Assignment 2

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***Abstract*—This document contains the solution for Assignment 2 (ICSE Class 12 Maths 2019 Q.12(a))**

Question 12(a) The volume of a closed rectangular metal box with a square base is 4096 cm^3. The cost of polishing the outer surface of the box is ₹4 per cm^2. Find the dimensions of the box at the minimum cost of polishing it.

Solution. Let the volume of the closed rectangular metal box be V.

Given that the volume of this box is 4096 cm^3.

$$\implies V = 4096cm^3 \tag{1}$$

Given that the box has a square base which means that the breadth and height of the box are equal. Let their value be a.

And let the value of length be b.

In this case, we have

$$V = a^2b \tag{2}$$

From (1) and (2), we have

$$a^2b = 4096 \tag{3}$$

Getting b in terms of a, we get

$$b = \frac{4096}{a^2} \tag{4}$$

In the second part of the question, it is given that the cost of polishing the outer surface of the box is ₹4 per cm^2

Let the total surface area of the box be S.

$$S = 2a^2 + 4ab \tag{5}$$

From (4), we have

$$S = 2a^2 + 4a(\frac{4096}{a^2}) \tag{6}$$
$$= 2a^2 + 4(\frac{4096}{a}) \tag{7}$$
$$= 2a^2 + \frac{16384}{a} \tag{8}$$
$$\therefore S = 2a^2 + \frac{16384}{a} \tag{9}$$

Now let $S = y = f(a)$

We need to find the a at where f(a) is minimum by using gradient descent method.

Let us find $\nabla f(a)$

$$\frac{dy}{da} = \frac{d}{da}\left(2a^2 + \frac{16384}{a}\right) \tag{10}$$
$$\implies f'(a) = 4a - \frac{16384}{a^2} \tag{11}$$

We will be able to find the corresponding a value of the minimum of $f(a)$ by iterating the following equation till $(f'(a_{k-1}))$ approaches zero.

$$a_k = a_{k-1} - (\alpha \times f'(a_{k-1})) \tag{12}$$

where a_{k-1} is initial assumed value/ previous obtained value

a_k is updated assumed value

α represents the step size we are taking according to the slope $(f'(a_{k-1}))$

At first, let us randomly choose a_{k-1} as 4. Then, $f'(4) = -1008$.

Since the slope is too far from zero and for manual purpose, we can take large step size. Hence let us choose α as 0.125

Lets go through couple of iterations

$$a_k = 4 - 0.125 \times (-1008) \tag{13}$$
$$= 130 \tag{14}$$
$$a_k = 130 - 0.125 \times (519.03053) \tag{15}$$
$$= 65.12118 \tag{16}$$
$$a_k = 65.12118 - 0.125 \times (256.62127) \tag{17}$$
$$= 33.04353 \tag{18}$$
$$a_k = 33.04353 - 0.125 \times (117.16874) \tag{19}$$
$$= 18.39744 \tag{20}$$
$$a_k = 18.39744 - 0.125 \times (25.1831) \tag{21}$$
$$= 15.24956 \tag{22}$$
$$a_k = 15.24956 - 0.125 \times (-9.4557) \tag{23}$$
$$= 16.43152 \tag{24}$$
$$a_k = 16.43152 - 0.125 \times (5.04345) \tag{25}$$
$$= 15.80109 \tag{26}$$
$$a_k = 15.80109 - 0.125 \times (-2.41709) \tag{27}$$
$$= 16.10322 \tag{28}$$
$$a_k = 16.10322 - 0.125 \times (1.23072) \tag{29}$$
$$= 15.94938 \tag{30}$$

Clearly, we observe that a_k is tending to 16 from both the left hand side as well as the right hand side. Hence the possible whole number at where the minimum of $f(a)$ exists is $a = 16$.

Put $a = 16$ in (9),

$$S_m = 2(16)^2 + \frac{16384}{16} \tag{31}$$
$$= 2(256) + 1024 \tag{32}$$
$$= 512 + 1024 \tag{33}$$
$$= 1536 \tag{34}$$
$$\therefore S_m = 1536cm^2 \tag{35}$$

Let the cost per unit area be c which is equal to ₹$4percm^2$

Let the minimum cost of polishing the metal box be C_m

$$\implies C_m = c \times S_m \tag{36}$$

From (35), we have

$$C_m = 4 \times 1536 \tag{37}$$
$$= 3072 \tag{38}$$

$\boxed{\therefore \text{The minimum cost of polishing the metal box is ₹3072}}$

TABLE I
DESIGN TABLE

Variable	**Formula/Value**	**Description**
c	₹4 per cm^2	Cost per unit area,Input
V	$4096cm^3$	Volume, Input
a	-	Breadth, Height
b	$\frac{V}{a^2}$	Length
S	$2a^2 + 4ab$	Total Surface Area
S_m	$\frac{dS}{da} ata = 16$	Output
C_m	$c \times S_m$	Output