

# Random Numbers

SADINENI ABHINAY - CS21BTECH11055

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**Abstract**—This manual provides solutions to the Assignment of Random Numbers

## 1 UNIFORM RANDOM NUMBERS

Let  $U$  be a uniform random variable between 0 and 1.

1.1 Generate  $10^6$  samples of  $U$  using a C program and save into a file called uni.dat .

**Solution:** Download the following files and execute the C program.

```
wget https://github.com/GovindaRohith/
  Assignments/blob/main/Randomnum/
  codes/1.1.c
wget https://github.com/GovindaRohith/
  Assignments/blob/main/Randomnum/
  codes/source.h
```

Download the above files and execute the following commands

```
$ gcc 1.1.c
$ ./a.out
```

1.2 Load the uni.dat file into python and plot the empirical CDF of  $U$  using the samples in uni.dat. The CDF is defined as

$$F_U(x) = \Pr(U \leq x) \quad (1.1)$$

**Solution:** The following code plots Fig. 1.2

```
wget https://github.com/GovindaRohith/
  Assignments/blob/main/Randomnum/
  codes/1.2.py
```

Download the above files and execute the following commands to produce Fig.1.2

```
$ python3 1.2.py
```

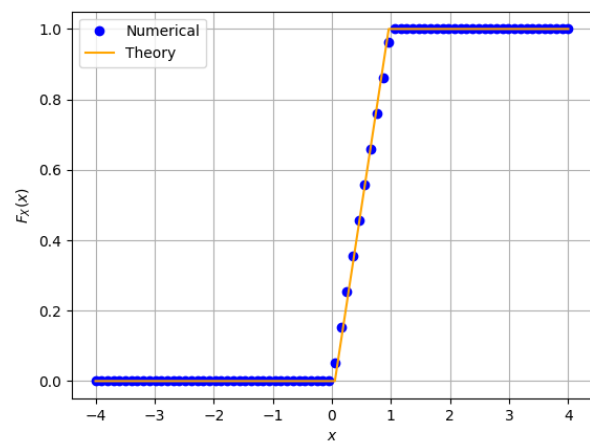


Fig. 1.2: The CDF of  $U$

1.3 Find a theoretical expression for  $F_U(x)$ .

**Solution:** Given  $U$  is a uniform Random Variable

$$p_U(x) = 1 \text{ for } 0 < x < 1 \quad (1.2)$$

$$F_U(x) = \int_{-\infty}^{\infty} p_U(x) dx \quad (1.3)$$

$$\Rightarrow F_U(x) = \begin{cases} 0 & x \leq 0 \\ x & 0 < x < 1 \\ 1 & x \geq 1 \end{cases} \quad (1.4)$$

1.4 The mean of  $U$  is defined as

$$E[U] = \frac{1}{N} \sum_{i=1}^N U_i \quad (1.5)$$

and its variance as

$$\text{var}[U] = E[U - E[U]]^2 \quad (1.6)$$

Write a C program to find the mean and variance of  $U$ .

**Solution:** Download the following files and execute the C program.

```
wget https://github.com/GovindaRohith/
Assignments/blob/main/Randomnum/
codes/1.4.c
wget https://github.com/GovindaRohith/
Assignments/blob/main/Randomnum/
codes/source.h
```

Download the above files and execute the following commands

```
$ gcc 1.4.c
$ ./a.out
```

1.5 Verify your result theoretically given that

$$E[U^k] = \int_{-\infty}^{\infty} x^k dF_U(x) \quad (1.7)$$

**Solution:**

$$\text{var}[U] = E[U - E[U]]^2 \quad (1.8)$$

$$\Rightarrow \text{var}[U] = E[U^2] - E[U]^2 \quad (1.9)$$

$$E[U] = \int_{-\infty}^{\infty} x dF_U(x) \quad (1.10)$$

$$E[U] = \int_0^1 x \quad (1.11)$$

$$\Rightarrow E[U] = \frac{1}{2} \quad (1.12)$$

$$E[U^2] = \int_{-\infty}^{\infty} x^2 dF_U(x) \quad (1.13)$$

$$E[U^2] = \int_0^1 x^2 dF_U(x) \quad (1.14)$$

$$\Rightarrow E[U^2] = \frac{1}{3} \quad (1.15)$$

$$\Rightarrow \text{var}[U] = \frac{1}{12} = 0.0833 \quad (1.16)$$

## 2 CENTRAL LIMIT THEOREM

2.1 Generate  $10^6$  samples of the random variable

$$X = \sum_{i=1}^{12} U_i - 6 \quad (2.1)$$

using a C program, where  $U_i, i = 1, 2, \dots, 12$  are a set of independent uniform random variables between 0 and 1 and save in a file called gau.dat

