Problem 3, A route to chaos

for February 09, 23.59, 2024

The dynamics of a population with adults cannibalising on their offspring can be described by the Ricker map

$$\eta_{\tau+1} = R\, \eta_\tau\, e^{-\alpha \eta_\tau}$$

where η_τ denotes the number of adults in generation $\tau = 1, 2, \ldots$, while R is related to the number of offspring produced by each adult and α is the incidence rate of cannibalism. The factor $e^{-\alpha \eta_\tau}$ describes the probability of offspring survival to maturity. In this task you will analyse the stability of the steady states of the Ricker map as a function of the parameter R for $\alpha = 0.01$ and $\eta_0 = 900$.

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a) Assume that R takes the values from 1 to 30 in steps of 0.1 and plot the bifurcation diagram of η against R as follows. For each value of R, run the model for 300 generations and plot the last 100 values of η_τ versus the value of R. The final plot should show the results for all values of R tested. Describe the result.

In[191]:=

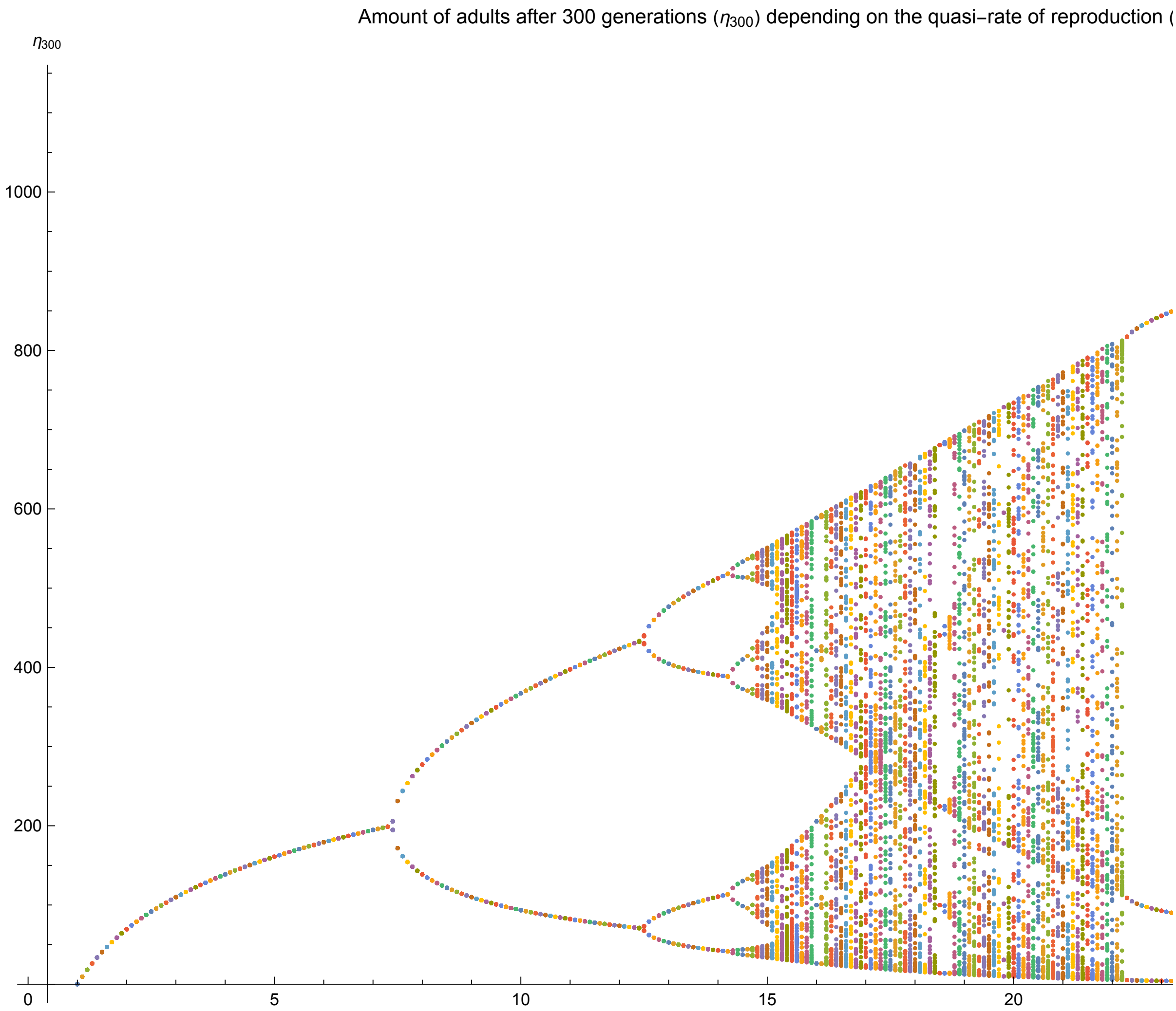
```
ClearAll["Global`*"];
MaxGeneration = 300;
η = 900;
α = 0.01;

dataPoints = Table[
    (* First data will be created using NestList and then the first 200 data points ar
    data = Drop[NestList[R*(#)*Exp[-(α*(#))] &, η, MaxGeneration], 200];
    (* Now the data which is a list of values is changed to a list of list with two el
    pairedData = Map[{R, #} &, data];
    {R, pairedData},
    {R, 1, 30, 0.1}];

ListPlot[dataPoints[[All, 2]],
 AxesLabel → {"R", "η300"},
 PlotRange → All,
 PlotLabel→"Amount of adults after 300 generations (η300) depending on the quasi-rate c
 ImageSize→{500,300}]
```

