12/06/24 | ECE 6320 / Final | Howes, Kade

1 - Thermostat Control Design (70 points)

The heat flow in a room can be modeled for control design using what is known as a lumped sum model with a resulting linear form

$\dot{x} = Ax + Bu + Ed,\ y = Cx,$

where x is the state vector, consisting of a number of temperature values, u is the input vector, corresponding to various energy inputs into the system, and d is the disturbance vector.

Consider the system contained in sys_1.mat. This system has a single disturbance vector, two control inputs, and five states. Assume that the goal is to keep $x_{d_3} = 20$.

A - Control Design

Before implementing the simulation, define a controller that will have the system converge to the desired value for x_{d_3} assuming that $d = 10$, but with the knowledge that the disturbance could vary linearly with respect to time. Use output feedback LQR for the control design with a desired maximum deviation for x_3 being 1, a maximum deviation for any integral states being 20, and a maximum deviation for all control being 0.5. No consideration should be given to other states in the cost. Clearly state the control law to be employed and all of the steps to create the control law. Prove that the closed-loop system will be globally asymptotically stable.

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}, \quad x_d = \begin{bmatrix} x_{1d} \\ x_{2d} \\ 20 \\ x_{3d} \\ x_{4d} \end{bmatrix}$$

1. Change of state

$$z = x - x_d$$

2. Find feedforward term (try $\dot{x}_d = 0$)

$$B u_{ff} + A x_d - \dot{x}_d = 0$$

$$\Rightarrow u_{ff} = \begin{bmatrix} 110.4972 - 9.8 u_{ff_2} \\ u_{ff_2} \end{bmatrix}, \text{ choose } u_{ff_2} = 0$$

$$\Rightarrow u_{ff} = \begin{bmatrix} 110.4972 \\ 0 \end{bmatrix} \Rightarrow x_d = \begin{bmatrix} 20 \\ 20 \\ 20 \\ 0.1105 \\ 20 \end{bmatrix}$$

3. Express system with full control (sanity check)

$$B \cdot u_{ff} + A x_d - \dot{x}_d = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \checkmark \Rightarrow \dot{z} = Az + B\hat{u}$$

4. Check controllability

$$\Gamma = [B \;\; AB \;\; A^2B \;\; A^3B \;\; A^4B] \Rightarrow \text{rank}(\Gamma) = 5$$

$\Rightarrow$ System is completely controllable

5. Use pole placement (LQR)

Integral Control

a. Design Augmented State (x_3 has ramp disturbance)

$\Rightarrow$ Need two integrals

$$\dot{z} = Ax + B\hat{u}$$

let $\sigma_1 = \int z_3 \, d\tau$

note: $z_3 = x_3 - 20$

$$\Rightarrow \dot{\sigma}_1 = \overset{C_1}{[0\,0\,1\,0\,0]}\, z$$

let $\sigma_2 = \iint z_3 \, d\tau$

$$\Rightarrow \dot{\sigma}_2 = \int \sigma_1 \, d\tau = \overset{C_2}{[1]}\, \sigma_1$$

$$\xi := \begin{bmatrix} z \\ \sigma_1 \\ \sigma_2 \end{bmatrix}$$

$$\Rightarrow \dot{\xi} = \overset{\bar{A}}{\begin{bmatrix} A & 0 & 0 \\ C_1 & 0 & 0 \\ 0 & C_2 & 0 \end{bmatrix}} \begin{bmatrix} z \\ \sigma_1 \\ \sigma_2 \end{bmatrix} + \overset{\bar{B}}{\begin{bmatrix} B \\ 0 \\ 0 \end{bmatrix}} \hat{u}$$

b. Use LQR for pole placement

$$Q = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{12} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/20^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1/20^2 \end{bmatrix}, \quad R = \begin{bmatrix} \frac{1}{0.5^2} & 0 \\ 0 & \frac{1}{0.5^2} \end{bmatrix}$$

checks: 1. $Q = C_c^T C_c$ ✓

When $C_c = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & .05 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & .05 \end{bmatrix}$

2. $(\bar{A}, C_c)$ is completely observable (c.o.)

$$\Omega_c = \begin{bmatrix} C_c \\ C_c\bar{A} \\ \vdots \\ C_c\bar{A}^6 \end{bmatrix} \Rightarrow \text{rank}(\Omega_c) = 7$$

$\rightarrow$ C.O. ✓

c. use lqr to choose K

$$K = \begin{bmatrix} 5.19 & 69.66 & 176.75 & 2.02 & 78.06 & 4.10 & .025 \\ 0.85 & 46.33 & 22.84 & 0.32 & 74.61 & 0.31 & .0004 \end{bmatrix}$$

