function, so that high frequencies go to low frequencies
  - The FT of a Gaussian is a Gaussian.

The convolution theorem

- The Fourier transform of the convolution of two functions is the product of their Fourier transforms

  - The inverse Fourier transform of the product of two Fourier transforms is the convolution of the two inverse Fourier transforms

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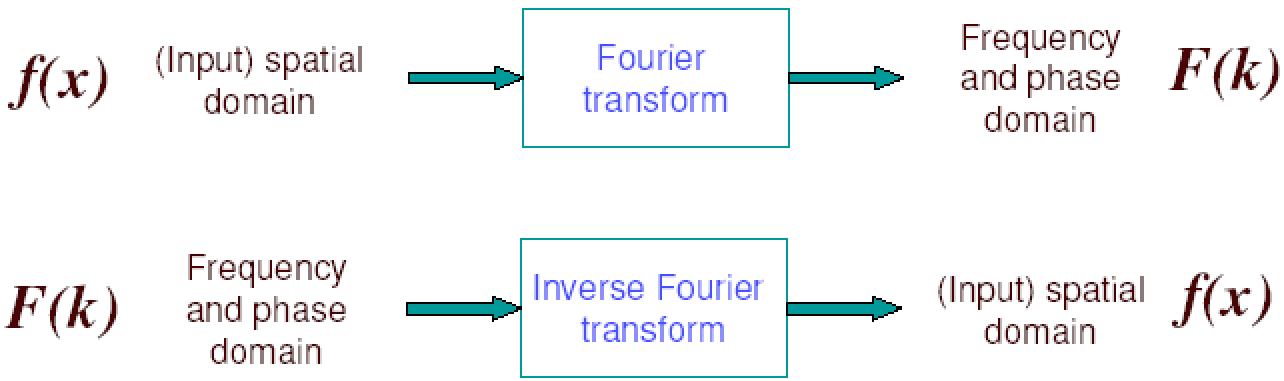
Fourier Transform

Basic idea



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Fourier Transform

Convolution property

$$f(x) * g(x) \xrightarrow{\text{FT}} F(k) \times G(k)$$

Convolution in *input* domain = **Multiplication** in *frequency* domain

$$f(x) \times g(x) \xrightarrow{\text{FT}} F(k) * G(k)$$

Multiplication in *input* domain = **Convolution** in *frequency* domain

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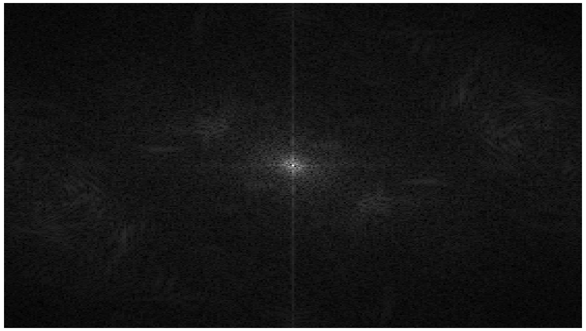
2D convolution theorem example

