Assignment 12

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LaTeX

Outline

Question

7-32(Papoullis):

The random variables X and Y are uncorrelated with zero mean and $\sigma_x = \sigma_y = \sigma$. Show that if$z = x + iy$ then

$$f_z(Z) = f(x, y) = \frac{1}{2\pi\sigma^2} e^{-(x^2+y^2)/2\sigma^2} = \frac{1}{\pi\sigma_Z^2} e^{-|Z^2|/\sigma_z^2}$$

$$\Phi_Z(\Omega) = \exp\left\{-\frac{1}{2}(\sigma^2 u^2 + \sigma^2 v_Z^2)\right\} = \exp\left\{-\frac{1}{4}\sigma_Z^2 \left|\Omega\right|^2\right\}$$

Solution(a)

Since X and Y are independent

$$\text{Variance of } Z = X + iY \tag{1}$$

$$Var(Z) = E(\left|Z - E(Z)^2\right|) = E(|Z|^2) - E(|Z|)^2 \tag{2}$$

$$M_Z(s) = E(e^{-sZ}) = E(e^{-s(x+iY)}) \tag{3}$$

$$E(e^{-sZ}) = E(e^{-sX})E(e^{-siY}) \tag{4}$$

$$\implies E(|Z|) = E(X) + iE(Y) = 0 \tag{5}$$

Contd.

$$M_{|Z|^2}(s) = E(e^{-s|Z|^2}) = E(e^{-sX^2})E(e^{-sY^2}) \tag{6}$$

$$E(e^{-s|Z|^2}) = \left(1 + sE(X^2) + \frac{s^2E(X^4)}{2!} + \dots\right)\left(1 + sE(Y^2) + \frac{s^2E(Y^4)}{2!} + \dots\right) \tag{7}$$

$$E(|Z|^2) = E(X^2) + E(Y^2) \tag{8}$$

Substitute Equations (8,5) in Equation(2)

$$f_Z(Z) = \frac{1}{2\pi\sigma^2}e^{-(x^2+y^2)/2\sigma^2} = \frac{1}{\pi\sigma_Z^2}e^{-|Z^2|/\sigma_z^2} \tag{9}$$

Solution(b)

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$$\Phi_Z(\Omega) = \Phi_X(u)\Phi_Y(v) \tag{10}$$

$$\Phi_Z(\Omega) = e^{-\frac{1}{2}\sigma^2(u^2+v^2)} \tag{11}$$

$$\Phi_Z(\Omega) = e^{-\frac{1}{2}\sigma^2|\Omega^2|} \tag{12}$$

$$\implies \boxed{\Phi_Z(\Omega) = e^{-\frac{1}{4}\sigma_Z^2|\Omega^2|}} \tag{13}$$