6. Express full control, $\hat{u} = -K\xi$

$$u = u_{ff} + \hat{u} = u_{ff} - K\xi = u_{ff} - K\left(\begin{bmatrix} z \\ \sigma_1 \\ \sigma_2 \end{bmatrix}\right) = u_{ff} - K\left(\begin{bmatrix} x - x_d \\ \sigma_1 \\ \sigma_2 \end{bmatrix}\right)$$

where K is above (step 5), $u_{ff} = \begin{bmatrix} 110.4972 \\ 0 \end{bmatrix}$, $x_d = \begin{bmatrix} 20 \\ 20 \\ 20 \\ 0.1105 \\ 20 \end{bmatrix}$

Proving closed-loop system is G.A.S.

we know $\dot{z} = Ax + B\hat{u}$

$$\Rightarrow \dot{\xi} = \overbrace{\begin{bmatrix} A & 0 & 0 \\ C_1 & 0 & 0 \\ 0 & C_2 & 0 \end{bmatrix}}^{\bar{A}} \begin{bmatrix} z \\ \sigma_1 \\ \sigma_2 \end{bmatrix} + \overbrace{\begin{bmatrix} B \\ 0 \\ 0 \end{bmatrix}}^{\bar{B}} \hat{u}$$

and $\hat{u} = -K\xi$, so closed loop dynamics are

$\longrightarrow \dot{\xi} = (\bar{A} - \bar{B}K)\xi$, so if $\bar{A} - \bar{B}K$ is a stability matrix, then closed loop system is G.A.S.

Check $\text{eig}(\bar{A} - \bar{B}K) = \left\{ \begin{matrix} -3.67, -0.036, -0.0175, -0.0072 \pm 0.009j, \\ -0.0023, -0.0012 \end{matrix} \right\}$

$\Rightarrow$ closed loop system is G.A.S. since $Re(\lambda) < 0 \ \forall \lambda \in \text{eig}(\bar{A} - \bar{B}K)$

B - Observer Design

Mathematically define an observer that would work with the dynamics. State the dynamic equation for your observer, $\dot{\hat{x}}$. Create an aggregate state that includes a change of state, integrators, and the observer error and show a proof that the closed-loop aggregate dynamics are asymptotically stable. Numerically define each feedback matrix and show that the aggregate closed loop system has appropriate eigenvalues. Use lqr with Q having 10's on the diagonals and R having 1's on the diagonal for the observer design.

Hints:

- Integrator states are not estimated (i.e., they should not show up in $\hat{x}$)
- Your aggregate dynamics will consist of two dynamic components - the true state/change of state (including the integrators) and the error $(x - \hat{x})$

Define observer dynamics

$$\dot{\hat{x}} = A\hat{x} + Bu + L(y - C\hat{x}), \quad e = x - \hat{x} \Rightarrow x = e - \hat{x} \Rightarrow \hat{x} = x - e$$

Creating aggregate state: $z = x - x_d$, $\xi = \begin{bmatrix} z \\ \sigma_1 \\ \sigma_2 \end{bmatrix}$

$$\mathcal{X} := \begin{bmatrix} \xi \\ e \end{bmatrix} = \begin{bmatrix} z \\ \sigma_1 \\ \sigma_2 \\ e \end{bmatrix}$$

$\dot{\xi}$:

$$\dot{\xi} = \bar{A}\xi + B\hat{u} = \bar{A}\xi - \bar{B}K\hat{\xi} \longleftarrow \hat{\xi} := \begin{bmatrix} \hat{z} \\ \sigma_1 \\ \sigma_2 \end{bmatrix} := \begin{bmatrix} \hat{x} - x_d \\ \sigma_1 \\ \sigma_2 \end{bmatrix}$$

$$= \bar{A}\begin{bmatrix} x - x_d \\ \sigma_1 \\ \sigma_2 \end{bmatrix} - \bar{B}K\begin{bmatrix} x - e - x_d \\ \sigma_1 \\ \sigma_2 \end{bmatrix} = \bar{A}\xi - \bar{B}K\xi + \bar{B}K\begin{bmatrix} e \\ 0 \\ 0 \end{bmatrix}$$

$$\dot{\xi} = (\bar{A} - \bar{B}K)\xi + \bar{B}K\begin{bmatrix} e \\ 0 \\ 0 \end{bmatrix} = (\bar{A} - BK)\xi + \bar{B}\bar{K}e$$

$K \in \mathbb{R}^{2\times 7}$

$T \in \mathbb{R}^{7\times 5}$

$\Rightarrow \bar{K} \in \mathbb{R}^{2\times 5}$

where $\bar{K} = K\overset{T}{\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}}$