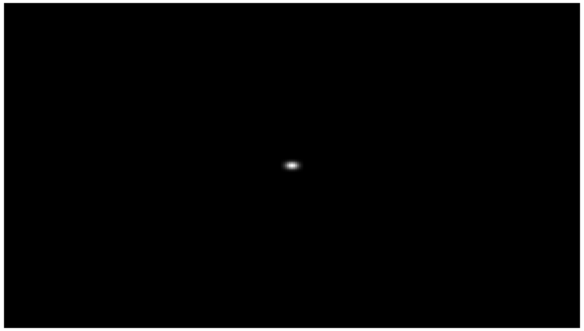
$f(x,y)$

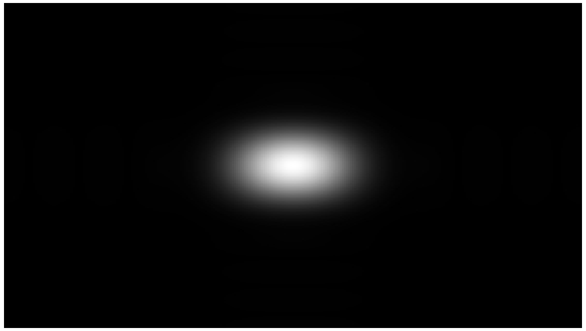


$|F(s_x,s_y)|$

$*$ $\times$

$h(x,y)$

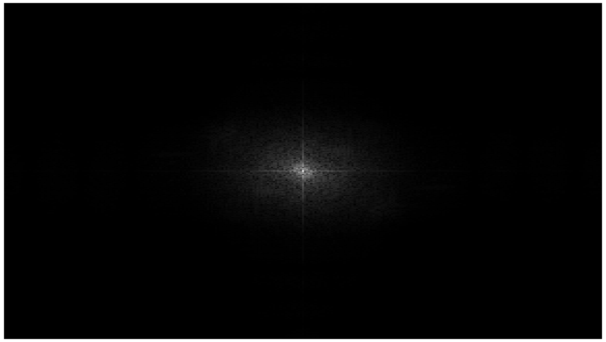




$|H(s_x,s_y)|$

$\Downarrow$ $\Downarrow$

$g(x,y)$



$|G(s_x,s_y)|$

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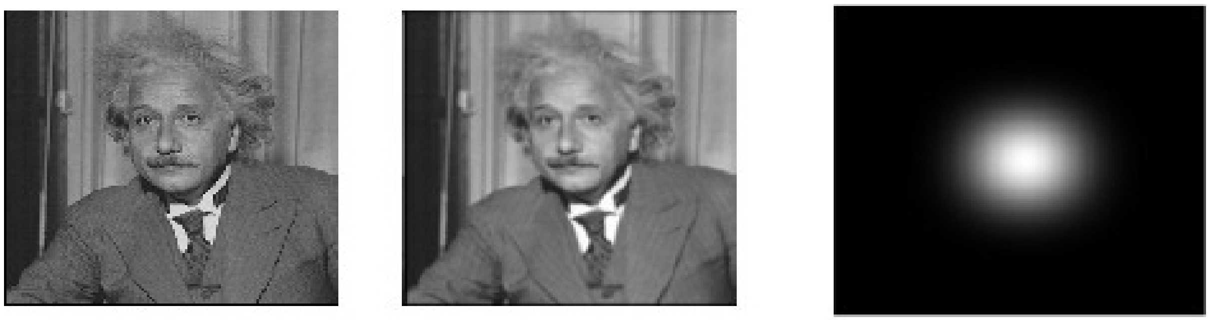
Low pass / Band pass / High pass filters

low-pass:

