2. For Differential Equation
$(D + 2.5) y_0(t) = 0$ with $y_0(0) = 3$

a) Find and plot analytical solution for $0 \le t \le 10$

$\lambda + 2.5 = 0 \qquad \lambda = -2.5$

$y_0(t) = Ce^{-2.5t}$ initial: $3 = C$

$\boxed{y_0(t) = 3e^{-2.5t}}$ for $t \geq 0$.

plot on matlab

b) program plot on matlab

c) Comparison

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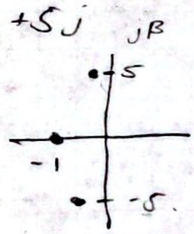
3. For Differential Equation
$(D^3 + 0.6D^2 + 25.1125D + 2.5063) y_0(t) = 0$
with initial conditions $y_0(0) = 1.5$, $\dot{y}_0(0) = 2$, $\ddot{y}_0(0) = -1$

a) Find and plot analytical solution
Using online cubic root calculator as approved!

$\lambda = -0.1$, $\lambda = -0.25 - 5j$, $\lambda = -0.25 + 5j$

roots,



$y_0(t) = Ce^{-0.1t} + Ae^{-0.25t}\cos(5t + \theta)$

$\dot{y}_0(t) = -0.1Ce^{-0.1t} - 0.25Ae^{-0.25t}\cos(5t+\theta) - 5Ae^{-0.25t}\sin(5t+\theta)$

$\ddot{y}_0(t) = 0.01Ce^{-0.1t} + 0.0625Ae^{-0.25t}\cos(5t+\theta) + 1.25Ae^{-0.25t}\sin(5t+\theta)$
$+ 1.25Ae^{-0.25}\sin(5t+\theta) - 25Ae^{-25t}\cos(5t+\theta)$

$\Rightarrow 1.5 = C + A\cos(\theta)$

$2 = -0.1C - 0.25A\cos(\theta) - 5A\sin(\theta)$

$-1 = 0.01C + 0.0625A\cos(\theta) + 1.25A\sin(\theta) + 1.25A\sin(\theta) - 25A\cos(\theta)$

Using waveform from program, I know $C = 1.5$ and $\theta = \frac{\pi}{2}$

so $2 = -0.1(1.5) - 5A \Rightarrow A = -0.43$

$C = 1.5 \quad A = -0.43 \quad \theta = \frac{\pi}{2}$ ← (checked with online calculator as well)

$\boxed{y_0(t) = 1.5e^{-0.1t} - 0.43e^{-0.25t}\cos(5t + \frac{\pi}{2})}$

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b) Put 3rd order Diff Eq into state-space form
let $x_1(t) = y_0(t)$, $x_2(t) = \dot{y}_0(t)$, $x_3(t) = \ddot{y}_0(t)$

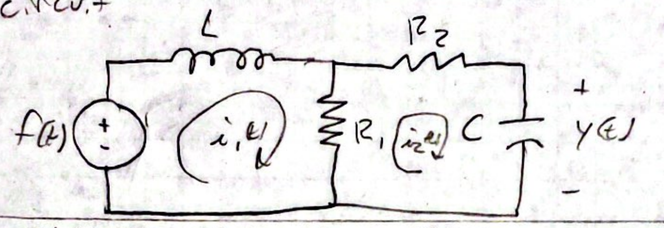
$\dot{x}(t) = Ax(t) \qquad A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2.5063 & -25.1125 & -0.6 \end{bmatrix}$

$\dot{x}(t) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2.5063 & -25.1125 & -0.6 \end{bmatrix} x(t)$

c) Program solution and plot

d) Compare

4. For circuit



$R_1 = 1\ k\Omega$
$R_2 = 22\ k\Omega$
$C = 10\ \mu F$
$L = 5\ H$

a) Determine Diff Eq relating input $f(t)$ to output $y(t)$

$$f(t) = L D i_1(t) + R_1(i_1(t) - i_2(t))$$

$$0 = R_2(i_2(t)) + \frac{1}{CD} i_2(t) + R_1(i_2(t) - i_1(t))$$

$$\Rightarrow f(t) = (LD + R)i_1(t) - R_1 i_2(t)$$

$$0 = -R_1 i_1(t) + \left(\frac{1}{CD} + R_1 + R_2\right) i_2(t)$$

$$\Rightarrow \begin{bmatrix} LD + R_1 & -R_1 \\ -R_1 & \frac{1}{CD} + R_1 + R_2 \end{bmatrix} \begin{bmatrix} i_1(t) \\ i_2(t) \end{bmatrix} = \begin{bmatrix} f(t) \\ 0 \end{bmatrix}$$

