

AI1110-Assignment 2

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(ICSE-12-2017) Question 1

(ii) if $y - 2x - k = 0$ touches the conic $3x^2 - 5y^2 = 15$, find the value of k

Solution :

The given equation (Hyperbola) is

$$3x^2 - 5y^2 - 15 = 0 \quad (0.0.1)$$

which can be written as

$$\mathbf{x}^T \mathbf{V} \mathbf{x} + 2\mathbf{u}^T \mathbf{x} + f = 0 \quad (0.0.2)$$

$$\mathbf{V} = \begin{pmatrix} 3 & 0 \\ 0 & -5 \end{pmatrix}, \mathbf{u} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, f = -15 \quad (0.0.3)$$

The Given line Can be written as

$$L : \mathbf{x} = \mathbf{A} + \lambda \mathbf{m} \quad (0.0.4)$$

$$\text{Choose } \mathbf{A} = \begin{pmatrix} 0 \\ k \end{pmatrix} \quad (0.0.5)$$

$$\mathbf{m} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \quad (0.0.6)$$

$$\Rightarrow \mathbf{x} = \begin{pmatrix} \lambda \\ k + 2\lambda \end{pmatrix} \quad (0.0.7)$$

Substituting the line in the conic equation we get the following quadratic equation

$$17\lambda^2 + 20k\lambda + 5k^2 + 15 = 0 \quad (0.0.8)$$

To be a tangent the above equation should have only root so

$$400k^2 - 4(17)(5k^2 + 15) = 0 \quad (0.0.9)$$

$$\Rightarrow k^2 - 17 = 0 \quad (0.0.10)$$

$$\boxed{k = \pm\sqrt{17}} \quad (0.0.11)$$

