Newton's Second Law

Introduction

One way to study the dynamics of a system in which there is motion is to measure the kinematics of the system and see if there is any acceleration. If there is acceleration, this implies that there is a net (unbalanced) force on some part of the system. If there is no acceleration, then this means that either there are no forces on the system, or that the forces within the system are balanced.

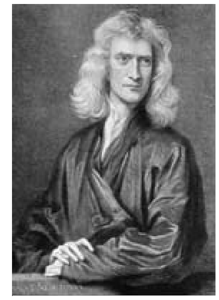


Fig. 1: Newton (*NASA*)

Newton

Sir Isaac Newton was not the first person to study and measure the motion of objects. As we stated in previous activities, there have been others like Aristotle and Galileo who did this before him. Nor was he the first person who tried to model motion with equations that would allow for predictive behavior. However, he was the first person to clearly state the basic laws of motion that allow us to analyze all systems. These three laws, known as Newton's Laws of Motion, are

1. An object at rest or in a state of constant motion will remain in that state until acted upon by an unbalanced force.
2. The net force on an object is proportional to its acceleration, with the proportionality constant being called the mass, i.e. $\vec{F}_{net} = m\vec{a}$.
3. For every force on an object, there is an equal and opposite force produced by the object but acting on the object that has originated the first force.

The first of these laws is a restatement of one of Galileo's discoveries. While seemingly obvious, especially to us now, it is a very powerful statement. It says that an object that is noticed to be accelerating must have a net force acting upon it, even if the manner of the force is not readily noticeable. For instance, if one were to swing a rock on the end of a string in a circle, it would be obvious that the acceleration of the rock was due to the force exerted on it by the string, which is being accelerated by the hand holding it. However, what of an electron that is above the Earth's atmosphere that also moves in a circular orbit? Since it is accelerating, we know that there must be some net force on it, even though there is nothing in contact with it (it turns out that the force operating on it is electromagnetism). Or what of a distant star that is going around in an elliptical orbit, seemingly by itself? We know that some other object must be applying a force on it, and by studying its motion, we might find its "neighbor" that is causing it to move thusly, even if its "neighbor" is a black hole that cannot be observed directly.

The second law is often quoted as only $\vec{F}_{net} = m\vec{a}$, but this simplified version misses out on the fact that this law defines the term mass, which is one of the more misunderstood terms in science. Many people confuse mass for weight, which is understandable since even scientists still persist in saying silly things like "one kilogram is equal to 2.25 pounds". But a fuller description of Newton's Second Law shows that the mass is the proportionality constant between the applied force and the resulting acceleration. Weight is merely the force that comes about because the gravitational attraction between two objects. It is only important to know this when this one type of force is present. Mass, on the other hand, is always an important factor, as it will determine the acceleration when any type of force is present.

Possibly more misunderstood is the last of Newton's laws. It is used in many contexts where it should not, like economics, politics, and advertising, and is almost always used inappropriately. Even when it is used in the context of physics and the dynamics of objects, it is often misused. The reason is that the agents and receivers of the two forces involved in a situation are often left out, as when we state it merely as "For every force, there is an equal and opposite force." If one does not realize that these two forces act on different objects, then they might not think that acceleration is never possible, as there will never be a net force if every force creates an equal opposing force.

Theory and Model

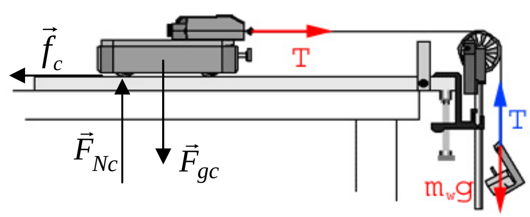
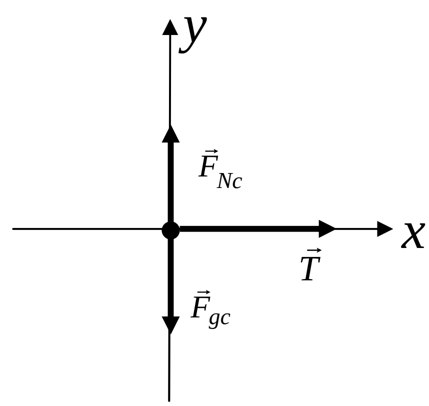


Fig. 2: Diagram of lab setup

In this week's activity, we are going to test the validity of Newton's Second Law using a cart attached to weights via a pulley (Figure 2). Because gravity is allowed to operate on the weights, they will supply a force to the hanging weight that is equal to $\vec{F}_{gw} = m_W \vec{g}$. This force will result in a force acting on the cart via the tension T in the string. If the pulley is lossless (frictionless) the tension in the string will be the same at both ends of the pulley and the hanging mass and the cart will accelerate at the same magnitude. The forces acting on the cart are the gravitational pull of the earth on the cart, which is often called the weight of the cart ($\vec{F}_{gc}$), the normal force exerted by the track on the cart ($\vec{F}_{Nc}$), the pull of the string $\vec{T}$, and the friction between the cart and the track ($\vec{f}_c$). The design of the cart and track are such that the friction is quite small and can be neglected. Using m_c for the mass of the cart and $\vec{a}_c$ for the acceleration of the cart, we can write Newton's 2nd law of motion for the cart as:



$$\vec{T} + \vec{F}_{gc} + \vec{F}_{Nc} = m_c \vec{a}_c$$

Since we cannot add vectors when they are not in the same direction, we write this equation along the x and y directions:

Along the x-direction (horizontal): $\quad T_x + F_{gcx} + F_{Ncx} = m_c a_{cx}$

Along the y-direction (vertical): $\quad T_y + F_{gcy} + F_{Ncy} = m_c a_{cy}$

To be able to solve these equations, we do two things. The first is to examine what is actually happening to the cart, and how is it accelerating. The second is to try to identify the components of each of the forces acting on the cart.