**Solution:** Download the following files and execute the C program.

```
wget https://github.com/GovindaRohith/
Assignments/blob/main/Randomnum/
codes/2.1.c
wget https://github.com/GovindaRohith/
Assignments/blob/main/Randomnum/
codes/source.h
```

Download the above files and execute the following commands

```
$ gcc 2.1.c
$ ./a.out
```

2.2 Load gau.dat in python and plot the empirical CDF of  $X$  using the samples in gau.dat. What properties does a CDF have?

**Solution:** The CDF of  $X$  is plotted in Fig. 2.2 using the code below

```
wget https://github.com/GovindaRohith/
Assignments/blob/main/Randomnum/
codes/2.2.py
```

Download the above files and execute the following commands to produce Fig.2.2

```
$ python3 2.2.py
```

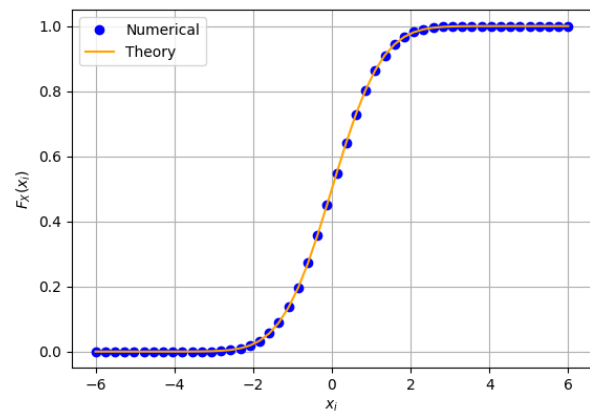


Fig. 2.2: The CDF of  $X$

Some of the properties of CDF

a)  $\lim_{x \rightarrow \infty} F_X(x) = 1$

- b)  $F_X(x)$  is non decreasing function.  
 c) Symmetric about one point.

2.3 Load gau.dat in python and plot the empirical PDF of  $X$  using the samples in gau.dat. The PDF of  $X$  is defined as

$$p_X(x) = \frac{d}{dx} F_X(x) \quad (2.2)$$

What properties does the PDF have?

**Solution:** The PDF of  $X$  is plotted in Fig. 2.3 using the code below

```
wget https://github.com/GovindaRohith/
Assignments/blob/main/Randomnum/
codes/2.3.py
```

Download the above files and execute the following commands to produce Fig.2.3

```
$ python3 2.3.py
```

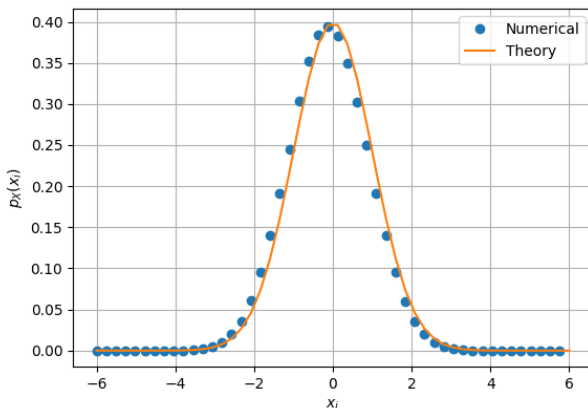


Fig. 2.3: The PDF of  $X$

Some of the properties of the PDF:

- a) Symmetric about  $x = \mu$  in this case  
 b) Decreasing function for  $x > \mu$  and increasing for  $x < \mu$  and attains maximum at  $x = \mu$   
 c) Area under the curve is unity.

