

Assignment 9

Govinda Rohith Y

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Question

2-24(Papoullis):

Box 1 contains 1000 bulbs of which 10% are defective. Box 2 contains 2000 bulbs which 5% are defective. Two bulbs are picked from a randomly selected box.

- (a) Find the probability that both bulbs are defective.
- (b) Assuming that both are defective, find the probability that they came from box 1.

Denote Random Variables

Assign events to random variables

Denote the random variables $Y \in \{0, 1\}$. and $X \in \{0, 1, 2\}$ Events are described in Table 1:

Variable	Event
$X = 0$	Picking two defective bulbs
$X = 1$	Picking one defective bulbs
$X = 2$	Picking zero bulbs
$Y = 0$	Picking up Box 1
$Y = 1$	Picking up Box 2

Table 1

Given data

Represent the given data

Event	Probability
$\Pr(Y = 0)$	0.5
$\Pr(Y = 1)$	0.5
$\Pr(X = 0 Y = 0)$	$\frac{11}{1110}$
$\Pr(X = 0 Y = 1)$	$\frac{99}{39980}$

Table 2

Solution(a)

Solution

$\Pr(X = 0)$ denotes the probability that both bulbs are defective.

From Total probability theorem

$$\Pr(X = 0) = \sum_{i=0}^1 \Pr(Y = i) \Pr(X = 0|Y = i) \quad (1)$$

$$\implies \boxed{\Pr(X) \approx 0.006193} \quad (2)$$

Solution(b)

Solution

$\Pr(Y = 0|X = 0)$ denotes two bulbs are picked from box 1 assuming both are defective.

From Bayes theorem

$$\Pr(Y = 0|X = 0) = \frac{\Pr(Y = 0) \Pr(X = 0|Y = 0)}{\Pr(X)} \quad (3)$$

$$\implies \boxed{\Pr(Y = 0|X = 0) \approx 0.8} \quad (4)$$