Users guide to SPHERE: GPU based discrete element method software

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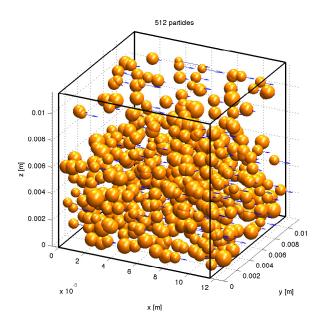
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Abstract

This document is the official documentation for the SPHERE discrete element modelling software. It presents the theory behind the discrete element method (DEM), the structure of the software C source code, and the Matlab configuration methods for handling input- and output data.

SPHERE is developed by Anders Damsgaard Christensen under supervision of David Lundbek Egholm and Jan A. Piotrowski, all of the department of Geology, University of Aarhus, Denmark. This document is a work in progress, and is still in an early, unfinished state. This document is typeset with \LaTeX standard) to make space for handwritten notes.



Revision history

Date	Doc. version	SPHERE version	Description	
2010-12-06	0.1	Beta 0.03	Initial draft	

Contents

1	Introduction								
2	Discrete element method theory								
3	SPHERE source code structure								
	3.1 The SPHERE algorithm	. 5							
	3.1.1 Host and device memory types	. 6							
	3.2 The main loop	. 8							
	3.3 Performance	. 8							
	3.3.1 Particles and computational time	. 8							
	3.3.2 Parallel computing	. 8							
	3.3.3 Compute profiler results	. 8							
	3.4 Compilation	. 8							
4	Model setup	8							
	4.1 Creating the particles	. 8							
	4.2 Introducing heterogenieties	. 9							
	4.3 Periodic boundary conditions	. 10							
5	DEM simulation using SPHERE	10							
6	Data analysis in MATLAB	10							
	6.1 Exporting plots	. 11							
	6.2 Importing data	. 12							
A	Folder structure	14							
В	SPHERE source code variables	15							
\mathbf{C}	Parameter structure in MATLAB	16							
D	O Sample: initsetup.m								
Re	eferences	20							

1 Introduction

The SPHERE-software is used for three-dimensional discrete element method (DEM) particle simulations. The source code is written in C, and compiled by the user. The main computations are performed on the graphics processing unit (GPU) using NVIDIA's general purpose parallel computing architecture, CUDA.

The ultimate aim of the SPHERE software is to simulate soft-bedded subglacial conditions, while retaining the flexibility to perform simulations of granular material in other environments. The requirements to the host computer are:

- UNIX, Linux or Mac OS X operating system.
- GCC, the GNU compiler collection.
- A CUDA-enabled GPU with compute capability 1.1 or greater¹.
- The CUDA Developer Drivers and the CUDA Toolkit².

For simulation setup and data handling, a MATLAB installation of a recent version is essential. There is however no requirement of MATLAB on the computer running the SPHERE calculations, i.e. model setup and data analysis can be performed on a separate device. Command examples in this document starting with the symbol '\$' are executed in the terminal of the operational system, and '>>' means execution in MATLAB. All numerical values in this document, the source code, and the configuration files are typeset with strict respect to the SI unit system.

2 Discrete element method theory

The discrete element method (or distinct element method) was initially formulated by Cundall and Strack (1979). It simulates the physical behavior and interaction of discrete, unbreakable particles, with their own mass and inertia, under the influence of e.g. gravity and boundary conditions such as moving walls. By discretizing time into small time steps ($\Delta t \approx 10^{-8} \text{s}$) eulerian integration of Newton's second law of motion is used to predict the new position and kinematic values for each particle from the previous sums of forces. This lagrangian approach is ideal for simulating discontinuous materials, such as granularities. The complexity of the computations is kept low by representing the particles as spheres, which keeps contact-searching algorithms simple.

3 SPHERE source code structure

The source code is located in the sphere/src/ folder, and named sphere.cu. After compiling the SPHERE binary (see sub-section 3.4), the procedure of a creating and handling a simulation is typically arranged in the following order:

¹See http://www.nvidia.com/object/cuda_gpus.html for an official list of NVIDIA CUDA GPUs.

