Assignment 11

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Outline

Question

6-55(Papoullis):

Let X represent the number of successes and Y the number of failures of n independent Bernoulli trials with p representing the probability of success in anyone trial. Find the distribution of $Z = X - Y$. Show that $E\{z\} = n(2p - 1)$, $Var\{Z\} = 4np(1 - p)$.

Solution(a)

Solution(a)

Let

$$X = k \tag{1}$$

$$\implies Y = n - k \tag{2}$$

$$Z = X - Y \tag{3}$$

$$\implies \boxed{Z = 2X - n} \tag{4}$$

So the Z can take values $\{-n, -(n-2), \cdots, n\}$

$\implies$ Z is Binomial distribution.

Solution (a)

Contd.

$$\Pr\left(\{Z = z\}\right) = \Pr\left(\{2X - n = z\}\right) \Pr\left(\{X = \frac{n+z}{2}\}\right) \tag{5}$$

$$\Pr\left(\{Z = z\}\right) = \binom{n}{n + z/2} p^{(n+z)/2} q^{(n-z)/2} \tag{6}$$

Solution (b)

Solution(b)

$$E(Z) = E(2X - n) \tag{7}$$

$$E(Z) = 2np - n \tag{8}$$

$$\implies \boxed{E(Z) = n(2p - 1)} \tag{9}$$

Solution(c)

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$$Var(Z) = E((Z - \mu_Z^2)^2) \tag{10}$$

$$Var(Z) = 4E((X - np)^2) \tag{11}$$

$$Var(Z) = 4Var(x) \tag{12}$$

$$\implies \boxed{Var(Z) = 4npq} \tag{13}$$