$y = Cx$

$\dot{e}$:

$$\dot{e} = \dot{x} - \dot{\hat{x}} = Ax + \cancel{Bu} - (A\hat{x} + \cancel{Bu} + L(y - C\hat{x}))$$

$$= Ax - A\hat{x} - LC(x - \hat{x}) = (A - LC)e$$

$$\dot{x}: \quad \dot{x} = \begin{bmatrix} \dot{\xi} \\ \dot{e} \end{bmatrix} = \overbrace{\begin{bmatrix} \bar{A} - \bar{B}K & \bar{B}\bar{K} \\ 0 & A - LC \end{bmatrix}}^{M} \begin{bmatrix} \xi \\ e \end{bmatrix}$$

Since M is block upper triangular, the eigenvalues of M are the eigenvalues of each of it's block diagonals. In other words:

$$eig(M) = \{eig(\bar{A} - \bar{B}K), eig(A - LC)\}.$$

So, if $\bar{A} - \bar{B}K$ and $A - LC$ are both stability matrices, then the closed loop system is asymptotically stable.

Note: $\bar{A} - \bar{B}K$ is already a stability matrix and we will choose L in the next step s.t. $A - LC$ is a stability matrix

Choosing L

1. Check observability

$$\Omega = \begin{bmatrix} C \\ \vdots \\ CA^4 \end{bmatrix} \Rightarrow \text{rank}(\Omega) = 5 \Rightarrow \text{Completely Observable} \checkmark$$

2. LQR

$$Q_L = \begin{bmatrix} 10 & 0 & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 & 0 \\ 0 & 0 & 10 & 0 & 0 \\ 0 & 0 & 0 & 10 & 0 \\ 0 & 0 & 0 & 0 & 10 \end{bmatrix}, \quad R_L = [1]$$

$$L = lqr(A^T, C^T, Q_L, R_L)^T = \begin{bmatrix} 2.6329 \\ 0.7227 \\ 1.8471 \\ 0.0106 \\ 3.0250 \end{bmatrix}$$

3. checking $A-LC$ is a stability matrix

$$eig(A-LC) = \{-4.47, -0.38, -0.02, -0.04, -0.009\}$$

✓ $(A-LC)$ is a stability matrix since $Re(\lambda) < 0 \ \forall \lambda \in eig(A-LC)$