²Obtainable free of charge from http://developer.nvidia.com/object/cuda_3_2_downloads.html

- 1. Setup of particle assemblage, physical properties and conditions in MATLAB, described in section 4, page 8.
- 2. Execution of SPHERE software, which simulates the particle behavior as a function of time, as a result of the conditions initially specified in MATLAB. Described in section 5, page 10.
- 3. Inspection, analysis, interpretation and visualization of SPHERE output in MATLAB. Described in section 6, page 10.

3.1 The SPHERE algorithm

The SPHERE-binary is launched from the system terminal by passing the simulation ID as an input parameter, e.g.: ./sphere simulation_test_04. The sequence of events in the program is the following:

- 1. System check, including search for NVIDIA CUDA compatible devices.
- 2. Initial data import from binary Matlab file.
- 3. Allocation of memory for all host variables (particles, grid, etc.).
- 4. Continued import from binary MATLAB file.
- 5. Memory allocation of device memory.
- 6. Transfer of data from host to device variables.
- 7. Initialization of CUDPP radix sort configuration.
- 8. OpenGL initialization.
- 9. Status and data written to <simulation_ID>.status.dat and <simulation_ID>.output0.bin, both located in output/ folder.
- 10. Main loop (while time.current <= time.total):
 - (a) CUDA thread synchronization point.
 - (b) calcHash<<<,>>>(...): Particle-grid hash value calculation.
 - (c) CUDA thread synchronization point.
 - (d) cudppSort(...): CUDPP radix sort of particle-grid hash array.
 - (e) cudaMemset(...): Writing zero value (Oxffffffff) to empty grid cells.
 - (f) reorderParticles<<<,>>>(...): Reordering of particle arrays, based on sorted particle-grid-hash values.
 - (g) CUDA thread synchronization point.
 - (h) cudaBindTexture(...): Binding of textures (position, linear velocity, angular velocity, particle radii).
 - (i) interact<<<,>>>(...): For each particle: Search of contacts in neighbor cells, processing of optional collisions and updating of resulting forces (F_{res}) . Values are read from textures, but written to read/write device memory arrays.

- (j) CUDA thread synchronization point.
- (k) integrate<<<,>>>(...): Updating of spatial degrees of freedom by eulerian integration.
- (1) cudaUnbindTexture(...): Unbinding of textures.
- (m) Update of timers and loop-related counters (e.g. time.current)
- (n) If file output interval is reached:
 - i. Optional write of data to output binary (<simulation_ID>.output#.bin).
 - ii. Update of <simulation ID>.status#.bin.
- (o) Return to point 10a, unless time.current >= time.total, in which case the program continues to point 11.
- 11. Liberation of device and host memory.
- 12. End of program.

The length of the computational time steps (time.dt) is calculated via equation 1, where length of the time intervals is defined by:

$$\Delta t = 0.5 \times \min\left(\sqrt{\frac{\rho R^2}{K}}\right) \tag{1}$$

where ρ is the particle material density, R is particle radius, and K is bulk modulus. This equation ensures that the strain signal (traveling at the speed of sound) is resolved twice while traveling through the smallest particle.

3.1.1 Host and device memory types

A full, listed description of the SPHERE source code variables can be found in appendix B, page 15. There are four types of memory types employed in the SPHERE source code, with different characteristics and physical placement in the system (figure ??). Three-dimensional variables (e.g. spatial vectors in E^3) are stored as float4 arrays, since float4 read and writes can be coalesced, while float3's cannot. This alone yields a $\sim 20\times$ performance boost, even though it involves 25% more (unused) data.

Host memory is the main random-access computer memory (RAM), i.e. read and write memory accessible by CPU processes, but inaccessible by CUDA kernels executed on the device.

Device memory is the main, global device memory. It resides off-chip on the GPU, in the form of up to 2GB DRAM.

Constant memory

Textures

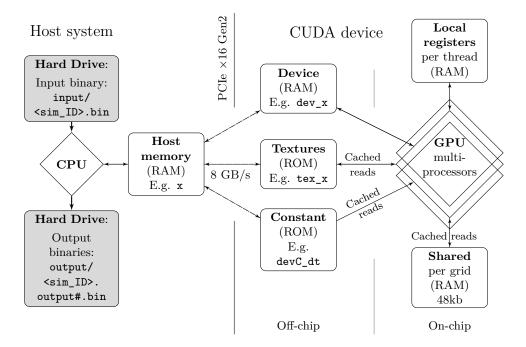


Figure 1: Flow chart of system memory types and communication paths. RAM: Random-Access Memory (read + write), ROM: Read-Only Memory. Specified communication path bandwidth on test system (2010 Mac Pro w. Quadro 4000 noted.

