Assignment

Govinda Rohith Y
CS21BTECH11062

CONTENTS

***Abstract*—This manual provides solutions to the Assignment of Random Numbers**

I. UNIFORM RANDOM NUMBERS

Let U be a uniform random variable between 0 and 1.

I.1 Generate 10^6 samples of U using a C program and save into a file called uni.dat .
Solution: Download the following files and execute the C program.

```
wget https://github.com/GovindaRohith/
    Assignments/blob/main/Randomnum/
    codes/1.1.c
wget https://github.com/GovindaRohith/
    Assignments/blob/main/Randomnum/
    codes/source.h
```

I.2 Load the uni.dat file into python and plot the empirical CDF of U using the samples in uni.dat. The CDF is defined as

$$F_U(x) = \Pr\left(U \leq x\right) \tag{1}$$

Solution: The following code plots Fig. I.2

```
wget https://github.com/GovindaRohith/
    Assignments/blob/main/Randomnum/
    codes/1.2.py
```

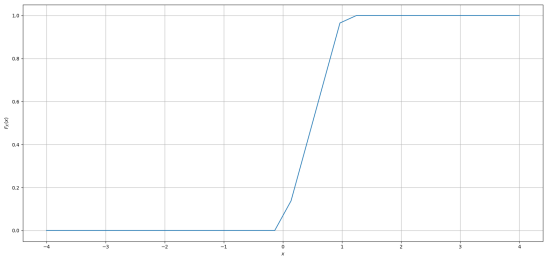

Fig. I.2. The CDF of U

I.3 Find a theoretical expression for $F_U(x)$.
Solution: Given U is a uniform Random Variable

$$p_U(x) = 1 \text{ for} \tag{2}$$

$$F_U(x) = \int_{-\infty}^{\infty} p_U(x)dx \tag{3}$$

$$\implies F_U(x) = x \tag{4}$$

I.4 The mean of U is defined as

$$E\left[U\right] = \frac{1}{N}\sum_{i=1}^{N} U_i \tag{5}$$

and its variance as

$$\text{var}\left[U\right] = E\left[U - E\left[U\right]\right]^2 \tag{6}$$

Write a C program to find the mean and variance of U.
Solution: Download the following files and execute the C program.

```
wget https://github.com/GovindaRohith/
    Assignments/blob/main/Randomnum/
    codes/1.4.c
wget https://github.com/GovindaRohith/
    Assignments/blob/main/Randomnum/
    codes/source.h
```

I.5 Verify your result theoretically given that

$$E\left[U^k\right] = \int_{-\infty}^{\infty} x^k dF_U(x) \tag{7}$$

Solution:

$$\text{var}\left[U\right] = E\left[U - E\left[U\right]\right]^2 \tag{8}$$

$$\implies \text{var}\left[U\right] = E\left[U^2\right] - E\left[U\right]^2 \tag{9}$$

$$E\left[U\right] = \int_{-\infty}^{\infty} x dF_U(x) \tag{10}$$

$$E\left[U\right] = \int_{0}^{1} x \tag{11}$$

$$\implies \boxed{E\left[U\right] = \frac{1}{2}} \tag{12}$$

$$E\left[U^2\right] = \int_{-\infty}^{\infty} x^2 dF_U(x) \tag{13}$$

$$E\left[U^2\right] = \int_{0}^{1} x^2 dF_U(x) \tag{14}$$

$$\implies E\left[U^2\right] = \frac{1}{3} \tag{15}$$

$$\implies \boxed{\text{var}\left[U\right] = \frac{1}{12} = 0.0833} \tag{16}$$

II. Central Limit Theorem

II.1 Generate 10^6 samples of the random variable

$$X = \sum_{i=1}^{12} U_i - 6 \tag{17}$$

using a C program, where $U_i, i = 1, 2, \ldots, 12$ are a set of independent uniform random variables between 0 and 1 and save in a file called gau.dat

Solution: Download the following files and execute the C program.

```
wget https://github.com/GovindaRohith/Assignments/blob/main/Randomnum/codes/2.1.c
wget https://github.com/GovindaRohith/Assignments/blob/main/Randomnum/codes/source.h
```

II.2 Load gau.dat in python and plot the empirical CDF of X using the samples in gau.dat. What properties does a CDF have?

