The UMAP Journal

Vol. 28, No. 2

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Dept. of Mathematics and Computer Science

Carl Mendelson
Dept. of Geology

Yaffa Grossman
Dept. of Biology

Beloit College
700 College St.
Beloit, WI 53511–5595
campbell@beloit.edu

Introduction

This year's ICM was marred by the disqualification of two teams. Their papers would have been judged Outstanding—except that the papers weren't their work. The papers included numerous entire paragraphs from other sources without any acknowledgment. The teams presented as their own the work of others: They utterly failed to distinguish where the sources' words ended and the team's began.

The Rule

The very first contest rule for the ICM (and the MCM) is

> 1. Teams may use any inanimate source of data or materials—computers, software, references, web sites, books, etc., however all sources used must be credited. Failure to credit a source will result in a team being disqualified from the competition.

The UMAP Journal 28 (1) (2007) 93–98.

What to Do

We reproduce below advice and examples that we give to our classes, modified slightly for ICM/MCM teams. Further examples and advice are offered by Hacker [2007].

When you research a topic for a paper, project, or presentation, you collect information from books, journal and magazine articles, lecture notes, Web pages, and other sources.

You then construct your own statements about a topic, and occasionally you may want to use direct quotations from a text. Either way, there must be

- a *citation*: the source of the information must be acknowledged in the text of your work, and
- a *reference*: bibliographic information must be provided at the end of the document or in a footnote, depending on the style book used in the discipline.

Some Guidelines

As you work on a topic, ideas initially provided by an author or colleague become very familiar. Often they become so familiar that you may begin to think of them as your own! To avoid omissions or errors in acknowledging the sources of ideas, take thorough notes as you research. These notes should include bibliographic information and page numbers (or bookmarked URLs) for finding the information again.

Here are some guidelines:

- Widely known facts do not need attribution ("The U.S. population has increased substantially over the last 50 years.")
- Your references do not need to include sources that are not cited in your paper.
- Lines of reasoning (such as the outline of a mathematical model) taken or adapted from others must be credited, even if you do not use any wording from the source.
- All data sets, equations, figures, photos, graphs, and tables must be credited with their sources.
- It is OK to quote a source directly. To do that correctly,
  - either use quotation marks or (particularly for a long quotation) indent and set off the quotation from the rest of the text,
  - cite the source (including page number),
  - make sure you include among your references the full bibliographic data for the reference, and

   – get the quotation right.

- It's OK to summarize an author's thoughts. You still must cite the source, but in addition you must avoid close paraphrases. It is not acceptable to adopt an author's phrases, sentences, or sentence structure, even if you substitute synonyms or modify a few words. Use your own words and your own syntax. The best way to do that is not to have the source text in front of you when you write.

Instructive Examples

Here is an excerpt from Benton [1989, 4]:

> *The fin-back* Dimetrodon *was able to keep warm by orienting its "sail" perpendicular to the direction of sunlight.*

The following examples should help you understand the nature of plagiarism. All examples use the citation style of this *Journal*, which is the publication outlet for Outstanding ICM/MCM papers. Further style details of the *Journal* are at Campbell [2007]. (Of course, if you are preparing a non-contest paper for a different journal, you should acquaint yourself with its style policy and follow that.)

1. *The fin-back* Dimetrodon *was able to keep warm by orienting its "sail" perpendicular to the direction of sunlight [Benton 1989].*

   Why is this plagiarism? Since the statement is a direct quotation, attribution (to Benton) is not enough; you also need quotation marks (and many style manuals require you to cite the page number). Here's the proper way to write this example:

   > *"The fin-back* Dimetrodon *was able to keep warm by orienting its 'sail' perpendicular to the direction of sunlight" [Benton 1989, 4].*

2. *Benton [1989] claimed that the fin-back* Dimetrodon *was able to keep warm by orienting its "sail" perpendicular to the direction of sunlight.*

   This is properly attributed to Benton, but it still is plagiarism. Tell why.

3. *The sail-back* Dimetrodon *could keep warm by orienting her "sail" perpendicular to the sunlight.*

   There is no attribution. This is plagiarism, despite alteration of a few words.

4. *The sail-back* Dimetrodon *could keep warm by orienting her "sail" perpendicular to the sunlight [Benton, 1989].*

   This is almost exactly the same as 3). This time, there is attribution of the source, though it is not clear that the twice-used "apt term" of "sail" is Benton's and not the writer's. Also, although a few words have been changed from the original, the sentence structure is identical. Of course, the writer could continue to change the words until the result was almost completely different from the original. Hacker [2007] clearly indicates that this example is plagiarism.

How to Avoid Plagiarism

Develop your own approach. Don't rely on another author to dictate the organization of your report. If you adopt that organization, you might be tempted to adopt the sentence structure as well, and maybe even the wording—and this can lead to plagiarism. How do you develop your own approach?

- read about your subject,
- develop an outline,
- establish a set of topics to discuss,
- take notes from sources, and
- write those notes on index cards, one per topic.

You'll end up with a collection of note cards, each of which has ideas from five or six authors. You will then see connections among the ideas of a number of different authors; these new connections will allow you to express your ideas (and those of the authors) from a unique viewpoint, a viewpoint that requires a sentence structure and a vocabulary that differ from those of the original author(s). One way to do this is to read one of your note cards, then turn it over and write your own thoughts on the matter.

For example, suppose that your paper is to have a section on the thermal physiology of mammals and mammal-like reptiles:

> *Mammals are truly warm-blooded: they have specific physiologic mechanisms that maintain their body within a narrow range of temperatures. Mammal-like reptiles probably exploited a variety of strategies; for example,* Dimetrodon *may have used its "sail" as a thermoregulatory device [Benton 1989, 4].*

Here the statement about warm-bloodedness in mammals does not need attribution—warm-bloodedness in mammals is common knowledge. In addition, the language has been changed substantially, so quotation marks are not necessary. However, the specific point about *Dimetrodon* needs attribution, particularly since the word "sail" is an "apt term" that you did not invent but is Benton's.

Application to the ICM/MCM

The contest rule is simple and explicit. Given the unpleasant circumstances this year, however, the Contest Director will review it with a revision in mind to urge team members to review guidelines such as those given above. An important new policy in the ICM / MCM will be:

> *At the discretion of the judges, papers will be checked for originality of content and proper attribution of sources used. Any paper with an unattributed direct quotation or paraphrase, uncredited line of reasoning from another source, or uncredited figure or table will be disqualified. The team advisor will be informed of the disqualification, and the plagiarism policy of the institution may be invoked.*

In particular, all citations should include specific page numbers or URLs in the reference. Any references obtained electronically should specifically state so and include the exact URL or other source reference (not just say, for example, the generic page `http://stats.bls.gov`). For example, if your source is a journal article but you acquired it electronically (e.g., finding it on the Web or through JSTOR or another periodical database), you should give both the journal reference and the page-specific URL; doing so is an aid to the reader in locating a copy, which is the purpose of full bibliographic information in references.

References

Benton, M.J. 1989. [This reference is fictitious!]

Campbell, Paul J. 2007. Guide for authors. *The UMAP Journal* 28 (1): 91–92.

Hacker, Diana. 2007. *A Writer's Reference Sixth Edition with Writing in the Disciplines*. New York: Bedford / St. Martin's.

About the Author

Paul Campbell is Professor of Mathematics and Computer Science at Beloit College, where he was Director of Academic Computing from 1987 to 1990. He has been the editor of *The UMAP Journal of Undergraduate Mathematics and Its Applications* since 1984.

Carl Mendelson is Professor of Geology at Beloit College. He teaches Earth history, paleontology, and planetary geology. He is interested in early life on Earth and other planets.

Yaffa Grossman is Associate Professor of Biology at Beloit College and has chaired the Environmental Studies program since 2003. She received the James R. Underkofler Award for Excellence in Undergraduate Teaching Award in 2005. She is an author of PEACH, a computer simulation model of peach tree growth and development. (Photo by Greg Anderson.)

Modeling Forum

Results of the 2007 Interdisciplinary Contest in Modeling

Chris Arney, ICM Co-Director
Division Chief, Mathematical Sciences Division
Program Manager, Cooperative Systems
Army Research Office
PO Box 12211
Research Triangle Park, NC 27709–2211
David.Arney1@arl.army.mil

Introduction

A record total of 273 teams from five countries spent a weekend in February working on an applied modeling problem involving managing and promoting organ transplantation in the 9th Interdisciplinary Contest in Modeling (ICM). This year's contest began on Thursday, Feb. 8 and ended on Monday, Feb. 12, 2007. During that time, teams of up to three undergraduate or high school students researched, modeled, analyzed, solved, wrote, and submitted their solutions to an open-ended interdisciplinary modeling problem involving public health policy concerning organ transplants. After the weekend of challenging and productive work, the solution papers were sent to COMAP for judging. Two top papers, judged to be Outstanding by the expert panel of judges, appear in this issue of *The UMAP Journal*.

COMAP's Interdisciplinary Contest in Modeling and its sibling contest, the Mathematical Contest in Modeling, involve students working in teams to find and report a solution to an open problem. Centering its educational philosophy on mathematical modeling, COMAP supports the use of mathematical tools to explore real-world problems. It serves society by developing students as problem solvers in order to become better informed and prepared as citizens, contributors, consumers, workers, and community leaders.

The UMAP Journal 28 (2) (2007) 99–116.

This year's public-health problem was challenging in its demand for teams to utilize many aspects of science and mathematics in their modeling and analysis. The problem required teams to understand the basic science of organ transplantation and understand and model the complex policy issues associated with the health and psychological issues of organ failure and treatment in order to advise the Congress and the Department of Health Services on how to manage the nation's transplant network. In order to accomplish their tasks, the students had to consider many difficult and complex issues. Political, social, psychological, and technological issues had to be considered and analyzed along with several challenging requirements needing scientific and mathematical analysis. The problem also included the ever-present requirements of the ICM to use thorough data analysis, research, creativity, approximation, precision, and effective communication. The author of the problem was Paul J. Campbell, Professor of Mathematics and Computer Science at Beloit College. The problem came from a seminar of his on the mathematics behind the popular TV series *Numb3rs*. A commentary from Dr. Campbell appears in this issue of *The UMAP Journal*.

All members of the 273 competing teams are to be congratulated for their excellent work and dedication to scientific modeling and problem solving. The judges remarked that this year's problem was challenging with many interdisciplinary tasks.

Next year, we will continue with the public health theme for the contest problem. Teams preparing for the 2008 contest should consider reviewing interdisciplinary topics in the area of public health modeling and analysis.

Start-up funding for the ICM was provided by a grant from the National Science Foundation (through Project INTERMATH) and COMAP. Additional support is provided by the Institute for Operations Research and the Management Sciences (INFORMS) and IBM.

The Kidney Exchange Problem

Transplant Network

Despite the continuing and dramatic advances in medicine and health technology, the demand for organs for transplantation drastically exceeds the number of donors. To help this situation, the U.S. Congress passed the National Organ Transplant Act in 1984, establishing the Organ Procurement and Transplantation Network (OPTN) to match organ donors to patients with organ needs. Even with all this organizational technology and service in place, there are nearly 94,000 transplant candidates in the U.S. waiting for an organ transplant and this number is predicted to exceed 100,000 very soon. The average wait time exceeds three years—double that in some areas, such as large cities. Organs for transplant are obtained either from a cadaver queue or from living donors. The keys for the effective use of the cadaver queue are cooperation and good communication throughout the network. The good news is that the system is functioning and more and more donors (alive and deceased) are identified and used each year with record numbers of transplants taking place every month. The bad news is that the candidate list grows longer and longer. Some people think that the current system with both regional and national aspects is headed for collapse with consequential failures for some of the neediest patients. Moreover, fundamental questions remain: Can this network be improved and how do we improve the effectiveness of a complex network like OPTN? Different countries have different processes and policies,; which of these work best? What is the future status of the current system?

Task 1

For a beginning reference, read the OPTN Website (`http://www.optn.org`) with its policy descriptions and data banks (`http://www.optn.org/data` and `http://www.optn.org/latestData/viewDataReports.asp`). Build a mathematical model for the generic U.S. transplant network(s). This model must be able to give insight into the following: Where are the potential bottlenecks for efficient organ matching? If more resources were available for improving the efficiency of the donor-matching process, where and how could they be used? Would this network function better if it was divided into smaller networks (for instance at the state level)? And finally, can you make the system more effective by saving and prolonging more lives? If so, suggest policy changes and modify your model to reflect these improvements.

Task 2

Investigate the transplantation policies used in a country other than the U.S. By modifying your model from Task 1, determine if the U.S. policy would be improved by implementing the procedures used in the other country. As members of an expert analysis team (knowledge of public health issues and network science) hired by Congress to perform a study of these questions, write a one-page report to Congress addressing the questions and issues of Task 1 and the information and possible improvements you have discovered from your research of the different country's policies. Be sure to reference how you used your models from Task 1 to help address the issues.

Focusing on Kidney Exchange

Kidneys filter blood, remove waste, make hormones, and produce urine. Kidney failure can be caused by many different diseases and conditions. People with end-stage kidney disease face death, dialysis (at over $60,000/yr), or the hope for a kidney transplant. A transplant can come from the cadavers of an individual who agreed to donate organs after death, or from a live donor. In the U.S., about 68,000 patients are waiting for a kidney from a deceased donor, while each year only 10,000 are transplanted from cadavers and 6,000 from living individuals (usually relatives of the patients). Hence the median wait for a matching kidney is three years—unfortunately, some needy patients do not survive long enough to receive a kidney.

There are many issues involved in kidney transplantation—the overall physical and mental health of the recipient, the financial situation of the recipient (insurance for transplant and post-operation medication), and donor availability (is there a living donor willing to provide a kidney?). The transplanted kidney must be of a compatible ABO blood type. The 5-year survival of the transplant is enhanced by minimizing the number of mismatches on six HLA markers in the blood. At least 2,000 would-be-donor/recipient pairs are thwarted each year because of blood-type incompatibility or poor HLA match. Other sources indicate that over 6,000 people on the current waiting list have a willing but incompatible donor. This is a significant loss to the donor population and worthy of consideration when making new policies and procedures.

An idea that originated in Korea is that of a kidney exchange system, which can take place either with a living donor or with the cadaver queue. One exchange is *paired-kidney donation*, where each of two patients has a willing donor who is incompatible, but each donor is compatible with the other patient; each donor donates to the other patient, usually in the same hospital on the same day. Another idea is *list paired donation*, in which a willing donor, on behalf of a particular patient, donates to another person waiting for a cadaver kidney; in return, the patient of the donor-patient pair receives higher priority for a compatible kidney from the cadaver queue. Yet a third idea is to expand the paired kidney donation to 3-way, 4-way, or a circle (n-paired) in which each donor gives to the next patient around the circle. On November 20, 2006, 12 surgeons performed the first-ever 5-way kidney swap at Johns Hopkins Medical Facility. None of the intended donor-recipient transplants were possible because of incompatibilities between the donor and the originally intended recipient. At any given time, there are many patient-donor pairs (perhaps as many as 6,000) with varying blood types and HLA markers. Meanwhile, the cadaver queue receives kidneys daily and is emptied daily as the assignments are made and the transplants performed.

Task 3

Devise a procedure to maximize the number and quality of exchanges, taking into account the medical and psychological dynamics of the situation. Justify in what way your procedure achieves a maximum. Estimate how many more annual transplants your procedure will generate, and the resulting effect on the waiting list.

Strategies

Patients can face agonizing choices. For example, suppose a barely compatible—in terms of HLA mismatches—kidney becomes available from the cadaver queue. Should they take it or wait for a better match from the cadaver queue or from an exchange? In particular, a cadaver kidney has a shorter half-life than a live donor kidney.

Task 4

Devise a strategy for a patient to decide whether to take an offered kidney or to even participate in a kidney exchange. Consider the risks, alternatives, and probabilities in your analysis.

Ethical Concerns

Transplantation is a controversial issue with both technical and political issues that involve balancing what is best for society with what is best for the individual. Criteria have been developed very carefully to try to ensure that people on the waiting list are treated fairly, and several of the policies try to address the ethical concerns of who should go on to the list or who should come off. Criteria involved for getting on or coming off the list can include diagnosis of a malignant disease, HIV infection or AIDS, severe cardiovascular disease, a history of non-compliance with prior treatment, or poorly controlled psychosis. Criteria used in determining placement priority include: time on the waiting list, the quality of the match between donor and recipient, and the physical distance between the donor and the recipient. As a result of recent changes in policy, children under 18 years of age receive priority on the waiting list and often receive a transplant within weeks or months of being placed on the list. The United Network for Organ Sharing Website recently (Oct. 27, 2006) showed the age of waiting patients as:

Under 18:	748
18 to 34:	8,033
35 to 49:	20,553
50 to 64:	28,530
65 and over:	10,628

One ethical issue of continual concern is the amount of emphasis and priority on age to increase overall living time saved through donations. From a statistical standpoint, since age appears to be the most important factor in predicting length of survival, some believe kidneys are being squandered on older recipients.

Political Issues

Regionalization of the transplant system has produced political ramifications (e.g., someone may desperately need a kidney and is quite high on the queue, but his or her deceased neighbor's kidney still can go to an alcoholic drug dealer 500 miles away in a big city). Doctors living in small communities, who want to do a good job in transplants, need continuing experience by doing a minimum number of transplants per year. However, the kidneys from these small communities frequently go to the hospitals in the big city and, therefore, the local doctors cannot maintain their proficiency. This raises the question, perhaps transplants should be performed only in a few large centers, by a few expert and experienced surgeons? But would that be a fair system and would it add or detract from system efficiency?

Many other ethical and political issues are being debated. Some of the current policies can be found at `http://www.unos.org/policiesandbylaws/policies.asp?resources=true` . For example, recent laws have been passed in the U.S. that forbid the selling or mandating the donation of organs, yet there are many agencies advocating for donors to receive financial compensation for their organ. The state of Illinois has a new policy that assumes everyone desires to be an organ donor (presumed consent) and people must opt out if they do not. The Department of Health and Human Services Advisory Committee on Organ Transplantation is expected to recommend that all states adopt policies of presumed consent for organ donation. The final decision on new national policies rests with the Health Resources and Services Administration within the U.S. Department of Health and Human Services.

Task 5

Based on your analysis, do you recommend any changes to these criteria and policies? Discuss the ethical dimensions of your recommended exchange procedure and your recommended patient strategy (Tasks 3 and 4). Rank order the criteria you would use for priority and placement, as above, with rationale as to why you placed each where you did. Would you consider allowing people to sell organs for transplantation? Write a one-page paper to the Director of the U.S. Health Resources and Services Administration with your recommendations.

Task 6

From the potential donor's perspective, the risks in volunteering involve assessing the probability of success for the recipient, the probability of survival of the donor, the probability of future health problems for the donor, the probability of future health risks (such as failure of the one remaining kidney), and the post-operative pain and recovery. How do these risks and others affect the decision of the donor? How do perceived risks and personal issues (phobias, irrational fears, misinformation, previous experiences with surgery, level of altruism, and level of trust) influence the decision to donate? If entering a list paired network rather than a direct transplant to the relative or friend, does the size n of the n-paired network have any effect on the decision of the potential donor? Can your models be modified to reflect and analyze any of these issues? Finally, suggest ways to develop and recruit more altruistic donors.

(Note: None of the data files, including the data in the appendices, are included in this article. They are available on the COMAP Website, at `http://www.comap.com/undergraduate/contests/mcm/contests/2007/problems/`.)