In examining the motion of the cart, we can note that it is moving only along the horizontal. This leads us to realize that $a_{cy} = 0$.

To identify the components of each of the forces, we draw a free body diagram. That is a coordinate system with all forces acting on the cart drawn with their tails at the origin. The free body diagram for the cart is shown in Fig. 3. We now rewrite the equations for the vertical and horizontal based on the information we can get from the free body diagram:

Along the x-direction (horizontal): $T + 0 + 0 = m_c a_c$ (eq. 1)

Along the y-direction (vertical): $0 - m_c g + F_{Nc} = 0$ (eq. 2)

In the case of the hanging weight, the only forces acting on it are the gravitational pull of the earth on the weight ($\vec{F}_{gw}$), the pull of the string $\vec{T}$. Newton's second law of motion is then:

$$\vec{T} + \vec{F}_{gw} = m_w \vec{a}_w$$

Since the motion and all the forces acting on the hanging weight are along the vertical, we write this equation along the vertical:

Along the y-direction (vertical): $T - m_w g = m_w a_w$

Since the magnitude of acceleration of both objects is the same, and since the hanging weight in accelerating downwards, we replace a_w by ($- a_c$). Consequently:

Along the y-direction (vertical): $T - m_w g = -m_w a_c$ (eq. 3)

By combining equations 1, 2, and 3, you should be able to (do it) find: $a_c = \dfrac{m_w g}{m_w + m_c}$

During the activity, we will use different masses of weights and measure the corresponding velocities of the cart. If friction is negligible, the plot of the velocity of the cart versus the time should yield a straight line whose slope, the acceleration, is given by the above equation.

Procedure

Figure 2 shows the set up for this activity. We will be applying a constant force to a dynamics cart to pull it across the track. The constant force will be provided by weights that are attached to a string that is suspended over a pulley. As gravity pulls these weights downward, it will pull the cart forward. This motion will be measured and transferred to the computer by a photogate built into the pulley.

1. Make sure the track is level by adjusting the levelling screws until the cart remains at rest when positioned on the track.
2. Measure the mass of the dynamics cart.

   Mass of the Cart = ________ g Mass of the bar weight = _______ g

3. Connect one end of the string to the dynamics cart and the other to a mass hanger. Place the cart on the track and run the string over the pulley. Pull the cart as far back along the track so that the mass hanger almost touches the pulley. Hold the hanger in place above the ground to prevent the system from moving. Place masses on the mass holder until there is a total of 50 gm.
4. Turn the pulley so that the photogate beam of the Smart Pulley is "unblocked" (light-emitting diode (LED) on the photogate is off).
5. Click the "REC" button to begin data recording.
6. Release the glider so it can be pulled by the falling mass hanger. Data recording will begin when the SP photogate is first blocked.
7. Stop the data recording just before the mass hanger reaches the floor by clicking the "STOP" button. Have someone in the group stop the glider from rebounding while the "STOP" button is pushed.
8. Change the mass on the hanger and repeat the data acquisition procedures for total masses in 50 gram increments up to 250 grams.

	Run 1	Run 2	Run 3	Run 4	Run 5
Total Cart Mass					
Hanging Mass					

9. After this, add the bar weights to the cart and repeat the above procedure starting at 50 grams on the hanger

	Run 6	Run 7	Run 8	Run 9	Run 10
Total Cart Mass					
Hanging Mass					

Data Analysis

From the computer, we will have tables of times versus velocity for each run. Since the acceleration should be constant, a plot of velocity versus time should yield a straight line whose slope is the acceleration. Record these slopes on the activity sheet below, and then compare them with the theoretical acceleration using the above equation.

	Run 1	Run 2	Run 3	Run 4	Run 5	Run 6	Run 7	Run 8	Run 9	Run 10
Measured Acceleration										
Theoretical Acceleration										
Percent Eror										

For each of the data runs, determine the hanging weight F_g (in Newtons), the tension in the string T (in Newtons) and the net force acting on the cart.

	Run 1	Run 2	Run 3	Run 4	Run 5	Run 6	Run 7	Run 8	Run 9	Run 10
F_g (Newtons)										
T (Newtons)										
Net Force (Newtons)										

1. How does the net force and the hanging weight F_g compare? Explain.

2. How does the net force and the tension in the string T compare? Explain.

3. What are the possible sources of random errors in this experiment? How have you attempted to account for them?

4. What are the possible sources of systematic errors in this experiment? Are their effects noticeable? If so, is the error large?

5. During the derivation of the acceleration equation, the frictional force between the cart and the track were neglected. Does this appear to be a valid approximation for the experiment? Are there certain experimental configurations where this approximation becomes less valid? Physically explain why or why not.

6. How would the experimental results have changed if the cart was moved from a level track to one with an uphill gradient?