Optimization Assignment - 2

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Problem Statement **-Show that the altitude of the right circular cone of maximum volume that can be inscribed in a sphere of radius R is 4R/3**

Solution

Let r=5
volume of cone is given V = $\frac{1}{3}\pi R^2$h
h=r+$\sqrt{r^2 - R^2}$

$$V = \frac{1}{3}\pi R^2 r + \frac{1}{3}\pi R^2\sqrt{r^2 - R^2} \tag{1}$$

$$V' = \frac{2}{3}\pi Rr + \frac{2\pi Rr^2 - 3\pi R^3}{3\sqrt{r^2 - R^2}} \tag{2}$$

Gradient descent

$$\implies V = \frac{1}{3}\pi R^2 r + \frac{1}{3}\pi R^2\sqrt{r^2 - R^2} \tag{3}$$

$$x_{n+1} = x_n + \alpha\nabla V \tag{4}$$

$$\implies x_{n+1} = x_n + \alpha\left(\frac{2}{3}\pi Rr + \frac{2\pi Rr^2 - 3\pi R^3}{3\sqrt{r^2 - R^2}}\right) \tag{5}$$

Taking $x_0 = r - 1, \alpha = 0.001$ and precision = 0.00000001, values obtained using python are:

$$\boxed{\text{Maxima} = 155.14037795505135} \tag{6}$$

$$\boxed{\text{Maxima Point} = 4.71404516335876} \tag{7}$$

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