AI1110-Assignment 2

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(ICSE-12-2017)Question 1
(ii) if $y-2x-k=0$ touches the conic $3x^2-5y^2=15$,find the value of k

Solution :

The given equation (Hyperbola)is

$$3x^2-5y^2-15=0 \tag{0.0.1}$$

which can be written as

$$\mathbf{x}^\top\mathbf{V}\mathbf{x}+2\mathbf{u}^\top\mathbf{x}+f=0 \tag{0.0.2}$$

$$\mathbf{V}=\begin{pmatrix}3 & 0\\ 0 & -5\end{pmatrix},\mathbf{u}=\begin{pmatrix}0\\0\end{pmatrix},f=-15 \tag{0.0.3}$$

The Given line Can be written as

$$L:\mathbf{x}=\mathbf{A}+\lambda\mathbf{m} \tag{0.0.4}$$

$$\text{Choose }\mathbf{A}=\begin{pmatrix}0\\k\end{pmatrix} \tag{0.0.5}$$

$$\mathbf{m}=\begin{pmatrix}1\\2\end{pmatrix} \tag{0.0.6}$$

$$\implies \mathbf{x}=\begin{pmatrix}\lambda\\k+2\lambda\end{pmatrix} \tag{0.0.7}$$

Substituting the line in the conic equation we get the following quadratic equation

$$17\lambda^2+20k\lambda+5k^2+15=0 \tag{0.0.8}$$

To be a tangent the above equation should have only root so

$$400k^2-4(17)(5k^2+15)=0 \tag{0.0.9}$$

$$\implies k^2-17=0 \tag{0.0.10}$$

$$\boxed{k=\pm\sqrt{17}} \tag{0.0.11}$$