sphere Documentation

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This is the official documentation for the sphere discrete element modelling software. This document aims at guiding the installation process, documenting the usage, and explaining the relevant theory.

sphere is developed by Anders Damsgaard as part as his Ph.D. project, under supervision of David Lunbek Egholm and Jan A. Piotrowski, all of the Department of Geoscience, Aarhus University, Denmark. The author welcomes interested third party developers. This document is a work in progress.

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ONE

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1.1 Introduction

The sphere-software is used for three-dimensional discrete element method (DEM) particle simulations. The source code is written in C++, CUDA C and Python, and is compiled by the user. The main computations are performed on the graphics processing unit (GPU) using NVIDIA's general purpose parallel computing architecture, CUDA. Simulation setup and data analysis is performed with the included Python API.

The ultimate aim of the sphere software is to simulate soft-bedded subglacial conditions, while retaining the flexibility to perform simulations of granular material in other environments.

The purpose of this documentation is to provide the user with a walk-through of the installation, work-flow, dataanalysis and visualization methods of sphere. In addition, the sphere internals are exposed to provide a way of understanding of the discrete element method numerical routines taking place.

Note: Command examples in this document starting with the symbol \$ are meant to be executed in the shell of the operational system, and >>> means execution in Python. IPython is an excellent, interactive Python shell.

All numerical values in this document, the source code, and the configuration files are typeset with strict respect to the SI unit system.

1.1.1 Requirements

The build requirements are:

- A Nvidia CUDA-supported version of Linux or Mac OS X (see the CUDA toolkit release notes for more information)
- GNU Make
- CMake, version 2.8 or newer
- The GNU Compiler Collection (GCC)
- The Nvidia CUDA toolkit, version 5.0 or newer

In Debian GNU/Linux, these dependencies can be installed by running:

```
$ sudo apt-get install build-essential cmake nvidia-cuda-toolkit
```

Unfortunately, the Nvidia Toolkit is shipped under a non-free license. In order to install it in Debian GNU/Linux, add non-free archives to your /etc/apt/sources.list.

The runtime requirements are:

A CUDA-enabled GPU with compute capability 1.1 or greater.

· A Nvidia CUDA-enabled GPU and device driver

Optional tools, required for simulation setup and data processing:

- Python
- Numpy
- · Matplotlib
- Python bindings for VTK
- · Imagemagick
- ffmpeg. Soon to be replaced by avconv!

In Debian GNU/Linux, these dependencies can be installed by running:

```
$ sudo apt-get install python python-numpy python-matplotlib python-vtk \
imagemagick libav-tools
```

sphere is distributed with a HTML and PDF build of the documentation. The following tools are required for building the documentation:

- Sphinx
 - sphinxcontrib-programoutput
- Doxygen
- Breathe
- dvipng
- TeX Live, including pdflatex

In Debian GNU/Linux, these dependencies can be installed by running:

```
$ sudo apt-get install python-sphinx python-pip doxygen dvipng \
          python-sphinxcontrib-programoutput texlive-full
$ sudo pip install breathe
```

Git is used as the distributed version control system platform, and the source code is maintained at Github. sphere is licensed under the GNU Public License, v.3.

Note: All Debian GNU/Linux runtime, optional, and documentation dependencies mentioned above can be installed by executing the following command from the doc/ folder:

```
$ make install-debian-pkgs
```

1.1.2 Obtaining sphere

The best way to keep up to date with subsequent updates, bugfixes and development, is to use the Git version control system. To obtain a local copy, execute:

```
$ git clone git@github.com:anders-dc/sphere.git
```

1.1.3 Building sphere

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sphere is built using cmake, the platform-specific C/C++ compilers, and nvcc from the Nvidia CUDA toolkit.

If you plan to run sphere on a Kepler GPU, execute the following commands from the root directory:

```
$ cmake . && make
```

If you instead plan to execute it on a Fermi GPU, change set (GPU_GENERATION 1) to set (GPU_GENERATION 0 in CMakeLists.txt.

In some cases the CMake FindCUDA module will have troubles locating the CUDA samples directory, and will complain about helper_math.h not being found.

In that case, modify the CUDA_SDK_ROOT_DIR variable in src/CMakeLists.txt to the path where you installed the CUDA samples, and run cmake . && make again. Alternatively, copy helper_math.h from the CUDA sample subdirectory common/inc/helper_math.h into the sphere src/directory, and run cmake and make again. Due to license restrictions, sphere cannot be distributed with this file.

After a successfull installation, the sphere executable will be located in the root folder. To make sure that all components are working correctly, execute:

```
$ make test
```

All instructions required for building sphere are provided in a number of Makefile's. To generate the main sphere command-line executable, go to the root directory, and invoke CMake and GNU Make:

```
$ cmake . && make
```

If successfull, the Makefiles will create the required data folders, object files, as well as the sphere executable in the root folder. Issue the following commands to check the executable:

```
$ ./sphere --version
```

The output should look similar to this:

Command u'../../sphere -version' failed: [Errno 2] No such file or directory

The build can be verified by running a number of automated tests:

```
$ make test
```

The documentation can be read in the reStructuredText-format in the doc/sphinx/ folder, or in the HTML or PDF formats in the folders doc/html and doc/pdf.

Optionally, the documentation can be built using the following commands:

```
$ cd doc/sphinx
$ make html
$ make latexpdf
```

To see all available output formats, execute:

```
$ make help
```

1.1.4 Updating sphere

To update your local version, type the following commands in the sphere root directory:

```
$ git pull && cmake . && make
```

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1.1.5 Work flow

After compiling the sphere binary, the procedure of a creating and handling a simulation is typically arranged in the following order:

- Setup of particle assemblage, physical properties and conditions using the Python API.
- Execution of sphere software, which simulates the particle behavior as a function of time, as a result of the conditions initially specified in the input file.
- Inspection, analysis, interpretation and visualization of sphere output in Python, and/or scene rendering using the built-in ray tracer.