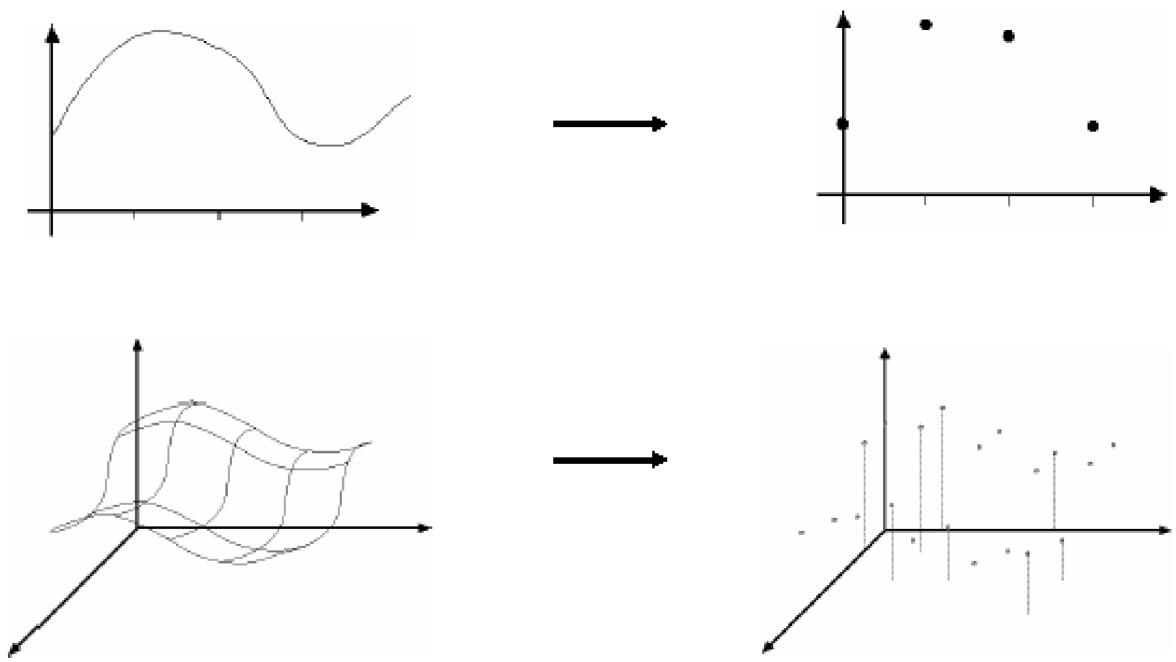


band-pass:

what's high-pass?

Sampling

Sampling an input image



The Sampling operation takes a continuous function and replaces it with a vector of values, consisting of the function's values at a set of sample points

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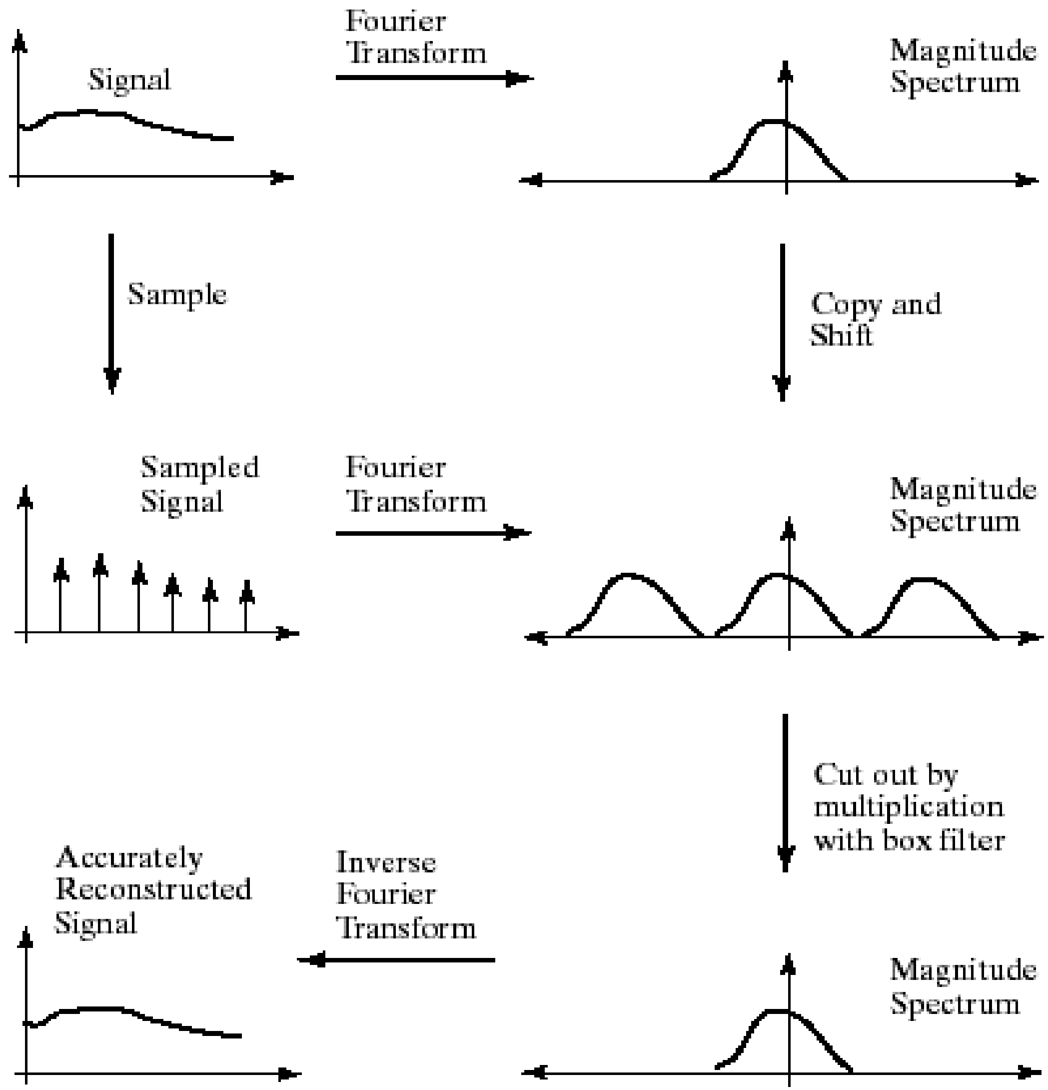
Sampling



