Random Numbers

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Contents

***Abstract*—This manual provides solutions to the Assignment on Random Numbers**

1 Uniform Random Numbers

Let U be a uniform random variable between 0 and 1.

1.1 Generate 10^6 samples of U using a C program and save into a file called uni.dat .
Solution: Download the following files and execute the C program.

```
wget https://github.com/GovindaRohith/
    Assignments/blob/main/Randomnum/
    codes/1.1.c
wget https://github.com/GovindaRohith/
    Assignments/blob/main/Randomnum/
    codes/source.h
```

Download the above files and execute the following commands

```
$ gcc 1.1.c
$ ./a.out
```

1.2 Load the uni.dat file into python and plot the empirical CDF of U using the samples in uni.dat. The CDF is defined as

$$F_U(x) = \Pr(U \le x) \tag{1.1}$$

Solution: The following code plots Fig. 1.2

```
wget https://github.com/GovindaRohith/
    Assignments/blob/main/Randomnum/
    codes/1.2.py
```

Download the above files and execute the following commands to produce Fig.1.2

```
$ python3 1.2.py
```

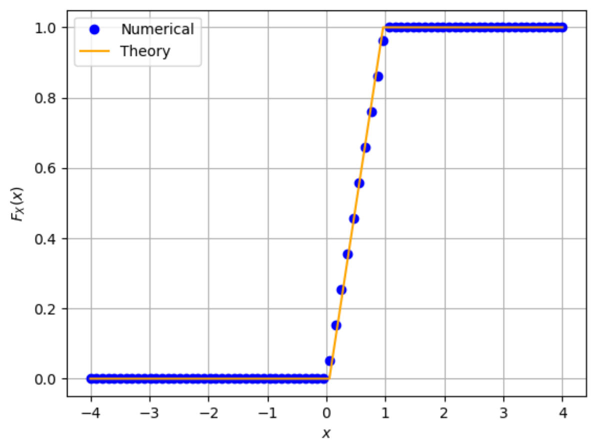

Fig. 1.2: The CDF of U

1.3 Find a theoretical expression for $F_U(x)$.
Solution: Given U is a uniform Random Variable

$$p_U(x) = 1 \text{ for} \tag{1.2}$$

$$F_U(x) = \int_{-\infty}^{\infty} p_U(x)dx \tag{1.3}$$

$$\boxed{\implies F_U(x) = \begin{cases} 0 & x \le 0 \\ x & 0 < x < 1 \\ 1 & x \ge 1 \end{cases}} \tag{1.4}$$

1.4 The mean of U is defined as

$$E[U] = \frac{1}{N}\sum_{i=1}^{N} U_i \tag{1.5}$$

and its variance as

$$\text{var}[U] = E[U - E[U]]^2 \tag{1.6}$$

Write a C program to find the mean and variance of U.
Solution: Download the following files and execute the C program.

```
wget https://github.com/GovindaRohith/
    Assignments/blob/main/Randomnum/
    codes/1.4.c
wget https://github.com/GovindaRohith/
    Assignments/blob/main/Randomnum/
    codes/source.h
```

Download the above files and execute the following commands

```
$ gcc 1.4.c
$ ./a.out
```

1.5 Verify your result theoretically given that

$$E\left[U^k\right] = \int_{-\infty}^{\infty} x^k dF_U(x) \tag{1.7}$$

Solution:

$$\text{var}[U] = E[U - E[U]]^2 \tag{1.8}$$

$$\implies \text{var}[U] = E\left[U^2\right] - E[U]^2 \tag{1.9}$$

$$E[U] = \int_{-\infty}^{\infty} x dF_U(x) \tag{1.10}$$

$$E[U] = \int_0^1 x \tag{1.11}$$

$$\implies \boxed{E[U] = \frac{1}{2}} \tag{1.12}$$

$$E\left[U^2\right] = \int_{-\infty}^{\infty} x^2 dF_U(x) \tag{1.13}$$

$$E\left[U^2\right] = \int_0^1 x^2 dF_U(x) \tag{1.14}$$

$$\implies E\left[U^2\right] = \frac{1}{3} \tag{1.15}$$

$$\implies \boxed{\text{var}[U] = \frac{1}{12} = 0.0833} \tag{1.16}$$

2 Central Limit Theorem

2.1 Generate 10^6 samples of the random variable

$$X = \sum_{i=1}^{12} U_i - 6 \tag{2.1}$$

using a C program, where $U_i, i = 1, 2, \ldots, 12$ are a set of independent uniform random variables between 0 and 1 and save in a file called gau.dat
Solution: Download the following files and execute the C program.

```
wget https://github.com/GovindaRohith/
    Assignments/blob/main/Randomnum/
    codes/2.1.c
wget https://github.com/GovindaRohith/
    Assignments/blob/main/Randomnum/
    codes/source.h
```