3.2 The main loop

The SPHERE software calculates particle movement and rotation based on the forces applied to it, by application of Newton's law of motion (Newton's second law with constant particle mass: $F_{\rm net} = m \cdot a_{\rm cm}$). This is done in a series of algorithmic steps, see list on page 5. The steps are explained in the following sections with reference to the SPHERE-source file; sphere.cu. The intent with this document is *not* to give a full theoretical background of the methods, but rather how the software performs the calculations.

3.3 Performance

3.3.1 Particles and computational time

With an increasing amount of particles, obviously more calculations have to be performed each time step, which directly translates to an increasing computational time.

The size of the computational timestep length is fixed at a sufficiently low value, so that the particle response is calculated several times while the speed of sound (the strain signal) travels through even the smallest particle, see equation 1. If there is a large variety of particle sizes, the particle with the smallest radius determines dt for all calculations.

3.3.2 Parallel computing

The DISC code is heavily parallized, i.e. it does carries out multiple calculations simultaneously, utilizing the GPU.

3.3.3 Compute profiler results

3.4 Compilation

SPHERE is supplied with a makefile which helps the compilation process. Open a terminal, go to the src/subfolder and type make. The GNU Make will return the parameters passed to the individual CUDA and GNU compilers (nvcc and gcc). The resulting binary file (sphere) is placed in the SPHERE root folder.

4 Model setup

In Matlab, enter the mfiles/-directory as the current folder (example: >> cd /code/sphere/mfiles/). For each new experiment setup, it might be a good approach to copy the software root-folder, thus saving past configurations and data.

4.1 Creating the particles

To begin with, the model surroundings and the particles are created. Particle sizes are randomly drawn from either a *log-normal-* or *uniform* distribution. These particles are situated in a fixed grid-like setup.

Using the MATLAB Editor or another editor, open the file mfiles/initsetup.m. Here you set:

- The filename of the output file (fn)
- The number of particles (p.np)
- The particle size distribution (logn: log-normal or uni: uniform), including
 - Mean size (p.m)Variance (p.v)
- Physical parameters (params.g: gravity, p.rho: density, p.k_n: normal stiffness, p.k_s: shear stiffness), etc...)
- Time parameters (time.dt: computational time step length, time.current: starting time, time.total: total simulation time, time.file_dt: interval between output generation)

After any modifications the file needs to be saved. Write the specified model setup using the following command in MATLAB:

```
>> initsetup
```

This will write the binary file for SPHERE to read, display a histogram of the particle size distribution, and show a figure of the model setup.

4.2 Introducing heterogenieties

Until now, all grains in the model have the same physical properties. Heterogenieties can be introduced in the same way as the coloring above. The find()-function in MATLAB can be used to set different properties on particles, that fulfill a specific requirement, e.g. are contained between geometric border-values in the coordinate system in the initial setup.

Please note, that these deviant values need to be set after the general physical values in template.m. The following is an example, that assigns new values for cohesion (C) and angle of internal friction (ϕ) for the bottom 0.02 m particles:

```
\label{eq:weak-bottom-layer} \% \text{Heterogenieties} \\ \% \text{Weak bottom layer} \\ \textbf{y} = \textbf{p.x}(:,2); \\ \textbf{I} = \text{find}(\textbf{y} < 0.02); \\ \% \text{Particles defined as quantity I} \\ \textbf{phi} = \textbf{1}; \\ \% \text{Angle of internal friction} \\ \textbf{DEM.particles.friction}(\textbf{I}) = \text{tan}(\text{phi*pi}/180)*\text{ones}(\text{length}(\leftarrow \textbf{I}),1); \\ \textbf{C} = \textbf{10}; \\ \% \text{Cohesion} \\ \textbf{DEM.particles.cohesion}(\textbf{I}) = \textbf{C*ones}(\text{length}(\textbf{I}),1); \\ \textbf{DEM.particles.matflag}(\textbf{I}) = \textbf{2}; \\ \end{cases}
```

Here, the affected particles are given a different matflag-value, for visual identification of the particles with different properties.