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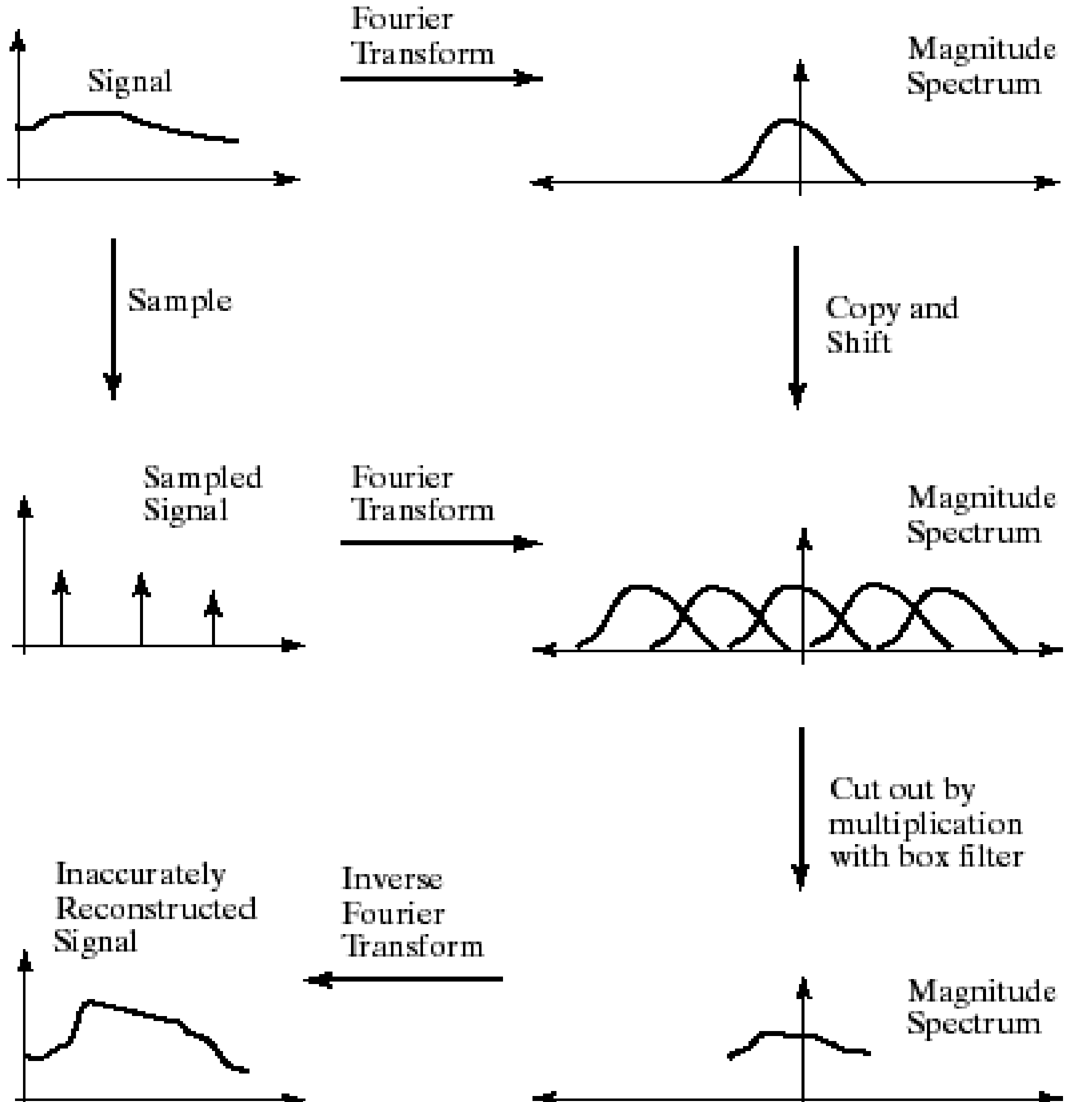
Sampling



