Assignment 12

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Outline

Question

7-32(Papoullis):

The random variables X and Y are uncorrelated with zero mean and $\sigma_x = \sigma_y = \sigma$. Show that if$z = x + iy$ then

$$f_z(Z) = f(x, y) = \frac{1}{2\pi\sigma^2} e^{-(x^2+y^2)/2\sigma^2} = \frac{1}{\pi\sigma_Z^2} e^{-|Z^2|/\sigma_z^2}$$

$$\Phi_Z(\Omega) = \exp\left\{-\frac{1}{2}(\sigma^2 u^2 + \sigma^2 v_Z^2)\right\} = \exp\left\{-\frac{1}{4}\sigma_Z^2 \left|\Omega\right|^2\right\}$$

Solution(a)

Since X and Y are independent

$$\sigma_Z^2 = E(\left|Z^2\right|) = E(X^2 + Y^2) \tag{1}$$

$$\sigma_Z^2 = \sigma_X^2 + \sigma_Y^2 = 2\sigma^2 \tag{2}$$

$$M_Z(t) = E(e^{tZ}) = E(e^{t(x+iY)}) \tag{3}$$

$$E(e^{tZ}) = E(e^{tX})E(e^{tiY}) \tag{4}$$

$$\text{But,} M_X(t) = E(e^{tX}) = \int_{-\infty}^{\infty} e^{tx} f_X(X)dx \tag{5}$$

Contd.

$$\int_{-\infty}^{\infty} e^{tZ} f_Z(Z) dZ = \int_{-\infty}^{\infty} e^{tX} f_X(X) dX \times \int_{-\infty}^{\infty} e^{tY} f_Y(Y) dY \tag{6}$$

$$\int_{-\infty}^{\infty} e^{tZ} f_Z(Z) dZ = \left(\int_{-\infty}^{\infty} e^{tX} \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-X^2}{2\sigma^2}} dx \right)^2 \tag{7}$$

$$f_Z(Z) = \frac{1}{2\pi\sigma^2} e^{-(x^2+y^2)/2\sigma^2} = \frac{1}{\pi\sigma_Z^2} e^{-|Z|^2/2\sigma^2} \tag{8}$$

Solution(b)

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$$\Phi_Z(\Omega) = \Phi_X(u)\Phi_Y(v) \tag{9}$$

$$\Phi_Z(\Omega) = e^{-\frac{1}{2}\sigma^2(u^2+v^2)} \tag{10}$$

$$\Phi_Z(\Omega) = e^{-\frac{1}{2}\sigma^2|\Omega^2|} \tag{11}$$

$$\implies \boxed{\Phi_Z(\Omega) = e^{-\frac{1}{4}\sigma_Z^2|\Omega^2|}} \tag{12}$$