

Assignment 11

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Question

6-55(Papoullis):

Let X represent the number of successes and Y the number of failures of n independent Bernoulli trials with p representing the probability of success in anyone trial. Find the distribution of $Z = X - Y$. Show that $E\{z\} = n(2p - 1)$, $Var\{Z\} = 4np(1 - p)$.

Solution(a)

Solution(a)

Let

$$X = k \quad (1)$$

$$\implies Y = n - k \quad (2)$$

$$Z = X - Y \quad (3)$$

$$\implies \boxed{Z = 2X - n} \quad (4)$$

So the Z can take values $\{-n, -(n-2), \dots, n\}$

$\implies Z$ is Binomial distribution.

Solution (a)

Contd.

$$\Pr(Z = z) = \Pr(2X - n = z) \Pr\left(X = \frac{n+z}{2}\right) \quad (5)$$

$$\Pr(Z = z) = \binom{n}{n+z/2} p^{(n+z)/2} q^{(n-z)/2} \quad (6)$$

Solution (b)

Solution(b)

$$E(Z) = E(2X - n) \quad (7)$$

$$E(Z) = 2np - n \quad (8)$$

$$\Rightarrow \boxed{E(Z) = n(2p - 1)} \quad (9)$$

Solution(c)

Solution(c)

$$\text{Var}(Z) = E((Z - \mu_Z^2)^2) \quad (10)$$

$$\text{Var}(Z) = 4E((X - np)^2) \quad (11)$$

$$\text{Var}(Z) = 4\text{Var}(x) \quad (12)$$

$$\implies \boxed{\text{Var}(Z) = 4npq} \quad (13)$$