In the plot we can see fractal behaviour, with infinite period doubling. Each point on the map corresponds to a cycle starting with a fixed point (1-point cycle) until $R \approx 7.4$ then a 2-point cycle start and goes over to a 4-point cycle around $R \approx 12.4$. This doubling goes on until R_∞ is reached where the period is infinite. If one overshoots this R_∞ the full period doubling pattern reappears in small windows of the plot ($\approx 22 < R < 25.1$).

Out[ ]=



b) Plot the population dynamics η_τ versus τ (where $\tau = 0, \ldots, 40$) for four values of R. Choose four representative values of R where η_τ has a stable fixed point, a 2-point cycle. a 3-point cycle, and a 4-point cycle. Describe the dynamics observed for the four values of R.

I found a stable fixed point for the value of $R =$ 5, a 2-point cycle for $R = 10$, a 3-point cycle for $R = 23$ and a 5-point cycle for $R = 13$.

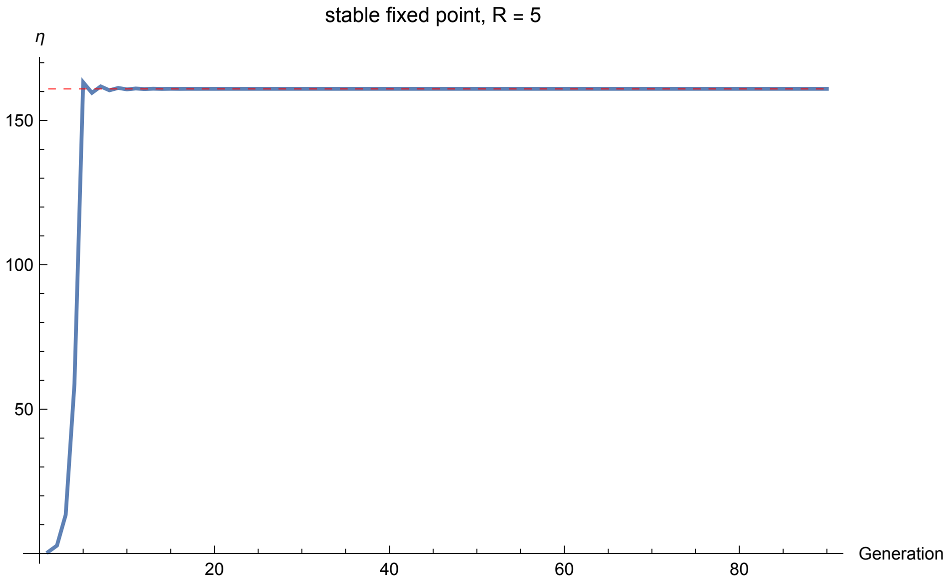
In[236]:=

```
ClearAll["Global`*"];
η = 900;
α = 0.01;
R = 5;

dataPoints = Table[
    data = Drop[NestList[R*(#)*Exp[-(α*(#))] &, η, MaxGeneration], MaxGeneration];
    pairedData = MapIndexed[{First[#2] + MaxGeneration - Length[data], #1} &, data];
    pairedData,
    {MaxGeneration, 1, 90, 1}];

ListPlot[Flatten[dataPoints, 1],
 AxesLabel → {"Generation", "η"},
 PlotRange → All,
 PlotLabel → "stable fixed point, R = 5",
 Joined → True,
  Epilog → {
  Red, Dashed,Line[{{1, 160.9}, {90, 160.9}}]},
  ImageSize→{500,300}
  ]
```

