Magnetic Domain-Wall Racetrack Memory

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abstract.

1 Introduction

1.1 Theme

The means of storing information for computer applications is evolving over time. The two widely used techniques to store information are Magnetic hard disk drives and solid-state random-access memory. On the one hand HDD is cheaper than RAM to store information, On the other hand, HDD is slower than RAM due to large rotating disk. The architecture of computer is greatly simplified if there were a single memory storage device with low cost of the HDD but High performance and reliability of solid-state RAM.

This requirement leads us to a new approach called *racetrack memory* which is an efficient way to store and access the information in a fast way. The racetrack memory consists of a ferromagntic nanowire through which data can be moved and can be read and written by fixed reading and writing elements. The material are made of electrons with spins. Applying external magnetic field results in spin alignment. Successively spins with the same alignment form a so-called domain. Domains which are next to each other are magnetized in the opposite direction and seperated by a domain-wall. The domain-wall is a region between the domains in which a smooth transition between the opposite magnetizations takes place.

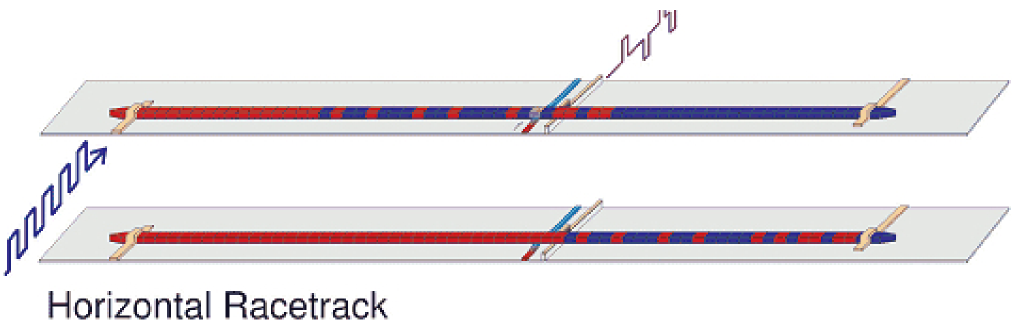


Abbildung 1: Horizontal racetrack with reading/writing element

When current is passed through the nanowire, the current becomes spin polarized and carries the spin angular momentum. When the spin polarized current is passed through a domain wall the current transfers the spin angular momentum to domain wall thereby applying the torque to the moments in the DW which results in motion of the wall.

1.2 Mathematical description

The configuration of the spins is described by the magnetization vector $\vec{\mathbf{m}}$ which is given by the Landau-Lifshitz equation

$$\vec{\mathbf{m}} \times \partial_t \vec{\mathbf{m}} + \alpha \partial_t \vec{\mathbf{m}} + \nabla E_{LL}(\vec{\mathbf{m}}) \left(1 - \vec{\mathbf{m}}\vec{\mathbf{m}}^T\right) = h \left(1 - \vec{\mathbf{m}}\vec{\mathbf{m}}^T\right). \tag{1}$$

To obtain the quantity of interest $\vec{\mathbf{m}}$ we have to solve equation (1). Solving (1) is equivalent to minimizing the energy functional given by

$$E_{LL}[\vec{\mathbf{m}}] = E_a[\vec{\mathbf{m}}] + E_e[\vec{\mathbf{m}}] + E_s[\vec{\mathbf{m}}] + E_z[\vec{\mathbf{m}}]. \tag{2}$$

The energy functional is the summation of four different energies influencing the spin configuration.

1. **Anisotropy Energy**
   The structure of the cristalline lattice initiates an prefered orientation for the spins. For an nanowire we can model it as an uniaxial material, that means that it has only one prefered direction. Choosing this prefered direction is one part of our simulation in which we are going to find out which direction minimizes our total energy. For example choosing m_1 as preferred direction, the effect of the structure is given by the equation

$$E_a[\vec{\mathbf{m}}] = \frac{K_u}{M_s^2} \int_\Omega \left(m_2^2 + m_3^2\right) d\mathbf{x} \tag{3}$$

   with K_u as a material parameter (in units of J/m^3) and M_s being the constant length of the vector field $\vec{\mathbf{m}}$.