1.2 Discrete element method

The discrete element method (or distinct element method) was initially formulated by Cundall and Strack (1979). It simulates the physical behavior and interaction of discrete, unbreakable particles, with their own mass and inertia, under the influence of e.g. gravity and boundary conditions such as moving walls. By discretizing time into small time steps, explicit integration of Newton's second law of motion is used to predict the new position and kinematic values for each particle from the previous sums of forces. This Lagrangian approach is ideal for simulating discontinuous materials, such as granular matter. The complexity of the computations is kept low by representing the particles as spheres, which keeps contact-searching algorithms simple.

1.3 Fluid simulation and particle-fluid interaction

A new and experimental addition to *sphere* is the ability to simulate a mixture of particles and a Newtonian fluid. The fluid is simulated using an Eulerian continuum approach, using a custom CUDA solver for GPU computation. This approach allows for fast simulations due to the limited need for GPU-CPU communications, as well as a flexible code base.

The following sections will describe the theoretical background, as well as the solution procedure and the numerical implementation.

1.3.1 Derivation of the Navier Stokes equations with porosity

Following the outline presented by Limache and Idelsohn (2006), the continuity equation for an incompressible fluid material is given by:

$$\nabla \cdot \boldsymbol{v} = 0$$

and the momentum equation:

$$\rho \frac{\partial \boldsymbol{v}}{\partial t} + \rho(\boldsymbol{v} \cdot \nabla \boldsymbol{v}) = \nabla \cdot \boldsymbol{\sigma} + \rho \boldsymbol{f}$$

Here, v is the fluid velocity, ρ is the fluid density, σ is the Cauchy stress tensor, and f is a body force (e.g. gravity). For incompressible Newtonian fluids, the Cauchy stress is given by:

$$\sigma = -pI + \tau$$

p is the fluid pressure, I is the identity tensor, and τ is the deviatoric stress tensor, given by:

$$\boldsymbol{\tau} = \nu \nabla \boldsymbol{v} + \nu (\nabla \boldsymbol{v})^T$$

By using the following vector identities:

$$\nabla \cdot (p\mathbf{I}) = \nabla p$$
$$\nabla \cdot (\nabla \mathbf{v}) = \nabla^2 \mathbf{v}$$
$$\nabla \cdot (\nabla \mathbf{v})^T = \nabla (\nabla \cdot \mathbf{v})$$

the deviatoric component of the Cauchy stress tensor simplifies to the following, assuming that spatial variations in the viscosity can be neglected:

$$= -\nabla p + \nu \nabla^2 \mathbf{v}$$

Since we are dealing with fluid flow in a porous medium, additional terms are introduced to the equations for conservation of mass and momentum. In the following, the equations are derived for the first spatial component. The solution for the other components is trivial.

The porosity value (in the saturated porous medium the volumetric fraction of the fluid phase) denoted ϕ is incorporated in the continuity and momentum equations. The continuity equation becomes:

$$\frac{\partial \phi}{\partial t} + \nabla \cdot (\phi \mathbf{v}) = 0$$

For the x component, the Lagrangian formulation of the momentum equation with a body force f becomes:

$$\frac{D(\phi v_x)}{Dt} = \frac{1}{\rho} \left[\nabla \cdot (\phi \boldsymbol{\sigma}) \right]_x + \phi f_x$$

In the Eulerian formulation, an advection term is added, and the Cauchy stress tensor is represented as isotropic and deviatoric components individually:

$$\frac{\partial(\phi v_x)}{\partial t} + \boldsymbol{v} \cdot \nabla(\phi v_x) = \frac{1}{\rho} \left[\nabla \cdot (-\phi p \boldsymbol{I}) + \phi \boldsymbol{\tau} \right]_x + \phi f_x$$

Using vector identities to rewrite the advection term, and expanding the fluid stress tensor term:

$$\frac{\partial(\phi v_x)}{\partial t} + \nabla \cdot (\phi v_x v) - \phi v_x (\nabla \cdot v) = \frac{1}{\rho} \left[-\nabla \phi p \right]_x + \frac{1}{\rho} \left[-\phi \nabla p \right]_x + \frac{1}{\rho} \left[\nabla \cdot (\phi \tau) \right]_x + \phi f_x$$

Spatial variations in the porosity are neglected,

$$\nabla \phi := 0$$

and the pressure is attributed to the fluid phase alone (model B in Zhu et al. 2007 and Zhou et al. 2010). The divergence of fluid velocities is defined to be zero:

$$\nabla \cdot \boldsymbol{v} := 0$$

With these assumptions, the momentum equation simplifies to:

$$\frac{\partial(\phi v_x)}{\partial t} + \nabla \cdot (\phi v_x v) = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \frac{1}{\rho} \left[\nabla \cdot (\phi \tau) \right]_x + \phi f_x$$

The remaining part of the advection term is for the x component found as:

$$\nabla \cdot (\phi v_x v) = \begin{bmatrix} \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \end{bmatrix} \begin{bmatrix} \phi v_x v_x \\ \phi v_x v_y \\ \phi v_x v_z \end{bmatrix} = \frac{\partial (\phi v_x v_x)}{\partial x} + \frac{\partial (\phi v_x v_y)}{\partial y} + \frac{\partial (\phi v_x v_z)}{\partial z}$$