Out[241]=



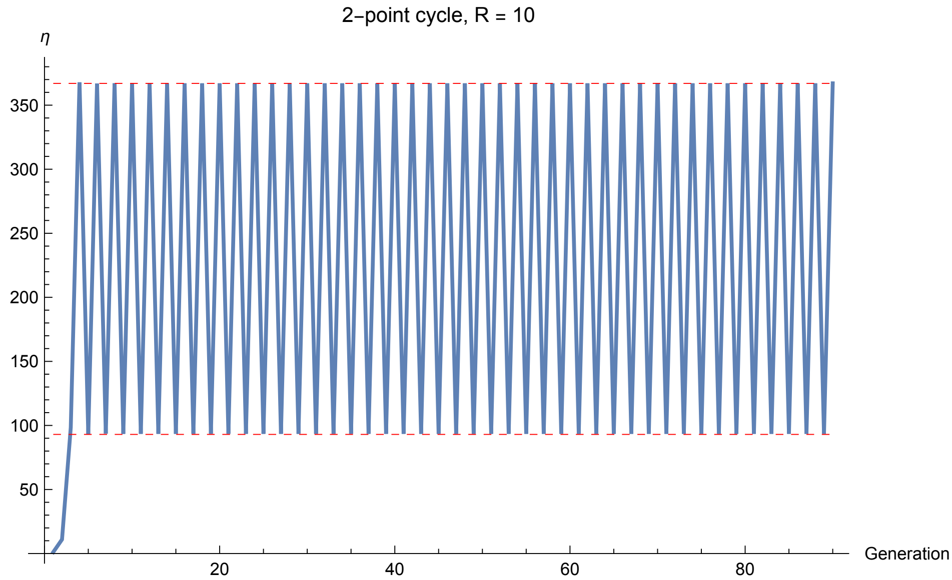
In[230]:=

```
ClearAll["Global`*"];
η = 900;
α = 0.01;
R = 10;

dataPoints = Table[
    data = Drop[NestList[R*(#)*Exp[-(α*(#))] &, η, MaxGeneration], MaxGeneration];
    pairedData = MapIndexed[{First[#2] + MaxGeneration - Length[data], #1} &, data];
    pairedData,
    {MaxGeneration, 1, 90, 1}];

ListPlot[Flatten[dataPoints, 1],
 AxesLabel → {"Generation", "η"},
 PlotRange → All,
 PlotLabel → "2-point cycle, R = 10",
 Joined → True,
  Epilog → {
  Red, Dashed, Line[{{1, 93}, {90, 93}}],Line[{{1, 367}, {90, 367}}]},
  ImageSize→{500,300}
  ]
```

Out[235]=



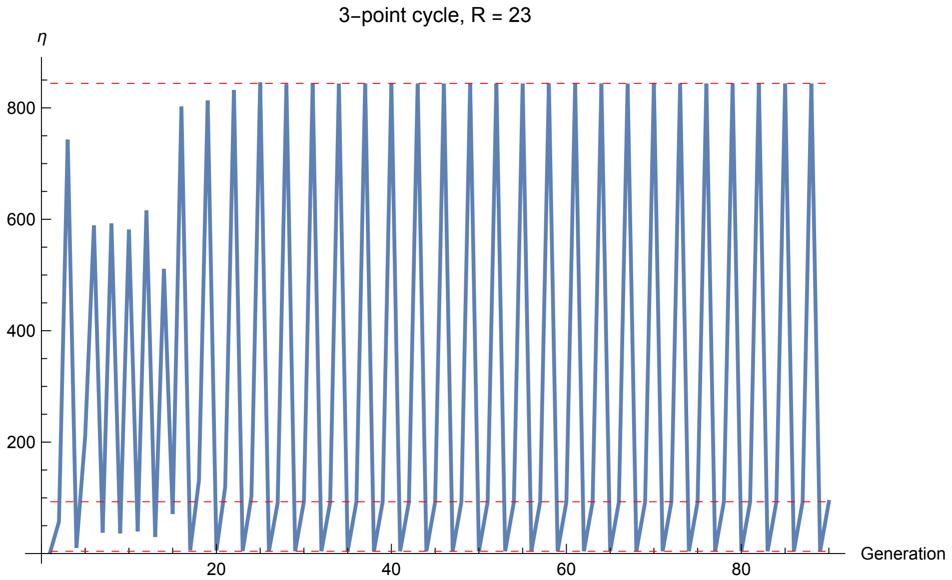
In[248]:=

```
ClearAll["Global`*"];
η = 900;
α = 0.01;
R = 23;

dataPoints = Table[
    data = Drop[NestList[R*(#)*Exp[-(α*(#))] &, η, MaxGeneration], MaxGeneration];
    pairedData = MapIndexed[{First[#2] + MaxGeneration - Length[data], #1} &, data];
    pairedData,
    {MaxGeneration, 1, 90, 1}];

ListPlot[Flatten[dataPoints, 1],
 AxesLabel → {"Generation", "η"},
 PlotRange → All,
 PlotLabel → "3-point cycle, R = 23",
 Joined → True,
  Epilog → {
  Red, Dashed, Line[{{1, 844}, {90, 844}}],Line[{{1, 93}, {90, 93}}], Line[{{1, 4}, {9
  ImageSize→{500,300}
  ]
```