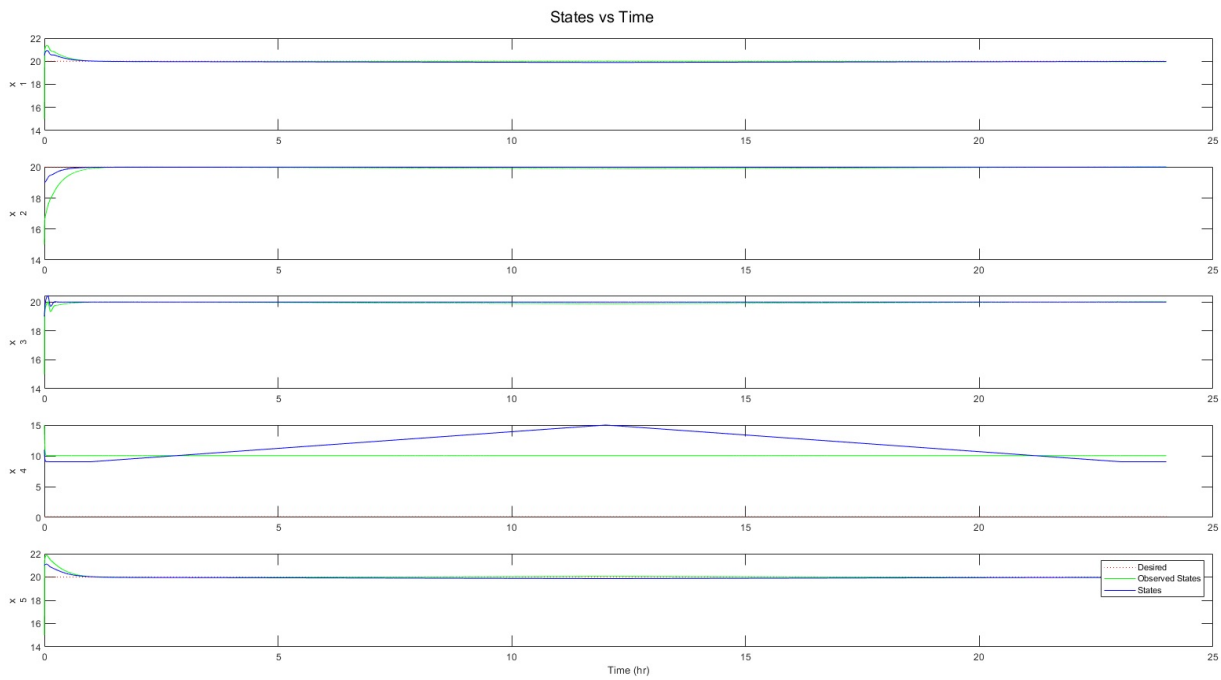
C - Simulation

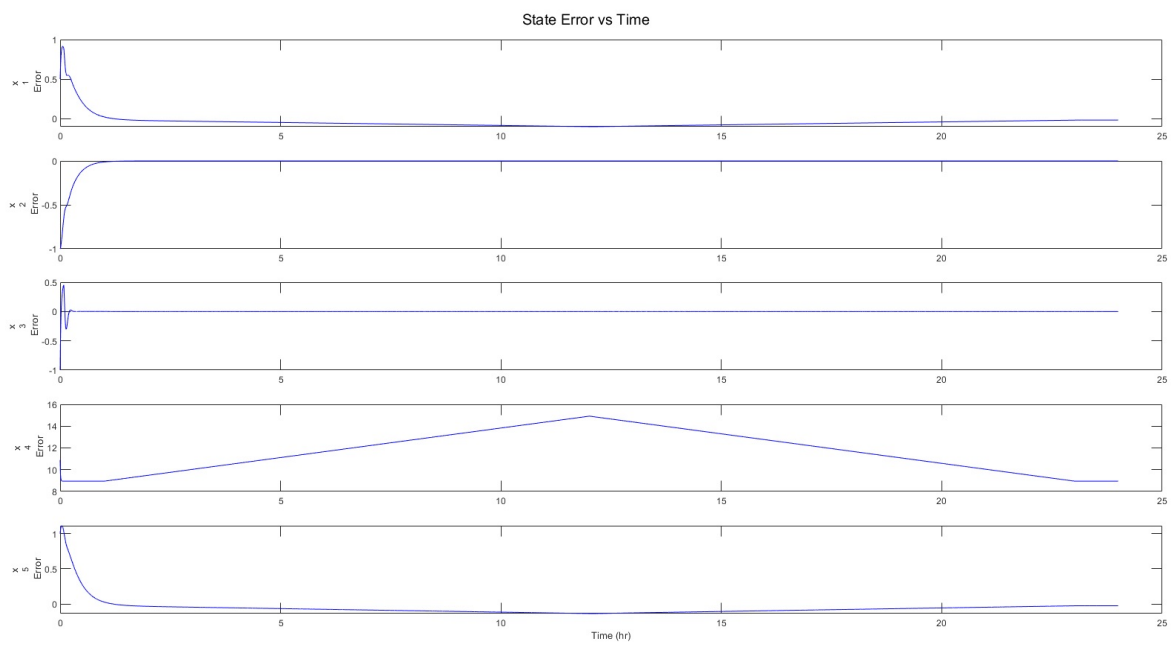
You will now simulate the controlled system. Note that the disturbance actually corresponds to the outside temperature and will vary throughout the day. Use the controller from part A without any knowledge of the true perturbations (i.e., you will lose all points if you cheat and design the control based on the actual values of d in the simulation).

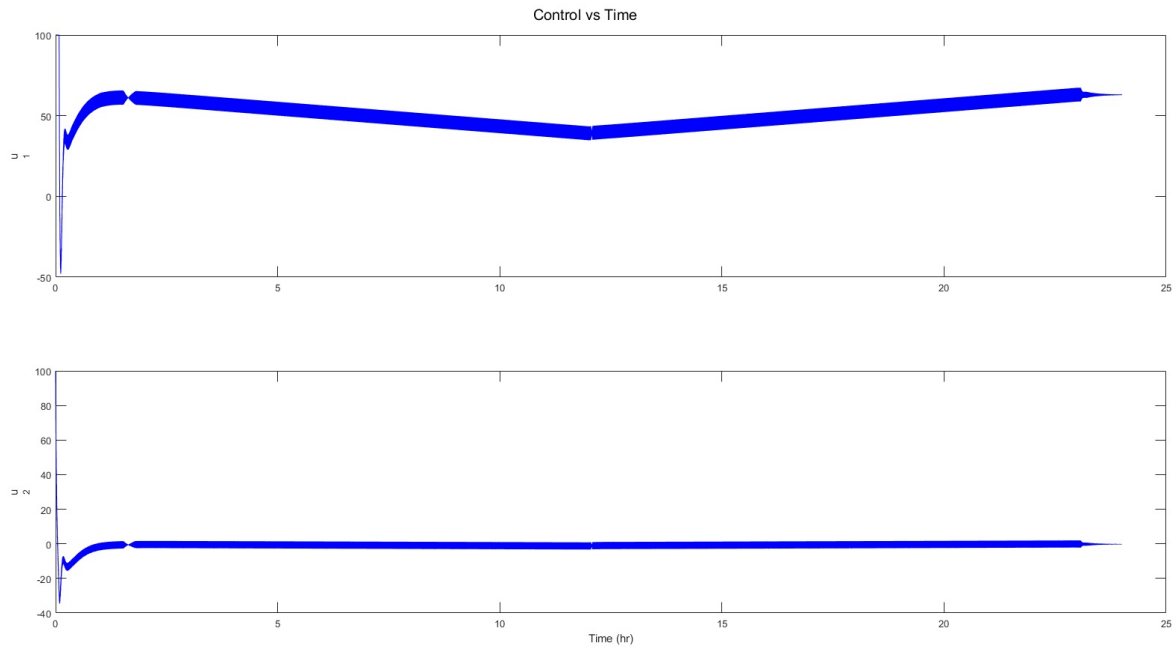
Show the following plots:

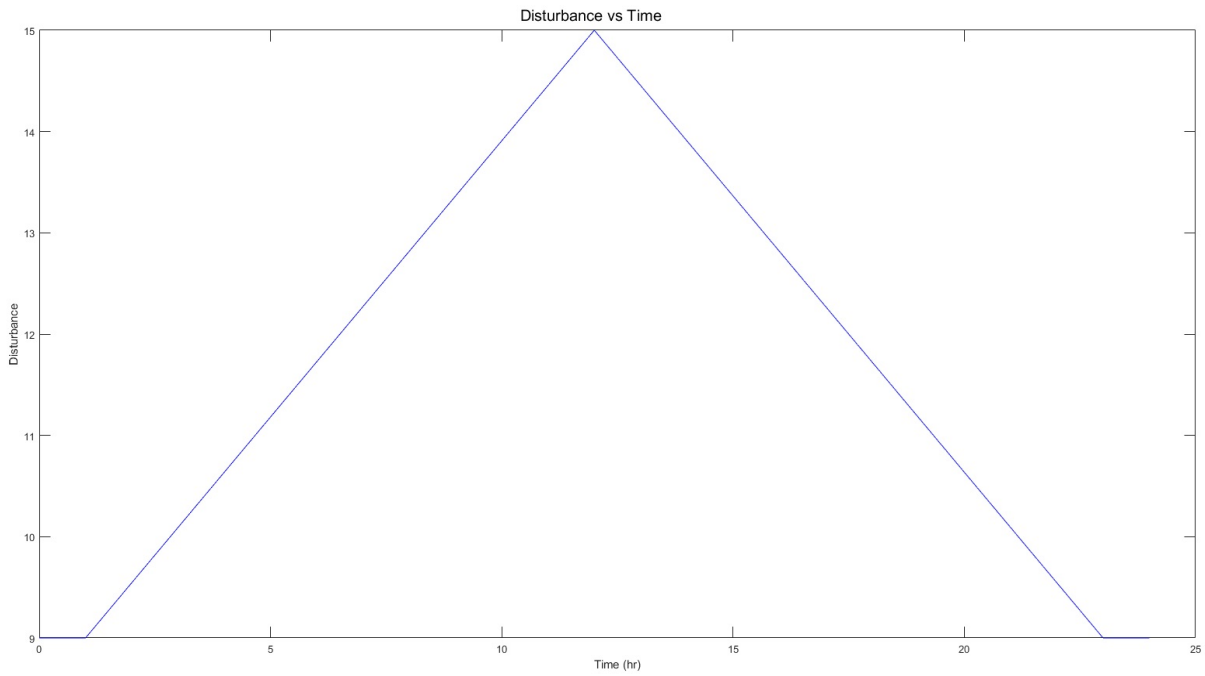
- Actual state, observer state, and desired state overlayed for all five states (i.e., one subplot per state, each subplot containing the actual state, the estimate for that state, and the desired value for that state)
- Error vs time for all five states
- Control vs time
- Disturbance vs time

Comment on the convergence and any deviations from the desired values. Explain why these deviations may occur.









Comments: It is important to notice that state x3 converges to its desired values even though the other states don't necessarily (state x2 appears to converge, but this just happened with how the disturbance affects the dynamics, as no integral states were added for x2). This is expected because if there is any disturbance in the other states that aren't modeled in the A or B matrices, my control law doesn't take those into account. However, I implemented two integral states (a single integral on the error of x3 and a double integral on the error of the state x3). These integral states force the error of state x3 to 0, thus explaining why it is the only state that converges to its desired and thus its error (plot 2) converges to 0 as well.

Additionally, it can be seen from the plots above that the difference of the states from their desired value (error plot 2) corresponds to the shape and time of the disturbance (plot 4). This makes sense as the dynamics of the disturbance are not modeled in the control law.