where $i_2(t) = CDy(t)$

$$\Rightarrow i_2(t) = \frac{\begin{vmatrix} LD + R_1 & f(t) \\ -R_1 & 0 \end{vmatrix}}{\begin{vmatrix} LD + R_1 & -R_1 \\ -R_1 & \frac{1}{CD} + R_1 + R_2 \end{vmatrix}} = \frac{R_1 f(t)}{(LD + R_1)(\frac{1}{CD} + R_1 + R_2) - R_1^2}$$

$$\Rightarrow \left[\left(\frac{1}{CD} + R_1 + R_2\right)(LD + R_1) - R_1^2\right] CDy(t) = R_1 f(t)$$

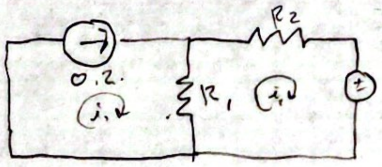
$$= \left[\frac{L}{C} + LDR_1 + LDR_2 + \frac{R_1}{CD} + \cancel{R_1^2} + R_1R_2 - \cancel{R_1^2}\right] CDy(t) = R_1 f(t)$$

$$= \left[D^2(LR_1C + LR_2C) + D(R_1R_2C + L) + R_1\right] y(t) = R_1 f(t)$$

$$= \left[D^2 + D\frac{(R_1R_2C + L)}{(LR_1C + LR_2C)} + \frac{(R_1)}{(LR_1C + LR_2C)}\right] y(t) = R_1 f(t)$$

$$= \boxed{\left[D^2 + 195.652D + 869.565\right] y(t) = R_1 f(t)}$$

b) Determine initial condition for $\dot{y}_0(t)$ if $i_1(0) = 0.2\ A$ and $y(0) = 5\ V$



$$i_2(0) = C\dot{y}(0)$$

$$R_2(i_2(0)) + 5 + R_1(i_2(0) - 0.2) = 0$$

$$i_2(0) = \frac{0.2R_1 - 5}{R_1 + R_2} = 0.00848$$

$$\boxed{\dot{y}(0) = \frac{i_2(0)}{C} = 847.826\ \frac{V}{s}}$$

4 cont.

c) Determine analytical solution for the zero-input response

$$[D^2 + 195.652D + 869.565]y(t) = 0$$

$$\frac{-195.652 \pm \sqrt{195.652^2 - 4(869.565)}}{2}$$

$$\lambda = -4.5503, -191.1019$$

$y_0(0) = 5$

$\dot{y}_0(0) = 847.826$

$$y_0(t) = C_1 e^{-4.5503t} + C_2 e^{-191.1019t}$$

$$\dot{y}_0(t) = -4.5503 C_1 e^{-4.5503t} - 191.1019 C_2 e^{-191.1019t}$$

$$\Rightarrow \quad 5 = C_1 + C_2$$
$$847.826 = -4.5503 C_1 - 191.1019 C_2$$

$$\Rightarrow \begin{bmatrix} 1 & 1 \\ -4.5503 & -191.1019 \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 847.826 \end{bmatrix}$$

$$\Rightarrow \quad C_1 = 9.\bar{6} \qquad C_2 = -4.\bar{6}$$

$$\Rightarrow \boxed{y_0(t) = 9.\bar{6} e^{-4.5503t} - 4.\bar{6} e^{-191.1019t}}$$

d) Represent the differential equation in state space form

let $x_1(t) = y_0(t)$, $x_2(t) = \dot{y}_0(t)$

$$\dot{x}(t) = A x(t) \qquad A = \begin{bmatrix} 0 & 1 \\ -869.565 & -195.652 \end{bmatrix}$$

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ -869.565 & -195.652 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

e) Program solution and analytical solution plots

f) Compare analytical and numerical

g) Suppose the circuit has non-linear element, such as dependent source. Describe how analytical and numerical solution would change

First the important part is "non-linear" element that means we could not use either of our standard methods for solving. Because there are resistors in our original circuit, the output will eventually decay down to 0. It is expected to follow an exponential decay, but not with the same rate as it would be without any real world dependent sources.