Out[253]=



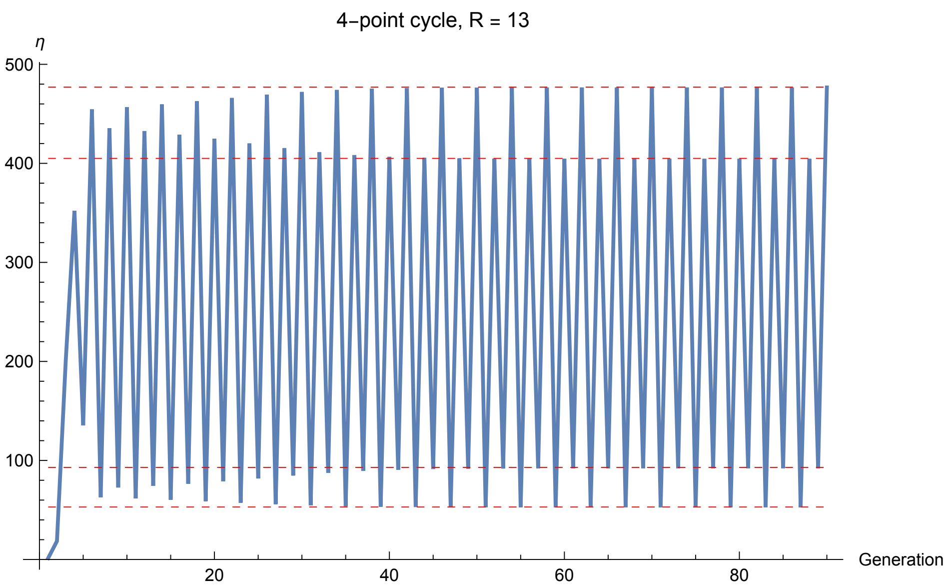
In[272]:=

```
ClearAll["Global`*"];
η = 900;
α = 0.01;
R = 13;

dataPoints = Table[
    data = Drop[NestList[R*(#)*Exp[-(α*(#))] &, η, MaxGeneration], MaxGeneration];
    pairedData = MapIndexed[{First[#2] + MaxGeneration - Length[data], #1} &, data];
    pairedData,
    {MaxGeneration, 1, 90, 1}];

ListPlot[Flatten[dataPoints, 1],
 AxesLabel → {"Generation", "η"},
 PlotRange → All,
 PlotLabel → "4-point cycle, R = 13",
 Joined → True,
  Epilog → {
  Red, Dashed, Line[{{1, 477}, {90, 477}}],Line[{{1, 93}, {90, 93}}], Line[{{1, 53}, {
  ImageSize→{500,300}
  ]
```

Out[277]=



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c) Investigating the plot in subtask a), at which value R_1 of R does the population dynamics bifurcate from a stable equilibrium to a stable 2-point cycle? At which value R_2 does the dynamics bifurcate to a stable 4-point cycle?

From the plot of a) it looks like that for $R_1 \approx 7.4$ or 7.5 the bifurcation happens and goes over to a stable 2-point cycle, while the 4-point cycle appears for $R_2 \approx 12.4$ or 12.4.

d) By making zooms and refining the R-grid, make a rough estimate of R_∞, the first parameter value where the period-doubling bifurcation has repeated an infinite number of times. Explain how you come to this estimate of R_∞.