2 - Linear Control Design (30 points)

Consider the system (A,B,C) and desired state, x_d, defined in sys2.mat. Do the following (Turn in a file called "linear_control_design.m" with any work that you completed, make sure to show all the steps of your work in the pdf)

A- Minimal System

Define a system $(\hat{A}, \hat{B}, \hat{C})$ that realizes the same transfer function as (A,B,C) but has the least number of states possible. Display these values in the pdf and turn in a .mat file titled, "prob2a.mat" that has "Ahat", "Bhat", and "Chat" values.

Check controllability/observability:

$\text{rank}(\Gamma) = 5 \Rightarrow$ not completely controllable

$\text{rank}(\Omega) = 4 \Rightarrow$ not completely observable

Kalman Decomposition:

From textbook page 217, I used minreal() to compute the kalman decomposition transform matrix T.

note: T is inverse how we did in class

Then, $\hat{A} = T \cdot A \cdot T^{-1}$

$$= \begin{bmatrix} \boxed{\begin{matrix} 3.98 & 1.1 & -1.5 \\ -0.01 & 2.93 & 0.61 \\ -0.08 & 0.98 & 1.08 \end{matrix}} & \begin{matrix} 0 & 0 & 0.93 & 8.11 \\ 0 & 0 & -1.72 & -3.64 \\ 0 & 0 & 3.99 & 10.43 \end{matrix} \\ \begin{matrix} -2.04 & -12.35 & -9.18 \\ 0.996 & 1.52 & -0.14 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{matrix} & \begin{matrix} -1 & -9.66 & -20.97 & -6.19 \\ 0 & 1 & 1.23 & -5.75 \\ 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & -2 \end{matrix} \end{bmatrix}$$

$\hat{A}_{11}$

$$\hat{B} = T \cdot B = \begin{bmatrix} \boxed{\begin{matrix} 0.96 & 0.24 & 0.58 & 0.63 \\ -0.69 & 0.38 & -0.29 & -0.02 \\ 0.89 & 0.02 & 0.93 & -0.02 \end{matrix}} \\ \begin{matrix} -4.63 & -5.35 & -2.61 & 2.90 \\ -2.0 & 0.31 & -0.94 & -0.52 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{matrix} \end{bmatrix}$$

$\hat{B}_1$

$$\hat{C} = CT^{-1} =$$

$$= \begin{bmatrix} \boxed{\begin{matrix} 1.63 & 1.64 & -0.49 \\ 1.54 & -0.89 & -1.23 \\ 0.03 & -0.07 & 1.03 \end{matrix}} & \begin{matrix} 0 & 0 & 1.52 & 3.48 \\ 0 & 0 & -0.28 & 0.51 \\ 0 & 0 & 0.74 & 2.72 \end{matrix} \end{bmatrix}$$

$\hat{C}_1$

So, a minimum realization for (A, B, C) as $(\hat{A}, \hat{B}, \hat{C})$ is:

$$\hat{B} = \begin{bmatrix} 0.96 & 0.24 & 0.58 & 0.63 \\ -0.69 & 0.38 & -0.29 & -0.02 \\ 0.89 & 0.02 & 0.93 & -0.02 \end{bmatrix}$$

$$\hat{A} = \begin{bmatrix} 3.98 & 1.1 & -1.5 \\ -0.01 & 2.93 & 0.61 \\ -0.08 & 0.98 & 1.08 \end{bmatrix}$$

$$\hat{C} = \begin{bmatrix} 1.63 & 1.64 & -0.49 \\ 1.54 & -0.89 & -1.23 \\ 0.03 & -0.07 & 1.03 \end{bmatrix}$$

B- Design a Controller

Design a controller that could be used to control the system to x_d. If it is not possible to design a controller, state why. Continue to design the controller even if it is not possible completing the remaining steps of the control design as if it had not failed. Use *place()* for your control design with poles at *-[1, 2, 3, ... m]* where *m* is the number of poles needing to be placed. State the complete control law and resulting closed loop eigenvalues. Also turn in a .mat file titled, *"prob2b.mat"* with the values "uff" and "K".