4.3 Periodic boundary conditions

For simulating mechanisms that involve large horizontal strains, as in a ringshear apparatus, it is a good approach to make the left and right borders cyclic. That means that a particle disappearing out of the right side will reappear in the same vertical position in the left side. Particles can affect each other with contact forces in the same way. This has the advantage that the total number of particles can be kept low in relation to a very wide model setup.

The cyclic borders are in MATLAB enabled by the command DEM.rm.periflag = 1. In this configuration, only the top and bottom walls are needed. A specific normal-pressure is applied in pascal $(Pa^{-1} = \frac{N}{m})$ (denote the value with a negative sign for downward oriented pressure). The shearing motion is applied by fixing the bottom particles to a horizontal velocity of zero (0), and moving the uppermost particles at a constant horizontal velocity (e.g. $0.1\frac{m}{s}$). The particles are not fixed in the vertical directions, since particles still need to be able to pass each other by volumetric expansion.

5 DEM simulation using SPHERE

After the initial model preferences have been set up in MATLAB and the binary input file is written, the DEM-calculations can be initialized from the terminal. Specify the input file name as a parameter for SPHERE:

```
$ ./sphere simulation_name
```

In Matlab, the state of the calculations can be checked with:

```
>> status('simulation_name')
```

The basics of the DEM algorithm, used in SPHERE, is described in section ??, page ??. While SPHERE is running, output binary files are placed in the output/folder with the format simulation_name.output#.bin. On UNIX systems, the CPU usage of active processes can be monitored with the command 'top'. GPU usage is monitored using e.g. the NVIDIA Compute Visual Profiler.

6 Data analysis in Matlab

A number of preconfigured visualization methods are featured in show.m. It shows a figure of the particle assemblage, created on base of the data from a specific output-file. This output file is selected with the command show('file',#), where the second parameter is the number of the output file (the numbering starts from zero (0)). Even though the SDEM calculations (./start-script) have not been completed, the latest output file can be visualized in MATLAB, as long as at least one output-file has been generated:

```
>> show('file','latest')
```

A series of graphs, including total kinetic energy, dissipation energy, number of particles and time steps as functions of time can be displayed with:

```
>> show('energy')
```

When defining a wall-position as a function of pressure, not velocity, all wall velocities can be displayed with:

```
>> show('wallspeed')
```

And if the wall-position is a function of velocity, not pressure, the wall pressures can be displayed with:

```
>> show('wallforce')
```

The total accumulated horizontal displacement of the particles:

```
>> show('sheardisp')
```

If simulating a shearing motion, it may be desired to create a graph, that shows the horizontal velocity of each particle. This is done by using the 'veloprofile'-option in show(), for example:

```
>> show('file','latest','veloprofile')
```

Additional arguments can be used in the show-command, to visualize various parameters, i.e. pressure or strain for each particle. The colorbar-function adds a bar explaining the values and colors:

```
>> show('file','latest','field','pressure')
>> colorbar
>> show('file','latest','field','eplast')
>> colorbar
```

For displaying the colorbar with a fixed span:

```
>> show('file','latest','field','eplast','clim',[0;5]) >> colorbar
```

6.1 Exporting plots

The MATLAB function exportplots(mode) is included for automatically plotting a range of output files, and exporting them as image files. It plots the raw DEM-data from the last run experiment with show() and saves the output as image files. Open mfiles/exportplots.m, and check the folder, filename and graphics format settings. After saving, call the function in MATLAB:

```
>> exportplots()
```

This creates an image for each output file in the output/directory, and with the default settings, saves them in the folder flic/. During the process, MATLAB will display a waitbar, showing the progress. If needed, the export may be halted by clicking the 'Cancel' button in the waitbar window. <u>Do not</u> press

Ctrl-C; after killing the export this way, a restart of MATLAB is needed before repeating the command. If, for some reason, the code crashes wile running, the waitbar is forcibly closed by:

```
>> set(0, 'ShowHiddenHandles', 'on')
>> delete(get(0, 'Children'))
```

To export the plots with particles colored according to the elastic plasticity or pressure:

```
>> exportplots('eplast')
>> exportplots('pressure')
```

To create a plot over the velocity profile for each output file, and exporting it:

```
>> exportplots('veloprofile')
```

To export all normal-, eplast-, pressure and veloprofiles as images use the command exportall(), which creates a series of images (combi) containing the four view modes simultaneously:

```
>> exportall(1)
```

When the goal of the export is to create an animation in the avi or mpegformat, I recommend exporting to the jpeg-format, which can be chosen in exportplots.m. In most video editing software the png-format is only poorly, if at all, supported.