Download the above files and execute the following commands

```
$ gcc 2.1.c
$ ./a.out
```

2.2 Load gau.dat in python and plot the empirical CDF of X using the samples in gau.dat. What properties does a CDF have?
Solution: The CDF of X is plotted in Fig. 2.2 using the code below

```
wget https://github.com/GovindaRohith/
    Assignments/blob/main/Randomnum/
    codes/2.2.py
```

Download the above files and execute the following commands to produce Fig.2.2

```
$ python3 2.2.py
```

Some of the properties of CDF
a) $\lim_{x\to\infty} F_X(x) = 1$
b) $F_X(x)$ is non decreasing function.
c) Symmetric about one point.

2.3 Load gau.dat in python and plot the empirical PDF of X using the samples in gau.dat. The PDF of X is defined as

$$p_X(x) = \frac{d}{dx} F_X(x) \tag{2.2}$$

What properties does the PDF have?
Solution: The PDF of X is plotted in Fig. 2.3 using the code below

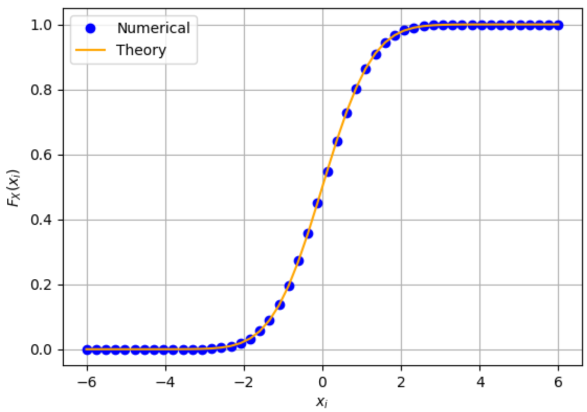

Fig. 2.2: The CDF of X

```
wget https://github.com/GovindaRohith/
     Assignments/blob/main/Randomnum/
     codes/2.3.py
```

Download the above files and execute the following commands to produce Fig.2.3

```
$ python3 2.3.py
```

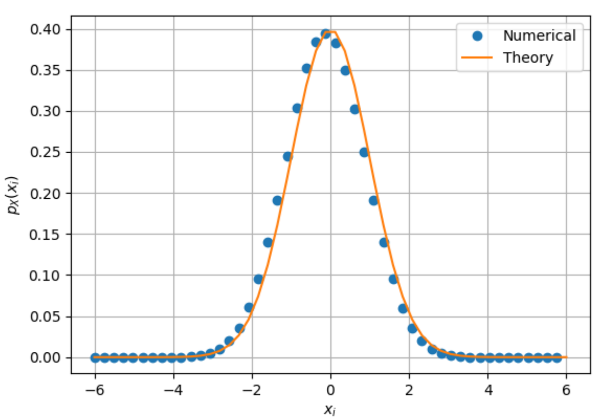

Fig. 2.3: The PDF of X

Some of the properties of the PDF:

a) Symmetric about $x = \mu$ in this case
b) Decreasing function for $x > \mu$ and increasing for $x < \mu$ and attains maximum at $x = \mu$
c) Area under the curve is unity.

2.4 Find the mean and variance of X by writing a C program.
Solution: Download the following files and execute the C program.

```
wget https://github.com/GovindaRohith/
     Assignments/blob/main/Randomnum/
     codes/2.4.c
wget https://github.com/GovindaRohith/
     Assignments/blob/main/Randomnum/
     codes/source.h
```

Download the above files and execute the following commands

```
$ gcc 2.4.c
$ ./a.out
```

2.5 Given that

$$p_X(x) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right), -\infty < x < \infty, \quad (2.3)$$

repeat the above exercise theoretically.

Solution:

1) CDF is given by

$$F_X(x) = \int_{-\infty}^{\infty} p_X(x)dx \quad (2.4)$$

$$\boxed{F_X(x) = 1} \quad (2.5)$$

2) Mean is given by

$$E(x) = \int_{-\infty}^{\infty} xp_X(x)dx \quad (2.6)$$

$$\implies \boxed{E(x) = 0} \quad (2.7)$$

3) Variance is given by

$$\text{var}\,[U] = E(U^2) - (E(U))^2 \quad (2.8)$$

$$Ex^2 = \int_{-\infty}^{\infty} x^2 p_X(x)dx \quad (2.9)$$

$$= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} x^2 exp\left(-\frac{x^2}{2}\right)dx \quad (2.10)$$

$$= \frac{1}{\sqrt{2\pi}}\left(x\int xexp\left(-\frac{x^2}{2}\right)dx\right) \quad (2.11)$$