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Sampling

Low-pass filtering before sampling

- The minimum frequency at which we must sample a signal in order to be able to fully reconstruct it called the **Nyquist frequency**.
- Nyquist frequency = 2 times the maximum frequency contained in the waveform.
- The message of the FT is that high frequencies lead to trouble with sampling.
- Solution: suppress high frequencies before sampling
  - multiply the FT of the signal with something that suppresses high frequencies
  - or convolve with a low-pass filter
- Common solution: use a Gaussian
  - multiplying FT by Gaussian is equivalent to convolving image with Gaussian.

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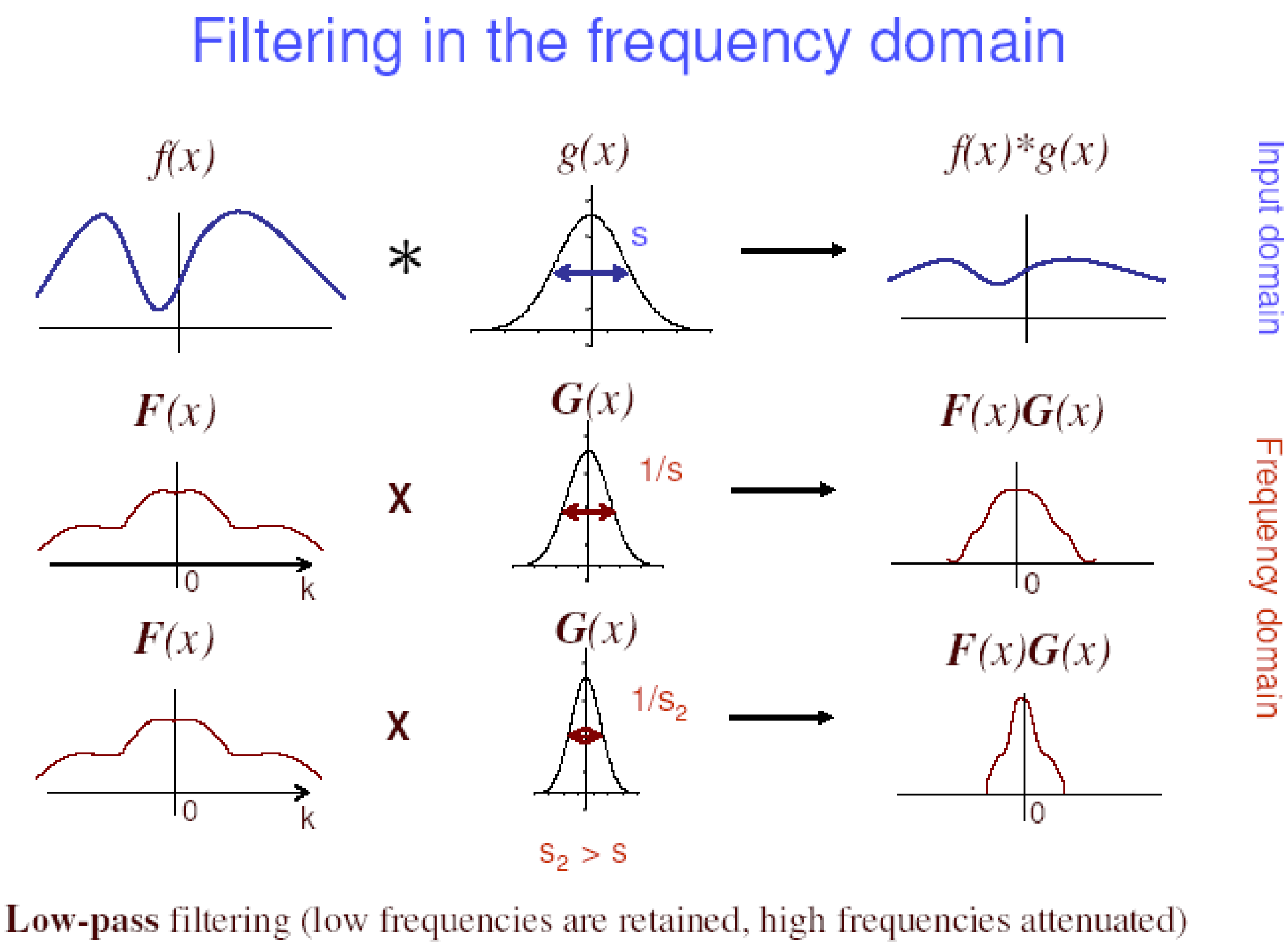
Fourier Transform



