Quiz on Infinite Series

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For each series, write out the first three or four terms, then indicate whether the series converges or diverges and give reasons. For a geometric series or a p-series, you are allowed to state reasons for convergence or divergence briefly (but do state them). Where you are applying tests for convergence, you need to state the supporting information needed to see that the test applies.

1.
$$\sum_{n=2}^{\infty} \frac{1}{n^3+1}$$

$$\frac{1}{9} + \frac{1}{28} + \frac{1}{65} \ldots$$

Compare this with
$$\sum_{n=2}^{\infty} \frac{1}{n^3},$$
which we know converges as it is a p-series with $p > 1$.

Each term $\frac{1}{n^3+1}$ is nonnegative and less than the corresponding term $\frac{1}{n^3}$ in the convergent series $\sum_{n=2}^{\infty} \frac{1}{n^3}$, so $\sum_{n=2}^{\infty} \frac{1}{n^3+1}$ also converges by the Comparison Test.

2.

$$\sum_{n=2}^{\infty} \frac{1}{n^3-1}$$

$$\frac{1}{7}+\frac{1}{26}+\frac{1}{63}\dots$$

Compare this with

$$\sum_{n=2}^{\infty} \frac{1}{n^3},$$

which we know converges as it is a p-series with $p > 1$.

Unfortunately, each term $\frac{1}{n^3+1}$ is *greater than* the corresponding term $\frac{1}{n^3}$ in the convergent series $\sum_{n=2}^{\infty} \frac{1}{n^3}$, so we have to use a different kind of comparison. We attempt the Limit Comparison Test.

$$\lim_{n\to\infty} \frac{\frac{1}{n^3}}{\frac{1}{n^3-1}} = \lim_{n\to\infty} \frac{n^3-1}{n^3} = 1$$

so by the limit comparison test $\sum_{n=2}^{\infty} \frac{1}{n^3}$

and $\sum_{n=2}^{\infty} \frac{1}{n^3-1}$ have the same convergence behaviour (they either both converge or both diverge) and we know that the first one converges, so the second one does also.

3.

$$\sum_{n=1}^{\infty}(-1)^{n-1}\frac{1}{\sqrt{n}}$$

(also answer, does this series converge absolutely?)

$$\frac{1}{1}-\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}-\frac{1}{\sqrt{4}}\cdots$$

Notice the alternating signs. This suggests the Alternating Series Test. For the Alternating Series Test, there is a checklist of things to say. The terms alternate between positive and negative terms. The absolute values of the terms, $\frac{1}{\sqrt{n}}$, decrease each time n increases. The limit of the absolute values of the terms $\lim_{n\to\infty}\frac{1}{\sqrt{n}}$ is zero. So the series converges by the alternating series test. **When you use a test you must mention its name.**

Does the series converge absolutely? The series

$$\sum_{n=1}^{\infty}\frac{1}{\sqrt{n}}$$

of absolute values of terms of this series diverges (it is a p-series with $p=\frac{1}{2}<1$) so the alternating series does not converge absolutely.

4.

$$\sum_{n=0}^{\infty}\frac{2n}{n!}$$

If you see a factorial you are almost certainly looking at a Ratio Test problem.

$$\lim_{n\to\infty} \frac{\frac{2(n+1)}{(n+1)!}}{\frac{2n}{n!}} = \lim_{n\to\infty} \frac{2(n+1)n!}{2n(n+1)!}$$
$$= \lim_{n\to\infty} \frac{(n+1)n!}{n(n+1)!} = \lim_{n\to\infty} \frac{(n+1)n!}{n(n)!(n+1)} = \lim_{n\to\infty} \frac{(n+1)}{n(n+1)}$$
$$= \lim_{n\to\infty} \frac{1}{n} = 0 < 1$$

so this series converges by the Ratio Test.