Assignment 2

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***Abstract*—This document contains the solution for Assignment 2 (ICSE Class 12 Maths 2019 Q.12(a))**

Question 12(a) The volume of a closed rectangular metal box with a square base is 4096 cm^3. The cost of polishing the outer surface of the box is ₹4 per cm^2. Find the dimensions of the box at the minimum cost of polishing it.

Solution. Let the volume of the closed rectangular metal box be V.

Given that the volume of this box is 4096 cm^3.

$$\implies V = 4096cm^3 \tag{1}$$

Given that the box has a square base which means that the breadth and height of the box are equal. Let their value be a.

And let the value of length be b.

In this case, we have

$$V = a^2b \tag{2}$$

From (1) and (2), we have

$$a^2b = 4096 \tag{3}$$

Getting b in terms of a, we get

$$b = \frac{4096}{a^2} \tag{4}$$

In the second part of the question, it is given that the cost of polishing the outer surface of the box is ₹4 per cm^2

Let the total surface area of the box be S.

$$S = 2a^2 + 4ab \tag{5}$$

From (4), we have

$$S = 2a^2 + 4a(\frac{4096}{a^2}) \tag{6}$$

$$= 2a^2 + 4(\frac{4096}{a}) \tag{7}$$

$$= 2a^2 + \frac{16384}{a} \tag{8}$$

$$\therefore S = 2a^2 + \frac{16384}{a} \tag{9}$$

For minimum cost we must have minimum surface area.

Now, for minimum value of surface area, we must have

$$\frac{dS}{da} = 0 \tag{10}$$

$$\frac{dS}{da} = 4a - \frac{16384}{a^2} \tag{11}$$

From (10), we have

$$4a - \frac{16384}{a^2} = 0 \tag{12}$$

$$\implies 4a = \frac{16384}{a^2} \tag{13}$$

$$\implies a^3 = 4096 \tag{14}$$

$$\implies a = 16 \tag{15}$$

Taking the second derivative of S, we get

$$\frac{d^2S}{da^2} = 4 + \frac{32768}{a^3} \tag{16}$$

Clearly (16) is positive for all positive values of a. S has a minimum at a = 16

Put $a = 16$ in (9),

$$S_m = 2(16)^2 + \frac{16384}{16} \tag{17}$$

$$= 2(256) + 1024 \tag{18}$$

$$= 512 + 1024 \tag{19}$$

$$= 1536 \tag{20}$$

$$\therefore S_m = 1536cm^2 \tag{21}$$

Let the cost per unit area be c which is equal to ₹$4 per cm^2$

Let the minimum cost of polishing the metal box be C_m

$$\implies C_m = c \times S_m \tag{22}$$

From (21), we have

$$C_m = 4 \times 1536 \tag{23}$$
$$= 3072 \tag{24}$$

$\therefore$The minimum cost of polishing the metal box is ₹3072

TABLE I
DESIGN TABLE

Variable	**Formula/Value**	**Description**
c	₹4 per cm^2	Cost per unit area,Input
V	$4096cm^3$	Volume, Input
a	-	Breadth, Height
b	$\frac{V}{a^2}$	Length
S	$2a^2 + 4ab$	Total Surface Area
S_m	$\frac{dS}{da} ata = 16$	Output
C_m	$c \times S_m$	Output