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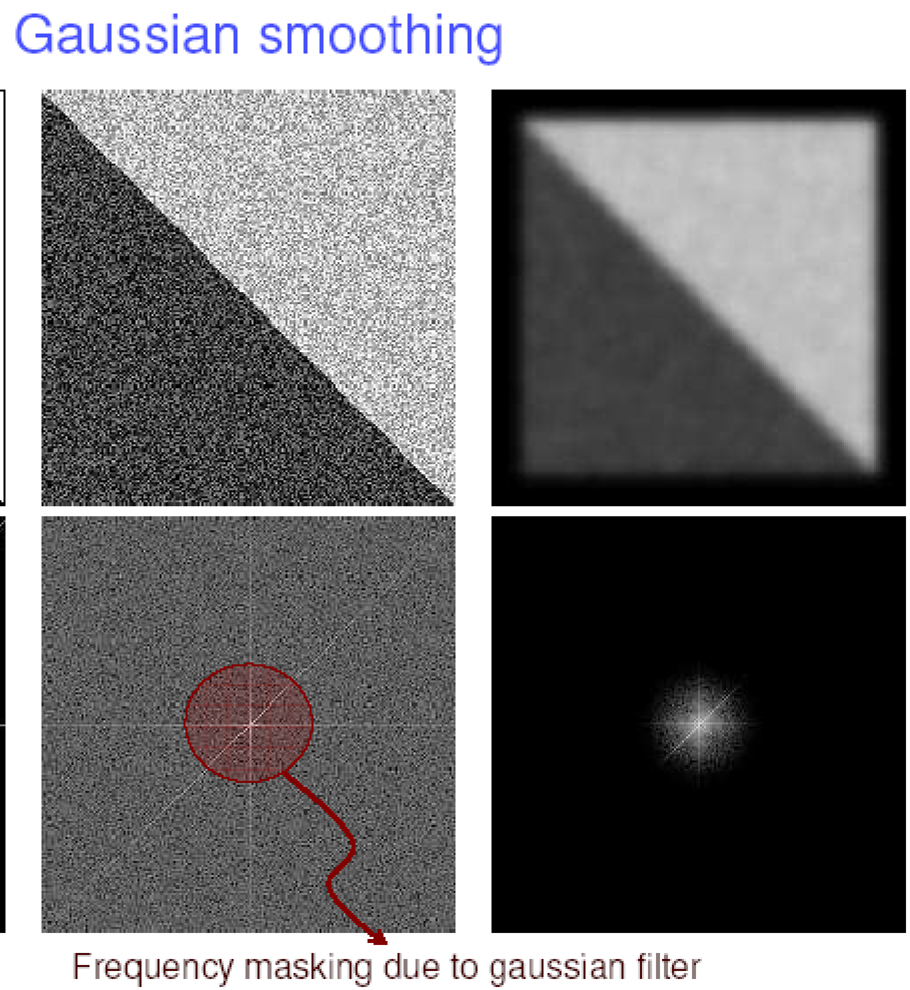
Fourier Transform