2. **Exchange Energy**
   This includes the special characteristic of a ferromagnetic material, that one spin experiences an exchange field by the spins directly next to it, which aligns them in the same direction. It is described by the equation

$$E_e[\vec{\mathbf{m}}] = \frac{C_{ex}}{M_s^2} \int_\Omega |\nabla \vec{\mathbf{m}}|^2 d\mathbf{x} \tag{4}$$

   The C_{ex} is the exchange constant in units of J/m^3.

3. **Stray Field Energy**
   The stray field energy originates from the fact that angular momentum carries a magnetic dipole moment which induces a dipole magnetic field, the *self-induced field*. The sum of elementary dipole fields creates the stray fields of the sample. One computes this energy using the formula

$$E_s[\vec{\mathbf{m}}] = \frac{\mu_0}{2} \int_\Omega \vec{\mathbf{m}} \cdot \nabla U d\mathbf{x}, \tag{5}$$

   where $\mu_0 = 4\pi \cdot 10^{-7} N/A^2$ is the magnetic permeability of vacuum. To this end, it is necessary to compute the potential U given by

$$U(\mathbf{x}) = \int_\Omega \nabla N(\mathbf{x} - \mathbf{y}) \cdot \vec{\mathbf{m}}(\mathbf{y}) d\mathbf{y} \tag{6}$$

   which is the solution of the laplace equation in $3D$. In the formulation of U, N is the Newtonian potential

$$N(\mathbf{x}) = -\frac{1}{4\pi} \frac{1}{|\mathbf{x}|}. \tag{7}$$

4. **Zeeman Energy**
   This energy is the effect of an external field $\mathbf{H}_e$ applied so that the spins tend to align with it

$$E_Z[\vec{\mathbf{m}}] = -\mu_0 \int_\Omega \mathbf{H}_e \cdot \vec{\mathbf{m}} d\mathbf{x}. \tag{8}$$

2 Strategy and Initial Concerns

Our goal is to develop a numerical tool to study the structure of the domain walls. We have referred (reference to garcia2007) and we have carefully examined the numerical procedure that was presented in the paper.
In general, the Landau-Lifshitz-Equation has to be solved in order to compute the magnetization vector and observe the phenomenon of domain-walls. Since we wanted to consider a long thin nanowire, we were forced to reduce the dimension of the domain from 3D to 1D. To start with, we aimed to compute the steady-state solution for the magnetization vector in our 1D domain. This problem is equivalent to a minimization of the energy $E_{LL}(\vec{\mathbf{m}})$ and can be solved by a fixed point iteration.
Let $M = \{\vec{\mathbf{m}}_i\}_{i\in I}$ be the set of unknowns after discretization of $\vec{\mathbf{m}}$. To solve (1) we used the fixed point iteration scheme proposed by (garcia2007, p. 17)

$$\vec{\mathbf{m}}_i^{n+1} = \frac{h_i^n}{|h_i^n|}. \tag{9}$$

Here, h_i^n denotes the approximation to $\nabla E_{LL}[\vec{\mathbf{m}}]$ after the n-th iteration given by

$$\mathbf{h}_i = -q(m_{i,2}\mathbf{e}_2 + m_{i,3}\mathbf{e}_3) + \epsilon\nabla_h^2\vec{\mathbf{m}}_i - \nabla u_i + \mathbf{h}_e. \tag{10}$$

Equation (10) and the constants $q = 2K_u/(\mu_0 M_s^2)$ and $\epsilon = 2C_{ex}/(\mu_0 s^2 L^2)$ are in a dimensionless form. The derivatives have been approximated by using the finite difference scheme (central difference scheme).
When applying the scheme we encountered several issues. Most of the issues dealt with the dimensional reduction of the stray energy term ∇u_i. To compute u it is necessary to solve the poisson/laplace equation on the $\mathbb{R}^3$ space. The solution can be computed as a convolution of the Newtonian potential and the magnetization vector, and can thus be represented as a three dimensional integral. The next section contains an outline of the dimensional reduction of this convolution and possible next steps. For the results displayed in Section 4 we used the dimensional reduction to a 1D thin film as in (garcia2004).

Furthermore, we had issues with the fixed point iteration mostly due to oscillations introduced by the exchange energy term. To fix this and to construct a more robust iteration, we used a pseudo time stepping scheme leading to the following term:

$$\vec{\mathbf{m}}_i^{n+1} = \vec{\mathbf{m}}_i^n + \Delta t\frac{h_i^n}{|h_i^n|}.$$

This modified scheme converged properly and yielded reliable results.

