Archimedes Principle

History

Archimedes of Syracuse (c. 287 BC – c. 212 BC) was a Greek mathematician, physicist, engineer, inventor, and astronomer. Although few details of his life are known, he is regarded as one of the leading scientists in classical antiquity. Among his advances in physics are the foundations of hydrostatics, statics, and an explanation of the principle of the level. He is credited with designing innovative machines, including siege engines and the screw pump that bears his name.

The most widely known anecdote about Archimedes tells of how he invented a method for determining the volume of an object with an irregular shape. According to Vitruvius, a crown had been made for King Hiero II, who had supplied the pure gold to be used, and Archimedes was asked to determine whether some silver had been substituted by the dishonest goldsmith. Archimedes had to solve the problem without damaging the crown, so he could not melt it down into a regularly shaped body in order to calculate its density. While taking a bath, he noticed that the level of the water in the tub rose as he got in, and realized that this effect could be used to determine the volume of the crown. For practical purposes water is incompressible, so the submerged crown would displace an amount of water equal to its own volume. By dividing the mass of the crown by the volume of water displaced, the density of the crown could be obtained. This density would be lower than that of gold if cheaper and less dense metals had been added. Archimedes then took to the streets naked, so excited by his discovery that he had forgotten to dress, crying "Eureka!" (Greek: "εὕρηκα!," meaning "I have found it!"). The test was conducted successfully, proving that silver had indeed been mixed in.

The story of the golden crown does not appear in the known works of Archimedes. Moreover, the practicality of the method it describes has been called into question, due to the extreme accuracy with which one would have to measure the water displacement. Archimedes may have instead sought a solution that applied the principle known in hydrostatics as Archimedes' principle, which he describes in his treatise *On Floating Bodies*. This principle states that a body immersed in a fluid experiences a buoyant force equal to the weight of the fluid it displaces. Using this principle, it would have been possible to compare the density of the golden crown to that of solid gold by balancing the crown on a scale with a gold reference sample, then immersing the apparatus in water. The difference in density between the two samples would cause the scale to tip accordingly. Galileo considered it "probable that this method is the same that Archimedes followed, since, besides being very accurate, it is based on demonstrations found by Archimedes himself.

We can write Archimedes principle as: $F_{bouyant} = mg$, where m is the mass of the displaced water. Since density (ρ) is equal to mass/volume, we can also write the principle as: $F_{bouyant} = (\rho_{water})Vg$, where V is the volume displace and the density of water is given by ρ_{water} = 1 gram/milliliter.

Part One – Density of Metal

For every basic measurement we make, there is an associated uncertainty. For a measurement coming from a ruler the uncertainty is roughly equal to the smallest notch on the ruler; i.e. if the smallest notch on the ruler is one mm = 0.1 cm, then a measurement of 40 cm should REALLY look like:

Distance = 40.0 cm ± 0.1 cm   or Distance = 0.400 m ± 0.001 m

Similarly, for mass measurements the uncertainty is roughly equal to the smallest digit we can read out on the scale. For the graduated cylinder the uncertainty is given directly on the cylinder itself.

Let us first use the equipment to measure the density of the metal weights found in the common items bin, as well as its experimental uncertainty. The density is given by:

$$\rho = m/V \qquad \text{(Equation 1)}$$

Choose the largest weight which will fit inside the tall narrow graduated cylinder. Measure the mass of the weight on the scale, and record the data here, along with its uncertainty:

Mass =   Uncertainty in mass =

Now measure the volume of the weight by filling the graduated cylinder with water until it is halfway full. Record the volume of water before and after submerging the weight into the graduated cylinder using the string. The difference between these two is the volume of the weight itself.

Volume before: Volume after:

Volume of weight:

The uncertainty in the volume measurement is given on the graduated cylinder itself. Record the uncertainty in the volume measurement below:

Uncertainty in volume measurement:

We now have the volume of the weight; you can plug this volume into equation (1) to find the density of the weight. Record the density in the space below:

In order to deal with the uncertainties in the density equation and find the uncertainty in the overall density we use the formula:

$(\text{Density Uncertainty}/\text{Density})^2 = (\text{Mass Uncertainty}/\text{Mass})^2 + (\text{Volume Uncertainty}/\text{Volume})^2$

Think of this as a special Pythagoras-type ($a^2 + b^2 = c^2$) rule for combining the uncertainties of two measurements into one combined uncertainty. Use this formula to calculate the uncertainty in the density of the weight in the space below:

Once you have the calculated the measured density of the weight AND its measured uncertainty you can compare it to a table of densities of standard metals (Table 1).