$$- \frac{1}{\sqrt{2\pi}}\int\int\left(xexp\left(-\frac{x^2}{2}\right)\right)dx.dx \quad (2.12)$$

$$= \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} exp\left(-\frac{x^2}{2}\right)dx \quad (2.13)$$

$$= \frac{\sqrt{2\pi}}{\sqrt{2\pi}} \quad (2.14)$$

$$= 1 \implies \boxed{\text{var}\,[U] = 1} \quad (2.15)$$

3 From Uniform to Other

3.1 Generate samples of

$$V = -2\ln(1-U) \tag{3.1}$$

and plot its CDF.
Solution: Download the following files and execute the C program.

```
wget https://github.com/GovindaRohith/
    Assignments/blob/main/Randomnum/
    codes/3.1.c
wget https://github.com/GovindaRohith/
    Assignments/blob/main/Randomnum/
    codes/source.h
```

Download the above files and execute the following commands

```
$ gcc 3.1.c -lm
$ ./a.out
```

The CDF of V is plotted in Fig. 3.1 using the code below

```
wget https://github.com/GovindaRohith/
    Assignments/blob/main/Randomnum/
    codes/3.1pyth.py
```

Download the above files and execute the following commands to produce Fig.3.1

```
$ python3 3.1pyth.py
```

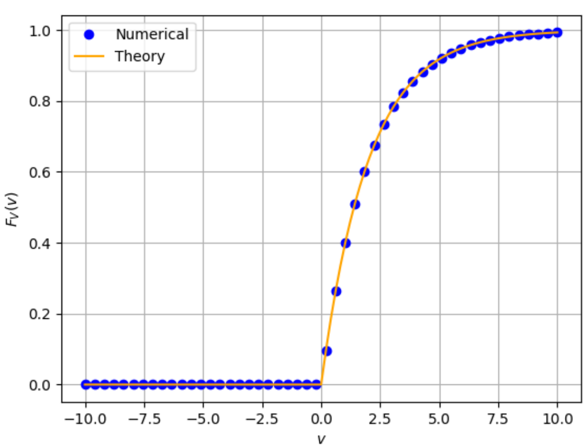

Fig. 3.1: The CDF of V

3.2 Find a theoretical expression for $F_V(x)$.
Solution: If Y = g(X), we know that $F_Y(y) = F_X(g^{-1}(y))$, here

$$V = -2\ln(1-U) \tag{3.2}$$

$$1-U = e^{\frac{-V}{2}} \tag{3.3}$$

$$U = 1-e^{\frac{-V}{2}} \tag{3.4}$$

$$F_V(x) = F_U(1-e^{\frac{-x}{2}}) \tag{3.5}$$

$$\implies F_V(x) = \begin{cases} 0 & x < 0 \\ 1-e^{\frac{-x}{2}} & x \geq 0 \end{cases} \tag{3.6}$$

4 Triangular Distribution

4.1 Generate

$$T = U_1 + U_2 \tag{4.1}$$

Solution: Download the following files and execute the C program.

```
wget https://github.com/GovindaRohith/
    Assignments/blob/main/Randomnum/
    codes/4.1.c
wget https://github.com/GovindaRohith/
    Assignments/blob/main/Randomnum/
    codes/source.h
```

Download the above files and execute the following commands

```
$ gcc 4.1.c
$ ./a.out
```

4.2 Find the CDF of T.
Solution: The CDF of T is plotted in Fig. 4.2 using the code below

```
wget https://github.com/GovindaRohith/
    Assignments/blob/main/Randomnum/
    codes/4.5cdf.py
```

Download the above files and execute the following commands to produce Fig.4.2

```
$ python3 4.5cdf.py
```

4.3 Find the PDF of T.
Solution: The PDF of T is plotted in Fig. 4.2 using the code below

```
wget https://github.com/GovindaRohith/
    Assignments/blob/main/Randomnum/
    codes/4.5pdf.py
```

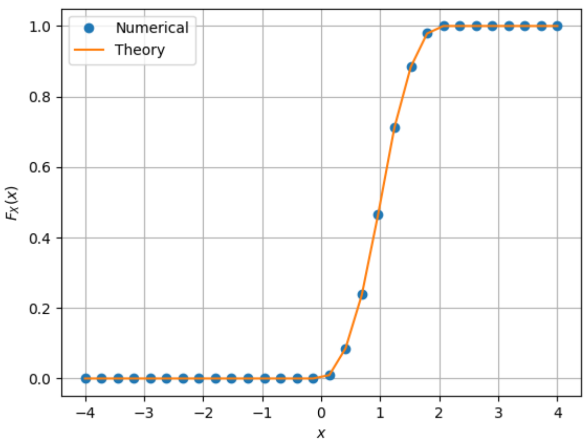

Fig. 4.2: The CDF of T

Download the above files and execute the following commands to produce Fig.4.2

```
$ python3 4.5pdf.py
```

