Assignment

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CS21BTECH11062

CONTENTS

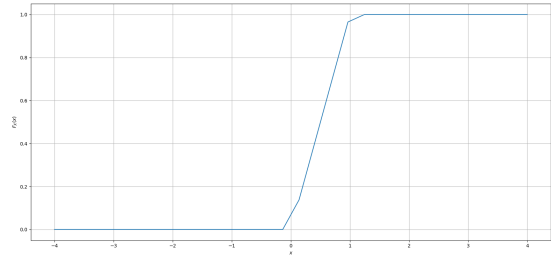*Abstract*—**This manual provides solutions to the Assignment of Random Numbers**

Fig. I.2. The CDF of U

I. UNIFORM RANDOM NUMBERS

Let U be a uniform random variable between 0 and 1.

I.1 Generate 10^6 samples of U using a C program and save into a file called uni.dat .
Solution: Download the following files and execute the C program.

> wget https://github.com/GovindaRohith/
> Assignments/blob/main/Randomnum/
> codes/1.1.c
> wget https://github.com/GovindaRohith/
> Assignments/blob/main/Randomnum/
> codes/source.h

Download the above files and execute the following commands
a) $ gcc 1.1.c
b) $ ./a.out

I.2 Load the uni.dat file into python and plot the empirical CDF of U using the samples in uni.dat. The CDF is defined as

$$F_U(x) = \Pr(U \le x) \tag{1}$$

Solution: The following code plots Fig. I.2

> wget https://github.com/GovindaRohith/
> Assignments/blob/main/Randomnum/
> codes/1.2.py

Download the above files and execute the following commands to produce Fig.I.2
a) $ python3 1.2.py

I.3 Find a theoretical expression for $F_U(x)$.
Solution: Given U is a uniform Random Variable

$$p_U(x) = 1 \text{ for} \tag{2}$$

$$F_U(x) = \int_{-\infty}^{\infty} p_U(x)dx \tag{3}$$

$$\implies F_U(x) = x \tag{4}$$

I.4 The mean of U is defined as

$$E[U] = \frac{1}{N}\sum_{i=1}^{N} U_i \tag{5}$$

and its variance as

$$\text{var}[U] = E[U - E[U]]^2 \tag{6}$$

Write a C program to find the mean and variance of U.
Solution: Download the following files and execute the C program.

> wget https://github.com/GovindaRohith/
> Assignments/blob/main/Randomnum/
> codes/1.4.c
> wget https://github.com/GovindaRohith/
> Assignments/blob/main/Randomnum/
> codes/source.h

Download the above files and execute the following commands

a) $ gcc 1.4.c
b) $ ./a.out

I.5 Verify your result theoretically given that

$$E\left[U^k\right] = \int_{-\infty}^{\infty} x^k dF_U(x) \tag{7}$$

Solution:

$$\text{var}\left[U\right] = E\left[U - E\left[U\right]\right]^2 \tag{8}$$

$$\implies \text{var}\left[U\right] = E\left[U^2\right] - E\left[U\right]^2 \tag{9}$$

$$E\left[U\right] = \int_{-\infty}^{\infty} x dF_U(x) \tag{10}$$

$$E\left[U\right] = \int_{0}^{1} x \tag{11}$$

$$\implies \boxed{E\left[U\right] = \frac{1}{2}} \tag{12}$$

$$E\left[U^2\right] = \int_{-\infty}^{\infty} x^2 dF_U(x) \tag{13}$$

$$E\left[U^2\right] = \int_{0}^{1} x^2 dF_U(x) \tag{14}$$

$$\implies E\left[U^2\right] = \frac{1}{3} \tag{15}$$

$$\implies \boxed{\text{var}\left[U\right] = \frac{1}{12} = 0.0833} \tag{16}$$

II. CENTRAL LIMIT THEOREM

II.1 Generate 10^6 samples of the random variable

$$X = \sum_{i=1}^{12} U_i - 6 \tag{17}$$

using a C program, where $U_i, i = 1, 2, \ldots, 12$ are a set of independent uniform random variables between 0 and 1 and save in a file called gau.dat

Solution: Download the following files and execute the C program.

wget https://github.com/GovindaRohith/
    Assignments/blob/main/Randomnum/
    codes/2.1.c

wget https://github.com/GovindaRohith/
    Assignments/blob/main/Randomnum/
    codes/source.h

Download the above files and execute the following commands

a) $ gcc 2.1.c
b) $ ./a.out

II.2 Load gau.dat in python and plot the empirical CDF of X using the samples in gau.dat. What properties does a CDF have?
Solution: The CDF of X is plotted in Fig. II.2 using the code below

wget https://github.com/GovindaRohith/
    Assignments/blob/main/Randomnum/
    codes/2.2.py

Download the above files and execute the following commands to produce Fig.II.2
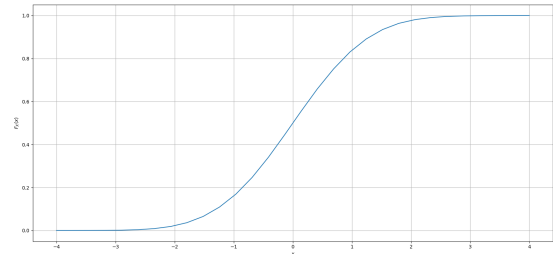
a) $ python3 2.2.py

Fig. II.2. The CDF of X

Some of the properties of CDF

a) $F_X(x)$ is non decreasing function.
b) Symmetric about one point.

