

Assignment

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CONTENTS

I	Uniform Random Numbers	1
II	Central Limit Theorem	2
III	From Uniform to Other	3

Abstract—This manual provides solutions to the Assignment of Random Numbers

I. UNIFORM RANDOM NUMBERS

Let U be a uniform random variable between 0 and 1.

I.1 Generate 10^6 samples of U using a C program and save into a file called uni.dat .

Solution: Download the following files and execute the C program.

```
wget https://github.com/GovindaRohith/
Assignments/blob/main/Randomnum/
codes/1.1.c
wget https://github.com/GovindaRohith/
Assignments/blob/main/Randomnum/
codes/source.h
```

I.2 Load the uni.dat file into python and plot the empirical CDF of U using the samples in uni.dat. The CDF is defined as

$$F_U(x) = \Pr(U \leq x) \quad (1)$$

Solution: The following code plots Fig. I.2

```
wget https://github.com/GovindaRohith/
Assignments/blob/main/Randomnum/
codes/1.2.py
```

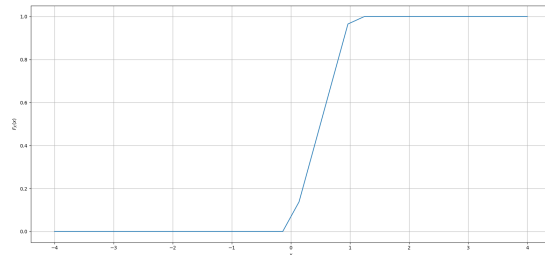


Fig. I.2. The CDF of U

I.3 Find a theoretical expression for $F_U(x)$.

Solution: Given U is a uniform Random Variable

$$p_U(x) = 1 \text{ for } 0 \leq x \leq 1 \quad (2)$$

$$F_U(x) = \int_{-\infty}^{\infty} p_U(x) dx \quad (3)$$

$$\implies F_U(x) = x \quad (4)$$

I.4 The mean of U is defined as

$$E[U] = \frac{1}{N} \sum_{i=1}^N U_i \quad (5)$$

and its variance as

$$\text{var}[U] = E[U - E[U]]^2 \quad (6)$$

Write a C program to find the mean and variance of U .

Solution: Download the following files and execute the C program.

```
wget https://github.com/GovindaRohith/
Assignments/blob/main/Randomnum/
codes/1.4.c
wget https://github.com/GovindaRohith/
Assignments/blob/main/Randomnum/
codes/source.h
```

I.5 Verify your result theoretically given that

$$E[U^k] = \int_{-\infty}^{\infty} x^k dF_U(x) \quad (7)$$

Solution:

$$\text{var}[U] = E[U - E[U]]^2 \quad (8)$$

$$\Rightarrow \text{var}[U] = E[U^2] - E[U]^2 \quad (9)$$

$$E[U] = \int_{-\infty}^{\infty} x dF_U(x) \quad (10)$$

$$E[U] = \int_0^1 x \quad (11)$$

$$\Rightarrow E[U] = \frac{1}{2} \quad (12)$$

$$E[U^2] = \int_{-\infty}^{\infty} x^2 dF_U(x) \quad (13)$$

$$E[U^2] = \int_0^1 x^2 dF_U(x) \quad (14)$$

$$\Rightarrow E[U^2] = \frac{1}{3} \quad (15)$$

$$\Rightarrow \text{var}[U] = \frac{1}{12} = 0.0833 \quad (16)$$

II. CENTRAL LIMIT THEOREM

II.1 Generate 10^6 samples of the random variable

$$X = \sum_{i=1}^{12} U_i - 6 \quad (17)$$

using a C program, where $U_i, i = 1, 2, \dots, 12$ are a set of independent uniform random variables between 0 and 1 and save in a file called gau.dat

Solution: Download the following files and execute the C program.

```
wget https://github.com/GovindaRohith/
Assignments/blob/main/Randomnum/
codes/2.1.c
wget https://github.com/GovindaRohith/
Assignments/blob/main/Randomnum/
codes/source.h
```

II.2 Load gau.dat in python and plot the empirical CDF of X using the samples in gau.dat. What properties does a CDF have?