The Results

The 273 solution papers were coded at COMAP headquarters so that names and affiliations of the authors were unknown to the judges. Each paper was then read preliminarily by "triage" judges at the U.S. Military Academy at West Point, NY. At the triage stage, the summary, the model description, and overall organization are the primary elements in judging a paper. Final judging by a team of modelers, analysts, and subject-matter experts took place in March. The judges classified the 273 submitted papers as follows:

	Outstanding	Meritorious	Honorable Mention	Successful Participation	Total
Kidney Exchange	2	42	169	58	273

The two papers that the judges designated as Outstanding appear in this special issue of *The UMAP Journal*, together with commentaries by the author and the final judges. We list those two Outstanding teams and the Meritorious teams (and advisors) below. The complete list of all participating schools, advisors, and results is provided in the **Appendix**.

Outstanding Teams

Institution and Advisor	**Team Members**
"Optimizing the Effectiveness of Organ Allocation"	
Duke University	Matthew Rognlie
Durham, NC	Peng Shi
David Kraines	Amy Wen
"Analysis of Kidney Transplant System Using Markov Process Models"	
Princeton University	Jeff Tang
Princeton, NJ	Yue Yang
Ramin Takloo	Jingyuan Wu

Meritorious Teams (38)

Anhui University, China (Mingsheng Chen)
Asbury College, Wilmore, KY (Duk Lee)
Beijing Jiao Tong University, China (4 teams) (Yuchuan Zhang) (Zhouhong Wang)
(Hong Zhang—-2 teams)
Beijing Language and Culture University, China (Rou Song)
Beijing University of Posts and Telecommunications, China (Hongxiang Sun)
Berkshire Community College, Pittsfield, MA (Andrew Miller)
Cambridge University, England (Thomas Duke)
Chengdu University of Technology, China (Yuan Yong)
Duke University, Durham, NC (2 teams) (Anita Layton) (Fernando Schwartz)
East China University of Science & Technology, China (Lu Xiwen)
Fudan University, China (Yuan Cao)
Harbin Institute of Technology, China (2 teams) (Shouting Shang) (Jiqyun Shao)
Hunan University, China (Chuanxiu Ma)
Nanjing University, China (Xu LiWei)
Olin College, Needham, MA (Burt Tilley)
Peking University Health Science Center, China (Zhang Xia)
PLA University of Science and Technology, China (2 teams) (Yao Kui) (Zheng Qin)
Rice University, Houston, TX (Robert Hardt)
South China University of Technology, China (Shen-Quan)
Southeast University, China (Zhiqiang Zhang)
Shandong University, China (3 teams) (Baodong Liu) (Jianliang Chen) (Huang Shuxiang)
Shandong University at Weihai, China (Huaxiang Zhao)
United States Military Academy, West Point, NY (Joseph Lindquist)
University of Electronics Science, China (3 teams) (Du Hongfei) (Lihui Wang) (Li Mingqi)
University of Geosciences, China (2 teams) (Guangdong Huang – 2 teams)
Xidian University, China (2 teams) (Xuewen Mu) (Xiaogang Qi)
Zhejiang Gongshang, University, China (Ding Zhengzhong)
Zhejiang University, China (Yong Wu)
Zhejiang University City College, China (Waibin Huang)
Zuhai University, China (Zhiwei Wang)

Awards and Contributions

Each participating ICM advisor and team member received a certificate signed by the Contest Directors and the Head Judge. Additional awards were presented to the Princeton University team from the Institute for Operations Research and the Management Sciences (INFORMS).

Judging

Contest Directors

Chris Arney, Division Chief,
Mathematical Sciences Division, Army Research Office,
Research Triangle Park, NC

Joseph Myers, Dept. of Mathematical Sciences, U.S. Military Academy,
West Point, NY

Associate Director

Richard Cassady, Dept. of Industrial Engineering, University of Arkansas,
Fayetteville, AR

Judges

Kathleen Crowley, Dept. of Psychology, College of Saint Rose, Albany, NY

Michael Matthews, Dept. of Behavioral Sciences, U.S. Military Academy,
West Point, NY

Shawnie McMurran, Dept. of Mathematics, California State University,
San Bernardino, CA

Bret McMurran, Dept. of Economics, Chaffey Community College,
Rancho Cucamonga, CA

Frederick Rickey, Dept. of Mathematical Sciences, U.S. Military Academy,
West Point, NY

Frank Wattenberg, Dept. of Mathematical Sciences, U.S. Military Academy,
West Point, NY

Triage Judges

Dept. of Mathematical Sciences, U.S. Military Academy, West Point, NY:

Eric Brown, Gabriel Costa, Jong Chung, Rachelle DeCoste, Amy Erickson, Keith Erickson, Edward Fuselier, Greg Graves, Michael Harding, Alex Heidenberg, Josh Helms, Anthony Johnson, Thomas Kastner, Jerry Kobylski, Ian McCulloh, Barbara Melendez, Thomas Messervey, Fernando Miguel, Jason Miseli, Joseph Myers, Mike Phillips, Todd Retchless, Wiley Rittenhouse, Mick Smith, Heather Stevenson, Rodney Sturdivant, Krista Watts, Robbie Williams, Erica Slate Young

Dept. of Behavioral Sciences, U.S. Military Academy, West Point, NY:

Mike Matthews

Acknowledgments

We thank:

- the Institute for Operations Research and the Management Sciences (INFORMS) for its support in judging and providing prizes for the winning team;
- IBM for their support for the contest;
- all the ICM judges and ICM Board members for their valuable and unflagging efforts;
- the staff of the U.S. Military Academy, West Point, NY, for hosting the triage and final judgings.

Cautions

To the reader of research journals:

Usually a published paper has been presented to an audience, shown to colleagues, rewritten, checked by referees, revised, and edited by a journal editor. Each of the student papers here is the result of undergraduates working on a problem over a weekend; allowing substantial revision by the authors could give a false impression of accomplishment. So these papers are essentially au naturel. Light editing has taken place: minor errors have been corrected, wording has been altered for clarity or economy, style has been adjusted to that of *The UMAP Journal*, and the papers have been edited for length. Please peruse these student efforts in that context.

To the potential ICM Advisor:

It might be overpowering to encounter such output from a weekend of work by a small team of undergraduates, but these solution papers are highly atypical. A team that prepares and participates will have an enriching learning experience, independent of what any other team does.

Editor's Note

As usual, the Outstanding papers were longer than we can accommodate in the *Journal*, so space considerations forced me to edit them for length. It was not possible to include all of the many tables and figures.

In editing, I endeavored to preserve the substance and style of the papers, especially the approach to the modeling.

—Paul J. Campbell, Editor

Appendix: Successful Participants

KEY:
P = Successful Participation
H = Honorable Mention
M = Meritorious
O = Outstanding (published in this special issue)

INSTITUTION	CITY	ADVISOR	C
CALIFORNIA			
Harvey Mudd College	Claremont	Darryl Yong	H,P
		Susan Martonosi	H
COLORADO			
University of Colorado	Colorado Springs	Radu C. Cascaval	P
	Denver	Gary A. Olson	H
ILLINOIS			
Monmouth College (Phys)	Monmouth	Christopher G. Fasano	P
IOWA			
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Editor's Note

For team advisors from China, I have endeavored to list family name first.

Abbreviations for Organizational Unit Types (in parentheses in the listings)

(none)	Mathematics	M; Pure M; Applied M; Computing M; M and Computer Science; M and Computational Science; M and Information Science; M and Statistics; M, Computer Science, and Statistics; M, Computer Science, and Physics; Mathematical Sciences; Applied Mathematical and Computational Sciences; Natural Science and M; M and Systems Science; Applied M and Physics
Bio	Biology	B; B Science and Biotechnology; Biomathematics; Life Sciences
Bus	Business	B; B Management; B and Management; B Administration
Chm	Chemistry	C; Applied C; C and Physics; C, Chemical Engineering, and Applied C
CS	Computer	C Science; C and Computing Science; C Science and Technology; C Science and (Software) Engineering; Software; Software Engineering; Artificial Intelligence; Automation; Computing Machinery; Science and Technology of Computers
Econ	Economics	E; E Mathematics; Financial Mathematics; E and Management; Financial Mathematics and Statistics; Management; Business Management; Management Science and Engineering
Eng	Engineering	Civil E; Electrical Eng; Electronic E; Electrical and Computer E; Electrical E and Information Science; Electrical E and Systems E; Communications E; Civil, Environmental, and Chemical E; Propulsion E; Machinery and E; Control Science and E; Mechanisms; Mechanical E; Electrical and Info E; Materials Science and E; Industrial and Manufacturing Systems E
Info	Information	I Science; I and Computation(al) Science; I and Calculation Science; I Science and Computation; I and Computer Science; I and Computing Science; I Engineering; I and Engineering; Computer and I Technology; Computer and I Engineering; I and Optoelectronic Science and Engineering
Phys	Physics	P; Applied P; Mathematical P; Modern P; P and Engineering P; P and Geology; Mechanics; Electronics
Sci	Science	S; Natural S; Applied S; Integrated S; School of S
Software	Software	
Stat	Statistics	S; S and Finance; Mathematical S; Probablity and S; S and Actuarial

Optimizing the Effectiveness of Organ Allocation

Matthew Rognlie
Peng Shi
Amy Wen

Duke University
Durham, NC

Advisor: David Kraines

Introduction

The first successful organ transplant occurred in 1954,a kidney transplant between twin brothers in Boston [Woodford 2004]. Since then, although the number of transplants per year has steadily risen, the number of organ donors has not kept up with demand [Childress and Liverman 2006] (**Figure 1**).

To ensure equitable distribution of available organs, Congress passed the National Organ Transplant Act in 1984. The act established the Organ Procurement and Transplantation Network (OPTN), a regionalized network for organ distribution [Conover and Zeitler 2006]. In 2000, the U.S. Department of Health and Human Services (HHS) implemented a additional policy called the *Final Rule*, which ensured that states could not interfere with OPTN policies that require organ sharing across state lines [Organ Procurement . . . 1999].

The organ matching process involves many factors, whose relative importance depends on the type of organ involved. These include compatibility, region, age, urgency of patient, and waitlist time [Organ Procurement . . . 2006]. Although most countries use the same basic matching processes, systems vary in their emphasis on particular parameters [Transplantation Society . . . 2002; UK Transplant 2007; Doxiadis et al. 2004; De Meester et al. 1998].

In 2006, kidneys comprised 59% of all organs transplanted [Organ Procurement . . . 2007]. In determining compatibility in kidney transplants, doctors look at:

- **ABO blood type:** The ABO blood type indicates the presence of two types of antigens, A and B, present in the patient's body. Antigens are foreign molecules or substances that trigger an immune response. People with blood

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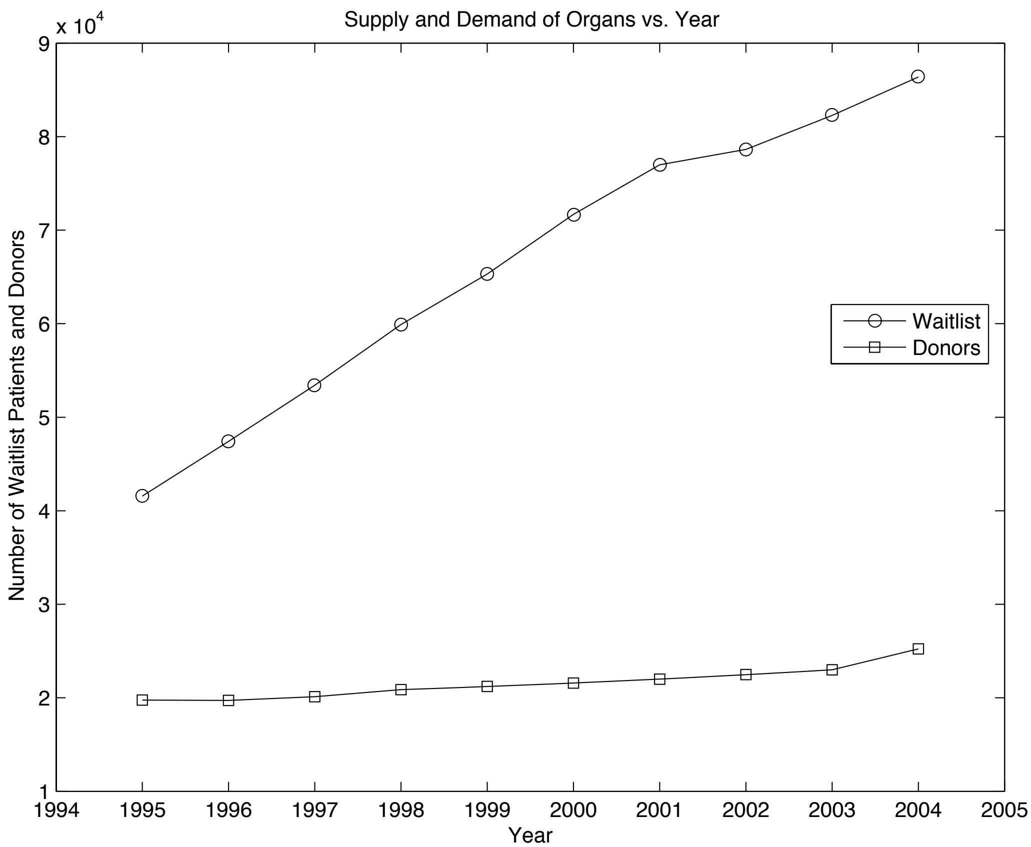

Figure 1. Number of transplants and cadaveric organ donors. Source: OPTN Annual Report 2005 [U.S. Organ Procurement . . . 2005].

type A have antigen A in their body, people with blood type B have antigen B, people with blood type AB have both, and people with blood type O have neither. A person with blood type AB can receive an organ from anyone, a person with blood type A or B can receive an organ from a person of blood type O or the same blood type, and a person with blood type O can receive an organ only from someone with type O blood.

- **Human Leukocyte Antigens (HLA):** HLA indicates a person's tissue type, whose most important components are the A, B, and DR antigens. Each antigen consists of two alleles, and matching all six components results in a significantly increased success rate for kidney transplants. Patients with mismatched components, however, can still survive for many years [U.S. Organ Procurement . . . 2005].

- **Panel Reactive Antibody (PRA):** PRA is a blood test, measuring the percentage of the U.S. population that blood samples are likely to react with. It tests for the presence of antibodies, proteins that bind to foreign molecules [University of Maryland . . . 2004a]. Blood can become more sensitive due to previous transplants, blood transfusions, or pregnancies [Duquesnoy 2005].

Kidney transplants are common partly because kidneys, unlike most other organs, can be safely obtained from live donors. In fact, live-donor kidneys are more effective than cadaveric kidneys, with longer half-lives and lower rejection rates [Gentry et al. 2005]. Over 75% of living donors in 2004 were related (parents, siblings, spouses) to the transplant recipients [Childress and Liverman 2006]. However, some people willing to donate to an intended recipient cannot because of blood type or HLA incompatibility, leaving over 30% of patients without a suitable kidney transplant [Segev et al. 2005]. One solution is kidney paired donation (KPD), which matches two incompatible donor-recipient pairs where the donor of each pair is compatible with the recipient of the other, satisfying both parties [Ross et al. . Another is list paired donation (LPD), where a recipient receives higher priority on the waitlist if an associated donor gives to another compatible recipient on the waitlist [Gentry et al. 2005].

We incorporate all these factors in modeling the various aspects of transplantation. First, we focus on the U.S. network and produce a generic model of the processes that impact the number of people on the waitlist, the number of transplants, and the length of wait time. To illustrate our model, we use data specific to kidney transplants, and also examine the policy of Eurotransplant for ideas on improving the current U.S. system. We then construct a model of list paired donation to determine how to maximize the number of exchanges while maintaining compatibility. Finally, we analyze the implications of our model for patient and donor decisions, taking note of important ethical and political issues.

Generic U.S. Transplant Network

Overview

We model the generic U.S. transplant network as a rooted tree (growing downward). The root represents the entire network, and its immediate children represent the regions. Each node represents some kind of organization, whether an Organ Procurement Center, a state organization, or a interstate region. At each node, there is a patient wait list, the concatenation of the wait list of the node's children.

We approximate the network's functioning as a discrete-time process, in which each time step is one day with four phases:

- In phase I, patients are added to the leaf nodes. We approximate the rate of wait list addition by a Poisson process; doing so is valid because we can reasonably assume that the arrivals are independent, identically distributed, and approximately constant from year to year. Suppose that this number of candidates added to the wait list at time t is addition$_t$, then we model

$$Pr(\text{addition}_{t+1} - \text{addition}_t = k) = \frac{e^{-\lambda}\lambda^k}{k!}.$$

For the rate constant λ, we use the number of organ applicants in a given year, $\lambda \approx$ (number of new applicants)$/365.25$.

- In phase II, we add cadaver organs to the leaf nodes. As with patients, we model cadaver arrivals as a Poisson process, with rate the average number of cadaver organs added in a given year.

- In phase III, we allocate organs based on *bottom-up priority rules*. A bottom-up priority rule is a recursive allocation process propagated up from the bottom of the tree, which requires any organ-patient match to meet some minimum priority standard. For example, for kidney allocation, the first priority rule is to allocate kidneys to patients who match the blood type and HLA profile exactly. Within this restriction, OPTN dictates that kidneys be allocated locally first, then regionally, then nationally. In our model, this corresponds to moving from the leaves up the tree. Matched organ-patient pairs undergo transplantation, which has a success rate dependent on the quality of the match. (In later sections, we explore the success rate as also a function of the experience of the doctors at the center and the quality of the kidney.)

- In phase IV, we simulate the death of patients on the waiting list. We treat the death rate k of a patient as a linear function $aT + b$ of the person's wait time T. Hence, calculating from time 0, a person's chance of survival to time T is $e^{-kT} = e^{-(aT+b)T}$.

Under this mathematical model, our problem becomes finding a good tree structure and an appropriate set of bottom-up priority rules.

Simulation

To study this model, we average results over many simulations of the kidney transplant network. Our simulation works as follows: At every time round, in phase I, we generate a number according to the Poisson distribution of the number of new candidates. For each new patient added, we randomly generate the person's race and age according to data on race and age distributions. Using the person's race, we generate the person's blood type and HLA makeup, according to known distributions, and the patient's PRA, based on probabilities published by the OPTN.

Similarly, in Phase II, we generate a list of donor organs according to known distributions of blood type and HLA makeup. Moreover, we record where the organ was generated, so we can study the effect of having to move the organ before transplantation, the time for which lowers its quality.

In Phase III, we implement recursive routines that traverse the tree from the bottom up, following the OPTN system for kidneys. To model the success rate of an operation, we use the statistics published by the OPTN; our main method of determining whether an operation is successful is the number of

HLA mismatches. Success is affected by the sensitivity of the person, measured by the person's PRA, which we model by adding a linear term to the success rate. Moreover, we reduce the success rate by 5 percentage points if the organ is not procured from the same center as the patient; 5% is the average effect on the success rate of increasing the delay by 10–20 hrs, according to OPTN data.

In Phase IV, we regress the coefficients a and b of the previous section, and use this formula to calculate the probability of death.

To adjust the parameters for this model, we use the OPTN national data for the national active wait list for cadaver kidneys from 1995 to 2004 and feed into our model the number of donations for each year.