2.4 Find the mean and variance of  $X$  by writing a C program.

**Solution:** Download the following files and execute the C program.

```
wget https://github.com/GovindaRohith/
Assignments/blob/main/Randomnum/
codes/2.4.c
wget https://github.com/GovindaRohith/
Assignments/blob/main/Randomnum/
codes/source.h
```

Download the above files and execute the following commands

```
$ gcc 2.4.c
$ ./a.out
```

2.5 Given that

$$p_X(x) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right), -\infty < x < \infty, \quad (2.3)$$

repeat the above exercise theoretically.

**Solution:**

1) CDF is given by

$$F_X(x) = \int_{-\infty}^{\infty} p_X(x) dx \quad (2.4)$$

$$F_X(x) = 1 \quad (2.5)$$

2) Mean is given by

$$E(x) = \int_{-\infty}^{\infty} x p_X(x) dx \quad (2.6)$$

$$\Rightarrow E(x) = 0 \quad (2.7)$$

3) Variance is given by

$$\text{var}[U] = E(U^2) - (E(U))^2 \quad (2.8)$$

$$\Rightarrow \text{var}[U] = \sqrt{2} \quad (2.9)$$

### 3 FROM UNIFORM TO OTHER

3.1 Generate samples of

$$V = -2 \ln(1 - U) \quad (3.1)$$

and plot its CDF.

**Solution:** Download the following files and execute the C program.

```
wget https://github.com/GovindaRohith/
Assignments/blob/main/Randomnum/
codes/3.1.c
wget https://github.com/GovindaRohith/
Assignments/blob/main/Randomnum/
codes/source.h
```

Download the above files and execute the following commands

```
$ gcc 3.1.c -lm
$ ./a.out
```

The CDF of  $V$  is plotted in Fig. 3.1 using the code below

```
wget https://github.com/GovindaRohith/
Assignments/blob/main/Randomnum/
codes/3.1pyth.py
```

Download the above files and execute the following commands to produce Fig.3.1

```
$ python3 3.1pyth.py
```

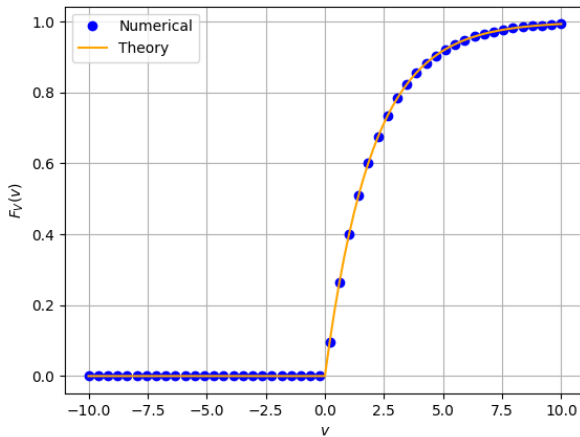


Fig. 3.1: The CDF of  $V$

3.2 Find a theoretical expression for  $F_V(x)$ .

**Solution:** If  $Y = g(X)$ , we know that  $F_Y(y) = F_X(g^{-1}(y))$ , here

$$V = -2 \ln(1 - U) \quad (3.2)$$

$$1 - U = e^{\frac{-V}{2}} \quad (3.3)$$

$$U = 1 - e^{\frac{-V}{2}} \quad (3.4)$$

$$F_V(x) = F_U(1 - e^{\frac{-x}{2}}) \quad (3.5)$$

$$\Rightarrow F_V(x) = \begin{cases} 0 & x < 0 \\ 1 - e^{\frac{-x}{2}} & x \geq 0 \end{cases} \quad (3.6)$$

#### 4 TRIANGULAR DISTRIBUTION

4.1 Generate

$$T = U_1 + U_2 \quad (4.1)$$

**Solution:** Download the following files and execute the C program.

```
wget https://github.com/GovindaRohith/
Assignments/blob/main/Randomnum/
codes/4.1.c
```

```
wget https://github.com/GovindaRohith/
Assignments/blob/main/Randomnum/
codes/source.h
```

Download the above files and execute the following commands

```
$ gcc 4.1.c
$ ./a.out
```

