MATRICES

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FWC22032 IITH Future Wireless Communication (FWC) Assignment 5

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Contents

1 Problem

Find the equation of the circle whose radius is 5 and which touches the circle x2+y2-2x-4y-20=0 at the point (5, 5).

2 Construction

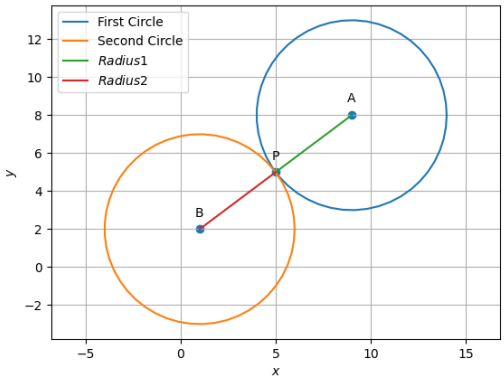

Figure of construction

The steps for constructing above figure are :

1. Generate a circle1 of radius $r1$ with centre $\mathbf{B}$
2. the circle2 whose radius $r2$ is 5 and which touches circle1 at the point $\mathbf{P}$
3. By using Section formula find the center of the cicle2 point $\mathbf{A}$
4. Find the equations of the circle2 by using standard equation of conics

3 Solution

Circle equation : $x^2 + y^2 - 2x - 4y - 20 = 0$
The standard equation of the conics is given as :

$$\mathbf{x}^\top \mathbf{V}\mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \tag{1}$$

The given circle can be expressed as conics with parameters

$$\mathbf{V} = \mathbf{I} \tag{2}$$

$$\mathbf{u} = -\begin{pmatrix}1\\2\end{pmatrix} \tag{3}$$

$$f = -20 \tag{4}$$

Radius and Centre are

$$r_1 = \sqrt{\mathbf{u}^\top \mathbf{u} - f} \tag{5}$$

$$\mathbf{B} = -u \tag{6}$$

The input parameters for this construction are

Symbol	Value	Description
$\mathbf{B}$	$\begin{pmatrix}1\\2\end{pmatrix}$	Centre of circle1
r_2	5	radius of circle2
$\mathbf{P}$	$\begin{pmatrix}5\\5\end{pmatrix}$	Point P

$$\mathbf{P} = \frac{\mathbf{A}(r_2) + \mathbf{B}(r_1)}{(r_1) + (r_2)} \tag{7}$$

$$2\mathbf{P} - \mathbf{B} = \mathbf{A} \tag{8}$$

to find "f2" from radius formula taking "A" as center we get

$$f_2 = \mathbf{u}^\top \mathbf{u} + r^2 \tag{9}$$

The standard equation of the conics is given as :

$$\mathbf{x}^\top \mathbf{V}_2 \mathbf{x} + 2\mathbf{u}_2^\top \mathbf{x} + f_2 = 0 \tag{10}$$

$$\mathbf{V}_2 = \mathbf{I} \tag{11}$$

$$\mathbf{A} = -u_2 \tag{12}$$