Results of the Basic Model

To quantify the quality of a network, we use a set of objective functions, which represent various ideas about the desirability of policy outcomes. For these functions, let

x be the number of "healthy" patients to receive a successful transplant each year,

y be the corresponding number of "sick" patients (those with some terminal illness or serious medical condition) to receive a successful transplant,

a be the average age of the transplant recipients, and

m be the maximum wait time in the queue.

We examine the following objective functions:

$x + y$: This is simply the number of successful transplants per year.

$(100 - a) \times (x + y)$: This considers the premise that transplants are more valuable when given to young recipients.

$x + 0.5y$: This is a stylized adoption of the idea that transplants given to terminally ill recipients are less valuable.

$(x + y)/\max(9, m)$: This incorporates queue wait time.

We also include a proposed tradeoff between big and small centers:

- In a big center, the doctors are more experienced. We simulate this by decreasing the success rate of operations at centers that do not perform a threshold number of operations per year.
- With small centers, kidneys are allocated on a more local basis, which minimizes deterioration of organs in transportation. We simulate this by applying a penalty when kidneys are moved to larger regional centers, and also when kidneys are moved between centers.

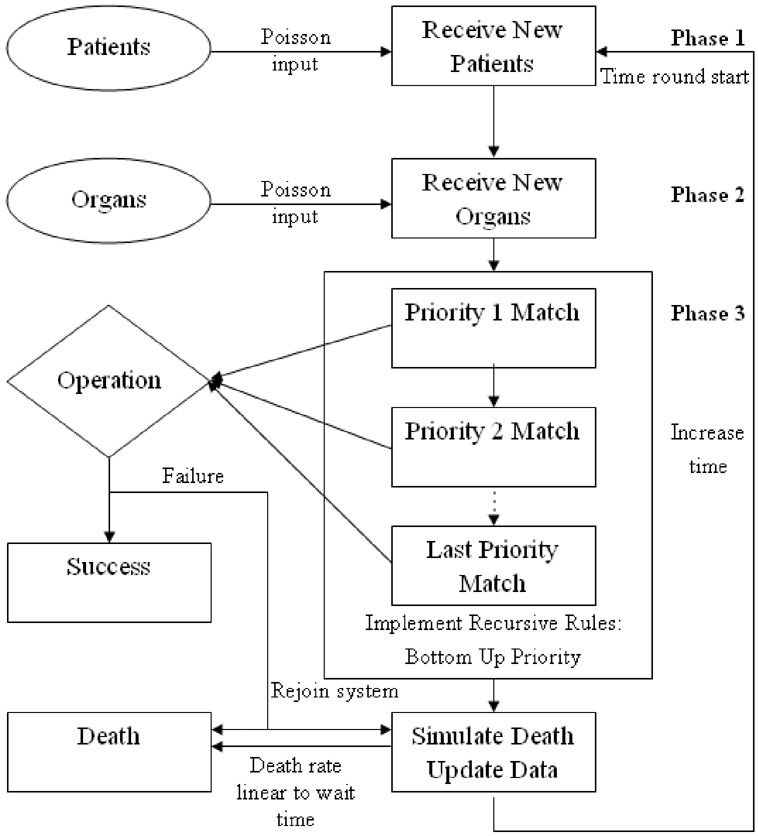

Figure 2. Flowchart for simulation.

Summary of Assumptions

- Arrivals in the waiting queue, both of cadaver donors and needy patients, are independent and randomly distributed
- The generic U.S. transplant network can be simulated as a rooted tree
- Death rate can be approximated as a linear function of time on the waiting list.

Other Countries' Transplantation Policies

We researched the policies of other countries, such as China, Australia, and the United Kingdom; they differ little from the U.S. policy. China uses organs from executed prisoners, which we do not believe to be ethical. We decided that the policies of Eurotransplant have the best groundwork: People analyze their policy each year, tweaking the waiting-time point system.

The Eurotransplant policy does not emphasize regions as much, with the maximum number of points for distance being 300. In contrast, the number of points received for zero HLA mismatch is 400. The Eurotransplant policy also has greater emphasis on providing young children with a kidney match, giving children younger than 6 years an additional 1095 waiting-time points.

We implemented the Eurotransplant policy in our model to see if that policy could also benefit the U.S., but we found little difference.

Utilizing Kidney Exchanges

A promising approach for kidney paired exchange is to run the maximal matching algorithm over the graph defined by the set of possible exchanges. However, this approach takes away from the autonomy of patients, because it requires them to wait for enough possible pairs to show up before performing the matching, and sometimes it may require them to take a less than perfect matching.

We sought to improve this supposedly "optimal solution" by implementing list paired donation in our model.

According to each patient's phenotypes, we calculate the expected blood types of the person's parents and siblings, and make that the person's contribution to the "donor pool." In other words, the person brings to the transplant network an expected number r of potential donors. We then make the patient perform list paired donation with the topmost person in the current queue who is compatible in blood type to the donor accompanying the new patient. According to our research, kidneys from live donors are about 21% better than cadaver kidneys in terms of success rate. Thus, it is in the cadaver-list person's best interest to undergo this exchange.

We find that for any value of r from 0.2 to 2, list paired donation drastically decreases the length of the waitlist, by factors as large as 3, and makes the queue size stabilize (**Figure 3**).

Patient Choices

What should a patient do when presented with the opportunity for a kidney? The decision is not clear-cut; for instance, if the patient is offered a poorly matched kidney now, but a well-matched kidney is likely to arrive in a reasonable time, the patient should perhaps wait. We examine the this tradeoff.

We assume that a patient who has already received a kidney transplant may not receive another in the future. While this is not always true, it suffices for the purposes of our model, since we posit a choice between accepting a "lesser" kidney today and a better kidney later. (When a patient receives a second kidney transplant after the first organ's failure, there is no reason to expect a better organ, since the patient cannot immediately return to the top of the cadaver kidney queue, and live donors are likely to be more reluctant after a previous failure.)

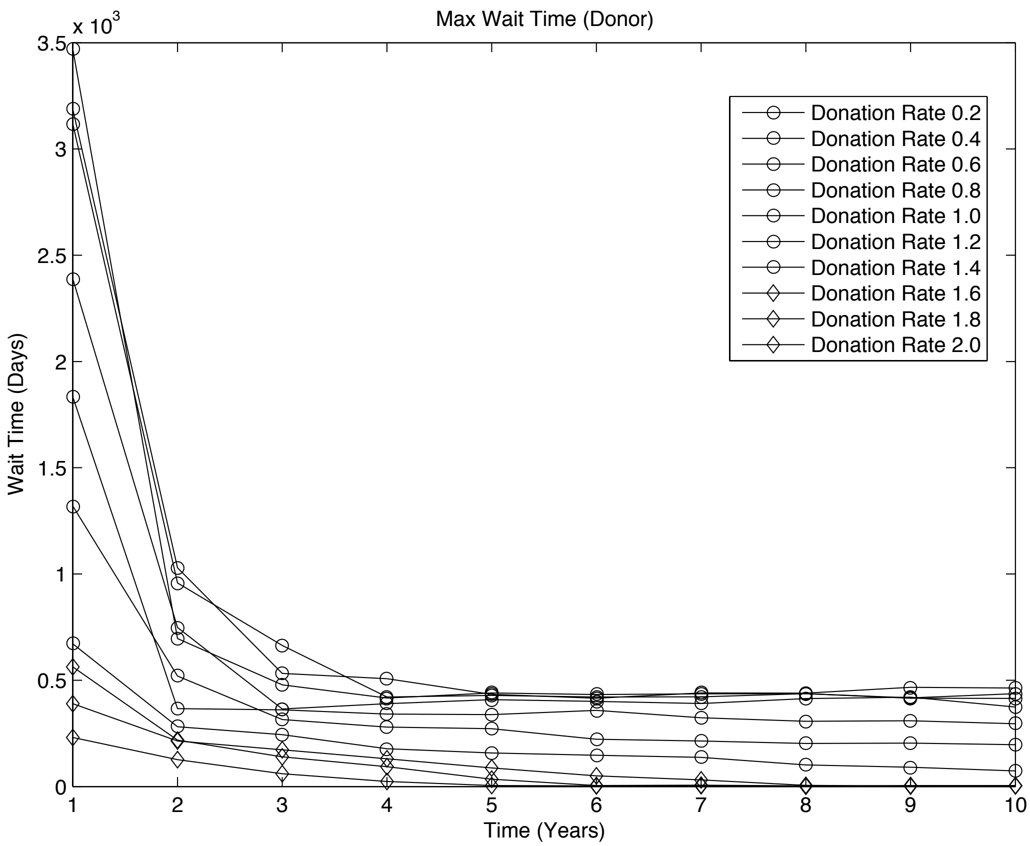

Figure 3. Wait time (in days) for various values of donation rate r, with list paired donation, applied over time to the current waitlist.

We assume that patients want to maximize expected years of life.

Let there be a current transplant available to the patient; we call this the *immediate alternative* and denote it by $\mathcal{A}_0$. The patient and doctor have some estimate of how this transplant will affect survival; we assume that they have a *survival function* $s_0(0, t)$ that describes chance of being alive at time t after the transplant. We further assume that this survival function is continuous and has limit zero at infinity: In other words, the patient is neither strangely prone to die in some infinitesimal instant nor capable of living forever.

The patient also has a set of possible future transplants, which we call *future alternatives* and write as $(\mathcal{A}_1, \mathcal{A}_2, \dots, \mathcal{A}_n)$. Each future alternative $\mathcal{A}_i$ also has a corresponding survival function $s_i(t_0, t)$, where t_0 is the starting time of transplant and t is the current time. We assume that there is a constant probability p_i that alternative $\mathcal{A}_i$ will become available at any time. While this is not completely true, we include it to make the problem manageable: More complicated derivations would incorporate outside factors whose complexity would overwhelm our current framework. Finally, if the patient opts for a future alternative and delays transplant, survival is governed by a *default survival function* s_d.

Summary of Assumptions

- The patient can choose either a transplant now (the immediate alternative $\mathcal{A}_0$), or from a finite set of transplants $(\mathcal{A}_1, \mathcal{A}_2, \dots, \mathcal{A}_n)$ in the hypothetical future (the future alternatives).
- Each alternative has a corresponding survival function $s_i(t_0, t)$, which describes the chance of survival until time t as a function of t and transplant starting time t_0. Survival functions have value 1 at time 0, are continuous, and have limit zero at infinity.
- Each future alternative $\mathcal{A}_i$ has a corresponding constant probability p_i of becoming available at any given time. Hence, the probability at time t of the alternative not yet having become available is $e^{-p_i t}$.
- A default survival function $s_d(t)$ defines the chance of survival when there has not yet been a transplant. To maintain continuity, $s_d(t_0) = s_i(t_0, t)$.
- The patient can have only one transplant.
- The patient attempts to maximize expected lifespan; in case of a tie in expected values, the patient chooses an option that provides a kidney more quickly.

The survival functions must behave consistently; they cannot become wildly better or worse-performing relative to each other. We propose a formal definition to capture this concept.

> *A* separable *survival function $s_i(t_0, t)$ is one that can be expressed as the product of two functions, one a function of only t_0 and the other a function of only $t - t_0$:*
> $$s_i(t_0, t) = a_i(t_0) b_i(t - t_0).$$
> *We stipulate that $b(0) = 1$. In a* separable set *of survival functions, all functions are individually separable with the same function $a(t_0)$.*

Is it be reasonable to assume that for any patient, the set of survival functions is separable? It is not an entirely natural condition, and indeed there are cases where it does not seem quite right—for instance, when some t_0 is high, so that higher values of t approach extreme old age, where survival decreases rapidly and the patient is less likely to survive than the product of a and b predicts. But in this case, the absolute error is small anyway: $a(t_0)$ accounts for the probability of survival that stems from waiting for a kidney until time t_0, and thus if t_0 is large, $a(t_0)b(t - t_0)$ is likely to be quite tiny as well.

Moreover, separability is intuitively reasonable for modeling the effects of a delayed kidney donation. The function $a(t_0)$ measures the decrease in survival rate that results from waiting for an organ transplant. This *should* be consistent across all survival functions for a given patient; we express this notion in the concept of a separable set. Meanwhile, the factor $b(t - t_0)$ accounts for the decrease in survival during the time $(t - t_0)$ spent with the new kidney.

Consequently, we assume that our survival functions are separable. This will lead us to an explicit heuristic for lifespan-maximizing decisions, which is the goal of this section.

For $\mathcal{A}_i$ and $\mathcal{A}_j$ two future alternatives in a separable set, we assign an order according to:

$$\int_0^\infty b_i(t)\, dt \le \int_0^\infty b_j(t)\, dt \longleftrightarrow \mathcal{A}_i \le \mathcal{A}_j$$

We turn to the derivation of an lifespan-maximizing strategy. Such a strategy, when presented with alternative $\mathcal{A}_i$ at time t_0, will either accept or wait for other alternatives. In fact:

Theorem. *If a patient's alternatives form a separable set, then the optimal strategy is either to accept an alternative $\mathcal{A}_i$ at all times t_0 or to decline it at all times t_0. If the patient declines $\mathcal{A}_i$, then the patient must decline all alternatives less than or equal to $\mathcal{A}_i$ in the order relation defined above. Similarly, if the patient accepts $\mathcal{A}_j$, then the patient must accept all alternatives greater than or equal to $\mathcal{A}_i$.*

Proof: The patient will accept the alternative or probabilistic bundle of alternatives that the patient's survival functions indicate gives the greatest lifespan. For alternative $\mathcal{A}_i$, the expected lifespan beyond time t_0 is

$$\int_0^\infty s_i(t_0, t)\, dt.$$

Suppose that a patient at time 0 declines this alternative in favor of some optimal set of future alternatives. Furthermore, suppose that this set includes some alternative $\mathcal{A}_k$ such that $\mathcal{A}_k \le \mathcal{A}_i$. Then the expected lifespan from this set is

$$\left(\sum_j p_j + p_k\right) \int_0^\infty \exp\left[-\left(\sum_j p_j + p_k\right) t_0\right] a(t_0) \int_0^\infty \frac{\left(\sum_j p_j b_j(t) + p_k b_k(t)\right)}{\sum_j p_j + p_k}\, dt\, dt_0,$$

where j ranges over all alternatives $\mathcal{A}_j$ in the optimal set except $\mathcal{A}_k$. This double integral does not mix integration variables and is therefore equal to a product of two integrals:

$$\left(\sum_j p_j + p_k\right) \int_0^\infty \exp\left[-\left(\sum_j p_j + p_k\right) t_0\right] a(t_0)\, dt_0 \int_0^\infty \frac{\sum_j p_j b_j(t) + p_k b_k(t)}{\sum_j p_j + p_k}\, dt.$$

Since $\mathcal{A}_k$ is less than or equal to $\mathcal{A}_i$, and $\mathcal{A}_i$ was declined in favor of the set of alternatives that we are examining, the presence of the k term in the weighted average under the right integrand lowers the value of the average. The previous expression is thus less than

$$\left(\sum_j p_j + p_k\right) \int_0^\infty \exp\left[-\left(\sum_j p_j + p_k\right) t_0\right] a(t_0)\, dt_0 \int_0^\infty \frac{\sum_j p_j b_j(t) + p_k b_k(t)}{\sum_j p_j}\, dt.$$

Using integration by parts on the left, we finally get:

$$\left[1+\int^{\infty}\exp\left[-\left(\textstyle\sum_j p_j+p_k\right)t_0\right]a(t_0)\,dt_0\right]\left[\int_0^{\infty}\frac{\sum_j p_j b_j(t)+p_k b_k(t)}{\sum_j p_j}\,dt\right]$$
$$<\left[1+\int_0^{\infty}\exp\left[\left(-\textstyle\sum_j p_j\right)t_0\right]a(t_0)\,dt_0\right]\left[\int_0^{\infty}\frac{\sum_j p_j b_j(t)}{\sum_j p_j}\,dt\right].$$

The expression on the right is strictly larger than our starting expression, but it is is also equal (as inverse integration by parts shows) to the expected lifespan for the same optimal set of alternatives except *without* $\mathcal{A}_k$. This is a contradiction: By removing $\mathcal{A}_k$ from our "optimal" set, we have found a bundle with longer expected lifespan, indicating that the original set was not truly optimal. Our assumption that $\mathcal{A}_k$ is part of the optimal set is therefore false; in general, this means that when alternative $\mathcal{A}_i$ is declined for an optimized set of future alternatives, no alternative less than or equal to $\mathcal{A}_i$ can be in that set.

An analogous argument proves the opposite result: When alternative $\mathcal{A}_i$ is taken, all alternatives greater than or equal to $\mathcal{A}_i$ must also be taken when possible.

That the choice to accept or decline a given alternative is independent of the time of decision now follows immediately. With separable survival functions, the only difference between the expected lifetimes of alternatives and optimal sets over a time interval from t_1 to t_2 is the constant ratio $a(t_2)/a(t_1)$, which does not alter the direction of the inequality sign. □

This theorem immediately implies a heuristic for an optimal strategy:

Heuristic for Finding an Optimal Strategy over Separable Survival Functions:

1. Determine the set of possible alternatives and the separable survival functions accompanying each.

2. Use the order relation given earlier to put the alternatives in order from $\mathcal{A}_1$ to $\mathcal{A}_n$, with $\mathcal{A}_1$ lowest and $\mathcal{A}_n$ highest.

3. Start with alternative $\mathcal{A}_n$.

4. Label the current alternative $\mathcal{A}_k$. Determine whether the expected value for a set including all alternatives $\mathcal{A}_{k-1}$ and greater is higher than the expected value for the set of alternatives at and above $\mathcal{A}_k$.

5. If yes, move down to $\mathcal{A}_{k-1}$ and repeat the previous step, unless you are already at $\mathcal{A}_0$. In that case, it is optimal to take all alternatives available, in particular, the immediate alternative.

6. If no, the optimal strategy is to take all alternatives from $\mathcal{A}_k$ to $\mathcal{A}_n$, but none smaller.

Ethical and Political Ramifications

It is reasonable to question kidney assignment to a patient who is less likely, for whatever reason, to benefit fully from the transplant's impact on lifespan.

We incorporate these situations into our model by altering the objective function for a particular class of patients. We simply alter the "returns" that determine how we measure success in the first place. This is a clean and efficient way to incorporate both practical (diseased people are not likely to benefit much from organ transplants) and moral ("save the kids!") judgments into our model.

