AI1110: Assignment 3

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Outline

Question

Two groups are competing for the position on the Board of directors of a corporation. The probabilities that the first and the second groups will win are 0.6 and 0.4 respectively. Further, if the first group wins, the probability of introducing a new product is 0.7 and the corresponding probability is 0.3 if the second group wins. Find the probability that the new product introduced was by the second group.

Solution

Let random variables $X, Y \in \{0, 1\}$ denote the following events in Table (1)

Event	Description
$X = 0$	First Group Wins
$X = 1$	Second group wins
$Y = 0$	Group Introduces New Product

Table 1: Description of events

Input probabilities

The following are the input probabilities as given in the question:

Probability	**Value**
$\Pr(X=0)$	0.6
$\Pr(X=1)$	0.4
$\Pr(Y=0\|X=0)$	0.7
$\Pr(Y=0\|X=1)$	0.3
$\Pr(X=1\|Y=0)$	?

Table 2: Input probabilities

Solving..

The required probability is given by:

$$\Pr(X = 1|Y = 0) \tag{1}$$

$$= \frac{\Pr(Y = 0, X = 1)}{\Pr(Y = 0)} \tag{2}$$

$$= \frac{\Pr(Y = 0|X = 1)\Pr(X = 1)}{\sum_{i=0}^{1}\Pr(X = i, Y = 0)} \tag{3}$$

$$= \frac{\Pr(Y = 0|X = 1)\Pr(X = 1)}{\sum_{i=0}^{1}\Pr(Y = 0|X = i)\Pr(X = i)} \tag{4}$$

Answer

On substituting the values from Table (2) we get:

$$\Pr(Y = 0|X = 1) = \frac{0.4 \times 0.3}{0.6 \times 0.7 + 0.4 \times 0.3} \tag{5}$$

$$= \frac{12}{54} \tag{6}$$

$$= \frac{2}{9} \approx 0.222 \tag{7}$$