AI1110: Assignment 5

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Outline

Question

Consider the following three events: (a) At least 1 six is obtained when six dice are rolled, (b) at least 2 sixes are obtained when 12 dice are rolled, and (c) at least 3 sixes are obtained when 18 dice are rolled. Which of these events is more likely?

Solution (a)

Possible outcomes when a die is rolled once are:

$$\{1, 2, 3, 4, 5, 6\}$$

In the first event, six dice are rolled.
Let $X = i$ denote the event where six occurs i times on the dice
$i \in \{0, 1, 2, ..., n\}$
Let $n = 6$ in this case
Let the probability of obtaining at least one six be $P_n(X \geqslant 1)$ where $n = 6$

$$P_6(X \geqslant 1) = 1 - P_6(X = 0) \tag{1}$$

$$= 1 - {}^6C_0(1-p)^0 p^{6-0} \tag{2}$$

$$= 1 - {}^6C_0(\frac{1}{6})^0(\frac{5}{6})^6 \tag{3}$$

$$= 1 - (\frac{5}{6})^6 = 0.66511 \tag{4}$$

Solution (b)

In the second event, twelve dice are rolled.
Now we have $n = 12$
Let the probability of obtaining at least two sixes be $P_n(X \geqslant 2)$ where $n = 12$

$$P_{12}(X \geqslant 2) = 1 - P_{12}(X \leq 1) \tag{5}$$
$$= 1 - P_{12}(X = 0) - P_{12}(X = 1) \tag{6}$$
$$= 1 - {}^{12}C_0 (\frac{1}{6})^0 (\frac{5}{6})^{12} - {}^{12}C_1 (\frac{1}{6})^1 (\frac{5}{6})^{11} \tag{7}$$
$$= 1 - (\frac{5}{6})^{11} (\frac{17}{6}) = 0.61867 \tag{8}$$

Solution (c)

In the second event, eighteen dice are rolled.
Now we have $n = 18$
Let the probability of obtaining at least two sixes be $P_n(X \geqslant 3)$ where $n = 18$

$$P_{18}(X \geqslant 3) = 1 - P_{18}(X \leq 2) \tag{9}$$

$$= 1 - P_{18}(X = 0) - P_{18}(X = 1) - P_{18}(X = 2) \tag{10}$$

$$= 1 - {}^{18}C_0(\frac{1}{6})^0(\frac{5}{6})^{18} - {}^{18}C_1(\frac{1}{6})^1(\frac{5}{6})^{17} - {}^{18}C_2(\frac{1}{6})^2(\frac{5}{6})^{16} \tag{11}$$

$$= 1 - (\frac{5}{6})^{18} - 18(\frac{1}{6})(\frac{5}{6})^{17} - 153(\frac{1}{)}36(\frac{5}{6})^{16} = 0.59078 \tag{12}$$

Conclusion

Clearly,
$P_6(X \geqslant 1) > P_{12}(X \geqslant 2) > P_{18}(X \geqslant 3)$
From the above inequality, we conclude that the getting at least one six when the die is rolled six times is the most likely.