6.2 Importing data

All parameters and values from the simulation results can be imported into the MATLAB workspace. In MATLAB, make sure the current folder is the mfiles/directory inside the DISC folder. The DEM structure from the current configuration is loaded by DEM = DEMload(), which reads input/DEM.mat. The resulting variable structure imported into the MATLAB workspace, and consists of a hierarchy of variables, as mapped out in appendix C. While DEMload() loads the initial model setup, the MATLAB function DEMloadfnr(fnr) loads particle configurations from output file with number fnr.

The following is a number of examples, showing possible ways to visualize particle speed³. The following series of commands loads all particle positions and creates a three-dimensional plot with speed on the z-axis from output file #25:

```
>> [p, grid, time, params] = freadbin('../output/', 5);  
>> x=p.x(:,1);  
>> y=p.x(:,2);  
>> z=sqrt(p.vel(:,1).^2+p.vel(:,2).^2);  
>> plot3(x,y,z,'.')
```

³Inspired by: http://geology.wlu.edu/connors/primers/Surfaces_and_Grids_in_Matlab/Surfaces_and_Grids_in_Matlab.htm

Or as a triangular surface for better visualization; lone points distributed in a 3D space, visualized in 2D, are difficult to comprehend visually:

```
>> tri=delaunay(x,y);
>> trisurf(tri,x,y,z)
```

Or as a (smoothed) surface with interpolated data point values in between particles:

```
>> res=100; %Resolution in x- and y-direction >> rangeX=min(x): (max(x)-min(x))/res: max(x); >> rangeY=min(y): (max(y)-min(y))/res: max(y); >> [X,Y]=meshgrid(rangeX,rangeY); >> Z=griddata(x,y,z,X,Y); >> surf(X,Y,Z)
```

To make a contour plot of the gridded data:

```
>> resZ=100; %Resolution in z-direction 
>> rangeZ=\min(z):(\max(z)-\min(z))/\text{resZ}:\max(z); 
>> contourf(X,Y,Z,rangeZ);
```

Combining contours and surfaces:

```
>> hold on
>> surf(X,Y,Z,'EdgeColor','none','FaceColor','interp','←
    FaceLighting','phong')
>> contour3(X,Y,Z,rangeZ,'k')
>> figure(gcf)
```

Another example: Arrow field of particle velocities:

```
>> s=0.02; %Velocity scaling factor
>> quiver(p.x(:,1), p.x(:,2),...
p.vel(:,1)*s, p.vel(:,2)*s,'AutoScale','off');
>> axis([0 0.2 0 0.11])
```

Appendix

A Folder structure

SPHERE sub directory and file description:

- 'cudaprofiler/': Folder for saving CUDA Compute Visual Profiler project data.
- 'doc/': Folder containing documentation
 - 'graphics/': Documentation image files
 - 'sphere-doc.pdf': This document
 - 'sphere-doc.tex': LATEX source code for this document
 - Auxillary LATEX files
- 'flic/': Folder for image creation plugins (not included)
 - 'eps/': EPS image format directory
 - 'gif/': GIF image format directory
 - 'png/': PNG image format directory
- 'include/': Directory for preprocessor-files, utilized in sphere.cu.
 - 'nrutil.h': Numerical recipes header file.
 - 'random.c': Contains the function ran0(...) that returns a random number.
- 'input/': Folder for simulation configuration binaries.
- 'mfiles/': Files which contain functions available to MATLAB.
 - 'bubbleplot.m': bubbleplot(p) plots particles in structure p using bubbleplot3.m
 - 'bubbleplot3.m': bubbleplot(x,y,z,r) Produces a three-dimensional bubble plot of spheres with coordinates (x,y,z) and radius (r). Created by Peter Bodin, obtained through Mathworks File Exchange⁴.
 - 'freadbin.m': [p, grid, time, params] = freadbin(path, fn) reads binary file named fn in directory path, and returns MATLAB structures (appendix C).
 - 'fwritebin.m': fwritebin(path, fn, p, grid, time, params) writes MAT-LAB structures (appendix C) to binary file named fn in directory path.
 - 'initsetup.m': initsetup() creates particles in grid-like arrangement.
 The number of particles and the PSD is specified. Calling the function creates a .bin-file in the input/-folder.
 - 'status.m': status('simulation_name') writes the status of the current model run to the command window.
- 'output/': Folder containing output .bin-files. For each output time-step (time.file_dt), a new simulation_name.output#.bin is created, where # is the output time step number. Simultaneously, the status-file is updated.
 - 'simulation_name.logfile.txt': Text file containing eventual error messages from the computation.
 - 'simulation_name.output#.bin': Main SPHERE output data.
 - 'simulation_name.status.dat': File showing the completed calculation progress in percentage, and time steps.
- 'src/': Folder containing SPHERE source code.