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Sampling

Sub-sampling an Image with a factor of two

Apply a low-pass filter to the original image
(a Gaussian with a σ of between one and two pixels is usually an acceptable choice)

Create a new image whose dimensions on edge are half those of the old image

Set the value of the i,j th pixel of the new image to the value of the 2i, 2j th pixel of the filtered image

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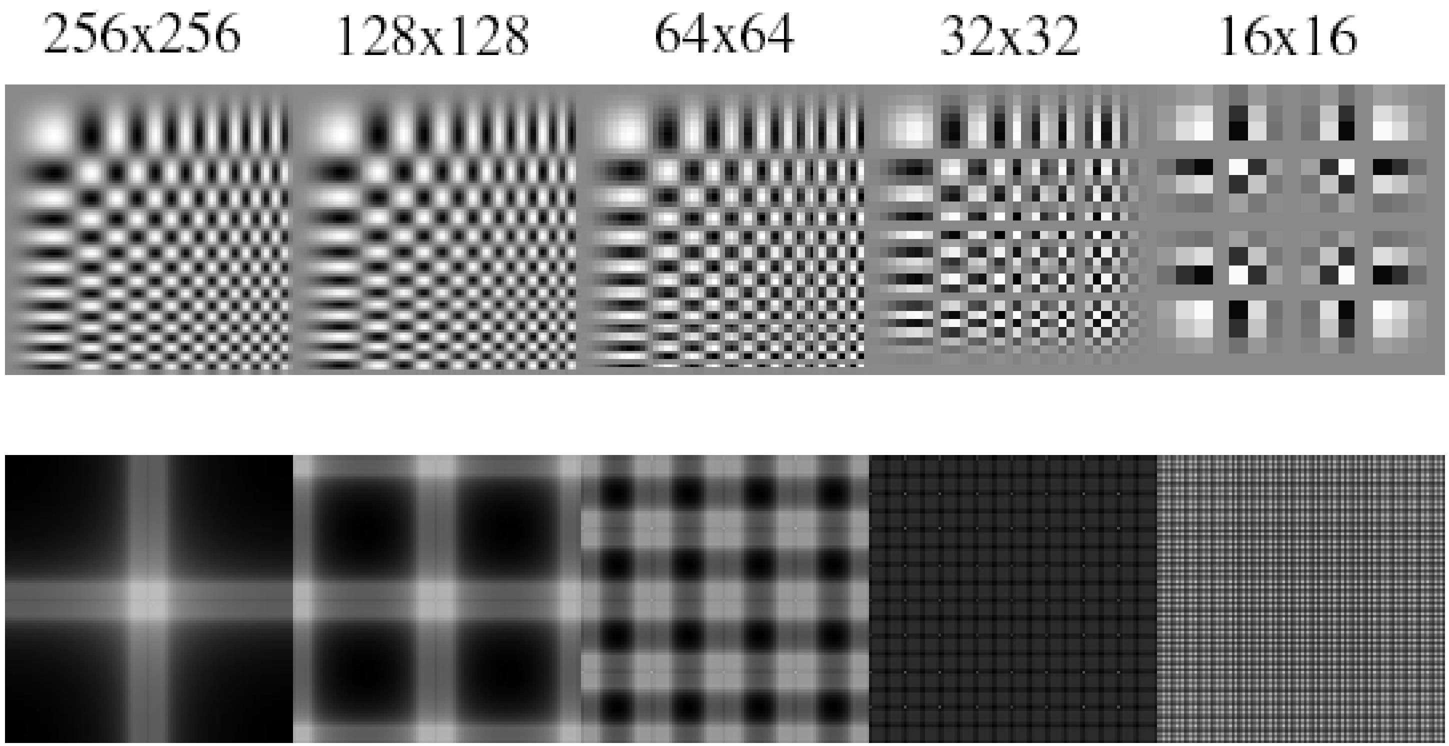
Sampling

Sampling without smoothing. Top row shows the images, sampled at every second pixel to get the next; bottom row shows the magnitude spectrum of these images.