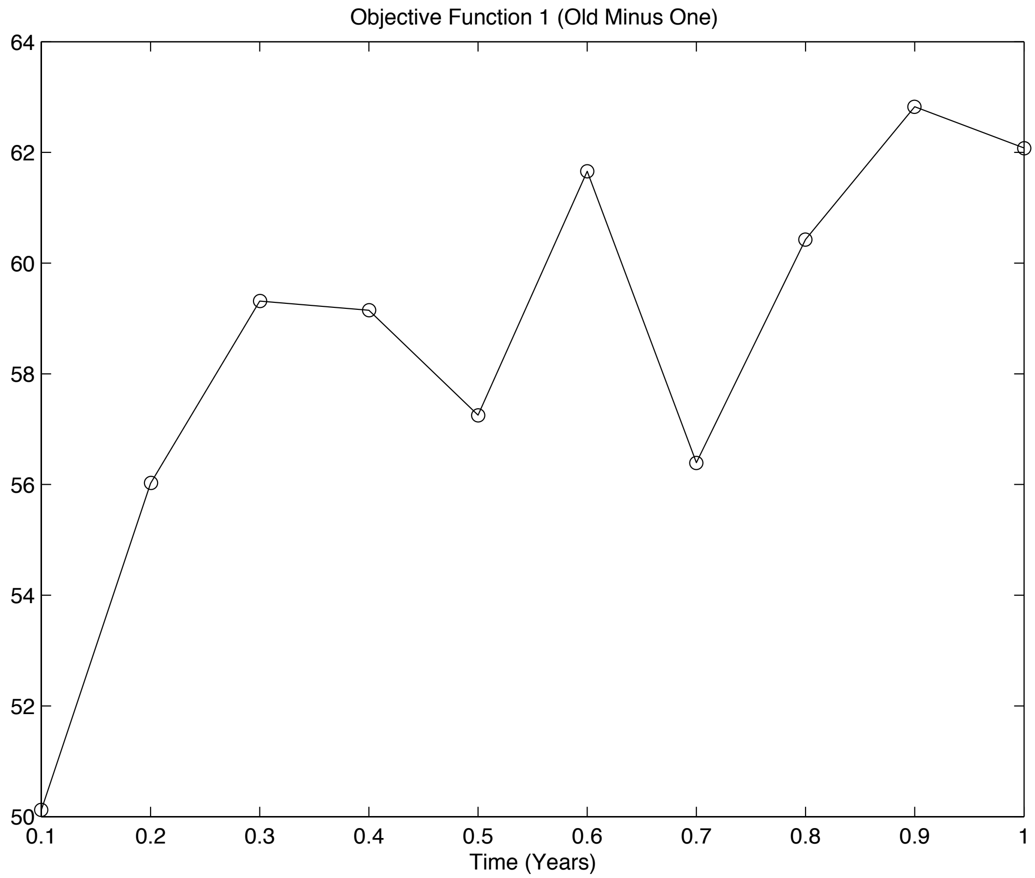

Figure 4. Results of policy of subtracting one point from the objective function of those above 60.

That transplantation is an enormously complex medical procedure, demanding dedicated facilities and experienced doctors, raises questions about the location of the surgery. Should we always ship kidneys to large, well-established medical centers, which may be more consistent in performing the operation? Or should we make transplants mostly local, so that all facilities become more experienced and maintain proficiency?

To reflect these concerns, we add two new and important wrinkles to our model.

- First, we introduce a "doctor experience function," which maps greater experience in transplant surgery to greater success and consistency in performing the procedure. Although it is impossible to pinpoint a precise analytical relationship between the number of transplants performed and success in performing them, we use regression on an OPTN data set to identify a rough linear function, and examine the effect of several other functions as well.

- Currently in the United States, fewer than half of cadavers with the potential to provide kidneys are used as donors. This is due to a system that relies heavily on *family wishes*: In most situations, doctors will defer to family members on the decision not to donate organs, even when there is preexisting affirmation of desire to donate from the deceased individual.

  One dramatic improvement, already implemented in some countries and since early this year in the State of Illinois, is *presumed consent*. Under presumed consent, every individual (possibly with limited restrictions) is assumed to have given consent for postmortem organ donation. An individual opposed to the prospect must explicitly "opt out." In many other countries, such a system has dramatically increased the pool of cadaveric kidneys available for transplant; in Austria, for instance, availability is nearly equal to demand. The result in the U.S. would not be quite so favorable; the Austrian system "benefits" from a high rate of traumatic road deaths.

In our model of living kidney donation, donors are limited to individuals with a substantial relationship to the patient: spouse, siblings, parents, children, and close friends. Excluding the black market in commercial kidneys, this setup accurately reflects the situation in the U.S. today, and in an ideal world it would be enough to provide high-quality living organs to those in need. But while list paired donation dramatically improves the usefulness of this system, its dependence on a limited pool of kidneys prevents full distribution. Moreover, some related donors decide against donation, further narrowing the supply for matching schemes.

Multiple studies identify financial disincentives as some of the main barriers to donation. Surprisingly, donors are sometimes liable for a portion of the medical costs of their procedure (and its consequences). Meanwhile, they often lose income, as they generally cannot work for some period of time during and after the transplant. And although exact figures on the number of potential donors dissuaded by these costs are difficult to obtain, sources suggest possible percentages as high as 30%.

What are the solutions? First, an authority (most likely the government) could provide for the full medical expenses of the operation, along with insurance for any future health consequences.

Some observers have proposed an even more radical reform: legalizing trade in human organs and creating "kidney markets" to ensure supply. A worldwide black market already exists in live-donor kidneys, offering some insights into how a legal system might work. The verdict is unclear; researchers have been both surprised by the quality of black-market transplants and appalled by them. Arguably, however, concerns about quality and safety in an legal organ-trading system are misplaced, since there is little reason to believe that a regulated market (with transplants conducted by well-established centers) would be any worse than the general kidney transplant system.

More controversial is the *ethical* propriety of compensation from organs. By banning all "valuable considerations" in exchange for organs, the National Organ Transplant Act of 1984 expressed a widespread sense that any trade in organs is ethically appalling. Many commentators assert that it would inevitably lead to exploitation and coercion of the poor. At the same time, others claim that markets in organs are morally *obligatory*: If these markets are the only way to save lives, they must be implemented. We do not take a firm position on the ethics of this question but recommend studies of it.

Donor Decision

When considering donating a kidney, a potential donor takes into account many factors: the risk to self, the risk of future health issues, personal issues, and the chance of transplantation success.

Immediate Risk to Donor

Especially when the recipient of the kidney has no relation to the donor, the risk to the donor is of greatest importance. After all, they are putting their lives at risk when they are not otherwise in any danger of dying themselves. Of course, steps are taken to ensure that the donor is healthy enough to undergo a successful operation. At many institutions, the criteria for exclusion of potential living kidney donors include kidney abnormalities, a history of urinary tract infection or malignancy, extremely young or old age, and obesity. In addition, the mortality rate around the time of the operation is only about 0.03% [Jones et al. 1993].

Future Health Concerns

There have been suggestions that the early changes that result from the removal of the kidney, increase in glomerular filtration rate and renal blood flow, may lead to insufficiency of kidney function later on. However, a study of 232 kidney transplant patients, with a mean follow-up time of 23 years, demonstrated that if the remaining kidney was normal, survival was identical to that in the overall population [Jones et al. 1993]. In fact, another study suggests that kidney donors live longer than the age-matched general population, most likely due to the bias that occurs in the selection process [Ramcharan and Matas

2002]. Therefore, there is not a higher risk for development of kidney failure in the long run for kidney transplant donors.

Psychological Issues

There may, however, be future psychological issues stemming from depression. According to donor reports from a follow-up conducted by the University of Minnesota, 4% were dissatisfied from their donation experience, with non-first-degree relatives and donors whose recipient died within a year of transplant more likely to say that they regretted their decision and wish that they had not donated [Johnson et al. 1999]. To reduce this percentage, a more careful selection based on a rigorous psychosocial evaluation should be conducted.

Personal Issues

Some potential donors believe that they would incur the costs for the operation, discouraging them from following through with donating [United Network ... 2007]. This is simply not true. Also, a survey of 99 health insurance organizations found that kidney donation would not affect an insured person's coverage and a healthy donor would be offered health insurance, so money is not a problem [Spital and Kokmen 1996]. However, money is lost from time away from work, and time away from home is also another significant contributor to the decision of a potential donor. Another personal issue that might deter potential donors is their attitude towards surgery in general, which may be affected by irrational fears or previous experiences. Such potential donors would be unable to cope psychologically with surgery, in spite of knowledge of the high probability of success.

Relationship

The relationship between the potential donor and the intended recipient is also a key factor. Close family members and the spouses of the recipients are three times as willing to participate in paired donation compared to other potential donors [Waterman et al. 2006]. Although it is hard to quantify, there also appears to be a significant number of altruistic donors. Most potential donors would not want to participate in nondirected donation, since it would not benefit their intended recipients, but 12% of potential donors were extremely willing to donate to someone they did not know [Waterman et al. 2006]. Since the risk/benefit ratio is much lower for living anonymous donors (LAD), there are concerns that such donors are ! psychologically unstable. However, studies conducted by the British Columbia Transplant Society indicate otherwise. About half of the potential LADs who contacted their center and completed extensive assessments that looked at psychopathology and personality disorder met rigorous criteria to be donors [Henderson et al 2003].

Across the different donor-exchange programs, there is a association between the willingness of potential donors to participate and how likely they thought their intended recipient would receive a kidney. With kidney paired donation, willingness to participate is 64% [Waterman et al. 2006]. For a compatible donor-recipient pair, the willingness for a direct transplant would be arguably closer to 100%. So we hypothesize that as the size n of a kidney exchange increases, potential donors become less willing to participate, because of the increasing chance for error in one of the swaps and the increasing difficulty of coordination. Potential donors would not wish to go through so much trouble without certainty of acquiring a kidney for their recipient, especially since they are giving up one of their own kidneys. For list paired donation, the data are inconclusive; in our model, we hypothesized that a random percentage of a predefined donor base would become actual organ donors.

There are essentially four basic models of systems that deal with incentives for live donors:

- **Market compensation model**, which is based on a free-market system in which the laws of supply and demand regulate the monetary price for donating a kidney.
- **Fixed compensation model**, where all donors are paid a fixed amount regardless of market value, for any trouble caused by the donation.
- **Expense reimbursement model**, which covers only the expenses incurred by the donor, such as travel and childcare costs, that are related to the transplant process.
- **No-compensation model**, the current system in the U.S., which forces an altruistic donor to cover his or her own expenses [Israni et al. 2005].

The market compensation model guarantees that the demand for kidneys will be met, as seen from Iran's organ market [President's Council on Bioethics 2006]. But it discourages altruistic donors, since they gain only monetary and no altruistic benefit. Also, the large demand for kidneys would likely drive the price up, causing ethical concerns about kidneys becoming a commodity available only to the rich. To encourage more altruistic donors, we argue that the expense reimbursement model is the best approach. This model allows altruistic donors to volunteer for the transplantation procedure without worrying about financial costs, and, unlike the fixed compensation model, prevents donors from making a profit from their donation. When there is an opportunity for profit, there is a risk of developing a market for organs, which many would argue is unethical. Still, we believe that given the enormous potential for increase in the kidney donor pool, it is important to investigate the possible results from compensation and incentive-based systems.

Conclusion

We believe in the absolute necessity of implementing a list paired donation system. Its dramatic positive effect on outcomes for kidney patients in need of replacement organs is remarkable and cannot be ignored.

We have also found several other less striking results.

- The basic importance of **geography**. High transportation time leads to deterioration of the organs being transferred; in poorly designed systems, this occurs even when there are plausible and equally valuable local routes for transmission.
- We recognize the importance of **age and disease stage** in allocating kidneys. When these factors are incorporated into our objective function, altering the point system for allocation decisions becomes important.
- It is critical to reflect the **objectives** of the transplant system. Without a well-verified and established relationship between what we include in our allocation decisions and our moral and ethical bases for judgment, we will always be dissatisfied.

Evaluation of Solutions

Strengths

Our main model's strength is its enormous flexibility. For instance, the distribution network can acquire many different structures, from a single nationally-run queue to a heavily localized and hierarchical system. Individuals, represented as objects in C++ code, are made to possess a full range of important attributes, including blood type, HLA type, PRA level, age, and disease. Including all these factors into a single, robust framework, our model enables realistic simulation of kidney allocation but remains receptive to almost any modification.

This strength allows us to make substantive conclusions about policy issues, even without extensive data sets. By varying parameters, allocation rules, and our program's objective function—all quite feasible within the structure—we can examine the guts of policymaking: the ethical principles underlying a policy, the implementation rules designed to fulfill them, and the sometimes nebulous numbers that govern the results.

Weaknesses

Although we list the model's comprehensive, discrete simulation as a strength, it is (paradoxically) also the most notable weakness. Our results lack clear illustrative power; data manipulated through a computer program cannot achieve the same "aha" effect as an elegant theorem. Indeed, there is a fundamental tradeoff here between realism and elegance, and our model arguably veers toward overrealism.

Second, our model demands greater attention to numbers. While its general structure and methodology are valid, the specific figures embedded in its code are not airtight. For instance, the existing literature lacks consensus on the importance of HLA matching, possibly because developments in immunogenic drugs are changing the playing field too rapidly. Our use of parameters derived from OPTN data cannot guarantee numerical accuracy.

Third, and perhaps most fundamentally, the bulk of our simulation-based analysis hinges on the "objective functions" that we use to evaluate the results. This raises a basic question: What "good" *should* the kidney allocation system maximize? We attempt to remedy this problem by including multiple objective functions.

Appendices

Appendix I: Letter to Congress

We have undertaken an extensive examination of organ procurement and distribution networks, evaluating the results of differences in network structure on the overall success of the transplant system. In particular, we took a close look at the impact of two representative schemes for kidney allocation. First, we evaluated a simple model consisting of one nationally-based queue, where kidney allocation decisions are made without regard for individual region. Second, we looked at a more diffuse system with twenty regionally-based queues. We simulated the trade-off in effectiveness between the two systems by including a "distance penalty," which cut success rates for organs that were transported between different regions. This was implemented to recognize the role that cold ischemia, which is necessary for long-distance transportation of organs, plays in hurting transplant outcomes.

Higher levels of this parameter hurt the single-queue system, which transports its kidneys a noticeably greater average distance. Indeed, we found that for essentially all values of the distance penalty parameter, the multiple-queue system was superior. This is because the regionalized system in our simulation, modeled directly on the American system, uses geography to allocate organs when there is a "tie": when the organ has many similarly optimal potential destinations. This approach has no apparent downside, while minimizing inefficiency tied to unnecessary organ transportation. We recommend that you preserve the current regionally-based allocation system. Additionally, if you desire to allocate additional funds to improve the organ distribution system, we suggest that you support the streamlining of organ transportation.

We also compared the American OPTN organ allocation system to the analogous Eurotransplant system. Both use rubrics that assign "points" for various characteristics important to their matching goals. They differ, however, in their scheme of point assignment; while the contrasts are largely technical, they have real impacts on the overall welfare of transplant patients. Although the systems were relatively close in effectiveness overall, our simulations identified the American point system as slightly superior. Therefore, we recommend you preserve the main points underlying the OPTN point allocation scheme.

Appendix II: Letter to the Director of the U.S. HRSA

We write having undertaken an extensive simulation-based review of the various political and ethical questions underlying decisions about organ allocation, and of policies for increasing live and cadaveric organ donation. First, we implemented a portion of code that represents the value of transplant center and doctor experience in improving donation outcomes. When we input a sizable experience effect, which does not appear to exist from the empirical data at this point, we found that a centrally based allocation and treatment system became substantially more effective than a more diffuse, multiple-queue based model. Given the lack of evidence that there is actually such an "experience" effect–the main available data, which comes from the Organ Procurement and Transportation Network, does not appear to indicate one–we advise caution before changing the system to reflect this theoretical result.

We also examined the usefulness of including heavy weights for age and terminal illness in the kidney allocation system. In general, we concluded that such measures are both justified and effective. Although blood type, HLA match, and PRA are still the most fundamental factors to consider, we believe that the extreme difference in effectiveness of helping people of different ages and health status justifies substantial inclusion of those factors in the allocation process.

Finally, we examined literature and existing research as they relate to the possibility of changing our system of consent and compensation for donation. We recommend implementing a "presumed consent" system for cadaveric organs, which will automatically tally deceased individuals as donors unless they have specifically opted out of the system. We also recommend exploring, but not necessary implementing, the possibility of some sort of compensation for live kidney donation. While we are mindful of widespread ethical concerns about the practice, we believe that the extreme demand for kidneys should prompt us to consider all alternatives. Specifically, we suggest a pilot program of light to moderate compensation for live kidney donors, and a thorough review of the outcome and change in incentives for those involved.

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Duke ICM 2007 team members Matt Rognlie, Amy Wen, and Peng Shi, wearing T-shirts with "nice numbers," and advisor David Kraines.

Analysis of Kidney Transplant System Using Markov Process Models

Jeffrey Y. Tang
Yue Yang
Jingyuan Wu

Princeton University
Princeton, NJ

Advisor: Ramin Takloo-Bighash

Summary

Abstract: We use Markov processes to develop a mathematical model for the U.S. kidney transplant system. We use both mathematical models and computer simulations to analyze the effect of certain parameters on transplant waitlist size and investigate the effects of policy changes on the model's behavior.

Our results show that the waitlist size is increasing due to the flooding of new waitlist members and insufficient deceased donor and living donor transplants available. Possible policy changes to improve the situation include presumed consent, tightening qualifications for joining the waitlist, and relaxing the requirements for accepting deceased donors.

We also evaluate alternative models from other countries that would reduce the waitlist, and examine the benefits and costs of these models compared with the current U.S. model. We analyze kidney paired exchange along with generic n-cycle kidney exchange, and use our original U.S. model to evaluate the benefits of incorporating kidney exchange.

We develop a model explaining the decisions that potential recipients face concerning organ transplant, then expand this consumer decision theory model to explain the decisions that potential organ donors face when deciding whether to donate a kidney.

We finally consider an extreme policy change—the marketing of kidneys for kidney transplants—as a method of increasing the live-donor pool to reduce waitlist size.

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Introduction

The American organ transplant system is in trouble: Waitlist size is increasing ; as of February 2007, 94,000 candidates were waiting for a transplant, among them 68,000 waiting for kidneys. We create a mathematical model using a Markov process to examine the effects of parameters on waitlist size and to investigate the effects of policy changes. Possible policy changes to improve the situation include assuming that all people are organ donors unless specifically specified (presumed consent), tightening qualifications for joining the waitlist, and relaxing the requirements for accepting deceased donors.

We evaluate alternative models from other countries that could reduce the waitlist, and examine the benefits and costs of these models compared with the current U.S. model. We analyze the Korean kidney paired exchange along with the generic n-cycle kidney exchange, and use our original U.S. model to evaluate the benefits of incorporating the kidney exchange. The Korean model increases the incoming rate of live donors, which is preferable because live-donor transplants lead to higher life expectancy. However, this policy alone cannot reverse the trend in waitlist size.

We also develop a model explaining the decisions that potential recipients face concerning organ transplant. We expand this consumer decision theory model to explain the decisions that potential organ donors face when deciding whether or not to donate a kidney. Finally, we consider an extreme policy change-the marketing of kidneys for kidney transplants as a method of increasing the live donor pool to reduce waitlist size. We consider two economic models: one in which the government buys organs from willing donors and offsets the price via a tax, and one in which private firms are allowed to buy organs from donors and offer transplants to consumers at the market-equilibrium price.

Task 1: The U.S. Kidney Transplant System

Background: Kidney Transplants

- **Blood Type**: Recipient and donor must have compatible blood types (**Table 1**).

- **HLA**: Recipient and donor must have few mismatches in the HLA antigen locus. Because of diverse allelic variation, perfect matches are rare. Mismatches can cause rejection of the organ.

- **PRA**: PRA is a blood test that measures rejection to human antibodies in the body. The value is between 0 and 99, and its numerical value indicates the percent of the U.S. population that the blood's antibodies reacts with. High PRA patients have lower success rates among potential donors[U so it is more difficult to locate donate matches for them (**Table 2**).

Table 1.
Compatible blood types [American National Red Cross 2006].

