



## Acceleration Due to Gravity

### Introduction

One of the more famous explanations as to why and how things fall was given to the world by the Greek scholar Aristotle (384-322 B.C.). He believed in the concept of teleology, which is the philosophy that the explanation of an occurrence was found in its final causes. Applied to the motion of objects, Aristotle believed that object always wanted to return to their beginnings where they could be with like objects, i.e. a rock that was from the ground wanted to return to the ground to be amongst other rocks.

Aristotle did not stop at just giving a reason for why things fall down. He also went on to describe the motion. In particular, he stated that heavier object fall faster than lighter objects. For instance, he posited that an object that was twice as heavy as another object would fall twice as fast. This, too, was accepted by the public, based somewhat on the authority of Aristotle and somewhat on observations. For instance, if you drop a light piece of paper and a heavy rock at the same time, the rock hits the ground well before the paper.

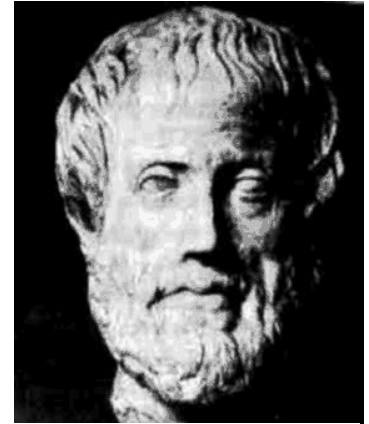


Fig. 1: Aristotle (NIH)



Fig. 2: Galileo (Sustermans)

Today, we know that the cause of this phenomenon is air resistance. However, it was not until the early 1600's that Galileo Galilei showed that this theory of heavier objects falling faster was wrong. He did this by running experiments wherein the weight of the object did not affect the rate at which objects fall. The experiments that Galileo performed to study gravity were done with a ball rolling down an inclined plane, rather than a ball falling through the air. The reason for this was quite simple: Galileo wanted to time how long it took the ball to move a given distance, and the clocks of his time did not allow for accurate measurements on such short time scales as a second or so. By having the balls roll down an inclined plane, Galileo lengthened the time over which the balls moved. Using the most accurate pendulum and water clocks of his time, he was able to make fairly accurate measurements of the elapsed time.

These measurements allowed Galileo to do more than to show that Aristotle was wrong. He was able to test and discern mathematical relationships between various factors, which led to the first accurate equations about the motions of objects. What Galileo found was that, under the influence of gravity, the distance that a ball travels from rest is proportional to the square of the amount of time that the ball is allowed to travel. Mathematically, we write this as

$$\Delta x = kt^2$$

where  $\Delta x$  is the distance traveled,  $t$  is the elapsed time from rest, and  $k$  is a proportionality constant. Knowing that the average velocity of an object is related to the displacement by  $\mathbf{v}_{\text{avg}} = \Delta \mathbf{x}/t$  and that the final velocity of an object is related to the acceleration (for an object starting at rest) by  $\mathbf{v}_{\text{final}} = \mathbf{a}t$ , Galileo deduced that

$$\Delta x = \frac{1}{2}at^2$$

This leads to the following equations for describing the behavior of a freely falling object:

$$x(t) = x_i + v_i t + \frac{1}{2}at^2 \quad \text{and} \quad v_f(t) = at + v_i$$

where the acceleration  $\mathbf{a}$  is equal to the gravitational field constant  $\mathbf{g}$ . The magnitude of  $g$  is approximately equal to (it varies depending on several factors, mainly the altitude)  $9.8 \text{ m/s}^2$  and its direction is downwards.

## Experiment

The equations above are only valid under the assumption that there is no air resistance. This presents some problems, as we happen to live on a planet that has air. Even though the effect is small, it is bound to affect our results.

The experiment is broken in two parts. In the first, we will be using commercially available photogates from Pasco, Inc. These photogates work by passing an infrared beam across an opening to a receiver unit. If something crosses the opening, it stops the beam from reaching the receiver, which causes the photogate to send a signal to the computer to signify that an object has entered the area. When the object leaves the opening, the beam is once again picked up by the receiver, which causes the photogate to send another signal to the computer to let it know that the object has left the area. In this manner, the computer is able to time the movement of the object.

To investigate the motion due to gravity, we will be dropping a transparent ruler that has opaque stripes placed on it at regular intervals (5.0 cm). As each stripe passes through the gate, the computer will note the time it takes the opaque stripe to pass through the light gate. At the conclusion of the experiment, it will display these arrival times, along with the position of each stripe on the ruler. We will then analyze this data and calculate the acceleration due to gravity.

In the second part of the experiment, we will use video analysis to collect position data of a ball that is tossed upwards and use the data to find its acceleration.



Fig. 3: Ruler (Pasco)

### Procedure (part 1)

1. Plug the photogate into the digital input #1 on the Pasco interface. Make sure that the interface is turned on by checking that the green light is on.
2. Position the photogate close to the edge of the table so that the ruler can fall through the photogate. Place a piece of foam rubber below the photogate so that the ruler does not directly hit the floor.
3. Start the DataStudio software and select Create Experiment. This will open a new window titled "Experiment Setup".
4. Select Channel 1. This will be the farthest left of the circled sensors in the "Experiment Setup" window. In the new window that opens, select "Photogate & Picket Fence". You are now ready to perform the experiment and collect data.
5. Place the lower edge of the picket fence so that it is just above the photogate. Click the Start button in DataStudio and drop the picket fence, taking care so that the fence drops freely without contacting the photogate or stand. After the fence strikes the foam rubber, click stop.
6. Repeat step 5 four more times with the ruler starting at different heights above the photogate.

