PHYSICS **Standing Waves**

Name:	Teammates:

Introduction

Fig. 1: Tacoma Narrows Bridge

For a spring mass system or a pendulum, when released from a position away from the equilibrium position, the position, velocity, and acceleration of the mass are all described by single sinusoidal functions.

$$x(t) = A\cos(\omega t + \varphi)$$
$$v(t) = -A\omega\sin(\omega t + \varphi)$$
$$a(t) = -\omega^2 x(t) = -\omega^2 A\cos(\omega t + \varphi)$$

Because the motion repeats at one particular frequency, called harmonic, the system is referred to as a simple harmonic oscillator (SHO). It is noteworthy that the motion of a great many systems, such as the shock absorbers in cars or a wrecking ball on the end of a crane, can be modeled fairly well by SHO.

This is not to say that all systems that oscillate can by modeled by a single SHO. Take, for instance, the motion of a pinecone on the end of a branch of a tree. The small branch to which the cone is attached operates very much like a pendulum, which is a SHO. However, this branch is attached to larger branches, which are themselves attached to the tree trunk. All of these branches and trunks by themselves operate like SHO's. When coupled together to form a tree, the motion of the cone will be quite complicated, as each SHO will have its own distinct frequency at which it will want to oscillate. Our arms act in a somewhat similar manner to this. The forearm and the humerous operate as two pendula that are attached at the elbow, albeit with the limitation that the forearm is not free to swing completely backward relative to the humerous due to bicep tendons.

Driven Oscillators and Resonance

Of course, the effects of friction and drag will dampen any oscillatory behavior. Some systems have very small retarding forces acting upon them, and will oscillate for a long time without much decay in their motion others will quickly stop oscillating. This dampening effect can be counteracted if the force that starts the oscillation of the system is also periodic. Of particular importance is when this "driving" force is operating at a frequency equal to the frequency at which the system naturally oscillates. Any person who has ridden a swing knows this. You can go higher and higher on the swing if someone pushes you from behind at the exact same frequency as the natural oscillation frequency of the swing. In some cases, a similar effect can be achieved by driving the system at a frequency that is an integer multiple of the natural frequency.

When a system is driven at its natural frequency we say that it is in resonance. If the size of the driving force is large enough compared to the damping forces, the amplitude of the oscillation will continually grow as the force "builds" upon its own actions. If the amplitude gets too large, one of several things can happen. For one, the natural frequency of the oscillation can change, which will cause the amplitude to stop growing as the system is no longer in resonance. Another thing that can happen is that the damping forces increase to a point where they exactly offset the driving force, causing the amplitude to settle down to a given value. Lastly, the amplitude of the oscillation can get so large that the system breaks. A great example of this was the collapse of the Tacoma Narrows Bridge in 1940. The bridge, which was prone to vibrating up and down, began oscillating violently one day when it was driven into resonance by winds. Eventually, the entire bridge collapsed. You can watch a video of the collapse at http://www.youtube.com/watch?v=j-zczJXSxnw.

Strings – Both Ends Clamped

One of this week's activities looks at a system that is very similar to the Tacoma Narrows Bridge: a taut string. Both of these systems involve an extended, thin mass that is stretched between two supports. At equilibrium, the mass forms a straight line between the supports. If the mass is pulled away from equilibrium, there will be a restoring force to bring it toward equilibrium. The wave speed in the string is then given by

$$v = \sqrt{\frac{T}{m/L}}$$

where T is the tension in the string and m is the mass of the string and L its length.

