## Objectives

There are several objectives to this lab exercise. The first objective is to learn a formal method for dealing with uncertainties in a physics calculation. The second objective is to learn to measure the velocity and acceleration of a moving object with their respective uncertainties. The third objective is to use an inclined plane to measure 'g', the acceleration due to gravity; at the end of the lab you will compare your value of 'g' and its uncertainty to the accepted value.

The equipment you will need for this lab is: a pinball and track, a stopwatch (any device that can record time down to tenths of a second will work), one measuring tape or ruler (the ruler on the track will work or you can use one of the meter sticks at the back of the lab), a set of calipers, and a set of shims to incline the track. You will also need a large earth-sized planet under your feet to produce the gravitational field that will move the cart down the inclined plane; this has been provided for you.

## Part I - Measuring velocity

The tracks have a ruler attached to them; we will use the groove on the track which is closest to the ruler to roll the ball.

Method: Place the pinball in the groove at the edge of the track furthest from the bumper. Measure the distance from the pinball to the bumper end of the track and call this distance "Track Distance".

Record the distance here: Track Distance = _____

Now give the pinball a light push so that it rolls down the track towards the bumper and using the stopwatch record the time it takes to reach the bumper; call this time "Time of Motion". It's best to do the time measurement three times and take the average.

Record the times: Time of Motion 1 = _____
Time of Motion 2 = _____
Time of Motion 3 = _____
Average Time of Motion = _____

Now calculate the velocity of the pinball by dividing the Track Distance by the Average Time of Motion and record the results below:

What units does the velocity measurement have?

What do you think some sources of uncertainty might be in this part of the experiment?

## Part II - Propagation of Uncertainties

The final question in Part I asks you to think about uncertainties in a very informal and qualitative way. Now we will make a first attempt at QUANTIFYING uncertainties.

For every basic measurement we make, there is an associated uncertainty. For a measurement coming from a ruler the uncertainty is roughly equal to the smallest notch on the ruler; i.e. if the smallest notch on the ruler is one mm = 0.1 cm, then a measurement of 40 cm should REALLY look like:

Distance = 40.0 cm ± 0.1 cm    or Distance = 0.400 m ± 0.001 m

The 0.1 cm is the uncertainty in the measurement of 40 cm. Similarly for a time measurement, a person may only be able to make a measurement with a stop watch down to the nearest 0.2 s, in this case a time measurement of 2.4 s should REALLY look like:

Time = 2.4 s ± 0.2 s

When combining these to form a velocity we obtain:

Velocity = Distance / Time = (0.400 m ± 0.001 m)/(2.4 s ± 0.2 s)

In order to deal with the uncertainties in the equation and find the uncertainty in the overall velocity we use the formula:

(Velocity Uncertainty/Velocity)$^2$ = (Distance Uncertainty/Distance)$^2$ + (Time Uncertainty/Time)$^2$

Think of this as a special Pythagoras-type ($a^2 + b^2 = c^2$) rule for combining the uncertainties of two measurements into one combined uncertainty. In the example above

(Velocity Uncertainty/Velocity)$^2$ = (0.001 m / 0.400 m)$^2$ + (0.2 s/ 2.4 s)$^2$
                                                        = 0.000006 + 0.007 ≈ 0.007
(Velocity Uncertainty/Velocity) =  √(0.007) = 0.083

And since Velocity = Distance / Time = 0.400 m /2.4 s = 0.17 m/s
Then Velocity Uncertainty = 0.083 × (Velocity) = 0.083 × 0.17 m/s = 0.01 m/s
And we can express the final velocity with its uncertainty as v = 0.17 m/s ± 0.01 m/s

You can apply this method of propagation of errors to your velocity measurement from Part I.

What do you think the uncertainty is in your distance measurement? Record it below:
Uncertainty in Track Distance:_____

What do you think the uncertainty is in your time measurement? Record it below:
Uncertainty in Time of Motion:_____

Now use the method described on the previous page to calculate the velocity you measured in Part I **with its uncertainty**.

Does this uncertainty seem reasonable? If the pinball were moving faster what happens to the relative uncertainty in the velocity measurement?

## Part III – Acceleration of an object down an Inclined Plane

Now we will tilt the apparatus using the shims and try rolling the pinball down the groove in the track. In order to tilt the track, use the shims provided and place one shim under each side of the track. Use the calipers to measure the heights of the shims; if you don't know how to use Vernier calipers please ask your instructor. Record the results below:

Height of shim 1:_____
Height of shim 2:_____
Average height of shims:_____

What do you think the uncertainty is on the average height of the the shims? Record it below:

Uncertainty on average height of shims:_____

Now place the shims side by side under the side of the track furthest from the bumper. Place the pinball at the top end of the groove beside the ruler. When you release the pinball from this point it how far does it roll until it reaches the end of the groove?. What do you think the uncertainty is on this distance? Record these below:

Distance of motion of pinball:_____
Uncertainty in distance of motion:_____

Now release the pinball and use the stop watch to time how long it takes to roll to the bottom of the track. Take three trials and record the times below:

Time of motion 1:_____
Time of motion 2:_____
Time of motion 3:_____
Average time of motion:_____

What do you think the uncertainty is on this average time of motion? Record it below:

Uncertainty in average time of motion:_____

In Parts I and II of the lab the cart moved at a roughly constant velocity along the track. But in this case the pinball starts from rest and picks up speed as it rolls down the track; this type of motion is called uniform acceleration. We will discuss acceleration in greater detail later in the class, but for now you can use the formula:

Acceleration = 2 × (Distance of Motion) / (Time of Motion)$^2$

Calculate your acceleration below:

What are the units of your acceleration measurement?

