Assignment 13

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Outline

Question

9-31(Papoullis):

Show that if

$$S = \int_0^{10} x(t)dt \text{ then } E(S^2) = \int_{-10}^{10} (10 - |\tau|)R_X(\tau)d\tau$$

Find the mean and variance of S if $E(x(t)) = 8, R_X(\tau) = 64 + 10^{-2|\tau|}$

Solution (a)

Solution (a)

The moment of S is equal to moments of (Since x(t) is WSS)

$$S = \int_0^{10} x(t)dt = \int_{-5}^{5} x(t)dt \tag{1}$$

$$\text{Let, } h(t) = u(t) - u(t-10) \tag{2}$$

$$y(t) = x(t) * h(t) = \int_0^{10} x(t-\tau)d\tau \tag{3}$$

$$\text{Let, } b = t - \tau \tag{4}$$

Contd...

Continued

$$y(t)\Big|_{t=10} = \int_0^{10} x(b)db \tag{5}$$

$$\text{But, } E(S^2) = \int_{-\infty}^{\infty} S_X(f)df \tag{6}$$

Where S_X is power spectral density and defined as

$$S_X(f) = \int_{-\infty}^{\infty} R_X(\tau)d\tau \tag{7}$$

Contd...

Continued

$$E(S^2) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} R_X(t_1 - t_2)dt_1 dt_2 \tag{8}$$

$$E(S^2) = \int_{-5}^{5}\int_{-5}^{5} R_X(t_1 - t_2)dt_1 dt_2 \tag{9}$$

$$\implies \boxed{E(S^2) = \int_{-10}^{10}(10 - |\tau|)R_X(\tau)d\tau} \tag{10}$$

Solution (b)

Solution (b)

$$s = \int_0^{10} x(t)dt \tag{11}$$

$$\implies E(s) = \int_0^{10} E(x(t))dt \tag{12}$$

$$\text{Given } E(x(t)) = 8 \tag{13}$$

$$\implies \boxed{E(s) = 80} \tag{14}$$

Contd...

Contd

$$Var(s) = E(S - E(S)^2) = E(S^2) - E(S)^2 \tag{15}$$

$$\sigma^2 = 2\int_0^{10}(10-\tau)(64+10e^{-2\tau})d\tau - 80^2 \tag{16}$$

$$\implies \boxed{\sigma^2 \approx 9.5} \tag{17}$$