Solution: The CDF of X is plotted in Fig. II.2 using the code below

```
https://github.com/GovindaRohith/Assignments/blob/main/Randomnum/codes/2.2.py
```

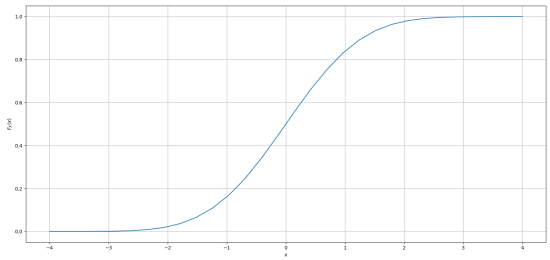

Fig. II.2. The CDF of X

Some of the properties of CDF

a) $F_X(x)$ is non decreasing function.
b) Symmetric about one point.

II.3 Load gau.dat in python and plot the empirical PDF of X using the samples in gau.dat. The PDF of X is defined as

$$p_X(x) = \frac{d}{dx} F_X(x) \tag{18}$$

What properties does the PDF have?

Solution: The PDF of X is plotted in Fig. II.3 using the code below

```
wget https://github.com/GovindaRohith/Assignments/blob/main/Randomnum/codes/2.3.py
```

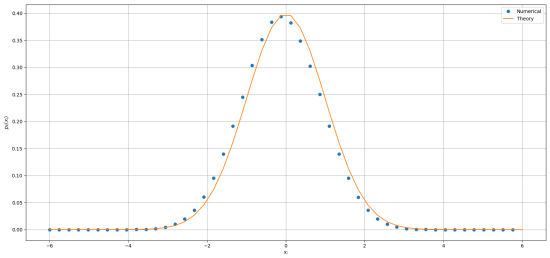

Fig. II.3. The PDF of X

Some of the properties of the PDF:

a) Symmetric about $x = \mu$
b) decreasing function for $x < \mu$ and increasing for $x > \mu$ and attains maximum at $x = \mu$
c) Area under the curve is unity.

II.4 Find the mean and variance of X by writing a C program.
Solution: Download the following files and execute the C program.

```
wget https://github.com/GovindaRohith/
    Assignments/blob/main/Randomnum/
    codes/2.4.c
wget https://github.com/GovindaRohith/
    Assignments/blob/main/Randomnum/
    codes/source.h
```

II.5 Given that

$$p_X(x) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right), -\infty < x < \infty, \tag{19}$$

repeat the above exercise theoretically.
Solution:

1) CDF is given by

$$F_X(x) = \int_{-\infty}^{\infty} p_X(x)dx \tag{20}$$

$$\boxed{F_X(x) = 1} \tag{21}$$

2) Mean is given by

$$E(x) = \int_{-\infty}^{\infty} xp_X(x)dx \tag{22}$$

$$\implies \boxed{E(x) = 0} \tag{23}$$

3) Variance is given by

$$\text{var}\left[U\right] = E(U^2) - (E(U))^2 \tag{24}$$

$$\implies \boxed{\text{var}\left[U\right] = \sqrt{2}} \tag{25}$$

III. From Uniform to Other

III.1 Generate samples of

$$V = -2\ln\left(1-U\right) \tag{26}$$

and plot its CDF.
Solution: Download the following files and execute the C program.

```
wget https://github.com/GovindaRohith/
    Assignments/blob/main/Randomnum/
    codes/3.1.c
wget https://github.com/GovindaRohith/
    Assignments/blob/main/Randomnum/
    codes/source.h
```

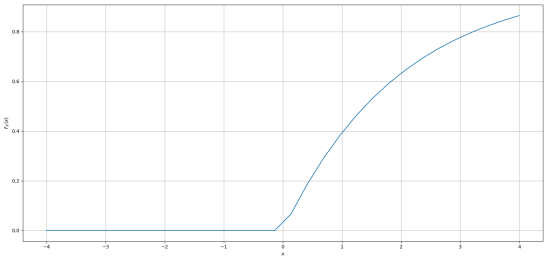

Fig. III.1. The PDF of X

The CDF of V is plotted in Fig. III.1 using the code below

```
wget https://github.com/GovindaRohith/
    Assignments/blob/main/Randomnum/
    codes/2.3.py
```

III.2 Find a theoretical expression for $F_V(x)$.
Solution:

$$V = -2\ln\left(1-U\right) \tag{27}$$

$$\implies E(V) = -2\ln(1-E(U)) \tag{28}$$

$$\int_{-\infty}^{x} xp_V(x)dx = -2\ln\left(1-\frac{x^2}{2}\right) \tag{29}$$

Differentiating the above equation

$$p_V(x) = \frac{4}{2-x^2} \tag{30}$$

$$\implies F_V(x) = \int_0^x \frac{4dx}{2-x^2} \tag{31}$$

$$\boxed{F_V(x) = \sqrt{2}\ln\frac{\sqrt{2}+x}{\sqrt{2}-x}} \tag{32}$$