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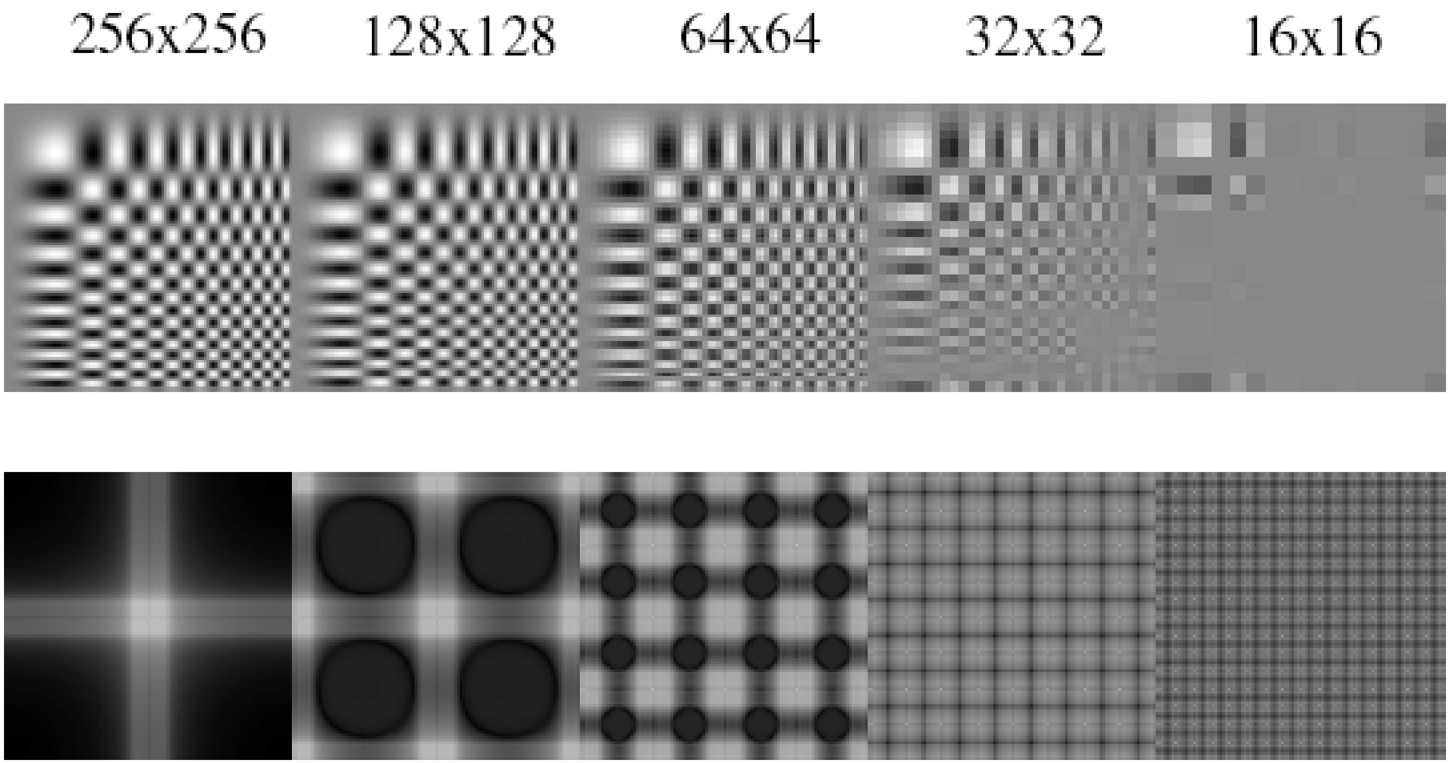
Sampling

Sampling with smoothing. Top row shows the images. We get the next image by smoothing the image with a Gaussian with sigma 1 pixel, then sampling at every second pixel to get the next; bottom row shows the magnitude spectrum of these images.



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