

# Assignment 9

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## Question

### 2-24(Papoullis):

Box 1 contains 1000 bulbs of which 10% are defective. Box 2 contains 2000 bulbs which 5% are defective. Two bulbs are picked from a randomly selected box.

- (a) Find the probability that both bulbs are defective.
- (b) Assuming that both are defective, find the probability that they came from box 1.

# Denote Random Variables

## Assign events to random variables

Denote the random variables  $X_1, X_2, X_3 \in \{0, 1\}$ . Events are described in Table 1:

Variable	Event
$X_1 = 0$	Picking two defective bulbs from Box 1
$X_1 = 1$	Not picking two defective bulbs from Box 1
$X_2 = 0$	Picking two defective bulbs from Box 2
$X_2 = 1$	Not picking two defective bulbs from Box 2
$X_3 = 0$	Picking up Box 1
$X_3 = 1$	Picking up Box 2

Table 1

# Given data

Represent the given data

Event	Probability
$\Pr(X_3 = 0)$	0.5
$\Pr(X_3 = 1)$	0.5
$\Pr(X_1 = 0)$	$\frac{11}{1110}$
$\Pr(X_2 = 0)$	$\frac{99}{39980}$

Table 2

# Solution(a)

## Solution

$\Pr(Y = X_1 + X_2)$  denotes the probability that both bulbs are defective.

From Total probability theorem

$$\Pr(Y) = \sum_{i=0}^1 \Pr(X_3 = i) \Pr(X_{i+1} = 0) \quad (1)$$

$$\implies \boxed{\Pr(Y) \approx 0.006193} \quad (2)$$

# Solution(b)

## Solution

$\Pr(X_1 = 0|Y = X_1 + X_2)$  denotes two bulbs are picked from box 1 assuming both are defective.

From Bayes theorem

$$\Pr(X_1 = 0|Y) = \frac{\Pr(X_3 = 0) \Pr(Y|X_3 = 0)}{\Pr(Y)} \quad (3)$$

$$\Rightarrow \boxed{\Pr(X_1 = 0|Y) \approx 0.8} \quad (4)$$