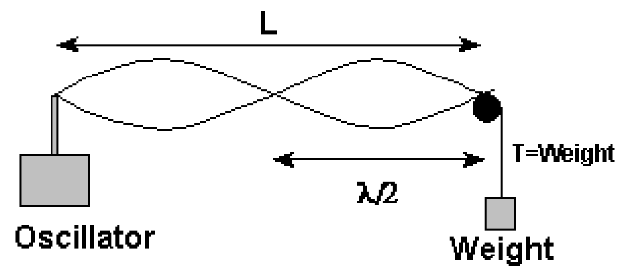


Fig. 2: Diagram of string system

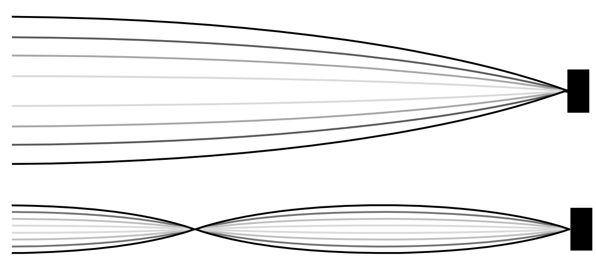
In this case, the string is fixed at each end to the equilibrium point. This causes the waves to reflect off of the ends. If there is no forcing in the system, damping will cause the waves to eventually die out and the string eventually returns to equilibrium. If the system is forced at a particular frequency, it will create waves of a particular wavelength. If the wavelength is not a half-integer multiple of the length L of the string, then the waves going in both directions will *constructively and destructively* add in *a constantly changing fashion* and nothing special is observed. Yet, under the correct *forcing* frequencies, resonant waves can build up. This occurs when half of the wavelength of the waves is an integer multiple of the length of the string. The waves give the appearance of the string not moving, thus the name "standing wave". For this condition, the relationship between the length of the string and the wavelength of the waves is

$$L = \frac{n\lambda_n}{2}\text{, where n = 1,2,3,.....}$$

Since v = $f\lambda$, the frequency at which the string must be forced is given by: $f_n = \frac{nv}{2L}$

where v is the velocity of the waves and n = 1,2,3,.....

Metal Strips – One End Clamped



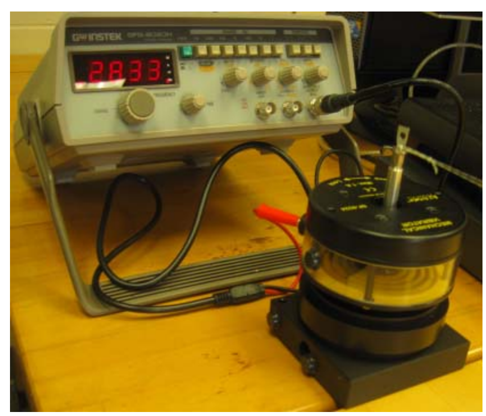
In the second part of the lab, we will deal with metal strips that are fixed only on one end. Under the correct forcing frequencies, resonant waves can also build up on the strip. This occurs when a quarter of the wavelength of the waves is an odd integer multiple of the length of the string. For this condition, the relationship between the length of the string and the wavelength is

$$L = \frac{(2n+1)\lambda_n}{4}\text{, where n = 0, 1,2,3,.....}$$

The frequency at which the string must be forced is given by: $f_n = \frac{(2n+1)v}{4L}$

where v is the velocity of the waves and n = 0,1,2,3,.....

Procedure



In this lab, the forcing frequencies will be provided to the string, spring or strip by a *Mechanical Vibrator*. The driver shaft of the vibrator can be tied to a string, strip, or spring via provided "banana plugs". The frequency and amplitude of the vibrations are regulated through the use of a *Function Generator*. The image to the right shows a typical setup of a Mechanical Vibrator connected to a Function Generator.

The Function Generator

The image to the right shows the *Function Generator* used in this lab along with labels pointing to the controls that you will need to use.

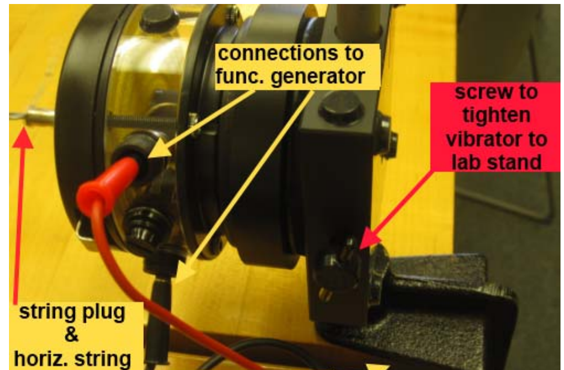
- You will need to use the BNC connector at the bottom right of the generator (and labeled *OUTPUT*) to connect to the vibrator.
- Set the AMP knob to its maximum value.
- Press the ~ button to ensure that the signal fed to the vibrator is sinusoidal.
- Press the *100* button to ensure that the range of frequencies we use is in the 100 range.
- You control the value of the frequency by rotating both coarse and fine frequency knobs. The value you select is displayed by the red LED display.

