Standing Waves

Name:	Teammates:

Introduction

Fig. 1: Tacoma Narrows Bridge

For a spring mass system or a pendulum, when release from a position away from the equilibrium position, the position, velocity, and acceleration of the mass are all described by single sinusoidal functions.

$$x(t) = A\cos(\omega t + \varphi)$$
$$v(t) = -A\omega \sin(\omega t + \varphi)$$
$$a(t) = -\omega^2 x(t) = -\omega^2 A\cos(\omega t + \varphi)$$

Because the motion repeats at one particular frequency, called harmonic, the system is referred to as a simple harmonic oscillator (SHO). It is noteworthy that the motion of a great many systems, such as the shock absorbers in cars or a wrecking ball on the end of a crane, can be modeled fairly well by SHO.

This is not to say that all systems that oscillate can by modeled by a single SHO. Take, for instance, the motion of a pinecone on the end of a branch of a tree. The small branch to which the cone is attached operates very much like a pendulum, which is a SHO. However, this branch is attached to larger branches, which are themselves attached to the tree trunk. All of these branches and trunks by themselves operate like SHO's. When coupled together to form a tree, the motion of the cone will be quite complicated, as each SHO will have its own distinct frequency at which it will want to oscillate. Our arms act in a somewhat similar manner to this. The forearm and the humerous operate as two pendula that are attached at the elbow, albeit with the limitation that the forearm is not free to swing completely backward relative to the humerous due to bicep tendons.

Driven Oscillators and Resonance

Of course, the effects of friction and drag will dampen any oscillatory behavior. Some systems have very small retarding forces acting upon them, and will oscillate for a long time without much decay in their motion others will quickly stop oscillating after they are pulled from equilibrium. This dampening effect can be counteracted if the force that starts the oscillation of the system is also periodic. Of particular importance is when this "driving" force is operating at a frequency equal to the frequency at which the system naturally likes to oscillate. Any person who has ridden a swing knows this. You can go higher and higher on the swing if someone pushes you from behind at the exact same frequency as the natural oscillation frequency of the swing. In some cases, a similar effect can be achieved by driving the system at a frequency that is an integer multiple of the natural frequency.

Such a system that is driven at its natural frequency is said to be in resonance. If the size of the driving force is large enough compared to the damping forces, the amplitude of the oscillation will continually grow as the force "builds" upon its own actions. If the amplitude gets too large, one of several things can happen. For one, the natural frequency of the oscillation can change, which will cause the amplitude to stop growing as the system is no longer in resonance. Another thing that can happen is that the damping forces increase to a point where they exactly offset the driving force, causing the amplitude to settle down to a given value. Lastly, the amplitude of the oscillation can get so large that the system breaks. A great example of this was the collapse of the Tacoma Narrows Bridge in 1940. The bridge, which was prone to vibrating up and down, began oscillating violently one day when it was driven into resonance by winds. Eventually, the entire bridge collapsed.

Strings

This week's activity looks at a system that is very similar to the Tacoma Narrows Bridge: a taut string. Both of these systems involve an extended, thin mass that is stretched between two supports. At equilibrium, the mass forms a straight line between the supports. If the mass is pulled away from equilibrium, there will be a restoring force to bring it toward equilibrium. The wave speed in the string is then given by

$$v = \sqrt{\frac{T}{m/L}}$$

where T is the tension in the string and m is the mass of the string and L its length.

In our system, the string is fixed at each end to the equilibrium point. This causes the waves to reflect off of the ends. If there is no forcing in the system, damping will cause the waves to eventually die out and the string eventually returns to equilibrium. If the system is forced at a particular frequency, it will create waves of a particular wavelength. If the wavelength is not a half-integer multiple of the length L of the string, then the waves going in both directions will constructively and destructively add in a constantly changing fashion and nothing special is observed. However, under the correct forcing frequencies, resonant waves can build up on the string where half of the wavelength of the waves is an integer multiple of the length of the string. The waves give the appearance of the string not moving much, thus the name "standing wave". For this condition, the relationship between the length of the string and the wavelength of the waves is

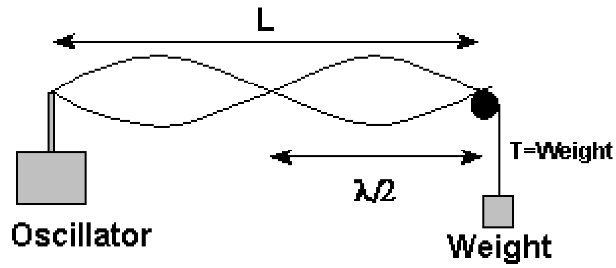

Fig. 2: Diagram of string system

$$L = \frac{n\lambda_n}{2}\text{, where n = 1,2,3,.....}$$

Since v = $f\lambda$, the frequency at which the string must be forced is given by

$$f_n = \frac{nv}{2L}$$

where v is the velocity of the waves and n = 1,2,3,.....

Procedure

The apparatus should be set up as in Figure 2. The weights on the end permit the tension in the string to be changed. By changing the frequency of the oscillator, you will be able to find a setting at which standing resonnant waves are set up in the string system. Since this is not a perfect theoretical system, there will be a range of frequencies over which a resonant condition has apparently been established. However, the correct resonant frequency will be the one at which the amplitude of the waves is at a maximum.

1. Connect the input to the oscillator to the "OUTPUT" of the Pasco interface.
2. Start up the DataStudio software, and open the file "P11_standing".
3. Make sure that the Signal Generator is set for 2 V and 5 Hz.
4. Start with a .75 m length of string and 5.0 gram mass on the end of the string.

Mass per unit length of string (M/L) = ________ kg/m

5. Click the On button on the Signal Generator, which should cause the oscillator to begin working. Change the frequency until you establish a resonant condition and then count the number of antinodes. If the number is greater than 1, lower the frequency until a new resonant condition has been established at which only 1 antinode exist. This is the lowest resonance frequency for this system. Record this frequency.
6. Beginning at this frequency, increase the frequency of oscillation until you have found the first 4 resonance frequencies.
7. After this, increase the mass to 10.0 grams and again find the first 5 resonant frequencies.

Mass	Length	f_1	f_2	f_3	f_4	Theoretical f_1	Percent error in f_1

8. Return the weight back to 5 grams and increase the string length to 1.50 m. Find the first 4 resonant frequencies.

Mass	Length	f_1	f_2	f_3	f_4	Theoretical f_1	Percent error in f_1

9. Answer the following questions.

A. What are the random errors in this experiment? Do they appear to be large or small?

B. What are the systematic errors in this experiment? Are your results consistent with these errors?

C. What is the relationship between the fundamental frequency (f_1) and the harmonics (f_2, f_3 and f_4)?

D. What is the effect of changing the length of the string on fundamental frequency (f_1)?

E. What is the effect of changing the tension of the string on fundamental frequency (f_1)?

F. How would the standing wave pattern be different if one end of the string was free to move (not bound)?