The deviatoric stress tensor is in this case symmetrical, i.e. $\tau_{ij} = \tau_{ji}$, and is found by:

$$\begin{split} &\frac{1}{\rho} \left[\nabla \cdot (\phi \boldsymbol{\tau}) \right]_x = \frac{1}{\rho} \left[\left[\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right] \phi \begin{bmatrix} \tau_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \tau_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \tau_{zz} \end{bmatrix} \right]_x \\ &= \frac{1}{\rho} \left[\begin{array}{ccc} \frac{\partial (\phi \tau_{xx})}{\partial x} + \frac{\partial (\phi \tau_{xy})}{\partial y} + \frac{\partial (\phi \tau_{xz})}{\partial z} \\ \frac{\partial (\phi \tau_{yx})}{\partial x} + \frac{\partial (\phi \tau_{yy})}{\partial y} + \frac{\partial (\phi \tau_{zz})}{\partial z} \\ \frac{\partial (\phi \tau_{zx})}{\partial x} + \frac{\partial (\phi \tau_{zy})}{\partial y} + \frac{\partial (\phi \tau_{zz})}{\partial z} \end{array} \right]_x = \frac{1}{\rho} \left(\frac{\partial (\phi \tau_{xx})}{\partial x} + \frac{\partial (\phi \tau_{xy})}{\partial y} + \frac{\partial (\phi \tau_{xz})}{\partial z} \right) \end{split}$$

In a linear viscous fluid, the stress and strain rate $(\dot{\epsilon})$ is linearly dependent, scaled by the viscosity parameter ν :

$$\tau_{ij} = 2\nu \dot{\epsilon}_{ij} = \nu \left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right)$$

With this relationship, the deviatoric stress tensor components can be calculated as:

$$\tau_{xx} = 2\nu \frac{\partial v_x}{\partial x} \qquad \tau_{yy} = 2\nu \frac{\partial v_y}{\partial y} \qquad \tau_{zz} = 2\nu \frac{\partial v_z}{\partial z}$$

$$\tau_{xy} = \nu \left(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x} \right)$$

$$\tau_{xz} = \nu \left(\frac{\partial v_x}{\partial z} + \frac{\partial v_z}{\partial x} \right)$$

$$\tau_{yz} = \nu \left(\frac{\partial v_y}{\partial z} + \frac{\partial v_z}{\partial y} \right)$$

The above formulation of the fluid rheology assumes identical bulk and shear viscosities. The derivation of the equations for the other spatial components is trivial.

1.3.2 Porosity estimation

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1.3.3 Solution procedure by operator splitting

The partial differential terms in the previously described equations are found using finite central differences. Modifying the operator splitting methodology presented by Langtangen et al. (2002), the predicted velocity v^* after a finite

time step Δt is found by explicit integration of the momentum equation.

$$\begin{split} \frac{\Delta(\phi v_x)}{\Delta t} + \nabla \cdot (\phi v_x \boldsymbol{v}) &= -\frac{1}{\rho} \frac{\Delta p}{\Delta x} + \frac{1}{\rho} \left[\nabla \cdot (\phi \boldsymbol{\tau}) \right]_x + \phi f_x \\ & \quad \quad \downarrow \\ \phi \frac{\Delta v_x}{\Delta t} + v_x \frac{\Delta \phi}{\Delta t} + \nabla \cdot (\phi v_x \boldsymbol{v}) &= -\frac{1}{\rho} \frac{\Delta p}{\Delta x} + \frac{1}{\rho} \left[\nabla \cdot (\phi \boldsymbol{\tau}) \right]_x + \phi f_x \end{split}$$

We want to isolate Δv_x in the above equation in order to project the new velocity.

$$\phi \frac{\Delta v_x}{\Delta t} = -\frac{1}{\rho} \frac{\Delta p}{\Delta x} + \frac{1}{\rho} \left[\nabla \cdot (\phi \boldsymbol{\tau}) \right]_x + \phi f_x - v_x \frac{\Delta \phi}{\Delta t} - \nabla \cdot (\phi v_x \boldsymbol{v})$$

$$\Delta v_x = -\frac{1}{\rho} \frac{\Delta p}{\Delta x} \frac{\Delta t}{\phi} + \frac{1}{\rho} \left[\nabla \cdot (\phi \boldsymbol{\tau}) \right]_x \frac{\Delta t}{\phi} + \Delta t f_x - v_x \frac{\Delta \phi}{\phi} - \nabla \cdot (\phi v_x \boldsymbol{v}) \frac{\Delta t}{\phi}$$

The term β is introduced as an adjustable, dimensionless parameter in the range [0; 1], and determines the importance of the old pressure values in the solution procedure (Langtangen et al. 2002). A value of 0 corresponds to Chorin's projection method originally described in Chorin (1968).

$$v_x^* = v_x^t + \Delta v_x$$

$$v_x^* = v_x^t - \frac{\beta}{\rho} \frac{\Delta p^t}{\Delta x} \frac{\Delta t}{\phi^t} + \frac{1}{\rho} \left[\nabla \cdot (\phi^t \boldsymbol{\tau}^t) \right]_x \frac{\Delta t}{\phi} + \Delta t f_x - v_x^t \frac{\Delta \phi}{\phi^t} - \nabla \cdot (\phi^t v_x^t \boldsymbol{v}^t) \frac{\Delta t}{\phi^t}$$

Here, Δx denotes the cell spacing. The velocity found (v_x^*) is only a prediction of the fluid velocity at time $t + \Delta t$, since the estimate isn't constrained by the continuity equation:

$$\frac{\Delta \phi^t}{\Delta t} + \nabla \cdot (\phi^t v^{t+\Delta t}) = 0$$

The divergence of a scalar and vector can be split:

$$\phi^t \nabla \cdot \boldsymbol{v}^{t+\Delta t} + \boldsymbol{v}^{t+\Delta t} \cdot \nabla \phi^t + \frac{\Delta \phi^t}{\Delta t} = 0$$

The predicted velocity is corrected using the new pressure (Langtangen et al. 2002):

$$oldsymbol{v}^{t+\Delta t} = oldsymbol{v}^* - rac{\Delta t}{
ho}
abla \epsilon \quad ext{where} \quad \epsilon = p^{t+\Delta t} - \beta p^t$$