3 Our work

3.1 Reduced model for the Stray Field Energy

Our reduced model of a thin nanowire will be derived in this section.
The Stray Field H_s has a (magnetostatic) potential U:

$$H_s = -\nabla U.$$

The Maxwell equations lead to a Poisson equation for the potential:

$$\nabla^2 U = \begin{cases} \nabla \cdot M, & \text{in } \Omega \\ 0, & \text{else} \end{cases}$$

and jump conditions along the boundary:

$$\begin{aligned} [U]_{|\partial\Omega} &= 0 \\ \left[\frac{\partial U}{\partial \nu}\right]_{|\partial\Omega} &= -M \cdot \nu. \end{aligned}$$

The solution is given by:

$$U(x) = \int_\Omega \nabla N(x-y) \cdot M(y)\, \mathrm{d}x$$

where $N(x) := \frac{-1}{4\pi}\frac{1}{|x|}$ is the Newtonian potential.
With obvious calculations one can pull out the gradient:

$$\begin{aligned} U(x) &= \int_\Omega \nabla N(x-y) \cdot M(y)\, \mathrm{d}y \\ &= \nabla \cdot (N * M)(x). \end{aligned}$$

The 1D domain will be approximated by a 3D domain parameterized by δ:

$$\Omega = \Omega_\delta = \left\{ (x_1, r\cos\phi, r\sin\phi) \in \mathbb{R}^3 \mid \quad x_1 \in [0,1],\ 0 \le r < \delta,\ \phi \in [0, 2\pi) \right\}.$$

This domain is used for the convolution:

$$(N * M)(x) = \int_{\Omega_\delta} N(x-y) \cdot M(y)\, \mathrm{d}y\,.$$

A translation along the x_1-axis is useful:

$$\begin{aligned} z &:= y - (x_1, 0, 0)^T \\ \Omega_x &:= \Omega - (x_1, 0, 0)^T \end{aligned}$$

and a rotation around the x_1-axis.
The magnetization is assumed to be only x_1 dependent: $M(x) = M(x_1)$.
The convolution integral along the r variable leads to:

$$(N * M)(x) = \int_{-x_1}^{1-x_1} \mathrm{d}z_1 \int_0^{2\pi} \mathrm{d}\tilde{\phi}\, M(z_1 + x_1) \int_0^\delta \mathrm{d}r\, \frac{r}{\sqrt{(r-r_0)^2 + \alpha}}$$

$$\int_0^\delta \mathrm{d}r\, \frac{r}{\sqrt{(r-r_0)^2 + \alpha}} = \sqrt{\alpha(t^2+1)}\Big|_{t_0}^{t_\delta} + r_0 \operatorname{arsinh} t \Big|_{t_0}^{t_\delta}$$

with the following auxiliary variables

$$\begin{aligned}
x &= (x_1, R\cos\varphi, R\sin\varphi)^T \\
z &= (z_1, r\cos\phi, r\sin\phi)^T \\
\tilde{\phi} &= \varphi - \phi \\
r_0 &:= R\cos\tilde{\phi} \\
\alpha &:= R^2 + z_1^2 - r_0^2 \\
t(r) &:= \frac{r - r_0}{\sqrt{\alpha}}.
\end{aligned}$$

The angular integration might lead to factors of the complete elliptic integral of first and second kind. This should be look upon further.

4 Simulation and Results

Using the previously explained methods and schemes the Landau-Lifshitz-Equation was solved for the magnetization vector in the steady state solution, i.e. the minimum energy solution. The following parameters were applied:

$$\begin{aligned} q &= 1 \\ \epsilon &= 10^{-3} \\ \Delta t &= 2.5 \cdot 10^{-2} \end{aligned} \tag{11}$$

First of all, the different energy terms in the free energy functional were investigated individually for their physical meaning. For all individual cases, a random initial magnetization was assumed as in figure 2. As boundary conditions we chose in vectors pointing in opposite directions.