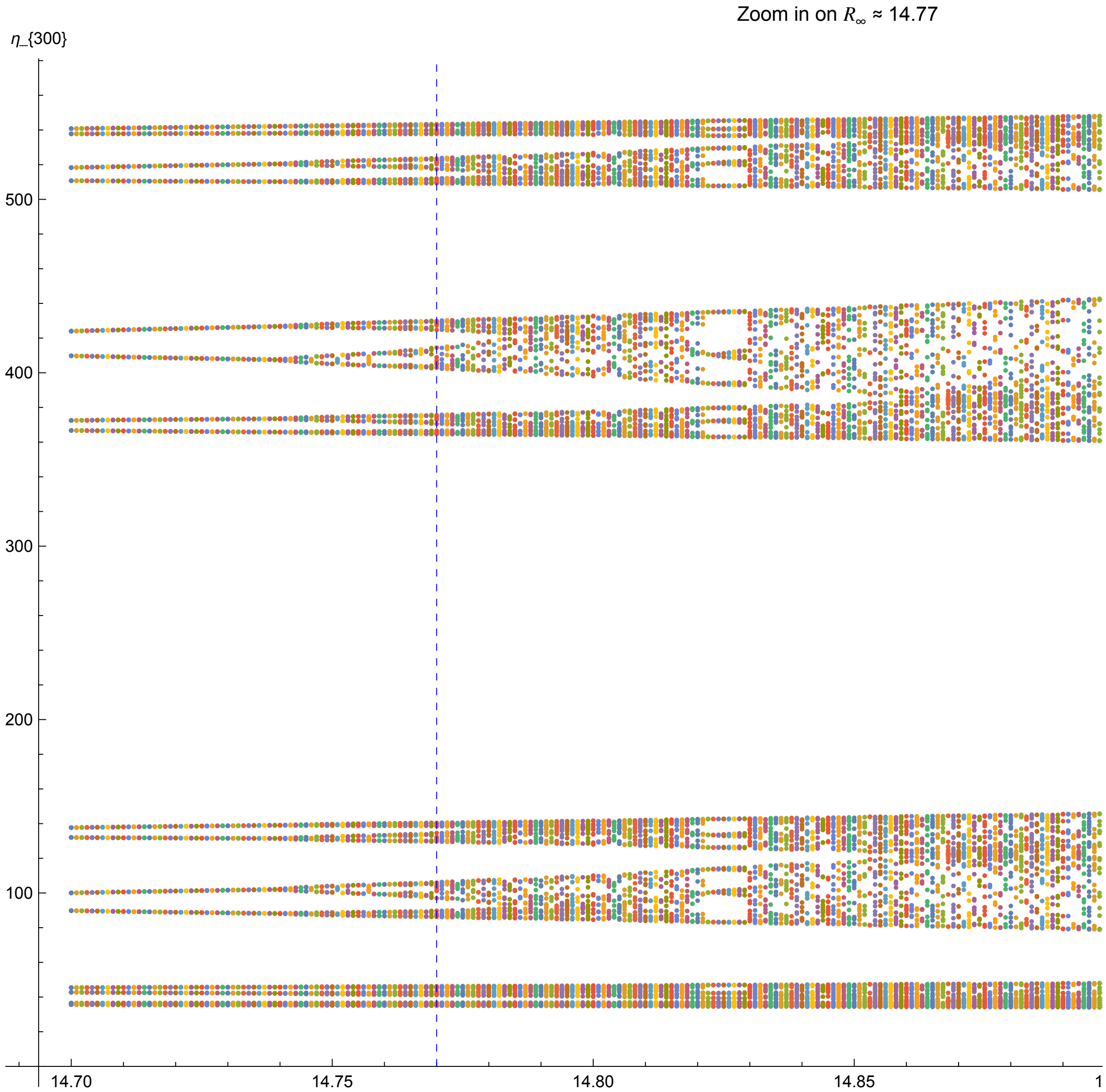
In[278]:=

```
ClearAll["Global`*"];
MaxGeneration = 300;
η = 900;
α = 0.01;

dataPoints = Table[
    (* First data will be created using NestList and then the first 200 data points ar
    data = Drop[NestList[R*(#)*Exp[-(α*(#))] &, η, MaxGeneration], 200];
    (* Now the data which is a list of values is changed to a list of list with two el
    pairedData = Map[{R, #} &, data];
    {R, pairedData},
    {R, 14.7, 15, 0.001}];

ListPlot[dataPoints[[All, 2]],
 AxesLabel → {"R", "η_{300}"},
 PlotRange → All,
 PlotLabel → "Zoom in on R∞≈14.77",
 Epilog → {Blue, Dashed, Line[{{14.77, 0}, {14.77, 600}}]},
 ImageSize→{500,300}
]
```

Out[283]=



My guesstimate for the R_∞ is around $R \approx 14.77$. This because that is where the pockets close for the first time. When they close the period doubling has happened an infinite amount of times.