The above formulation of the future velocity is put into the continuity equation:

$$\Rightarrow \phi^t \nabla \cdot \left(\boldsymbol{v}^* - \frac{\Delta t}{\rho} \nabla \epsilon \right) + \left(\boldsymbol{v}^* - \frac{\Delta t}{\rho} \nabla \epsilon \right) \cdot \nabla \phi^t + \frac{\Delta \phi^t}{\Delta t} = 0$$

$$\Rightarrow \phi^t \nabla \cdot \boldsymbol{v}^* - \frac{\Delta t}{\rho} \phi^t \nabla^2 \epsilon + \nabla \phi^t \cdot \boldsymbol{v}^* - \nabla \phi^t \cdot \nabla \epsilon \frac{\Delta t}{\rho} + \frac{\Delta \phi^t}{\Delta t} = 0$$

$$\Rightarrow \frac{\Delta t}{\rho} \phi^t \nabla^2 \epsilon = \phi^t \nabla \cdot \boldsymbol{v}^* + \nabla \phi^t \cdot \boldsymbol{v}^* - \nabla \phi^t \cdot \nabla \epsilon \frac{\Delta t}{\rho} + \frac{\Delta \phi^t}{\Delta t}$$

The pressure difference in time becomes a Poisson equation with added terms:

$$\Rightarrow \nabla^2 \epsilon = \frac{\nabla \cdot \boldsymbol{v}^* \rho}{\Delta t} + \frac{\nabla \phi^t \cdot \boldsymbol{v}^* \rho}{\Delta t \phi^t} - \frac{\nabla \phi^t \cdot \nabla \epsilon}{\phi^t} + \frac{\Delta \phi^t \rho}{\Delta t^2 \phi^t}$$

The right hand side of the above equation is termed the forcing function f, which is decomposed into two terms, f_1 and f_2 :

$$f_1 = \frac{\nabla \cdot \boldsymbol{v}^* \rho}{\Delta t} + \frac{\nabla \phi^t \cdot \boldsymbol{v}^* \rho}{\Delta t \phi^t} + \frac{\Delta \phi^t \rho}{\Delta t^2 \phi^t}$$
$$f_2 = \frac{\nabla \phi^t \cdot \nabla \epsilon}{\phi^t}$$

During the Jacobi iterative solution procedure f_1 remains constant, while f_2 changes value. For this reason, f_1 is found only during the first iteration, while f_2 is updated every time. The value of the forcing function is found as:

$$f = f_1 - f_2$$

Using second-order finite difference approximations of the Laplace operator second-order partial derivatives, the differential equations become a system of equations that is solved using iteratively using Jacobi updates. The total number of unknowns is $(n_x - 1)(n_y - 1)(n_z - 1)$.

The discrete Laplacian (approximation of the Laplace operator) can be obtained by a finite-difference seven-point stencil in a three-dimensional, cubic grid with cell spacing Δx , Δy , Δz , considering the six face neighbors:

$$\begin{split} \nabla^2 \epsilon_{i_x,i_y,i_z} \approx \frac{\epsilon_{i_x-1,i_y,i_z} - 2\epsilon_{i_x,i_y,i_z} + \epsilon_{i_x+1,i_y,i_z}}{\Delta x^2} + \frac{\epsilon_{i_x,i_y-1,i_z} - 2\epsilon_{i_x,i_y,i_z} + \epsilon_{i_x,i_y+1,i_z}}{\Delta y^2} \\ + \frac{\epsilon_{i_x,i_y,i_z-1} - 2\epsilon_{i_x,i_y,i_z} + \epsilon_{i_x,i_y,i_z+1}}{\Delta z^2} \approx f_{i_x,i_y,i_z} \end{split}$$

Within a Jacobi iteration, the value of the unknowns (ϵ^n) is used to find an updated solution estimate (ϵ^{n+1}) . The solution for the updated value takes the form:

$$\epsilon_{i_x,i_y,i_z}^{n+1} = \frac{-\Delta x^2 \Delta y^2 \Delta z^2 f_{i_x,i_y,i_z} + \Delta y^2 \Delta z^2 (\epsilon_{i_x-1,i_y,i_z}^n + \epsilon_{i_x+1,i_y,i_z}^n) + \Delta x^2 \Delta z^2 (\epsilon_{i_x,i_y-1,i_z}^n + \epsilon_{i_x,i_y+1,i_z}^n) + \Delta x^2 \Delta y^2 (\epsilon_{i_x,i_y,i_z-1}^n + \epsilon_{i_x,i_y,i_z-1}^n) + \Delta x^2 \Delta y$$

The difference between the current and updated value is termed the normalized residual:

$$r_{i_x,i_y,i_z} = \frac{(\epsilon_{i_x,i_y,i_z}^{n+1} - \epsilon_{i_x,i_y,i_z}^{n})^2}{(\epsilon_{i_x,i_y,i_z}^{n+1})^2}$$

Note that the ϵ values cannot be 0 due to the above normalization of the residual.

The updated values are at the end of the iteration stored as the current values, and the maximal value of the normalized residual is found. If this value is larger than a tolerance criteria, the procedure is repeated. The iterative procedure is ended if the number of iterations exceeds a defined limit.

After the values of ϵ are found, they are used to find the new pressures and velocities:

$$\bar{p}^{t+\Delta t} = \beta \bar{p}^t + \epsilon$$

$$\bar{\boldsymbol{v}}^{t+\Delta t} = \bar{\boldsymbol{v}}^* - \frac{\Delta t}{\rho} \nabla \epsilon$$

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1.3.4 Numerical implementation

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1.4 Python API

The Python module sphere is intended as the main interface to the sphere application. It is recommended to use this module for simulation setup, simulation execution, and analysis of the simulation output data.