4.2 Find the CDF of  $T$ .

**Solution:** The CDF of  $T$  is plotted in Fig. 4.2 using the code below

```
wget https://github.com/GovindaRohith/
Assignments/blob/main/Randomnum/
codes/4.5cdf.py
```

Download the above files and execute the following commands to produce Fig.4.2

```
$ python3 4.5cdf.py
```

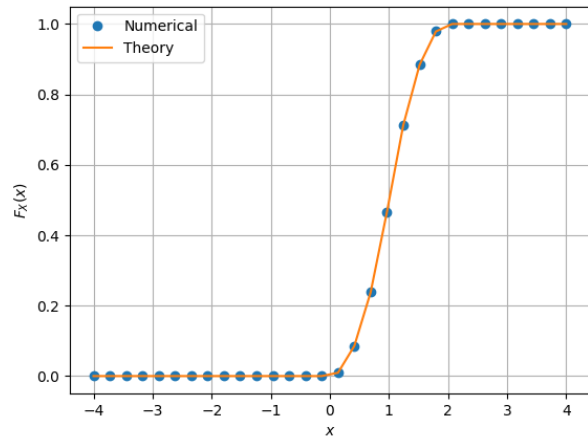


Fig. 4.2: The CDF of  $T$

4.3 Find the PDF of  $T$ .

**Solution:** The PDF of  $T$  is plotted in Fig. 4.2 using the code below

```
wget https://github.com/GovindaRohith/
Assignments/blob/main/Randomnum/
codes/4.5pdf.py
```

Download the above files and execute the following commands to produce Fig.4.2

```
$ python3 4.5pdf.py
```

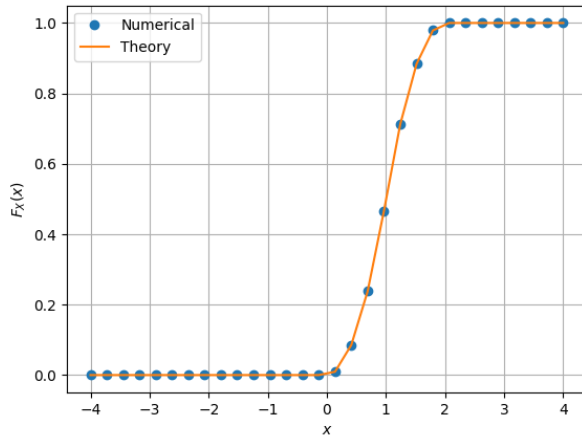


Fig. 4.3: The CDF of  $T$

$$\Rightarrow F_T(t) = \begin{cases} 0 & t < 0 \\ \frac{t^2}{2} & 0 \leq t \leq 1 \\ 2t - 1 - \frac{t^2}{2} & 1 < t \leq 2 \\ 1 & t > 2 \end{cases}$$

4.5 Verify your results through a plot

**Solution:** The Results are verified in the plots Fig 4.2 and Fig 4.3

4.4 Find the Theoretical Expression for the PDF and CDF of  $T$

**Solution:**

$$T = U_1 + U_2 \quad (4.2)$$

$$\Rightarrow p_T(t) = \int_{-\infty}^t p_{U_1}(x)p_{U_1}(y)dx \quad (4.3)$$

$$\text{As, } p_{U_1}(x) = p_{U_1}(y) = p_U(u) \quad (4.4)$$

$$\Rightarrow p_T(t) = \int_{-\infty}^t p_U(u)p_U(t-u)du \quad (4.5)$$

a) Theoretical PDF

i)  $t \leq 1$

$$p_T(t) = \int_0^t p_U(t-u)du \quad (4.6)$$

$$\Rightarrow p_T(t) = \int_0^t du = t \quad (4.7)$$

ii)  $t > 1$

$$p_T(t) = \int_0^1 p_U(t-u)du \quad (4.8)$$

$$\Rightarrow p_T(t) = \int_{t-1}^1 du = 2 - t \quad (4.9)$$

$$\Rightarrow P_T(t) = \begin{cases} t & 0 \leq t \leq 1 \\ 2 - t & 1 < t \leq 2 \\ 0 & t < 0 \text{ or } t > 2 \end{cases}$$

b) Theoretical CDF

$$F_T(t) = \int_{-\infty}^t p_T(u)du \quad (4.10)$$