Substance	Density (grams/milliliter)
Aluminum	2.70
Brass	8.40
Copper	8.92
Gold	19.3
Iron	7.86
Lead	11.3
Platinum	21.4
Silver	10.5
Tin	7.30
Uranium	19.1

Which metals have densities which are consistent (i.e. are within the margins of error) of your density measurement?

Which metals have densities which are inconsistent with your density measurement?

Part Two – Bouyant Force

Now we will use Archimedes principle to make a measurement of 'g', the acceleration due to gravity on the surface of the earth. We will use the formula:

$g = F_{bouyant} / m_{displaced}. = (F_{non\text{-}submerged} - F_{submerged}) / (\rho_{water} \cdot V)$ (Equation 2)

where $F_{non\text{-}submerged}$ and $F_{submerged}$ are the forces measured by the spring with the weight out of the water and submerged in the water, V is the volume of the weight, and ρ_{water} is given as 1000 kg/m^3.

The difference between the two forces is the buoyant force. Record the two measured forces below along with their uncertainties:

Now you can record $F_{bouyant}$ along with its measured uncertainty. For the uncertainty in $F_{bouyant}$: take the uncertainty from either the $F_{non\text{-}submerged}$ or $F_{submerged}$ measurements (they should be the same in any case).

$F_{bouyant}$ =

Now use equation 2 to calculate g in the space below. Use the volume you measured in Part One; it is best to convert the volumes to cubic meters by remembering that $(1\ m)^3 = (1 \times 100\ cm)^3 = 10^6\ cm^3 = 10^6$ milliliters.

Using the method developed in part 1, take the uncertainty in the mass and buoyant force measurements and find the uncertainty in the measurement of g from the uncertainties in the force and volume measurements in the space below. Don't worry about uncertainty in the density of water; for this lab we will take it to be exactly 1 gram/milliliter or 1000 kg/m^3.

Is your result consistent with the normally accepted value of 9.8 m/s^2?

Part Three –Densities of Floating Objects

Now we will use the Archimedes Principle to measure the density of a floating object. Use the corks provided, along with the short, wide-mouthed graduated cylinder. Don't use the tall narrow-mouthed cylinder or the cork may become lodged inside.

First use the scale to measure the masses of various corks, along with their uncertainties. Use 3 corks of different sizes, and be sure to include the largest cork in your measurements.

Mass of cork 1 (and uncertainty) =

Mass of cork 2 (and uncertainty) =

Mass of cork 3 (and uncertainty) =

Now use the graduated cylinder to measure the volumes of the corks, along with their uncertainties. You will have to use your finger or a pen/pencil to completely submerge the cork in each case. Remember to include the uncertainty in the volume measurement.

Volume of cork 1 (and uncertainty) =

Volume of cork 2 (and uncertainty) =

Volume of cork 3 (and uncertainty) =

Using the formulae from Part One, calculate the densities and the uncertainties in the density measurement for the three corks in the space below:

What happens to the uncertainty in the density measurement as the size of the cork increases?

The density of cork is usually between 0.2 and 0.25 g/ml. Are your measurements consistent with this range?

Part Four –Bouyant Forces on Floating Objects

Now we will take the largest cork and measure the buoyant force on it while it is floating in the water. Take the large graduated cylinder and measure the volume of the water before and after placing the largest cork in it.

Volume without cork: Volume with cork:

Record the difference in these two volumes below, along with the uncertainty from the graduated cylinder:

Difference in volumes: Uncertainty:

The buoyant force on the cork is given by:

$F_{bouyant} = (\rho V)\, g$ (Equation 3)

where ρ is the density of water and V is the volume of the displaced water (the difference in volumes measured above. Taking $\rho = 1000\ kg/m^3$ and $g = 9.8\ m/s^2$, use Equation 3 to calculate $F_{bouyant}$:

For the error in $F_{bouyant}$ take the same relative error as you had in the volume measurement:

(Uncertainty in $F_{bouyant}$) / ($F_{bouyant}$) = (Uncertainty in Volume)/(Volume of displaced water)

Calculate the uncertainty in $F_{bouyant}$ in the space below:

For a cork which is floating on the surface, the upward force ($F_{bouyant}$) should be equal in magnitude to the downward force of gravity on the cork (mg). Using the mass of the cork measured in part 3, calculate the force of gravity on the cork in the space below:

Are the measurements of the two forces consistent with each other?