⁴http://www.mathworks.com/matlabcentral/fileexchange/8231-bubbleplot3

B SPHERE source code variables

Variables without a prefix are located in the host memory. Variables with the prefix 'dev_' are stored in the read & write device memory, 'tex_' means data is stored in the read-only texture memory on the device.

Name	\mathbf{Type}	Description
x[i]{.x,.y,.z}	float4	Particle coordinates. Returns the coordinates of
$dev_x[i]\{.x,.y,.z\}$		particle i, optionally only the value in dimension
$\texttt{tex_x[i]}\{.\texttt{x,.y,.z}\}$		x, y or z. E.g. x[124].x returns x-axis placement
		of particle 124, read from host memory.

Table 1: Some important variables in **sphere.cu**. The integer **i** refers to the particle number. Curly brackets ({}) indicate optional, exclusive extensions.

C Parameter structure in MATLAB

The DEM structure from the current configuration is loaded into MATLAB by the command [p, grid, time, params] = freadbin(path, fn), which reads the specified binary file. The resulting variable structures imported into the MATLAB workspace are the following:

p.	np	Number of particles
	psd	Particle size distribution
	m	Mean particle radius
	v	Particle radius variance
	radius	Radius of all particles
	x	Particle positions in E^3 (x,y,z)
	vel	Particle linear velocities
	angvel	Particle angular velocities
	vel0	Fixed linear particle velocitis
	force	Sum of forces upon each particle
	torque	Sum of torque upon each particle
	rho	Particle material densities
	k_n	Particle normal stiffnesses
	k_s	Particle shear stiffnesses
	mu	Inter-particle contact friction coefficients
	C	Inter-particle tensile strengths (cohesion)
	E	Young's modulus for each particle
	K	Bulk modulus for each particle
	nu	Poisson's ratio for each particle
time.	dt	Computational time step length
	current	Current time
	total	Total simulation time
	${\tt file_dt}$	Interval between output file generation
	$step_count$	Current step count
grid.	nd	Number of dimensions
_	origo	Cartesian space origo
	num	Number of grid cells in each dimension
	L	Grid length in each dimension
params.	g	Gravitational acceleration
_	global	Equals 1: Particle mechanical parameters are global,
		equals 0: Particle mechanical parameters are individual
		and set in the p structure.

 Table 2:
 MATLAB variable structure

The values are written using the corresponding command fwritebin(path, fn, p, grid, time, params).