To calculate the uncertainty in your acceleration measurement we will use a method similar to that discussed in Part II, with the modification that the relative uncertainty in the time measurement will count twice as much as the relative uncertainty in the distance measurement. This is because time is squared in the formula for acceleration. The formula for calculating the uncertainty in the acceleration will look like this:

$$\text{(Acceleration Uncertainty/Acceleration)}^2 = \text{(Distance Uncertainty/Distance)}^2 + 2 \times \text{(Time Uncertainty/Time)}^2$$

Use this formula to calculate the uncertainty in your acceleration measurement in the space below:

Does this uncertainty seem reasonable?

How does the uncertainty in the acceleration measurement compare with the uncertainty in the velocity measurement? Is it more or less accurate (as a percentage error)?

Now repeat the acceleration measurement using two shims (i.e. on shim on top of the other) under each side of the track. As before release the pinball and use the stop watch to time how long it takes to roll to the bottom of the track. Repeat three times and record these times and their uncertainties below:

Time of motion 1:_____

Time of motion 2:_____

Time of motion 3:_____

Average time of motion:_____

Uncertainty in average time of motion:_____

You can calculate the new acceleration using the formula:
Acceleration = 2 × (Distance of Motion) / (Average Time of Motion)$^2$
Calculate your new acceleration in the space below:

You can calculate the new uncertainty in the acceleration using the formula:

$$\text{(Acceleration Uncertainty/Acceleration)}^2 = \text{(Distance Uncertainty/Distance)}^2 + 2 \times \text{(Time Uncertainty/Time)}^2$$

Calculate the new uncertainty in the acceleration in the space below:

How does the uncertainty in this measurement compare with the uncertainty in the previous acceleration measurement? Is it more accurate or less accurate (as a percentage error)?

## Part IV – Determine 'g' – the acceleration due to gravity

You can now use your acceleration measurements to determine 'g', the acceleration due to gravity. As we will see later on in the course, for an object sliding down an inclined plane, the acceleration down the plane is equal to the acceleration due to gravity 'g' multiplied by the sine of the angle at which the plane is inclined to the horizontal (see below):
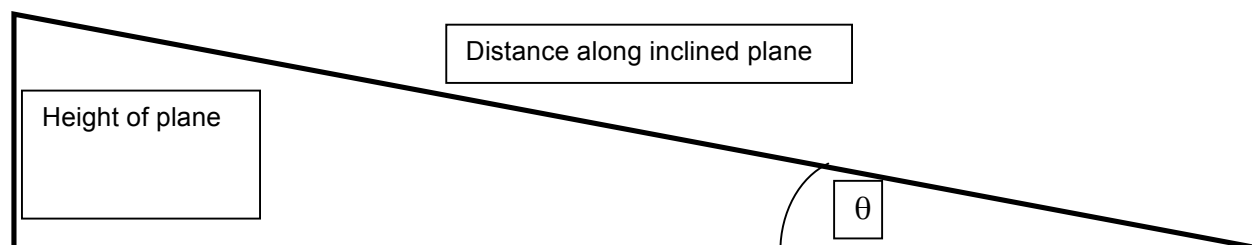


The sine of the angle $\theta$ can be expressed as:      $\sin \theta$ = Height/Distance
Thus, the acceleration down the plane can be written as:      $a = g (\sin \theta) = g \times h / d$
Where 'a' is the acceleration down the plane, 'h' is the height, and 'd' is the distance.
Rearrange this to solve for g:      $g = a \times d / h$

This is the formula for the case of an object sliding down an inclined plane, but in our case we have a rolling sphere so we need to add a factor 7/5 to this equation (the reason for this will be explained when we get to the section on rolling motion in Chapter 10 of your textbook)

Thus for the rolling pinball:      $g = 7/5\ a \times d / h$

Since you have made measurements of 'a', 'd' and 'h' in the previous section, you now can solve for 'g' using the data with only one shim under each side of the ramp. Solve for 'g' in the space below:

Now solve for 'g' using the data you collected with two shims under each side of the ramp in the space below:

How do your two values compare with each other?

How to they compare with the accepted value of g = 9.81 m/s$^2$?

Now calculate the uncertainty in your values of 'g'. You can use the formula:

('g' uncertainty / 'g')$^2$ = ('a' uncertainty / 'a')$^2$ + ('h' uncertainty / 'h')$^2$ + ('d' uncertainty / 'd')$^2$

Use the space below to calculate the uncertainty in your first value of 'g' (from the data with only one set of shims)

Use the space below to calculate the uncertainty in your second value of 'g' (from the data with two sets of shims)

Is the difference between the two values of 'g' you calculated less than the sum of their uncertainties? If so then the measurements can be said to be consistent with each other given the accuracy of the experiment.

Is the difference between your first value of 'g' and the accepted value less than the uncertainty? What about for your second value of 'g'? Are they consistent with the accepted value?

Do you think this is a useful way of measuring 'g'?

Do you know who first performed this experiment?