In order to use the API, the file sphere.py must be placed in the same directory as the Python files.

1.4.1 Sample usage

Below is a simple, annotated example of how to setup, execute, and post-process a sphere simulation. The example is also found in the python/folder as collision.py.

```
#!/usr/bin/env python
   Example of two particles colliding.
   Place script in sphere/python/ folder, and invoke with 'python collision.py'
   # Import the sphere module for setting up, running, and analyzing the
   # experiment. We also need the numpy module when setting arrays in the sphere
   # object.
   import sphere
10
   import numpy
11
12
13
14
   ### SIMULATION SETUP
15
   # Create a sphere object with two preallocated particles and a simulation ID
16
   SB = sphere.Spherebin(np = 2, sid = 'collision')
17
18
   SB.radius[:] = 0.3 # set radii to 0.3 m
19
   # Define the positions of the two particles
22
   SB.x[0, :] = numpy.array([10.0, 5.0, 5.0])
                                                  # particle 1 (idx 0)
   SB.x[1, :] = numpy.array([11.0, 5.0, 5.0]) # particle 2 (idx 1)
23
24
   # The default velocity is [0,0,0]. Slam particle 1 into particle 2 by defining
25
   # a positive x velocity for particle 1.
26
   SB.vel[0, 0] = 1.0
   # let's disable gravity in this simulation
29
   GRAVITY = numpy.array([0.0, 0.0, 0.0])
30
31
   # Set the world limits and the particle sorting grid. The particles need to stay
32
   # within the world limits for the entire simulation, otherwise it will stop!
   SB.initGridAndWorldsize(q = GRAVITY, margin = 5.0)
35
  # Define the temporal parameters, e.g. the total time (total) and the file
   # output interval (file_dt), both in seconds
37
   SB.initTemporal(total = 2.0, file_dt = 0.1)
38
```

1.4. Python API

```
# Save the simulation as a input file for sphere
   SB.writebin()
41
42
   # Using a 'dry' run, the sphere main program will display important parameters.
43
44
   # sphere will end after displaying these values.
   SB.run(dry = True)
45
46
47
   ### RUNNING THE SIMULATION
48
49
   # Start the simulation on the GPU from the sphere program
   SB.run()
51
52
53
   ### ANALYSIS OF SIMULATION RESULTS
54
55
   # Plot the system energy through time, image saved as collision-energy.png
56
57
   sphere.visualize(SB.sid, method = 'energy')
58
   # Render the particles using the built-in raytracer
59
   SB.render()
60
61
   # Alternative visualization using ParaView. See the documentation of
62
  # ''Spherebin.writeVTKall()'' for more information about displaying the
  # particles in ParaView.
   SB.writeVTKall()
```

The full documentation of the sphere Python API can be found below.

1.4.2 The sphere module

```
class sphere. Spherebin (np=1, nd=3, nw=1, sid='unnamed', fluid=False) Class containing all sphere data.
```

Contains functions for reading and writing binaries, as well as simulation setup and data analysis. Most arrays are initialized to default values.

Parameters

- np(int) The number of particles to allocate memory for (default = 1)
- **nd** (*int*) The number of spatial dimensions (default = 3). Note that 2D and 1D simulations currently are not possible.
- **nw** (*int*) The number of dynamic walls (default = 1)
- **sid** (*str*) The simulation id (default = 'unnamed'). The simulation files will be written with this base name.
- **fluid** (*bool*) Setup fluid simulation (default = False)

```
 \begin{tabular}{ll} {\bf addParticle} (x, radius, xysum=array([ \ 0., \ 0.]), vel=array([ \ 0., \ 0., \ 0.]), fixvel=array([ \ 0.]), \\ force=array([ \ 0., \ 0., \ 0.]), angpos=array([ \ 0., \ 0., \ 0.]), angvel=array([ \ 0., \ 0., \ 0.]), \\ torque=array([ \ 0., \ 0., \ 0.]), es\_dot=array([ \ 0.]), es=array([ \ 0.]), ev\_dot=array([ \ 0.]), \\ ev=array([ \ 0.]), p=array([ \ 0.])) \\ \end{tabular}
```

Add a single particle to the simulation object. The only required parameters are the position (x) and the radius (radius).

Parameters

- **x** (*numpy.array*) A vector pointing to the particle center coordinate.
- radius (float) The particle radius
- vel (numpy.array) The particle linear velocity (default = [0,0,0])
- **fixvel** (*float*) Fix horizontal linear velocity (0: No, 1: Yes, default=0)
- angpos (numpy.array) The particle angular position (default = [0,0,0])
- angvel (numpy.array) The particle angular velocity (default = [0,0,0])
- **torque** (numpy.array) The particle torque (default = [0,0,0])
- **es_dot** (*float*) The particle shear energy loss rate (default = 0)
- **es** (float) The particle shear energy loss (default = 0)
- ev_dot (*float*) The particle viscous energy rate loss (default = 0)
- ev(float) The particle viscous energy loss (default = 0)
- \mathbf{p} (*float*) The particle pressure (default = 0)

adjustUpperWall (z_adjust=1.1)

Included for legacy purposes, calls adjustWall with idx=0

adjustWall (idx, adjust=1.1)

Adjust grid and dynamic wall to max. particle position

bond (i, i)

Create a bond between particles i and j

bondsRose (imgformat='pdf')

Visualize strike- and dip angles of the bond pairs in a rose plot.

bulkPorosity()

Calculate and return the bulk porosity

consolidate (deviatoric_stress=10000.0, periodic=1)

Setup consolidation experiment. Specify the upper wall deviatoric stress in Pascal, default value is 10 kPa.

createBondPair (i, j, spacing=-0.1)

Bond particles i and j. Particle j is moved adjacent to particle i, and oriented randomly. @param spacing (float) The inter-particle distance prescribed. Positive values result in a inter-particle distance, negative equal an overlap. The value is relative to the sum of the two radii.

currentDevs()

Return current magnitude of the deviatoric normal stress

Initialize particle parameters to default values. Radii must be set prior to calling this function.

energy (method)

Calculate the sum of the energy components of all particles.

forcechains (lc=200.0, uc=650.0, outformat='png', disp='2d')

Visualizes the force chains in the system from the magnitude of the normal contact forces, and produces an image of them. @param lc: Lower cutoff of contact forces. Contacts below are not visualized (float) @param uc: Upper cutoff of contact forces. Contacts above are visualized with this value (float) @param outformat: Format of output image. Possible values are 'interactive', 'png', 'epslatex', 'epslatex-color' @param disp: Display forcechains in '2d' or '3d' Warning: Will segfault if no contacts are found.