Recipient blood type	Donor red blood cells must be:							
AB+	O−	O+	A−	A+	B−	B+	AB−	AB+
AB−	O−		A−		B−		AB−	
A+	O−	O+	A−	A+				
A−	O−		A−					
B+	O−	O+			B−	B+		
B−	O−				B−			
O+	O−	O+						
O−	O−							
In U.S. population:	7%	38%	6%	34%	2%	9%	1%	3%

Table 2.
Relationship between PRA and transplant waiting time [University of Maryland . . . 2007].

Peak PRA	Proportion of waiting list	Median waiting time to transplant (days)
0–19	60%	490
20–79	21%	1,042
80+	19%	2,322

Explanation of Model

The Organ Procurement and Transplantation Network's (OPTN) priority system for assigning and allocating kidneys is used as the core model for the current U.S. transplantation system [Organ Procurement . . . 2006]. The OPTN kidney network is divided into three levels: the local level, the regional level, and the national level. There are 270 individual transplant centers distributed throughout the U.S. [Dept. of Health and Human Services 2007], organized into 11 geographic regions.

The priority system for allocation of deceased-donor kidneys to candidates on the waitlist takes into account proximity of recipient to donor, recipient wait time, and match to donor, with location carrying greater weight, according to a point system [Organ Procurement . . . 2006]:

- **Wait time points** A candidate receives one point for each year on the waiting list. A candidate also an additional fraction of a point based on rank on the list: With n candidates on the list, the rth-longest-waiting candidate gets $1 - (r-1)/n$ points. So, for example, the longest-waiting candidate ($r = 1$) gets one additional point, the newest arrival on the list ($r = n$) gets $1/n$ additional points.

- **Age points** The young receive preferential treatment because their expected lifetime with the transplant is greater. Children below 11 years of age get 4 additional points, and those between 11 and 18 get 3 additional points.

- **HLA mismatch points** Because there are two chromosomes, the possible number m of mismatches in the donor-recipient (DR) locus of the HLA sequence is 0, 1, or 2. A candidate-donor pair gets $2 - m$ points.

Model Setup

We model the entry and exit of candidates from the waitlist with a continuous-time Markov birth/death process [Ross 2002]. It accommodates reduction of the waitlist size (arrivals of living donors and deceased donors and deaths and recoveries of waitlist candidate) and waitlist additions.

- In 2006, 29,824 patients were added to the kidney transplant waitlist, while 5,914 transplants had living donors, so $5914/(29824 + 5914) \approx 17\%$ of incoming patient cases have a willing compatible living donor.
- The procedure for allocating deceased-donor kidneys is [Organ Procurement . . . 2006, 3.5, 3–16ff]:
  - First, match the donor blood type with compatible recipient blood types. The only exceptions are:
    - Type O donors must be donated to type O recipients first, and:
    - Type B donors must be donated to type B recipients first.
  - Perfect matches (same blood type and no HLA mismatch) receive first priority.
  - If a kidney with blood type O or B has no perfect-matching candidates in the above procedure, then the pool is reopened for all candidates.
  - In the 17% of cases of no a perfect match with any recipient [Wikipedia 2007], then sort by PRA value (higher priority to high PRA; high PRA means low compatibility, which likely means being on the waitlist for a long time), then by regional location of the kidney, then by points in the point allocation system.

Summary of Markov Process

Let N_t be a random variable indicating the number of people in the waitlist at time t. The properties of N_t can be generalized in **Figure 1**, where

- Each arrow represents a possible event at the current state (N).
- The rate at which each event occurs is exponentially distributed.

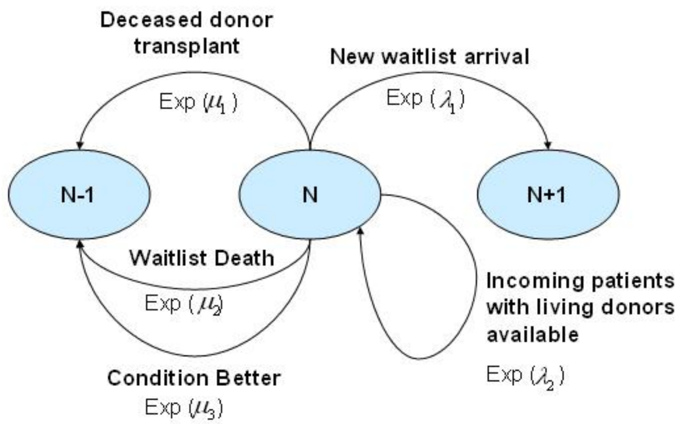

Figure 1. Markov process model of waitlist.

- After the event occurs, by memorylessness of the exponential distribution, the time is reset to zero, as if nothing has happened.
- Wait time is assumed zero for compatible live donor transplants.
- Because there are so many local centers (270), we simplify our model to consider the region (of which there are 11) as the lowest level of waitlist candidates.
- Candidates who become medically unfit surgery are removed from the wait list and in our model are classified as deaths.
- Candidates whose conditions improve enough are removed from the waitlist. Both these people and those recovering from surgery have exponential remaining lifetime with mean 15 years.
- We use the parameter values in **Table 3**, which come from the OPTN database using values from 2006.

Table 3.
Means of exponential distributions.

Symbol	Rate	Mean
λ_1	new waitlist arrivals	81.7 d
λ_2	incoming patients with living donors available	16.2 d
$\lambda_3 = \lambda_1 + \lambda_2$	total incoming patients (independent RVs)	81.7 + 16.2 = 97.9 d
μ_1	arrivals of deceased donor transplants	26.9 d [Norman 2005]
μ_2	waitlist deaths	27.0 d
μ_3	waitlist condition improves per day	2.4 d
$\mu_4 = \mu_1 + \mu_2 + \mu_3$	waitlist departures (independent RVs)	26.9 + 27.0 + 2.4 = 56.3 d
T_{AB}	time of life after surgery [European Medical Tourism 2007]	0, if candidate dies; 15 y with transplant.

Analysis of Model

Our two variables to indicate strength of model strategy are the number of people in the waitlist (or the number of people who get transplants) and optimizing the matches so as to maximize lifetime after receiving a transplant.

Efficient Allocation of Kidney Transplants

We build a new model to take into account the effects of both distance and optimal match. A kidney arriving at a center can be given to the best matching candidate at that center, the best in the region, or the best in the country.

Of 10,000 candidate recipients, on average 37 are from the center, 873 are from the region outside the center, and 9,090 are from the nation outside the region. Using a uniform distribution on $(0,1)$, we randomly assign scores to each of the 10,000, rank them by score, and take the highest rank at each level. We iterate this process 10,000 times and find the average rank of the top candidate in each area (**Table 4**).

Table 4.

Average quality of top candidate in each area.

	Probability that top candidate is in this group			Average rank (from bottom) of top candidate among 10,000
Center	$\frac{1}{270}$	$=$	0.37%	9739.7
Region outside center	$\frac{1}{11}-\frac{1}{270}$	$=$	8.72%	9989.7
Nation outside region	$\frac{10}{11}$	$=$	90.90%	9999.9

Transportation of the kidney can lead to damage, because of time delay in transplanting. Thus, we posit a damage function f that depends on the location of the recipient: lower in the center, slightly higher in the region but outside the center, and even higher in the country but outside the region, i.e.,

$$f(\text{local}) < f(\text{regional}) < f(\text{national}).$$

Let us assume that when a kidney arrives in a center, it goes to the center, the region, or outside the region with probabilities a_1, a_2, and a_3. Let G be the weighted score for the kidney, with

$$\begin{aligned} G \;=\; & a_1 \cdot \big(1 - f(\text{local})\big) \cdot \text{score}_{\text{local}} + a_2 \cdot \big(1 - f(\text{regional})\big) \cdot \text{score}_{\text{regional}} \\ & + a_3 \cdot \big(1 - f(\text{national})\big) \cdot \text{score}_{\text{national}}. \end{aligned} \quad \textbf{(1)}$$

and expected value

$$\begin{aligned} E(G) \;=\; & a_1 \cdot \big(1 - f(\text{local})\big) \cdot 9739.7 + a_2 \cdot \big(1 - f(\text{regional})\big) \cdot 9989.7 \\ & + a_3 \cdot \big(1 - f(\text{national})\big) \cdot 9999.9. \end{aligned} \quad \textbf{(2)}$$

Optimizing G as a function of the a_i is a linear programming problem, but we cannot solve it without assessing the damage function for different regions.

Minimizing the Waitlist

There are some who argue that the wait time assignment is too lax and leads to unfair waitlists. In the current system, urgency is specifically stated as have no effect on the points used to determine who receives a transplant [Organ Procurement ... 2006]. A patient is permitted to join the waitlist (in more than one region, even) when kidney filtration rate falls below a particular value or when dialysis begins. Getting on the waitlist as early as possible helps "pad" the points for waiting time. A patient not yet on dialysis can afford to wait longer yet may receive a kidney sooner than others joining later who have more urgent need. Urgency has no effect on a patient's rank for receiving a kidney. A possible solution is to tighten the conditions for joining the waitlist, so that that a patient's wait time begins at dialysis. This policy would slow the rate of growth of the waitlist, at the expense of more waitlisted patients dying.

A strategy to increase the rate of deceased-donor arrival, already policy in Illinois, is to presume that everyone desires to be an organ donor unless they specifically opt out.

Figure 2 shows the field space of combinations from rates for these two policies.

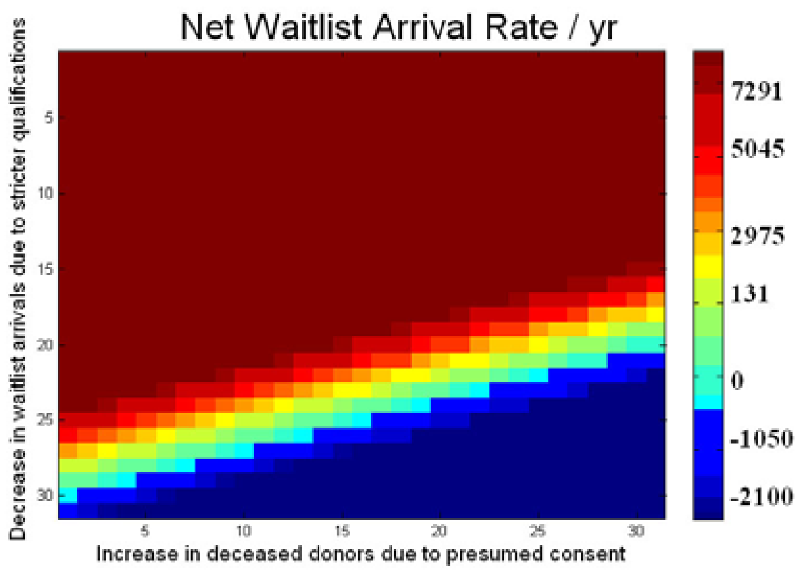

Figure 2. Net waitlist arrival rate per year.

Using both strategies could make net waitlist arrival rate negative, for example, if waitlist arrivals can be decreased by 25% and donor size by 17%.

Model Strengths

- The Markov process, with exponentially distributed entry / exit times, makes calculations simple.

- Minimizing the waitlist depends on only two variables.
- The model incorporates HLA values, PRA distributions, no-mismatch probabilities, region distribution, and blood-type distribution and compatibility requirements.
- The model is compatible with alternative strategies, such as a paired exchange system.

Model Weaknesses

- Remaining lifetime after surgery should be adjusted, since an exponential distribution for remaining lifetime is appropriate only until a certain age.
- The model cannot account for patients'. We assume that all patients offered a kidney take it if the HLA value is reasonable, which may not be the case.
- The model does not make distinctions for race and socioeconomic status. Different races have differing wait times [Norman 2005, 457].
- We assume independence of random variables, so that increasing or decreasing parameters will not affect other parameters.
- Our emphasis on waitlist size neglects waitlist time; another approach would be to try to minimize waitlist wait time.

Tasks 2 and 3: Kidney Paired Exchange

Background

As noted at the University of Chicago Hospitals, "In 10 to 20 percent of cases at the Hospitals, patients who need a kidney transplant have family or friends who agree to donate, but the willing donor is found to be biologically unsuited for that specific recipient" [Physicians propose . . . 1997].

In the simple kidney paired exchange system (**Figure 3**), there are two pairs of patient / donor candidates. Each donor is incompatible with the intended patient but compatible with the other patient. Surgery is performed simultaneously in the same hospital on four people, with two kidney removals and two kidney transplants.

However, for not all patient-donor pairs will there be a mutually compatible partner pair. In such a case, it is possible for the cycle to expand to n patient-donor pairs, with each donor giving to a compatible stranger patient (**Figure 4**). Since such an exchange requires at least $2n$ surgeons at the same hospital, higher-order exchanges are less desirable on logistic grounds.

A kidney paired exchange program does not affect the intrinsic model outlined for Task 1. The only change when live incompatible pairs get swapped is

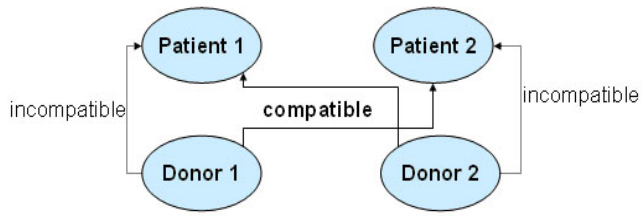

Figure 4. Kidney paired exchange system.

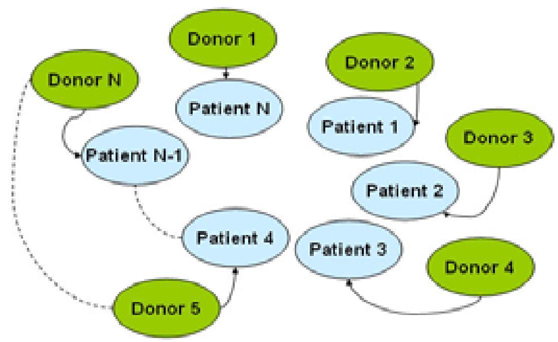

Figure 5. n-way kidney exchange.

that the rate of candidates entering the waitlist is reduced. However, for those in the waitlist, the same procedure is still being used.

- Kidney paired exchanges have higher priority than larger cycles, for logistical reasons.
- Recipients must receive a transplant in the region in which they are on the waitlist (this reduces travel time).
- Exchanges with no mismatch are prioritized over exchanges with mismatch.

Waiting time for an exchange is assumed to be 0, as was live donor matches in Task 1. After an exchange, all individuals involved are removed from the pools of donors and recipients.

We use Region 9 as a sample region to test our model. Region 9 has a waitlist (6058) similar to the average waitlist per region, and 909 candidates (15%) have willing but incompatible donors. We ran our simulation 100 times and computed averages. **Table 5** shows extrapolation of the results nationwide.

Analysis

In 2006, there were 26,689 kidney transplants nationwide, including only a few kidney paired exchanges. The approximately 9,656 additional transplants yearly indicated in **Table 5** would have been a 36% increase and would have reduce the waitlist correspondingly by 14%, from 69,983 to 60,327.

Table 5.
Averaged results of repeated simulations of multiple-pair transplant exchange nationwide (extrapolated from Region 9 data).

Kind of match	Transplants	Percentage
2-way no mismatch	2	
2-way non-perfect	9,646	
3-way no mismatch	0	
3-way non-perfect	8	
Total transplants	9,656	92%
Candidates with willing but incompatible donor	10,497	

Another option is to consider multiple exchanges for all donor-recipient pairs in a particular center. This minimizes the travel time required for the patients, while improving the computational power of the search algorithm. A center has on average 259 candidates, of whom 39 have willing but incompatible donors available. For this sample size, we get on average 25 transplants (65%), compared to 92% under exchange at the regional level. Furthermore, the proportion of high-quality transplants is also smaller. The benefits of a center-only exchange system are personal and psychological: Patients live close to the surgery location, which means better support from both family and familiar physicians.

Task 4: Patient Choice Theory

Suppose a patient is offered a barely compatible kidney from the cadaver queue. There are two options:

- take the bad-match kidney immediately, or
- wait for a better match,
  - from the cadaver queue or
  - from a paired exchange.

We consider two cases: without paired exchange and with it.

Model 1: Decision Scenario without Paired Exchange

Of transplants with poorly matched kidneys, 50% fail after 5–7 years. So we assume that the lifetime after a poorly matched kidney transplant is exponentially distributed with mean 6 years [Norman 2005, 458].

We translate data of **Table 6** on survival probabilities to exponential variables with mean λ by solving $P(\text{survive } t \text{ years}) = e^{-\lambda t}$.

Table 6.
Rates for patient survival [National Institute of Diabetes and Digestive and Kidney Diseases 2007].

Time (y)	Dialysis		Live-donor transplant		Cadaver transplant	
	p	λ	p	λ	p	λ
1	.774	0.256	.977	0.0233	.943	0.0587
2	.632	0.229	.959	0.0209	.907	0.0209
5	.315	0.231	.896	0.0220	.819	0.0399
10	.101	0.229	.753	0.0284	.591	0.0526
Avg.		**0.236**		**0.0237**		**0.0500**

We first diagram the wait strategy for the scenario of waiting on dialysis for a deceased-donor kidney, with no kidney paired exchange (**Figure 5**).

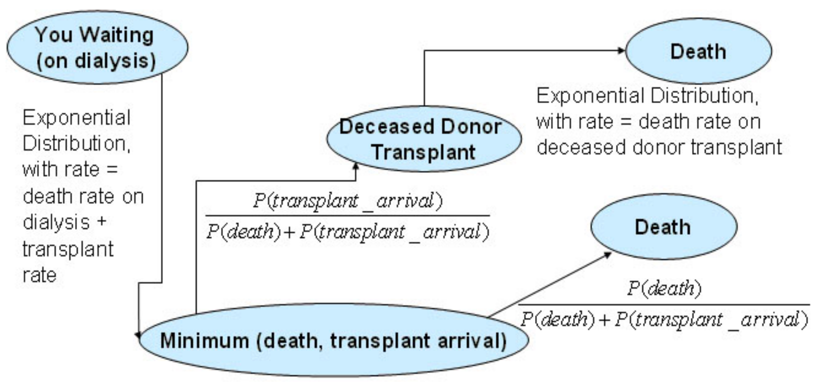

Figure 6. Wait strategy with no paired exchange.

We then calculate expected remaining lifetime with this strategy. We use

$$P(\text{deceased-donor transplant}) = \frac{\text{deceased-donor transplants}}{\text{waitlist}} = \frac{10659}{75711} = .140,$$

using 2006 data [Organ Procurement ... 2007][1]. We have

$$E(\text{lifetime}) = \frac{0.236}{0.236+0.140}\left(\frac{1}{0.236+0.140}\right) + \frac{0.140}{0.236+0.140}\left(\frac{1}{0.236+0.140}+\frac{1}{0.050}\right) \approx 10 \text{ years.}$$

If instead the patient chooses to undergo immediate transplant with a bad match deceased-donor kidney, then remaining lifetime is exponentially distributed with rate 0.167, so

$$E(\text{lifetime}) \approx 6.0 \text{ years.}$$

[1] Author note: In hindsight, a better measure is probably to use the **Table 2** median waiting times to calculate average waiting time (2.66 years). Using the assumption of exponential distribution, we have that probability of an arrival of a deceased-donor kidney in one year is $e^{(}-2.66) \approx .07$.

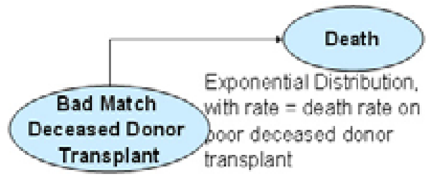

Figure 7. Transplant strategy with no paired exchange.