II.3 Load gau.dat in python and plot the empirical PDF of X using the samples in gau.dat. The PDF of X is defined as

$$p_X(x) = \frac{d}{dx} F_X(x) \tag{18}$$

What properties does the PDF have?
Solution: The PDF of X is plotted in Fig. II.3 using the code below

wget https://github.com/GovindaRohith/
    Assignments/blob/main/Randomnum/
    codes/2.3.py

Download the above files and execute the following commands to produce Fig.II.3
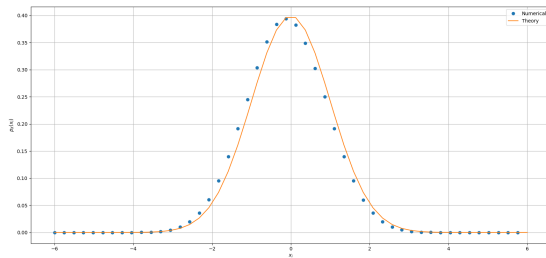
a) $ python3 2.3.py

Fig. II.3. The PDF of X

Some of the properties of the PDF:
a) Symmetric about $x = \mu$
b) decreasing function for $x < \mu$ and increasing for $x > \mu$ and attains maximum at $x = \mu$
c) Area under the curve is unity.

II.4 Find the mean and variance of X by writing a C program.
Solution: Download the following files and execute the C program.

> wget https://github.com/GovindaRohith/
> Assignments/blob/main/Randomnum/
> codes/2.4.c
> wget https://github.com/GovindaRohith/
> Assignments/blob/main/Randomnum/
> codes/source.h

Download the above files and execute the following commands
a) $ gcc 2.4.c
b) $ ./a.out

II.5 Given that

$$p_X(x) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right), -\infty < x < \infty,$$
$$(19)$$

repeat the above exercise theoretically.
Solution:
1) CDF is given by

$$F_X(x) = \int_{-\infty}^{\infty} p_X(x)dx \qquad (20)$$

$$\boxed{F_X(x) = 1} \qquad (21)$$

2) Mean is given by

$$E(x) = \int_{-\infty}^{\infty} x p_X(x)dx \qquad (22)$$

$$\implies \boxed{E(x) = 0} \qquad (23)$$

3) Variance is given by

$$\text{var}\,[U] = E(U^2) - (E(U))^2 \qquad (24)$$

$$\implies \boxed{\text{var}\,[U] = \sqrt{2}} \qquad (25)$$

III. FROM UNIFORM TO OTHER

III.1 Generate samples of

$$V = -2\ln(1 - U) \qquad (26)$$

and plot its CDF.
Solution: Download the following files and execute the C program.

> wget https://github.com/GovindaRohith/
> Assignments/blob/main/Randomnum/
> codes/3.1.c
> wget https://github.com/GovindaRohith/
> Assignments/blob/main/Randomnum/
> codes/source.h

Download the above files and execute the following commands
a) $ gcc 3.1.c -lm
b) $ ./a.out
The CDF of V is plotted in Fig. III.1 using the code below

> wget https://github.com/GovindaRohith/
> Assignments/blob/main/Randomnum/
> codes/3.1pyth.py

Download the above files and execute the following commands to produce Fig.III.1
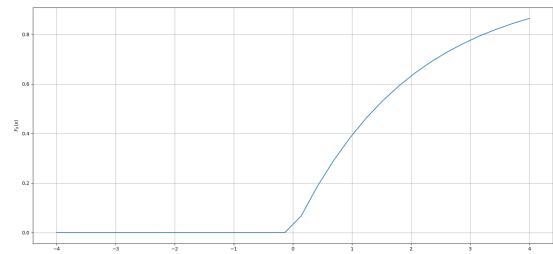a) $ python3 3.1pyth.py

Fig. III.1. The PDF of X

III.2 Find a theoretical expression for $F_V(x)$.

Solution: If Y = g(X), we know that $F_Y(y) = F_X(g^{-1}(y))$, here

$$V = -2\ln(1 - U) \qquad (27)$$

$$1 - U = e^{\frac{-V}{2}} \qquad (28)$$

$$U = 1 - e^{\frac{-V}{2}} \qquad (29)$$

$$F_V(X) = F_U(1 - e^{\frac{-X}{2}}) \qquad (30)$$

when , $0 \le 1 - e^{\frac{-X}{2}} \le 1$

$$0 \le e^{\frac{-X}{2}} \le 1 \qquad (31)$$

$$X \ge 0, \text{So,} \qquad (32)$$

$$F_V(X) = 1 - e^{\frac{-X}{2}}, X \ge 0 \qquad (33)$$

$$F_V(X) = 0, X < 0 \qquad (34)$$