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forcechainsRose(lower limit=0.25)

Visualize strike- and dip angles of the strongest force chains in a rose plot.

generateBimodalRadii (r_small=0.005, r_large=0.05, ratio=0.2, verbose=True)

Draw random radii from two distinct sizes.

Parameters

- **r_small** (*float*) Radii of small population [m], in]0;r_large[
- r large (float) Radii of large population [m], in]r small;inf[
- ratio (float) Approximate volumetric ratio between the two populations (large/small).

generateRadii (psd='logn', radius_mean=0.00044, radius_variance=8.8e-09, histogram=True)

Draw random particle radii from a selected probability distribution. The larger the variance of radii is, the slower the computations will run. The reason is two-fold: The smallest particle dictates the time step length, where smaller particles cause shorter time steps. At the same time, the largest particle determines the sorting cell size, where larger particles cause larger cells. Larger cells are likely to contain more particles, causing more contact checks.

Parameters

- psd (str) The particle side distribution. One possible value is logn, which is a log-normal probability distribution, suitable for approximating well-sorted, coarse sediments. The other possible value is uni, which is a uniform distribution from radius_mean-radius_variance to radius_mean+radius_variance.
- radius_mean (float) The mean radius [m] (default = 440e-6 m)
- radius variance (*float*) The variance in the probability distribution [m].

See also: generateBimodalRadii().

initFluid(nu=0.00089)

Initialize the fluid arrays and the fluid viscosity

initGrid()

Initialize grid suitable for the particle positions set previously. The margin parameter adjusts the distance (in no. of max. radii) from the particle boundaries.

initGridAndWorldsize (*g*=*array*([0., 0., -9.80665]), *margin*=2.0, *periodic*=1, *contactmodel*=2) Initialize grid suitable for the particle positions set previously. The margin parameter adjusts the distance (in no. of max. radii) from the particle boundaries.

```
initGridPos (g=array([ 0., 0., -9.80665]), gridnum=array([12, 12, 36]), periodic=1, contact-
model=2)
```

Initialize particle positions in loose, cubic configuration. Radii must be set beforehand. xynum is the number of rows in both x- and y- directions.

```
initRandomGridPos (g=array([ 0., 0., -9.80665]), gridnum=array([12, 12, 32]), periodic=1, con-
tactmodel=2)
```

Initialize particle positions in loose, cubic configuration. Radii must be set beforehand. xynum is the number of rows in both x- and y- directions.

```
initRandomPos (g=array([ 0., 0., -9.80665]), gridnum=array([12, 12, 36]), periodic=1, contact-
model=2)
```

Initialize particle positions in loose, cubic configuration. Radii must be set beforehand. xynum is the number of rows in both x- and y- directions.

```
initTemporal (total, current=0.0, file_dt=0.05, step_count=0)
```

Set temporal parameters for the simulation. Particle radii, physical parameters, and the optional fluid grid need to be set prior to these.

plotConvergence (format='png')

Plot the convergence evolution in the CFD solver. The plot is saved in the output folder with the file name <simulation ID>-conv.<format>.

Parameters format (*str*) – The plot file type (default = 'png')

plotFluidDiffAdvPresZ()

Compare contributions to the velocity from diffusion and advection at top boundary, assuming the flow is 1D along the z-axis, phi = 1, and dphi = 0. This solution is analog to the predicted velocity and not constrained by the conservation of mass.

plotFluidPorositiesY (iteration=-1, y=-1, outformat='png')

Plot the porosity values from the simulation. If iteration is -1 (default value), the last output file will be shown. If the y value is -1, the center x,z plane will be rendered

plotPrescribedFluidPressures (format='png')

Plot the prescribed fluid pressures through time that may be modulated through the class parameters p_mod_A, p_mod_f, and p_mod_phi.

```
porosities (outformat='pdf', zslices=16)
```

Plot porosities with depth

```
porosity (slices=10, verbose=False)
```

Calculate the porosity as a function of depth, by averaging values in horizontal slabs. Returns porosity values and depth

random2bonds (ratio=0.3, spacing=-0.1)

Bond an amount of particles in two-particle clusters @param ratio: The amount of particles to bond, values in]0.0;1.0] (float) @param spacing: The distance relative to the sum of radii between bonded particles, neg. values denote an overlap. Values in]0.0,inf[(float). The particles should be initialized beforehand. Note: The actual number of bonds is likely to be somewhat smaller than specified, due to the random selection algorithm.

```
readbin (targetbin, verbose=True, bonds=True, devsmod=True, esysparticle=False)
```

Reads a target sphere binary file.

```
See also writebin(), readfirst(), readlast(), readsecond(), and readstep().
```

Parameters

- targetbin (str) The path to the binary sphere file
- **verbose** (*bool*) Show diagnostic information (default = True)
- **bonds** (*bool*) The input file contains bond information (default = True). This parameter should be true for all recent sphere versions.
- **devsmod** (*bool*) The input file contains information about modulating stresses at the top wall (default = True). This parameter should be true for all recent sphere versions.
- **esysparticle** (*bool*) Stop reading the file after reading the kinematics, which is useful for reading output files from other DEM programs. (default = False)

readfirst (verbose=True)

Read the first output file from the ../output/ folder, corresponding to the object simulation id (self.sid).

