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Numerical Methods for PDE

Homework 1

Given Problem:

Write a computer program that generates a Taylor table, and determines the coefficients of a difference approximation. The program should be written in such a way that derivatives of arbitrary order (i.e. m in eq. (1)) can be handled, and also different stencil sizes (i.e. p, q) can be selected.

Test the implementation by generating the coefficients (a, b, c, d, e) corresponding to the best possible (i.e. highest order consistent) difference approximation

$$\frac{au_{i-2} + bu_{i-1} + cu_i + du_{i+1} + eu_{i+2}}{h^2} = u_i'' + \mathcal{O}(h^r).$$

Solution

From the above mentioned equation, the following details can be inferred

The lower stencil size is -2

And the higher stencil size 2

The order of derivative 2

The coefficients values for the given problem as follows:

a= -0.083333

b= 1.333333

c=-2.500000

d=1.33333

e=-0.083333

Order of accuracy:

There are several methods available to find the order of accuracy of a difference approximation, the method which I used is as follows:

With the known values of Taylor series coefficients(for say, here a, b, c, d, e) , error values for each order of term in Taylor series can be formed by multiplying the coefficient with respective Taylors series and by summing values of respective order term in each Taylor series.

(Please refer the attached C program output)

From the <values of summation of error for each order term>, One can see the non zero values after certain order of term, is for the given stencil size and order of derivative. Since we are doing this calculation for second order derivative the order of accuracy can be found by the formula

The Order of accuracy= First non zero term in the summation of each respective column in the Taylors table or (The leading order error term) – Oder of derivative (m)

For instance, given stencil size is -2 to 2 and order of derivative is 2. The non-zero value starts from sixth Order term (Please refer the following C program output) so **order of accuracy is 4.**

Similarly it can be calculated for other Stencil sizes and different order of derivatives simply by looking at values of summation of error for each order term.

C:\Users\CHRIST\Desktop\ccode_edilbert.c.exe

```
Enter order of derivative m
2
Enter the value of lower stencil  p
2
Enter the value of higher stencil q
2

The augumented matrix
1.000000        1.000000        1.000000        1.000000        1.000000        0.000000
-2.000000       -1.000000       0.000000        1.000000        2.000000        0.000000
2.000000        0.500000        0.000000        0.500000        2.000000        1.000000
-1.333333       -0.166667       0.000000        0.166667        1.333333        0.000000
0.666667        0.041667        0.000000        0.041667        0.666667        0.000000



The Coefficients are:

z0=-0.083333
z1=1.333333
z2=-2.500000
z3=1.333333
z4=-0.083333

The Taylors table
-0.083333       0.166667        -0.166667       0.111111        -0.055556       0.022222        -0.007407       0.002116        -0.000529
1.333333        -1.333333       0.666667        -0.222222       0.055556        -0.011111       0.001852        -0.000265       0.000033
-2.500000       -0.000000       -0.000000       -0.000000       -0.000000       -0.000000       -0.000000       -0.000000       -0.000000
1.333333        1.333333        0.666667        0.222222        0.055556        0.011111        0.001852        0.000265        0.000033
-0.083333       -0.166667       -0.166667       -0.111111       -0.055556       -0.022222       -0.007407       -0.002116       -0.000529



The  summation of Error values for each terms in Taylors Table is
0  Order term= -0.000000
1  Order term= -0.000000
2  Order term= 1.000000
3  Order term= -0.000000
4  Order term= -0.000000
5  Order term= -0.000000
6  Order term= -0.011111
7  Order term= -0.000000
8  Order term= -0.000992

Accuracy of approximation is 4



Process returned 0 (0x0)   execution time : 2.162 s
```

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