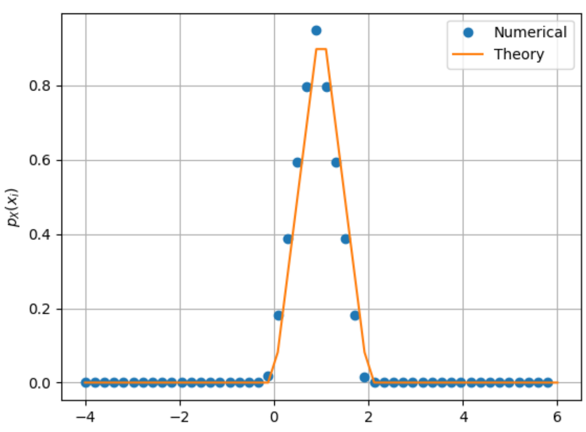

Fig. 4.3: The PDF of T

4.4 Find the Theoreotical Expression for the PDF and CDF of T
Solution:

$$T = U_1 + U_2 \tag{4.2}$$

$$\implies p_T(t) = \int_{-\infty}^{t} p_{U1}(x)p_{U2}(y)dx \tag{4.3}$$

$$\text{As,} p_{U1}(x) = p_{U1}(y) = p_U(u) \tag{4.4}$$

$$\implies p_T(t) = \int_{-\infty}^{t} p_U(u)p_U(t-u)du \tag{4.5}$$

a) Theoretical PDF

i) $t \leq 1$

$$p_T(t) = \int_0^t p_U(t-u)du \tag{4.6}$$

$$\implies p_T(t) = \int_0^t du = t \tag{4.7}$$

ii) $t > 1$

$$p_T(t) = \int_0^1 p_U(t-u)du \tag{4.8}$$

$$\implies p_T(t) = \int_{t-1}^1 du = 2-t \tag{4.9}$$

$$\implies \boxed{P_T(t) = \begin{cases} t & 0 \leq t \leq 1 \\ 2-t & 1 < t \leq 2 \\ 0 & t < 0 \text{ or } t > 2 \end{cases}}$$

b) Theoretical CDF

$$F_T(t) = \int_{-\infty}^{t} p_T(u)du \tag{4.10}$$

$$\implies \boxed{F_T(t) = \begin{cases} 0 & t < 0 \\ \frac{t^2}{2} & 0 \leq t \leq 1 \\ 2t - 1 - \frac{t^2}{2} & 1 < t \leq 2 \\ 1 & t > 2 \end{cases}}$$

4.5 Verify your results through a plot
Solution: The Results are verfied in the plots Fig 4.2 and Fig 4.3

5 Guasssian to Other

5.1 Generate equiprobable $X \in \{1, -1\}$. **Solution:** Download the following files and execute the C program.

```
wget https://github.com/GovindaRohith/
    Assignments/blob/main/Randomnum/
    codes/5.1.c
wget https://github.com/GovindaRohith/
    Assignments/blob/main/Randomnum/
    codes/source.h
```

Download the above files and execute the following commands

```
$ gcc 5.1.c
$ ./a.out
```

5.2 Generate

$$Y = AX + N, \quad (5.1)$$

Where A=5 dB and $N \sim \mathcal{N}(0,1)$
Solution: :

```
wget https://github.com/GovindaRohith/
    Assignments/blob/main/Randomnum/
    codes/5.2.c
wget https://github.com/GovindaRohith/
    Assignments/blob/main/Randomnum/
    codes/source.h
```

Download the above files and execute the following commands

```
$ gcc 5.2.c
$ ./a.out
```

5.3 Plot Y using a scatter plot.
Solution: The CDF of V is plotted in Fig. 5.5 using the code below

```
wget https://github.com/GovindaRohith/
    Assignments/blob/main/Randomnum/
    codes/5.3.py
```

Download the above files and execute the following commands to produce Fig.5.5

```
$ python3 5.3.py
```

