# MATRIX ANALYSIS USING PYTHON


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## Contents



## 1 Problem

The foci of the ellipse $\frac{x^2}{16}+\frac{y^2}{b^2}=1$ and the hyperbola $\frac{x^2}{144}$-$\frac{y^2}{81}=\frac{1}{25}$ coincide . Then the value of $b^2$

## 2 Construction

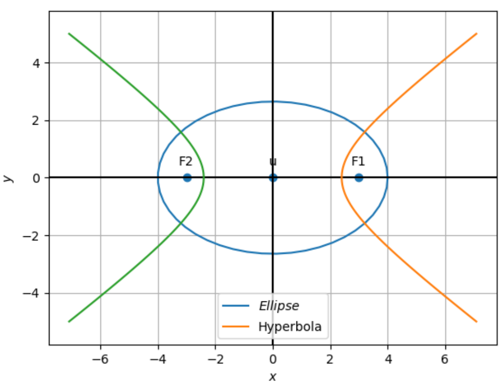


Figure of construction

## 3 Solution

Hyperbola equation :

$$\frac{x^2}{144}-\frac{y^2}{81}=\frac{1}{25} \tag{1}$$

The standard equation of the conics is given as :

$$\mathbf{x}^\top \mathbf{V}\mathbf{x}+2\mathbf{u}^\top \mathbf{x}+f=0 \tag{2}$$

The given ellipse can be expressed as conics with parameters

$$\lambda_1=25/144, \lambda_2=-25/81 \tag{3}$$

$$\mathbf{V}=\begin{pmatrix}\lambda_1 & 0\\ 0 & \lambda_2\end{pmatrix}, \mathbf{u}=\begin{pmatrix}0\\0\end{pmatrix}, f=-1 \tag{4}$$

Eccentricity:

$$e=\sqrt{1-\frac{\lambda_1}{\lambda_2}} \tag{5}$$

$$\implies e=15/12 \tag{6}$$

Foci:

$$f0=-f, \mathbf{e}_1=\begin{pmatrix}1\\0\end{pmatrix} \tag{7}$$

$$\mathbf{F}_1=e\sqrt{\frac{|f_0|}{\lambda_2\left(1-e^2\right)}}\mathbf{e}_1 \tag{8}$$

$$\mathbf{F}_2=-e\sqrt{\frac{|f_0|}{\lambda_2\left(1-e^2\right)}}\mathbf{e}_1 \tag{9}$$

$$\implies \mathbf{F}_1=\begin{pmatrix}3\\0\end{pmatrix}, \mathbf{F}_2=\begin{pmatrix}-3\\0\end{pmatrix} \tag{10}$$

Ellipse equation :

$$\frac{x^2}{16}+\frac{y^2}{b^2}=1 \tag{11}$$

The standard equation of the conics is given as :

$$\mathbf{x}^\top \mathbf{V}_1\mathbf{x}+2\mathbf{u}_1^\top \mathbf{x}+f_1=0 \tag{12}$$

The given ellipse can be expressed as conics with parameters

$$\lambda_3=b^2, \lambda_4=16 \tag{13}$$

$$\mathbf{V}_1=\begin{pmatrix}\lambda_3 & 0\\ 0 & \lambda_4\end{pmatrix}, \mathbf{u}_1=\begin{pmatrix}0\\0\end{pmatrix}, f_1=-16b^2 \tag{14}$$

Eccentricity:

$$e=\sqrt{1-\frac{\lambda_3}{\lambda_4}} \tag{15}$$

$$e=\sqrt{1-\frac{b^2}{16}} \tag{16}$$

Foci:

$$f_0=-f_1, \mathbf{e}_1=\begin{pmatrix}1\\0\end{pmatrix} \tag{17}$$

$$\mathbf{F}_4=e\sqrt{\frac{|f_0|}{\lambda_4\left(1-e^2\right)}}\mathbf{e}_1 \tag{18}$$

$$\mathbf{F}_5=-e\sqrt{\frac{|f_0|}{\lambda_4\left(1-e^2\right)}}\mathbf{e}_1 \tag{19}$$

Then equate the $\mathbf{F}_1$ $=\mathbf{F}_4$we get the $b^2$

$$b^2=\lambda_4-(\mathbf{F}_1^T\mathbf{e}_1)^2 \tag{20}$$