AI1110: Assignment 10

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Outline

Question

Show that

$$\mathrm{R}_{xy}(\lambda) = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} x(t+\lambda)y(t)dt$$

iff

$$\lim_{T\to\infty} \frac{1}{2T} \int_{-2T}^{2T}(1 - \frac{|\tau|}{2T})E\{x(t+\lambda+\tau)y(t+\tau)x(t+\lambda)y(t)\}d\tau = R_{xy}^2(\lambda)$$

Solution

If $z(t) = x(t+\lambda)y(t)$, then

$$C_{zz}(\tau) = E\{x(t+\lambda+\tau)y(t+\tau)x(t+\lambda)y(t)\}d\tau - R_{xy}^2(\lambda) \tag{1}$$

Now we know that a process x(t) with autocovariance $C(\tau)$ is mean-ergodic iff

$$C_{zz}(\tau) = \frac{1}{2T}\int_{-2T}^{2T} C(\tau)(1 - \frac{|\tau|}{2T})d\tau \tag{2}$$

$$= \frac{1}{T}\int_{0}^{2T} C(\tau)(1 - \frac{|\tau|}{2T})d\tau \longrightarrow_{T\to\infty} 0 \tag{3}$$

The given process is mean-ergodic

$\implies R_{xy}(\lambda) = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} x(t+\lambda)y(t)dt$