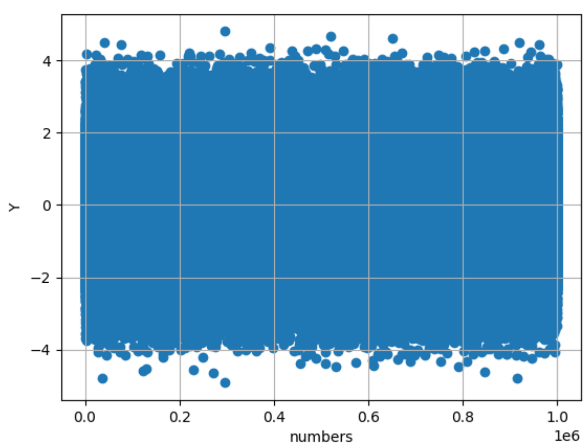

Fig. 5.5: The Scatter Plot

5.4 Guess how to estimate X from Y.
Solution: :

i) If $Y < 0$ then probably $X = -1$
ii) If $Y > 0$ then probably $X = 1$

5.5 Find

$$P_{e|0} = \Pr\left(\hat{X} = -1 | X = 1\right) \quad (5.2)$$

and

$$P_{e|1} = \Pr\left(\hat{X} = 1 | X = -1\right) \quad (5.3)$$

Solution: :

```
wget https://github.com/GovindaRohith/
    Assignments/blob/main/Randomnum/
    codes/5.5.c
wget https://github.com/GovindaRohith/
    Assignments/blob/main/Randomnum/
    codes/source.h
```

Download the above files and execute the following commands to get the result

```
$ gcc 5.5.c
$ ./a.out
```

$$P_{e|0} = 0.499033 \quad (5.4)$$
$$P_{e|1} = 0.500138 \quad (5.5)$$

5.6 Find P_e assuming that X has equiprobable symbols.
Solution:

$$P_e = P(X = 1)P_{e|0} + P(X = -1)P_{e|1} \quad (5.6)$$

Since X is equiprobable

$$P(X = 1) = P(X = -1) = 0.5 \quad (5.7)$$
$$\implies P_e = \frac{P_{e|0} + P_{e|1}}{2} \quad (5.8)$$
$$\implies \boxed{P_e = 0.499585} \quad (5.9)$$

5.7 Verify by plotting the theoretical P_e with respect to A from 0 to 10 dB.
Solution: :

$$P_{e|0} = \Pr\left(\hat{X} = -1 | X = 1\right) \quad (5.10)$$
$$P_{e|0} = \Pr\left(AX + N < 0 | X = 1\right) \quad (5.11)$$
$$P_{e|0} = \Pr\left(N < -A\right) \quad (5.12)$$
$$P_{e|0} = \int_{-\infty}^{-A} \frac{e^{\frac{-x^2}{2}}}{\sqrt{2\pi}} \quad (5.13)$$
$$P_{e|0} = \int_{A}^{\infty} \frac{e^{\frac{-x^2}{2}}}{\sqrt{2\pi}} \quad (5.14)$$
$$P_{e|0} = Q_N(A) \quad (5.15)$$
$$\text{Similarly, } P_{e|1} = Q_N(A) \quad (5.16)$$

```
wget https://github.com/GovindaRohith/
    Assignments/blob/main/Randomnum/
    codes/5.6.py
```

Download the above files and execute the following commands to produce Fig.5.5

```
$ python3 5.6.py
```

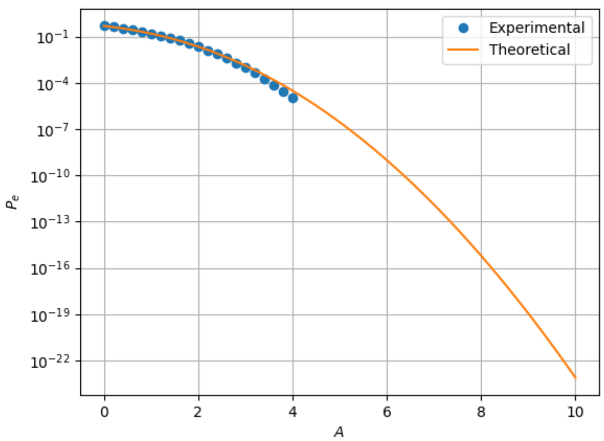

Fig. 5.5: $P_e(A)$ with semilog-y axis

5.8 Now, consider a threshold δ while estimating X from Y. Find the value of δ that maximizes the theoretical P_e.
Solution: To estimate X from Y, we now consider the following:

$$X = \begin{cases} 1, & Y > \delta \\ -1, & Y < \delta \end{cases} \tag{5.17}$$

Therefore,

$$P_{e|0} = \Pr\left(\hat{X} = -1 | X = 1\right) \tag{5.18}$$
$$= \Pr\left(AX + N < \delta | X = 1\right) \tag{5.19}$$
$$\implies P_{e|0} = \Pr\left(N < \delta - A\right) \tag{5.20}$$
$$= \int_{-\infty}^{\delta - A} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx \tag{5.21}$$
$$= \int_{A-\delta}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx \tag{5.22}$$
$$\implies P_{e|0} = Q_N(A - \delta) \tag{5.23}$$