Parameters verbose (*bool*) – Display diagnostic information (default = True)

```
See also readbin(), readlast(), readsecond(), and readstep().
```

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```
readlast (verbose=True)
```

Read the last output file from the ../output/ folder, corresponding to the object simulation id (self.sid).

Parameters verbose (*bool*) – Display diagnostic information (default = True)

```
See also readbin(), readfirst(), readsecond(), and readstep().
```

readsecond(verbose=True)

Read the second output file from the ../output/ folder, corresponding to the object simulation id (self.sid).

Parameters verbose (*bool*) – Display diagnostic information (default = True)

```
See also readbin(), readfirst(), readlast(), and readstep().
```

readstep (step, verbose=True)

Read a output file from the .../output/ folder, corresponding to the object simulation id (self.sid).

Parameters

- **step** (*int*) The output file number to read, starting from 0.
- **verbose** (*bool*) Display diagnostic information (default = True)

```
See also readbin(), readfirst(), readlast(), and readsecond().
```

render (*method='pres'*, *max_val=1000.0*, *lower_cutoff=0.0*, *graphicsformat='png'*, *verbose=True*) Render all output files that belong to the simulation, determined by sid.

run (*verbose=True*, *hideinputfile=False*, *dry=False*, *valgrind=False*, *cudamemcheck=False*)

Execute sphere with target project

setFluidPressureModulation (A, f, phi=0.0)

Set the parameters for the sine wave modulating the fluid pressures at the top boundary. Note that a cos-wave is obtained with phi=pi/2.

Parameters

- A (*float*) Fluctuation amplitude [Pa]
- **f** (*float*) Fluctuation frequency [Hz]
- **phi** (*float*) Fluctuation phase shift (default=0.0)

```
shear (shear strain rate=1, periodic=1)
```

Setup shear experiment. Specify the upper wall deviatoric stress in Pascal, default value is 10 kPa. The shear strain rate is the shear length divided by the initial height per second.

```
sheardisp(outformat='pdf', zslices=32)
```

Show particle x-displacement vs. the z-pos

shearstrain()

Calculates and returns the current shear strain (gamma) value of the experiment.

shearvel()

Calculates and returns the shear velocity (gamma_dot) of the experiment

status()

Show the current simulation status

thinsection_x1x3 (x2='center', graphicsformat='png', cbmax=None, arrowscale=0.01, velarrowscale=1.0, slipscale=1.0, verbose=False)

Produce a 2D image of particles on a x1,x3 plane, intersecting the second axis at x2. Output is saved as '<sid>-ts-x1x3.txt' in the current folder.

An upper limit to the pressure color bar range can be set by the cbmax parameter.

The data can be plotted in gnuplot with: gnuplot> set size ratio -1 gnuplot> set palette defined (0 "blue", 0.5 "gray", 1 "red") gnuplot> plot '<sid>-ts-x1x3.txt' with circles palette fs transparent solid 0.4 noborder

```
torqueScript (email='adc@geo.au.dk', email_alerts='ae', walltime='24:00:00', queue='qfermi', cudapath='/com/cuda/4.0.17/cuda', spheredir='/home/adc/code/sphere', workdir='/scratch')
```

Create job script for the Torque queue manager for the binary

```
triaxial (wvel=-0.001, deviatoric_stress=10000.0)
```

Setup triaxial experiment. The upper wall is moved at a fixed velocity in m/s, default values is -0.001 m/s (i.e. downwards). The side walls are exerting a deviatoric stress

```
uniaxialStrainRate (wvel=-0.001, periodic=1)
```

Setup consolidation experiment. Specify the upper wall velocity in m/s, default value is -0.001 m/s (i.e. downwards).

voidRatio()

Returns the current void ratio

```
writeFluidVTK (folder='../output/', verbose=True)
```

Writes a VTK file for the fluid grid to the ../output/ folder by default. The file name will be in the format fluid-<self.sid>.vti. The vti files can be used for visualizing the fluid in ParaView.

The fluid grid is visualized by opening the vti files, and pressing "Apply" to import all fluid field properties. To visualize the scalar fields, such as the pressure, the porosity, the porosity change or the velocity magnitude, choose "Surface" or "Surface With Edges" as the "Representation". Choose the desired property as the "Coloring" field. It may be desirable to show the color bar by pressing the "Show" button, and "Rescale" to fit the color range limits to the current file. The coordinate system can be displayed by checking the "Show Axis" field. All adjustments by default require the "Apply" button to be pressed before regenerating the view.

The fluid vector fields (e.g. the fluid velocity) can be visualizing by e.g. arrows. To do this, select the fluid data in the "Pipeline Browser". Press "Glyph" from the "Common" toolbar, or go to the "Filters" mennu, and press "Glyph" from the "Common" list. Make sure that "Arrow" is selected as the "Glyph type", and "Velocity" as the "Vectors" value. Adjust the "Maximum Number of Points" to be at least as big as the number of fluid cells in the grid. Press "Apply" to visualize the arrows.

If several data files are generated for the same simulation (e.g. using the writeVTKall() function), it is able to step the visualization through time by using the ParaView controls.

Parameters

- **folder** (*str*) The folder where to place the output binary file (default (default = '../out-put/')
- **verbose** (*bool*) Show diagnostic information (default = True)