Solution: The CDF of X is plotted in Fig. II.2 using the code below

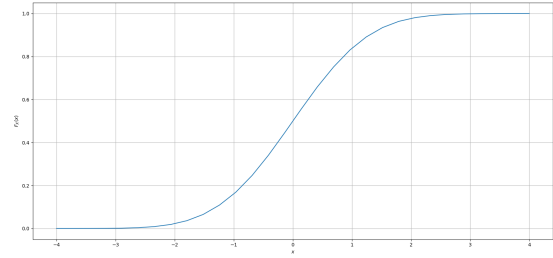


Fig. II.2. The CDF of X

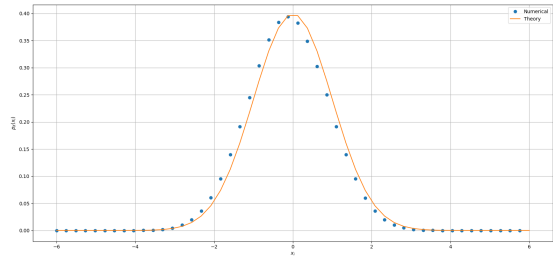


Fig. II.3. The PDF of X

<https://github.com/GovindaRohith/Assignments/blob/main/Randomnum/codes/2.2.py>

Some of the properties of CDF

- $F_X(x)$ is non decreasing function.
- Symmetric about one point.

II.3 Load gau.dat in python and plot the empirical PDF of X using the samples in gau.dat. The PDF of X is defined as

$$p_X(x) = \frac{d}{dx} F_X(x) \quad (18)$$

What properties does the PDF have?

Solution: The PDF of X is plotted in Fig. II.3 using the code below

```
wget https://github.com/GovindaRohith/
Assignments/blob/main/Randomnum/
codes/2.3.py
```

Some of the properties of the PDF:

- Symmetric about $x = \mu$
- decreasing function for $x < \mu$ and increasing for $x > \mu$ and attains maximum at $x = \mu$
- Area under the curve is unity.

II.4 Find the mean and variance of X by writing a C program.

Solution: Download the following files and execute the C program.

```
wget https://github.com/GovindaRohith/
Assignments/blob/main/Randomnum/
codes/2.4.c
wget https://github.com/GovindaRohith/
Assignments/blob/main/Randomnum/
codes/source.h
```

II.5 Given that

$$p_X(x) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right), -\infty < x < \infty, \quad (19)$$

repeat the above exercise theoretically.

Solution:

1) CDF is given by

$$F_X(x) = \int_{-\infty}^{\infty} p_X(x) dx \quad (20)$$

$$\boxed{F_X(x) = 1} \quad (21)$$

2) Mean is given by

$$E(x) = \int_{-\infty}^{\infty} xp_X(x) dx \quad (22)$$

$$\Rightarrow \boxed{E(x) = 0} \quad (23)$$

3) Variance is given by

$$\text{var}[U] = E(U^2) - (E(U))^2 \quad (24)$$

$$\Rightarrow \boxed{\text{var}[U] = \sqrt{2}} \quad (25)$$

III. FROM UNIFORM TO OTHER

III.1 Generate samples of

$$V = -2 \ln(1 - U) \quad (26)$$

and plot its CDF.

Solution: The CDF is as follows **Solution:** Download the following files and execute the C program.

```
wget https://github.com/GovindaRohith/
Assignments/blob/main/Randomnum/
codes/3.1.c
wget https://github.com/GovindaRohith/
Assignments/blob/main/Randomnum/
codes/source.h
```

The CDF of V is plotted in Fig. III.1 using the code below

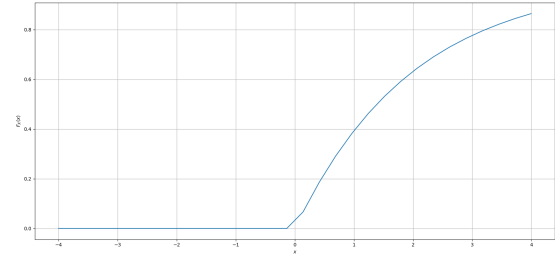


Fig. III.1. The PDF of X

```
wget https://github.com/GovindaRohith/
Assignments/blob/main/Randomnum/
codes/2.3.py
```

III.2 Find a theoretical expression for $F_V(x)$.

Solution:

$$V = -2 \ln(1 - U) \quad (27)$$

$$\Rightarrow E(V) = -2 \ln(1 - E(U)) \quad (28)$$

$$\int_{-\infty}^x xp_V(x) dx = -2 \ln\left(1 - \frac{x^2}{2}\right) \quad (29)$$

Differentiating the above equation

$$p_V(x) = \frac{4}{2 - x^2} \quad (30)$$

$$\Rightarrow F_V(x) = \int_0^x \frac{4dx}{2 - x^2} \quad (31)$$

$$\boxed{F_V(x) = \sqrt{2} \ln \frac{\sqrt{2} + x}{\sqrt{2} - x}} \quad (32)$$