Similarly,

$$P_{e|1} = Q_N(A + \delta) \tag{5.24}$$
$$P_e = P_{e|0} \Pr\left(X = 1\right) + P_{e|1} \Pr\left(X = -1\right) \tag{5.25}$$
$$= \frac{Q_N(A - \delta) + Q_N(A + \delta)}{2} \tag{5.26}$$

Differentiating the above equation wrt δ:

$$0 = \frac{d}{d\delta}\left(\frac{Q_N(A - \delta) + Q_N(A + \delta)}{2}\right) \tag{5.27}$$
$$= \frac{1}{2}\left(\frac{1}{\sqrt{2\pi}} e^{-\frac{(\delta - A)^2}{2}} - \frac{1}{\sqrt{2\pi}} e^{-\frac{(A+\delta)^2}{2}}\right) \tag{5.28}$$
$$\implies (\delta - A)^2 = (\delta + A)^2 \tag{5.29}$$
$$\implies \boxed{\delta = 0} \tag{5.30}$$

5.9 Repeat the above exercise when

$$p_X(0) = p \tag{5.31}$$

Solution: Using Eq. (5.25), we have:

$$P_e = P_{e|0} p + P_{e|1}(1 - p) \tag{5.32}$$
$$= pQ_N(A - \delta) + (1 - p)Q_N(A + \delta) \tag{5.33}$$

Differentiating as before, we get:

$$0 = p\frac{1}{\sqrt{2\pi}} e^{-\frac{(\delta - A)^2}{2}} - (1 - p)\frac{1}{\sqrt{2\pi}} e^{-\frac{(A+\delta)^2}{2}} \tag{5.34}$$

$$e^{\frac{(\delta+A)^2 - ((\delta - A))^2}{2}} = \frac{1 - p}{p} \tag{5.35}$$
$$\implies \boxed{\delta = 0} \tag{5.36}$$

5.10 Repeat the above exercise using the MAP criterion.
Solution: Assume that

$$\Pr\left(X = -1\right) = p \tag{5.37}$$
$$\Pr\left(X = 1\right) = (1 - p) \tag{5.38}$$

From Total Probability Theorem, we have:

$$p_Y(y) = p_{Y|X=-1}(y|-1)\Pr(X=-1) + p_{Y|X=1}(y|1)\Pr(X=1) \tag{5.39}$$

$$p_Y(y) = p \times p_{(-A+N)}(y) \tag{5.40}$$

$$+ (1-p) \times p_{(A+N)}(y) \tag{5.41}$$

Now, $p_{(-A+N)}$ is just the pdf of a shifted normal distribution, and therefore:

$$p_Y(y) = p\frac{e^{-\frac{(y+A)^2}{2}}}{\sqrt{2\pi}} + (1-p)\frac{e^{-\frac{(y-A)^2}{2}}}{\sqrt{2\pi}} \tag{5.42}$$

To use the MAP criterion, we must find $p_{X|Y}(x|y)$. To do this, we use the Theorem of Conditional Probability:

$$p_{X|Y}(x|y) = \frac{p_{Y|X}(y|x) \times p_X(x)}{p_Y(y)} \tag{5.43}$$

When $X = 1$, we have:

$$p_{X|Y}(1|y) = \frac{p_{Y|X}(y|1) \times p_X(1)}{p_Y(y)} \tag{5.44}$$

$$= \frac{(1-p)\frac{e^{-\frac{(y-A)^2}{2}}}{\sqrt{2\pi}}}{p\frac{e^{-\frac{(y+A)^2}{2}}}{\sqrt{2\pi}} + (1-p)\frac{e^{-\frac{(y-A)^2}{2}}}{\sqrt{2\pi}}} \tag{5.45}$$

$$= \frac{(1-p)e^{2yA}}{p + (1-p)e^{2yA}} \tag{5.46}$$

Similarly, when $X = -1$, we get:

$$p_{X|Y}(-1|y) = \frac{p}{p + (1-p)e^{2yA}} \tag{5.47}$$

Therefore, when $p_{X|Y}(1|y) > p_{X|Y}(-1|y)$, we have:

$$\frac{(1-p)e^{2yA}}{p + (1-p)e^{2yA}} > \frac{p}{p + (1-p)e^{2yA}} \tag{5.48}$$

$$e^{2yA} > \frac{p}{(1-p)} \tag{5.49}$$

$$y > \frac{1}{2A}\ln\frac{p}{(1-p)} \tag{5.50}$$

Therefore, when Eq. (5.50), we can assert that $X = 1$, and $X = -1$ otherwise. Now, consider when $p = \frac{1}{2}$. We have:

$$y > \frac{1}{2A}\ln\frac{p}{(1-p)} \tag{5.51}$$

$$= \frac{1}{2A}\ln 1 \tag{5.52}$$

$$= 0 \tag{5.53}$$

Therefore, when $y > 0$, we choose $X = 1$, and we choose $X = -1$ otherwise.

