Assignment 9

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Outline

Question

2-24(Papoullis):

Box 1 contains 1000 bulbs of which 10% are defective. Box 2 contains 2000 bulbs which 5% are defective. Two bulbs are picked from a randomly selected box.

(a) Find the probability that both bulbs are defective.

(b) Assuming that both are defective, find the probability that they came from box 1.

Denote Random Variables

Assign events to random variables

Denote the random variable $X \in \{0, 1\}$ and $Y \in \{0\}$. Events are described in Table 1:

Variable	Event
X=0	Picking Box 1
X=1	Picking Box 2
Y=0	Picking two defective bulbs

Table 1

Given data

Represent the given data

Event	Probabilty
$\Pr(X = 0)$	0.5
$\Pr(X = 1)$	0.5
$\Pr(Y = 0 \mid X = 0)$	$\frac{11}{1110}$
$\Pr(Y = 0 \mid X = 1)$	$\frac{99}{39980}$

Table 2

Solution(a)

Solution

$\Pr(Y = 0)$ denotes the probability that both bulbs are defective.

From Total probability theorem

$$\Pr(Y = 0) = \sum_{j=0}^{1} \Pr(X = j) \Pr(Y = 0 | X = j) \tag{1}$$

$$\implies \boxed{\Pr(Y = 0) \approx 0.006193} \tag{2}$$

Solution(b)

Solution

$\Pr(X = 0|Y = 0)$ denotes two bulbs are picked from box 1 assuming both are defective.

From Bayes theorem

$$\Pr(X = 0|Y = 0) = \frac{\Pr(X = 0)\Pr(Y = 0|X = 0)}{\Pr(Y = 0)} \tag{3}$$

$$\implies \boxed{\Pr(X = 0|Y = 0) \approx 0.8} \tag{4}$$