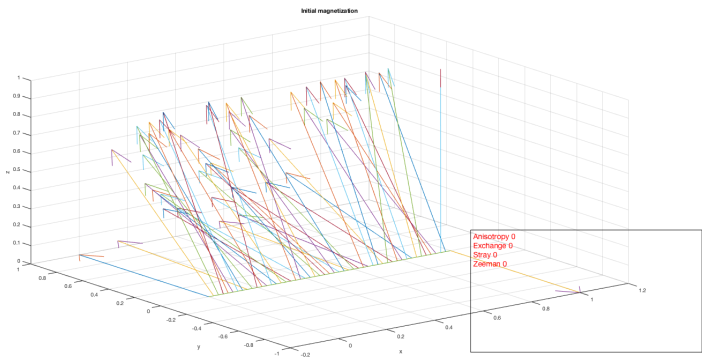
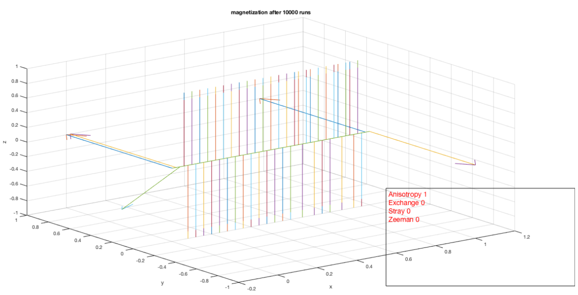
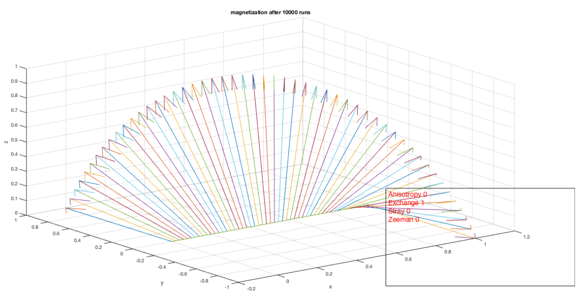


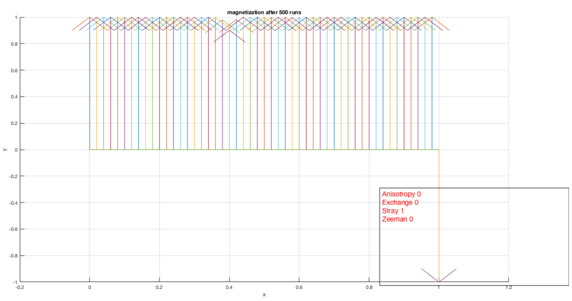
Abbildung 2: Randomly initialized magnetization vectors



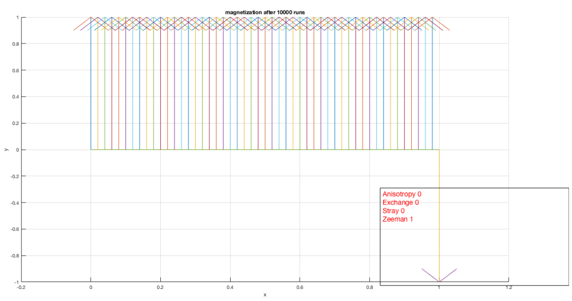
(a) Anisotropic Energy



(b) Exchange Energy



(c) Stray Energy



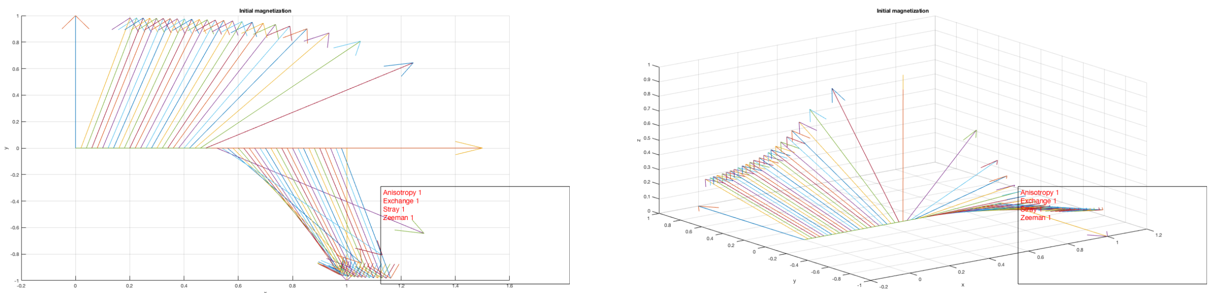
(d) Zeeman Energy

Abbildung 3: Single Energy Term Solutions

Figure 3 depicts the impact of applying a single force term to the initial condition as in figure 2. In figures (a), (c) and (d) it is possible to see, that the minimum energy is reached, when the magneti-

zation points towards a parametrized preferred direction.
When the exchange energy (b) is applied however, i.e. the laplacian operator in every direction, we can see a very smooth transition from one to the other boundary condition.

