



## Mass-Spring Motion

Name:

Teammates:

### Introduction

Consider the case of a mass on a frictionless table that is attached to the end of a spring (Fig 1). If the mass is displaced by a very small amount so that the spring is either compressed or extended, the force it experiences is given by:

$$F = -kx$$

where  $x$  is the displacement of the mass from the equilibrium position and  $k$ , the spring constant. The spring constant is determined by such factors as the type and thickness of material used in the spring. This equation has two very distinctive qualities. The first of these has to do with the direction of the force. If the mass is displaced in the positive  $x$ -direction, then the force is in the negative direction. If the mass is displaced in the negative  $x$ -direction, then the force becomes positive. Therefore, at all times, the force of the spring on the mass is toward the equilibrium position of  $x = 0$ . The second of these has to do with the magnitude of the force. As the mass is displaced a further distance from equilibrium, the force increases in a linear fashion. This means that the strongest force on the mass will be experienced while the mass is farthest from equilibrium and the weakest when it is near the equilibrium point. Hence, we expect the velocity to be changing the most while the mass is far away from equilibrium and very little as it is passing through the equilibrium point. The result of this is that the position of the mass will change as a function of time in a sinusoidal manner,

$$x = x_{\max} \sin\left(\frac{2\pi t}{T} + \psi\right)$$

where  $T$  is the period of oscillations,  $x_{\max}$  corresponds to the greatest distance that the mass moves from equilibrium and  $\psi$  is what is called the phase angle.  $\psi$  is determined by the initial speed and position of the mass when it is released.

In this lab activity, we will focus on the period of oscillation of a mass on a spring that is hanging vertically (Fig 2). In this case, two forces act on the mass along the direction of motion, the spring force and the gravitational force. However, the effect of the gravitational force will be to just create a new equilibrium position, we have:

$$mg = ky_0$$

where  $m$  is the mass attached to the spring,  $g$  is the gravitational field constant,  $k$  is the spring constant, and  $y_0$  is the amount by which the spring has stretched due to  $m$ .

Note that we can find the spring constant  $k$  of a spring by attaching a variety of masses and measuring their corresponding  $y_0$ .

If we move the mass either upward or downward from this new equilibrium position, it will oscillate. The only difference to the equation for the position of the mass for a horizontal spring is that the displacement in this case should be measured relative to the new equilibrium position.

### Activity

For this activity, you will need a set of small cylindrical masses, four equal length springs, one table stand (with a pendulum attachment), and a timer.

1. For two of the provided small springs, measure the spring elongation ( $\Delta y$ ) due to various masses. **(Don't attach masses that will stretch the spring beyond its elasticity limit):**

#### Spring 1:

Hanging mass						
$\Delta y$						

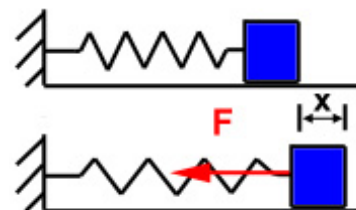


Fig 1: Mass-spring system

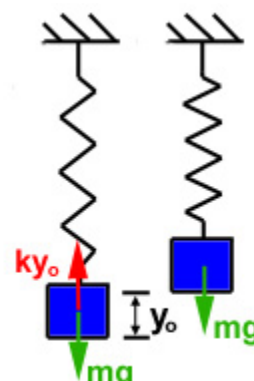


Fig 2: Vertical spring

**Spring 2:**

Hanging mass						
$\Delta y$						

2. Graph the obtained data, and use the graphs to obtain the spring constant  $k$  for both springs:

$k$  spring 1: \_\_\_\_\_ (N/m)

$k$  spring 2: \_\_\_\_\_ (N/m)

3. In addition to spring constant, determine two possible factors that might affect the value of the period of a spring-mass system:

Factor 1: Spring Constant Factor 2: \_\_\_\_\_ Factor 3: \_\_\_\_\_

4. For both springs, hang them vertically, pull them away from the equilibrium position, and measure their period of oscillation (**Don't stretch the spring beyond its elasticity limit – Remember: to measure the period, time 20 oscillations then use that data to determine the period**):

Spring constant		
Mass used		
Period (T)		

5. From this limited amount of data, what is the apparent relationship between “spring constant” and the period  $T$ ?

6. Discuss how you would test the effect of “Factor 2” on the period of the spring-mass system.

7. Using the provided supplies, run the experiment to test you hypothesis. Collect data, graph it.

Factor 2						
Period (T)						

8. If your graph does not result in a linear relationship, graph alternative relationships like  $T$  vs.  $(\text{factor } 3)^2$ ,  $T$  vs.  $(\text{factor } 3)^{0.5}$ ,  $T$  vs.  $1/(\text{factor } 3)$ , or  $T$  vs.  $1/(\text{factor } 3)^2$ .
9. What is then the relationship between “factor 2” and the period  $T$ ?

10. Discuss how you would test the effect of “Factor 3” on the period of the spring-mass system.

11. Using the provided supplies, run the experiment to test you hypothesis. Collect data (making sure to take the average of several measurements), graph it.

Factor 3						
Period (T)						

12. If your graph does not result in a linear relationship, try to identify the relationship by graphing alternative relationships.
13. What is then the relationship between “factor 3” and the period  $T$ ?

A. What do you conclude, what are the factors that affect the period of a spring-mass system?

B. Knowing that the theoretical period of a spring is given by:

$$T = 2\pi\sqrt{\frac{m}{k}}$$

Does your data agree with that equation?

C. How would you use the data that you have collected in testing factor 1 or factor 2 to find the spring constant k?

D. What are the sources of error in this experiment?