1. Change of state

$$z = x - x_d$$

2. Find feed-forward term

$$B u_{ff} + A x_d - \dot{x}_d = 0$$

$$\Rightarrow \quad u_{ff} = \begin{bmatrix} -2 \\ -1 \\ -4 \\ -7 \end{bmatrix}$$

3. Express system with full control (sanity check)

$$u = u_{ff} + \hat{u} \Rightarrow \dot{z} = Ax + B\hat{u} \quad \checkmark$$

4. Check controllability

$$\text{rank}(\mathcal{C}) = 5 < 7 \Rightarrow \text{not completely controllable}$$

Check stabilizability : Do controllable decomposition

$$T_c = [\text{orth}(\mathcal{C}) \quad \text{null}(\mathcal{C}^T)], \quad \hat{A}_c = T^{-1} A T, \quad \hat{B}_c = T^{-1} B$$

$\hat{A}_{c11}$

$$\Rightarrow \hat{A}_c = \begin{bmatrix} 3.90 & 6.61 & 8.38 & 10.69 & 4.62 & 6.87 & -17.20 \\ 0.15 & 3.10 & -4.54 & -8.79 & 5.53 & -10.21 & 5.79 \\ 0.0002 & 0.003 & 0.45 & -1.53 & -2.27 & 2.12 & 12.90 \\ -0.0001 & -0.0008 & -0.30 & 0.14 & -1.28 & -5.30 & -5.57 \\ 0.0003 & 0.0007 & -0.14 & -0.40 & 0.40 & 2.91 & 1.02 \\ 0 & 0 & 0 & 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -2 \end{bmatrix}$$

$$\hat{B}_c = \begin{bmatrix} -4.03 & -4.29 & -2.29 & 1.94 \\ 1.91 & 3.14 & 1.09 & -2.17 \\ 1.96 & 0.77 & 1.23 & -0.44 \\ 0.02 & 0.41 & -0.39 & -0.48 \\ 0.16 & 0.19 & -0.03 & 0.43 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\hat{B}_{c_1}$

$\boxed{Re(\lambda) < 0 \ \forall \lambda}$

Since $\hat{A}_{c_{22}} = \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix} \Rightarrow eig(\hat{A}_{c_{22}}) = \{-2, -2\}$

and $\hat{A}_{c_{22}}$ is the uncontrollable portion of the system, the system is stabilizable

5. Pole Placement (using place())

$\hat{u} = -K_1 \hat{z}$ where $\hat{z} = T_c^{-1} z$

$K_1 = place(\hat{A}_{c_{11}}, \hat{B}_{c_1}, [-1, -2, -3, -4, -5])$

$$K_1 = \begin{bmatrix} -1.21 & -3.07 & 4.71 & 10.48 & -0.65 \\ -4.97 & -3.2 & -8.99 & -4.81 & 1.60 \\ 3.29 & 5.65 & -0.399 & -15.59 & -0.26 \\ -6.51 & -6.66 & -6.72 & -1.61 & 4.20 \end{bmatrix}$$

$$\hat{u} = -Kz \quad \text{where} \quad K = [K_1 \quad 0]\, T_c^{-1}$$

$$\Rightarrow \quad K = \begin{bmatrix} -7.11 & -0.01 & -4.14 & 5.83 & 5.91 & -1.94 & -1.73 \\ 9.47 & -1.77 & 3.89 & -2.93 & 3.82 & 2.16 & -2.39 \\ 6.43 & 0.51 & 2.79 & -8.40 & -10.83 & 2.46 & 6.50 \\ 8.47 & -3.17 & 2.97 & 1.18 & 7.50 & 1.72 & -0.47 \end{bmatrix}$$

6. Express full control

$$u = u_{ff} + \hat{u} = u_{ff} - K(x - x_d)$$

where K as above and $u_{ff} = \begin{bmatrix} -2 \\ -1 \\ -4 \\ -7 \end{bmatrix}$

Check closed loop eigenvalues

$$\text{eig}(A - BK) = \{-1, -2, -2, -2, -3, -4, -5\} \checkmark$$

C- Design an Observer

Design an observer that could be used to estimate the state of the system. If it is not possible to design an observer, state why. Continue to design the observer even if it is not possible completing the remaining steps of the observer design as if it had not failed. Use *place()* for your observer design with poles at *-[1, 2, 3, ... m]* where *m* is the number of poles needing to be placed. State the complete observer formulation and resulting eigenvalues of the observer error. Also turn in a .mat file titled, *"prob2c.mat"* with the value "L".

Check observability:

$\text{rank}(\Omega) = 4 < 7 \Rightarrow$ not completely observable

Check detectability: (No observable decomp)

$T_o = [\text{orth}(\Omega^T) \quad \text{null}(\Omega)]$, $\hat{A}_o = T^{-1}AT$, $\hat{C}_o = CT$

$\hat{A}_{o11}$

$$\Rightarrow \hat{A}_o = \left[\begin{array}{cccc|ccc} 4.05 & -0.08 & 0.08 & 0.0001 & 0 & 0 & 0 \\ 0.72 & 2.95 & -0.55 & -0.0005 & 0 & 0 & 0 \\ 0.134 & 0.08 & -2.01 & -0.002 & 0 & 0 & 0 \\ -0.44 & -0.66 & 9.07 & 1.01 & 0 & 0 & 0 \\ \hline -1.48 & -5.64 & -0.82 & 2.98 & -5.13 & -1.64 & 0.11 \\ -4.31 & 12.47 & 19.36 & -4.81 & 13.82 & 4.17 & 1.06 \\ 0.74 & 4.67 & -5.47 & -5.76 & 6.12 & 2.40 & -1.04 \end{array}\right]$$

