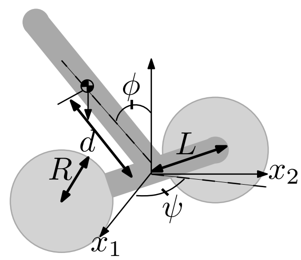

Figure 1: Shown is a diagram of the inverted pendulum robot with the symbols defined in Table 1.

Inverted Pendulum Robot

ECE/MAE 6320 Linear Controls

We utilize the model developed in (Kim, Kim, & Kwak, 2005) for an inverted pendulum robot. The state vector describing the inverted pendulum robot can be expressed as

$$x = \begin{bmatrix} x_1 & x_2 & v & \psi & \omega & \phi & \dot{\phi} \end{bmatrix}^T, \tag{1}$$

where x_1, x_2, v, and ψ are defined as before and ϕ is the tilt angle from the vertical, as depicted in Figure 1. This allows the dynamics of the system to be expressed as

$$\dot{x} = f(x, u) = \begin{bmatrix} v\cos(\psi) & v\sin(\psi) & \dot{v} & \omega & \ddot{\psi} & \dot{\phi} & \ddot{\phi} \end{bmatrix}^T \tag{2}$$

where $\dot{v}$, $\ddot{\psi}$, and $\ddot{\phi}$ are obtained from the following equations:

$$3(m_c + m_s)\dot{v} - m_s d\cos(\phi)\ddot{\phi} + m_s d\sin(\phi)(\dot{\phi}^2 + \omega^2) = -\frac{1}{R}(\alpha + \beta), \tag{3}$$

$$\big((3L^2 + \frac{1}{2R^2})m_c + m_s d^2\sin^2(\phi) + I_2\big)\ddot{\psi} + m_s d^2\sin(\phi)\cos(\phi)\omega\dot{\phi} = \frac{L}{R}(\alpha - \beta), \tag{4}$$

Table 1: This table defines the symbols used in the dynamics of the two-wheel inverted pendulum robot. The numeric values are given in (Kim et al., 2005).

Table of Symbols	
m_c	Mass of wheel
m_s	Mass of body
d	Distance from center of wheel axis to center of gravity
L	Half the distance between the wheels
R	Radius of wheels
I_2	Rotational inertia of the body about the ψ axis
I_3	Rotational inertia of the body about the axel
$\alpha.\ \beta$	Wheel Torques

$$m_s d \cos(\phi)\dot{v} + (-m_s d^2 - I_3)\ddot{\phi} + m_s d^2 \sin(\phi)\cos(\phi)\dot{\phi}^2 + m_s g d \sin(\phi) = \alpha + \beta, \tag{5}$$

and the symbols are defined in Table 1.

References

Kim, Y., Kim, S., & Kwak, Y. (2005). Dynamic analysis of a nonholonomic two-wheeled inverted pendulum robot. *Journal of Intelligent & Robotic Systems*, *44*(1), 25–46.