The expected remaining lifetime for the wait strategy is 4 years greater, so we recommend that strategy. It assumes that the patient is risk-neutral. Being on dialysis leads to an expected remaining lifetime of 4.2 years, which is less than the expected remaining lifetime for a bad-match kidney. The decision hinges on how much risk the patient is willing to take.

Model 2: Decision Scenario with Paired Exchange

This modified scenario leads to **Figure 7**. Since between 10% and 20% of patients have willing but incompatible donors [Physicians propose ... 1997], and lacking any better data, we use .15 as the probability of a kidney paired exchange being possible. Using similar calculations as before, we find the expected remaining lifetime for the wait strategy:

$$E(\text{lifetime}) \approx 19.5 \text{ years.}$$

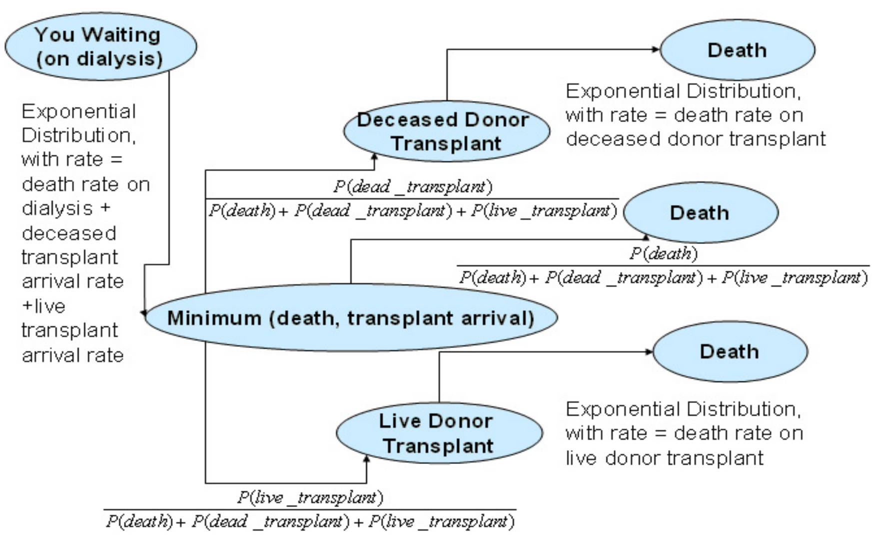

Figure 8. Wait strategy with paired exchange.

Model Strengths

- The model compares strategies numerically.

- Values for survival rate, rate of death, and rate of donor arrival are available from data, and these parameters can be easily adjusted for different scenarios.
- The model can be modified to accommodate other strategies, new categorizations of transplants (perhaps divide transplants into grades A, AA, AAA for quality).

Task 5: New Organ Market

Another method to increase the rate of incoming live donors is to implement a market allowing people to sell organs for transplantation. Currently, it is illegal in the U.S. to "transfer an organ for valuable consideration" [National Organ Transplant Act 1984]. There are two possible ways a market can work: government-managed (**Figure 8**) or using a public market (**Figure 9**).

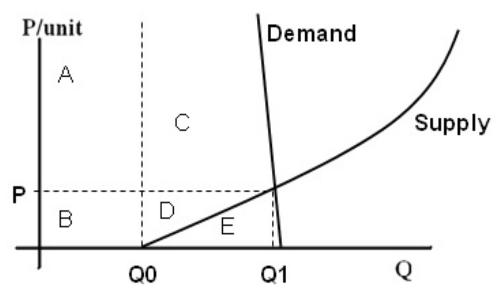

Figure 9. Government-managed organ-buying system.

Originally, Q_0 units of organs were available (via cadaver and living donor transplants). In the government-managed model, people sell their kidneys to the government at the market equilibrium price. The government pays $D + E$ for the kidney transplants, so the economy suffers a tax of $D + E$. However, the customers gain the consumer surplus of C and the suppliers gain the supplier surplus of D, so benefit of having more kidneys available to society is $C + D - (D + E) = C - E$. Because of the inelasticity of demand for kidney transplants, $C > E$, so $C - E > 0$. Therefore, a government-managed system would eliminate the waitlist because Q_1 would likely be greater than the total number of people on the waitlist. However, government-managed systems are known to be slow and inefficient [Krauss 2006], so people in this market would have long wait times for a transplant. The increase from Q_0 to Q_1 would be drastic, leading to a strain on hospitals and on the health care system.

A possible solution to the long wait times intrinsic to government-managed surgeries is to privatize the market and allow private companies to buy kidneys and sell surgeries. The Q_0 donors who originally donated for free would still be in the model (but we assume that they would still be uncompensated), so

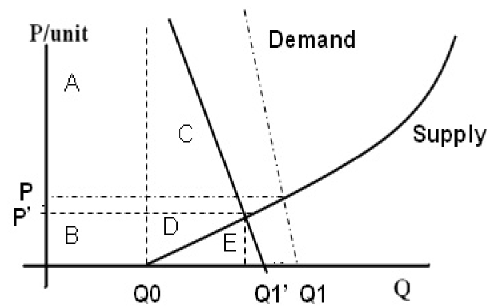

Figure 10. Free market for organs.

the companies buy organs only from the remaining supply curve. The market equilibrium is still the same coordinate, but this time the consumer surplus is increased by C and the supplier surplus by D, so the benefit to society is $C + D$ instead of $C - E$, and the free-market system is more efficient to society by $D + E$. Because the company's demand curve is less inelastic than the customer's demand curve, it is likely that companies will buy fewer kidneys, at a lower price, than the government would. However, companies may take advantage of the inelastic demand curves and try to make long-run profits.

A free-market system has benefits over the government-managed system; it is more efficient, and it would lead to more transplants if the government-managed system had longer wait times and inefficient allocations due to the bureaucratic difficulties of managing a nationwide kidney industry.

However, the free-market system also has disadvantages. Matches involving different races are less likely to lead to good transplants, because tissue-type gene sequences have different distributions by race and hence lower likelihood of compatibility across races. Race-associated differences in genetics could lead to race-specific markets (and prices) for kidneys.

Another possible protest against this market structure is ethical dilemmas regarding the selling of organs. The organ is a part of one's body, but questions arise whether one "owns" one's organs. One religious view would be that because the body is sacred, it would be wrong to sell one's body for monetary gain. Furthermore, introduction of a market for kidneys could lead some companies to try to start stemcell or nonliving organ farms, a dangerous step according to the realm of bioethics.

While a kidney market would increase efficiency and lead to more transplants, both the government-sponsored and the free-market versions could each provide major ethical problems.

Task 6: Potential Donor Decision Theory

We calculate the probability that donors will donate for various situations. We build a model similar to the patient's decision scenario, only now we consider the states of the world from a potential donor's viewpoint. We consider three cases: kidney donation to a loved one, to a random person, and to a random person in a kidney exchange system so that a loved one can receive a kidney transplant as well.

We first evaluate the case when the kidney could be donated to a loved one.

Donor Decision: Donate to a Loved One?

Strategy 1: Donate

Let C be a score assigned to the strategy when you donate a kidney to a loved one. Let C be a function of three random variables X, Y and Z, where

- X is the remaining lifetime of the recipient given a live donor transplant,
- Y is the remaining lifetime of the donor given a live donor transplant, and
- Z is the pain and depression value of the donor after donating a kidney.

The remaining lifetime of a recipient of a live-donor transplant is exponentially distributed with rate 0.0237. We also know that the perioperative mortality rate is 3 deaths per 10,000 donors (0.03%), and that 2% of donors encounter major complications [Najarian et al. 1992]. (Some donors experience depression or conflict with family members, but these problems are unrelated to the success of the transplantation [Liounis et al. 1988].) Thus, X, Y, and Z are as follows:

- X is exponentially distributed with rate $\lambda = 0.0237$;
- $Y = \begin{cases} 0, & \text{with probability } 0.03\%, \\ T_{\mathrm{N}}, & \text{with probability } 97.97\%, \\ T_{\mathrm{MC}}, & \text{with probability } 2\%, \end{cases}$

  where T_{N} is the random variable for remaining lifetime of a normal person, and T_{MC} is the random variable for remaining lifetime of a person with major complications of a donor from kidney transplant;
- Z is the numerical value for amount of depression, conflict, and anger that results from donating kidney.

Hence C from this example is given by:

$$C = a_1 X + a_2 Y - a_3 Z,$$

where a_1, a_2, and a_3 are weights for how important each variable is. These weights reflect the emphasis on each variable by the given donor.

$$\begin{aligned} E(C) &= a_1 E(X) + a_2 E(Y) - a_3 E(Z) \\ &= a_1 \cdot 42.19 + a_2 \cdot (0.0003 \cdot 0 + 0.9797 \cdot \overline{T_{\mathrm{N}}} + 0.02 \cdot \overline{T_{\mathrm{MC}}}) - a_3 \cdot \overline{Z}. \end{aligned}$$

Strategy 2: Don't Donate.

In this case, we notice several changes to the variables X, Y, and Z,

- There are two scenarios: Your loved one does not get a transplant and dies, or your loved one receives a transplant. In the first case, the time until your loved one dies is exponentially distributed with rate 0.236; and in the second case, the time until your loved one receives a transplant is exponentially distributed with rate 0.140 (same as in Task 4). Thus, the minimum of the two, which is the time until the first event occurs, is exponentially distributed with rate $(0.140 + 0.236) = 0.376$. Thus, we have

$$X = \min\{T_{\text{die}}, T_{\text{transplant}}\} \cdot P(\text{die}) \\ + \big(\min\{T_{\text{die}}, T_{\text{transplant}}\} + T_{\text{RLADS}}\big) \cdot P(\text{transplant}),$$

where T_{RLADS} is the remaining life after transplant from a deceased donor, which is exponentially distributed with mean 0.050. We know that $E(X) \approx 10.11$ years.

- $Y = T_n$.
- $Z = 0$.

It follows again from $C = a_1 X + a_2 Y - a_3 Z$ that

$$E(C) = a_1 E(X) + a_2 E(Y) - a_3 E(Z) = a_1 \cdot 10.11 + a_2 \cdot \overline{T_N} - a_3 \cdot 0.$$

The expected value of the benefit of the transplant strategy over the no-transplant strategy is

$$E\big[C(\text{transplant})\big] - E\big[C(\text{no transplant})\big] = \\ a_1 \cdot 32.08 + a_2 \cdot (-0.0203 \cdot \overline{T_{\text{N}}} + 0.02 \cdot \overline{T_{\text{MC}}}) - a_3 \cdot \bar{Z}.$$

The first component is positive, while the second and third are negative, since $\overline{T_{MC}} < \overline{T_N}$. The result can be either negative or positive, depending on the weights a_i.

Donor Decision: Donate to an Unknown Person?

In this case, the model stays exactly the same but the values of the a_i will be different.

Donor Decision: Donate via Paired Exchange?

We consider an N-pair exchange. We continue using the system provided in the previous example but must include more parameters:

- X_1 is the remaining lifetime of the loved-one recipient given a live donor transplant, and is exponentially distributed with rate $\lambda = 0.0237$;
- X_2 is the remaining lifetime of the $N-1$ stranger recipients who are each given a live donor transplant, and obviously $X_2 = (N-1)X_1$;
- Y is the remaining lifetime of the donor given a live donor transplant, same as in non-paired-exchange scenario; and
- Z is the pain and depression value of the donor after donating a kidney.

We then have

$$\begin{aligned} E(C) &= a_1 E(X_1) + a_2(N-1)E(X_2) + a_3 E(Y) - a_4 E(Z) \\ &= a_1 \cdot 42.19 + a_2 \cdot (N-1) \cdot 42.19 \\ &\quad + a_3 \cdot \big(.0003 \cdot 0 + 0.9797 \cdot \overline{T_N} + 0.02 \cdot \overline{T_{MC}} - a_4 \cdot \overline{Z}. \end{aligned}$$

For the no-transplant strategy, then in this case instead of one recipient being forced to wait for a donor, all N recipients must now wait, since no size $N-1$ cycle exists. Thus, we have:

- X_1 is the same as in the original X for the no-transplant, no-exchange strategy, but now transplants arrive faster because new live transplants are available. So

  $$\begin{aligned} X_1 = {} & \min\{T_{\text{die}}, T_{\text{DEADtransplant}}, T_{\text{LIVEtransplant}}\} \cdot P(\text{die}) \\ & + (\min\{T_{\text{die}}, T_{\text{DEADtransplant}}, T_{\text{LIVEtransplant}}\} + T_{\text{RLADS}}) \cdot P(\text{DEADtransplant}) \\ & + (\min\{T_{\text{die}}, T_{\text{DEADtransplant}}, T_{\text{LIVEtransplant}}\} + T_{\text{RLALS}}) \cdot P(\text{LIVEtransplant}), \end{aligned}$$

  where T_{RLAS} is the remaining life after surgery given that it is from a deceased donor. We know this to be exponentially distributed with mean 0.050, and $E(X_1) = 19.26$ years. This value is different from the no-exchange system because people on the waitlist will have a higher chance of receiving a transplant when the policy changes to permit and encourage exchanges.

- $X_2 = (N-1)X_1$ is the remaining lifetime of the $N-1$ stranger recipients who are each given a live donor transplant, and

  $$E(X_2) = (N-1)E(X_1) = (N-1) \cdot 19.26 \text{ years.}$$

- $Y = T_n$.
- $Z = 0$.

Hence we obtain

$$\begin{aligned} E(C) &= a_1 E(X_1) + a_2(N-1)E(X_2) + a_3 E(Y) - a_4 E(Z) \\ &= a_1 \cdot 19.26 + a_2 \cdot (N-1) \cdot 19.26 + a_3 \cdot \overline{T_N} - a_4 \cdot 0. \end{aligned}$$

Using both of the C values for transplant and no-transplant possibilities, we see that the A variable for choosing the transplant is: The expected value of the benefit of the transplant strategy over the no-transplant strategy is

$$E\big[C(\text{transplant})\big] - E\big[C(\text{no transplant})\big] =$$
$$a_1 \cdot 22.93 + a_2 \cdot (N-1) \cdot 22.93 + a_3 \cdot (-0.0203 \cdot \overline{T_N} + 0.02 \cdot \overline{T_{MC}}) - a_4 \cdot \bar{Z}.$$

We compare this value with $a_1 \cdot 32.08 + a_2 \cdot (-0.0203 \cdot \overline{T_N} + 0.02 \cdot \overline{T_{MC}}) - a_3 \cdot \bar{Z}$ for a no-exchange system.

The expected lifetime of the related patient has lower impact in the exchange model. This is because the related patient may receive exchange transplants in the future if you do not donate your organ through an exchange. While the effect of a_1 decreases, a new variable $a_2 \cdot (N-1) \cdot 42.19$ increases the probability that a donor decides to donate a kidney. This is because the donor feels responsible for increasing the lifetime for all N recipients in the size-N transplant exchange, because without that donor, none of the transplants would be possible. However, because the donor feels less attached to random recipients, we have $a_1 >> a_2$.

Model Strengths

The model

- provides a numerical value useful in gauging the probability that a donor decides to donate;
- is adjustable to any new system created;
- incorporates personal and psychological factors.

Model Weaknesses

Some variables and parameters are not independent, but our model assumes that the rates are independent.

Conclusions

After developing a model to understand the effects of components of the kidney transplant model, we have developed a list of solutions to the waitlist dilemma:

- **Tighten Waitlist Entry Requirement** Currently, patients join the waitlist when kidney filtration rate falls below a particular value or when dialysis begins. We recommend that only those whose conditions are at dialysis or worse should be allowed to join the waitlist. This change would lead to reduced inflow of waitlist candidates, dramatically improving the system.

- **Presumed Consent** Currently, those who wish to donate kidneys after death must have explicit documentation on hand when their bodies are retrieved. A new policy would assume that all deceased people are eligible for deceased-donor transplant, unless explicitly expressed otherwise. This change would dramatically increase the inflow of deceased donors.
- **Kidney Paired Exchange System** Many waitlist candidates have potential donors who cannot donate due to incompatible blood types or HLA. A kidney paired exchange system would match these people in a broad regional pool, identifying when donors can donate to the respective other paired recipient. This reduces the flow of incoming waitlist candidates.
- **Market Kidneys** We investigated government-sanctioned kidney purchases and a free market for kidneys. In both cases, the size of the waitlist diminishes with the number of live kidneys sold. However, a government bureaucracy could not handle the number of kidney transplants, so waiting time would increase for some, at least at first. In a free market, biological factors of kidney transplants could lead to discriminatory prices. Thus, marketing of kidneys is discouraged on the basis of parity.

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Judges' Commentary: The Outstanding Kidney Exchange Papers

Chris Arney
Division of Mathematical Sciences
Army Research Office
Research Triangle Park, NC
david.arney1@us.army.mil

Kathleen Crowley
Psychology Dept.
The College of Saint Rose
Albany, NY 12203

Introduction

Eight judges prepared for this year's ICM judging by studying the Kidney Exchange Problem and calibrating their criteria. When prepared for their task, they had the opportunity to read and compare an excellent set of creative and interesting papers. It is likely that many students and some advisors would find it surprising that the judges face challenges as complex as those tackled by the ICM contestants: The judges must determine how best to evaluate, grade, and score a myriad of papers as fairly and accurately as possible over a very short period of time. Their goal is to insure that awards are given to the best teams and that the papers published in *The UMAP Journal* represent the finest student work produced in the contest.

The papers were assessed in three key areas:

- effective use of current data and policies relevant to the U.S. organ transplant network to reveal supply and distribution bottlenecks and to identify means for producing improvements in the efficiency and fairness of the organ-donation network;
- application of an appropriate modeling process and appropriate use of the model to perform insightful and critical analysis; and

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- integration of mathematics, science, ethical principles, and common sense to render appropriate recommendations to the decision-makers.

Overall, the judges were impressed by the considerable efforts expended by the contestants and by the prodigious and sophisticated work produced by many of the teams. This year's problem was particularly challenging because it involved multiple tasks and the teams having to take on several diverse perspectives, ranging from mathematics and sciences to ethics and social policy.

The Problem

The issues raised in the Kidney Exchange Problem are real, and the problem is timely. Organ transplant problems and opportunities are often discussed in the news, and there is an ongoing stream of new proposals seeking to improve organ transplant management. Many of these proposals are being debated by both the U.S. Congress and members of the executive branch of the government. A wide array of professional researchers, staffers, and analysts are engaged in work on many of the same questions, challenges, and procedures that the teams had to address over the contest weekend. Some of these professional working groups are also engaged in advocacy—writing letters to members of Congress or to executives in the Department of Health and Human Services, processes mirrored by two of the required tasks faced by the contestants.

The data needed for the problem were readily available on the Internet. And while there are a number of articles and even fully developed models in the literature, the problem required more than mere research: Real, robust, creative modeling and critical interdisciplinary thinking were needed to complete successfully the required analyses. Several disciplines in addition to mathematical modeling had to be included for full consideration of this problem; ethical, medical, sociological, political, cultural, and psychological perspectives were all essential components to the development of complete and creative solutions. The dilemmas surrounding the issues of organ donation are undeniably and genuinely interdisciplinary in nature, and they have global relevance. Useful solutions to these challenges clearly require robust modeling if the current inefficiencies of present-day organ-donation systems are to be reduced or eliminated.

Judging Criteria

In the end, the judges selected two papers to be designated as Outstanding. Both of these efforts, and another special paper, will be considered later in this article. The judges organized the evaluative rubric into five categories (listed below), and we use this framework to summarize the judges' perspectives and determinations.