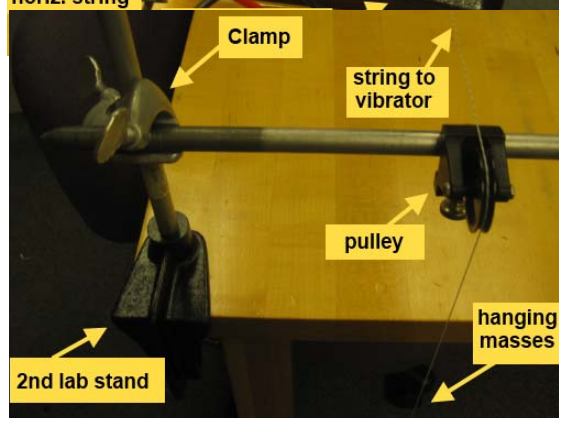
The Mechanical Vibrator

- Make sure you connect the "banana" ends of the BNC cable to the vibrator as shown in the Figure.
- Also ensure that the vibrator is unlocked.

Strings – Both Ends Clamped

1. Start by connecting two lab stands to the ends of the lab bench. The first lab stand should be connected the end of the bench closest to the computer. That is where you should position your function generator as well.
2. Slide the Function Generator through the shaft of the lab stand as shown.
3. Tighten the screw on the vibrator to ensure that the vibrator is secured.
4. Insert the banana plug (available inside the vial) into the driver shaft of the vibrator.
5. Position the string on the "banana plug" as shown in the Figure.
6. Pull the string horizontally to the other end of the bench.
7. Attach the other lab stand, clamp and pulley as shown in the Figure.
8. Attach a 100 g mass to the hanging end of the string.
9. Adjust the position of the pulley and the stand so that the string is horizontal and straight.
10. Make sure all stands, pulley and vibrator are held tight.
11. Turn both frequency knobs (coarse and fine) fully counter-clockwise.
12. Turn the function generator on by pressing on the power button (it is colored green).
13. Measure the length of the vibrating portion of the string (note: the string length at this stage should be approximately 2.0 m):





String Length = ________ m

14. Change the frequency until you establish a resonant condition and then count the number of antinodes. If the number is greater than 1, lower the frequency until a new resonant condition has been established at which only 1 antinode exist. This is the lowest resonance frequency for this system. Record this frequency f_1.
15. Beginning at this frequency f_1, increase the frequency of oscillation until you have found the first 4 resonance frequencies. *Could you have predicted these higher three frequencies?*
16. For each or obtained frequencies, *measure* the corresponding wavelength and enter the data in the following table:

Hanging Mass	Length	f_1	λ_1	f_2	λ_2	f_3	λ_3	f_4	λ_4
100 g									

A. From the data that you have obtained, what is the relationship between the fundamental frequency (f_1) and the harmonics (f_2, f_3 and f_4)?

17. Going back to f_1, by adding 50 g at a time, while not exceeding 400g, find the resonant frequency for each of the hanging masses.
18. For each or obtained frequencies, *measure* the corresponding wavelength and enter the data in the following table:
19. Determine how you would graph this data to show whether or not your data agrees with the theory provided in the introduction to this lab.

Hanging Mass	Length	f_1	λ_1
100 g			
150 g			
200 g			
250 g			
300 g			
350 g			
400 g			

