AI1110-Assignment 2

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(ICSE-12-2017)Question 1

(ii) if $y - 2x - k = 0$ touches the conic $3x^2 - 5y^2 = 15$,find the value of k

Solution :

The Given Conic(Hyperbola) can be written as

$$\mathbf{x}\mathbf{V}\mathbf{x}^{\mathbf{T}} + f\mathbf{I} = \mathbf{O} \qquad (0.0.1)$$

Where

$$\mathbf{x} = \begin{pmatrix} x & y \end{pmatrix} \qquad (0.0.2)$$

$$\mathbf{V} = \begin{pmatrix} 3 & 0 \\ 0 & -5 \end{pmatrix} \qquad (0.0.3)$$

$$f = -15 \qquad (0.0.4)$$

$$\mathbf{I} = \begin{pmatrix} 1 \end{pmatrix} \qquad (0.0.5)$$

$$\mathbf{O} = \begin{pmatrix} 0 \end{pmatrix} \qquad (0.0.6)$$

The Given line Can be written as

$$L : \mathbf{x} = \mathbf{A} + \lambda\mathbf{m} \qquad (0.0.7)$$

$$\text{Choose } \mathbf{A} = \begin{pmatrix} 0 \\ k \end{pmatrix} \qquad (0.0.8)$$

$$\mathbf{m} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \qquad (0.0.9)$$

$$\implies \mathbf{x} = \begin{pmatrix} \lambda \\ k + 2\lambda \end{pmatrix} \qquad (0.0.10)$$

Substituting the line in the conic equation we get the following quadratic equation

$$17\lambda^2 + 20k\lambda + 5k^2 + 15 = 0 \qquad (0.0.11)$$

To be a tangent the above equation should have only root so

$$400k^2 - 4(17)(5k^2 + 15) = 0 \qquad (0.0.12)$$

$$\implies k^2 - 17 = 0 \qquad (0.0.13)$$

$$\boxed{k = \pm\sqrt{17}} \qquad (0.0.14)$$