- **Quality of the Executive Summary:** Most teams demonstrated that they knew that it was important to provide a good executive summary. Moderately successful efforts only summarized the requirements and stated their recommendations. The more successful efforts also provided a logical link between the research issues, the models, and the recommendations.

- **Scientific Knowledge and Application:** Many papers demonstrated their significant knowledge of organ transplant science by providing well-written reports of the technical and social issues inherent to the transplant process and the organ distribution network. Many also provided excellent summaries of the widely differing issues regarding the role of government, public health policy, ethics, psychology, international culture, and social issues involved in the procurement and distribution of organs. It was clear that this problem was both very complex and demanding. Although most papers addressed the basic issues of the transplant systems, some papers addressed the complexities of this capacious issue better than others, through use of creative models and insightful analyses. The least successful papers were those that did not go beyond reporting information from the Internet or journal articles. Papers in this category sometimes included unnecessary or irrelevant scientific information, and they sometimes failed to fully integrate and / or document the information they presented. These kinds of papers did not fall into the highest-level categories. Some of the moderately successful papers were rather disjointed and apparently had the science section written by one part of the team and the modeling and analysis sections written by another. Although these sections may have been quite good when considered in isolation, they typically were not well-integrated and therefore did not present a strong synthesis of key elements. The most successful papers used the scientific knowledge as a basis for their models and their subsequent analysis as the basis for their policy recommendations. Almost every team was able to cite an enormous amount of information from the open literature and clearly had used Internet sources fully and effectively. But the stronger teams not only gathered an abundance of information, they examined international procedures and ideas to suggest potential improvements to the U.S. organ donation network that sometimes stumbles along under current public policies and social constraints. In other words, these papers demonstrated that the authors had an understanding that network functionality was critically important in the design of an outstanding solution to this problem.

- **Modeling:** The most effective papers made their assumptions explicitly from the scientific foundation that they developed to build their models. As one judge noted, some of the models appeared to be hammers looking for nails—making some models so complex that the entire report was devoted to developing the model without devoting time to any thoughtful analysis or meaningful recommendations. Some teams demonstrated their abilities to make appropriate model refinements in the follow-on tasks that addressed the more interdisciplinary issues of ethics and politics. It was important that the modeling process was well formulated and robust; but unfortunately, some papers had wonderful models that offered little explanation of how the model functioned or provided little use of the results in the analysis.

- **Analysis/Reflection:** Successful papers discussed the ways in which their models were able to address the issues and tasks involved in improving the current organ donation system. The most effective modelers verified the sensitivity and robustness of their models. This problem asked many questions, and even a long weekend does not provide much time to perform all the tasks for this very complex and interdisciplinary model. The most effective papers, however, found the time to recommend new policies based on their analyses, and the policies that they recommended were fully justified by the model analysis they had performed. The weaker papers did not address the questions with effective mathematical modeling or simulation, but relied instead solely on Internet research. The papers that addressed the policy issues well seemed to show that the U.S. was not doing enough to increase the population of donors and provided plausible solutions that were verified or used by their models. The problem tasks led many teams to talk about ethical concerns; but to the dismay of the judges, many other teams did not include consideration of ethical concerns in either their models or in their analyses of the issues.

- **Communication:** The quality of writing in the reports this year seemed to have slipped a bit compared to papers in previous competitions. This decline in quality may have been a consequence of the unusually high number of tasks and requirements for this year's problem compared to those in previous years. Nevertheless, this year's most effective papers demonstrated clear organization throughout the modeling process by establishing logical connections between sections of the report. Good communicators also understood that well-selected graphics were a highly effective means for making their points. As to the length of the papers, succinct with adequate explanation was preferred. Long, rambling papers were judged to be less effective because they created the frustration of requiring the judges to read unnecessary details and irrelevancies. Some papers hinted of good analysis but lacked sufficient clarity in their presentation. These teams apparently reasoned better than they communicated and consequently their important ideas and good arguments were not made readily apparent to the readers. The strongest papers presented the problem, discussed the data, explained their analyses, and fully developed their mathematical models. The biggest difference between the stronger and weaker papers was whether or not they were able to inform the reader about what they did, and more importantly, how they did it. Clear presentations allowed the judge to comprehend the logic and reasoning of the successful modelers. The top papers artfully blended the scientific literature with humanistic concerns, strong argument, and elegant mathematics.

Analysis of the Outstanding Papers

The papers judged to be Outstanding shared several common elements: robust modeling, insightful analysis, effective communication, and a touch of creativity. What truly distinguished these papers from the less successful, however, was their passion for the problem. They demonstrated a desire not only to solve the tasks at hand, but also to improve the overall health and well-being of real-world patients who suffer from organ disease. It is not surprising that given the complexity of the problem, both of the Outstanding papers still contained some weaknesses.

"Optimizing the Effectiveness of Organ Allocation" from Duke University "quantitatively analyzes kidney allocation, possible improvements to live donation, and many other strategies to improve the current process" (Executive Summary). Their model showed how list-based pairing could dramatically increase live donations, and they creatively addressed both ethical and political considerations. This team's letters to Congress and Director of the Health Services were crisp and clear with solid, well-supported, recommendations.

The Princeton paper entitled "Complete Analysis of Kidney Transplant System using Markov Process and Computations Models" made excellent use of the team's model to investigate the effects of policy changes. The authors recommended presumed consent and paired-kidney exchange. They also modeled the psychology of donating through the use of a "consumer decision theory model to explain the decisions that potential donors face when deciding whether or not to donate a kidney" (Executive Summary). Their analysis of the marketing of kidneys led them to reject the idea of making organs available on the free market because of their astute analysis of the serious ethical concerns such a system raises, even though it could potentially supply many kidneys to the organ donation network.

One other paper, while not designated Outstanding, also deserves mention because of the high quality of its creativity and presentation style. "The Giving Tree" submitted by a team from Berkshire Community College, provided a model that was "delicately designed so that the best ethical practices compliment the most efficient strategy, all while remaining economically feasible" (Executive Summary). The team proposed the very creative concept of "mandate choice," in which potential donors are required to declare their donation preferences when they seek driver's licenses. This team also included incentive strategies to educate citizens to the benefits of organ donation. All and all, this was a highly notable paper and one of the first that we have so favorably received from a small two-year institution.

Plagiarism

Unfortunately, this year's contest was marred by the disqualification of strong papers because of improper referencing, over-reliance on the published work of others, and a failure to appropriately and fully acknowledge the use of sources. The contest rules state that failure to credit a source will result in disqualification from the competition, and we were disappointed that this had to be done at the conclusion of this year's competition.

The Joy of Interdisciplinary Modeling

Talk of modeling, science, mathematics, psychology, communication, summaries, transplants, HLA, PRA, TTCC, OPN, references, algorithms, and computer programs echoed in the air; and then intense discussion of scores, rubrics, and criteria ensued as the final decisions were made. All this happened as the eight final judges came together to evaluate the finalist papers. As judges and interdisciplinary problem-solvers, we were most happy when we found papers full of excellent modeling, detailed mathematics, scientific facts, deep analysis, informative graphics, interesting solutions, successful collaboration, and especially strong evidence of student passion. This approach made mathematics all it should be: exciting, relevant, and potentially transformative.

Conclusion

The judges congratulate all the members of the successful teams. We saw evidence that all teams recognized and struggled with the challenges of a real, large-scale, interdisciplinary problem and hope that all of the ICM participants learned from their experiences as a part of this process. We believe that participation in the ICM contributes to the development of contestants in their quest to become sophisticated and effective interdisciplinary modelers. The effort and creativity demonstrated by almost every team was inspiring, and many papers served to reveal clearly the power of interdisciplinary problem-solving. We look forward to both continued improvement in the quality of the contest reports and increasing interest and participation in the annual ICM.

Recommendations for Future Teams (with help from the triage and final judges)

- Spend as much time as you can on analysis of the model, not just its development. Do not just report the model output and data. If possible, summarize your analyses clearly in tabular or graphical form.
- Clear communication makes it easier to identify outstanding work. Check your equations to avoid typographical errors resulting in a relationship that is inconsistent with the written description. State your assumptions, limitations, and strengths, and be sure to integrate fully and appropriately your

research sources with your model. Do not allow the background research to stand alone, unrelated to the model that you propose. The science that you report should be relevant to the model, and the model should reflect the science.

- Evaluate your results and discuss their implications. Explain how your results compare to similar work in the literature.
- Keep in mind that simple explanations suffice. If you are doing something super-elegant or ultra-complex, do not lose the reader in super-ultra-elegant complexity. Overly complicated models are not good ones. A significant part of the art of modeling is choosing the most important factors and using appropriate science and mathematics to simplify the problem.
- Long papers are not necessarily good papers. If you cannot describe your models clearly and succinctly, then they probably are not good models.
- The final important reminder is that any material that comes from other sources, even if paraphrased, must be carefully and completely documented; it must be placed in quotation marks if taken verbatim from another source.

About the Authors

Chris Arney graduated from West Point and became an intelligence officer. His studies resumed at Rensselaer Polytechnic Institute with an M.S. (computer science) and a Ph.D. (mathematics). He spent most of his military career as a mathematics professor at West Point, before becoming Dean of the School of Mathematics and Sciences and Interim Vice President for Academic Affairs at the College of Saint Rose in Albany, NY. Chris has authored 20 books, written more than 100 technical articles, and given more than 200 presentations and 30 faculty development workshops. His technical interests include mathematical modeling, cooperative systems, and the history of mathematics and science; his teaching interests include using technology and interdisciplinary problems to improve undergraduate teaching and curricula; his hobbies include reading, mowing his lawn, and playing with his two Labrador retrievers. Chris is Director of the Mathematical Sciences Division of the Army Research Office, where he researches cooperative systems, particularly in information networks, pursuit-evasion modeling, and robotics. He is codirector of COMAP's Interdisciplinary Contest in Modeling and the editor for the *Journal*'s ILAP (Interdisciplinary Lively Applications Project) Modules.

Kathleen Crowley is a Professor of Psychology at the College of Saint Rose. Her Ph.D. in educational psychology was earned at the State University of New York at Albany. Her teaching interests include parenting, child development, gender development, and the history of psychology; and she has taught a variety of courses at both Saint Rose and the University of Hartford in these subjects. Kathleen's research interests involve gender development, the psychology of women, and teaching of psychology. Recently, she has written on psychology as it relates to the post-modern philosophy. Always interested in using the latest technologies, Dr. Crowley has taught courses online and often uses service learning to enhance her courses. In addition to teaching, she has served as acting dean of the School of Mathematics and Sciences at Saint Rose for several years and as chair and member of many faculty committees. Enjoying travel, theatre, and film, she is the mother of two boys and active in her campus community. Recently, she took trips to London and California and was on an American Psychological Association delegation to Vietnam, Cambodia, and Hong Kong. Her latest hobbies include reading, singing, dancing, and soaking in her hot tub.

Practitioner's Commentary: The Outstanding Kidney Exchange Papers

Sommer Gentry
Dept. of Mathematics
United States Naval Academy
Annapolis, MD 21402
gentry@usna.edu

Mathematical Models Can Influence Public Policy on the Organ Shortage

The topic of kidney allocation and the shortage of kidneys for transplantation was a timely choice for the Interdisciplinary Contest in Modeling. In the past few years, public policy on organ donation and allocation has been changing rapidly, often in response to conclusions drawn from increasingly sophisticated mathematical models. Already in 2007, numerical projections of the significant impact of kidney paired donation in this country have prompted federal legislative and judicial action, which was necessary to clarify the indeterminate legality of paired donation. Bills passed in the House and Senate, and a Department of Justice memorandum, state that paired donation does not violate the National Organ Transplantation Act's prohibition on giving organs for valuable consideration.

The United Network for Organ Sharing (UNOS) is charged with oversight of all transplantation in the United States, including the allocation of organs from deceased donors to recipients. UNOS provides voluminous and easily accessible data on transplants at its Website `http://www.unos.org`. Within the past five years, UNOS has completely redrawn allocation procedures for liver and lung transplants. Recently, after statistical evidence showed that recipients with a liver allocation score of 15 or lower had shorter expected lifespans if they received a transplant than if they did not, liver allocation policy was revised to ensure that liver transplants went only to recipients who receive a survival benefit. The Department of Health and Human Services has also urged the

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transplant community to eliminate geographical disparities in allocation of organs.

Organ allocation makes an appealing target for interdisciplinary research, and not just because of the very real influence that mathematical modeling can have in shaping public policy. An application that can improve health care using mathematics also has tremendous motivational value for students, and makes a compelling advertisement of the contributions of operations research and interdisciplinary approaches to the lay public.

The Outstanding Papers

The problem statement asks students to address a range of questions about public policy and individual decisions in organ transplantation. A strong paper, then, should clearly state what changes to national policy are being advocated, and should use well-documented and correct analytical models to substantiate its conclusions.

Both Outstanding papers begin with dynamical models of the deceased donor kidney waiting list. There are more than 70,000 people on the waiting list to receive a kidney transplant from a deceased donor, and every year more people are added to the list than are removed. The team from Princeton University used a Markov-chain birth-and-death process to represent the size of the queue. The team used actual UNOS data to calculate fixed probabilities of each transition event: a new waitlist arrival increments the size of the queue, while transplantation, recovery of kidney function, or death all decrement the size of the queue. They validated their model's predictions against real data for net waiting-list additions in 2006. Importantly, the team used this model to evaluate two policies numerically: presumed consent for organ donation, and restricting the population that could join the waiting list. The team estimated annual list growth under perturbed versions of their model with increased transplantation rates or decreased waitlist arrival rates, and focused on zero waiting-list growth as the outcome of interest.

The Duke University team created a detailed discrete-event simulation in C++ to track the size of the queue as these events unfolded. By making the likelihood of death in each time period depend on the length of time that each individual had been waiting, this team captured the likely increase in deaths on the waiting list as the list grows longer. This latter property suggests that the size of the queue will stabilize when deaths on the waiting list occur at a rate that balances new additions, but the team did not explore this possibility. Because this detailed model of the waiting list contains recipient-specific information, this group was able to implement the UNOS point system for allocating kidneys and compare it to the Eurotransplant point system. Unfortunately, the results of the comparison were hard to interpret, because some figures that the team provided lacked axis labels, had cryptic captions, and were not cited in the text.

This team ran into a familiar dilemma for operations researchers working in organ transplantation, namely, that the objective of organ allocation is ill-specified by the transplant community and has myriad reasonable formulations. In particular, should allocation try to maximize the expected number of life-years gained from transplants? If so, then African-Americans and people older than 40 will be effectively denied any opportunity for transplantation, because these groups have lower expected lifetimes after transplant. This issue is being debated because the Kidney Allocation Review Subcommittee of UNOS proposes to increase the weight given to net lifetime survival benefit in allocation decisions. Historically, fairness to disadvantaged subgroups has been included in UNOS kidney allocation objectives. As another example, because kidney recipients can wait indefinitely on dialysis before receiving their transplants, but no life-prolonging therapies are available for liver or heart recipients, liver and heart allocation favors severely ill patients who are most at risk of death without a transplant rather than those who will receive the largest survival benefit after transplant.

Some members of the transplant community view deceased-donor organs as a local resource because local residents are the donors and local professionals counsel families about donation. These stakeholders feel that allocating organs recovered in one geographical area to recipients in a different area is unfair. However, the "Final Rule" legislation of 1999 requires that UNOS allocate organs in a way that minimizes the geographically-dependent variance in waiting times, outcomes, or other performance measures [Organ Procurement and Transplantation Network 1999]. This goal not yet been achieved, because waiting times for deceased-donor kidneys can vary by a factor of three or four between regions. The disparity is such that some hopeful recipients register in multiple regions to decrease waiting time.

Geographical aspects of transplantation were addressed well in both Outstanding papers. The models explored a tradeoff between

- achieving a high HLA (human leukocyte antigen) match, to ensure better outcomes for recipients; and
- decreasing transport distances, to reduce in-transit cold ischemic time and the associated risk of injury to the kidney.

Both teams made a crucial observation that the total cold ischemic time could be reduced even for kidneys shipped long distances, simply by speeding up the pretransit allocation process. Thus, the teams recommended that time to placement for each organ should be reduced if possible. Stefanos Zenios has published an excellent analysis of a system that could effectively reduce placement time by offering lower-quality deceased donor organs only to those recipients who are likely to accept them, which could mean using broader geographic sharing.

Finally, both teams discussed in detail the practical aspects of transplantation that they chose to simplify in each model, and in what way the messy details might alter their results and conclusions. For instance, the Princeton University team reported that its model of donor decision making does not account for factors such as the recipient's blood type and geographic location that would affect the person's estimated lifetime if the person remains on the deceased-donor kidney waiting list. The Duke University team correctly pointed out a lack of consensus in the literature about whether HLA matching affects survival of the transplanted organ, a question that has caused difficulties in my own research.

Optimization and Kidney Paired Donation

The problem statement set out an array of different tasks for the teams, so the teams did well to address some topics in more detail than other topics. As it happened, neither of the Outstanding papers proposed optimization models for kidney paired donation.

About one-third of recipients who have a loved one willing to be a live donor will find that the donor is incompatible. Without paired donation, these available donors do not give an organ and the recipient is added to the waiting list. In kidney paired donation, a match is made between two such incompatible pairs, so that the donor of the first pair can give to the recipient of the second pair, and vice versa. More generally, paired donation can include an exchange of kidneys among more than two incompatible pairs. There have been more living kidney donors than deceased kidney donors in recent years, and so kidney paired donation represents one of the most promising avenues for increasing the number of kidneys available.

Embedded within paired donation is a fascinating combinatorial optimization problem with a rich history: Among a set of incompatible pairs, how can the largest number of transplants overall be arranged? This is a graph-theory problem known as *maximum matching*, and it was first solved by Jack Edmonds about 50 years ago. A paired donation graph has a vertex to represent each recipient and his incompatible donor, with edges connecting two vertices iff they are mutually compatible. For example, in **Figure 1** many pairs could exchange with pair E, but the only pair which could exchange with pair C is pair A.

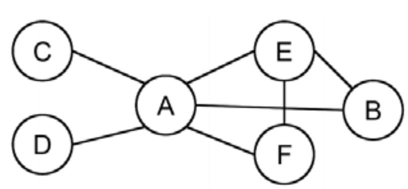

Figure 1. A kidney paired donation graph. Each vertex represents a recipient and the recipient's incompatible donor, and each edge indicates a mutually compatible exchange.

A *matching* is any subset of edges which does not contain any two edges incident on a single vertex. Some of the matchings on the graph of **Figure 1** are {AC, BE}, {AB, EF}, and {AB}. However, {AB, AC} is not a matching. To use maximum matching algorithms in practice, a paired donation system must allow a number of pairs to arrive before deciding which transplants should occur. If instead paired donations were performed as soon as possible, and if pair A and pair B arrived first, then only two transplants would occur for this group instead of the four that would otherwise be possible. Kim and Doyle [2006; 2007] have designed two interactive puzzles that allow students to solve tricky maximum matching problems and test out their favorite heuristics for the problem.

The team from Princeton University reported on a simulation of paired donation, but they did not comment on their matching algorithm. Their paper alluded to running a program every time a new pair arrives, which suggests that their method may not achieve the maximum number of transplants. Their results showed about 90% of incompatible pairs finding another pair with a complementary incompatibility. The best available results have match rates lower than 50%, but there was not sufficient detail provided to determine which modeling assumptions needed revision. Possibly the simulated blood types were not appropriately skewed towards the blood types likely to wind up in incompatible pairs.

