## 1

## AI1110-Assignment 2

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(ICSE-12-2017)Question 1

(ii) if y-2x-k=0 touches the conic  $3x^2-5y^2=15$ , find the value of k

## Solution:

The given equation (Hyperbola)is

$$3x^2 - 5y^2 - 15 = 0 ag{0.0.1}$$

which can be written as

$$\mathbf{x}^{\mathsf{T}}\mathbf{V}\mathbf{x} + 2\mathbf{u}^{\mathsf{T}}\mathbf{x} + f = 0 \qquad (0.0.2)$$

$$\mathbf{V} = \begin{pmatrix} 3 & 0 \\ 0 & -5 \end{pmatrix}, \mathbf{u} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, f = -15 \qquad (0.0.3)$$

The Given line Can be written as

$$L: \mathbf{x} = \mathbf{A} + \lambda \mathbf{m} \tag{0.0.4}$$

Choose 
$$\mathbf{A} = \begin{pmatrix} 0 \\ k \end{pmatrix}$$
 (0.0.5)

$$\mathbf{m} = \begin{pmatrix} 1\\2 \end{pmatrix} \tag{0.0.6}$$

$$\implies \mathbf{x} = \begin{pmatrix} \lambda \\ k + 2\lambda \end{pmatrix}$$
 (0.0.7)

Substituting the line in the conic equation we get the following quadratic equation

$$17\lambda^2 + 20k\lambda + 5k^2 + 15 = 0 ag{0.0.8}$$

To be a tangent the above equation should have only root so

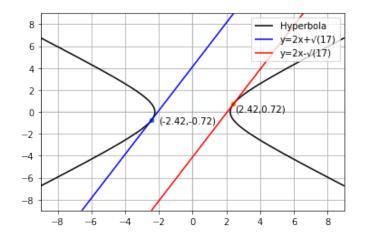


Fig. 0. Graph showing the hyperbola and the two tangents

$$400k^2 - 4(17)(5k^2 + 15) = 0 (0.0.9)$$

$$\implies k^2 - 17 = 0 \tag{0.0.10}$$

$$k = \pm \sqrt{17} \tag{0.0.11}$$