20. Make the graph. Does your data agree with the theory?

B. Based on your data, what is the relationship between the tension of the string and the fundamental frequency (f_1)?

21. Return the hanging mass to 100 g and answer the following:

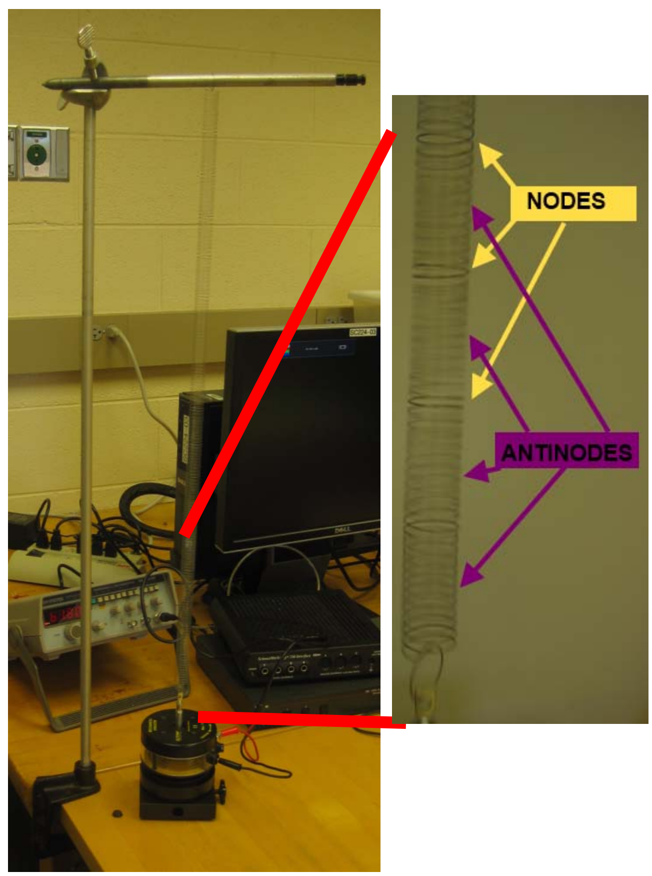
C. From the data you have gathered, and from your observations, can you calculate the new fundamental frequency if we limit the portion of the string that oscillates to 1.0 m? Make your calculations below, and then test it out.

Hanging Mass	Length	λ_1	f_1 calculated	f_1 measured
100 g	1.0 m			

D. Based on your data, what t is the relationship between the length of the string and the fundamental frequency (f_1)?

Springs – Both Ends Clamped

This part of the lab will allow you to observe longitudinal standing waves.



1. Start by connecting the lab stand, vibrator, and spring as shown in the image. The spring should be vertical and straight.
2. With the generator off, turn both frequency knobs (coarse and fine) fully counter-clockwise.
3. Turn the function generator on.
4. Change the frequency until you establish a resonant condition and then count the number of antinodes. Let us call the number of antinodes you counted N.

Number of antinodes N = ________

Corresponding frequency f_N = ________ Hz

Corresponding wavelength λ_N = ________ m

(Note: you might need to make an additional measurement to figure out the wavelength.)

5. Predict the resonant frequency f_{N+2}, for a number of antinodes equal to (N+2)

Predicted value of f_{N+2} = ________ Hz

6. Change the frequency of the function generator until you observe a number of antinodes equal to (N+2)

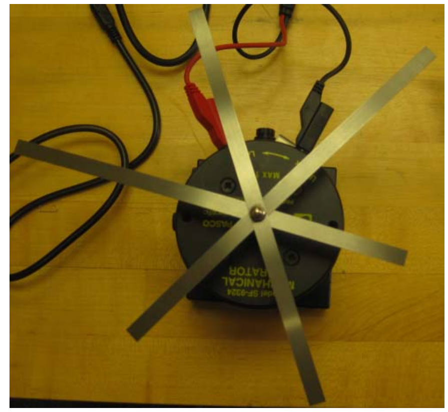
Observe value of f_{N+2} = ________ Hz

7. How do both values compare?

8. Use the data you have collected to find the velocity of the wave in the spring. v = ________ m/s

Strips – Only One End is Clamped

This part of the lab will allow you to observe standing waves on a system where one end is clamped while the other is free.



1. Start by connecting the vibrator, and strips as shown in the image.
2. Measure the length of each of the strips.
3. With the generator off, turn both frequency knobs (coarse and fine) fully counter-clockwise.
4. Turn the function generator on.
5. Slowly increase the frequency until you establish a resonant condition for one of the strips. It should be either the shortest or longest strip. Which one do you think it should be? Data from the first part of the lab should guide you to the answer.

6. Assuming that the strips make-up is the same, the speed of the wave on the strips should then be the same. We should then be able to predict the fundamental frequency of each of the strips.

	Strip 1	Strip 2	Strip 3	Strip 4	Strip 5	Strip 6
Length (m)						
λ_1 (m)						
Predicted: f_1 (Hz)						
Measured: f_1 (Hz)						
Percent Difference						

7. Change the frequency until you identify the fundamental frequency for each of the strips and add your data to the table above.
8. Do your predictions agree with your observations? Explain the discrepancies if any.

9. Now focusing on the longest strip, what is its fundamental frequency? f_1 = ________ Hz

10. Predict the wavelength and frequency for the next harmonic for this strip: λ = ________ m, f = ________ Hz

11. Change the frequency until you observe this harmonic for long strip: f = ________ Hz
12. Do your predictions agree with your observations? Explain the discrepancy if any.