List paired donation is a somewhat different approach whereby the living donor gives to a person on the deceased donor waiting list in return for moving the donor's intended recipient to the top of the waiting list. The team from Duke University made a strong claim that list paired donation alone might stabilize the queue size. However, deceased donor kidneys survive only half as long on average as live donor kidneys, so living paired donation is always preferable to list paired. Our own simulations show that list paired donation would not be an important contributor to transplantation rates if living paired donation were widely available [Gentry et al. 2005].

Mathematical simulations have been indispensable in demonstrating the impact of paired donation, because UNOS does not collect any data about the incompatible donors who come forward with recipients. The missing data can be reconstructed from known statistics using discrete event simulation. An interdisciplinary approach can be very successful in influencing public debate on these issues by offering detailed projections of the impact of new policies, as exemplified by some of the teams active in paired donation research. Extensive work in this arena has come from the team of Alvin Roth, Tayfun Sönmez, and Utku Ünver in cooperation with transplant surgeon Francis Delmonico; and I have enjoyed a very productive collaboration with my husband Dorry Segev, a transplant surgeon at Johns Hopkins.

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About the Author

Sommer Gentry studied applied mathematics and operations research at Stanford University and the Massachusetts Institute of Technology, receiving her Ph.D. in 2005. She was a Department of Energy Computational Science Graduate Fellow from 2001 to 2005. She is an Assistant Professor of Mathematics at the United States Naval Academy and a Research Associate in the Division of Transplantation at Johns Hopkins. Her research on maximizing paired donation to help ease the organ shortage has been profiled in *Time* and *Reader's Digest* and featured on the Discovery Channel. She serves as an adviser to both the United States and Canada in their efforts to create national paired donation registries.

Author's Commentary: The Outstanding Kidney Exchange Papers

Paul J. Campbell
Mathematics and Computer Science
Beloit College
Beloit, WI 53511
campbell@beloit.edu

We all use math every day . . .

[start of the opening sequence of episodes of the TV series *Numb3rs*]

Introduction

The 2007 ICM Kidney Exchange Problem arose from discussions in my one-semester-hour seminar in Spring 2006 on the mathematics behind the TV series *Numb3rs*. The specific inspirational episode was "Harvest," which deals with bringing poor Third-World people to the U.S. for black-market sale of their kidneys ["Lady Shelley 2006].

In the "Harvest" episode, one such donor dies after the operation and another potential donor is missing. The mathematician star of the series cites "optimization theory developed at Johns Hopkins University to determine the best matches between organ donors and recipients." He and his colleagues use the blood type and HLA-compatibility of the sister of the missing woman to try to identify potential recipients, despite (according to them) there being only a one-fourth chance that the sister matches the missing woman. The team first checks the database of patients registered to receive a kidney, draws a blank, and then realizes that they are probably looking for a patient "who cannot obtain an organ in the normal way—so they wouldn't be on any official list." (We find out later that the black-marketeer patient has a "blood disorder that disqualifies him from getting a transplant.") Fortunately, the FBI finds a "potential list of customers" (with blood data) on a suspect's computer, the list has

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a unique "positive match," and the missing girl is rescued just as she is to be operated on.

The remark about Johns Hopkins reminded me of earlier mention of that research in *SIAM News* [Cipra 2004]. Searching on the Web brought me quickly to the Web pages of Sommer Gentry that introduce her research on the mathematics behind optimizing kidney paired donation [2005].

Here I focus on algorithms for matching donors to kidneys, with particular focus on the work of Alvin E. Roth of the Harvard Business School and his associates.

The Problem

The problem involved a variety of tasks that spanned many interdisciplinary aspects—it is a very complex problem—and teams needed to be aware of the contest guideline that "Partial solutions are acceptable." (In fact, virtually all solutions were by nature partial.) Because of the number of tasks and their difficulty, I did not expect so many teams (273 of the 1222 completing the ICM/MCM) to tackle this problem. But then I also did not expect a 26% increase in participation in the contests, either. The proportion of teams selecting the ICM problem was about the same as in 2006.

Matching Kidneys to Patients

Kidneys from Cadavers

Kidneys become available as people die and must be transplanted very shortly after death. Thus, the problem is dynamic, and a priority scheme is needed to determine the recipient of such a cadaver kidney. Such a scheme could be provided by regulation (e.g., the Organ Procurement and Transplantation Network (OPTN) in the U.S.) or by compensation (e.g., a market for kidneys).

Living Donors

Suppose that we have a group of patients in need of kidneys and a group of altruistic living people each willing to donate a kidney to any patient.

We can model the situation as a bipartite graph, with patients in one part of the graph and donors in the other. An edge joins each donor to each patient for whom the kidney would be suitable. The graph has n vertices (patients plus donors) and m edges (corresponding to feasible donations). A greedy algorithm, using the concepts of *alternating path* and *augmenting path*, finds a matching with the most matches (called a *maximum (cardinality) matching*) in $\mathcal{O}(mn)$ time. Saip and Lucchesi [1993, 5] note other sequential algorithms offering different complexity, as well as parallel algorithms.

This matching "saves" the most patients possible but relies on the altruism of people to become donors.

Quality of Match

A refinement would be to assess the "value" of each match, in terms of a single number incorporating

- medical quality (match on ABO blood type, HLA markers, and Panel Reactive Antibody (PRA)),
- individual desirability (among other aspects, preferences for different quality kidneys and travel distance to the operation), and
- social desirability, perhaps in terms of QALY—quality-adjusted life years [Gold et al. 1996; Phillips and Thompson 2001] and a measure of equity (see Zenios et al. [2000]).

The value of the match can be incorporated into the model as a weight for the edge, and the graph-theoretic problem generalizes to finding a matching that

- maximizes the sum of the edge weights; or, alternatively,
- among all maximum cardinality matchings, maximizes the sum of the edge weights.

The reason to distinguish these two situations is that maximizing the sum of edge weights might result in fewer than the maximum possible number of matches: We might get better matches but "save" fewer patients.

The first kind of matching can be realized through the Hungarian algorithm of Kuhn (1955) in $\mathcal{O}(mn^2)$ time, and the second by an algorithm of Edmonds and Karp (1972) in $\mathcal{O}(mn \log n)$ time. Again, Saip and Lucchesi [1993, 5] note other sequential algorithms offering different complexity, as well as parallel algorithms.

Like a priority scheme for allocation of cadaver kidneys, incorporation of any measure of social desirability of a match is a political question. Mathematicians can only highlight the tradeoffs for various schemes.

Dynamism

Both the cadaver situation and the living donor situation are dynamic, in that the optimal matching may change (perhaps drastically) with entry or withdrawal of a donor or patient.

Kidney Paired Donation (KPD)

In kidney paired donation, a patient with a willing but incompatible donor is matched with another patient/donor pair such that the donor of each pair is compatible with the recipient of the other pair.

Matching Model

A bipartite graph is not an appropriate model, since the matching now demands that if donor/patient pair i donates to pair j, then j must donate to i. A general graph is called for. Gentry and Segev [2005] display such a graph with the vertices in a circle and edges as chords, which can be weighted.

The algorithms for finding optimal matchings in a bipartite graph require some tweaking to handle a general graph. Edmonds's algorithm (1965) finds a maximum cardinality matching in $\mathcal{O}(mn^2)$ time, while that of Blum (1990) finds one in $\mathcal{O}(mn^{1/2})$ time. An algorithm of Galil, Micali, and Gabow solves the edge-weighted problem in $\mathcal{O}(mn \log n)$ time [Saip and Lucchesi 1993, 14]. In the "Math Behind *Numb3rs*" seminar, we worked through the Edmonds algorithm and its proof but found that hand implementation of it on even small graphs—necessary for really understanding it—was unwieldy.

The idea for kidney paired donation originated with Rapaport [1986] and was first implemented in Korea [Park et al. 1991]. In the U.S., about 150 such transplants have been performed.

How Much Difference Could KPD Make?

How much difference could kidney paired donation make? In June 2007, there were 72,000 individuals awaiting kidneys, at 270 centers, an average of 270 per center [Organ Procurement ... 2007]. Of those 270, according to the team from Princeton University, 10%–15%, or 27 to 40, have a willing but incompatible donor. A simulation by Roth et al. [2005a] shows that in a population of 25 donor-patient pairs (where the pair may or may not be compatible), on average 12 patients receive a kidney from their own associated compatible donor, but an additional 4 could receive a kidney with paired donation. In a larger population of 100 donor-patient pairs, the corresponding numbers are 47 from their own donor and an additional 23 from paired donation. These data suggest that *one-third to one-half more live kidney transplants could take place with widespread implementation of kidney paired donation*.

Well, how many is that and how much difference would it make? In 2006, there were 17,100 transplants in all, of which 6,435 were live transplants; one-third to one-half more of the latter would be 2,100 to 3,200. That would not be enough to turn the tide: Over the course of 2006, the waiting list for a kidney grew by 6,100 (to 72,200), despite 4,200 leaving the list by dying [Organ Procurement ... 2007].

Donation Circles

The idea of kidney paired donation generalizes naturally to n-way circular exchanges, in which each donor-patient pair donates to another in the circle and in turn receives a donation from a pair in the circle. A few 3-way exchanges have taken place, and one 5-way exchange has been performed [Ostrov 2006].

Roth et al. [2005b] explain how 3-way exchanges offer further benefits beyond 2-way exchanges, and why going to 4-way exchanges has very limited further value (because of the rarity of the AB blood type). They calculate upper bounds, based on national data for blood types and PRA, for the effect of n-way exchanges. These bounds agree well with their simulations, which used "various integer programming techniques" for optimization in the case of greater than 2-way exchanges. In a population of 25 *incompatible* donor-patient pairs, on average 9 patients can receive a kidney via 2-way exchange, and 2 more via 3-way exchanges. In a population of 100 donor-patient pairs, the corresponding numbers are 50 via 2-way and an additional 10 via 3-way. Allowing 4-way or larger circles has negligible additional benefit.

In fact, Roth et al. prove under mild conditions—mainly, that the population of donor-patient pairs is large—the remarkable result that "4-way exchange suffices": If there is a matching with the maximum number of patient-donor pairs, with no restriction on the size of exchanges, then there is a matching involving the *same* pairs that uses only 2-way, 3-way, and 4-way exchanges. (Perhaps the 5-way exchange in 2006 could not have been reduced to smaller exchanges because of too few donor-patient pairs at that transplant center.)

A Kidney Is Like a House

Roth et al. [2004] cite an analogy between a housing market, as modeled by Shapley and Scarf [1974], and the "kidney transplant environment":

> [There are] n agents, each of whom is endowed with an indivisible good, a "house." Each agent has preferences over all the houses (including his own), and there is no money in the market, trade is feasible only in houses . . . [I]f we consider exchange only among patients with donors, the properties of the housing market model essentially carry over unchanged . . . .

Top Trading Cycles Algorithm

The authors note that Shapley and Scarf attribute to David Gale a particular algorithm for clearing such a market, called the *top trading cycle (TTC)* algorithm:

> Each agent points to her most preferred house (and each house points to its owner). Since the number of houses is finite and since each house has an owner, there is at least one cycle in the resulting directed graph. In each such cycle, the corresponding trades are carried out, i.e. each agent

in the cycle receives the house she is pointing to, and these agents and houses are removed from the market.

The remaining agents express new preferences and the procedure is iterated recursively. This system cannot be "gamed": The algorithm results in an allocation in which no coalition could have done better by trading among themselves, and it is in each agent's best interest to express true preferences [Roth 1982]. Roth et al. [2004] show that the TTC mechanism is the unique mechanism that is "individually rational, [Pareto-]efficient, and strategy-proof."

TTC has further applications to other important current problems of allocation or matching, such as college admissions, student placement, and school choice [Kesten 2004; Sönmez 2005].

Combining Cadavers and Living Donors

The top trading cycles algorithm would suffice for allocating kidneys among patients with willing but incompatible donors, via kidney paired exchange and kidney circles.

A complicating factor is that the kidney transplant environment also contains "unowned" cadaver kidneys. This situation corresponds to what Abdulkadiroğlu and Sönmez [1999] call the *housing allocation problem with existing tenants*—which, to the intrigue of the students in the *Numb3rs* seminar, was exactly the problem that they were facing at the time of our study: room draw for dorm rooms.

Abdulkadiroğlu and Sönmez critique the mechanism commonly used by colleges (including my institution, Beloit College), which they dub *random serial-dictatorship with squatting rights*. Under this system, a student may elect to keep their current dorm room ("squat" it) for next year and thus opt out of the dorm-room lottery. The major deficiency is that a student who foregoes keeping their current room and enters the lottery may wind up with a worse room.

You Request My House—I Get Your Turn

Abdulkadiroğlu and Sönmez consequently generalize the top trading cycles mechanism to a procedure that they call *you request my house—I get your turn (YRMH–IGYT)*. All students indicate their preferences, all are in the lottery, and turns are chosen at random. If a student whose turn comes wants your room (and you have not already had your turn), you get the very next turn before they get to choose. So you can always keep your room if all the "better" rooms are gone.

This seems like a great idea, one that could be completely automated; but the *Numb3rs* students and I had to reflect on the difficulties of changing a procedure well-established at the College. That procedure absorbs several days of staff time sitting for appointments with students coming to select rooms, not to mention students bolting out of classes to meet their appointments. We could see no good way to assess the likely level of improvement in overall student satisfaction, apart from just trying YRMH–IGYT for a year's room draw. However, Chen and Sönmez [2002; 2004] offer results of small-scale experiments.

At first, YRMH–IGYT would seem to be an easy sell to students: You can't be any worse off than you already are or could be under the current system. But it seemed to us that if all students had the same rankings for rooms, then the advantages of YRMH–IGYT accrue to students already in "good" rooms: If instead of "squatting" their current "good" room under the current system, they enter the YRMH–IGYT lottery and do better by taking "top" rooms, it is in some sense at the expense of other students for whom those "top" rooms are then not available. Yilmaz [2005] offers a further critique of the fairness of TTC; shows the incompatibility of fairness ("no justified envy"), individual rationality, and strategy-proofness; and offers his own algorithm.

Kidney Analogy: LPD

The kidney transplantation community independently invented YRMH–IGYT (dubbing it "indirect exchange," now more commonly known as *list paired donation (LPD)*): A patient's willing but incompatible donor donates to the highest-priority compatible patient on the cadaver waitlist; in return, the donor's intended recipient goes to the top of that list [Roth et al. 2004]. However, analogous to a student being reluctant to enter the dorm-room lottery for fear of losing their current "good" room, a donor may be unwilling to donate unless the donor's intended recipient gets a kidney at least as good as the donor's.

Roth et al. itemize the differences between the housing market and the kidney environment. The main difference is the dynamism that we mentioned earlier: No one knows when or what quality kidneys will become available on the cadaver queue, and such kidneys must be allocated and transplanted immediately. "Therefore, a patient who wishes to trade his donor's kidney in return for a priority in the cadaveric waiting list is receiving a *lottery* instead of a specific kidney" [2004].

Top Trading Cycles and Chains

Roth et al. introduce the *top trading cycles and chains (TTCC)* mechanism for kidney exchange, a recursive procedure that generalizes TTC. Each patient points toward a kidney or toward the cadaver queue, and each kidney points a paired recipient. In addition to cycles, this directed graph can also feature *w-chains*. A *w-chain* is a directed path on which kidneys and patients alternate and which starts at a kidney and ends at a patient pointing to the cadaver queue The result is a chain to the "waitlist," hence the "w" in the name "w-chain." Such chains correspond to generalized indirect exchanges: The kidneys can be allocated to their immediate successor patients, with the last patient getting a high place on the cadaver queue. As before with TTC, we can resolve and remove cycles; now we can also resolve and remove w-chains, perhaps preferably w-chains of maximal length (or perhaps not, for logistical reasons). But since a kidney or a patient can be part of several w-chains, there is a policy dimension needed to complete the algorithm, and Roth et al. discuss several conceivable chain-selection rules. They characterize TTCC using a particular class of rules as Pareto-efficient and TTCC with certain specific rules as strategy-proof.

Segev et al. [2005] and Gentry et al. [2005] examine optimal use of kidneys through kidney paired donation in association with list paired donation.

Future Prospects

Kidney paired donation, plus 3-way donations, plus TTCC, offer prospects for reducing the growth of the kidney waiting list and saving the vast cost of dialysis for patients on it.

Roth et al. [2004] offer simulation results that demonstrate for a population of 30 donor-candidate pairs:

- The rate of utilization of potential-donor kidneys goes from 55% to 69% with kidney paired exchange (this result roughly confirms that of the team from Duke University) and to 81–85% under the TTCC mechanism with varying chain-selection rules.
- TTCC decreases the average HLA mismatch from 4.8 to 4.2–4.3.
- The average cycle size is 2.5–3 pairs and the average w-chain size is 1.8–2 pairs.

Larger numbers of pairs (Roth et al. give results for 100 and 300) result in (on average) higher utilization rates, lower HLA mismatch, and longer cycles and chains.

Roth et al. go on to describe the advantages of TTCC over current kidney exchange programs.

Operations researchers (as some economists are, too) must always consider not just the problem of optimization but that of selling an improved solution. In particular, operations researcher Robert E.D. "Gene" Woolsey (Colorado School of Mines) cast as his First Law: "A manager would rather live with a problem he cannot solve than accept a solution he does not understand" [2003]. Implementing new kidney exchange procedures on a wide scale would require suitable legislation, coordination of databases, and education of patients and potential donors.

Finally, different blood types between willing donor and patient may not be an absolute obstacle in the future. In September 2006, a type-A wife donated to a type-B husband, a transplant that "was made possible by desensitization, a process that removed [rejection] antibodies from [the husband's] blood and kept them away with medication" [Wausau couple . . . 2006]. Segev et al. [2006] recommend pursuing desensitization, because

> even in a large-cohort live donor match, approximately half of the patients remain unmatched . . . . There is subsequently little additional benefit to placing difficult-to-match patients into a list exchange program.

What about a market for kidneys? The team from Princeton University explores the economics of this possibility. They dismiss the option of a government-managed program on the basis of their conviction that government management implies bureaucratic inefficiency and slowness; but surprisingly, after touting the relative advantages of a free market in kidneys, in their conclusion they reject that, too, because of the danger of discriminatory pricing. Roth [2007] reflects on general repugnance to certain economic efficiencies, including a market for kidneys.

Conclusion

I previously wrote about the potential and sources for using the *Numb3rs* TV series as a way to draw college students into appreciation of mathematics [Campbell 2006]. Instructors may find doing so more feasible with the advent in fall 2007 of a potential "textbook" for a course or seminar based on the series [Devlin and Lorden 2007].

My experience demonstrated to me that such a course also offers the instructor an opportunity to serendipitously expand horizons—to learn and engage with more mathematics as it manifests itself in life. I am glad that the TV series, its "Harvest" episode, and the course also led to the ICM Kidney Exchange Problem; I have enjoyed learning a great deal about an interdisciplinary congeries of mathematics, algorithms, medical practice, economics, public policy, and ethics.

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About the Author

Paul Campbell is professor of mathematics and computer science at Beloit College, where he was Director of Academic Computing from 1987 to 1990. He has been the editor of *The UMAP Journal of Undergraduate Mathematics and Its Applications* since 1984.