Please enter your name and uID below.

Name:

uID:

Collaborators, if any, and how you collaborated:

Submission notes

- Due at 11:59 pm (midnight) on Thursday, Nov 10th.
- Solutions must be typeset using one of the template files. For each problem, your answer must fit in the space provided (e.g. not spill onto the next page) *without* space-saving tricks like font/margin/line spacing changes.
- Upload a PDF version of your completed problem set to Gradescope.
- Teaching staff reserve the right to request original source/tex files during the grading process, so please retain these until an assignment has been returned.
- Please remember that for this problem set, you are allowed to collaborate in detail with your peers, as long as you cite them. However, you must write up your own solution, alone, from memory. If you do collaborate with other students in this way, you must identify the students and describe the nature of the collaboration. You are not allowed to create a group solution, and all work that you hand in must be written in your own words. Do not base your solution on any other written solution, regardless of the source.

1. (Semi-connected, 30pts)

   A directed graph G is *semi-connected* if, for every pair of vertices u and v, either u is reachable from v or v is reachable from u (or both).

   a. Give an example (either a picture or a list of vertices and edges) of a directed acyclic graph with a unique source that is *not* semi-connected.

   b. Describe a $O(V(V+E))$ algorithm[1] to determine whether a given *acyclic* graph is semi-connected. Describe the running time of your algorithm.

   c. Describe a $O(V+E)$ algorithm to determine whether a given *acyclic* graph is semi-connected. Justify why your algorithm works, and describe the running time of your algorithm.

[1] It's OK if your algorithm is faster than $O(V(V+E))$, it could even be the same algorithm as for (c), but it's easiest to come up with a brute force algorithm first.

2. (Max-Min Edges, 30pts)

   Let $G = (V, E)$ be an arbitrary connected undirected graph with weighted edges with unique values.

   a. Prove that for any cycle in G, the minimum spanning tree of G *excludes* the maximum-weight edge in that cycle.

   b. Prove or disprove: The minimum spanning tree of G includes the minimum-weight edge in *every* cycle in G.