```
writeVTK (folder='../output/', verbose=True)
```

Writes a VTK file with particle information to the .../output/ folder by default. The file name will be in the format <self.sid>.vtu. The vtu files can be used to visualize the particles in ParaView.

After opening the vtu files, the particle fields will show up in the "Properties" list. Press "Apply" to import all fields into the ParaView session. The particles are visualized by selecting the imported data in the "Pipeline Browser". Afterwards, click the "Glyph" button in the "Common" toolbar, or go to the "Filters" menu, and press "Glyph" from the "Common" list. Choose "Sphere" as the "Glyph Type", set "Radius"

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to 1.0, choose "scalar" as the "Scale Mode". Check the "Edit" checkbox, and set the "Set Scale Factor" to 1.0. The field "Maximum Number of Points" may be increased if the number of particles exceed the default value. Finally press "Apply", and the particles will appear in the main window.

The sphere resolution may be adjusted ("Theta resolution", "Phi resolution") to increase the quality and the computational requirements of the rendering. All adjustments by default require the "Apply" button to be pressed before regenerating the view.

If several vtu files are generated for the same simulation (e.g. using the :func:writeVTKall() function), it is able to step the visualization through time by using the ParaView controls.

Parameters

- **folder** (*str*) The folder where to place the output binary file (default (default = '../out-put/')
- **verbose** (*bool*) Show diagnostic information (default = True)

writeVTKall (verbose=True)

Writes a VTK file for each simulation output file with particle information and the fluid grid to the ../output/ folder by default. The file name will be in the format <self.sid>.vtu and fluid-<self.sid>.vti. The vtu files can be used to visualize the particles, and the vti files for visualizing the fluid in ParaView.

After opening the vtu files, the particle fields will show up in the "Properties" list. Press "Apply" to import all fields into the ParaView session. The particles are visualized by selecting the imported data in the "Pipeline Browser". Afterwards, click the "Glyph" button in the "Common" toolbar, or go to the "Filters" menu, and press "Glyph" from the "Common" list. Choose "Sphere" as the "Glyph Type", set "Radius" to 1.0, choose "scalar" as the "Scale Mode". Check the "Edit" checkbox, and set the "Set Scale Factor" to 1.0. The field "Maximum Number of Points" may be increased if the number of particles exceed the default value. Finally press "Apply", and the particles will appear in the main window.

The sphere resolution may be adjusted ("Theta resolution", "Phi resolution") to increase the quality and the computational requirements of the rendering.

The fluid grid is visualized by opening the vti files, and pressing "Apply" to import all fluid field properties. To visualize the scalar fields, such as the pressure, the porosity, the porosity change or the velocity magnitude, choose "Surface" or "Surface With Edges" as the "Representation". Choose the desired property as the "Coloring" field. It may be desirable to show the color bar by pressing the "Show" button, and "Rescale" to fit the color range limits to the current file. The coordinate system can be displayed by checking the "Show Axis" field. All adjustments by default require the "Apply" button to be pressed before regenerating the view.

The fluid vector fields (e.g. the fluid velocity) can be visualizing by e.g. arrows. To do this, select the fluid data in the "Pipeline Browser". Press "Glyph" from the "Common" toolbar, or go to the "Filters" mennu, and press "Glyph" from the "Common" list. Make sure that "Arrow" is selected as the "Glyph type", and "Velocity" as the "Vectors" value. Adjust the "Maximum Number of Points" to be at least as big as the number of fluid cells in the grid. Press "Apply" to visualize the arrows.

If several data files are generated for the same simulation (e.g. using the writeVTKall() function), it is able to step the visualization through time by using the ParaView controls.

Parameters verbose (*bool*) – Show diagnostic information (default = True)

writebin (folder='../input/', verbose=True)

Writes a sphere binary file to the ../input/ folder by default. The file name will be in the format <self.sid>.bin.

See also readbin().

Parameters

```
• folder (str) – The folder where to place the output binary file
                   • verbose (bool) – Show diagnostic information (default = True)
     zeroKinematics()
          Zero kinematics of particles
sphere.V sphere(r)
     Returns the volume of a sphere with radius r
sphere.cleanup(spherebin)
     Remove input/output files and images from simulation
sphere.convert (graphicsformat='png', folder='../img_out')
     Converts all PPM images in img out to graphicsformat, using ImageMagick
sphere.render(binary, method='pres', max_val=1000.0, lower_cutoff=0.0, graphicsformat='png', ver-
                   bose=True)
     Render target binary using the sphere raytracer.
sphere.run (binary, verbose=True, hideinputfile=False)
     Execute sphere with target binary as input
sphere.status(project)
     Check the status.dat file for the target project, and return the last file numer.
sphere.thinsectionVideo (project, out_folder='./', video_format='mp4', fps=25, qscale=1, bi-
                                trate=1800, verbose=False)
     Use ffmpeg to combine thin section images to animation. This function will start off by rendering the images.
sphere.torqueScriptParallel3(obj1, obj2, obj3, email='adc@geo.au.dk', email_alerts='ae',
                                       walltime='24:00:00',
                                                                      queue='qfermi',
                                                                                               cudap-
                                       ath='/com/cuda/4.0.17/cuda', spheredir='/home/adc/code/sphere',
                                       workdir='/scratch')
```

Create job script for the Torque queue manager for three binaries, executed in parallel. Returns the filename of the script

```
sphere.torqueScriptSerial3 (obj1, obj2, obj3, email='adc@geo.au.dk', email_alerts='ae', walltime='24:00:00', queue='qfermi', cudap-ath='/com/cuda/4.0.17/cuda', spheredir='/home/adc/code/sphere', workdir='/scratch')

Create job script for the Torque queue manager for three binaries
```

Create job script for the Torque queue manager for three omaries

```
sphere.video (project, out_folder='./', video_format='mp4', graphics_folder='../img_out/', graphics_format='png', fps=25, qscale=1, bitrate=1800, verbose=False)

Use ffmpeg to combine images to animation. All images should be rendered beforehand.
```

```
sphere.visualize (project, method='energy', savefig=True, outformat='png')

Visualize output from the target project, where the temporal progress is of interest.
```

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