Assignment 9

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***Abstract*—This document contains the solution for Assignment 9(Papoullis Text Book Chapter-2 Problems 2-24)**

2-24(Papoullis): Box 1 contains 1000 bulbs of which 10% are defective. Box 2 contains 2000 bulbs which 5% are defective. Two bulbs are picked from a randomly selected box.

(a) Find the probability that both bulbs are defective.
(b) Assuming that both are defective, find the probability that they came from box 1.

Solution:

Denote the random variable $X \in \{0, 1, 2\}$. Events are described in Table I:

Variable	Event
X=0	Picking Box 1
X=1	Picking Box 2
X=2	Picking two defective bulbs

TABLE I

Given data:

Event	Probabilty
$\Pr(X=0)$	0.5
$\Pr(X=1)$	0.5
$\Pr(X=2\|X=0)$	$\frac{11}{1110}$
$\Pr(X=2\|X=1)$	$\frac{99}{39980}$

TABLE II

(a) $\Pr(X=2)$ denotes the probability that both bulbs are defective.
From Total probability theorem

$$\Pr(X=2) = \sum_{j=0}^{1} \Pr(X=j)\Pr(X=2|X=j) \tag{1}$$

$$\implies \boxed{\Pr(X=2) \approx 0.006193} \tag{2}$$

(b) $\Pr(X=0|X=2)$ denotes two bulbs are picked from box 1 assuming both are defective.
From Bayes theorem

$$\Pr(X=0|X=2) = \frac{\Pr(X=0)\Pr(X=2|X=0)}{\Pr(X=2)} \tag{3}$$

$$\implies \boxed{\Pr(X=0|X=2) \approx 0.8} \tag{4}$$