### Data Analysis (part 1)

The data measured by the computer is the time at which each opaque picket after the first picket enters the photogate. If we imagine the edge of the first picket to be at  $x = 0$  cm, then the second picket is at 5.0 cm, and each subsequent picket is 5.0 cm further away than the previous. The table of data that we have for each run, then, consists of position-time pairs for the ruler. The differences in each run are due primarily to two factors: the amount of time that elapses between when we press start and when we release the ruler, and the initial velocity with which the ruler enters the photogate because we dropped it from a different height.

We can examine the position of the picket fence over time using the DataStudio software. In the Displays panel (located at the bottom left of the DataStudio interface), double click graph, then select Position -> Run #1 and click OK.

Looking at the plot, is the position versus time graph a straight line? Plot the other 4 Runs and examine them. Do all the plots have similar shapes?

Thinking about the equations of motion that describe free-falling objects, do you expect the position versus time graphs to be linear? Why or why not?

Using our position time pairs, we can reconstruct the average velocity of the picket fence between each time interval using the average velocity formula from class,

$$v_{avgn} = \frac{\Delta x}{\Delta t} = \frac{x_n - x_{n-1}}{t_n - t_{n-1}}$$

where  $v_{avg,n}$  is the average velocity over the nth interval. If the acceleration due to gravity is a constant, then we know that this average velocity over the time interval is equal to the instantaneous velocity at the midway point of the time interval. Thus, we can use the formula above as the instantaneous velocity if we merely post the velocity at the average time of the time interval. The average velocity for each interval is automatically calculated by the DataStudio software so we can now analyze it.

Now construct velocity versus time plots for each of the 5 runs just as you did for the position versus time. Are these graphs straight lines?

Again thinking about the equations of motion that govern free-falling objects, do you expect them to be straight lines? Why or why not?

For each velocity versus time plot, fit a linear trend to the plotted curve. This draws the best-fit line through the datapoints. To do this for each graph, click the Fit button at the top of each graph window and then select Linear Fit. This will display various information regarding the best-fit line displayed on the graph window.

For a velocity versus time graph, what does the slope represent? Again, think back to the equations of motion that govern free-falling objects.

For each Run, record the slope in the table and calculate the average slope. Then calculate the percent error for your slope using the below equation with the Accepted Value of  $9.80 \text{ m/s}^2$

$$\% \text{ error} = \frac{|\text{Accepted Value} - \text{Experimental Value}|}{\text{Accepted Value}} \times 100$$

	Run 1	Run 2	Run 3	Run 4	Run 5	Average Slope
Slope						

Percent Error:

1. What are the possible sources of random errors in this experiment? How have you attempted to account for them?
2. What are the possible sources of systematic errors in this experiment? Are their effects noticeable? If so, is the error large?

### Procedure (part 2)

1. Use the computer browser to go to: <http://physci.kennesaw.edu/tips/online/DoanneVideo/tossup.html>
2. Close the credits window by pressing on the X then press on the "Take Scale" button to the left of the video.
3. Type 1 m for the known distance, press continue, position the cursor at the bottom black strip of the shown vertical ruler, then press the left mouse button, then position the cursor at the top black strip of the shown vertical ruler, and press the left mouse button (each white strip is 10 cm long). Press continue.
4. Use the advance video button that is available at the bottom right of the video until you reach frame 16.
5. Position the cursor over the tossed ball (make sure to center the ball) then press the left mouse button. Note that this will result in position and time data being recorded and on the video advancing to the next frame. Note that since the video was recorded at 30 frames per second, the software uses that information to figure out the time for each of the frames.
6. Keep on collecting the position data for the ball until you reach frame 41.
7. Position the cursor over the data table, press the right mouse button, select "select all", press the right mouse button again and select copy.
8. Paste the copied data into a Microsoft Excel spreadsheet.
9. Delete the column that corresponds to the x-data since we are interested only in vertical motion.
10. Use Excel to calculate the velocity for each of the time intervals.

### Data Analysis (part 2)

Use Excel to plot the vertical position (y) versus time graph of the tossed ball. Is the position versus time graph a straight line?

Thinking about the equations of motion that describe free-falling objects, does the graph accurately represent the equation? Explain.

Use the graph to identify the different motion components of the ball (moving upwards, reaching the top most position, moving downwards).

Calculate the average velocity for each time interval using the equation given above for average velocity then construct a velocity versus time plot. Is this graph linear?

Again thinking about the equations of motion that govern free-falling objects, do you expect the graphs to be a straight line? Why or why not?

Fit the velocity graph to a linear trend. To do this, position the cursor over the data points, press the right mouse button and select "Add Trendline". Select "Linear", press the "Options" tab then tick "Display equation on chart" option. Press OK.

For a velocity versus time graph, what does the slope represent?

Use the velocity graph to identify the different motion components of the ball (moving upwards, reaching the top most position, moving downwards).

Was the slope of the velocity graph ever zero?

Did the slope of the velocity graph ever change? Explain why or why not?

Does the acceleration of the ball change at anytime?

Calculate the percent error for your slope using the below equation with the Accepted Value of  $9.80 \text{ m/s}^2$ .  
Percent Error:

1. What are the possible sources of random errors in this experiment? How have you attempted to account for them?
2. What are the possible sources of systematic errors in this experiment? Are their effects noticeable? If so, is the error large?