AI1110: Assignment 6

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Outline

Question

Show that the random variables x and y ate independent iff for any a and b:

$$E\{U(a-x)U(b-y)\} = E\{U(a-x)\}E\{U(b-y)\}$$

Solution

$$E\{U(a-x)\} = \int_{-\infty}^{\infty} U(a-x)f(x)dx \tag{1}$$

$$= \int_{-\infty}^{a} f(x)dx \tag{2}$$

$$= F_x(a) \tag{3}$$

Similarly,

$$E\{(U(b-y))\} = F_y(b) \tag{4}$$

$$E\{U(a-x)U(b-y)\} = \int_{-\infty}^{a}\int_{-\infty}^{b} f(x,y)dxdy \tag{5}$$

$$= F_{xy}(a,b) \tag{6}$$

Conclusion

Hence,

$$F_{xy}(a,b) = F_x(a)F_y(b) \tag{7}$$

$$E\{U(a-x)U(b-y)\} = E\{U(a-x)\}E\{U(b-y)\} \tag{8}$$