$\hookrightarrow \hat{A}_{o22}$

and

$$\Rightarrow \hat{C}_o = \left[\begin{array}{cccc|ccc} -2.52 & 0.13 & 3.64 & 0.003 & 0 & 0 & 0 \\ -0.95 & -2.0 & 6.21 & 0.0002 & 0 & 0 & 0 \\ -0.03 & 0.30 & 2.68 & 1.32 & 0 & 0 & 0 \end{array}\right]$$

$\hat{C}_{o1}$

$\hat{A}_{o22}$ is the unobservable portion and has eigenvalues:

$\text{eig}(\hat{A}_{o22}) = \{1, -2, -1\} \Rightarrow$ $\boxed{\text{Not detectable}}$

$\Rightarrow$ not possible to design observer that is guaranteed to converge, will continue to design observer anyways

Pole Placement (using place())

$\hat{z} = T_o^{-1} z$

$\Rightarrow L = T \begin{bmatrix} L_1 \\ 0 \end{bmatrix}$

$$L_1 = (\text{place}(\hat{A}_{o11}^T, \hat{C}_{o1}^T, [-1, -2, -3, -4]))^T$$

$$L_1 = \begin{bmatrix} -1.98 & -0.19 & 2.19 \\ 0.53 & -2.41 & -0.60 \\ 0.08 & 0.02 & 0.51 \\ -0.03 & 0.08 & 3.44 \end{bmatrix}$$

$$L = T_o \begin{bmatrix} L_1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1.77 & -0.83 & -2.25 \\ 0.07 & 0.15 & 1.65 \\ 0.88 & -0.42 & -1.13 \\ -0.13 & 0.82 & -1.14 \\ 0.51 & 2.10 & 0.16 \\ 0.13 & -0.30 & -0.13 \\ -0.12 & 0.52 & 2.61 \end{bmatrix}$$

where observer dynamics are:

$$\dot{\hat{x}} = A\hat{x} + Bu + L(y - C\hat{x})$$

$\Rightarrow$ closed loop observer error eigenvalues =

$$= \text{eig}(A - LC) = \{-1, -2, -3, -4, 1, -2, -1\}$$

3 - Evaluation (+5 points)

Provide a paragraph briefly describing how much time you spent on the final as well as rough time on each of the homework assignments. Also describe what portions of the final were especially difficult or easy.

Final:
This final took me around 8-9 hours to complete. I found it to be well-balanced honestly. The hardest part for me was for sure the 1st question of the two. Specifically, finding the rest of x_d was new to me, since we have always been given a full vector for x_d, not just a single value. Also, dealing with the aggregation of two additional integral states as well as the observer states all at the same time was one of the more difficult parts. I found the 2nd question to be a bit more straightforward, however I did struggle to figure out the Kalman decomposition at first, since we had only done really simple systems in class (the intersections of subspaces using MATLAB was not clear to me), so I didn't know how to get the correct T without just using the MATLAB built in function minreal(). Although with this function, this question was much simpler. I liked the MATLAB-based final project, where we get to see results of our work live in simulations and get the tools to help us through the project.

HW1: This homework took me a really long time, longer than most other weeks. Maybe around 10-12 hours. It felt very tedious and hard because my linear algebra skills were very weak before this class.

HW2: This linearization homework took me a longer time too, around 8-10 hours is my best guess. I remember it feeling tedious because I did it all by hand and also checked my answers in MATLAB, but didn't use MATLAB to accelerate the process much.

HW3: I don't remember this homework as well, but I believe it took me around 6 hours and was pretty average in terms of difficulty and time.

HW4: I believe this homework took me around 4-5 hours and was slightly less average in terms of difficulty and time. I found the discretization work repetitive after the first one, but pretty easy.

HW5: I believe this homework too me 4 hours to complete, and a lot of it was on the first problem. I found the stability characterization to be relatively straightforward.

HW6: This homework took me a long time relatively. Around 10 hours.

HW8: This homework was fairly average in terms of time to complete. Around 5 hours.

HW9: This homework was well-rounded and took me around 6 hours. The extra credit took me a couple of hours alone.

HW10: This homework took me around 7 hours. The extra credit simulation took me a lot of that time.

HW11: This homework took me around 3 hours. It was a shorter than average for me.

HW12: This homework was just above average in terms of time. It took me around 7 hours I believe.

HW13: This homework was very short. Around 30 minutes.

HW14: This homework took me about 2 hours all together.