D Sample: initsetup.m

```
1
     function initsetup()
    \% initsetup() creates a model setup of particles in 3D with cubic
 2
    % packing. Specify PSD and desired number of particles, and the
   \% function will determine the model dimensions, and fill the \leftrightarrow
 4
        space
    \% with a number of particles from a specified particle size \hookleftarrow
 5
         distribution.
 6
    close all;
 8
    %Simulation project name
 9
    simulation_name = '3dtest';
10
    %Physical, constant parameters
11
    params.g = 9.80665; %standard gravity, by definition 9.80665 m/s\leftarrow
12
13
    % No. of dimensions
14
15
    grid.nd = 3;
    grid.origo = [0 0 0]; %Coordinate system origo
16
17
18
    % Number of particles
19
   p.np = 5e2:
20
21
    % Create grid
    grid.num = ceil(nthroot(p.np,grid.nd)) * ones(grid.nd, 1); %
22
         Square/cubic
23
    p.np = grid.num(1)^grid.nd;
24
25 | %% Particle size distribution
   p.psd = 'logn'; %PSD: logn or uni
26
27
28 p.m = 440e-6; %Mean size
    \texttt{p.v} = \texttt{p.m} \! * \! 0.00001; \; \% \text{Variance}
29
30
    \%p.m = 22; \%Mean size
   \%p.v = p.m*0.001; \%Variance
31
32
33
    p.radius = zeros(1, p.np);
           = zeros(p.np, grid.nd);
34
    p.x
35
    if strcmp(p.psd, \lceil logn \rceil) %Log-normal PSD. Note: Keep variance \hookleftarrow
36
        small.
37
         mu = log((p.m^2)/sqrt(p.v+p.m^2));
38
         sigma = sqrt(log(p.v/(p.m^2)+1));
         p.radius = lognrnd(mu, sigma, 1, p.np); %Array of particle radii
39
40
    elseif strcmp(p.psd, 'uni') %Uniform PSD between rmin and rmax
        rmin = p.m - p.v*1e5;

rmax = p.m + p.v*1e5;
41
42
         %rmin = 0.1*dd; rmax = 0.4*dd;
43
         \texttt{p.radius} = (\texttt{rmax-rmin}) * \texttt{rand}(\texttt{p.np}, 1) + \texttt{rmin}; \; \% \text{Array of particle} \; \leftarrow
44
             radii
45
    end
46
47
    %Display PSD
48
    figure(2); hist(p.radius);
    title(['PSD: 'num2str(p.np) ' particles, m = ' num2str(p.m) ' m'
49
        1);
50
51
   % Other particle parameters
             = zeros(p.np, grid.nd); % Velocity vector
53 | p.vel
```

```
54 | p.angvel = zeros(p.np, grid.nd); % Angular velocity
     p.vel0 = zeros(p.np, grid.nd); % Pre-velocity vector
p.force = zeros(p.np, grid.nd); % Force vector
55
56
     \texttt{p.torque} \, = \, \underline{\texttt{zeros}} \, (\, \texttt{p.np} \, , \, \, \underline{\texttt{grid.nd}} \, ) \, ; \, \, \% \, \, \texttt{Torque} \, \, \, \texttt{vector}
57
58
     params.global = 1; \% 1: Physical parameters global, 0: individual \leftarrow
59
            per particle
60
               = 3600*ones(p.np,1);
                                                     % Density
     p.rho
                =\ 109680*{\tt ones}\,(\,{\tt p.np}\,,1\,)\;;
                                                     \% Normal stiffness
61
     p.k_n
                = 109680*ones(p.np,1); % Shear stifness
     p.k_s
                = 1*ones(p.np,1);
     p.mu
                                                     % Inter-particle contact friction ←
63
             coefficient
     p.C
                = 0*zeros(p.np,1);
                                                     % Inter-particle cohesion
                 =\ 10\,\mathtt{e9}*\mathtt{ones}\,(\,\mathtt{p}\,.\,\mathtt{np}\,,1\,)\;;
                                                     % Young's modulus
65
     p.E
66
     р.К
                 = 38e9*ones(p.np,1);
                                                     % Bulk modulus
                                                     % Poisson's ratio
67
                = 0.38*ones(p.np,1);
     p.nu
68
69
     %Time parameters
70
     time.dt
                            = 1e3*0.5*min(sqrt((p.rho(:).*p.radius(:).^2)./p. \leftarrow
          K(:))); %Computational delta t
71
     time.current
                          = 0.0;
                             = 1.0; %Total simulation time [s]
72
     time.total
73
     time.file_dt
                             = 0.1; %Interval between output#.bin generation [←
          \mathbf{s}
74
     time.step\_count = 0;
76
     % Calculate particle coordinates
     %Grid unit length. Maximum particle diameter determines grid size
77
     GU = 2*max(p.radius)*1.10; \% Ten percent margin