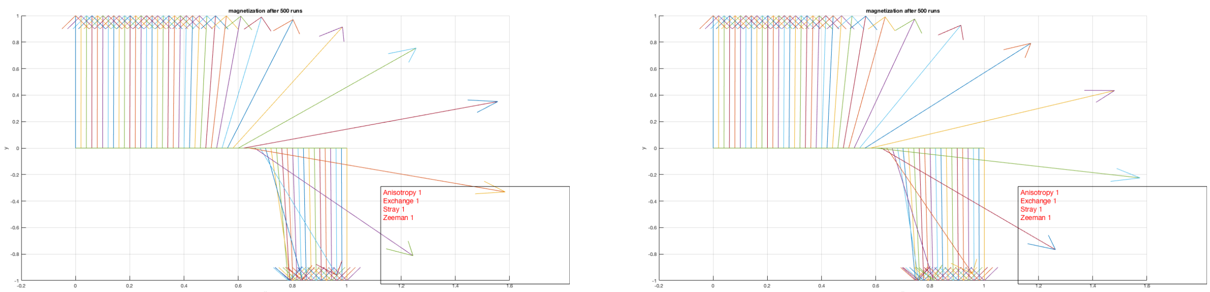
Finally it is aimed to investigate the development of Neel and Bloch domain-wall profiles as a steady state solution. To this end and initial solution was guessed using smooth functions and slightly perturbed in every direction, so as not to guess the correct solution by accident. These initial conditions can be seen in figurefig:initboundary.



(a) Neel Wall Initial Condition

(b) Bloch Wall Initial Condition

Abbildung 4



(a) Neel Wall

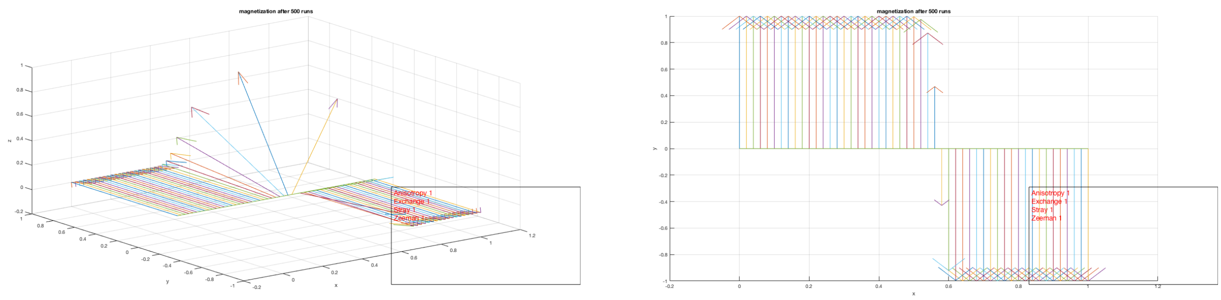
(b) Bloch Wall

Abbildung 5

These initial values propagate to the profiles seen in figure 5. The images look very similar, which we explained by the fact of using the equations reduced for a thin film. In other words, the Neel wall appears to be a more easily accessible local minimum. To actually obtain the profile of a Neel wall in this case we used the initial condition as in figure 4(b), only without the perturbation in x-Direction. With this we obtain the profile in figure 6.

5 Next course of action

We have come up with a new fixed point scheme to calculate the magnetization of a nanowire that minimizes the energy functional. This scheme is more reliable than the one suggested in [**?**]. However, we have computed only the steady state equation. Our next course of action is to compute the solution for the dynamic case, i.e. we want to develop a strategy to control and to move domain walls by an external magnetic field or by an external current. We assume that there is an upper bound for both external magnetic field and external current which we call the critical current and critical magnetic field, respectively. Since the information are encoded as magnetic spins it is crucial



(a) (b)

Abbildung 6: Actual Bloch Wall

to maintain the external fields below the critical value. If it exceeds then it will lead to a loss of information, i.e. the spins may flip inconsistently, the moment of domain wall will not be uniform.

Literatur