6 Guasssian to Other

6.1 Let $X_1 \sim \mathcal{N}(0,1)$ and $X_2 \sim \mathcal{N}(0,1)$.Plot the CDF and PDF of

$$V = X_1^2 + X_2^2 \tag{6.1}$$

Solution:
The CDF and PDF of V is plotted in Fig. 6.5 and 6.5 using the codes below

```
wget https://github.com/GovindaRohith/
    Assignments/blob/main/Randomnum/
    codes/6.1cdf.py
wget https://github.com/GovindaRohith/
    Assignments/blob/main/Randomnum/
    codes/6.1pdf.py
```

Download the above files and execute the following commands to produce Fig.6.5 and 6.5

```
$ python3 6.1cdf.py
$ python3 6.1pdf.py
```

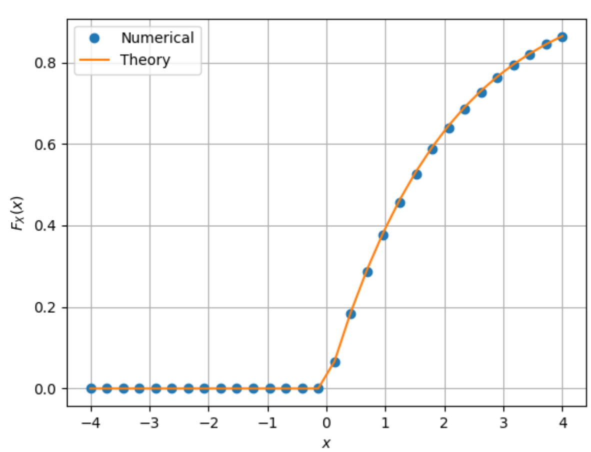

Fig. 6.5: The CDF of V

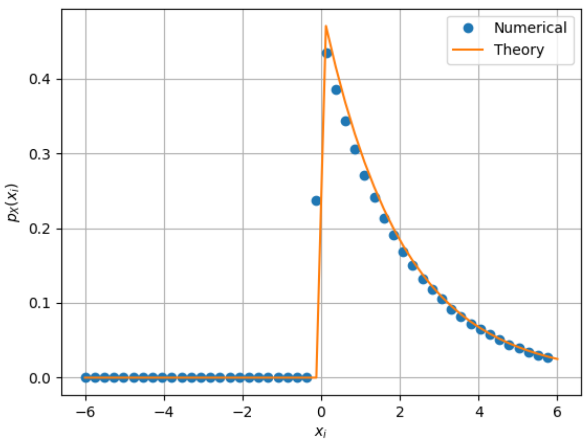

Fig. 6.5: The PDF of V

6.2 If

$$F_V(x) = \begin{cases} 1 - e^{-\alpha x} & x \geq 0 \\ 0 & x < 0, \end{cases} \tag{6.2}$$

find α.
Solution: Assuming X_1 and X_2 are i.i.d

$$X_1 = r\cos\Theta \tag{6.3}$$
$$X_2 = r\sin\Theta \tag{6.4}$$

The Jacobian matrix transforming r, Θ to X_1, X_2 is defined as

$$\mathbf{J} = \begin{pmatrix} \frac{\partial X_1}{\partial r} & \frac{\partial X_1}{\partial \Theta} \\ \frac{\partial X_2}{\partial r} & \frac{\partial X_2}{\partial \Theta} \end{pmatrix} \tag{6.5}$$
$$\implies \mathbf{J} = \begin{pmatrix} \cos\Theta & -r\sin\Theta \\ \sin\Theta & r\cos\Theta \end{pmatrix} \tag{6.6}$$
$$\implies \left|\mathbf{J}\right| = r \tag{6.7}$$
$$\tag{6.8}$$

$$f_{X_1,X_2}(x_1, x_2) = f_{X_1}(x_1) f_{X_2}(x_2) \tag{6.9}$$
$$\implies f_{X_1,X_2}(x_1, x_2) = \frac{e^{\frac{-x_1^2 - x_2^2}{2}}}{2\pi} \tag{6.10}$$
$$\implies f_{X_1,X_2}(x_1, x_2) = \frac{e^{\frac{-r^2}{2}}}{2\pi} \tag{6.11}$$
$$f_{R,\Theta}(r, \theta) = f_{X_1,X_2}(x_1, x_2)\left|\mathbf{J}\right| \tag{6.12}$$
$$\implies f_{R,\Theta}(r, \theta) = \frac{re^{\frac{-r^2}{2}}}{2\pi} \tag{6.13}$$
$$f_R(r) = \int_0^{2\pi} f_{R,\Theta}(r, \theta)\, d\theta \tag{6.14}$$
$$f_R(r) = \int_0^{2\pi} \frac{re^{\frac{-r^2}{2}}}{2\pi}\, d\theta \tag{6.15}$$
$$\implies \boxed{f_R(r) = re^{\frac{-r^2}{2}}} \tag{6.16}$$
$$F_V(r) = F_{X_1^2+X_2^2}(r) \tag{6.17}$$
$$\implies F_V(r) = F_{R^2}(r) \tag{6.18}$$