78
     %Model world side length (cubic arrangement)
grid.L = [GU*grid.num(1) GU*grid.num(2) GU*grid.num(3)];
79
80
81
     % Particle coordinates by filling grid.
82
83
     x = GU/2:GU:grid.L(1);
     y = GU/2:GU:grid.L(2);
84
     z = GU/2:GU:grid.L(3);
85
86
87
     % Wall coordinates
                                                                             , grid.L(1),0,\leftarrow
88
      walls = [0,0,0]
                                         , grid.L(1), 0, 0
                                        , 0,0,grid.L(3)
            grid.L(3)
                                                                            ; ... %Bottom wall←
              (x,z plane)
89
                    0, \operatorname{grid} . L(2), 0, \operatorname{grid} . L(1), \operatorname{grid} . L(2), 0, \operatorname{grid} . L(1), \operatorname{grid} . \hookrightarrow
                          \texttt{L(2)}\,, \texttt{grid}\,. \texttt{L(3)} \ , \ 0\,, \texttt{grid}\,. \texttt{L(2)}\,, \texttt{grid}\,. \texttt{L(3)} \ ; \ \dots \ \%\mathsf{Top} \ \hookleftarrow
                          wall (x,z plane)
                                         , 0, grid.L(2),0
                                                                              , grid.L(1), grid. \leftrightarrow
90
                                                , grid.L(1),0,0
                                                                                   ; ... %Front←
                          L(2),0
                           wall (x,y plane)
                    0\,,0\,,\mathrm{grid}\,.\,L\,(3)\quad,\quad 0\,,\mathrm{grid}\,.\,L\,(2)\,\,,\mathrm{grid}\,.\,L\,(3)\quad,\quad \mathrm{grid}\,.\,L\,(1)\,\,,\mathrm{grid}\,.\,\hookleftarrow
91
                          \texttt{L(2)}\,, \texttt{grid}\,.\, \texttt{L(3)} \ , \ \texttt{grid}\,.\, \texttt{L(1)}\,, 0\,, \texttt{grid}\,.\, \texttt{L(3)} \ ; \ \dots \ \% \\ \texttt{Back} \ \hookleftarrow
                           wall (x,y plane)
                                         , 0, grid.L(2),0
                                                                             , 0, \operatorname{grid} . L(2), \hookleftarrow
92
                          grid . L(3)
                                                    , 0,0,grid.L(3)
                                                                                        ; ... %↩
                           Left wall (y,z plane)
                     \operatorname{grid}.L(1) , 0, 0 \quad , \quad \operatorname{grid}.L(1) , \operatorname{grid}.L(2) , 0 \quad , \quad \operatorname{grid}.L(1) , \operatorname{grid}. \hookrightarrow
93
                          L(2), grid.L(3), grid.L(1), 0, grid.L(3)]; %Right \leftarrow
                           wall (y,z plane)
94
95
     % Plot
96
     figure (1);
97
     % %draw walls
99
          for i=1:6,
```

```
100
101
                       \begin{array}{l} \text{line} \ (\text{walls} \ (\text{i}, [4\ ,7])\ , \text{walls} \ (\text{i}, [5\ ,8])\ , \text{walls} \ (\text{i}, [6\ ,9])\ , \ ' \leftarrow \\ \text{linewidth} \ '\ , 2\ , \ '\text{color} \ '\ , \ '\text{k}')\ ; \\ \text{line} \ (\text{walls} \ (\text{i}, [10\ ,1])\ , \text{walls} \ (\text{i}, [11\ ,2])\ , \text{walls} \ (\text{i}, [12\ ,3])\ , \ ' \leftarrow \end{array}
102
103
                                linewidth',2,'color','k');
104
                   end:
105
          \begin{array}{ll} [\, {\tt X} \ {\tt Y} \ {\tt Z} \,] \ = \ \underset{{\tt X} = {\tt X} \, (\, : \, )}{\operatorname{meshgrid}} \, (\, {\tt x} \, , \, {\tt y} \, , \, {\tt z} \,) \, ; \\ {\tt X} = \! {\tt X} \, (\, : \, ) \, ; \ {\tt Y} = \! {\tt Y} \, (\, : \, ) \, ; \ {\tt Z} = \! {\tt Z} \, (\, : \, ) \, ; \end{array} 
106
107
108
109
          p.x = [X Y Z];
110
         \%Particle positions randomly modified by +/- 5 percent
111
112
         p.x = p.x .* (rand(p.np, grid.nd)*0.1 + 0.95);
113
         % Plot particles in bubble plot
114
         bubbleplot(p);
115
116
         % Write output binary
117
118
         \texttt{fwritebin('.../input/', [simulation\_name '.bin'], p, grid, time,} \leftarrow
                  params);
119
120
          \quad \text{end} \quad
```

References

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