AI1110: Assignment 9

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Outline

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Question

The input to the system

$$H(s) = \frac{1}{s^2 + 2s + 5}$$

is a WSS process x(t) with $E\{x^2(t)\} = 10$. Find $S_x(\omega)$ such that the average power $E\{y^2(t)\}$ of the resulting output $y(t)$ is maximum. (*Hint:* $|H(j\omega)|$ is maximum for $\omega = \sqrt{3}$)

Solution

In general

$$E\{y^2(t)\} = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_x(\omega)|H(\omega)|^2 d\omega \tag{1}$$

$$\leq |H(\omega_m)|^2 \frac{1}{2\pi}\int_{-\infty}^{\infty} S_x(\omega) d\omega \tag{2}$$

$$= E\{x^2(t)\}|H(\omega_m)|^2 \tag{3}$$

where $|H(\omega_m)|$ is the maximum of $|H(\omega)|$. In our case ,

$$|H(\omega)|^2 = \frac{1}{(5-\omega)^2 + (2\omega)^2} \tag{4}$$

From the hint, $|H(\omega)|$ is maximum at $\omega = \sqrt{3}$

Solution(Contd..)

Also $|H(\omega_m)|^2 = \frac{1}{16}$. Hence $E\{y^2(t)\} \leq \frac{10}{16}$ with the equality if $R_x(10) = 10\cos\sqrt{3}\tau$