$$\implies F_V(r) = \Pr\left(R^2 \leq x\right) \tag{6.19}$$
$$\implies F_V(r) = \Pr\left(R \leq \sqrt{x}\right) \tag{6.20}$$
$$\text{But } F_V(x) = \Pr\left(R \leq x\right) \tag{6.21}$$
$$\implies F_V(x) = \int_0^x f_R(r) dr \tag{6.22}$$
$$F_V(x) = 1 - e^{\frac{-x^2}{2}} \tag{6.23}$$
$$\implies F_V(r) = \begin{cases} 0 & r < 0 \\ 1 - e^{\frac{-r}{2}} & r \geq 0 \end{cases} \tag{6.24}$$

$$\implies \boxed{\alpha = \frac{1}{2}} \tag{6.25}$$

6.3 Plot the CDF and PDF of

$$A = \sqrt{V} \tag{6.26}$$

Solution: : For $r \geq 1$

$$F_A(r) = \Pr\left(A \leq r\right) \tag{6.27}$$
$$= \Pr\left(\sqrt{V} \leq r\right) \tag{6.28}$$
$$= \Pr\left(V \leq r^2\right) \tag{6.29}$$
$$= F_V(r^2) = 1 - e^{-\frac{r^2}{2}} \tag{6.30}$$

and so on differentiating

$$f_A(r) = re^{-\frac{r^2}{2}} \tag{6.31}$$

Thus, the CDF and PDF of A is given by

$$F_V(r) = \begin{cases} 1 - e^{-\frac{r^2}{2}} & r \geq 0 \\ 0 & r < 0 \end{cases} \tag{6.32}$$
$$f_V(r) = \begin{cases} re^{-\frac{r}{2}} & r \geq 0 \\ 0 & r < 0 \end{cases} \tag{6.33}$$

The CDF and PDF of A is plotted in Fig. 6.5 and 6.5 using the codes below

```
wget https://github.com/GovindaRohith/
    Assignments/blob/main/Randomnum/
    codes/6.3cdf.py
wget https://github.com/GovindaRohith/
    Assignments/blob/main/Randomnum/
    codes/6.3pdf.py
```

Download the above files and execute the following commands to produce Fig.6.5 and 6.5

```
$ python3 6.3cdf.py
$ python3 6.3pdf.py
```

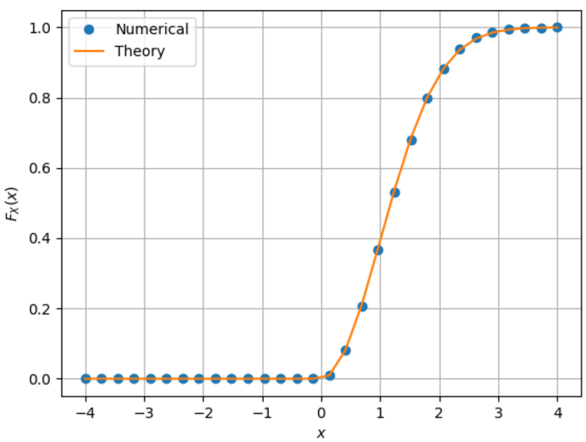

Fig. 6.5: The CDF of V

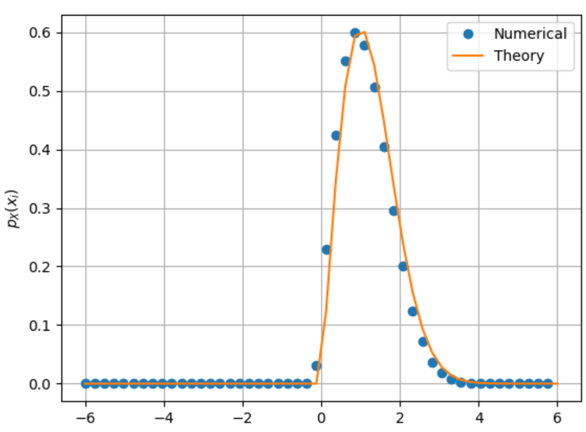

Fig. 6.5: The PDF of V